# The Consumer Never Rings Twice: Firms Compete for Search Share before Competing for Market Share 

Raluca Mihaela Ursu*<br>University of Chicago, Department of Economics<br>E-mail: rursu@uchicago.edu


#### Abstract

I model the idea that the fraction of consumers who search a certain firm is not exogenous, but rather is determined by previous interactions between a firm and a consumer. In particular, I consider a two period differentiated products duopoly model in which firms can affect the number of consumers who choose to visit them in the second period by choosing actions in the first period. I call this fraction a firm's search share. A firm's search share can be affected in multiple ways: through consumer learning, advertising, brand loyalty, etc. I choose a particularly simple example in which, when consumers search for products, they first visit the firm they purchased from in the previous period. Even in this simple setting, standard results from the search literature do not hold. More precisely, I prove two main results.

First, when consumer search share is endogenous, equilibrium prices are lower and consumer welfare higher than when firms are searched by an exogenous fraction of consumers, as is the case in most of the literature.

Second, when search share is endogenous, higher search costs lead to lower first period prices and thus to potentially higher consumer welfare. The basic intuition here is that when firms compete to remain in the consideration set of a consumer, higher search costs mean that consumers will choose a smaller consideration set in the second period, leading to more aggressive first period competition.


## 1 Introduction

At least since Stigler's (1961) seminal paper that opened the economic discussion on search, consumers are known to optimally choose not to become fully informed about all prices existent in the market at any time. It is understood that consumers have search costs that make it rational for them to stop searching before observing all firms' prices and that this allows firms to charge prices higher than the competitive level.

Stigler (1961) uses search to help explain the prevalence of price dispersion observed for seemingly homogenous products. Diamond (1971) observes that introducing only search frictions into a model with otherwise identical firms and consumers is not enough to obtain price dispersion. Moreover, he shows that regardless of the magnitude of search frictions introduced, a unique equilibrium arises at the monopoly price. Reinganum (1979) is the first to obtain price dispersion in a model of optimizing consumers and firms and she succeeds in doing so by introducing firm marginal cost heterogeneity. Other papers, also succeed in producing price dispersion by adding heterogeneity on the consumer side (see Varian, 1980; Salop and Stiglitz, 1982; Burdett and Judd, 1983 (only

[^0]ex-post heterogeneity); Stahl, 1989; Dana, 1994; Janssen and Moraga-Gonzalez, 2004; Janssen et el. 2011; Moraga-Gonzalez et al. 2014) or on both sides (Benabou 1993). On the empirical side, several papers find evidence of large search costs that lead consumers to search a very narrow set of firms. Honka (2014) reports that consumers searching for cars get an average of less than 3 price quotes. Even online, where search costs are thought of as being very small, consumers search for very few options before making a purchase. For example, De los Santos et al. (2012) report that consumers searching for books online check on average 1.2 books in 2002 and 1.3 books in 2004. Another set of empirical papers quantities the increase in prices due to consumers' search costs. For example, Goeree (2008) finds that in the US personal computer markets, firms charge on average markups of $19 \%$ over production costs, due to consumers' inability to become fully informed.

If it is indeed the case that consumers consider a very small set of firms before making a purchase, one might ask why firms are content that consumers do not see all available options before a purchase and thus happy to charge these consumers higher prices, instead of being concerned that these consumers may optimally choose to remain uninformed about their prices?

In the reality, firms make a lot of effort to make themselves visible to consumers. They have sales, they spend billions of dollars on advertising, they ask for consumers' friendship on Facebook or follow-ship on Twitter, etc. There are thus many examples in which firms do not seem content about consumers' unawareness of all options, but rather seem concerned about not being searched and thus willing to invest in drawing consumers' attention.

However, models so far fail to recognize the strategic aspect of consumer search. In other words, models ignore the possibility of firms taking actions to influence the number of consumers who will search them by competing to remain in a consumer's search set.

This paper attempts to fill this gap. I choose a particularly simple setting in which the history of observed prices affects the number of consumers who search a particular firm. I call this number a firm's search share. In particular, I consider the case in which, when consumers search for products, they first visit the firm they purchased from in the previous period. This way, prices in the previous period affect search decisions and firm's pricing in the current period. Even in this simple setting, standard results from the search literature do not hold. More precisely, I prove two main results.

First, when consumer search share is endogenous, equilibrium prices are lower and consumer welfare higher than when firms are searched by an exogenous fraction of consumers, as is the case in most of the literature.

Second, when search share is endogenous, higher search costs lead to lower first period prices and thus to potentially higher consumer welfare. The basic intuition here is that when firms compete to remain in the consideration set of a consumer, higher search costs mean that consumers will choose a smaller consideration set in the second period, leading to more aggressive first period competition.

## 2 Related Work

The current article draws heavily on the rich literature on consumer search. On the homogenous products side of this literature, consumers search for price information before making a purchase decision. Starting with Stigler's (1961) seminal contribution, search models have been advanced to rationalize observed price dispersion for homogenous products. Representative papers in the literature include Diamond (1971), Rothschild (1973), Salop and Stiglitz (1976), Reinganum (1979), Varian (1980), Burdett and Judd (1983),

Stahl (1989), Janssen and Moraga-Gonzalez (2004), Moraga-Gonzalez and Wildenbeest (2007), De los Santos et al. (2012).

Stigler (1961) introduces the idea that when consumers search for price information and search is costly, firms will charge different prices in equilibrium then in perfect information markets. Diamond's (1971) paper shows a stricking result: even with a continuum of identical firms and consumers, with any non-zero search costs, the unique equilibrium is the monopoly price. His paper shows that it is far from trivial to provide a theoretical model of price dispersion in a homogeneous good model. Papers that followed succeeded in obtaining equilibrium price dispersion with optimizing firms and consumers by introducing some form of heterogeneity, either on the firm side, (Reinganum, 1979), on the consumer side (Varian, 1980; Stahl, 1989; Janssen and Moraga-Gonzalez, 2004), or both (Benabou, 1993). These papers manage to avoid Diamond's paradox using some form of heterogeneity. For a more complete description of the search literature, please refer to the review by Baye, Morgan and Scholten (2005).

The search literature, though extensive, has so far ignored the possibility of firms influencing the number of consumers who will search them. More precisely, papers so far either assume that consumers search randomly or that each firm is visited by an equal fraction of consumers. Most relevant for the current study is the literature starting from Varian (1980) that introduces consumer heterogeneity into a search model. In particular, there are two types of consumers: either zero search cost consumers, the shoppers, or non-zero search cost consumers, the searchers. He shows that when modeled this way, Diamond's paradox is avoided and price dispersion occurs. This literature is relevant for our study because it thinks carefully about the types of consumers who search each firm and how firms might want to charge these types different prices. In fact, price dispersion arises from the tradeoff that firms face between charging low prices to consumers with zero search costs who become fully informed, and charging high prices to the rest. However, what this literature lacks is a model of how firms might influence the fraction of each type of consumer who searches them, instead of assuming that each firm receives an equal fraction of the informed and the uninformed consumers.

We already know that firms price lower when they expect to be searched by a large exogenous fraction of consumers. For example, prices may be lower during intensive shopping periods such as on holidays or weekends, when firms expect more consumers to walk into their stores and buy in higher quantities. Warner and Barsky (1995) document this phenomenon noting that "a significant number of markdowns are timed to occur when shopping intensity is exogenously high". In their model, when shopping intensity is exogenously high, consumers are more efficient shoppers, i.e. buy in higher quantities or buy several goods, so some of their search costs are shared across products. Because of this, firms perceive their demand to be more elastic, and optimally charge a lower markup. Although the literature on exogenous fluctuations in search share is well understood, models so far ignore the possibility of an endogenous search share, which is the subject of the current paper.

The branch of the search literature most relevant for the current study is that concerned with product differentiation (see for example, Wolinsky, 1986 and Anderson and Renault, 1999). In these models, consumers search both for price information and for a "good match" to the product or the firm. These models avoid the Diamond paradox since some consumers are not well matched with the initial firm and thus choose to visit another, leading to a pro-competitive effect of search.

More recently, a few papers, both theoretical (Arbatskaya 2007, Armstrong et al. 2009, Haan and Moraga-Gonzalez, 2011) and empirical (see Hortacsu and Syverson, 2004),
introduce advertising in the traditional product differentiation model. For example, Armstrong et al. (2009) extend the classical paper by Wollinsky (1986) to study the effect of prominence on equilibrium prices, profits and welfare. They show that when all consumers search one firm first, this firm will charge lower prices, but at the industry level, profits will be higher and consumer surplus lower.

Most relevant for the current study, Haan and Moraga-Gonzales (2011) model advertising as a tool for firms to influence the order in which they are visited by consumers. In particular, consumers visit firms in order of their saliency and firms affect their saliency by choosing how much to advertise. Higher advertising leads to a higher probability of a firm being searched first and thus to a higher search share. If a firm does not advertise, it is visited last, while if no firms advertise, firms are visited in a random order. They show that higher search costs decrease consumer welfare and firm profits when firms are identical in terms of their advertising technology. When they allow for firm heterogeneity in advertising costs, the more salient firms charge lower prices and as asymmetry increases, consumer welfare decreases and profits increase.

The current study takes Haan and Moraga-Gonzales (2011) as a starting point for the analysis. In particular, I extend their one period model to two periods and instead of advertising affecting the order in which firms are visited, I allow first period purchases to determine this order. Even though the setting is similar to that of Haan and MoragaGonzales (2011), the current model leads to very different predictions. More precisely, I show that equilibrium prices are lower and consumer welfare can be higher when search costs increase and that advertising can actually hurt consumers by allowing firms to charge higher first period prices.

Somewhat less related but still relevant to the current study is the paper by Robert and Stahl (1993) which explores the tradeoff between advertising and search as means for consumers to become informed. They consider a homogenous good model in which consumers' information is endogenously determined in an equilibrium with price dispersion. In particular, in their model, consumers can obtain information about price either through their own search, which is costly to them, or through advertising, which is costly to firms. They show that when the cost of advertising decreases, prices become competitive, but when search costs decrease, prices do not converge to marginal costs. The model thus shares Robert and Stahl's (1993) intuition that advertising can substitute for the information that consumers gather through their own search.

Another branch of the literature that is relevant for the current study is that on repeated search. Stigler (1961), in describing some of the determinants of search, makes the following remarks that pertain to the effect of price correlation over time on consumer search. He notes that

> If the correlation of asking prices of dealers in successive time periods is perfect (and positive!), the initial search is the only one that need be undertaken. If the correlation of successive prices is positive, customer search will be larger in the initial period than in subsequent periods.

Stigler claims as early as 1961 that there is a relation between expected prices and consumer search. More precisely, in the first part of the quote, he claims that if consumers expect price to stay the same across periods, they will invest in an initial search to find a long-term supplier for the product and thereby economize on search costs in future periods (this is the heart of Benabou's (1993) paper).

The second part of the quote is a bit different. It claims that if consumers expect prices to stay more or less the same, then consumers will search more early on. This claim is
related to Bagwell (1987) as well as the current study. Bagwell (1987) considers a monopoly model in which firms are privately informed about their marginal costs and find it optimal to set low introductory prices when they interact repeatedly with consumers. In his model, firms have introductory sales to signal to consumers that they have a low marginal cost of production. He shows that both separating and pooling equilibria can arise in this case, depending on whether low-cost or high-cost firms are thought to be rare. In a related working paper, Bagwell and Peters (1988) describe in passing a mechanism through which repeated interactions between firms and consumers can limit a firm's monopoly power when firms have constant returns to scale production. When firms sets prices and consumers make search decisions simultaneously, firms cannot charge the monopoly price (as in Diamond 1971) without being punished for this by consumers who will refuse to search them.

These papers as well as the two quotes above contain the basic idea that the inferences consumers make about future prices matter in that they determine how consumers will search and that consumer search in a repeated framework can limit a firm's monopoly power. These studies thus get close to the subject of the current paper in that they try to understand how expectations about prices may affect search and how search may limit rather than encourage firms' market power, as is the case in the most of the literature. However, none of the papers actually models the idea that firms can influence the number of consumer who search them through their actions, and thus they ignore the strategic aspect of the problem that I will address in this model.

My model is also related to the macroeconomic literature on asymmetric pricing in the face of marginal cost uncertainty (see for example, Benabou and Gertner 1993, Janssen, Pichler and Weidenholzer 2011, Yang and Ye 2008, Tappata 2009, Dana 1994). This literature on so called "rockets and feathers" provides a noncooperative model of why prices adjust more rapidly (like rockets) to increases in marginal costs of production, and slower (like feathers) to decreases in marginal costs.

Even though the research question of these papers is quite different from that of the current study, their mechanism is nevertheless relevant. For example, Tappata (2009) extends Varian's (1980) model to a setting of asymmetric pricing by maintaining the assumption that a fraction of consumers have zero search costs, while the rest have positive search costs, but allowing the number of consumers who choose to become informed to be determined endogenously. In other words, consumers with zero search costs are informed for sure, while out of those with non-zero search costs, only the fraction with low enough search costs compared to the expected reduction in price through search, will choose to search. Similarly, in a more stylized model, Yang and Ye (2008), consider three types of consumers: some have zero search costs and always search, some have very high search costs and never search (in their model this means that they choose a firm to purchase from at random), while others (called the critical consumers, have some search costs in between these two extremes and may choose to search depending on their beliefs about the firms' marginal cost type and thus their price. The number of consumers who search is then endogenously determined in their model as the fraction with low enough search costs (the third type of consumers) and pessimistic enough beliefs about the firms' cost type.

These papers go one step closer towards endogenizing a firm's search share by allowing the fraction of consumers who choose to search to be determined in equilibrium. This, however, does not determine the number of consumers who will search a particular firm in equilibrium, as these models still assume that out of those consumers who search, each firm is visited by an equal share of consumers.

In building my model, I also draw on the vast literature on switching costs. The seminal work of Klemperer (1987) shows that in a two period differentiated products duopoly model, when firms' second period profits depend on the fraction of "locked in" consumers from the first period and consumers are myopic, firms will compete more aggressively in the first period, leading to lower prices in that period than in the absence of switching costs. Subsequent work by Dube, Hitsch and Rossi (2009), Cabral (2009) and Doganoglu (2010), shows that markets with low switching costs can actually be more competitive then markets with no switching costs. For example, Doganoglu (2010) builds an infinite period overlapping generations model in which switching occurs along the equilibrium path and shows that prices are lower in a market with low switching costs than in a market without them. The basic intuition for this result is the following: when switching costs are low, an increase in switching costs leads firms to choose lower prices because future marginal profits decrease more than current marginal profits increase. As a result, firms prefer to decrease price and invest in acquiring new customers instead of exploiting their locked in consumers. One can understand the results of the current paper though the lens of this result. In my paper, firms have to invest in maintaining a high search share for future periods and this lowers prices in the first period.

Perhaps motivated by the results from the switching cost literature, in a recent working paper by Moraga-Gonzales et al. (2014), the authors identify a condition under which higher search costs can also lead to more competition in the market. Their basic insight is that higher search costs have two effects, instead of only one as the literature previously thought. On the one hand, when search costs increase fewer consumers who choose to visit a certain firm compare prices, thereby giving the firm an incentive to increase prices. They call this the intensive margin. On the other hand, on the extensive margin, fewer consumers in total will choose to search, which gives firms an incentive to lower prices to prevent these consumers from exiting the market without searching. The literature omitted this second channel because it usually assumed that the upper bound on search costs was low enough so that all consumers continued searching, thus the fraction of searching consumers could not shrink.

The basic idea outlined in this paper can also be found in the current study. More precisely, although I assume that consumers' search costs are low enough so that no consumer decides not to search at all, when search costs increase firms fear that they will not be searched. This is equivalent to the mechanism outlined in Moraga-Gonzales et al. (2014), i.e. firms are equally hurt if consumers decide to exit the market without searching or whether consumers decide to purchase without searching them.

In short, my paper draws insights from several branches of the search literature, most importantly the branch describing sequential search for differentiated products in which some form of saliency directs consumers' search. The paper proceeds as follows: in the next section, I present the general model and prove that an equilibrium exists, that second period prices increase in search costs, while first period prices decrease when search costs increase. In section 4, I provide focus on the uniform distribution, I am also to prove additional results and discuss welfare implications. Section 5 concludes.

## 3 The Model

I will present a very stylized model of firms competing to remain in a consumers' consideration set over time and of its effect on equilibrium prices and welfare in the market. I consider a particularly simple case in which first period actions affects equilibrium prices in the second period. More precisely, I posit that consumers, when deciding how to search in
the second period, will first visit the firm they purchased from in the first period. Though simple, I will show that this model leads to very different results than those found in the search literature.

Consider a two period differentiated products duopoly model with firms labelled A and B. Firms produce a differentiated products in both periods, at constant marginal cost normalized to zero for simplicity. The good is a new product and it is an experience good. There is a unit mass of consumers with inelastic demand for one unit of the product each period. Consumers are risk neutral and maximize expected utility. In the first period, all consumers search both firms, while in the second period they may use the information they possess about the market environment to economize on their search costs and only search one of the firms. Consumers search sequentially with costless recall. All consumers pay a common search cost equal to $c$. The market is covered each period, so all consumers prefer to buy from either firm regardless of the prices charged rather than choose the outside option (i.e. outside option gives utility $-\infty$.

Consumers in the first period are uncertain about the satisfaction level they will receive from consuming either product. However, they have different affinities for the two products due to ex-ante product differentiating between the two products. I model this in the typical Hotelling framework. Search costs in this period are zero.

Let $r_{i}$ be firm $i^{\prime}$ s price in period 1 , where $i \in\{A, B\}$. Suppose firm A is located at $l=0$ and firm B is located at $l=1$. Consumers are uniformly distributed along this line. A consumer located at $\eta \in[0,1]$ with zero search cost in the first period is aware of the prices charged by both firms and will purchase from firm A if $E\left(\epsilon_{A}\right)-\eta-r_{A}>E\left(\epsilon_{B}\right)-(1-\eta)-r_{B}$, where $\epsilon_{i}$ is the satisfaction level derived by the consumer from consuming product $i$. In the first period, the consumer expects this level to be the same at the two firms, but has an affinity towards one of the firms manifested by $\eta$. Another way to interpret this is as the good being horizontally differentiated in two dimensions: the first dimension, $\eta$ is observable before consumption, while $\epsilon_{i}$ is only observable after purchase and consumption. All consumer located such that $\eta<\frac{1+r_{B}-r_{A}}{2}$, will purchase from A, while the rest, purchase at B (i.e. the market is covered). Denote the fraction of consumers who purchase at A in the first period by $\sigma^{A}$ and with $\sigma^{B}=1-\sigma^{A}$, the fraction who purchase at B . I will call fraction $\sigma^{A}$ firm A's search share as these are the consumers who will begin their search in the second period by visiting firm A .

In the second period, consumers are certain about their satisfaction level at the firm their previously purchased from, but are still uncertain about their valuation of the alternative. Their initial affinities disappear and the only differentiation between the brands that remains is that due to the different information the consumers possess. We interpret this differentiation as a consumer's preference for what she already tried. Consumers now have non zero search costs equal to $c$ and have to decide which firms to visit and in what order. I assume that consumers who purchased at $i$ will start their search at $i$. More precisely, a consumer who purchased at firm $i$ experiences $\epsilon_{i}$ from consuming firm $i$ 's product and must decide whether to immediately purchase the same product again or whether to search firm $i$ 's competitor first. Valuation $\epsilon_{i}$ is a random draw from distribution $F(\epsilon)$ on $[\underline{\epsilon}, \bar{\epsilon}]$, with density $f(\epsilon)$ such that $f$ is log-concave. This is the same for both firms' products. I assume that the realization of $\epsilon$ cannot be too low so that consumers will prefer to buy at a firm, say firm B, in the first period, even though they prefer firm A, just to be able to start their search at firm B next period. This is basically an assumption on consistency of preferences.

### 3.1 Analysis

Let $r^{*}$ and $p^{*}$ denote the symmetric equilibrium prices in the first and second period, respectively. In order to compute the price equilibrium, I will consider the gains a firm obtains by deviating to prices $(r, p)$. Without loss of generality, suppose firm A deviates to such a pair. In this case, the total profits she obtains are given by

$$
\begin{equation*}
\Pi^{A}\left(r, r^{*}\right)=r \sigma^{A}\left(r, r^{*}\right)+\beta \Pi_{2}^{A}\left(\sigma^{A}\left(r, r^{*}\right)\right) \tag{1}
\end{equation*}
$$

where $\beta$ is the discount factor.
I will work backwards and compute the symmetric equilibrium in the second period and then proceed to describe the equilibrium in the first period.

### 3.2 Period 2

I look for a symmetric equilibrium $p^{*}$, given $\sigma^{A}$ which is determined in the first period ${ }^{1}$. Suppose firm B uses the equilibrium price, but firm A deviates to a price $p$. Suppose $\sigma^{A}$ consumers start by searching firm A in period 2 , while the remaining fraction, $1-\sigma^{A}$, start by visiting B in period 2. Now, let's derive firm A's demand.

Consider a consumer who visits firm A, observes utility $\epsilon_{A}-p$ and thinks about visiting B. She expects firm B to charge the equilibrium price $p^{*}$, in which case she will only gain by searching if $\epsilon_{B}-p^{*}>\epsilon_{A}-p$. Her gains from searching B are then given by

$$
\begin{equation*}
\operatorname{Gain}(B)=\int_{\epsilon_{A}-p+p^{*}}^{\bar{\epsilon}}\left(\epsilon_{B}-\epsilon_{A}+p-p^{*}\right) f\left(\epsilon_{B}\right) d \epsilon_{B} \tag{2}
\end{equation*}
$$

Let $x_{A}=\epsilon_{A}-p+p^{*}$. A consumer with search cost $c$, will only search firm $B$ if $\operatorname{Gain}(B)>c$. There exists a value $\hat{x}$ such that the consumer is indifferent between searching one more firm and stopping, where $\hat{x}$ solves

$$
\begin{equation*}
\int_{\hat{x}}^{\bar{\epsilon}}(\epsilon-\hat{x}) f(\epsilon) d \epsilon=c \tag{3}
\end{equation*}
$$

For $x_{A}<\hat{x}$, a consumer with search cost $c$ will want to search B as well.
Assume search costs, though positive, are small enough. To make this more precise, assume that even if firms charge the monopoly price, consumers are willing to make at least a first search. In other words, $\int_{p^{m}}^{1}\left(\epsilon-p^{m}\right) f(\epsilon) d \epsilon>c$.

Similarly, a consumer who starts searching at B will want to search firm A if $x_{B}<\hat{x}$, where $x_{B}=\epsilon_{B}$ and where $\hat{x}$ solves the same equation as above. This difference comes from the fact that the consumer who starts by searching at B expects firm A to also charge the equilibrium price $p^{*}$.

Now, we can compute firm A's expected demand. If a consumer starts by searching firm A, she will buy right away from A with probability $\operatorname{Pr}\left(x_{A}>\hat{x}\right)$,i.e. if she does not want to search firm B. Denote demand for these consumers by $q^{A}$, which is given by

$$
\begin{align*}
q^{A} & =\sigma^{A} \operatorname{Pr}\left(x_{A}>\hat{x}\right)=\sigma^{A} \operatorname{Pr}\left(\epsilon_{A}>\hat{x}+p-p^{*}\right) \\
& =\sigma^{A}\left[1-F\left(\hat{x}+p-p^{*}\right)\right] \tag{4}
\end{align*}
$$

[^1]Consumers who start by searching A may also search B and then return to purchase at A. This happens with probability $\operatorname{Pr}\left(x_{A}<\hat{x}, \epsilon_{A}-p>\epsilon_{B}-p^{*}, \epsilon_{A}>p\right)$. Denote demand from these consumers by $q^{A B}$ and hence

$$
\begin{align*}
q^{A B} & =\sigma^{A} \operatorname{Pr}\left(\epsilon_{A}-p+p^{*}<\hat{x}, \epsilon_{A}>\epsilon_{B}+p-p^{*}, \epsilon_{A}>p\right) \\
& =\sigma^{A} \operatorname{Pr}\left(\epsilon_{B}+p-p^{*}<\epsilon_{A}<\hat{x}+p-p^{*}, \epsilon_{A}>p\right) \\
& =\int_{p}^{\hat{x}+p-p^{*}} F\left(\epsilon_{A}-p+p^{*}\right) f\left(\epsilon_{A}\right) d \epsilon_{A} \tag{5}
\end{align*}
$$

Finally, a fraction $1-\sigma^{A}$ start their search at B. This consumer will also search firm A if $x_{B}<\hat{x}$, where $x_{B}=\epsilon_{B}$, since the consumer expects firm A to also charge $p^{*}$. This consumer will buy at A with probability $\operatorname{Pr}\left(x_{B}<\hat{x}, \epsilon_{A}-p>\epsilon_{B}-p^{*}, \epsilon_{A}>p\right)$. Denote demand from these consumers by $q^{B A}$

$$
\begin{align*}
q^{B A} & =\left(1-\sigma^{A}\right) \operatorname{Pr}\left(\epsilon_{B}<\hat{x}, \epsilon_{A}>\epsilon_{B}+p-p^{*}, \epsilon_{A}>p\right) \\
& =\left(1-\sigma^{A}\right)\left\{\operatorname{Pr}\left(\epsilon_{B}<\hat{x}\right) \operatorname{Pr}\left(\epsilon_{A}>\hat{x}+p-p^{*}\right)+\operatorname{Pr}\left(\epsilon_{B}+p-p^{*}<\epsilon_{A}<\hat{x}+p-p^{*}, \epsilon_{A}>p\right)\right\} \\
& =\left(1-\sigma^{A}\right)\left\{F(\hat{x})\left[1-F\left(\hat{x}+p-p^{*}\right)\right]+\int_{p}^{\hat{x}+p-p^{*}} F\left(\epsilon_{A}-p+p^{*}\right) f\left(\epsilon_{A}\right) d \epsilon_{A}\right\} \tag{6}
\end{align*}
$$

Simplifying, we obtain that profits in period 2 are given by

$$
\begin{aligned}
\Pi_{2}^{A} & =p\left(q^{A}+q^{A B}+q^{B A}\right) \\
& =p\left(\sigma^{A}\left[1-F\left(\hat{x}+p-p^{*}\right)\right]+\left(1-\sigma^{A}\right) F(\hat{x})\left[1-F\left(\hat{x}+p-p^{*}\right)\right]+\int_{p}^{\hat{x}+p-p^{*}} F\left(\epsilon-p+p^{*}\right) f(\epsilon) d(\epsilon 7)\right)
\end{aligned}
$$

Let $q=q^{A}+q^{A B}+q^{B A}$. Given, $\sigma^{A}$, which is determined in period 1 , we can solve for equilibrium prices in the second period. Differentiating second period profits with respect to $p$ gives $\frac{\partial \Pi_{2}^{A}}{\partial p}=q+p \frac{\partial q}{\partial p}=0$. Then

$$
\begin{array}{r}
\frac{\partial q}{\partial p}=-\sigma^{A} f\left(\hat{x}+p-p^{*}\right)-\left(1-\sigma^{A}\right) F(\hat{x}) f\left(\hat{x}+p-p^{*}\right) \\
-\int_{p}^{\hat{x}+p-p^{*}} f\left(\epsilon-p+p^{*}\right) f(\epsilon) d \epsilon+F(\hat{x}) f\left(\hat{x}+p-p^{*}\right)-F\left(p^{*}\right) f(p) \\
=-\left\{f\left(\hat{x}+p-p^{*}\right)\left[\sigma^{A}+\left(1-\sigma^{A}\right) F(\hat{x})-F(\hat{x})\right]+F\left(p^{*}\right) f(p)\right. \\
\left.+\int_{p}^{\hat{x}+p-p^{*}} f\left(\epsilon-p+p^{*}\right) f(\epsilon) d \epsilon\right\} \tag{8}
\end{array}
$$

$$
\begin{align*}
\frac{\partial \Pi_{2}^{A}}{\partial p}=\sigma^{A} & {\left[1-F\left(\hat{x}+p-p^{*}\right)\right]+\left(1-\sigma^{A}\right) F(\hat{x})\left[1-F\left(\hat{x}+p-p^{*}\right)\right]+\int_{p}^{\hat{x}+p-p^{*}} F\left(\epsilon-p+p^{*}\right) f(\epsilon) d \epsilon } \\
& -p\left(\sigma^{A} f\left(\hat{x}+p-p^{*}\right)\left[1-F\left(\hat{x}+p-p^{*}\right)\right]+F\left(p^{*}\right) f(p)+\int_{p}^{\hat{x}+p-p^{*}} f\left(\epsilon-p+p^{*}\right) f(\epsilon) d \epsilon\right) \tag{9}
\end{align*}
$$

Imposing symmetry $p=p^{*}$, we obtain

$$
\begin{align*}
\left.\frac{\partial \Pi_{2}^{A}}{\partial p}\right|_{p=p^{*}}=\sigma^{A} & {[1-F(\hat{x})]+\left(1-\sigma^{A}\right) F(\hat{x})[1-F(\hat{x})]+\int_{p^{*}}^{\hat{x}} F(\epsilon) f(\epsilon) d \epsilon } \\
& -p^{*}\left(\sigma^{A} f(\hat{x})[1-F(\hat{x})]+F\left(p^{*}\right) f\left(p^{*}\right)+\int_{p^{*}}^{\hat{x}} f(\epsilon)^{2} d \epsilon\right) \tag{10}
\end{align*}
$$

Setting the first order condition equal to zero, we can solve for the price that will prevail in equilibrium if an equilibrium exists:

$$
\begin{equation*}
p^{*}=\frac{\sigma^{A}[1-F(\hat{x})]+\left(1-\sigma^{A}\right) F(\hat{x})[1-F(\hat{x})]+\int_{p^{*}}^{\hat{x}} F(\epsilon) f(\epsilon) d \epsilon}{\sigma^{A} f(\hat{x})[1-F(\hat{x})]+F\left(p^{*}\right) f\left(p^{*}\right)+\int_{p^{*}}^{\hat{x}} f(\epsilon)^{2} d \epsilon} \tag{11}
\end{equation*}
$$

Using integration by parts, $\int_{p^{*}}^{\hat{x}} f(\epsilon)^{2} d \epsilon=\left.F(\epsilon) f(\epsilon)\right|_{p^{*}} ^{\hat{x}}-\int_{p^{*}}^{\hat{x}} F(\epsilon) f^{\prime}(\epsilon) d \epsilon=F(\hat{x}) f(\hat{x})-$ $F\left(p^{*}\right) f\left(p^{*}\right)-\int_{p^{*}}^{\hat{x}} F(\epsilon) f^{\prime}(\epsilon) d \epsilon$.

Also, using integration by parts we can rewrite $\int_{p^{*}}^{\hat{x}} F(\epsilon) f(\epsilon) d \epsilon=\left.F(\epsilon) F(\epsilon)\right|_{p^{*}} ^{\hat{x}}-$ $\int_{p^{*}}^{\hat{x}} f(\epsilon) F(\epsilon) d \epsilon$. Thus, $\int_{p^{*}}^{\hat{x}} F(\epsilon) f(\epsilon) d \epsilon=\frac{F(\hat{x})^{2}-F\left(p^{*}\right)^{2}}{2}$. We can then rewrite the numerator above as $\sigma^{A}[1-F(\hat{x})]+\left(1-\sigma^{A}\right) F(\hat{x})[1-F(\hat{x})]+\frac{1}{2}\left[F(\hat{x})^{2}-F\left(p^{*}\right)^{2}\right]$, which simplifies to $\frac{1-F\left(p^{*}\right)^{2}}{2}$ when $\sigma^{A}=1 / 2$.

Rewriting the expression for the equilibrium price, we can show the following result:
Proposition 1. An equilibrium exists in the second period in which the equilibrium price solves

$$
\begin{equation*}
p^{*}=\frac{\sigma^{A}[1-F(\hat{x})]+\left(1-\sigma^{A}\right) F(\hat{x})[1-F(\hat{x})]+\frac{1}{2}\left[F(\hat{x})^{2}-F\left(p^{*}\right)^{2}\right]}{\sigma^{A} f(\hat{x})+\left(1-\sigma^{A}\right) F(\hat{x}) f(\hat{x})-\int_{p^{*}}^{\hat{x}} F(\epsilon) f^{\prime}(\epsilon) d \epsilon} \tag{12}
\end{equation*}
$$

If in addition $f^{\prime} \geq 0$, the equilibrium is unique.
Proof. The proof is a slight modification of the proof of Proposition 1 in Haan and MoragaGonzales (2011). It is useful to rewrite the expression for the equilibrium price as

$$
\begin{equation*}
\frac{m-F\left(p^{*}\right)^{2}}{2 p^{*}}=k-\int_{p^{*}}^{\hat{x}} F(\epsilon) f^{\prime}(\epsilon) d \epsilon \tag{13}
\end{equation*}
$$

where $m=2 \sigma^{A}[1-F(\hat{x})]+2\left(1-\sigma^{A}\right) F(\hat{x})[1-F(\hat{x})]+F(\hat{x})^{2}$ in the numerator and $k=\sigma^{A} f(\hat{x})+\left(1-\sigma^{A}\right) F(\hat{x}) f(\hat{x})$ in the denominator. Here $m$ and $k$ are constants with respect to $p^{*}$.

To prove existence, I will show that as $p^{*} \rightarrow 0$, the LHS goes to infinity, while the RHS is finite, implying that $L H S>R H S$, while for $p^{*} \rightarrow \hat{x}, R H S<L H S$. Continuity will then imply that there must exist at least one $p^{*} \in(0, \hat{x})$ solving the above equation.

First, notice that the LHS is a positive valued function that decreases monotonically in $p^{*}$, while the RHS is also a positive value function, but that increases in $p^{*}$. When $p^{*} \rightarrow 0$, the numerator on the LHS is positive, while the denominator is zero, so the LHS goes to infinity. At the same time, the RHS is finite when $p^{*} \rightarrow 0$, implying that $L H S>R H S$.

If $p^{*} \rightarrow \hat{x}$, the RHS increases and is larger than the LHS if and only if

$$
\begin{align*}
\sigma^{A}[1-F(\hat{x})]+\left(1-\sigma^{A}\right) F(\hat{x})[1-F(\hat{x})] & <\hat{x}\left[\sigma^{A} f(\hat{x})+\left(1-\sigma^{A}\right) F(\hat{x}) f(\hat{x})\right]  \tag{14}\\
(1-F(\hat{x}))\left(\sigma^{A}+F(\hat{x})-\sigma^{A} F(\hat{x})\right) & <\hat{x} f(\hat{x})\left(\sigma^{A}+F(\hat{x})-\sigma^{A} F(\hat{x})\right)  \tag{15}\\
1-F(\hat{x}) & <\hat{x} f(\hat{x}) \tag{16}
\end{align*}
$$

To show that this condition is satisfied, consider the expression for monopoly profits. The monopoly price $p^{m}<\hat{x}^{2}$, solves max $p(1-F(p))$. The first order condition evaluated at the monopoly price is given by $1-F\left(p^{m}\right)-p^{m} f\left(p^{m}\right)=0$. If $f$ is log-concave, then $1-F$ is log-concave also. $1-F$ log-concave is equivalent to an increasing hazard rate, i.e. defining the hazard rate as $h=f /(1-F)$, note that $\frac{\partial \log (1-F)}{\partial p}=-f /(1-F)=-h$ and $\frac{\partial^{2} \log (1-F)}{\partial p^{2}}=-h^{\prime}$, which is negative only if $h^{\prime}>0$. Then, if $f$ is log-concave so that $1-F$ is also log-concave, then for $\hat{x}>p^{m}$, it must be that $1-F(\hat{x})-\hat{x} f(\hat{x})<0$.

Thus, for $p^{*} \rightarrow 0, L H S>R H S$, while for $p^{*} \rightarrow \hat{x}, R H S<L H S$. Continuity then implies that there must exist at least one $p^{*} \in(0, \hat{x})$ solving the above equation. This completes the existence proof.

To prove uniqueness we need that the RHS increases monotonically in $p^{*}$, which is satisfied if $f^{\prime} \geq 0$.

The next proposition shows that in the second period, prices increase in search costs, as is common in the literature.

Proposition 2. If $f^{\prime} \geq 0$ and $\sigma^{A} \geq 1 / 2^{3}$, then an increase in search costs $c$ increases equilibrium prices $p^{*}$ in the second period.

Proof. The equilibrium price in the second period depends on search costs only through $\hat{x}$. Also, $\hat{x}$ is related to search costs by $\int_{\hat{x}}^{\bar{\epsilon}}(\epsilon-\hat{x}) f(\epsilon) d \epsilon=c$. Thus, a higher level of search costs is equivalent to lower $\hat{x}$. Consider again the simplified expression for equilibrium profits

$$
\begin{equation*}
\frac{m-F\left(p^{*}\right)^{2}}{2 p^{*}}=k-\int_{p^{*}}^{\hat{x}} F(\epsilon) f^{\prime}(\epsilon) d \epsilon \tag{17}
\end{equation*}
$$

where $m=2 \sigma^{A}[1-F(\hat{x})]+2\left(1-\sigma^{A}\right) F(\hat{x})[1-F(\hat{x})]+F(\hat{x})^{2}$ and $k=\sigma^{A} f(\hat{x})+(1-$ $\left.\sigma^{A}\right) F(\hat{x}) f(\hat{x})$.

The LHS is decreasing in $p^{*}$ and it is decreasing in $\hat{x}$ if $\sigma^{A} \geq 1 / 2$. More precisely, differentiating LHS with respect to $\hat{x}$ gives

$$
\begin{align*}
\frac{\partial L H S}{\hat{x}} & =-2 \sigma^{A} f(\hat{x})+2\left(1-\sigma^{A}\right) f(\hat{x})(1-F(\hat{x}))-2(1-\sigma) F(\hat{x}) f(\hat{x})+2 F(\hat{x}) f(\hat{x}) \\
& =2 f(\hat{x})(1-F(\hat{x}))\left(1-2 \sigma^{A}\right) \tag{18}
\end{align*}
$$

where $f(\hat{x})(1-F(\hat{x}))>0$. Then, the LHS is increasing in $\hat{x}$ if and only if $\sigma^{A} \geq 1 / 2$. The RHS is increasing in $p^{*}$ since

$$
\begin{equation*}
\frac{\partial R H S}{p^{*}}=F\left(p^{*}\right) f^{\prime}\left(p^{*}\right) \tag{19}
\end{equation*}
$$

which is non-negative if $f^{\prime} \geq 0$.
Finally, the RHS is also increasing in $\hat{x}$ since

[^2]\[

$$
\begin{align*}
\frac{\partial R H S}{\hat{x}} & =\sigma^{A} f^{\prime}(\hat{x})+\left(1-\sigma^{A}\right)\left(f^{2}(\hat{x})+F(\hat{x}) f^{\prime}(\hat{x})\right)-F(\hat{x}) f^{\prime}(\hat{x})  \tag{20}\\
& =\sigma^{A} f^{\prime}(\hat{x})(1-F(\hat{x}))+\left(1-\sigma^{A}\right) f^{2}(\hat{x})>0 \tag{21}
\end{align*}
$$
\]

Thus, when $f^{\prime} \geq 0$ and $\sigma^{A} \geq 1 / 2$, higher $\hat{x}$ leads to lower equilibrium prices in the second period and thus higher search costs increase prices.

### 3.3 First Period

Consumers in the first period are uncertain about the satisfaction level they will receive from consuming either product. However, they have different affinities for the two products due to ex-ante product differentiating, denoted by $\eta$. I model these initial affinities in the typical Hotelling framework. Suppose firm A is located at $l=0$ and firm B is located at $l=1$. Consumers are uniformly distributed along this line. Search costs in this period are zero and thus consumers are aware of both prices.

A consumer located at $\eta \in[0,1]$ will purchase from firm A if $E\left(\epsilon_{A}\right)-\eta-r_{A}>$ $E\left(\epsilon_{B}\right)-(1-\eta)-r_{B}$, where $\epsilon_{i}$ is the satisfaction level derived by the consumer from consuming product $i$. In the first period, the consumer expects this level to be the same at the two firms, but has an affinity towards one of the firms manifested by $\eta$. Another way to interpret this is as the good being horizontally differentiated in two dimensions: the first dimension, $\eta$ is observable before consumption, while $\epsilon$ is only observable after purchase and consumption.

Let $r^{*}$ denote the symmetric equilibrium price in the first period. In order to compute the price equilibrium, I will consider the gains the deviating firm, firm A, obtains by charging price $r$. In this case, in period 1, firm A chooses price $r$ to maximize total profits, given by

$$
\begin{equation*}
\Pi^{A}\left(r, r^{*}\right)=\Pi_{1}^{A}\left(r, r^{*}\right)+\beta \Pi_{2}^{A}\left(\sigma^{A}\left(r, r^{*}\right)\right) \tag{22}
\end{equation*}
$$

where $\Pi_{1}^{A}\left(r, r^{*}\right)=r^{*} \sigma^{A}$.
All consumer located such that $\eta<\frac{1+r *-r}{2}$, will purchase from A. Thus,

$$
\begin{equation*}
\sigma^{A}=\frac{1+r^{*}-r}{2} \tag{23}
\end{equation*}
$$

The first order condition is given by

$$
\begin{equation*}
\frac{\partial \Pi^{A}}{\partial r}=\frac{\partial \Pi_{1}^{A}}{\partial r}+\beta \frac{\partial \Pi_{2}^{A}}{\sigma^{A}} \frac{\partial \sigma^{A}}{\partial r}=0 \tag{24}
\end{equation*}
$$

Interpreting $\epsilon_{i}$ as a taste for what the consumer has tried and assuming initial affinities are not too far off from $\epsilon_{i}$, it follows that rational consumers will not want to behave strategically in the following sense: buy from B even though A has a better deal for them in the first period only to be able to start their search at B. Instead, these assumptions insure that consumers who buy at $B$ will also prefer to start their search at $B$ as they somewhat prefer B.

It is now easy to see that firm A will set a lower first period price $r$ when second period profits depend on this price then in the case where firm A maximizes current period profits only. This it the subject of our first theorem.

Theorem 1. Equilibrium prices in the first period are lower when the firm's search share is endogenous rather than exogenous ${ }^{4}$
Proof. First note that $\frac{\partial \sigma^{A}}{\partial r}=-\frac{1}{2}<0$. If $\frac{\partial \Pi_{A}^{A}}{\sigma^{A}}>0$, it must be that $\frac{\partial \Pi_{1}^{A}}{\partial r}>q^{5}$. Compared to the case where second period profits are independent of $r$ and where $\frac{\partial \Pi_{1}^{A}}{\partial r}=0$, it must be that equilibrium prices are lower.

## 4 Uniform Framework

To obtain more precise intuition about the problem, let's consider next the case where values $\epsilon_{i}$ are distributed uniformly between $[0,1]$. In this case, $F(\epsilon)=\epsilon$ and $f(\epsilon)=1$. The density $f$ evidently satisfies log-concavity and $f^{\prime}=0$, so that all results so far hold. note also that log-concavity through $\frac{1-F(\hat{x})}{f(\hat{x})}<\hat{x}$ implies that $1-\hat{x}<\hat{x}$, so $\hat{x}>1 / 2$.

### 4.1 Second Period

The second period starts with a given level of $\sigma^{A}$. Given this level, firms have to choose prices that maximize second period profits. Equilibrium prices will then depend on a firm's search share, which will be determined in the first period.

Consider again firm A's profit function in the second period, simplified using the uniform distribution for $F$

$$
\begin{align*}
\Pi_{2}^{A} & =p\left(\sigma^{A}\left[1-\hat{x}-p+p^{*}\right]+\left(1-\sigma^{A}\right) \hat{x}\left[1-\hat{x}-p+p^{*}\right]+\int_{p}^{\hat{x}+p-p^{*}}\left(\epsilon-p+p^{*}\right) d \epsilon\right) \\
& =p\left(\sigma^{A}\left[1-\hat{x}-p+p^{*}\right]+\left(1-\sigma^{A}\right) \hat{x}\left[1-\hat{x}-p+p^{*}\right]+\frac{\hat{x}^{2}-p^{* 2}}{2}\right) \tag{25}
\end{align*}
$$

Taking the first order conditions and imposing symmetry yields the following equilibrium price, which note, depend on $\sigma^{A}$ :

$$
\begin{equation*}
p^{*}=\sqrt{\left[\sigma^{A}(\hat{x}-1)\right]^{2}+2\left[\sigma^{A}(1-\hat{x})+\hat{x}\right]}-\left[\sigma^{A}(1-\hat{x})+\hat{x}\right] \tag{26}
\end{equation*}
$$

where $\hat{x}=1-\sqrt{2 c}$ and thus $c \in[0,0.5]$. The reservation utility $\hat{x}$ is thus a decreasing function of the search costs, as can be seeing in the following graph:

[^3]

We can compute the monopoly price as the price that solves $\max p(1-p)$. It thus follows that the monopoly price equals $p^{m}=0.5$, as depicted in the picture above. Since we assumed that $p^{m}<\hat{x}$, we can see from the figure that this corresponds to low search costs, lower than $c=1 / 8$, i.e. the level of search costs that solve $0.5=1-\sqrt{2 c}$.

Equilibrium profits are then given by

$$
\begin{equation*}
\Pi_{2}^{*}=\left(\sigma^{A}(1-\hat{x})+\hat{x}\right)\left(\sigma^{A}(\hat{x}-1)+\hat{x}-\sqrt{\left[\sigma^{A}(\hat{x}-1)\right]^{2}+2\left[\sigma^{A}(1-\hat{x})+\hat{x}\right]}\right)^{2} \tag{27}
\end{equation*}
$$

From these two expressions, we can show the following:

- first, price in the second period is decreasing in $\hat{x}$ and thus increasing in search costs, i.e. $\frac{\partial p^{*}}{\partial \hat{x}}<0$, while $\frac{\partial p^{*}}{\partial c}>0$

- second, price in the second period is increasing in the search share $\sigma^{A}$ for $\hat{x}<\frac{1}{2}$ and high search costs, but decreasing in $\sigma^{A}$ for $\hat{x}>\frac{1}{2}$ and low search costs. Since, as assumed before, I focus on the case where $\hat{x}>p^{m}$, for this range, $\frac{\partial p^{*}}{\partial \sigma^{A}}<0$, as can be seen from the figure below:

- third, profits in the second period are increasing in the search share


Figure 1: Same result holds for all values of $\hat{x}$, but the lines become flatter as $\hat{x}$ increases and to ensure that the lines for low $\hat{x}$ do not appear vertical, I chose to limit the figure to only show three values of $\hat{x}$.

- fourth, profits in the second period are increasing in search costs for the relevant
region of low search costs $\left(c<1 / 8\right.$ when $\left.\hat{x}>p^{m}\right)$ :

- finally, quantity in the second period decreases in search costs


Thus, after considering only the second period, our results are standard in the literature: the higher the search costs of the consumer, the higher equilibrium prices. Also, the larger the share of consumers who start searching at a firm, the higher that firm's profits
and the lower its prices. These results change dramatically however, when we consider the first period where firms compete to maximize profits while taking into account how their choices in the first period affect search in the second period. As I will show next, when the firm's search share is endogenous, first period prices decrease with higher search costs.

### 4.2 First Period

In the first period, firms choose prices to maximize total profits, given by

$$
\begin{equation*}
\Pi^{A}\left(r, r^{*}\right)=\Pi_{1}^{A}\left(r, r^{*}\right)+\beta \Pi_{2}^{A}\left(\sigma^{A}\left(r, r^{*}\right)\right) \tag{28}
\end{equation*}
$$

where $\Pi_{1}^{A}\left(r, r^{*}\right)=r \sigma^{A}\left(r, r^{*}\right)$ where $\sigma^{A}\left(r, r^{*}\right)=\frac{1+r^{*}-r}{2}$.
Taking first order conditions and imposing symmetry, we can solve for the equilibrium first period price $r^{*}$ as

$$
\begin{equation*}
r^{*}=\frac{9}{20}\left(\hat{x}+5 \hat{x}^{2}+\hat{x}^{3}-\frac{\left(\hat{x}^{2}-1\right)\left(\hat{x}^{2}+6 \hat{x}+13\right)}{b}-\frac{43}{9}\right) \tag{29}
\end{equation*}
$$

where $b=\sqrt{\hat{x}^{2}+2 \hat{x}+5}$.
We can now show that even though second period prices are increasing in search costs, the opposite is true for first period prices. First period prices are increasing in $\hat{x}$ and thus decreasing in search costs.

In (symmetric) equilibrium $\sigma^{A}=\frac{1}{2}$, thus second period price simplifies to

$$
\begin{equation*}
p^{*}=\frac{1}{2}\left(\sqrt{\hat{x}^{2}+2 \hat{x}+5}-1-\hat{x}\right) \tag{30}
\end{equation*}
$$

While second period equilibrium profits are given by

$$
\begin{equation*}
\Pi_{2}^{*}=\frac{1}{4}(x+1)\left(\sqrt{\hat{x}^{2}+2 \hat{x}+5}-1-\hat{x}\right)^{2} \tag{31}
\end{equation*}
$$

Putting these results together, we can show the striking result that second period prices are increasing in search costs, but first period prices are decreasing in search costs. To do this, differentiate the two expressions for equilibrium prices to obtain

$$
\begin{equation*}
\frac{\partial p^{*}}{\partial \hat{x}}=\frac{\hat{x}+1}{4 \sqrt{\hat{x}^{2}+2 \hat{x}+5}}-\frac{1}{2} \tag{32}
\end{equation*}
$$

while the expression for $\frac{\partial r^{*}}{\partial \hat{x}}$ is more complicate ${ }^{6}$, but I will use the figure below to show that this derivative is actually positive and thus that $r^{*}$ is increasing in $\hat{x}$, but decreasing in search costs.

[^4]

Also, as is clear from the following figure, for search costs that are relevant for the current study, so for search costs lower than $1 / 8$, second period price is lower than the monopoly level, while the first period price is higher 7

[^5]

### 4.3 Welfare

We can easily see from the figure above that as search costs increase, prices in the first period increase slower than prices in the second period decrease. More precisely, the slope $p^{*}$ is flatter in absolute terms than the slope of $r^{*}$. It is thus likely that consumer welfare increases when search costs increase, because first period prices decreases more than do second period prices increases (and the second period is discounted by $\beta$, making the effect even stronger).

Short back of the envelope calculation shows this result.

$$
\left|r^{*}(0.08)-r^{*}(0.09)\right|=|0.9343-0.9241|=0.0102
$$

which is larger than

$$
\left|p^{*}(0.08)-p^{*}(0.09)\right|=|0.4806-0.4852|=0.0046
$$

Thus, consumers are made better off by an increase in search costs.


Figure 2: I cut the search cost lower so we can see the slopes better. First period profits is decreasing in search costs (expected this to have same shape as first period prices as a function of search costs), while the second period profits are increasing in search costs (saw this in previous graph better). However, first period profits are a lot larger, even though they are decreasing.

First period profits is decreasing in search costs (expected this to have same shape as first period prices as a function of search costs), while the second period profits are increasing in search costs (saw this in previous graph better). However, first period profits are a lot larger, even though they are decreasing. Also, the slope of first period profits is larger than that of second period profits. Thus, when search costs increase, second period profits increase, but by a lot less than first period profits decrease. Thus, the firm is made worse off by an increase in search costs.

Total welfare may thus stay constant, as search costs just transfer some of the surplus from firms to consumers.

## 5 Conclusion

In this paper, I have laid out the basic framework for thinking about how firms' prices may affect the way consumers decide to search in the future. I particular, I described a simple two period model in which consumers search both firms in the first period, observe their prices, and based on the inferences they make from these observed prices, decide how to optimally search in the second period. When the two periods are linked in this fashion, it is possible to show that equilibrium prices will be lower than if the fraction of consumers who decide to search each firm is exogenous. As a result, lower search costs may hurt consumers as they allow firms to charge higher prices in the second period.

## References

[1] Anderson, S. P., and R. Renault, Pricing, product diversity, and search costs: a Bertrand-Chamberlin-Diamond model, Rand Journal of Economics, Vol. 30(4), 719735, 1999.
[2] Armstrong, M., J. Vickers, and J. Zhou, Prominence and consumer search, Rand Journal of Economics, Vol. 40(2), 209-233, 2009.
[3] Arbatskaya, M., Ordered search, Rand Journal of Economics, Vol. 38, 119-127, 2007.
[4] Bagwell, K., Introductory price as a signal of cost in a model of repeat business, Review of Economic Studies, Vol. 54(3), 1987.
[5] Bagwell, K., and M. Peters, Dynamic monopoly power when search is costly, Northwestern Discussion Paper No. 772, 1988.
[6] Baye, M., J. Morgan, and P. Scholten, Information, search, and price dispersion, T. Hendershott (ed.) Handbook of Economics and Information Systems, Elsevier Press, Amsterdam, 2006.
[7] Benabou, R., Search market equilibrium, bilateral heterogeneity and repeat purchases, Journal of Economic Theory, Vol. 60(1), 140-158, 1993.
[8] Benabou, R. and R. Gertner, Search with learning from prices: does inflationary uncertainty lead to higher markups, Review of Economic Studies, Vol. 60(1), 69-94, 1993.
[9] Burdett, K. and K. L. Judd, Equilibrium price dispersion, Econometrica, Vol. 51(4), 955-69, 1983.
[10] Cabral, L.M.B., Small switching costs lead to lower prices, Journal of Marketing Research, Vol. 46 (August), 44951, 2009.
[11] Doganoglu, T., Switching costs, experience goods and dynamic price competition, Quantitative Marketing and Economics, Vol. 8(2) 167-205, 2010.
[12] Dube, J. P., G. J. Hitsch, and P. Rossi, Do switching costs make markets less competitive?, Journal of Marketing Research, Vol. 46(4), 435445, 2009.
[13] De Los Santos, B., A. Hortasu, and M. R. Wildenbeest, Testing models of consumer search using data on web browsing and purchasing behavior, American Economic Review, Vo. 102(6), 2955-2980, 2012.
[14] Dana, J. D., Learning in an equilibrium search model, International Economic Review, Vol. 35(3), 745-771, 1994.
[15] Diamond, P. A., A model of price adjustment, Journal of Economic Theory, Vol. 3(2), 156-168, 1971.
[16] Goeree, M. S., Limited information and advertising in the U.S. personal computer industry, Econometrics, Vol 76(5), 1017-1074, 2008.
[17] Haan, M.A. and J.L. Moraga-Gonzalez, Advertising for attention in a consumer search model, The Economic Journal, Vol. 121(552), 552-579, 2011.
[18] Honka, E., Quantifying Search and Switching Costs in the U.S. Auto Insurance Industry, Available at SSRN: http://ssrn.com/abstract=2023446 or http://dx.doi.org/10.2139/ssrn. 2023446
[19] Hortasu, A. and C. Syverson, Product differentiation, search costs and competition in the mutual fund industry: a case study of S\&P 500 index funds, Quarterly Journal of Economics, Vol. 119(2), 40356, 2004.
[20] Janssen, M. C. and J. L. Moraga-Gonzalez, Strategic pricing, consumer search and the number of firms, Review of Economics Studies, Vol. 71(4), 1089-1118, 2004.
[21] Janssen, M. C., P. Pichler, and S. Weidenholzer, Oligopolistic markets with sequential search and production cost uncertainty, Rand Journal of Economics, Vol. 42(3), 444-470, 2011.
[22] Klemperer, P., The competitiveness of markets with switching costs, Rand Journal of Economics, Vol. 18(1), 138-150, 1987.
[23] Matsumoto, B. and F. Spence, Price Beliefs and Experience: Do Consumers Beliefs Converge to Empirical Distributions with Repeated Purchases?, Working Paper, 2013.
[24] Moraga-Gonzalez, J.L, Z. Sandor, and M. Wildenbeest, Do higher search costs make markets less competitive?, Working paper, 2014.
[25] Reinganum, J. F., A simple model of equilibrium price dispersion, Journal of Political Economy, Vol. 87(4), 851-858, 1979.
[26] Robert, J., and D. O. Stahl, Informative price advertising in a sequential search model, Econometrica, Vol. 61 (3), 657-686, 1993.
[27] Salop, S. and E. Stiglitz, The theory of sales: a simple model of equilibrium price dispersion with identical agents, American Economic Review, Vol. 72(5), 1121-1130, 1982.
[28] Spence, F, Does Consumer Inexperience Generate Welfare Losses? Evidence from the Textbook Market, Working Paper, 2014.
[29] Stahl, D. O., Oligopolistic pricing with sequential consumer search, American Economic Review, Vol. 79(4), 700-712, 1989.
[30] Stigler, G.J., The Economics of Information, Journal of Political Economy, Vol. 69(3), 213-225, 1961.
[31] Tappata, M., Rockets and feathers: understanding asymmetric pricing, Rand Journal of Economics, Vol. 40(4), 673-687, 2009.
[32] Varian, H. R., A Model of Sales, American Economic Review, Vol. 70(4), 651-659, 1980.
[33] Warner, E. J, and R. Barsky, The timing and magnitude of ratio store markdowns: evidence from weekends and holidays, Quarterly Journal of Economics, Vol. 110(2), 321-352, 1995.
[34] Wolinsky, A, True monopolistic competition as a result of imperfect information, Quarterly Journal of Economics, Vol. 101(3), 493511, 1986.
[35] Yang, H. and L. Ye, Search with learning: understanding asymmetric price adjustments, Rand Journal of Economics, Vol. 39 (2), 547-564, 2008.


[^0]:    *I thank Ali Hortacsu, Richard Van Weelden and Pradeep Chintagunta for valuable suggestions and continuous advice.

[^1]:    ${ }^{1}$ Of course, $p^{*}$ depends on both $\sigma^{A}$ and $\sigma^{B}$, but since $\sigma^{B}=1-\sigma^{A}$, to simplify notation, I will only use $\sigma^{A}$ from now on in calculation, but it should be understood that both fractions affect the equilibrium.

[^2]:    ${ }^{2}$ The fact that the monopoly price is lower than $\hat{x}$ follows from the assumption that search costs are small enough so that even if the firm charges the monopoly price, consumers are willing to conduct a first search, i.e. $\int_{p^{m}}^{1}\left(\epsilon-p^{m}\right) f(\epsilon) d \epsilon>c$, while $\int_{\hat{x}}^{1}(\epsilon-\hat{x}) f(\epsilon) d \epsilon=c$.
    ${ }^{3}$ Of course, in any symmetric equilibrium, $\sigma^{A}=1 / 2$, so higher search costs will lead to higher equilibrium prices in the second period.

[^3]:    ${ }^{4}$ This proof similar in spirit to the argument used by Klemperer (1987) to show that first period prices in a market with switching costs and myopic consumers are lower than they otherwise would be if market share were not valuable in the second period.
    ${ }^{5}$ I did not prove that $\frac{\partial \Pi_{2}^{A}}{\sigma^{A}}>0$ for the general case yet, but it does hold for the uniform distribution example that I discuss in the next section

[^4]:    ${ }^{6}$ It equals,

    $$
    \frac{\partial r^{*}}{\partial \hat{x}}=-\frac{1143 x-288 x b^{3 / 2}+1008 b^{3 / 2}-864 b^{5 / 2}+1134 x^{2}+558 x^{3}+171 x^{4}+27 x^{5}-153}{20(4 b)^{3 / 2}}
    $$

    , where $b=\frac{x^{2}+2 x+5}{4}$.

[^5]:    ${ }^{7}$ The monopoly price is the price that maximizes $p[(1-p)+(1-F(p))]$ for both periods, which simplifies to maximizing $p(1-p)$ for uniform distribution in the first period as well as the second period, so $p^{m}=0.5$ for both periods.

