How to Persuade a Group: Simultaneously or Sequentially

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Abstract

How should a privately informed persuader optimally persuade a group of listeners if the listeners can investigate the persuader's message? Should he bring all the listeners together and persuade them simultaneously (public persuasion) or privately communicate with them sequentially (sequential persuasion)? The answer depends on the investigation costs of the listeners. Public persuasion tends to outperform sequential persuasion when the marginal investigation costs are not very large. The opposite can be true, if it is very costly for the listeners to verify the message reported by the speaker. This paper also shows that in the persuader-optimal equilibrium of either persuasion mode, the persuader pools extreme private information, while "truthfully reveals" his private information if it is moderate. An equilibrium with more equilibrium messages, which are the messages reported with positive probabilities on the equilibrium path, does not necessarily outperform one with fewer equilibrium messages. This is different from the finding in Crawford and Sobel (1982).

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1 Introduction

Persuasion is a common and important social and economic activity. It is common, because it arises as long as there exist conflicting interests among agents and one or more agents want to sway the opinions of others. It is very important, because in the fields of business, politics, and academic research, persuasive communication is influential and even crucial for the outcomes.

Persuasion usually involves a group of listeners. The examples below could help to illustrate the extensiveness and significance of multilistener persuasion in the real world:

- An entrepreneur would like to launch an existing project. But financing the project requires the investment of a group of venture capitalists. Each potential investor agrees to invest only if she believes that the project is good enough.

- A firm plans to launch a series of new products. To realize this business plan, the firm must persuade its suppliers to make specific investment in producing associated intermediate inputs. The suppliers are willing to do so only if they are convinced that the new products have great potentials so that their inputs will be highly demanded.

- To get his/her proposal approved, a politician usually needs to persuade several interest groups. The interest groups vote for this proposal only if they believe that it is beneficial to them.

The problem facing a persuader in a multilistener situation is undeniably more complex than that in a one-listener case. In a multilistener situation, the persuader not only needs to determine what information and the amount of information to share with the listeners as in a one-listener case, but also needs to consider the order of persuading the listeners, which is not an issue in the simpler case. However, most of the literature focus on the case of one speaker and one listener (e.g., Milgrom 1981; Crawford and Sobel 1982; Glazer and Rubinstein 2004, etc.). One-speaker/multilistener persuasion, which is more common in practice, has disproportionally attracted less attention. No paper has formally studied the impact of persuasion order on the persuader's payoff.

Important papers on multilistener persuasion include Farrell and Gibbons (1989), Goltsman and Pavlov (2011), Chakraborty and Harbaugh (2010), Koessler (2008), and Caillaud and Tirole (2007). The first three papers model persuasion as (costless, nonverifiable) cheap talk. Among them, Farrell and Gibbons (1989) and Goltsman and Pavlov (2011) focus on studying the differences between public communication and private communication. In their models, listeners are independent decision makers, that is, their payoffs are independent of the actions of others. Thus, persuasion order is irrelevant for the payoff of the persuader. Chakraborty and Harbaugh (2010) is devoted to analyzing the informativeness of persuasion which is modeled as a multidimensional cheap talk. The analysis of Koessler (2008) is similar to the first two papers, except that persuasion is modeled as communication through certifiable messages. Still, the order of persuasion is not part of the persuader's strategy.

Caillaud and Tirole (2007) touch the issue of persuasion order. They study a situation in which a project sponsor persuades a group of listeners to approve his project. They find that in optimality the persuader may selectively communicate with some of the listeners, and then uses the approvals of these listeners to sway the decisions of others. Concerning the order of persuasion, their results imply that the sponsor's expected payoff is independent of the order of persuasion as long as the number of listeners persuaded is the same. This result is essentially due to their assumption on the investigation technology that the listeners use to verify the evidence provided by the persuader. In their model the listeners by incurring a fixed cost can learn about the truth with probability 1, and every listener persuaded investigates the sponsor's evidence in equilibrium. So the expected probabilities of running the project are the same under different persuasion sequences when the same number of listeners are persuaded.

In the current paper, I am going to relax the assumption of Caillaud and Tirole (2007) on investigation technology and characterize the optimal order of persuasion from the persuader's perspective in a multilistener model. Our analysis is conducted by comparing two persuasion modes, public simultaneous persuasion and sequential persuasion. Public simultaneous persuasion (henceforth, public persuasion) is the case in which all the listeners are brought together and persuaded simultaneously by the persuader through a publicly observed message. Sequential persuasion is the case in which the persuader persuades the listeners one by one through private messages, and the listeners approached later could observe the decisions of the former listeners, but not their messages received. Moreover, different Caillaud and Tirole (2007), I assume that the persuader is privately informed and persuades listeners through "soft evidence" (costless and unsubstantiated message). Though these two assumptions are not key to the results on optimal persuasion order, they make the model better in fitting the examples above. The paper finds that the optimal persuasion order heavily depends on the investigation costs of the listeners. If the marginal costs of investigation are not too large, public persuasion tends to outperform sequential persuasion. The opposite can be true, if it is very costly for the listeners to investigate the true state.

The comparison between sequential schemes and simultaneous schemes has already been discussed in the voting literature. Dekel and Piccione (2000) emphasize that in a symmetric two-option environment, sequential voting may not outperform simultaneous voting at aggregating information. But Gershkov and Szentes (2009) shows that in a symmetric environment with costly information acquisition, the socially optimal voting scheme is sequential. However, this literature is quite different from that on persuasion, or strategic information transmission, because there is no communication involved before voting.

Besides the results on optimal persuasion modes, I also find that in the persuader-optimal equilibrium, there may be one or more listeners investigating; the persuader pools extreme private information, while "truthfully reveals" his private information if it is moderate, regardless of the number of investigators. The paper also shows that an equilibrium with more equilibrium messages, which are the messages reported on equilibrium path, does not necessarily outperform one with fewer equilibrium messages. This is different from the finding in Crawford and Sobel (1982).

The rest of this paper is organized as follows: Section 2 discusses persuasion with one listener, this case serves as a stepping stone for the analysis of multilistener case. Section 3 analyzes the equilibria of public persuasion and characterizes the optimal equilibrium. Section 4 analyzes sequential persuasion. Section 5 compares public persuasion with sequential persuasion, and shows how the comparison results depend on investigation costs. Section 6 concludes the paper.

2 One-listener Case

Though the focus of the paper is strategic persuasion addressed to multiple listeners, I would like to begin our analysis with the simpler one-listener case. In this section, we will discuss the equilibria of this case and characterize the optimal equilibrium. Analysis of this section is not redundant, as one will find that it not only sheds light on how persuasion works in the model, but also provides intuition for equilibria in the multilistener case.

2.1 Setup

There are two players, a persuader (he) and a listener (she). The persuader owns a project that he *always* prefers to launch. However, launching the project requires the investment of the listener. The payoff to the listener from financing the project is $\omega \in \Omega = [0, 1]$. ω is private information to the persuader. The listener is a risk-neutral expected payoff maximizer, and has an outside option which yields her payoff R. Thus, to invest in the project, the expected value of ω should be strictly larger than R. The listener has prior $f : \Omega \mapsto \mathcal{R}$ over ω, which is continuous and has full support on Ω. f is common knowledge. I assume that

$$E[\omega] < R < 1,$$

which implies that without further information on ω, the listener will not invest in the project.

Before the listener makes investment decision, the persuader has a chance to persuade, and the listener has the opportunity to investigate the project and acquire information on ω after being persuaded. In the persuasion stage, the persuader *costlessly* sends a message s to the listener. The listener forms a posterior over ω after receiving s. According to the posterior, she chooses her investigation intensity $\alpha \in [0, 1]$. α is the probability of identifying the true value of ω. With probability $(1-\alpha)$, the listener learns nothing from her investigation.[1] $\alpha = 0$ would mean that the listener does not investigate at all. The investigation cost associated with α is $c(\alpha)$ which satisfies $c(0) = 0$, $c'(0) = 0$, $c'(1) \geq 1$, and $c'' > 0$.[2]

The listener makes investment decision after the investigation result is revealed. One should note that if the true ω is not revealed, the belief of the listener is still the posterior formed after receiving s, and she makes investment decision based on the expected value of ω under this posterior.

2.2 Equilibrium Characterization

This subsection characterizes the equilibria of the one-listener case. Throughout the paper, I only consider pure-strategy equilibria. The equilibrium concept adopted is perfect Bayesian equilibrium (PBE).[3] Under this concept, an equilibrium strategy profile of the current model consists of the following elements,

1. Persuasion strategy of the persuader, $p : \Omega \mapsto S$, where S is the finite message space. This strategy specifies the message reported by each type ω of the persuader. For example, $p(\omega') = s'$ means that the persuader reports s', if his type is ω'.

2. The strategy of the listener includes two parts:

- Investigation strategy, $\alpha : S \mapsto [0, 1]$. This strategy specifies the investigation intensity of the listener for each message.

- Investment strategy. This strategy specifies the investment decision of the listener under each possible investigation outcome.

Under PBE, we need to specify a system of belief *consistent* with the equilibrium strategies. Beliefs on the equilibrium path can be easily specified using Bayes rule, but beliefs off the equilibrium path need more consideration. In an equilibrium of this game, two types of information sets may be reached with probability 0 (i.e., off equilibrium path), which are that (1) the listener receives a message not reported by any $\omega \in \Omega$ under the equilibrium persuasion strategy and (2) the listener finds, through her investigation, a value of ω which does not belong to the subset of Ω reporting the message she received. Information sets of type (2) only include one value of ω, so the listener assigns probability 1 to this value. In this model, PBE puts no restrictions on the beliefs on type (1) information sets, so we have pretty much freedom in specifying the beliefs

[1] It is equivalent to assuming that the listener receives a noise signal independent of the true value of ω.

[2] The assumption $c'(1) \geq 1$ is to exclude the corner solution $\alpha = 1$.

[3] In Crawford and Sobel (1982), Farrell and Gibbons (1989) and Goltsman and Pavlov (2011), the equilibrium concept adopted is Bayesian Nash equilibrium. The main difference between PBE and Bayesian Nash is that under PBE, we need to *properly* specify a belief system of the listener.

on such information sets. In the rest of equilibrium analysis, we will omit detailed discussion on equilibrium beliefs. Most of the time, the off-equibrium-path beliefs would be self-clear.

To begin with, let us examine the strategy of the listener. If the listener receives some message s *on the equilibrium path* under a persuasion strategy p, she will update her belief over ω to $f_p(\cdot|s)$ according to Bayes' rule,[4] i.e.,

$$f_p(\omega|s) = \begin{cases} 0, \text{ if } \omega \notin p^{-1}(s) \\ \frac{f(\omega)}{\int_{p^{-1}(s)} f(\hat{\omega})d\hat{\omega}}, \text{ if otherwise} \end{cases}, \forall \omega \in \Omega, \tag{1}$$

where $p^{-1}(s) \equiv \{\hat{\omega} \in \Omega : p(\hat{\omega}) = s\}$.

It is more convenient to characterize the strategy backwards, so we look at investment strategy first. Sequential rationality of PBE requires that when ω is identified, the listener invests if and only if ω is strictly larger than R; if no information about ω is learned, she invests in the project only if the expected payoff of investing under the posterior belief is strictly above R.

Suppose that the listener has posterior $f_p(\cdot|s)$ ($F_p(\cdot|s)$ as the CDF) after receiving message s. If she chooses α as the investigation intensity under message s, then the expected payoff of the listener is

$$Eu_p(\alpha|s) = \alpha \int_R^1 \omega f_p(\omega|s)\, d\omega + \alpha F_p(R|s) R + (1-\alpha) \max\{E_p[\omega|s], R\} - c(\alpha). \tag{2}$$

$Eu_p(\alpha|s)$ denotes the expected payoff of the listener when the persuasion strategy is p, she receives message s and chooses investigation level α. The first term of the expected payoff is the payoff to the listener when ω is identified to be larger than R and the listener invests. The second term is the payoff when ω is revealed to be smaller than R and the listener rejects the project. The third term is the payoff when the value of ω is not identified and the listener chooses whether to invest (obtains $E_p[\omega|s]$) or not invest (obtains R). $E_p[\omega|s]$ is the expected value of ω under posterior $f_p(\cdot|s)$. As an expected payoff maximizer, the listener would choose $\alpha \in [0,1]$ to maximize payoff (2). Thus, the sequentially rational investigation strategy satisfies

$$c'(\alpha_p(s)) = \int_R^1 \omega f_p(\omega|s)\, d\omega + F_p(R|s) R - \max\{E_p[\omega|s], R\}, \tag{3}$$

where $\alpha_p(s)$ represents the optimal investigation intensity under persuasion strategy p and message s. Since $\Omega = [0,1]$ and $R \in (0,1)$, the right-hand side of this equation is always between 0 and 1. With the assumptions on $c(\cdot)$, one should note that $\alpha_p(s)$ is *uniquely* defined by this equality,

[4] If $p^{-1}(s)$ only include finite number of ω, then the Bayes' rule in (1) is not applicable, because the denominator is 0. In this case, the posterior distribution is actually discrete; densities becomes probabilities. We assume that $\Pr_p(\omega|s) = f(\omega) / \sum_{\hat{\omega} \in p^{-1}(s)} f(\hat{\omega})$, if $\omega \in p^{-1}(s)$; $\Pr_p(\omega|s) = 0$, if otherwise. As we will see in the next subsection, this case can be assumed out without loss.

and $\alpha_p(s) \in [0,1)$.[5]

We assume that if the listener receives a message s' for which $p^{-1}(s')$ is empty, the listener's posterior induces her rejection without investigation. This assumption frees us from detailed discussion on off-equilibrium beliefs. Relaxing this assumption, i.e., allowing the listener to hold off-equilibrium beliefs that induces investigation, will not change the following equilibrium analysis, because, as one will see, an equilibrium that can be sustained by other off-equilibrium beliefs can always be sustained under this assumption.

Now we look at the persuasion strategy of the persuader. For any persuasion strategy p, the law of iterated expectation (LIE) gives

$$\sum_{s_i \in \{s \in S:\ p^{-1}(s) \neq \varnothing\}} E_p[\omega|s_i] \Pr(s_i|p) = E[\omega], \tag{4}$$

where $\Pr(s_i|p) = \int_{p^{-1}(s_i)} f(\hat{\omega})\, d\hat{\omega}$. Since $E[\omega] < R$, it is obvious from equation (4) that there must be some message s_i under strategy p such that $E_p[\omega|s_i] < R$ and $\Pr(s_i|p) > 0$. But the question of interest is, is it possible that there exists some s_j such that $E_p[\omega|s_j] > R$ in equilibrium? The answer is no. The intuition is that if there is such a message s_j under p, then no type of persuader will report the message s_i giving $E_p[\omega|s_i] < R$, because the persuader would prefer the listener to think high of his project! This would contradict $\Pr(s_i|p) > 0$. I formally state this result in the proposition below. The proof can be found in Appendix A.

Proposition 1 *In an equilibrium, for any message $s \in S$ that is reported by some type(s) of the persuader, there is $E_p[\omega|s] \leq R$.*

This proposition indicates that after receiving a message in equilibrium, if the listener does not acquire further information about the project, she will reject it. It is then natural for one to ask, do all these messages induce investigation in equilibrium? This can be true, but not necessarily. In the next subsection, we will demonstrate that the equilibria in which all the reported messages induce investigation are equivalent to the pooling equilibrium where any type of the persuader reports the same message. Below I provide an equilibrium showing that not all messages would induce investigation. In this equilibrium, the persuasion strategy is

$$p(\omega) = \begin{cases} s_1, \text{ if } \omega \in [0, \underline{\omega}] \cup (R, 1]; \\ s_2, \text{ if } \omega \in (\underline{\omega}, R]. \end{cases} \tag{5}$$

$\underline{\omega}$ is chosen properly such that $E[\omega|s_1] \leq R$. Existence of $\underline{\omega}$ is proved in Appendix B. For the listener, if message s_1 is received, she investigates the project with intensity $\alpha(s_1)$ satisfying $c'(\alpha(s_1)) = \int_R^1 \omega f_p(\omega|s_1)\, d\omega + F_p(R|s_1) R - R$, and invests only if she finds $\omega > R$; if message s_2

[5] Note that in the model, it is always not optimal to choose $\alpha = 1$ by the listener, because the marginal cost of investigation at $\alpha = 1$ in this case is $c'(1) \geq 1$, and the marginal benefit of investigation is less than 1 which is the highest possible value of ω.

is received, she declines the project without any investigation (i.e., $\alpha(s_2) = 0$). One can easily verify that this is indeed an equilibrium.

It is not possible to have all messages induce zero investigation intensity in equilibrium. Suppose that the listener receives a message s', the equilibrium investigation intensity for this message should be $\alpha(s')$ satisfying

$$
\begin{aligned}
c'(\alpha(s')) &= \int_R^1 \omega f_p(\omega|s')\, d\omega + F_p(R|s')\, R - R \qquad (6)\\
&= 1 - R - \int_R^1 F_p(\omega|s')\, d\omega,
\end{aligned}
$$

where the second equality is derived using integration by parts. This equation indicates that $\alpha(s')$ is zero if and only if $F_p(\omega|s') < 1$ for at most finite elements of $(R, 1]$, i.e., the set $p^{-1}(s') \cap (R, 1]$ has measure 0. Since $(R, 1] = \bigcup_{s\in S} p^{-1}(s) \cap (R, 1]$ and S is finite, there must exist a message s such that $p^{-1}(s) \cap (R, 1]$ has positive measure. Thus there must be at least one message inducing positive investigation.

One should be clear that in an equilibrium in which the listener adopts a sequentially rational investment strategy, if there are different levels of investigation intensity induced by different messages, then persuader with $\omega \in (R, 1]$ will always report the message inducing the highest investigation. The other types of persuader will be indifferent in reporting any message, as they can never be approved by the listener.

2.3 Equilibrium Simplification

In an equilibrium of the one-listener persuasion model, the number of messages which are reported by some type(s) of the persuader, $\#\{s \in S : p^{-1}(s) \neq \varnothing\}$, can be any finite natural number. The arbitrariness of the number of messages tends to make equilibrium analysis tedious and complex. However, the results of this subsection indicate that in searching for the optimal equilibrium which maximizes the persuader's expected payoff, we can focus on the equilibria in which $\#\{s \in S : p^{-1}(s) \neq \varnothing\} \leq 2$. (The rest of the paper calls $\#\{s \in S : p^{-1}(s) \neq \varnothing\}$ of an equibrium *the number of messages of the equilibrium.*) Specifically, this subsection will show that for any multiple-message equilibrium, one can always construct an equilibrium which has at most two messages, but generates the same *ex ante* expected payoffs for both the persuader and the listener as does the multiple-message equilibrium.

Suppose that in an equilibrium, there are multiple messages, $s_1, s_2, \ldots, s_I$, inducing positive investigation intensities of the listener. I define $S^+ \equiv \{s_1, s_2, \ldots, s_I\}$, so S^+ is the set of messages which induce investigation. The lemma below says that all the messages in S^+ induce the same level of investigation.

Lemma 2 *If $\alpha(s_i) > 0$ and $\alpha(s_j) > 0$ in a PBE, then $\alpha(s_i) = \alpha(s_j)$.*

Proof. One can simply prove this by contradiction. Suppose that $0 < \alpha(s_i) < \alpha(s_j) < 1$, for $s_i, s_j \in S^+$, $i \neq j$. Given that the listener invests if she finds $\omega > R$, which is required by the sequential rationality of PBE, no $\omega \in (R, 1]$ would report s_i, because reporting s_j gives higher probability of being identified from investigation, i.e., higher probability of running the project. Only the types with $\omega \leq R$ would report s_i, so $\alpha(s_i)$ should be 0 in equilibrium, which is a contradiction to the supposition that $\alpha(s_i) > 0$. ■

If there is a message in $S \backslash S^+$reported by some type(s) of persuader in equilibrium, definition of S^+indicates that this message induces 0 intensity of investigation, i.e., rejection without investigation. For the convenience of discussion, we name the set of such messages as $S^0 \ (\subset S \backslash S^+)$. All $\omega \in (R, 1]$ report messages in S^+in equilibrium and get positive probability of approval, they have no incentive to change their reports.

The rest of this subsection is devoted to the following proposition:

Proposition 3 *For the equilibrium in which S^+has more than two messages, we can construct an equilibrium having at most two messages, but generating the same payoffs for the persuader and the listener.*

Proof. Considering an equilibrium in which the persuasion strategy is p and $\#S^+ \geq 2$. The expression (6) implies that the equilibrium investigation strategy α should satisfy

$$c'(\alpha(s_i)) = \int_R^1 (\omega - R) f_p(\omega|s_i) \, d\omega, \ i = 1, \ldots, I. \tag{7}$$

For $\forall \omega \in \Omega$, the definition of $f_p(\omega|s_i)$ gives[6]

$$\begin{aligned} \sum_{i=1}^{I} f_p(\omega|s_i) \Pr(s_i|p) &= f(\omega) \\ &= \frac{f(\omega)}{\Pr(S^+|p)} \Pr\left(S^+|p\right), \text{ where } \Pr\left(S^+|p\right) = \sum_{i=1}^{I} \Pr(s_i|p) \\ &= f_p\left(\omega|S^+\right) \Pr\left(S^+|p\right). \end{aligned} \tag{8}$$

Let us modify the persuasion strategy of the persuader into one in which all types reporting messages in S^+ just report the message s_1 instead, other types reporting messages in S^0—if there is any—report a message, say s_0. Let $\tilde{p}$ denote this new persuasion strategy. Under $\tilde{p}$, according to

[6]If $p^{-1}(s_i)$ includes only finite number of elements, then we define $f_p(\omega|s_i) = \Pr_p(\hat{\omega}|s_i) \, d\hat{\omega}$, where $\Pr_p(\omega|s_i)$ is defined in a previous footnote. $\Pr(s_i|p) = \sum_{\omega \in p^{-1}(s_i)} f(\omega)$

equation (6), the optimal investigation intensity $\alpha_{\tilde{p}}(s_1)$ of the listener when receiving s_1 satisfies

$$\begin{aligned}
c'(\alpha_{\tilde{p}}(s_1)) &= \int_R^1 (\omega - R) f_{\tilde{p}}(\omega|s_1) d\omega \\
&= \int_R^1 (\omega - R) f_p\left(\omega|S^+\right) d\omega \\
&= \int_R^1 (\omega - R) \frac{\sum_{i=1}^I f_p(\omega|s_i) \Pr(s_i|p)}{\Pr(S^+|p)} d\omega \\
&= \sum_{i=1}^I \left(\int_R^1 (\omega - R) f_p(\omega|s_i) d\omega \right) \frac{\Pr(s_i|p)}{\Pr(S^+|p)} \\
&= c'(\alpha(s_i))
\end{aligned}$$

The second equality is due to the definition of $\tilde{p}$. The third equality is derived using equation (8). The last equality is based on Lemma 2 and (7) .This results implies that $\alpha_{\tilde{p}}(s_1) = \alpha(s_i)$. If there is any type of persuader reporting s_0 under $\tilde{p}$, then this type must have $\omega \leq R$. So it is optimal to have $\alpha_{\tilde{p}}(s_0) = 0$, which is the same as reporting a message in S^0 under p.

The persuasion strategy $\tilde{p}$, investigation strategy $\tilde{\alpha}$ with $\tilde{\alpha}(s_1) = \alpha_{\tilde{p}}(s_1)$ and $\tilde{\alpha}(s) = 0$ for $s \in S\backslash\{s_1\}$, and the sequentially rational investment strategy, which is to accept the project only if ω is identified to be strictly larger than R, constitute an equilibrium having at most two equilibrium messages. For $\forall\omega \in p^{-1}(S^+)$,the probability of success, which is $\tilde{\alpha}(s_1)$ for $\omega \in (R, 1]$ and 0 for $\omega \in [0, R]$, under this constructed strategy profile is the same as that under the original equilibrium. The same thing happens to all types reporting messages in S^0, if there is any. Therefore, from the perspective of the persuader, this modified strategy profile is equivalent to the original equilibrium.

The *ex ante* expected payoff of the listener under this newly constructed strategy is also the same as that under the original equilibrium. According to Lemma 2, first order condition (3) and the expression of expected payoff (2), we have

$$Eu_p(\alpha(s_i)|s_i) = Eu_p(\alpha(s_j)|s_j), \text{ for } s_i,\ s_j \in S^+. \tag{9}$$

That is, the expected payoff under each message in S^+ is identical in equilibrium. For a message in S^0, if S^0 is nonempty, the expected payoff of the listener is R, as she will reject without investigation. In the new equilibrium, since $\tilde{\alpha}(s_1) = \alpha(s_i)$, we can find that based on (3) and (2),

$$Eu_{\tilde{p}}(\tilde{\alpha}(s_1)|s_1) = Eu_p(\alpha(s_i)|s_i),\ s_i \in S^+. \tag{10}$$

If s_0 is reported by any type, the expected payoff of the listener is R. Therefore, we derive that

$$Eu_{\tilde{p}}\left(\tilde{\alpha}\left(s_1\right)|s_1\right)\Pr\left(s_1|\tilde{p}\right)+R\Pr\left(s_0|\tilde{p}\right)=\sum_{i=1}^{I}Eu_p\left(\alpha\left(s_i\right)|s_i\right)\Pr\left(s_i|p\right)+R\Pr\left(S^0|p\right),$$

i.e., the *ex ante* expected payoffs of the listener under these two equilibria are the same. ■

2.4 Optimal Equilibrium

This paper is interested in characterizing the *optimal equilibrium* for the persuader which maximizes the *ex ante* expected probability of launching the persuader's project. Discussion in the preceding subsection allows us to focus our analysis on equilibria consisting of no more than two messages.

Suppose that in an equilibrium, message s_1 under persuasion strategy p induces investigation intensity $\alpha\left(s_1\right)>0$. We know that all types with $\omega\in(R,1]$ will report s_1, and the investor will invest if she identifies them from investigation. Thus, the *ex ante* expected probability of running the project is $\alpha\left(s_1\right)\left[1-F\left(R\right)\right]$. Since $\left[1-F\left(R\right)\right]$ is independent of the choice of equilibrium, the optimal equilibrium should have the highest $\alpha\left(s_1\right)$. To simplify notation, I define $L\equiv p^{-1}\left(s_1\right)\cap[0,R]$ which is the set of ω lower than the outside option of the listener, R, and reporting s_1. I also define $\Pr\left(L\right)\equiv\int_L f\left(\hat{\omega}\right)d\hat{\omega}$ which is the probability measure of set L. According to equation (6),

$$\begin{aligned}c'\left(\alpha\left(s_1\right)\right)&=1-R-\int_R^1 F_p\left(\omega|s_1\right)d\omega\\&=1-R-\frac{\int_R^1 F\left(\omega\right)d\omega-\left[F\left(R\right)-\Pr\left(L\right)\right]\left(1-R\right)}{1-F\left(R\right)+\Pr\left(L\right)}.\end{aligned}$$

$\Pr\left(L\right)$ is the only equilibrium-dependent term that affects the value of $\alpha\left(s_1\right)$. Taking the derivative of $\alpha\left(s_1\right)$ with respect to $\Pr\left(L\right)$ yields

$$\frac{\mathrm{d}\alpha\left(s_1\right)}{\mathrm{d}\Pr\left(L\right)}=-\frac{1}{c''\left(\alpha\left(s_1\right)\right)}\frac{\int_R^1\left[1-F\left(\omega\right)\right]d\omega}{\left[1-F\left(R\right)+\Pr\left(L\right)\right]^2}<0. \tag{11}$$

So to achieve the highest level of $\alpha\left(s_1\right)$, we need to have $\Pr\left(L\right)$ as small as possible in equilibrium. Proposition 1 shows that there must be $E_p\left[\omega|s_1\right]\leq R$ in equilibrium. This condition puts a constraint for L as below

$$\int_L\frac{\omega f\left(\omega\right)}{1-F\left(R\right)+\Pr\left(L\right)}d\omega+\int_R^1\frac{\omega f\left(\omega\right)}{1-F\left(R\right)+\Pr\left(L\right)}d\omega\leq R,$$

which is equivalent to

$$\int_R^1 (\omega - R) f(\omega)\, d\omega \le R \Pr(L) - \int_L \omega f(\omega)\, d\omega. \tag{12}$$

The next proposition characterizes the optimal equilibrium.

Proposition 4 *In the optimal equilibrium, $p^{-1}(s_1) = [0, \bar{\omega}^*] \cup (R, 1]$, where $\bar{\omega}^*$ satisfied*

$$E[\omega | \omega \in [0, \bar{\omega}^*] \cup (R, 1]] = R.$$

That is, the set of ω reporting s_1 has the form $[0, \bar{\omega}^*] \cup (R, 1]$.

This seemingly surprising result is actually intuitive. In equilibrium, only a project with $\omega > R$ can be launched. Such a project gets the investment of the listener if and only if the value of ω is identified by the listener. Pooling the worse projects with the best ones gives the highest incentive for the listener to do investigation, thus maximizes the probability of identifying a project with $\omega > R$.

Proof. I first characterize the optimal equilibrium among the ones where $p^{-1}(s_1)$ has the form $[0, \bar{\omega}] \cup (R, 1]$, then I show that an arbitrary equilibrium cannot outperform it. Suppose in equilibrium $p^{-1}(s_1) = [0, \bar{\omega}] \cup (R, 1]$, $\bar{\omega} \le R$. Following the definition of L, $\Pr(L) = F(\bar{\omega})$. Based on (11) and (12), searching for the optimal equilibrium can be reformulated as the constrained minimization problem below

$$\min_{\bar{\omega} < R} \bar{\omega}$$

$$s.t. \int_0^{\bar{\omega}} \omega f(\omega)\, d\omega + \int_R^1 \omega f(\omega)\, d\omega \le R[1 - F(R) + F(\bar{\omega})].$$

I define function $g(\bar{\omega})$ as

$$g(\bar{\omega}) = R[1 - F(R) + F(\bar{\omega})] - \left[\int_0^{\bar{\omega}} \omega f(\omega)\, d\omega + \int_R^1 \omega f(\omega)\, d\omega\right].$$

So $g(\bar{\omega}) \ge 0$ means that $\bar{\omega}$ satisfies the constraint of the minimization problem above. Appendix B shows that g is monotone and $\exists$ unique $\bar{\omega}^* \in [0, R]$ such that $g(\bar{\omega}^*) = 0$. For $\forall \bar{\omega} < \bar{\omega}^*$, $g(\bar{\omega}) < 0$, the constraint will be violated. For $\forall \bar{\omega} > \bar{\omega}^*$, they are not the minimum of $\bar{\omega}$ satisfying the constraint. Hence $\bar{\omega}^*$ is the optimal solution.

Any other strategy profile whose persuasion strategy p achieves smaller $\Pr(L)$ than does $[0, \bar{\omega}^*] \cup (R, 1]$ cannot satisfy the constraint $E_p[\omega | s_1] \le R$, so does not form an equilibrium. Suppose $\hat{p}$ of a strategy profile has $\Pr\left(\hat{L}\right) < F(\bar{\omega}^*)$ where $\hat{L} = \hat{p}^{-1}(s_1) \cap [0, R]$. The monotonicity of $F(\bar{\omega})$ indicates that we can always find a $\bar{\omega}(\hat{L})$ such that $\Pr\left(\hat{L}\right) = F\left(\bar{\omega}(\hat{L})\right)$. Thus,

$F\left(\bar{\omega}(\hat{L})\right) < F\left(\bar{\omega}^*\right)$ and $\bar{\omega}(\hat{L}) < \bar{\omega}^*$. The monotonicity of g and definition of $\bar{\omega}^*$ determine that

$$g\left(\bar{\omega}(\hat{L})\right) < 0, \text{ i.e.,}$$

$$E\left[\omega|\omega \in [0, \bar{\omega}(\hat{L})] \cup (R, 1]\right] > R.$$

We conclude the proof by showing that $E_{\hat{p}}\left[\omega|s_1\right] = E\left[\omega|\omega \in \hat{p}^{-1}\left(s_1\right)\right] > R$. Taking the difference between $E\left[\omega|\omega \in [0, \bar{\omega}(\hat{L})] \cup (R, 1]\right]$ and $E_{\hat{p}}\left[\omega|s_1\right]$ yields

$$\begin{aligned} & E\left[\omega|\omega \in [0, \bar{\omega}(\hat{L})] \cup (R, 1]\right] - E\left[\omega|\omega \in \hat{p}^{-1}\left(s_1\right)\right] \\ = & \left[\int_0^{\bar{\omega}(\hat{L})} \omega f\left(\omega\right) d\omega - \int_{\hat{L}} \omega f\left(\omega\right) d\omega\right] \frac{1}{1 - F\left(R\right) + \Pr\left(\hat{L}\right)}. \end{aligned}$$

The sign of this difference is determined by the term in the square brackets. What below shows that it is nonpositive.

$$\begin{aligned} & \int_0^{\bar{\omega}(\hat{L})} \omega f\left(\omega\right) d\omega - \int_{\hat{L}} \omega f\left(\omega\right) d\omega \\ = & \int_{[0, \bar{\omega}(\hat{L}))\backslash \hat{L}} \omega f\left(\omega\right) d\omega - \int_{\hat{L}\backslash [0, \bar{\omega}(\hat{L}))} \omega f\left(\omega\right) d\omega \\ \leq & \bar{\omega}(\hat{L}) \cdot \Pr\left([0, \bar{\omega}(\hat{L})]\backslash \hat{L}\right) - \bar{\omega}(\hat{L}) \cdot \Pr\left(\hat{L}\backslash [0, \bar{\omega}(\hat{L})]\right) \\ = & 0, \text{ since } \Pr\left([0, \bar{\omega}(\hat{L})]\backslash \hat{L}\right) = \Pr\left(\hat{L}\backslash [0, \bar{\omega}(\hat{L})]\right). \end{aligned}$$

This means that $R < E\left[\omega|\omega \in [0, \bar{\omega}(\hat{L})] \cup (R, 1]\right] \leq E\left[\omega|\omega \in \hat{p}^{-1}\left(s_1\right)\right]$. So the constraint is also violated. The strategy profile with $\hat{p}$ is not an equilibrium. ■

3 Public Persuasion

In practice, situations of persuasion involving more than one listener are very common. However, analysis of one-listener case does not enable us to fully understand these situations, even if the listeners are independent decision makers (e.g., Farrell and Gibbons 1989; Goltsman and Pavlov 2011; Koessler 2008). The rest of this paper is devoted to discussing persuasion involving two heterogeneous listeners. The rest of our analysis is to examine, between simultaneous public persuasion and sequential persuasion, which mode of persuasion is better for the speaker.

For expositional purpose, I choose to start with public persuasion in which the listeners are persuaded through a publicly observable message. As one will see, public persuasion is a natural extension of the one-listener model; a lot of previous results have their counterparts in this case.

In sequential persuasion, the persuader has a new dimension in his persuasion strategy, the order of persuading the listeners.

The first subsection below sets up the model, the second subsection introduces a few equilibria with different characteristics. The last subsection characterizes the optimal equilibrium.

3.1 Setup

Now different from the single listener case, launching the persuader's project requires the investment of two listeners, listener 1 and listener 2. When both of them invest, the project is launched, and *each* listener gets payoff $\omega \in \Omega = [0, 1]$. If the project fails to get enough investment, i.e., at most one listener agrees to invest, listener i receives her reservation payoff R_i, $i \in \{1, 2\}$.

The value of ω of the project is still private to the persuader. The persuader would like to run the project regardless of the value of ω. But the risk-neutral listeners agree to invest only if the expected value of ω is strictly larger than their outside options. The listeners have common prior $f : \Omega \mapsto \mathcal{R}$ over ω. f is common knowledge. It is continuous and has full support on Ω. To make the analysis interesting, I assume that

$$E[\omega] < R_2 < R_1 < 1.$$

This assumption guarantees that no listener invests without further information beyond the prior, but when the project is "good" enough, it is profitable for them to invest. One should notice that listener 1 is pickier in investing.

The game also begins with a persuasion stage where the persuader sends a public message $s \in S$ to the listeners costlessly. Similar to the one-listener case, the persuasion strategy can be expressed as $p : \Omega \mapsto S$, where S is a finite message space. After receiving message s, the listeners simultaneously make decisions on investigation. The investigation strategy of i, $i \in \{1, 2\}$, is still a function $\alpha_i : S \mapsto [0, 1]$. Listener i pays cost $c_i(\alpha_i)$ for intensity α_i. $c_i(\cdot)$ satisfies $c_i(0) = 0$, $c_i'(0) = 0$, $c_i'(1) \geq 1$, and $c_i'' > 0$. If the true value of ω is learned after investigation, then listener i compares the value of ω with R_i to decide whether to invest or not. If the true state of the world is not revealed, then whether to invest depends on $E_p[\omega|s]$, the expected value of ω based on posterior $f_p(\cdot|s)$ which is formed following (1), and the strategy of the other listener. I assume that listeners' investigation outcomes and the investment decisions are all unobservable to each other.

We still use PBE as the equilibrium concept. In the rest of our equilibrium analysis, similar as before, we omit detailed discussion on equilibrium beliefs. Off-equilibrium beliefs will be specified when they are not self-clear.

3.2 Equilibria

The cheap-talk feature of the persuasion stage spawns numerous PBE's. It is impossible to exhaust them. But fortunately, the paper is able to show that in characterizing the optimal equilibrium of public persuasion, we can without loss of generality focus on a specific set of PBE's as we did in the one-listener case. I postpone this discussion to the next subsection.

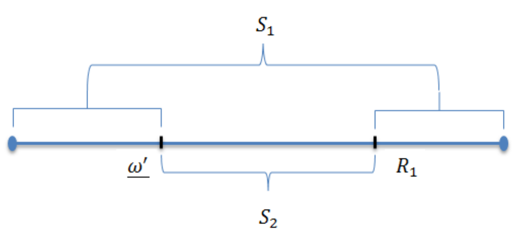
In this subsection, I describe several representative equilibria to illustrate how the players interact under public persuasion, and discuss the intuitions underlying. Analysis on these equilibria facilitates the proof in the next subsection.

1. Joint Rejection without Investigation

There is a class of trivial equilibria in which both listeners reject the project without investigation after receiving any message. That such equilibria could arise is essentially due to the fact that running the project requires the investment of both listeners; the rejection of one listener ruins the hope of running the project.

2. Unilateral Investigation

There is an equilibrium in which listener 2, the less picky listener, free rides on listener 1's investigation, while listener 1, the pickier listener, behaves just like she is in the one-listener case. The equilibrium strategy of the persuader, p, can be illustrated using the following figure,



If the project has $\omega \in [0, \underline{\omega}') \cup (R_1, 1]$, the persuader reports s_1; otherwise, he reports s_2. The equilibrium strategy of listener 1 is that under s_1, she investigates the project with intensity $\alpha_1(s_1)$ which is defined by

$$c_1'(\alpha_1(s_1)) = 1 - R_1 - \int_{R_1}^{1} F_p(\omega|s_1)\, d\omega,$$

and invests in the project only if ω is identified to be larger than R_1; if s_2 is received, she rejects the project without investigation. For listener 2, she agrees to invest without investigation when receiving s_1 and rejects the project when observing s_2.

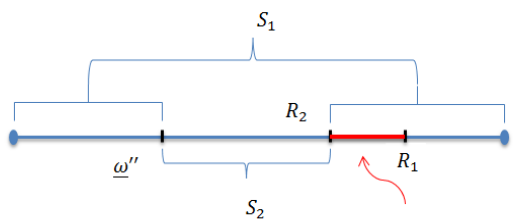
It is necessary to point out that $\underline{\omega}'$ is not arbitrary. The value of $\underline{\omega}'$ should guarantee that $E_p[\omega|s_1] \leq R_1$, according to Proposition 1. The existence of such $\underline{\omega}'$ can be proved using the method in Appendix B. The proof showing that this strategy profile is indeed an equilibrium is provided in Appendix C.

What is interesting about this equilibrium is that under message s_1, only listener 1 investigates, listener 2 free rides. Intuitively, this is because listener 1 is pickier and only agrees to invest when she finds ω is above R_1; doing investigation generates no benefit for listener 2.

3. Joint Investigation

One may have noticed that in the equilibrium above, a project with quality $\omega \leq R_1$ is never approved. Here I show that it is possible to have an equilibrium in which a project with $\omega \in (R_2, R_1]$ has positive probability to be launched. To have such an equilibrium exist, there must be $E[\omega|\omega > R_2] > R_1$, which means that preferences of the listeners are sufficiently aligned[7].

The strategy of the persuader in such an equilibrium, p, is as below,



If ω of the project belongs to the interval $(\underline{\omega}'', R_2]$, the persuader sends message s_2; if otherwise, the persuader sends message s_1 to the listeners.

The equilibrium strategy of listener 1 is that when s_1 is received, she investigates with intensity $\alpha_1(s_1)$, and rejects the project only if ω is revealed to be smaller than R_1; if s_2 is received, she rejects the project without investigation. For listener 2, if s_1 is received, she investigates with intensity $\alpha_2(s_1)$ and invests in the project only if ω is revealed to be above R_2; if s_2 is received, she rejects without investigation. Thus, there is joint investigation under message s_1, but no investigation under s_2. $\alpha_1(s_1)$ and $\alpha_2(s_1)$ are uniquely defined by the following system of equations

$$c_1'(\alpha_1(s_1)) = \alpha_2(s_1)\int_{R_2}^{R_1}(R_1-\omega) f_p(\omega|s_1)\,d\omega, \tag{13}$$
$$c_2'(\alpha_2(s_1)) = \int_{R_2}^{1}(\omega-R_2) f_p(\omega|s_1)\,d\omega - \alpha_1(s_1)\int_{R_2}^{R_1}(\omega-R_2) f_p(\omega|s_1)\,d\omega.$$

And $\alpha_1(s_1)$ must satisfy

$$\alpha_1(s_1) \leq \bar{\alpha}_{1p}(s_1) \equiv \frac{R_2 - E_p[\omega|s_1]}{\int_{R_1}^{1}\omega f_p(\omega|s_1)\,d\omega + F_p(R_1|s_1)R_2 - E_p[\omega|s_1]}. \tag{14}$$

The existence and uniqueness of $(\alpha_1(s_1), \alpha_2(s_1))$ satisfying equations in (13) will be discussed in

[7] This is either due to the closeness of R_1 and R_2, or resulted from the low probability of ω falling between R_1 and R_2.

the next subsection. Condition (14) means that the primitives and the choice of $\underline{\omega}''$ must guarantee that the investigation intensity of listener 1 is not too large, otherwise the strategy profile does not constitute an equilibrium, as it will not be optimal for listener 2 to reject when she learns nothing from investigation.

The rest of this subsection shows that the strategy profile is indeed an equilibrium if the primitives and $\underline{\omega}''$ satisfy condition (14). It is easy to show that under message s_1, when $E[\omega|\omega > R_2] > R_1$ and $\alpha_1(s_1) \leq \bar{\alpha}_{1p}(s_1)$ hold, the investment strategies of the two listeners specified above are optimal. Obviously, it is optimal for listener i to invest (reject) if she finds that $\omega > R_i$ ($\omega \leq R_i$). In the case that listener 1's investigation digs out no information on ω, it is optimal for her to invest, because given $E[\omega|\omega > R_2] > R_1$ and p,

$$\alpha_2 \int_{R_2}^{1} \omega f_p(\omega|s_1)\,d\omega + \alpha_2 F_p(R_2|s_1) R_1 + (1-\alpha_2) R_1 > R_1, \text{ for } \forall \alpha_2 \in (0,1).$$

It is optimal for listener 2 to reject when she fails to identify the value of ω, because given condition (14) and p,

$$\alpha_1(s_1) \int_{R_1}^{1} \omega f_p(\omega|s_1)\,d\omega + \alpha_1(s_1) F_p(R_1|s_1) R_2 + (1-\alpha_1(s_1)) E_p[\omega|s_1] \leq R_2,$$

the left-hand side of which is the expected payoff of listener 2 when she invests without finding out ω. Given the strategy of the other listener, listener i chooses intensity α_i to maximize her expected payoff. Let $Eu_{ip}(\alpha_i, \alpha_j(s_1)|s_1)$ denote the expected payoff of listener i from choosing α_i under message s_1, given that the intensity of listener $j \neq i$ is $\alpha_j(s_1)$. We have

$$\begin{aligned} Eu_{1p}(\alpha_1, \alpha_2(s_1)|s_1) &= \alpha_1 \left\{ \int_{R_1}^{1} [\alpha_2(s_1)\omega + (1-\alpha_2(s_1)) R_1] f_p(\omega|s_1)\,d\omega + F_p(R_1|s_1) R_1 \right\} \\ &\quad + (1-\alpha_1) \left[\alpha_2(s_1) \int_{R_2}^{1} \omega f_p(\omega|s_1)\,d\omega + \alpha_2(s_1) F_p(R_2|s_1) R_1 + (1-\alpha_2(s_1)) R_1 \right] \\ &\quad - c_1(\alpha_1), \end{aligned}$$

$$\begin{aligned} Eu_{2p}(\alpha_2, \alpha_1(s_1)|s_1) &= \alpha_1(s_1) \left[\alpha_2 \int_{R_1}^{1} \omega f_p(\omega|s_1)\,d\omega + \alpha_2 F_p(R_1|s_1) R_2 + (1-\alpha_2) R_2 \right] \\ &\quad + (1-\alpha_1(s_1)) \left[\alpha_2 \int_{R_2}^{1} \omega f_p(\omega|s_1)\,d\omega + \alpha_2 F_p(R_2|s_1) R_2 + (1-\alpha_2) R_2 \right] \\ &\quad - c_2(\alpha_2). \end{aligned}$$

The expected payoff of each listener is strictly concave in her investigation intensity, so the first order condition (F.O.C.) of expected payoff maximization is necessary and sufficient. Equations in (13) are just the F.O.C.'s for listener 1 and listener 2's maximization problems.

Under signal s_2, it is obvious that both listeners reject the project in optimality, as all the ω reporting s_2 are smaller than R_2. Given the strategy of the listeners, any type of the persuader has no incentive to deviate. Thus this profile is an equilibrium.

The equilibrium above is interesting, because the project with $\omega \in (R_2, R_1]$ has positive probability $[1 - \alpha_1(s_1)]\,\alpha_2(s_1)$ to be run, even though they are lower than the reservation value R_1 of listener 1. This result is essentially driven by the condition $E[\omega|\omega > R_2] > R_1$, which allows listener 1 to rely on listener 2's investment decision when she learns nothing from her investigation.

Under the condition $E[\omega|\omega > R_2] > R_1$, there might exist an even more surprising equilibrium where even a project with $\omega \in [0, R_2]$ has positive chance of being run. But the persuader pools all the types in such equilibrium, i.e., there is no persuasive communication at all between the persuader and the listeners. So I omit this case in the following analysis.

3.3 Equilibrium Simplification

The equilibria introduced above are very simple in terms of structure: for the first equilibrium, all the messages induce the same unappealing interaction between the listeners—joint rejection; for the other two equilibria, there are just two messages that could be reported in equilibrium.

Theoretically, due to the cheap-talk feature of the persuasion stage in our model, there could be arbitrarily finite number of equilibrium messages that induce various patterns of interaction between the listeners in a PBE. But in characterizing the optimal equilibrium of public persuasion, the findings below imply that without loss of generality we can focus on the PBE's whose structures are as simple as those in the preceding subsection.

To proceed, we first examine what patterns of listeners' interaction can be induced by messages in equilibrium. The lemma below tells us that there are just three of them.

Lemma 5 *In a PBE, a message on the equilibrium path must induce one of the following patterns of interaction between the listeners:*

1. *At least one of the listeners rejects the project without investigation.*
2. *Listener* 1 *conducts investigation, she rejects the project unless ω is identified to be above R_1, the reservation payoff of listener* 1*. Listener* 2 *approves the project without investigation.*
3. *Both listeners investigate the project, but listener* 2 *rejects the project unless ω is identified to be above R_2 , while listener* 1 *accepts the project unless ω is identified to be smaller than R_1.*

The proof for this lemma is not hard. Facing a message, one listener has four alternative strategies: to accept the project without investigation, to reject it without investigation, to investigate and accept it unless ω is identified to be smaller than her outside option, to investigate and reject it unless ω is identified to be larger than her outside option. Thus, there are $4 \times 4 = 16$ possible strategy profiles for this two-listener model. By ignoring the profile in which both listeners investigate and accept it unless ω is identified to be smaller than their outside options, one can show that only the profiles listed in the lemma are possible in a PBE.

Furthermore, I show in Appendix D that for an equilibrium which has multiple messages inducing the same strategy profile, one can construct another equilibrium which has only one message inducing that strategy profile, but generates the same expected probability of success for the persuader and the same expected payoffs for the listeners. Combining with Lemma 5, this result is formally stated in the following proposition.

Proposition 6 *A PBE with more than three equilibrium messages does not increase the players' payoffs over all PBE's with at most three equilibrium messages.*

With this proposition, in characterizing the optimal equilibrium of public persuasion, we can without loss of generality focus on PBE's with no more than three equilibrium messages and different messages inducing different interaction patterns.

3.4 Optimal Equilibrium

In this subsection, we characterize the equilibrium which maximizes the expected payoff of the persuader, i.e., the optimal equilibrium. Previous results enable us to focus our analysis on the equilibria with no more than three equilibrium messages and different messages inducing different interaction patterns. To further simplify the analysis, I assume that in an equilibrium the second and third patterns of interaction listed in Lemma 5 do not coexist. This assumption restricts our attention to equilibria with at most two messages reported in an equilibrium and only one of them inducing investigation.

It is obvious that in the optimal equilibrium there must be a message inducing investigation of the listeners. In an equilibrium with no message inducing investigation, all types of the project report the same message which leads to rejection of both listeners, so the probability of launching any type of the project is 0.

Let us first look at the class of equilibria in which listener 2 free rides in investigation under a message. In such equilibria, the strategic situations facing listener 1 and the persuader are essentially the same as those in the one-listener case, so the analysis in the one-listener case applies here. We use p^{PU} to denote the strategy of the persuader, where the superscript PU means that this persuasion strategy is for the case of unilateral investigation (U) under public persuasion (P), and s_1 to represent the message inducing investigation. According to the results in subsection 2.4, we can conclude that in the optimal equilibrium among this class of equilibria,

$p^{PU-1}(s_1) = [0, \bar{\omega}^{PU}] \cup (R_1, 1]$, where $\bar{\omega}^{PU}$ satisfies

$$\int_0^{\bar{\omega}^{PU}} \omega f(\omega)\, d\omega + \int_{R_1}^1 \omega f(\omega)\, d\omega = R_1 \left[1 - F(R_1) + F\left(\bar{\omega}^{PU}\right)\right].$$

The equilibrium investigation intensity $\alpha_1^{PU}(s_1)$ of listener 1 satisfies

$$c_1'\left(\alpha_1^{PU}(s_1)\right) = \frac{\int_{R_1}^1 (\omega - R_1) f(\omega)\, d\omega}{1 - F(R_1) + F(\bar{\omega}^{PU})}.$$

The *ex ante* expected probability of launching the project is

$$\alpha_1^{PU}(s_1)\left[1 - F(R_1)\right]. \tag{15}$$

Now let us examine the class of equilibria where the listeners jointly investigate under a message. Henceforth, we call such equilibria joint-investigation equilibria. Lemma 5 shows that in such equilibria listener 2 rejects the project if ω is not identified to be above R_2, while listener 1 accepts the project unless it is identified that $\omega < R_1$. To have such an equilibrium exist, a necessary condition is $E[\omega|\omega > R_2] > R_1$. Let p^{PJ} denote the strategy of the persuader in this class of equilibria, where P and J in the superscript respectively represent public persuasion and joint investigation. $s_1$still represents the message inducing investigation. It is clear that $\forall \omega \in (R_2, 1]$ reports s_1. According to the discussion in Equilibrium 3 of subsection 3.2, the equilibrium investigation intensities $\left(\alpha_1^{PJ}(s_1), \alpha_2^{PJ}(s_1)\right)$ of the listeners satisfy

$$\begin{aligned} c_1'\left(\alpha_1^{PJ}(s_1)\right) &= \alpha_2^{PJ}(s_1) \int_{R_2}^{R_1} (R_1 - \omega) f_{p^{PJ}}(\omega|s_1)\, d\omega, \\ c_2'\left(\alpha_2^{PJ}(s_1)\right) &= \int_{R_2}^1 (\omega - R_2) f_{p^{PJ}}(\omega|s_1)\, d\omega - \alpha_1^{PJ}(s_1) \int_{R_2}^{R_1} (\omega - R_2) f_{p^{PJ}}(\omega|s_1)\, d\omega, \end{aligned} \tag{16}$$

and $\alpha_1^{PJ}(s_1)$ must satisfy

$$\alpha_1^{PJ}(s_1) \le \bar{\alpha}_{1p^{PJ}}(s_1) \equiv \frac{R_2 - E_{p^{PJ}}[\omega|s_1]}{\int_{R_1}^1 \omega f_{p^{PJ}}(\omega|s_1)\, d\omega + F_{p^{PJ}}(R_1|s_1) R_2 - E_{p^{PJ}}[\omega|s_1]}.$$

We can prove that (16) has a unique solution. Define $b_i(\alpha_j)$, $i, j \in \{1, 2\}$, as

$$\begin{aligned} c_1'(d_1(\alpha_2)) &= \alpha_2 \int_{R_2}^{R_1} (R_1 - \omega) f_{p^{PJ}}(\omega|s_1)\, d\omega, \\ c_2'(d_2(\alpha_1)) &= \int_{R_2}^1 (\omega - R_2) f_{p^{PJ}}(\omega|s_1)\, d\omega - \alpha_1 \int_{R_2}^{R_1} (\omega - R_2) f_{p^{PJ}}(\omega|s_1)\, d\omega, \end{aligned}$$

and $B(\alpha_1, \alpha_2) = (b_1(\alpha_2), b_2(\alpha_1))$. By ignoring the constraint on α_1, $B(\alpha_1, \alpha_2)$ is a continuous mapping from $[0,1]\times[0,1]$ to $[0,1]\times[0,1]$, so according to Brouwer's fixed-point theorem, $B(\alpha_1, \alpha_2)$ has a fixed point (α_1^*, α_2^*) on $[0,1]\times[0,1]$. The fixed point is unique, because if there are two different fixed points (α_1^*, α_2^*) and $(\alpha_1^{*\prime}, \alpha_2^{*\prime})$ with $\alpha_1^* > \alpha_1^{*\prime}$, then the first equation implies $\alpha_2^* > \alpha_2^{*\prime}$, but the second equation implies $\alpha_2^* < \alpha_2^{*\prime}$, which is a conflict. If $\alpha_1^* \leq \bar{\alpha}_{1p^{PJ}}(s_1)$, then this fixed point is the profile of equilibrium intensities under s_1, i.e., $\left(\alpha_1^{PJ}(s_1), \alpha_2^{PJ}(s_1)\right) = (\alpha_1^*, \alpha_2^*)$. Otherwise, there does not exist an equilibrium.

We characterize the optimal equilibrium among joint-investigation equilibria. Since all $\omega \in (R_2, 1]$ must report s_1, $p^{PJ-1}(s_1)$ can be completely represented by $p^{PJ-1}(s_1) \cap [0, R_2]$, i.e., the set of ω which is (weakly) smaller than R_2 and reports s_1. To simplify notation, we define $L^{PJ} \equiv p^{PJ-1}(s_1) \cap [0, R_2]$. The definition of $f_{p^{PJ}}(\omega|s_1)$ enables us to rewrite (16) as below

$$\begin{aligned} c_1'\left(\alpha_1^{PJ}(s_1)\right) &= \alpha_2^{PJ}(s_1) \frac{\int_{R_2}^{R_1} (R_1 - \omega) f(\omega) d\omega}{1 - F(R_2) + \Pr(L^{PJ})}, \\ c_2'\left(\alpha_2^{PJ}(s_1)\right) &= \frac{\int_{R_2}^{1} (\omega - R_2) f(\omega) d\omega}{1 - F(R_2) + \Pr(L^{PJ})} - \alpha_1^{PJ}(s_1) \frac{\int_{R_2}^{R_1} (\omega - R_2) f(\omega) d\omega}{1 - F(R_2) + \Pr(L^{PJ})}, \end{aligned} \tag{17}$$

and the constraint becomes

$$\alpha_1^{PJ}(s_1) \leq \bar{\alpha}_{1p^{PJ}}(s_1) \equiv 1 - \frac{\int_{R_1}^{1} (\omega - R_2) f(\omega) d\omega}{\left[\int_{L^{PJ}} (R_2 - \omega) f(\omega) d\omega - \int_{R_2}^{R_1} (\omega - R_2) f(\omega) d\omega\right]}. \tag{18}$$

One can find that the composition of L^{PJ} determines the values of $\left(\alpha_1^{PJ}(s_1), \alpha_2^{PJ}(s_1)\right)$ and whether (18) is satisfied. Given that condition (18) is satisfied, the probability of launching the project is $\alpha_2^{PJ}(s_1)\left[1 - \alpha_1^{PJ}(s_1)\right]$ if $\omega \in (R_2, R_1]$ and $\alpha_2^{PJ}(s_1)$ if $\omega \in (R_1, 1]$. Thus, the optimal equilibrium of this class is the solution of the following maximization problem,

$$\begin{aligned} \max_{L^{PJ}} \quad & \alpha_2^{PJ}(s_1)\left[1 - \alpha_1^{PJ}(s_1)\right]\left[F(R_1) - F(R_2)\right] + \alpha_2^{PJ}(s_1)\left[1 - F(R_1)\right] \\ s.t. \quad & \text{Condition (18) is satisfied.} \end{aligned} \tag{19}$$

The proposition below helps us simplify the characterization of optimal L^{PJ} by narrowing down its form.

Proposition 7 *Without loss of generality, $p^{PJ}(s_1)$ in the optimal equilibrium of the joint-investigation equilibria can be restricted to the form $p^{PJ}(s_1) = [0, \bar{\omega}] \cup (R_2, 1]$, i.e., L^{PJ} can be restricted to the form of interval $[0, \bar{\omega}]$, where $\bar{\omega} \leq R_2$.*

Proof. One can find that if there is a set L^{PJ} making condition (18) satisfied, then we can always find an interval $[0, \bar{\omega}(L^{PJ})]$ where the cutoff $\bar{\omega}(L^{PJ})$ satisfies

$$F\left(\bar{\omega}\left(L^{PJ}\right)\right) = \Pr\left(L^{PJ}\right) \tag{20}$$

and gives

$$\int_{L^{PJ}} (R_2 - \omega) f(\omega)\, d\omega \leq \int_0^{\bar{\omega}\left(L^{PJ}\right)} (R_2 - \omega) f(\omega)\, d\omega. \tag{21}$$

The definition of $\bar{\omega}(L^{PJ})$ makes the interval $[0, \bar{\omega}(L^{PJ})]$ induce the same values of $\alpha_1^{PJ}(s_1)$ and $\alpha_2^{PJ}(s_1)$ as does L^{PJ}, due to (20) and (17), and also makes condition (18) satisfied, according to (18) and (21). Therefore, without loss of generality, we can just focus on the intervals in the form $[0, \bar{\omega}]$ to characterize the optimal L^{PJ}. That is, $p^{PJ}(s_1)$ can be restricted to the form $[0, \bar{\omega}] \cup (R_2, 1]$. ■

It is necessary to point out that joint-investigation equilibria may not exist, even if $E[\omega|\omega > R_2] > R_1$. For a persuasion strategy with $p^{PJ-1}(s_1) = L^{PJ} \cup (R_2, 1]$ and $p^{PJ-1}(s_2) = \Omega \backslash \left(L^{PJ} \cup (R_2, 1]\right)$, we can accordingly construct a strategy profile in which the strategies of the listeners under message s_1 satisfy (17) and those under message s_2, $p^{PJ-1}(s_2) \neq \varnothing$, are rejection. This strategy profile constitutes an equilibrium if and only if condition (18) is satisfied. The proof of Proposition 7 shows that if such a strategy profile is an equilibrium, then the strategy profile constructed by replacing L^{PJ} with $[0, \bar{\omega}(L^{PJ})]$ where $\bar{\omega}(L^{PJ})$ is defined by (20) also constitutes an equilibrium. Thus, in discussing the existence of joint-investigation equilibrium, we can focus on the strategy profiles with L^{PJ} being the form of $[0, \bar{\omega}]$. Let us replace L^{PJ} in (17) and (18) with $[0, \bar{\omega}]$ and define a function

$$\phi^{PJ}(c_1, c_2, R_1, R_2, f, \bar{\omega}) \equiv \alpha_1^{PJ}(s_1) - \bar{\alpha}_{1p^{PJ}}(s_1). \tag{22}$$

So condition (18) is satisfied if and only if $\phi^{PJ}(c_1, c_2, R_1, R_2, f, \bar{\omega}) \leq 0$. One can verify that $\alpha_1^{PJ}(s_1)$ solved from (17) is decreasing in $\bar{\omega}$ and $\bar{\alpha}_{1p^{PJ}}(s_1)$ is increasing in $\bar{\omega}$. Thus,

$$\min_{\bar{\omega} \leq R_2} \phi^{PJ}(c_1, c_2, R_1, R_2, f, \bar{\omega}) = \phi^{PJ}(c_1, c_2, R_1, R_2, f, R_2). \tag{23}$$

We define $\underline{\phi}^{PJ}(c_1, c_2, R_1, R_2, f) = \phi^{PJ}(c_1, c_2, R_1, R_2, f, R_2)$. Therefore, there exists a joint-investigation equilibrium if and only if

$$\underline{\phi}^{PJ}(c_1, c_2, R_1, R_2, f) \leq 0. \tag{24}$$

We say that a marginal cost function $\tilde{c}_i'$ is larger than another marginal cost function c_i' if $\tilde{c}_i'(\alpha) \geq c_i'(\alpha)$ for $\forall \alpha \in [0,1]$. According to (17), we can see that given c_2, R_1, R_2, f, existence condition (24) can be satisfied if the marginal cost function of listener 1, c_1', is large enough. This is because as c_1' becomes larger, $\alpha_1^{PJ}(s_1)$ becomes smaller, and $\phi^{PJ}(c_1, c_2, R_1, R_2, f, \bar{\omega})$ is smaller.[8]

[8] This can be proved by contradiction. Suppose $\alpha_1^{PJ}(s_1)$ is larger as c_1' becomes larger. Then the first equation

If c_1, R_1, R_2, f are fixed, (24) also holds, if c_2' is large enough. Therefore, it is safe to conclude that (24) will be satisfied if either c_1' or c_2' or both of them are large.

If condition (24) holds, then the maximization problem (19) has a solution. According to Proposition 7, we can focus on L^{PJ} of the form $[0, \bar{\omega}]$ in characterizing the optimal equilibrium. If condition (24) is satisfied with equality, then there is only one joint-investigation equilibrium in which $p^{PJ-1}(s_1) = \Omega$, and this equilibrium is optimal. If condition (24) is a strict inequality, then there is a set $[\underline{\bar{\omega}}, R_2]$ where $\underline{\bar{\omega}}$ satisfies $\phi^{PJ}(c_1, c_2, R_1, R_2, f, \underline{\bar{\omega}}) = 0$, and each $\bar{\omega}$ in this set is corresponding to a joint-investigation equilibrium. Since the objective function in (19) is continuous in $\bar{\omega}$ and $[\underline{\bar{\omega}}, R_2]$ is compact, there is an optimal $\bar{\omega}^{PJ}$.

To find the optimal equilibrium of public persuasion, we should compare the optimal equilibria of the two classes discussed above. It is hard to do a complete comparison between them. In this paper, we provide a local sufficient condition and a global sufficient condition for the optimal joint-investigation equilibrium to outperform.

Proposition 8 *When $E[\omega|\omega > R_2] > R_1$ and (24) hold, the optimal joint-investigation equilibrium is the optimal equilibrium of public persuasion if*

$$\varphi_2\left(\int_{R_1}^{1}(\omega - R_2) f(\omega)\, d\omega\right) \geq \varphi_1\left(\frac{\int_{R_1}^{1}(\omega - R_1) f(\omega)\, d\omega}{1 - F(R_1) + F(\bar{\omega}^{PU})}\right), \tag{25}$$

where φ_i is the inverse of c_i', $i = 1, 2$.

Proof. In the optimal joint-investigation equilibrium, the *ex ante* expected probability of launching the project is

$$\alpha_2^{PJ}(s_1)\left[1 - \alpha_1^{PJ}(s_1)\right][F(R_1) - F(R_2)] + \alpha_2^{PJ}(s_1)[1 - F(R_1)].$$

The expected probability of launching the project in the optimal unilateral-investigation equilibrium is expressed in (15). If $\alpha_2^{PJ}(s_1) \geq \alpha_1^{PU}(s_1)$, then the optimal joint-investigation equilibrium outperforms. From (17), we have

$$\begin{aligned} \alpha_2^{PJ}(s_1) &\geq \varphi_2\left(\frac{\int_{R2}^{1}(\omega - R_2) f(\omega)\, d\omega}{1 - F(R_2) + F(\bar{\omega}^{PJ})}\right) \\ &\geq \varphi_2\left(\int_{R_1}^{1}(\omega - R_2) f(\omega)\, d\omega\right). \end{aligned}$$

of (17) implies that $\alpha_2^{PJ}(s_1)$ should also increase. But the second equation requires that $\alpha_2^{PJ}(s_1)$ decrease. A contradiction. Therefore, $\alpha_1^{PJ}(s_1)$ is increasing in c_1'.

According to (15), there is

$$\alpha_1^{PU}(s_1) = \varphi_1 \left(\frac{\displaystyle\int_{R_1}^{1} (\omega - R_1) f(\omega) \, d\omega}{1 - F(R_1) + F(\bar{\omega}^{PU})} \right).$$

Condition (25) implies that $\alpha_2^{PJ}(s_1) \geq \alpha_1^{PU}(s_1)$. Thus, the optimal joint-investigation equilibrium is the optimal equilibrium of public persuasion. ■

This proposition gives a local sufficient condition for the optimal joint-investigation equilibrium to outperform. The sufficient condition provided by the next proposition puts more restrictions on the cost functions.

Proposition 9 *If $E[\omega|\omega > R_2] > R_1$ and c_1, c_2 satisfy (24) and*

$$\inf_{\alpha \in (0,1)} \frac{c_1'(\alpha)}{c_2'(\alpha)} \geq \frac{\displaystyle\int_{R_1}^{1} (\omega - R_1) f(\omega) \, d\omega}{[1 - F(R_1) + F(\bar{\omega}^{PU})] \displaystyle\int_{R_1}^{1} (\omega - R_2) f(\omega) \, d\omega}, \tag{26}$$

then the optimal joint-investigation equilibrium is the optimal equilibrium of public persuasion.

Proof. From (15) and (17) we can obtain that

$$\frac{c_1'\left(\alpha_1^{PU}(s_1)\right)}{c_2'\left(\alpha_2^{PJ}(s_1)\right)} \leq \frac{\displaystyle\int_{R_1}^{1} (\omega - R_1) f(\omega) \, d\omega}{[1 - F(R_1) + F(\bar{\omega}^{PU})] \displaystyle\int_{R_1}^{1} (\omega - R_2) f(\omega) \, d\omega}.$$

Based on condition (26), we can find that

$$\inf_{\alpha \in (0,1)} \frac{c_1'(\alpha)}{c_2'(\alpha)} \geq \frac{c_1'\left(\alpha_1^{PU}(s_1)\right)}{c_2'\left(\alpha_2^{PJ}(s_1)\right)}.$$

Thus, it must be that $\alpha_2^{PJ}(s_1) \geq \alpha_1^{PU}(s_1)$, which implies that the optimal joint-investigation equilibrium is the optimal equilibrium of public persuasion. ■

4 Sequential Persuasion

This section starts the discussion on sequential persuasion, the persuasion mode under which persuader approaches the two listeners sequentially. The basic setup of the model is the same as that in the public persuasion case, except that the timing of the game is changed as below:

1. The persuader observes the quality ω of the project, $\omega \in \Omega = [0,1]$.

2. Given the value of ω, the persuader decides the order of persuasion, i.e., whether to choose $\langle 1,2 \rangle$ or $\langle 2,1 \rangle$, where $\langle i,j \rangle$ means that i is persuaded first and j second.

3. Listener approached first by the persuader observes the order of persuasion, i.e., she knows that she is the first to be approached. At the same time, she receives a message from the persuader. Based on the order of persuasion, the message received, and her own belief, the listener makes decision on investigation and decides whether to invest in the project based on the investigation outcome.

4. If the first approached listener rejects the project, the game ends. Otherwise, the game continues, the persuader approaches the second listener and sends a message to her. The second listener observes the order of persuasion as well, but she cannot observe the message sent to the first listener, though she knows that the first listener accepts the project when she is approached. Based on the information she has and her belief, the second listener chooses her optimal investigation intensity and decides whether to invest or not when investigation result is revealed.

The specification of the game indicates that the strategy of the persuader includes three elements: (1) the order of persuasion, (2) the message sent to the first listener, (3) the message sent to the second listener. If we still use p to denote the strategy of the persuader, then p can be expressed as a mapping

$$p : \Omega \mapsto \{\langle 1,2 \rangle, \langle 2,1 \rangle\} \times S \times S.$$

For example, $p(\omega') = (\langle 2,1 \rangle, s, s')$ means that the persuader approaches listener 2 first and sends s to listener 2 and s' to listener 1. In the rest of this section, we use O to denote $\{\langle 1,2 \rangle, \langle 2,1 \rangle\}$ and o to represent either $\langle 1,2 \rangle$ or $\langle 2,1 \rangle$. The investigation strategies of the listeners are changed to

$$\alpha_i : O \times S \mapsto [0,1], i = 1,2$$

I still adopt PBE as the equilibrium concept.

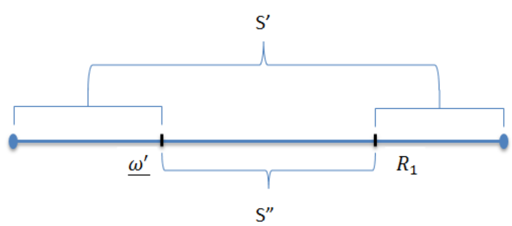
4.1 Equilibria

This subsection introduces a few representative equilibria of the sequential persuasion game. These equilibria will give us lots of insights on equilibrium analysis. We characterize the optimal equilibrium in the subsection after the next.

1. Unilateral Investigation

In public persuasion game, we introduced an equilibria in which after receiving a specific message, one listener investigates and the other listener free rides. We have a similar equilibrium under sequential persuasion. The equilibrium strategy of the persuader, p, and those of the listeners, α_1 and α_2, are specified respectively as below.

The persuader chooses to approach listener 1 first with persuasion strategy illustrated by the following figure. If the project has a $\omega \in [0, \underline{\omega}'] \cup (R_1, 1]$, then the persuader sends message s'; if otherwise, he sends message s''. When listener 1 approves the project, he sends the same message to listener 2. In this strategy, $\underline{\omega}'$ is properly chosen such that $E_p[\omega|s'] \leq R_1$.



For listener 1, her investigation strategy is independent of the order of persuasion and

$$\alpha_1 = \begin{cases} \arg\max_{\alpha} \alpha \int_{R_1}^{1} \omega f_p(\omega|s')\, d\omega + \alpha F_p(R_1|s') R_1 + (1-\alpha) R_1 - c_1(\alpha), & \text{if message is } s', \\ 0, & \text{if otherwise.} \end{cases}$$

She approves the project only if ω of the project is identified to be larger than R_1. The strategy of listener 2 is that she always rejects the project when she is persuaded first and always invests in the project regardless of the message received when she is approached second.

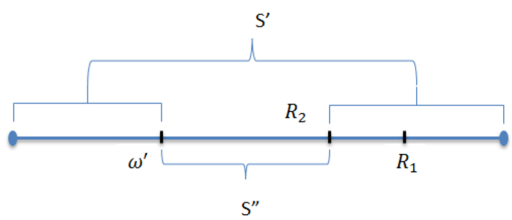
It is clear that this is an equilibrium, because for any type of the persuader, he has no incentive to change the order of persuasion and the message reported, given the strategies of the listeners. Given the strategy of the persuader and the systems of belief, the listeners' strategies are sequentially rational.

This equilibrium is interesting, because even though the persuader persuades the listeners in a manner completely different from public persuasion, the probability of getting the project approved is unchanged.

2. Joint Investigation

In the rest of this subsection, I introduce two equilibria in which both listeners investigate and all ω choose the same order of persuasion. But the orders of persuasion are different for these two equilibria. One will see that due to this difference, the two equilibria yield different expected probabilities of launching the project. It needs to point out that it is necessary to have $E[\omega|\omega > R_2] > R_1$ for the two equilibria to exist.

To begin with, I describe the equilibrium where listener 2 is always firstly persuaded. The persuasion strategy of the persuader facing listener 2 can be illustrated using the following figure. For the project with $\omega \in (\omega', R_2]$, he sends message s''; for the project with $\omega \in [0, \omega'] \cup (R_2, 1]$, he sends message s'. Conditional on that listener 2 is persuaded successfully, he sends the same message to listener 1.



For listener 1, her investigation strategy α_1 satisfies

$$\begin{aligned} c_1' \left(\alpha_1 \left(\langle 2,1\rangle, s'\right)\right) &= \frac{\int_{R_2}^{R_1} (R_1 - \omega) f(\omega) \, d\omega}{1 - F(R_2)}, \text{ and} \\ \alpha_1 (o, s_1) &= 0, \text{ if } o \neq \langle 2,1\rangle \text{ or } s_1 \neq s'. \end{aligned} \tag{27}$$

Under $(\langle 2,1\rangle, s')$, she accepts the project unless ω is identified to be smaller than R_1; under $o \neq \langle 2,1\rangle$ or $s_1 \neq s'$, she rejects without investigation. For listener 2, her investigation strategy α_2 satisfies

$$\begin{aligned} c_2' \left(\alpha_2 \left(\langle 2,1\rangle, s'\right)\right) &= \frac{\int_{R_2}^{1} (\omega - R_2) f(\omega) \, d\omega - \alpha_1 \left(\langle 2,1\rangle, s'\right) \int_{R_2}^{R_1} (\omega - R_2) f(\omega) \, d\omega}{1 - F(R_2) + F(\omega')}, \\ \alpha_2 (o, s_2) &= 0, \text{ if } o \neq \langle 2,1\rangle \text{ or } s_1 \neq s'. \end{aligned} \tag{28}$$

On observing $(\langle 2,1\rangle, s')$, she accepts the project only if ω is identified to be larger than R_2, and when observing $o \neq \langle 2,1\rangle$ or $s_1 \neq s'$, she rejects the project directly.

To have listener 2 reject the project when no information on ω is learned from investigation, there must be

$$R_2 \geq \alpha_1 \left(\langle 2,1\rangle, s'\right) \int_{R_1}^{1} \omega f_p (\omega | s') \, d\omega + \alpha_1 \left(\langle 2,1\rangle, s'\right) F_p (R_1 | s') R_2 + \left(1 - \alpha_1 \left(\langle 2,1\rangle, s'\right)\right) E_p [\omega | s'],$$

where the RHS is listener 2's expected payoff of approving the project when no further information on ω is revealed, given listener 1's strategy. So a necessary condition for this equilibrium is that

$$\alpha_1(\langle 2,1\rangle, s') \leq \bar{\alpha}_{1p}(\langle 2,1\rangle, s') \equiv \frac{R_2 - E_p[\omega|s']}{\int_{R_1}^{1} \omega f_p(\omega|s')\, d\omega + F_p(R_1|s') R_2 - E_p[\omega|s']}. \tag{29}$$

One may notice that this condition is similar to that in the joint-investigation equilibria of public persuasion. But $\alpha_1(\langle 2,1\rangle, s')$ is obviously larger than its counterpart in (13), if the value of ω' is the same for the two equilibria. Thus, this condition is "harder" to be satisfied under sequential persuasion than under public persuasion.

Now we look at another equilibrium where the persuader always approaches listener 1 first. For the type $\omega \in (\omega', R_2]$, he still sends message s'', and for the type $\omega \in [0, \omega'] \cup (R_2, 1]$, he sends message s'. Conditional on that listener 1 is successfully persuaded, he persuades listener 2 with the same message.

For listener 1, her investigation strategy can be expressed as

$$\alpha_1(o, s_1) = \begin{cases} > 0, \text{ if } o = \langle 1,2\rangle \text{ and } s_1 = s'; \\ 0, \text{ if otherwise.} \end{cases}$$

Under $(\langle 1,2\rangle, s')$, listener 1 approves the project unless ω is identified to be smaller than R_1; she rejects the project if $o \neq \langle 1,2\rangle$ or $s_1 \neq s'$. For listener 2, α_2 satisfies that

$$\alpha_2(o, s_2) = \begin{cases} > 0, \text{ if } o = \langle 1,2\rangle \text{ and } s_2 = s'; \\ 0, \text{ if otherwise.} \end{cases}$$

When $(\langle 1,2\rangle, s')$ is observed, she rejects the project unless its quality is identified to be larger than R_2; otherwise, she rejects the project. $\alpha_1(\langle 1,2\rangle, s')$ and $\alpha_2(\langle 1,2\rangle, s')$ are the solutions of the following system of equations

$$\begin{aligned} c_1'(\alpha_1(\langle 1,2\rangle, s')) &= \frac{\alpha_2(\langle 1,2\rangle, s')\left[\int_{R_2}^{R_1} (R_1 - \omega) f(\omega)\, d\omega\right]}{1 - F(R_2) + F(\omega')}, \\ c_2'(\alpha_2(\langle 1,2\rangle, s')) &= \frac{\int_{R_2}^{1} (\omega - R_2) f(\omega)\, d\omega - \alpha_1(\langle 1,2\rangle, s') \int_{R_2}^{R_1} (\omega - R_2) f(\omega)\, d\omega}{\alpha_1(\langle 1,2\rangle, s')[1 - F(R_1)] + (1 - \alpha_1(\langle 1,2\rangle, s'))[1 - F(R_2) + F(\omega')]}. \end{aligned} \tag{30}$$

Using Brouwer's fixed-point theorem, we can prove that there exists a solution to this system of equations on $[0,1] \times [0,1]$, but the solution may not be unique. (We will discuss the equilibrium selection when characterizing the optimal equilibrium.) Similar as the equilibrium above, there should be

$$\alpha_1(\langle 1,2\rangle, s') \leq \bar{\alpha}_{1p}(\langle 1,2\rangle, s') \equiv \frac{R_2 - E_p[\omega|s']}{\int_{R_1}^{1} \omega f_p(\omega|s')\,d\omega + F_p(R_1|s')\,R_2 - E_p[\omega|s']}, \tag{31}$$

in this equilibrium, so as that listener 2 would reject the project if she does not identify ω from her investigation.

In these two equilibria, one may have realized that the investigation intensities of the listeners under s' are different. This is because due to the change of persuasion order, the incentives of investigation for the listeners are changed, even though the pattern of the listeners' interaction is the same.

In the equilibrium where listener 1 is firstly persuaded, an interesting finding is that the investigation intensities of the two listeners are strategic complements under s'. In the first equation of (30), it is obvious that $\alpha_1(\langle 1,2\rangle, s')$ is increasing in $\alpha_2(\langle 1,2\rangle, s')$. $\alpha_2(\langle 1,2\rangle, s')$ is also increasing in $\alpha_1(\langle 1,2\rangle, s')$ in the second equation of (30), because by taking the derivative of $\alpha_2(\langle 1,2\rangle, s')$ with respect to $\alpha_1(\langle 1,2\rangle, s')$, we obtain $\frac{\mathrm{d}\alpha_2(\langle 1,2\rangle, s')}{\mathrm{d}\alpha_1(\langle 1,2\rangle, s')} > 0$. (See Appendix E for the proof.)

4.2 Second-stage Separating

In each equilibrium discussed above, after the first listener is successfully persuaded, all the types still report a same message to the second listener. It is natural for one to wonder whether a persuasion strategy involving separating at the second stage will make a difference, i.e., make the persuader better off or worse off. This subsection is devoted to answering this question. The following example shows that for the equilibria where both listeners investigate, a persuasion strategy involving separating at the second stage may change the payoff of the persuader over one without separating at the second stage.

In the second joint-investigation equilibrium where the persuader sends the same message to listener 2 after listener 1 approves the project, the equilibrium investigation intensities of the two listeners satisfy (30). Now we construct a new strategy profile. In this profile, $\forall\omega \in [0,\omega'] \cup (R_2, 1]$ still reports s' to listener 1 first. But if it is approved by listener 1, $\omega \in [0,\omega''] \cup (R_2, 1]$ where $\omega'' < \omega'$ reports message s''', while $\omega \in (\omega'', \omega']$ still reports s'. For listener 1, she adopts a similar strategy as in the second joint-investigation equilibrium. The only difference is that the magnitude of $\alpha_1(\langle 1,2\rangle, s')$ is changed. For listener 2, her investigation strategy satisfies

$$\alpha_2(o, s_2) = \begin{cases} > 0, \text{ if } o = \langle 1,2\rangle \text{ and } s_2 = s'''; \\ 0, \text{ if otherwise.} \end{cases}$$

When $(\langle 1,2\rangle, s''')$ is observed, she rejects the project unless its quality is identified to be larger than R_2; otherwise, she reject the project. $\alpha_1(\langle 1,2\rangle, s')$ and $\alpha_2(\langle 1,2\rangle, s''')$ are the solutions of the following system of equations,

$$
\begin{aligned}
c_1'\left(\alpha_1\left(\langle 1,2\rangle, s'\right)\right) &= \frac{\alpha_2\left(\langle 1,2\rangle, s'''\right)\left[\int_{R_2}^{R_1}\left(R_1-\omega\right) f(\omega)\, d\omega\right]}{1-F\left(R_2\right)+F\left(\omega'\right)}, \qquad (32)\\
c_2'\left(\alpha_2\left(\langle 1,2\rangle, s'''\right)\right) &= \frac{\int_{R_2}^{1}\left(\omega-R_2\right) f(\omega)\, d\omega-\alpha_1\left(\langle 1,2\rangle, s'\right)\int_{R_2}^{R_1}\left(\omega-R_2\right) f(\omega)\, d\omega}{\alpha_1\left(\langle 1,2\rangle, s'\right)\left[1-F\left(R_1\right)\right]+\left(1-\alpha_1\left(\langle 1,2\rangle, s'\right)\right)\left[1-F\left(R_2\right)+F\left(\omega''\right)\right]},
\end{aligned}
$$

where $\alpha_1\left(\langle 1,2\rangle, s'\right)$ should satisfy

$$
\alpha_1\left(\langle 1,2\rangle, s'\right) \leq \bar{\alpha}_{1p}\left(\langle 1,2\rangle, s'''\right) \equiv \frac{R_2-E_p\left[\omega \mid s'''\right]}{\int_{R_1}^{1} \omega f_p\left(\omega \mid s'''\right) d\omega+F_p\left(R_1 \mid s'''\right) R_2-E_p\left[\omega \mid s'''\right]}. \qquad (33)
$$

(32) has a solution on $[0,1]\times[0,1]$. It is easy to verify that if condition (31) is satisfied and ω'' is close to ω' such that (33) is satisfied, then the profile constitutes an equilibrium. We can find that $\alpha_2\left(\langle 1,2\rangle, s'''\right) > \alpha_2\left(\langle 1,2\rangle, s'\right)$ in the second joint-investigation equilibrium and $\alpha_1\left(\langle 1,2\rangle, s'\right)$ increases.

4.3 Optimal Equilibrium

To characterize the optimal equilibrium of sequential persuasion, I apply the same scheme as in public persuasion. To proceed, one should notice that the sequential rationality of PBE requires that listener i accept the project if ω is identified to be larger than R_i. Based on this requirement and the analysis in Appendix F, we can find that only the following investigation scenarios can confront a subset of Ω on the equilibrium path,[9]

Listener 1 is persuaded first:

1. Listener 1 refuses to invest in the project without any investigation;
2. Listener 1 investigates the project and refuses to invest unless ω is identified to be larger than R_1. Listener 2 approves the project when the persuader approaches her after getting the approval of listener 1;
3. Listener 1 investigates the project and agrees to invest unless ω is identified to be smaller than R_1. Listener 2 investigates the project and refuses to invest unless ω is identified to be larger than R_2 when the persuader approaches her with some messages. If the persuader approaches her with other messages, listener 2 rejects the project without investigation .

[9] The beliefs and strategies of the listeners on the off-equilibrium path are specified to sustain these equilibrium behaviours. We omit detailed discussion on them.

Listener 2 is persuaded first:

1. Listener 2 refuses to invest in the project without any investigation;

2. Listener 2 agrees to invest in the project without investigation. Listener 1 investigates the project when the persuader approaches her with some specific messages and rejects the project unless ω is identified to be larger than R_1. If the persuader approaches listener 1 with other messages, she rejects the project without investigation.

3. Listener 2 investigates the project and rejects it unless its ω is identified to be larger than R_2. Listener 1 investigates the project when the persuader persuades her with some specific messages, and accepts the project unless ω is identified to be smaller than R_1. Listener 1 rejects the project when the persuader persuades her with other messages.

Though only the few scenarios specified above can happen on equilibrium path, there may be multiple messages inducing the same scenario in an equilibrium. Using the technique simplifying the set of equilibria of public persuasion (see Proposition 6 and Appendix D), we can also simplify the set of equilibria of sequential persuasion. The result is that if all the messages inducing the same action of a listener at either the first stage or the second stage are "united" to a single message and the strategy of the listener is changed accordingly, the derived strategy profile is still an equilibrium and is *ex ante* equivalent to the original equilibrium from both the persuader's and the listeners' perspectives. This result enables us to, without loss of generality, focus on equilibria in which there are no two different messages inducing the same action of a listener in characterizing the optimal equilibrium. We summarize this result in the following proposition.

Proposition 10 *A PBE with multiple messages inducing the same action of a listener cannot increase the players' payoffs over all PBE's where different messages induce different actions of listeners.*

Whether all the scenarios described above could coexist in an equilibrium, i.e., whether they can all be induced in the same equilibrium with positive probability? The answer is no. The discussion in Appendix F shows for each order of persuasion, Scenario 2 and Scenario 3 could not coexist. In current analysis, I further assume that Scenario 3 under one order of persuasion does not coexist with Scenario 2 under another order of persuasion. This assumption is similar to that we made in characterizing the optimal equilibrium of public persuasion. Therefore, in characterizing the optimal equilibrium of sequential persuasion, we only need to consider four types of equilibria: (1) Scenario 2 of either $\langle 1,2\rangle$ or $\langle 2,1\rangle$ happens in equilibrium, all other messages induce direct reject; (2) Scenario 3 of either $\langle 1,2\rangle$ or $\langle 2,1\rangle$ arises in equilibrium, all other messages induce direct rejection; (3) Scenario 2 of both $\langle 1,2\rangle$ and $\langle 2,1\rangle$ happen in equilibrium, all other messages, if any, induce direct rejection; (4) Scenario 3 of both $\langle 1,2\rangle$ and $\langle 2,1\rangle$ are induced inequilibrium, all other messages, if any, induce direct rejection.

Let us analyze the optimal equilibria of type (1) and type (3) equilibria. It is easy to see that for any type (3) equilibrium, we can always find a type (1) equilibrium which generates the same expected payoffs for the persuader and the listeners. Thus, without loss of generality, we can only characterize the optimal equilibrium of type (1). Since listener 1 is the only investigator, the strategic situations facing the persuader and listener 1 are exactly the same at those in the unilateral-investigation equilibria of public persuasion. Thus, it is straightforward that the optimal equilibrium of type (1) equilibria yields the same expected payoff to the persuader as that shown in (15).

Now we characterize the optimal equilibria of type (2). To begin with, we look at the equilibria where Scenario 3 of $\langle 2,1\rangle$ is induced. For convenience, we call such equilibria $\langle 2,1\rangle -$*type (2) equilibria*. Let p^{SJ} be the strategy of the persuader, where superscripts S and J represent sequential persuasion and joint investigation respectively, s_2 be the (unique) message inducing listener 2 to investigate and s_1 the (unique) message inducing listener 1 to investigate. One should be clear that $\forall\ \omega \in (R_2, 1]$ would report s_2 and s_1. Define $L^{SJ}_{\langle 2,1\rangle} = p^{SJ-1}(\langle 2,1\rangle, s_2) \cap [0, R_2]$ where $p^{SJ-1}(\langle 2,1\rangle, s_2) = \bigcup_{s\in S} p^{SJ-1}(\langle 2,1\rangle, s_2, s)$. Thus, $p^{SJ-1}(\langle 2,1\rangle, s_2)$ is the set of ω approaching listener 2 first with message s_2 and $L^{SJ}_{\langle 2,1\rangle}$ is the subset of $p^{SJ-1}(\langle 2,1\rangle, s_2)$ with all of its elements being smaller than R_2. The equilibrium intensities of investigation should satisfy

$$c_1'(\alpha_1(\langle 2,1\rangle, s_1)) = \frac{\int_{R_2}^{R_1}(R_1-\omega) f(\omega)\,d\omega}{1-F(R_2)}, \tag{34}$$

$$c_2'(\alpha_2(\langle 2,1\rangle, s_2)) = \frac{\int_{R_2}^{1}(\omega-R_2) f(\omega)\,d\omega - \alpha_1(\langle 2,1\rangle, s_1)\int_{R_2}^{R_1}(\omega-R_2) f(\omega)\,d\omega}{1-F(R_2)+\Pr\left(L^{SJ}_{\langle 2,1\rangle}\right)}. \tag{35}$$

$$\alpha_1(\langle 2,1\rangle, s_1) \leq \bar{\alpha}_{1p^{SJ}}(\langle 2,1\rangle, s_2) = \frac{R_2 - E_{p^{SJ}}[\omega|s_2]}{\int_{R_1}^{1}\omega f_{p^{SJ}}(\omega|s_2)\,d\omega + F_{p^{SJ}}(R_1|s_2)\,R_2 - E_{p^{SJ}}[\omega|s_2]}. \tag{36}$$

$\alpha_1(\langle 2,1\rangle, s_1)$ in (34) is independent of $L^{SJ}_{\langle 2,1\rangle}$, because only $\omega \in (R_2, 1]$ could get approved by listener 2 and they all report s_1 to listener 1.

If constraint (36) is satisfied, the *ex ante* expected probability of launching the project is

$$\alpha_2(\langle 2,1\rangle, s_2)(1-\alpha_1(\langle 2,1\rangle, s_1))[F(R_1)-F(R_2)] + \alpha_2(\langle 2,1\rangle, s_2)[1-F(R_1)].$$

Since $\alpha_1(\langle 2,1\rangle, s_1)$ is fixed, the expected probability above is maximized if we has the maximal $\alpha_2(\langle 2,1\rangle, s_2)$. $\alpha_2(\langle 2,1\rangle, s_2)$ is decreasing in $\Pr\left(L^{SJ}_{\langle 2,1\rangle}\right)$, thus the optimal $\alpha_2(\langle 2,1\rangle, s_2)$ is obtained by solving the following problem

$$\min_{L^{SJ}_{\langle 2,1\rangle}} \Pr\left(L^{SJ}_{\langle 2,1\rangle}\right) \tag{37}$$

$$s.t.\ \alpha_1\left(\langle 2,1\rangle, s_1\right) \leq \bar{\alpha}_{1p^{SJ}}\left(\langle 2,1\rangle, s_2\right) \equiv 1 - \frac{\displaystyle\int_{R_1}^{1} (\omega - R_2) f(\omega)\, d\omega}{\left[\displaystyle\int_{L^{SJ}_{\langle 2,1\rangle}} (R_2 - \omega) f(\omega)\, d\omega - \int_{R_2}^{R_1} (\omega - R_2) f(\omega)\, d\omega\right]}.$$

The technique adopted in the proof of Proposition 7 is applicable for solving this problem. The result is that the optimal $L^{SJ}_{\langle 2,1\rangle}$ is equal to $\left[0, \bar{\omega}^{SJ}_{\langle 2,1\rangle}\right]$ where $\bar{\omega}^{SJ}_{\langle 2,1\rangle}$ makes the constraint in the problem binding. One should notice that there may not exist a solution to this problem. Let $L^{SJ}_{\langle 2,1\rangle}$ be replace by $[0, \bar{\omega}]$ in the constraint and define

$$\phi^{SJ}_{\langle 2,1\rangle}\left(c_1, c_2, R_1, R_2, f, \bar{\omega}\right) \equiv \alpha_1\left(\langle 2,1\rangle, s_1\right) - \bar{\alpha}_{1p^{SJ}}\left(\langle 2,1\rangle, s_2\right).$$

Since $\alpha_1\left(\langle 2,1\rangle, s_1\right)$ is independent of $\bar{\omega}$ and $\bar{\alpha}_{1p^{SJ}}\left(\langle 2,1\rangle, s_2\right)$ is increasing in $\bar{\omega}$,

$$\min_{\bar{\omega}} \phi^{SJ}_{\langle 2,1\rangle}\left(c_1, c_2, R_1, R_2, f, \bar{\omega}\right) = \phi^{SJ}_{\langle 2,1\rangle}\left(c_1, c_2, R_1, R_2, f, R_2\right).$$

Use $\underline{\phi}^{SJ}_{\langle 2,1\rangle}\left(c_1, c_2, R_1, R_2, f\right)$ to denote $\phi^{SJ}_{\langle 2,1\rangle}\left(c_1, c_2, R_1, R_2, f, R_2\right)$. Thus, problem (37) has a solution if and only if

$$\underline{\phi}^{SJ}_{\langle 2,1\rangle}\left(c_1, c_2, R_1, R_2, f\right) \leq 0. \tag{38}$$

For a type (2) equilibrium where Scenario 3 of $\langle 1,2\rangle$ is induced, we call it $\langle 1,2\rangle$ *−type (2) equilibrium* for convenience. In such an equilibrium, suppose p^{SJ} is the persuasion strategy, s_1 and s_2 are the messages inducing investigation of listener 1 and listener 2 respectively, the equilibrium investigation intensities should satisfy

$$\begin{aligned} c_1'\left(\alpha_1\left(\langle 1,2\rangle, s_1\right)\right) &= \frac{\alpha_2\left(\langle 1,2\rangle, s_2\right)\left[\displaystyle\int_{R_2}^{R_1} (R_1 - \omega) f(\omega)\, d\omega\right]}{1 - F(R_2) + \Pr\left(L_1^{SJ}\right)}, \\ c_2'\left(\alpha_2\left(\langle 1,2\rangle, s_2\right)\right) &= \frac{\displaystyle\int_{R_2}^{1} (\omega - R_2) f(\omega)\, d\omega - \alpha_1\left(\langle 1,2\rangle, s_1\right)\int_{R_2}^{R_1} (\omega - R_2) f(\omega)\, d\omega}{\alpha_1\left(\langle 1,2\rangle, s_1\right)\left[1 - F(R_1)\right] + \left(1 - \alpha_1\left(\langle 1,2\rangle, s_1\right)\right)\left[1 - F(R_2) + \Pr\left(L_2^{SJ}\right)\right]}, \end{aligned} \tag{39}$$

where $L_1^{SJ} \equiv p^{SJ-1}\left(\langle 1,2\rangle, s_1\right) \cap [0, R_2]$ and $L_2^{SJ} \equiv p^{SJ-1}\left(\langle 1,2\rangle, s_1, s_2\right) \cap [0, R_2]$, $p^{SJ-1}\left(\langle 1,2\rangle, s_1\right) = \bigcup_{s\in S} p^{SJ-1}\left(\langle 1,2\rangle, s_1, s\right)$ is the set of ω reporting s_1 to listener 1 and $p^{SJ-1}\left(\langle 1,2\rangle, s_1, s_2\right)$ is the set of ω reporting s_1 to listener 1 and s_2 to listener 2. Also, there should be

$$\alpha_1(\langle 1,2\rangle, s_1) \leq \bar{\alpha}_{1p^{SJ}}(\langle 1,2\rangle, s_2) \equiv \frac{R_2 - E_{p^{SJ}}[\omega|s_2]}{\int_{R_1}^{1} \omega f_{p^{SJ}}(\omega|s_2)\, d\omega + F_{p^{SJ}}(R_1|s_2) R_2 - E_{p^{SJ}}[\omega|s_2]}. \quad (40)$$

One should be clear that $(R_2, 1] \subset p^{SJ-1}(\langle 1,2\rangle, s_1, s_2)$. The optimal equilibrium of this class of equilibria is characterized by solving the following problem[10]

$$\max_{L_1^{SJ},\, L_2^{SJ}} \alpha_2(\langle 1,2\rangle, s_2)[1 - F(R_1)] + \alpha_2(\langle 1,2\rangle, s_2)[1 - \alpha_1(\langle 1,2\rangle, s_1)][F(R_1) - F(R_2)] \quad (41)$$

$$s.t.\ L_2^{SJ} \subset L_1^{SJ}, \text{ and}$$

$$\alpha_1(\langle 1,2\rangle, s_1) \leq \bar{\alpha}_{1p^{SJ}}(\langle 1,2\rangle, s_2) \equiv 1 - \frac{\int_{R_1}^{1} (\omega - R_2) f(\omega)\, d\omega}{\left[\int_{L_2^{SJ}} (R_2 - \omega) f(\omega)\, d\omega - \int_{R_2}^{R_1} (\omega - R_2) f(\omega)\, d\omega\right]}.$$

The first constraint is implied by the definitions of L_1^{SJ} and L_2^{SJ}.

The proposition below is a counterpart of Proposition 7. It helps us simplify the characterization of optimal L_1^{SJ} and L_2^{SJ} by narrowing down their forms.

Proposition 11 *Without loss of generality, p^{SJ} in the optimal $\langle 1,2\rangle$ −type (2) equilibrium can be restricted to the form $p^{SJ-1}(\langle 1,2\rangle, s_1) = [0, \bar{\omega}_1] \cup (R_2, 1]$ and $p^{SJ-1}(\langle 1,2\rangle, s_1, s_2) = [0, \bar{\omega}_2] \cup (R_2, 1]$, i.e., L_1^{SJ} and L_2^{SJ} can be restricted to the form of intervals $[0, \bar{\omega}_1]$ and $[0, \bar{\omega}_2]$, where $\bar{\omega}_2 \leq \bar{\omega}_1 \leq R_2$.*

Proof. The technique of proving this is similar to that adopted in Proposition 7.

Suppose that L_1^{SJ} and L_2^{SJ} solve the problem (41). Now we choose $0 \leq \bar{\omega}(L_2^{SJ}) \leq \bar{\omega}(L_1^{SJ}) \leq R_2$ such that $\Pr\left(L_1^{SJ}\right) = F\left(\bar{\omega}(L_1^{SJ})\right)$ and $\Pr\left(L_2^{SJ}\right) = F\left(\bar{\omega}(L_2^{SJ})\right)$. This pair of $F\left(\bar{\omega}(L_1^{SJ})\right)$ and $F\left(\bar{\omega}(L_2^{SJ})\right)$ will derive the same values of $\alpha_1(\langle 1,2\rangle, s_1)$ and $\alpha_2(\langle 1,2\rangle, s_2)$ as do $\Pr\left(L_1^{SJ}\right)$ and $\Pr\left(L_2^{SJ}\right)$, so the only thing we need to check is whether the constraints are still satisfied. It is obvious that the first constraint is satisfied. The LHS of the second constraint is unchanged after replacing $\Pr\left(L_1^{SJ}\right)$ and $\Pr\left(L_2^{SJ}\right)$ with $F\left(\bar{\omega}(L_1^{SJ})\right)$ and $F\left(\bar{\omega}(L_2^{SJ})\right)$. The only term on the right-hand side related to L_1^{SJ} and L_2^{SJ} is $\int_{L_2^{SJ}} (R_2 - \omega) f(\omega)\, d\omega$. It is easy to show that

$$\int_{L_2^{SJ}} (R_2 - \omega) f(\omega)\, d\omega - \int_0^{\bar{\omega}(L_2^{SJ})} (R_2 - \omega) f(\omega)\, d\omega \leq 0.$$

[10] In the case that (39) has multiple solutions, we do the problem for each of these solutions satisfying constraint (40), and then choose the optimal one among them.

Therefore, the RHS of the second constraint increases when we change from L_2^{SJ} to $\left[0, \bar{\omega}(L_2^{SJ})\right]$. Therefore, the optimal L_1^{SJ} and L_2^{SJ} can always be expressed in the form of intervals $\left[0, \bar{\omega}(L_1^{SJ})\right]$ and $\left[0, \bar{\omega}(L_2^{SJ})\right]$. ■

This proposition enables us to replace $\Pr\left(L_1^{SJ}\right)$ and $\Pr\left(L_2^{SJ}\right)$ in (39) with $F(\bar{\omega}_1)$ and $F(\bar{\omega}_2)$, and the maximization problem now can be changed to

$$\max_{\bar{\omega}_2 \leq \bar{\omega}_1 \leq R_2} \alpha_2(\langle 1,2\rangle, s_2)\left[1 - F(R_1)\right] + \alpha_2(\langle 1,2\rangle, s_2)\left[1 - \alpha_1(\langle 1,2\rangle, s_1)\right]\left[F(R_1) - F(R_2)\right] \quad (42)$$

$$s.t.\ \alpha_1(\langle 1,2\rangle, s_1) \leq 1 - \frac{\int_{R_1}^{1}(\omega - R_2) f(\omega)\, d\omega}{\left[\int_0^{\bar{\omega}_2}(R_2 - \omega) f(\omega)\, d\omega - \int_{R_2}^{R_1}(\omega - R_2) f(\omega)\, d\omega\right]}.$$

We analyze the condition for (42) to have a solution. According to (39), we can find that $\alpha_1(\langle 1,2\rangle, s_1)$ is decreasing in $\bar{\omega}_2, \bar{\omega}_1$, so $\alpha_1(\langle 1,2\rangle, s_1)$, the LHS of the constraint in (42), reaches its minimum at $\bar{\omega}_1 = \bar{\omega}_2 = R_2$.The RHS of the constraint is increasing in $\bar{\omega}_2$, so its maximum is achieved at $\bar{\omega}_2 = R_2$. Similar as before, we define

$$\phi_{\langle 1,2\rangle}^{SJ}(c_1, c_2, R_1, R_2, f, \bar{\omega}_1, \bar{\omega}_2) \equiv \alpha_1(\langle 1,2\rangle, s_1) - \bar{\alpha}_{1p^{SJ}}(\langle 1,2\rangle, s_2). \quad (43)$$

The monotonicity of $\alpha_1(\langle 1,2\rangle, s_1)$ and $\bar{\alpha}_{1p^{SJ}}(\langle 1,2\rangle, s_2)$ in $\bar{\omega}_1$ and $\bar{\omega}_2$ implies that

$$\min_{\bar{\omega}_2 \leq \bar{\omega}_1 \leq R_2} \phi_{\langle 1,2\rangle}^{SJ}(c_1, c_2, R_1, R_2, f, \bar{\omega}_1, \bar{\omega}_2) \equiv \phi_{\langle 1,2\rangle}^{SJ}(c_1, c_2, R_1, R_2, f, R_2, R_2).$$

We use $\underline{\phi}_{\langle 1,2\rangle}^{SJ}(c_1, c_2, R_1, R_2, f)$ to represent $\phi_{\langle 1,2\rangle}^{SJ}(c_1, c_2, R_1, R_2, f, R_2, R_2)$. So the set of equilibria satisfying the constraint in (42) is non-empty if and only if

$$\underline{\phi}_{\langle 1,2\rangle}^{SJ}(c_1, c_2, R_1, R_2, f) \leq 0. \quad (44)$$

The objective function of (42) is continuous in $(\bar{\omega}_1, \bar{\omega}_2)$. When (44) is satisfied, the set of $(\bar{\omega}_1, \bar{\omega}_2)$ satisfying the constraint is compact, problem (42) has a solution. Similar as the discussion in the case of public persuasion, (44) will be satisfied if either c_1' or c_2' or both of them are large.

Comparing the optimal $\langle 2,1\rangle$ −type (2) equilibrium with the optimal $\langle 1,2\rangle$ −type (2) equilibrium, we can find that the latter one is always better for the persuader. The proof is simple. Suppose that the former one exists, i.e., problem (37) has a solution $\bar{\omega}_{\langle 2,1\rangle}^{SJ} \leq R_1$, then we can always find a $\langle 1,2\rangle$ −type (2) equilibrium with $\bar{\omega}_1 = \bar{\omega}_2 = \bar{\omega}_{\langle 2,1\rangle}^{SJ}$ outperforms it. Because, according to (34), (35), and (39), we have $\alpha_1(\langle 1,2\rangle, s_1) < \alpha_1(\langle 2,1\rangle, s_1)$ and $\alpha_2(\langle 1,2\rangle, s_2) > \alpha_2(\langle 2,1\rangle, s_2)$ when $\bar{\omega}_1 = \bar{\omega}_2 = \bar{\omega}_{\langle 2,1\rangle}^{SJ}$, the expected probability of launching the project is higher in the $\langle 1,2\rangle$ −type (2) equilibrium. This result implies that in characterizing the optimal equilibrium of sequential persuasion, we can ignore the $\langle 2,1\rangle$ −type (2) equilibria, as it is never optimal.

We can also ignore the type (4) equilibria in characterizing the optimal equilibrium of sequential persuasion. This is because according to the analysis and intuition provided in the preceding paragraph, a type (4) equilibrium can always be outperformed by a $\langle 1,2\rangle$ −type (2) equilibrium. So a type (4) equilibrium can never be optimal.

All the results above finally indicate that in determining the optimal sequential persuasion equilibrium, we only need to focus on the optimal type (1) equilibrium and optimal $\langle 1,2\rangle$ −type (2) equilibrium. This implies that the analysis and results will be quite similar to those in public persuasion. The two propositions below are respective the counterparts of Proposition 8 and Proposition 9.

Proposition 12 *When $E[\omega|\omega > R_2] > R_1$ and (44) hold, the optimal $\langle 1,2\rangle$ −type (2) equilibrium is the optimal equilibrium of sequential persuasion if*

$$\varphi_2\left(\int_{R_2}^{1}(\omega - R_2) f(\omega)\, d\omega\right) \geq \varphi_1\left(\frac{\int_{R_1}^{1}(\omega - R_1) f(\omega)\, d\omega}{1 - F(R_1) + F(\bar{\omega}^{PU})}\right), \tag{45}$$

where φ_i is the inverse of c_i', $i = 1, 2$.

Proposition 13 *If $E[\omega|\omega > R_2] > R_1$ and c_1, c_2 satisfy (44) and*

$$\inf_{\alpha\in(0,1)} \frac{c_1'(\alpha)}{c_2'(\alpha)} \geq \frac{\int_{R_1}^{1}(\omega - R_1) f(\omega)\, d\omega}{[1 - F(R_1) + F(\bar{\omega}^{PU})]\int_{R_2}^{1}(\omega - R_2) f(\omega)\, d\omega}, \tag{46}$$

then the optimal joint-investigation equilibrium is the optimal equilibrium of public persuasion.

The proofs for these two propositions are quite similar to those of Proposition 8 and Proposition 9, so we can omit them.

5 Public Persuasion vs. Sequential Persuasion

All the work above is a preparation for answering the central question of this paper: which mode of persuasion, public persuasion or sequential persuasion, outperforms the other from the persuader's perspective? To answer this question, we compare the optimal equilibria of these two persuasion modes. The following analysis will demonstrate that the comparison is not deterministic. That is, there is no mode of persuasion that is always better than the other. Our analysis will focus on discussing how the investigation costs affect the relative performance of these two persuasion modes.

According to (17) in public persuasion and (39) in sequential persuasion, we can find that

$$\phi^{PJ}(c_1, c_2, R_1, R_2, f, \bar{\omega}) < \phi^{SJ}_{\langle 1,2\rangle}(c_1, c_2, R_1, R_2, f, \bar{\omega}, \bar{\omega}),$$

where ϕ^{PJ} and $\phi^{SJ}_{\langle 1,2\rangle}$ are defined in (22) and (43) respectively, because $\alpha_1^{PJ}(s_1)$ is smaller than $\alpha_1(\langle 1,2\rangle, s_1)$ and $\bar{\alpha}_{1p^{PJ}}(s_1)$ is the same as $\bar{\alpha}_{1p^{SJ}}(\langle 1,2\rangle, s_2)$ when $\bar{\omega}_1 = \bar{\omega}_2 = \bar{\omega}$. This implies that

$$\underline{\phi}^{PJ}(c_1, c_2, R_1, R_2, f) < \underline{\phi}^{SJ}_{\langle 1,2\rangle}(c_1, c_2, R_1, R_2, f), \tag{47}$$

$\underline{\phi}^{PJ}(c_1, c_2, R_1, R_2, f)$ and $\underline{\phi}^{SJ}_{\langle 1,2\rangle}(c_1, c_2, R_1, R_2, f)$ are the values of ϕ^{PJ} and $\phi^{SJ}_{\langle 1,2\rangle}$ when $\bar{\omega} = R_2$.

The comparison between these two persuasion modes can be divided into the following cases:

Case 1: $0 < \underline{\phi}^{PJ}(c_1, c_2, R_1, R_2, f)$ **or** $E[\omega|\omega > R_2] \leq R_1$

In this case, joint-investigation equilibria in public persuasion and type (2) equilibria in sequential persuasion do not exist. Because $E[\omega|\omega > R_2] > R_1$ is necessary for these equilibria to exist and $0 < \underline{\phi}^{PJ}(c_1, c_2, R_1, R_2, f)$, combining with (47), makes the existence conditions (24) and (44) violated. Thus, only unilateral-investigation equilibria in public persuasion and type (1) equilibria in sequential persuasion could arise in this case. Previous analysis shows that the optimal equilibria of them are equivalent from the persuader's perspective.

Case 2: $E[\omega|\omega > R_2] > R_1$ **and** $\underline{\phi}^{PJ}(c_1, c_2, R_1, R_2, f) \leq 0 < \underline{\phi}^{SJ}_{\langle 1,2\rangle}(c_1, c_2, R_1, R_2, f)$

According to our previous analysis, this case happens when c_1' and/or c_2' are large such that $\underline{\phi}^{PJ}(c_1, c_2, R_1, R_2, f) \leq 0$, but not too large such that $\underline{\phi}^{SJ}_{\langle 1,2\rangle}(c_1, c_2, R_1, R_2, f) \leq 0$. In this case, public persuasion cannot be dominated by sequential persuasion.

Since $E[\omega|\omega > R_2] > R_1$ and $\underline{\phi}^{PJ}(c_1, c_2, R_1, R_2, f) \leq 0$, joint-investigation equilibria in public persuasion exist, and they have an optimum. But because $0 < \underline{\phi}^{SJ}_{\langle 1,2\rangle}(c_1, c_2, R_1, R_2, f)$, existence condition (44) is violated, there are only type (1) (type (3)) equilibria in sequential persuasion. If the optimal unilateral-investigation equilibrium in public persuasions outperform that of joint-investigation equilibria, then public persuasion and sequential persuasion are still equivalent to each other, because the optimal unilateral-investigation equilibrium in public persuasion and optimal type (1) equilibrium in sequential persuasion yield the same payoff for the persuader. If the optimal joint-investigation equilibrium is the optimal public persuasion equilibrium, then public persuasion outperforms sequential persuasion. This is the case if the conditions in either Proposition 8 or Proposition 9 are satisfied.

Case 3: $E[\omega|\omega > R_2] > R_1$ **and** $\underline{\phi}^{SJ}_{\langle 1,2\rangle}(c_1, c_2, R_1, R_2, f) \leq 0$

This is case is true if c_1' and/or c_2' are very large. In the two cases above, sequential persuasion cannot be better than public public. In this case, we are going to show that this is not always the case.

It is obvious that sequential persuasion can outperform public persuasion when the optimal $\langle 1,2\rangle$ −type (2) equilibrium yields higher expected probability of launching the project than the optimal joint-investigation equilibrium in public persuasion. This can be true if c_1'' is very large everywhere and c_2'' is very small everywhere. Because when c_1'' is very large, i.e., c_1' is very steep, and c_2'' is very small, i.e., c_2' is very flat, according to (17) and (39), $\alpha_1^{PJ}(s_1)$ in the optimal joint-investigation equilibrium is extremely close to $\alpha_1(\langle 1,2\rangle, s_1)$ in the optimal $\langle 1,2\rangle$ −type (2) equilibrium, but $\alpha_2^{PJ}(s_1)$ is much smaller than $\alpha_2(\langle 1,2\rangle, s_2)$. The expected probability of launching the project is thus larger under the optimal $\langle 1,2\rangle$ −type (2) equilibrium than under the optimal joint-investigation equilibrium.

6 Concluding Remarks

This paper builds up a one-persuader/multilistener model of persuasion. The major features of the model are that (1) the persuader is privately informed about the quality of his project; (2) he communicates with the listeners about project's quality through "soft evidence"; (3) the listeners can costly investigate the project, and the investigation intensities are variable.

The objective of the paper is to find which mode of persuasion is optimal from the persuader's perspective. Two modes of persuasion are considered in the analysis, public persuasion and sequential persuasion. In comparing these two modes, the paper finds that the optimal persuasion order heavily depends on the investigation costs of the listeners. If the marginal costs of investigation are low, public persuasion tends to outperform sequential persuasion. The opposite can be true, if it is very costly for the listeners to investigate the true state, especially when listener 1's marginal investigation cost increases very fast in her investigation intensity, while listener 2's marginal investigation cost increases very slowly in her investigation intensity.

Appendix

A. Proof of Proposition 1

Proof. We first show that if s is reported by some type(s) of persuader and $E_p[\omega|s] > R$, it is impossible to have $\alpha_p(s)$, the optimal investigation intensity of the listener after receiving s under persuasion strategy p, strictly smaller than 1 in equilibrium. Suppose $\alpha_p(s) < 1$. Equation (4) and $E[\omega] < R$ jointly imply that there must be some message s' under strategy p such that $E_p[\omega|s'] < R$ and $\Pr(s'|p) > 0$. Since $E_p[\omega|s'] < R$, there must exist a $\omega' \in p^{-1}(s')$ such that $\omega' < R$. For this ω', the probability of launching the project is 0 by reporting message s', because if it is identified, the listener will reject it, as $\omega' < R$; if it is not identified, it will be rejected as well, as $E_p[\omega|s'] < R$. If ω' reports s instead, the probability of getting investment is $[1-\alpha_p(s)] > 0$. Therefore, persuader with $\omega < R$ never reports s'. This is a contradiction to $E_p[\omega|s'] < R$. Thus, if $E_p[\omega|s] > R$ in equilibrium, it is only possible to have $\alpha_p(s) = 1$. According to (2), we know that given $E_p[\omega|s] > R$, $\alpha_p(s) = 1$ if and only if

$$
\begin{aligned}
\int_R^1 \omega f_p(\omega|s)\,d\omega + F_p(R|s)\,R - E_p[\omega|s] - c'(1) &\geq 0\text{, or equivalently,}\\
\int_R^1 \omega f_p(\omega|s)\,d\omega + F_p(R|s)\,R &\geq E_p[\omega|s] + c'(1)\\
&> R+1\text{, since } E_p[\omega|s] > R \text{ and } c'(1) \geq 1\\
&> F_p(R|s)\,R + \int_R^1 \omega f_p(\omega|s)\,d\omega.
\end{aligned}
$$

A contradiction again. Therefore, $E_p[\omega|s] > R$ is not possible for any equilibrium message. ■

B. Existence of $\underline{\omega}$

The persuasion strategy of the persuader which specifies $p(\omega) = s_1$ for $\omega \in [0,\underline{\omega}) \cup (R,1]$ implies that

$$
\begin{aligned}
E_p[\omega|s_1] &= \int_0^{\underline{\omega}} \frac{\omega f(\omega)}{1-\int_{\underline{\omega}}^R f(\omega)\,d\omega}d\omega + \int_R^1 \frac{\omega f(\omega)}{1-\int_{\underline{\omega}}^R f(\omega)\,d\omega}d\omega\\
&= \frac{\int_0^{\underline{\omega}} \omega f(\omega)\,d\omega + \int_R^1 \omega f(\omega)\,d\omega}{F(\underline{\omega}) + [1-F(R)]}.
\end{aligned}
$$

So $E_p[\omega|s_1] \leq R$ is equivalent to $\frac{\int_0^{\underline{\omega}} \omega f(\omega)d\omega + \int_R^1 \omega f(\omega)d\omega}{F(\underline{\omega})+[1-F(R)]} \leq R$. This inequality can be rearranged as

$$
RF(\underline{\omega}) + R[1-F(R)] \geq \int_0^{\underline{\omega}} \omega f(\omega)\,d\omega + \int_R^1 \omega f(\omega)\,d\omega.
$$

Define function $g(\underline{\omega}) = RF(\underline{\omega}) + R[1 - F(R)] - \left[\int_0^{\underline{\omega}} \omega f(\omega)\, d\omega + \int_R^1 \omega f(\omega)\, d\omega\right]$. $g(R) > 0$, so the continuity of g implies that there exists $\underline{\omega} < R$ such that $g(\underline{\omega}) \geq 0$. Actually, $\underline{\omega}$ is not unique, because due to the continuity of f (i.e., differentiability of F), we have

$$g(0) < 0 \text{ and } \frac{\mathrm{d}g(\underline{\omega})}{\mathrm{d}\underline{\omega}} = (R - \underline{\omega}) f(\underline{\omega}) > 0 \text{ for } \underline{\omega} < R.$$

So there exists a $\tilde{\underline{\omega}}$ such that $g(\tilde{\underline{\omega}}) = 0$. Any $\underline{\omega} \in [\tilde{\underline{\omega}}, R]$ can satisfy that $g(\underline{\omega}) \geq 0$.

C. Verifying the Equilibrium of Unilateral Investigation

To examine that this is indeed a PBE, let us first look at the strategy of listener 1. Given the strategies of the persuader and listener 2, if listener 1 receives message s_1 and she chooses investigation intensity α_1, her payoff is

$$Eu_{1p}(\alpha_1, \alpha_2(s_1)|s_1) = \alpha_1 \int_{R_1}^1 \omega f_p(\omega|s_1)\, d\omega + \alpha_1 F_p(R_1|s_1) R_1 + (1-\alpha_1)R_1 - c_1(\alpha_1),$$

given that $E_p[\omega|s_1] \leq R_1$. F.O.C. of payoff maximization tells us that $\alpha_1(s_1)$ must satisfy the following equation,

$$c_1'(\alpha_1(s_1)) = 1 - R_1 - \int_{R_1}^1 F_p(\omega|s_1)\, d\omega > 0.$$

Verifying $\alpha_1(s_2) = 0$ is straightforward. Thus, listener 1's strategy is a best response to those of the persuader and listener 2.

For listener 2, given the strategies of the persuader and listener 1, if s_1 is received, her expected payoff of choosing intensity α_2 is

$$\begin{aligned} Eu_{2p}(\alpha_2, \alpha_1(s_1)|s_1) &= \alpha_2 \left\{\int_{R_1}^1 [\alpha_1(s_1)\omega + (1-\alpha_1(s_1))R_2] f_p(\omega|s_1)\, d\omega + F_p(R_1|s_1) R_2\right\} \\ &\quad + (1-\alpha_2)\left[\alpha_1(s_1)\int_{R_1}^1 \omega f_p(\omega|s_1)\, d\omega + \alpha_1(s_1) F_p(R_1|s_1) R_2 + (1-\alpha_1(s_1)) R_2\right] \\ &\quad - c_2(\alpha_2) \\ &= \alpha_1(s_1)\int_{R_1}^1 \omega f_p(\omega|s_1)\, d\omega + \alpha_1(s_1) F_p(R_1|s_1) R_2 + (1-\alpha_1(s_1)) R_2 - c_2(\alpha_2). \end{aligned}$$

Note that the second term in expected payoff is the payoff of listener 2 when no information about true ω is learned. Given the strategy of listener 1, it is optimal for listener 2 to invest when she has no further information about ω, because

$$R_2 < \alpha_1(s_1)\int_{R_1}^1 \omega f_p(\omega|s_1)\, d\omega + \alpha_1(s_1) F_p(R_1|s_1) R_2 + (1-\alpha_1(s_1)) R_2,$$

where R_2 is her payoff under rejection, the RHS is her expected payoff under acceptance.

The derivative of $Eu_{2p}(\alpha_2, \alpha_1(s_1)|s_1)$ with respect to α_2 is $c_2'(\alpha_2) < 0$, so it is optimal for listener 2 to exert no investigation. Consequently, it is optimal for her to accept the project under message s_1. It is easy to verify that the strategy of listener 2 under message s_2 is also optimal.

Given the strategies of both listeners, it is clear that the strategy of the persuader is optimal.

D. Proofs for Proposition 6

1. Rejection without Investigation

Suppose that in an equilibrium there is a set $S_1 = \left\{s_1^1, s_2^1, \ldots, s_{n_1}^1\right\}$ of messages which induce at least one of the listeners to reject the project without any investigation. We can construct another strategy profile where keeping other things unchanged, all the types of the persuader that reports a message in S_1 now report s_1^1 instead, and both listeners reject the project without any investigation when they receive message s_1^1.

It is easy to see that the proposed strategy profile constitutes an equilibrium. In this new strategy profile, all the types reporting a message in S_1 before still have zero probability of success. Every other type of the persuader has no incentive to change their reports, as all reports generate the same probability of launching the project as in the original equilibrium. Under message s_1^1, it is optimal for a listener to reject, given that the other rejects.

It is easy to see that all the players's payoffs are unchanged. Each type of the project in the new equilibrium has the same probability of being launched as in the original equilibrium, so the payoff of the persuader is unchanged. For the listeners, their payoffs under s_1^1 are their reservation payoffs and are the same as those under a message in S_1 in the original equilibrium.

2. Unilateral Investigation

Lemma 5 informs us that only listener 2 can free ride in equilibrium. Suppose that in an equilibrium, there is a set $S_2 = \left\{s_1^2, s_2^2, \ldots, s_{n_2}^2\right\}$ of messages, each message in which induces that listener 1 investigates and accepts the project only if its ω is identified to be above R_1; listener 2 approves the project without any investigation. The strategic situation facing listener 1 under each message in S_2 is exactly the same as that facing a listener with reservation payoff R_1 in the one-listener case. Thus, the proof in subsection 2.3 can be applied here. That is, we can find a strategy profile identical to this equilibrium, except that all the types reporting a message in S_2 report s_1^2 instead, the behavior of each listener on receiving s_1^2 is the same as that on receiving $s_i^2 \in S_2$. The new strategy profile is an equilibrium.

The expected payoff of the persuader is unchanged in the new equilibrium, as the project of each ω has the same probability of being launched. Similar to that in the proof of Proposition 3, we can show that listener 1's expected payoff is unchanged. The payoff of listener 2 under message s_i^2, $i \in \{1, 2, \ldots, n_2\}$, in the original equilibrium is

$$\alpha_1\left(s_i^2\right) \int_{R_1}^{1} \omega f_p\left(\omega | s_i^2\right) d\omega + \left[1 - \alpha_1\left(s_i^2\right)\left(1 - F_p\left(R_1 | s_i^2\right)\right)\right] R_2.$$

In the new equilibrium, let $\hat{p}$ and $\hat{\alpha}_1$ denote the strategies of the persuader and listener 1, the expected payoff of listener 2 under s_1^2 is

$$\begin{aligned} & \hat{\alpha}_1\left(s_1^2\right)\int_{R_1}^{1}\omega f_{\hat{p}}\left(\omega|s_1^2\right)d\omega+\left[1-\hat{\alpha}_1\left(s_1^2\right)\left(1-F_{\hat{p}}\left(R_1|s_1^2\right)\right)\right]R_2 \\ = & \sum_{i=1}^{n_2}\left\{\alpha_1\left(s_i^2\right)\int_{R_1}^{1}\omega f_p\left(\omega|s_i^2\right)d\omega+\left[1-\alpha_1\left(s_i^2\right)\left(1-F_p\left(R_1|s_i^2\right)\right)\right]R_2\right\}\frac{\Pr\left(s_i^2|p\right)}{\Pr\left(S_2|p\right)}. \end{aligned}$$

Thus, the *ex ante* payoff of listener is unchanged. If listener 2 is risk-averse, it is obvious that her expected payoff will be even higher in the new equilibrium.

3. Joint Investigation

Suppose that in an equilibrium, there is a set $S_3=\left\{s_1^3,s_2^3,\ldots,s_{n_3}^3\right\}$ of messages, each message in which would induce both listeners to investigate the project. Listener 2 rejects the project unless its ω is identified to be larger than R_2, but listener 1 rejects the project only if ω is identified to be smaller than R_1. For each s_i^3, there must be $E\left[\omega|\omega\in p^{-1}\left(s_i^3\right)\cap\left(R_2,1\right]\right]>R_1$, otherwise listener 1 will not accept the project in the case that she learns nothing from investigation.

It is easy to show that for any $s_i^3\in S_3$, $p^{-1}\left(s_i^3\right)\cap\left(R_2,R_1\right]\neq\phi$ and $p^{-1}\left(s_i^3\right)\cap\left(R_1,1\right]\neq\phi$. Suppose that the first intersection is empty, then listener 1 will not investigate, as the project accepted by listener 2 must have $\omega>R_1$. If the second intersection is empty, then listener 1 rejects the project directly.

Let α_1 and α_2 be the investigation strategies of listener 1 and listener 2 respectively. Under a message $s_i^3\in S_3$, $\alpha_1\left(s_i^3\right)$ and $\alpha_2\left(s_i^3\right)$ are uniquely defined by

$$\begin{aligned} c_1'\left(\alpha_1\left(s_i^3\right)\right) &= \alpha_2\left(s_i^3\right)\int_{R_2}^{R_1}\left(R_1-\omega\right)f_p\left(\omega|s_i^3\right)d\omega, \\ c_2'\left(\alpha_2\left(s_i^3\right)\right) &= \int_{R_2}^{1}\left(\omega-R_2\right)f_p\left(\omega|s_i^3\right)d\omega-\alpha_1\left(s_i^3\right)\left[\int_{R_2}^{R_1}\left(\omega-R_2\right)f_p\left(\omega|s_i^3\right)d\omega\right], \end{aligned} \tag{48}$$

where

$$\alpha_1\left(s_i^3\right)\leq\bar{\alpha}_{1p}\left(s_i^3\right)=\frac{R_2-E_p\left[\omega|s_i^3\right]}{\int_{R_1}^{1}\omega f_p\left(\omega|s_i^3\right)d\omega+F_p\left(R_1|s_i^3\right)R_2-E_p\left[\omega|s_i^3\right]}.$$

The coexistence of the messages in S_3 in equilibrium implies that

$$\alpha_1\left(s_i^3\right)=\alpha_1\left(s_j^3\right),\ \alpha_2\left(s_i^3\right)=\alpha_2\left(s_j^3\right),\forall s_i^3,s_j^3\in S_3,i\neq j.$$

This gives us the following equation

$$
\begin{aligned}
c_1'\left(\alpha_1\left(s_i^3\right)\right) &= \alpha_2\left(s_i^3\right)\sum_{k=1}^{n_3}\left[\int_{R_2}^{R_1}\left(R_1-\omega\right)f_p\left(\omega|s_k^3\right)d\omega\right]\frac{\Pr\left(s_k^3|p\right)}{\Pr\left(S_3|p\right)} \qquad (49)\\
&= \alpha_2\left(s_i^3\right)\int_{R_2}^{R_1}\left(R_1-\omega\right)\sum_{k=1}^{n_3}\left[\frac{f_p\left(\omega|s_k^3\right)\Pr\left(s_k^3|p\right)}{\Pr\left(S_3|p\right)}\right]d\omega\\
c_2'\left(\alpha_2\left(s_i^3\right)\right) &= \sum_{k=1}^{n_3}\left[\int_{R_2}^{1}\left(\omega-R_2\right)f_p\left(\omega|s_k^3\right)d\omega\right]\frac{\Pr\left(s_k^3|p\right)}{\Pr\left(S_3|p\right)}\\
&\quad -\alpha_1\left(s_i^3\right)\sum_{k=1}^{n_3}\left[\int_{R_2}^{R_1}\left(\omega-R_2\right)f_p\left(\omega|s_i^3\right)d\omega\right]\frac{\Pr\left(s_k^3|p\right)}{\Pr\left(S_3|p\right)}\\
&= \int_{R_2}^{1}\left(\omega-R_2\right)\sum_{k=1}^{n_3}\left[\frac{f_p\left(\omega|s_k^3\right)\Pr\left(s_k^3|p\right)}{\Pr\left(S_3|p\right)}\right]d\omega\\
&\quad -\alpha_1\left(s_i^3\right)\int_{R_2}^{R_1}\left(\omega-R_2\right)\sum_{k=1}^{n_3}\left[\frac{f_p\left(\omega|s_i^3\right)\Pr\left(s_k^3|p\right)}{\Pr\left(S_3|p\right)}\right]d\omega
\end{aligned}
$$

Now we consider a strategy profile in which everything is the same as in the equilibrium above except that all types of the persuader reporting messages in S_3 report the message s_1^3, and under message s_1^3, the strategies of listener 1 and listener 2 are same as those under a message s_i^3 in the original equilibrium. The analysis below shows that this strategy profile is an equilibrium.

Let $\hat{p}$, $\hat{\alpha}_1$, and $\hat{\alpha}_2$ denote the strategies of the persuader, listener 1, and listener 2 respectively. Under message s_1^3, $\hat{\alpha}_1\left(s_1^3\right)$ and $\hat{\alpha}_2\left(s_1^3\right)$ should satisfy

$$
\begin{aligned}
c_1'\left(\hat{\alpha}_1\left(s_1^3\right)\right) &= \hat{\alpha}_2\left(s_1^3\right)\int_{R_2}^{R_1}\left(R_1-\omega\right)f_{\hat{p}}\left(\omega|s_1^3\right)d\omega,\\
c_2'\left(\hat{\alpha}_2\left(s_1^3\right)\right) &= \int_{R_2}^{1}\left(\omega-R_2\right)f_{\hat{p}}\left(\omega|s_1^3\right)d\omega-\hat{\alpha}_2\left(s_1^3\right)\left[\int_{R_2}^{R_1}\left(\omega-R_2\right)f_{\hat{p}}\left(\omega|s_1^3\right)d\omega\right].
\end{aligned}
$$

The definitions of $\hat{p}$ and $f_{\hat{p}}\left(\omega|s_1^3\right)$ gives us

$$
f_{\hat{p}}\left(\omega|s_1^3\right)=\sum_{k=1}^{n_3}\left[\frac{f_p\left(\omega|s_k^3\right)\Pr\left(s_k^3|p\right)}{\Pr\left(S_3|p\right)}\right],\ \text{for } \forall\omega\in\Omega.
$$

Since the solution to (48) is unique, there must be $\left(\hat{\alpha}_1\left(s_1^3\right),\hat{\alpha}_2\left(s_1^3\right)\right)=\left(\alpha_1\left(s_i^3\right),\alpha_2\left(s_i^3\right)\right)$.

This does not complete the proof. We still need to show that

$$
\hat{\alpha}_1\left(s_1^3\right)<\bar{\alpha}_{1\hat{p}}\left(s_1^3\right)=\frac{R_2-E_{\hat{p}}\left[\omega|s_1^3\right]}{\int_{R_1}^{1}\omega f_{\hat{p}}\left(\omega|s_1^3\right)d\omega+F_{\hat{p}}\left(R_1|s_1^3\right)R_2-E_{\hat{p}}\left[\omega|s_1^3\right]}.
$$

Based on the expressions of $\bar{\alpha}_{1p}(s_i^3)$, we can find

$$\begin{aligned}
\bar{\alpha}_{1\hat{p}}\left(s_1^3\right) &= \frac{\sum_{i=1}^{n_3}\left[R_2 - E_p\left[\omega|s_i^3\right]\right]\frac{\Pr\left(s_i^3|p\right)}{\Pr(S_3|p)}}{\sum_{i=1}^{n_3}\left[\int_{R_1}^{1}\omega f_p\left(\omega|s_i^3\right)d\omega + F_p\left(R_1|s_i^3\right)R_2 - E_p\left[\omega|s_i^3\right]\right]\frac{\Pr\left(s_i^3|p\right)}{\Pr(S_3|p)}} \\
&= \sum_{i=1}^{n_3}\lambda\left(s_i^3\right)\bar{\alpha}_{1p}\left(s_i^3\right), \text{ where} \\
\lambda\left(s_i^3\right) &= \frac{\left[\int_{R_1}^{1}\omega f_p\left(\omega|s_i^3\right)d\omega + F_p\left(R_1|s_i^3\right)R_2 - E_p\left[\omega|s_i^3\right]\right]\Pr\left(s_i^3|p\right)}{\sum_{i=1}^{n_3}\left[\int_{R_1}^{1}\omega f_p\left(\omega|s_i^3\right)d\omega + F_p\left(R_1|s_i^3\right)R_2 - E_p\left[\omega|s_i^3\right]\right]\Pr\left(s_i^3|p\right)}, \sum_{i=1}^{n_3}\lambda\left(s_i^3\right) = 1.
\end{aligned}$$

So $\hat{\alpha}_1(s_1^3) < \bar{\alpha}_{1\hat{p}}(s_1^3)$.

Applying the same proof scheme as before, it is not hard to show that the *ex ante* expected payoffs of the persuader and listeners are unchanged.

E. Proof for Strategic Complementarity between Intensities

$$\begin{aligned}
\frac{\mathrm{d}\alpha_2(\langle 1,2\rangle, s')}{\mathrm{d}\alpha_1(\langle 1,2\rangle, s')} &= \frac{\left\{\begin{array}{c}\int_{R_1}^{1}(\omega - R_2)f(\omega)d\omega\cdot[F(R_1) - F(R_2) + F(\omega')] \\ -\int_{R_2}^{R_1}(\omega - R_2)f(\omega)d\omega\cdot[1 - F(R_1)]\end{array}\right\}}{c_2''(\alpha_2(\langle 1,2\rangle, s'))\left\{\begin{array}{c}\alpha_1(\langle 1,2\rangle, s')[1 - F(R_1)] \\ +(1 - \alpha_1(\langle 1,2\rangle, s'))[1 - F(R_2) + F(\omega')]\end{array}\right\}^2} \\
&\geq \frac{\left\{\begin{array}{c}(R_1 - R_2)[1 - F(R_1)][F(R_1) - F(R_2) + F(\omega')] \\ -(R_1 - R_2)[F(R_1) - F(R_2)][1 - F(R_1)]\end{array}\right\}}{c_2''(\alpha_2(\langle 1,2\rangle, s'))\left\{\begin{array}{c}\alpha_1(\langle 1,2\rangle, s')[1 - F(R_1)] \\ +(1 - \alpha_1(\langle 1,2\rangle, s'))[1 - F(R_2) + F(\omega')]\end{array}\right\}^2} \\
&= \frac{(R_1 - R_2)[1 - F(R_1)]F(\omega')}{c_2''(\alpha_2(\langle 1,2\rangle, s'))\left\{\begin{array}{c}\alpha_1(\langle 1,2\rangle, s')[1 - F(R_1)] \\ +(1 - \alpha_1(\langle 1,2\rangle, s'))[1 - F(R_2) + F(\omega')]\end{array}\right\}^2} \\
&> 0.
\end{aligned}$$

F. Possible Equilibrium Investigation Scenarios

PBE requires that listener $i \in \{1, 2\}$ accept the project if ω is identified to be larger than R_i.

The discussion is not as easy as that in public persuasion, even if we impose the above assumption. Because in public persuasion, there is just one stage of communication, but in sequential persuasion there are two. Types reporting a same message at one stage may not behave in a same way at the other stage. In the following analysis, I focus on an arbitrary set of types who send a same message to the first listener. I analyze what strategies of the second listener can be induced by this set.

In the investigation stage, each listener chooses among two types of strategies: (1) Investigate with positive intensity, (2) Do not investigate, i.e., investigate with 0 intensity. For the convenience of discussion, I use IN and NI to denote these two types of strategies respectively. In the stage of making investment decision, given the assumption of dominant strategy, each listener also has only two alternative strategies, which are (1) Reject the project unless ω is identified to be larger than R_i, and (2) Accept the project unless ω is identified to be smaller than R_i. I use R and A as abbreviations for these two alternatives respectively. Therefore, each listener in equilibrium may behave in one of the four possible ways which are $IN + R$, $IN + A$, $NI + R$ and $NI + A$. Since NI leads to no identification of the true state, $NI + R$ and $NI + A$ can be interpreted as rejection without investigation and acceptance without investigation. Below I analyze that for a set of types inducing the first listener to behave in one of the four ways, what strategies of the second listener will be confronted by those who get approval of the first listener in equilibrium.

To begin with, I consider the cases where listener 1 is firstly persuaded. We will see if a subset T of Ω induces one of the four following strategies of listener 1, what strategies of listener 2 will be confronted by the ones of T surviving from listener 1's scrutiny.

1. $IN + R$

   If T induces this strategy of listener 1, then it must be the case that both $T \cap (R_1, 1]$ and $T \cap [0, R_1]$ are nonempty, otherwise listener 1 will not investigate. Also, it is obvious that only the elements of T with $\omega > R_1$ can possibly survive from this strategy of listener 1. If they survive, what strategies of listener 2 will be confronted by them? Clearly, it is not possible that all of them confront $NI + R$, because if so, listener 1 will not investigate T at all in optimality. If these survivors are not mixed with elements of Ω with $\omega \leq R_2$ when they approach listener 2, the optimal strategy of listener 2 is $NI + A$. What if they are mixed with such $\omega \leq R_2$ types? To answer this question, one should first examine whether the types with $\omega \leq R_2$ can get approval of listener 1. If so, the strategies of listener 1 confronted by these types must be $IN + A$ and/or $NI + A$. But it is not possible to have any subset of Ω induce these strategies in equilibrium while T induces $IN + R$, because if $IN + A$ or $NI + A$ can be induced, then all the member of T will deviate, as $IN + A$ and $NI + A$ can yield them higher probabilities of success than does $IN + R$. Therefore, it is impossible that survivors of T are mixed with $\omega \leq R_2$ at the second stage of persuasion. This implies that if listener 1 exerts $IN + R$, survivors will confront $NI + A$ by listener 2. If another message can induce a different strategy of listener 1, this induced strategy can only be $NI + R$.

2. $IN + A$

Similar to the case above, it is impossible that all the elements of T surviving from this strategy confront $NI + R$ by listener 2, because if that is the case, listener 1 will deviate from investigation. Also, it must be that both $T \cap (R_1, 1]$ and $T \cap [0, R_1]$ have positive measures, otherwise listener 1 has no incentive to investigate. In the subsections 4.1 and 4.2, the equilibria of joint investigation already show that $IN + R$ and $NI + R$ of listener 2 can be induced by some messages in the second stage of persuasion. Now we examine whether $IN + A$ and $NI + A$ can be induced. To proceed, let α_1 denote the investigation intensity of listener 1. If $IN + A$ is induced by some message, then no message in equilibrium induces $IN + R$, $NI + R$ or $NI + A$ at the second stage of persuasion, as the first two give lower probability of success to any type than does $IN + A$, and $NI + A$ gives higher probability of success to the types with $\omega \le R_2$ than does $IN + A$. Furthermore, one can verify that when $IN + A$ is induced at the second stage, every $\omega \le R_2$ in T or mimicking the report of T can get positive probability of success, and no other strategy of listener 1 can be induced by other messages. Is it possible that $NI + A$ arises at the second stage of persuasion? The answer is no. Suppose that it arises, then it is easy to verify that $IN+A$, $IN+R$ and $NI+R$ of listener 2 can not be induced by any message, and also no message could induce any other strategy of listener 1. This indicates that T includes all the types approaching listener 1 first. Moreover, one should note that $T \cap [0, R_2]$ have zero measure, because otherwise it is not optimal for listener 2 to exert $NI + A$. Therefore, it must be the case that all the members of $[0, R_2]$ approach listener 2 first, and get a higher probability of success than mimicking the report of T which yields them $(1 - \alpha_1)$. The only combination of strategies of listener 2 and listener 1 that gives $[0, R_2]$ positive probability of success and can arise in equilibrium is that both listeners adopt $IN + A$[11]. Suppose that the investigation intensities of listener 1 and listener 2 are respectively α_1' and α_2'. The probability of success for $[0, R_2]$ is $(1 - \alpha_1')(1 - \alpha_2')$. $(1 - \alpha_1')(1 - \alpha_2') \ge (1 - \alpha_1)$ implies that $(1 - \alpha_1') > (1 - \alpha_1)$, which indicates the all the types in $(R_2, R_1]$ can get higher payoff from mimicking the behavior of $[0, R_2]$ than staying with T, i.e., $T \cap [0, R_1]$ should have zero measure in equilibrium. This is a contradiction. Thus it is impossible that $NI + A$ of listener 2 arises at the second stage of persuasion.

The strategy $IN + A$ of listener 2 may be induced in equilibrium. According to the discussion above, if $IN + A$ of listener 2 can be induced at the second stage, then all the types approaching listener 1 first must confront the strategy profile composed of $IN+A$ of listener 1 and $IN + A$ of listener 2. Let α_1'' and α_2'' be the investigation intensities of listener 1 and listener 2 respectively. In equilibrium, if there are some types choosing to persuade listener 2 first, based on the discussion below in cases 2 and 3 where listener 2 is firstly persuaded,

[11] The combination of $IN + A$ by Listener 1 and $NI + A$ by Listener 2 and combination of $IN + A$ by Listener 2 and $NI + A$ by Listener 1 can also yield positive probability of success for $[0, R_2]$, but these combinations cannot happen in equilibrium facing $[0, R_2]$, as the listener adopting $NI + A$ has incentive to deviate.

these types must confront a strategy profile composed of $IN + A$ of both listeners. In this profile where listener 2 is approached first, the investigation intensities of listener 1 and listener 2 should also be α_1'' and α_2''. Thus, such an equilibrium is equivalent to one where all types of the persuader persuade the listeners in the same order and induce both listeners to exert $IN + A$, from the persuader's viewpoint. However, an equilibrium with all types of the persuader persuading the listeners in the same order and inducing both listeners to exert $IN + A$ is equivalent to a pooling equilibrium where all types report the same message and induce listeners to adopt $IN+A$. This completely pooling equilibrium involve no persuasive communication, so we exclude it from our further analysis.

3. $NI + A$

   If the report of T induces listener 1 to adopt $NI+A$, then it should be true that in equilibrium no message at the second stage of persuasion inducing listener 2 to exert $NI + A$, because otherwise any type mimicking the behavior of T at the first stage and reporting the message inducing $NI + A$ at the second stage can yield probability 1 of success. Also, it is clear that $IN+A$ of listener 2 cannot be induced when $IN+R$ and/or $NI+R$ can be induced by some message, as no $\omega \leq R_2$ will report the messages inducing $IN + R$ and/or $NI + R$. Suppose that $IN + A$ arises at the second stage, then it must be true that some positive measure of $\omega \leq R_2$ survived from the first-stage persuasion. This positive measure of $\omega \leq R_2$ necessarily confronted $NI + A$ of listener 1, because all other strategies of listener 1, even if they can be induced, yields lower probability of success. But it is never optimal for listener 1 to adopt $NI + A$ facing a message including some of $\omega \leq R_2$, given that listener 2 adopts $IN + A$. We can use similar argument to exclude $IN + R$ of listener 2. So it is only possible that $NI+R$ of listener 2 can be induced at the second stage. This implies that $T \subset [0, R_2]$. Given this strategy of listener 2 and that T induces $NI + A$ of listener 1, the only other strategy of listener 1 that can be induced in the same equilibrium is $NI + R$. In our equilibrium analysis, we are going to replace the strategy profile composed of $NI + A$ of listener 1 and $NI + R$ of listener 2 which is induced by a subset of $[0, R_2]$ by $NI + R$ of listener 1. This replacement will facilitate our analysis without changing any result.

4. $NI + R$

   If T confronts $NI + R$, the game ends. It is possible that in the same equilibrium other strategies of listener 1 can be induced by some other messages. Specifically, $IN + R$ of listener 1 can be induced and listener 2 exerts $NI + A$ for any message reported. $IN + A$ of listener 1 can be induced as well. In this case, listener 2 exerts $IN + R$ and $NI + R$ to message reported. $NI + A$ of listener 1 can also arise when listener 2 adopts $NI + R$ at the second stage to any message. But these three cases cannot coexist in an equilibrium.

Now it is time to move to the cases where listener 2 is firstly persuaded. The pattern of the discussion for these cases is the same as above. We will see if a subset T of Ω induces one of the four following strategies of listener 2, what strategies of listener 1 will be confronted by the ones of T surviving from listener 2's scrutiny.

1. $IN + R$

   The set T inducing this strategy should satisfy that $T \cap (R_2, 1]$ and $T \cap [0, R_2]$ both have positive measures, otherwise Listener 2 makes no effort in investigation. In Subsection 4.2, I already provide an equilibrium where at the second stage of persuasion $IN + A$ of listener 1 is induced. If $IN + A$ of listener 1 is induced by some message (which means this message is reported by a positive measure of $\omega \leq R_1$), then no message would induce $IN + R$, $NI + R$ and $NI + A$ of listener 1, because $IN + A$ gives all types with $\omega \leq R_1$ higher probability of success than the first two strategies and gives the types with $\omega \leq R_1$ lower probability of success. Thus coexistence will lead to inconsistency. It is also impossible to have $IN + R$, $NI + R$ and $NI + A$ all induced by some messages in an equilibrium, because $NI + A$ yields higher probability of success to any types than does $IN + R$ and $NI + R$. Suppose that $NI + A$ of listener 1 is induced, then it must be that all the ones of T surviving from the investigation of listener 2 have $\omega > R_1$. So it should be the case that $T \cap (R_2, R_1]$ has zero measure. Since it is impossible to have $IN + A$ and $NI + A$ of listener 2 induced by some messages while $IN + R$ is induced by T, $(R_2, R_1]$ must approach listener 1 first in equilibrium and obtains a probability of success no less than that by mimicking the report of T. According to the analysis above, this is only if $(R_2, R_1]$ confront the combination of $IN + A$ by listener 1 and $IN + A$ by listener 2, or the combination of $IN + A$ by listener 1 and $IN + R$ by listener 2. If the former one is the case, then $\omega \leq R_2$ in T would deviate, as mimicking the reports of $(R_2, R_1]$ yields positive payoff. If the later case is true, then $\omega > R_1$ would have incentive to mimic the reports of $(R_2, R_1]$. Therefore, it is proved that $NI + A$ of listener 1 cannot arise in equilibrium. If $IN + R$ and/or $NI + R$ can be induced at the second stage, then the optimal response of listener 2 is to delegate the investigation job to listener 1 whose is pickier. Thus it is not possible to have them induced.

2. $IN + A$

   If this strategy is induced by T, then it must be that $T \cap (R_2, 1]$ and $T \cap [0, R_2]$ both have positive measures. At the second stage, it is impossible to have $NI + A$ of listener 1 induced, because if it is induced, $(R_2, R_1]$ must be included in T and report a message inducing $NI + A$ of listener 1 at the second stage, as this yields them probability 1 of success. The behavior of $(R_2, R_1]$ leads listener 1 to deviate from $NI + A$. The strategies $IN + R$, $NI + R$ and $IN + A$ of listener 2 cannot be induced by different messages in equilibrium, because $IN + R$ and $NI + R$ yield lower probability of success for any type than does $IN + A$. It is not possible that only $IN + R$ and/or $NI + R$ are induced in equilibrium, because if so, it is not optimal for listener 2 to do any investigation at the first stage. It is only possible that $IN + A$ of listener 1 is induced at the second stage.

3. $NI + A$

   If the report of T induces listener 2 to adopt $NI+A$, then it should be true that in equilibrium no message at the second stage of persuasion inducing listener 1 to exert $NI + A$, because otherwise any type mimicking the behavior of T at the first stage and reporting the message inducing $NI + A$ at the second stage can yield probability 1 of success. Also, it is clear that $IN + A$ of listener 1 cannot be induced when $IN + R$ and/or $NI + R$ can be induced by some message, as no one will report the messages inducing $IN + R$ and/or $NI + R$. Suppose that $IN + A$ arises at the second stage and the investigation intensity of listener 1 is α_1, then one should note that $T \cap [0, R_2]$ have zero measure, because otherwise it is not optimal for listener 2 to exert $NI + A$. Therefore, it must be the case that all the members of $[0, R_2]$ approach listener 1 first, and get a higher probability of success than mimicking the report of T which yields them $(1 - \alpha_1)$. The only combination of strategies of listener 1 and listener 2 that gives $[0, R_2]$ positive probability of success and can arise in equilibrium is that both listeners adopt $IN + A$. Suppose that the investigation intensities of listener 1 and listener 2 are respectively α_1' and α_2'. The probability of success for $[0, R_2]$ is $(1 - \alpha_1')(1 - \alpha_2')$. $(1 - \alpha_1')(1 - \alpha_2') \geq (1 - \alpha_1)$ implies that $(1 - \alpha_1') > (1 - \alpha_1)$, which indicates the all the types in $(R_2, R_1]$ can get higher payoff from mimicking the behavior of $[0, R_2]$ than staying with T, i.e., $T \cap [0, R_1]$ should have zero measure in equilibrium. This is a contradiction. Thus it is impossible that $IN + A$ of listener 1 arises at the second stage of persuasion. It is only possible that $IN + R$ and/or $NI + R$ are induced by some messages.

4. $NI + R$

   If T confronts $NI + R$, the game ends. It is possible that in the same equilibrium other strategies of listener 2 can be induced by some other messages. Specifically, $NI + A$ of listener 2 can be induced and listener 1 exerts $IN + R$ and $NI + R$ for any message reported. $IN + R$ of listener 2 can be induced as well. In this case, listener 1 exerts $IN + A$ to message reported. But these two cases cannot coexist in an equilibrium.

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