Stochastic Stability of Backward-induction Equilibrium in Adaptive Play with Mistakes

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Abstract

We consider a variation of adaptive play with mistakes (Young, 1993) in extensive-form games of perfect information, and view adaptive play as a selection mechanism and mistakes as mutations in an evolutionary process. For each player in the extensive-form game, there is a large population of individuals playing pure strategies in that player's role. The selection mechanism requires that in every period each individual in each population adopt a current best-response strategy. A state is stochastically stable if its long-run relative frequency of occurrence is bounded away from zero as the mutation rate decreases to zero. We show examples of finite stopping games where the backward induction-equilibrium component is not stochastically stable for large populations. We then give some sufficient conditions for stochastic stability in this evolutionary process, and show that the transition between any two Nash equilibrium components in an extensive-form game may take a very long time.

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Extended Abstract

An equilibrium is stochastically stable if it is robust against persistent perturbations. In [6], Young introduces the model of adaptive play with mistakes, where each individual, randomly drawn from a large group for each player's role, chooses a best-reply strategy based on a sample of information about what other individuals have done in the past. The perturbations are modeled by occasional mistakes made by individuals. Young then studies the invariant distribution of the generated stationary Markov chain of this dynamic process, and defines the stochastically stable states as those occurring with nonnegligible probability when the mistake rate is arbitrarily small. In particular, he shows the stochastic stability of risk-dominant equilibrium in 2×2 coordination games. In the present paper, we would like to extend this model of adaptive play in extensive-form games of perfect information. Can we understand equilibria in extensive-form games in a dynamic framework, and the transition process between Nash equilibria triggered by small stochastic perturbations? Is the backward-induction equilibrium component always the maximum stochastically stable set in any extensive-form game of perfect information?

This paper also continues the study initiated in [4], where we discuss the equilibrium refinement of stochastic stability in two-player extensive-form games of perfect information in an evolutionary context. We may view adaptive play as a selection mechanism, and mistakes as mutations in an evolutionary process. Thus, a state is *stochastically stable* if its long-term relative frequency of occurrence is bounded away from zero as the mutation rate decreases to zero. The model is adapted from Hart [2] and Gorodeisky [1]. Given an extensive-form game, we consider an associated population game where for each player there is a large population of individuals playing pure strategies in that player's role. In every period, one random individual in each population is chosen. With very small probability, a mutation happens and the individual picks a strategy randomly. When a selection occurs with high probability, the individual picks a best-reply strategy against the current distribution of all other populations. We need the population structure in [2] for mixed-strategy Nash equilibria in extensive-form games, which is not necessary for the study of pure-strategy Nash equilibria in coordination games. In a population game, each individual plays a pure strategy, but the proportion of those individuals playing a pure strategy s in the population represents the probability proportion of s in the mixed strategy. Moreover, as in [2], we would like to put more inertia into the model than in [6] in the sense that, if an individual

is currently playing a best-reply strategy, then she will not change her strategy in the next period if the selection happens. In this way, there is no “drift” caused by the selection mechanism when the state is already a Nash equilibrium.

We do not, however, give a characterization theorem of the stochastic stability for extensive-form games in [4]. Indeed, as we comment there, the generated Markov chain can be subtle and the dynamics of distribution of populations needs various combinatorial results. For a simplified evolutionary framework for extensive-form games of perfect information, we suggest modifying the model such that in each period every individual may change her strategy, which is a “faster” evolutionary process, but nevertheless consistent with the model in [6]. In such a setup, we can reach a clearer result under certain conditions.

This is in fact an extreme case discussed by Kandori, Mailath, and Rob [3]. They consider a *Darwinian* property, where the proportion of a population using the current best-reply strategy increases in each period. Their result on 2×2 coordination games is robust with respect to any possible increase of individuals currently using a best-reply strategy. For extensive-form games of perfect information, we focus here on the special case where in each period every individual can adopt their current best-reply strategy when selection takes effect.

The study of this fast evolutionary process is also closely related to continuous-time approximate best-response dynamics in extensive-form games, discussed in [5]. There, an ϵ-best-response dynamic is such that each player best responds to a strategy profile in the ϵ-neighborhood of the true strategy profile at that time. We show in [5] that along any interior approximate best-response trajectory, the evolving state is close to the set of Nash equilibria most of the time. However, [5] does not say in which neighborhood of the Nash equilibrium component the evolving state spends the most time in the long run. In fact, this depends on the exact perceived strategy profiles that each player best responds to in the dynamic. We may perturb this process to a discrete-time stochastic dynamic in a population game where the stochastic perturbation is consistent with some probability distribution, e.g., uniform distribution. Our fast evolutionary model is a simple mechanism for studying the long-run distribution in the state space.

This fast evolutionary process is the main topic of the present paper. The result in [4] that only Nash equilibria can be stochastically stable for any population size still holds. We can further show that, from any state that is not a Nash equilibrium, it takes only finitely many periods, independent of population size, to enter the set of Nash equilibria. Hence, we only need to study the transitions between Nash equilibrium components. As in [4], we may focus on the transition between Nash equilibrium components under the best-reply dynamics triggered by just one mutation, when the mutation rate is low. In terms of [6], the resistance of such a transition is 1. Here, we call such a best-reply process triggered by one mutation a *one-mutation transition*. Also as in [6], this dynamic process can be simplified by a stationary Markov chain with states of those equilibrium components. If each transition between components is a one-mutation transition, then the computation of the invariant distribution of this Markov chain is indeed much simpler than the one for the model in [4].

A finite stopping game is an extensive-form game where each player at each node has at most one move to continue the game. We concentrate on finite stopping games in this paper, as they capture the basic tree property while being free of dynamics of strategies involving moves at multiple branches. We show some different stability results from the ones in [4]. In particular, we show examples where the backward-induction equilibrium component is not stochastically stable for large populations. Hence, the fast evolutionary dynamic is not analogous to the dynamic in the model in [4], but just involves less transition time.

In a fast evolutionary process, we have to address the choice problem when there are multiple best-reply strategies for an individual in a period. Of course, different choices may affect the dynamic trajectory, especially when the individuals are responding to mutants. We here consider the case that each best-reply pure strategy is assigned with positive probability. As in [7], we may also concentrate on a specific case of a uniform distribution in the set of best-reply strategies. This is a natural response when a player faces complete uncertainty. Indeed, the notion of risk dominance in a 2×2 symmetric game is generated from the situation where a player assigns equal probability to both pure strategies of the other player.

While we can give some sufficient conditions for stochastic stability in this simplified model, it turns out that the stability result can be very sensitive to payoff vectors. There exists an extensive-form game such that when populations increase to infinity the probability of a one-mutation transition between any two equilibrium components is approaching zero, and any other possible transition takes a very long time in expectation.

References

[1] Z. Gorodeisky (2006), Evolutionary stability for large populations and backward induction, *Mathematics of Operations Research* 31, 369–380.

[2] S. Hart (2002), Evolutionary dynamics and backward induction, *Games and Economic Behavior* 41, 227–264.

[3] M. Kandori, G. Mailath, and R. Rob (1993), Learning, mutation, and long-run equilibrium in games, *Econometrica* 61, 29–56.

[4] Z. Xu, Stochastic stability in finite extensive-form games of perfect information, SSE/EFI Working Paper Series in Economics and Finance, No. 743.

[5] Z. Xu, Convergence of best-response dynamics in extensive-form games, SSE/EFI Working Paper Series in Economics and Finance, No. 745.

[6] H. P. Young (1993), The evolution of conventions, *Econometrica* 61, 57–84.

[7] H. P. Young (1998), *Individual Strategy and Social Structure*, Princeton University Press.