Information in contests

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Abstract

We show that private (public) information on contestants' types leads to strictly greater expected aggregate effort if the dichotomous distribution of types is non-degenerate and strictly skewed towards low (high) types. If partial information censoring is possible, expected aggregate effort is maximum with public information only in the case of a symmetric contest with high-type contestants, regardless of the distribution of types.

JEL Classification: C72 - D72 - D82

1 Introduction

The comparison of public and private information on players' types has a long seminal history in the economic literature, as it drives disclosure or concealment policies.[1] In auctions, the so-called linkage principle suggested by Milgrom and Weber (1982) states that public information on players' valuations is the expected-revenue-maximizing policy. Ottaviani and Prat (2001) show that the logic and the result of the linkage principle extend to monopolies in which buyers' types are to be revealed. However, public information is not the best policy in every setting. In principal-agent models, a privately informed "principal generically does strictly better than when the agent knows her information" (see Maskin and Tirole (1990)). The literature on contests is almost silent about this comparison, particularly for a tractability problem, which we discuss and sidestep here.[2] We provide conditions under which the expected aggregate effort is greater in a public or in a private information contest.

The idea of a contest usually traces back to the seminal contributions of Tullock (1967, 1980). A fixed prize is to be raffled, and contestants compete for it by simultaneously exerting efforts that are equally valuable to society.[3] The highly competitive nature of such a game makes contestants' knowledge about other contestants crucially affect their efforts. Analysis of private information Tullock contests requires the presence of a stochastic element in the game. We let contestants'

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[1]Type is a general parameter of the player's utility function. Often, type is the marginal cost or the valuation.

[2]Footnote 11 discusses some partial answers provided to date by the literature.

[3]The fact that the objective function is the expected aggregate effort exerted by contestants comes from the rent-seeking nature of a contest. Although this assumption is by far the most common in the literature, we propose and discuss in Section 5 other meaningful specifications.

private information be their own *type*.[4] Contestants' type can be either high or low with unequal probabilities.[5] We restrict the number of contestants to two, which is both the minimum number for a meaningful competition and the maximum number for avoiding endogenous participation issues. We find that the skewness of the distribution of types drives the choice of optimal information regime (private versus public information on types). In particular, if the distribution of types is strictly skewed - and non-degenerate - towards high (low) types, then public (private) information is optimal. We now explain the intuition behind this result which is driven by a novel interaction of second-order beliefs and strategic substitutability/complementarity; we use Pri for private information and Pub for public information here and throughout the paper.

Unevenness of types reduces efforts, as the low-type gives up hope of winning, and the high-type does not face much competition.[6] Hence, if two high-types or two low-types show up, we wish we had implemented Pub to prevent contestants from thinking that they are competing in an uneven contest. If instead a high-type and a low-type show up, we wish we had implemented Pri to conceal the unevenness of types and hence to lead contestants to exert a large amount of effort. Therefore, finding the optimal information regime boils down to comparing these two opposite forces from an *ex-ante* point of view.[7] We analyze the sign of the expected *net* benefits of Pri - i.e., benefits of Pri to the sum of efforts in the case of an uneven contest, minus losses of Pri to the sum of efforts in the case of an even contest - for different skewnesses of the distribution of types. First, think about a population in which high-types are very rare. Then, it is very likely to have two low-types who are basically aware of competing in an even contest, and in this case, Pub poorly affects efforts. However, say a high-type shows up, then her reasoning under Pri is "I basically know that I am against a low-type, but I better exert **more** effort than I would exert if facing a low-type under Pub because my low-type rival thinks that she is competing against another low-type, and therefore she will not give up hope and exert high effort". This interaction of second order beliefs and strategic complementarity of the action of the high-type *biases* upward the effort of the high-type under Pri. This *upward bias* fades away as high-types become less rare in the population because situations of misperception of rival's type as described above become less likely. Second, in a population in which low-types are rare, a low-type under Pri exerts **less** effort than if she is against a high-type under Pub because her high-type rival thinks that she is

[4] Contestants' type is their marginal cost of exerting effort or equivalently their prize valuation and it can be interpreted as skills. For the role of information revelation about the number of contestants, see Fu et al. (2011), Lim and Matros (2009), and Myerson and Warneryd (2006), and about contestants' performance in a dynamic setting, see Aoyagi (2010). Information acquisition by contestants is analyzed by Kovenock, Morath, and Munster (2010) and Szech (2011) in all-pay auctions and by Denter, Morgan, and Sisak (2011) and Yildirim (2005) in imperfectly discriminating contests.

[5] It will be clear that a diffuse prior would make the comparison between private and public information trivial.

[6] If I plays chess against Garry Kasparov, I exert little effort as I have very little chance of winning, but also Garry Kasparov exerts little effort, as he does not need much effort to be confident of winning.

[7] If the information policy was to be taken ex-post - i.e., if the information regime is chosen after observing the realization of types - and contestants were aware of this, a rational contestant would correctly interpret Pri as a sign that she is competing in an uneven contest, and hence she would perfectly infer her opponent's type regardless of the information observed. Therefore, ex-post information disclosure plays no role in affecting efforts, and contestants exert Pub-equilibrium-efforts. If Pri or Pub is chosen ex-ante, there is no such updating of beliefs. An interesting exercise is to check the robustness of this "unraveling result" (see Milgrom (1981) for the first of these types of results leading to full awareness of rival's types) to a more general n-dimensional type-space, in which there is more than one contingency of uneven contest. This robustness check is beyond the scope of this paper. We just point out that choosing the information regime ex-ante is never dominated by choosing it ex-post, as ex-ante Pri and Pub are possible, whereas ex-post contestants exert Pub-equilibrium-efforts. In other words, if a social planner was capable of choosing whether to decide ex-ante or ex-post her information policy, she would never have strict preference for an ex-post decision.

in an even contest, therefore exerting high effort, and this deters the low-type from exerting effort (strategic substitutability). This *downward bias* in the effort of the low-type under Pri fades away as low-types become less rare in the population.

Therefore, the expected net benefits of Pri are positive (negative) due to the upward (downward) bias when the population is skewed towards high (low) types. In case of a symmetric distribution of types, the upward and the downward biases balance out, and the expected aggregate effort is the same under the two information regimes. If skewness is maximum - i.e., a degenerate population with certainty of low or high types - the above-mentioned biases become irrelevant, as there is certainty (and contestants' awareness) of an even contest, and hence information does not affect efforts. These results lead to the optimal information regime depicted in Figure 1 as a function of the probability of a high-type in the population.

Comparing the expected aggregate effort in Pub and Pri information regimes naturally raises the following question: can we do better than under Pri or Pub? More specifically, can we extract more rent by inducing Pub or Pri under some realizations of types only? In the words of Milgrom and Weber (1982), "one might wonder whether it would be better still to censor information sometimes". We find that a specific form of such contingent information regime does indeed maximize the expected aggregate effort for any dichotomous distribution of types. This result shows that the optimality of Pri or Pub found in the above analysis drastically changes if a larger mechanism-space is possible as an information regime. In particular, we show that the optimal censoring is "Pri if two high-types and Pub otherwise", regardless of the distribution of types. The intuition behind this result is as follows. Pub if two high-types and Pri if two low-types or a high-type and a low-type makes the high (low) type always (never) capable of inferring her opponent's type. Hence, the expected aggregate effort:

- benefits from strategic complementarity: under Pri, a high-type faces with certainty a low-type who does not lose hope because she thinks she might be in an even contest - i.e., against another low-type.

- avoids the losses from strategic substitutability: under Pri, a low-type never faces a high-type who thinks of (possibly) being in an even contest - i.e., against another high-type.

The endogeneity of information in contests fits a number of potential applications, of which we acknowledge three:

1. A country's government wants to foster technological breakthroughs and decides to run an innovation contest. When choosing the rules of the contest (number of prizes, timing, etc.), the government can, for example, commit to disclose (or not) on the contest's website the list of participants' names, curricula (if individuals), balance sheets (if firms), etc..

2. A legislator mandates transparency (public information) or anonymity (private information) in public procurements aiming at market stimulation. If the market is sufficiently large or new, it is reasonable to think that tenderers do not know each other unless told.

3. Transparency in lobbying is a key issue currently at the center of many policy debates. Does a policy maker expect greater aggregate lobbying effort if lobbyists know or do not know each other? Note that if instead of a policy maker, we think of a central governmental authority, then lobbying efforts should be seen as wasteful, and hence the suitable social goal would be to minimize aggregate effort.[8]

[8]It is trivial to read our results in terms of aggregate effort minimization.

The disclosure of information about other contestants that would otherwise be ignored by contestants has several other consequences aside from directly affecting contestants' efforts: it may 1) trigger communication among contestants,[9] 2) affect the external visibility of contestants,[10] and 3) affect contestants' willingness to participate in the contest. We abstract away from all of these issues and focus here on the direct effects on efforts of making contestants' private information publicly available to other contestants. We do not claim that direct effects on efforts are more important than the above-mentioned ones.

In addition to analyzing the optimal information regime, another contribution of this paper is a step forward in the analysis of private-information Tullock contests with a dichotomous type-space that we claim will have positive spillovers on other research questions. The literature has imposed simplifications to this model to overcome the lack of tractability, which is why the apparently simple question posed in this paper has not yet been answered in our general framework.[11] We sidestep the problem of finding a closed-form solution for the equilibrium efforts by means of the following method: we present a novel property that is common to every equilibrium of this type of private-information game (Proposition 1), and we show that this property suffices for all of the findings of this paper. This property reads as follows. A certain type, say $\bar{\theta}$, exerts her maximum effort when there is public-information and the rival is another $\bar{\theta}$. Hence, in private-information her effort is a percentage of her maximum effort. The property says that this percentage weighted by the probability of $\bar{\theta}$ minus the probability of two $\bar{\theta}$-contestants (maximum-effort contingency) is constant across types. In other words, the private-information equilibrium efforts, relative to public-information ones, are linearly related in every equilibrium. Although this property of equilibrium efforts is not of straightforward interpretation, we conjecture that it will be useful for other studies. This conjecture is based on the fact that using properties of the equilibrium players' relative actions when lacking a closed-form solution is a well-established method in other fields. For instance, in auction theory, the difficulty of finding equilibrium bids in a first-price auction with asymmetric bidders has been partially overcome by retrieving some properties of bidding behaviors, which has proved useful to address several research questions; Kirkegaard (2009) draws important conclusions from analyzing the ratio of bidders' payoffs, bypassing the need for closed-form equilibrium bids.

Section 2 spells out the model. Section 3 compares information regimes (Pri versus Pub), whereas Section 4 allows for contingent information regimes. Section 5 concludes. All proofs are in the Appendix.

[9] We point out that communication among contestants might have both negative and positive effects on competition: negative as it facilitates collusion and positive as it facilitates a better understanding of the problem at hand ("The purpose of exchanging information is to improve the understanding of Government requirements and industry capabilities, thereby allowing potential offerors to judge whether or how they can satisfy the Government's requirements, and enhancing the Government's ability to obtain quality supplies and services, including construction, at reasonable prices, and increase efficiency in proposal preparation, proposal evaluation, negotiation, and contract award", see FAR 15.201 (b))

[10] Again, the external visibility could be positive (fostering contestants' visibility in the market) or negative (jeopardizing contestants' anonymity or exposing them to media pressure).

[11] Denter, Morgan and Sisak (2011) consider one-sided private-information - one contestant's type is common knowledge, and the other's is private information -, which admits a closed-form solution. Hurley and Shogren (1998) analyze numerical simulations. Fey (2008) studies the case of a diffuse prior; as is clear from the above discussion, the prior's skewness drives the main result, and hence the case of a diffuse prior leads to trivial answers to our research question. Dubey (2013) studies the case of dichotomous effort space.

2 Model

Two risk-neutral contestants exert efforts e_1 and e_2, and each has a probability of winning a prize equal to

$$p_i(e_i, e_j) = \begin{cases} 0 & \text{if } e_i = e_j = 0 \\ \frac{e_i^r}{e_i^r + e_j^r} & \text{otherwise} \end{cases} \tag{1}$$

with $i, j = 1, 2$, $j \neq i$, and $r \in (0, 1]$. Contestant i is of type θ_i, which is independently drawn from the distribution of types

$$\theta_i = \begin{cases} h & \text{with prob. } p \\ l & \text{with prob. } 1 - p \end{cases} \tag{2}$$

with $h > l > 0$, and $p \in [0, 1]$.[12] Everything but the realized θs is common-knowledge. The cost of effort is linear and the marginal cost equals the inverse of the contestant's type.[13] Let the prize equal 1 wlog. Hence, the expected utility of contestant i is

$$E\left[u_i(e_i, e_j)\, |\mathcal{R}\right] = E\left[\frac{e_i^r}{e_i^r + e_j^r}\, |\mathcal{R}\right] - \frac{e_i}{\theta_i} \tag{3}$$

where $\mathcal{R}$ is the commonly-known information regime. In Section 3, we compare the expected aggregate effort under the two regimes: private information (Pri) and public information (Pub). Pri leads contestants' actions to be based on their own type and the prior (2). Pub leads contestants' actions to be based on their own type and their rival's type. In Section 4, we analyze the expected aggregate effort under a contingent information regime - i.e., only some realizations of types are made public information. After observing their own types - and in the case of Pub, their rival's type as well - contestants simultaneously exert effort, and the winner is determined according to (1).

3 Public information versus private information

In this section, we analyze the implications of private information ($\mathcal{R} = Pri$) and public information ($\mathcal{R} = Pub$) on the expected aggregate effort. A well-known result of Tullock contests under Pub is that equilibrium efforts are given by

$$e_{hh} = \frac{rh}{4}, \quad e_{hl} = \frac{rh^{r+1}l^r}{(h^r + l^r)^2}, \quad e_{lh} = \frac{rl^{r+1}h^r}{(h^r + l^r)^2}, \quad e_{ll} = \frac{rl}{4} \tag{4}$$

where we denoted by $e_{\theta_i\theta_j}$ the equilibrium effort of type θ_i (aware of being) competing against a rival of type θ_j.[14]

Although the model is apparently straightforward, the equilibrium efforts under Pri are derived from a system of equations (see (6) in the Appendix) that generally lacks a closed-form solution.[15]

[12] We rule out from the beginning the possibility of $h = l$ or $l = 0$, as in the former case, information would play no role, and in the latter case, marginal cost would not be defined.

[13] Note that an equivalent specification would be to model types as prize valuations.

[14] See, for example, Nti (1999) for a derivation of this result.

[15] See Fey (2008) for equilibrium effort in case $p \in \{0, \frac{1}{2}, 1\}$.

In fact, the existence itself of a (pure-strategy) equilibrium was not established until recently (see Einy et al. (2013) for a general existence theorem). We outflank the need for a closed-form solution by showing in Proposition 1 that equilibrium efforts under Pri relative to those under Pub and an even contest - i.e., the maximum effort a certain type exerts (see Lemma 6) - are linearly related in equilibrium. This property of equilibrium efforts suffices for our purpose of finding the optimal information regime, both in this section and in the next one. We denote by e_{θ_i} the equilibrium effort of type θ_i under Pri. Note that e_{θ_i}, on the contrary of $e_{\theta_i \theta_j}$, does depend on p.

Proposition 1 *The following holds in equilibrium:*

$$p\frac{e_h}{e_{hh}} - p^2 = (1-p)\frac{e_l}{e_{ll}} - (1-p)^2 \tag{5}$$

A well-known result in Pub contests - that can be seen directly in (4) - is that the ratio of efforts equals the ratio of contestants' types - i.e., $e_{hl}/e_{lh} = h/l$.[16] Equation (5) shows that this result extends to Pri contests only in case of a diffuse prior - i.e., $p = 1/2$. Together (4) and (5) are sufficient to prove the primary result of this section.

Theorem 2 *Expected aggregate effort is strictly greater under private (resp. public) information if the distribution of types is non-degenerate and strictly skewed towards low (resp. high) types. Expected aggregate effort is equal under private and public information if and only if the prior is degenerate (i.e., $p \in \{0, 1\}$) or diffuse (i.e., $p = \frac{1}{2}$).*

See Figure 1 for a graphical depiction of Theorem 2. The remainder of this section builds intuition for Theorem 2. In Figure 2, we plot the equilibrium efforts in Pri and Pub as functions of p.[17] The thick lines are the efforts in Pub (e_{hh}, e_{hl}, e_{lh}, and e_{ll}), whereas the thin ones are the efforts in Pri (e_h and e_l).

First, note that $e_{hh} > e_{hl}$ and $e_{ll} > e_{lh}$ because contestants exert more effort in even contests than in uneven contests. Moreover, e_h increases in p (likelihood of being in an even contest for the high-type), and it converges to e_{hh} as $p \to 1$. Nevertheless, as $p \to 0$, e_h does not converge to e_{hl} because in Pri a high-type knows that she competes against a low-type who believes that she is against another low-type. Hence, the high-type competes with a low-type who does not give up hope and competes fiercely. Therefore, the high-type increases her effort with respect to e_{hl} (strategic complementarity). Similarly, e_l decreases in p and as $p \to 1$ it converges to a lower level than e_{lh} because a low faces a high who fights fiercely because she thinks she faces another high. Hence, the low-type decreases her effort with respect to e_{lh} (strategic substitutability). These are the two biases mentioned in the Introduction, and they both fade away as p moves closer to $1/2$. We now use these findings to answer the main question: does Pub or Pri induce more expected aggregate effort?

We visualize the benefits and the losses of Pri with respect to Pub in Figure 2. For a given p, the **losses** of Pri occur when the contest is even - two high-types with probability p^2 or two low-types with probability $(1-p)^2$ (see Table 1) -, and they can be visualized as the vertical distances between e_{hh} and e_h, and between e_{ll} and e_l, which *are not* affected by the biases. As p moves from 0 to 1, the losses of Pri move from just considering the difference between e_{ll} and e_l

[16] Proposition 2 in Corchón (2000) proves that this result holds under homogeneity of degree zero of the contest success function (together with mild regularity assumptions).

[17] All of the plots are created using the command *Root* in *Mathematica*® 8.0 on the polynomial in e_h-only obtained from the simplification of (6).

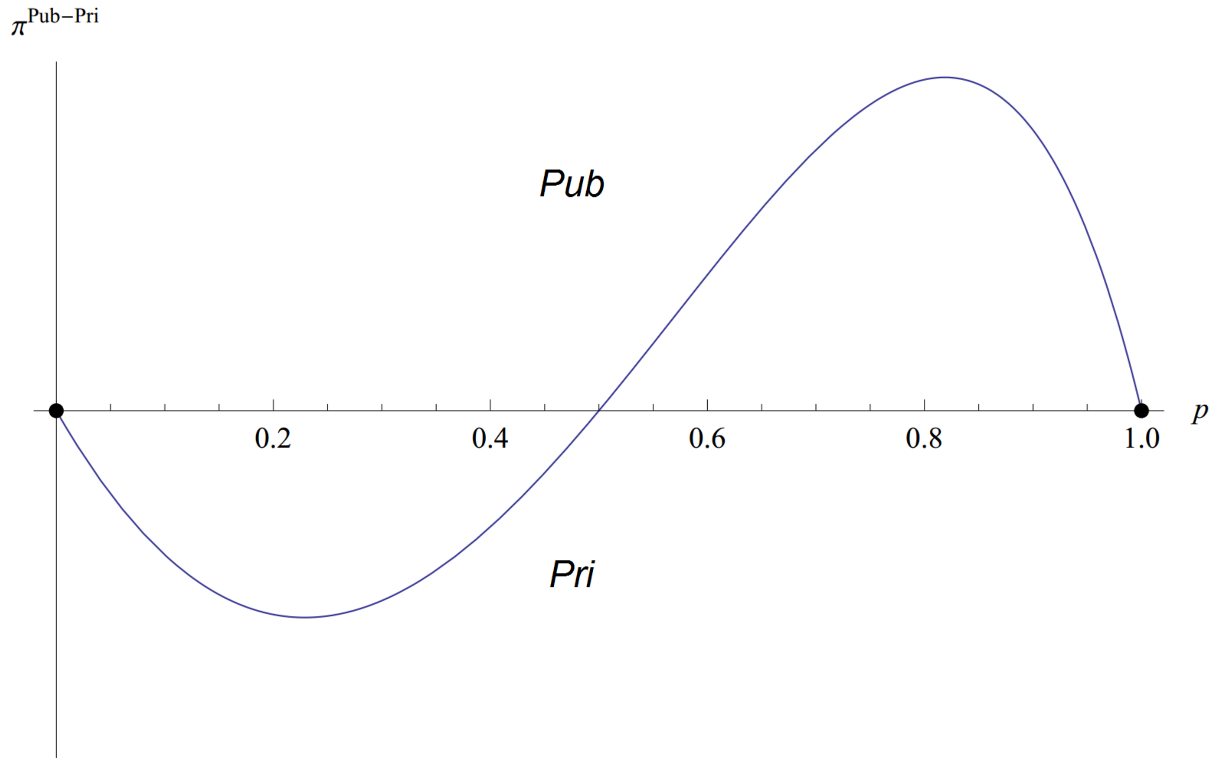

Figure 1: Optimal information regime: expected sum of efforts under Pub minus expected sum of efforts under Pri as function of p assuming $r = 1$, $l = 1$ and $h = 2$.

to just considering the difference between e_{hh} and e_h. Therefore, the losses of Pri are a concave parabola that takes value 0 if $p = 0$ or $p = 1$. For a given p, the **benefits** of Pri occur when the contest is uneven - one high and one low with probability $2p(1-p)$ - and they can be visualized as the vertical distance between e_h and e_{hl}, and between e_l and e_{lh}, both of which *are* affected by the biases. In particular, if p is sufficiently small, the upward bias in the benefits of Pri due to strategic complementarity prevails on the downward bias due to strategic substitutability, and vice-versa if p is large. Hence, the benefits of Pri are a concave parabola (that takes value 0 if $p = 0$ or $p = 1$) skewed towards low ps. Symmetric losses of Pri and asymmetric benefits of Pri yield the result of Theorem 2.

Likelihood	Contingency	Ex-post optimal information regime
p^2	$\{h,h\}$	Pub
$(1-p)^2$	$\{l,l\}$	Pub
$2p(1-p)$	$\{h,l\}$	Pri

Table 1: Possible realizations of contestants' types.

4 Contingent information regime

In Section 3, we compared the expected aggregate effort committing to either full public-information or to full-private information. In this section we assume a stronger commitment power; information regime can make types public under some realizations of types only. Therefore, $\mathcal{R} \in \{Pub, Pri\}^3$

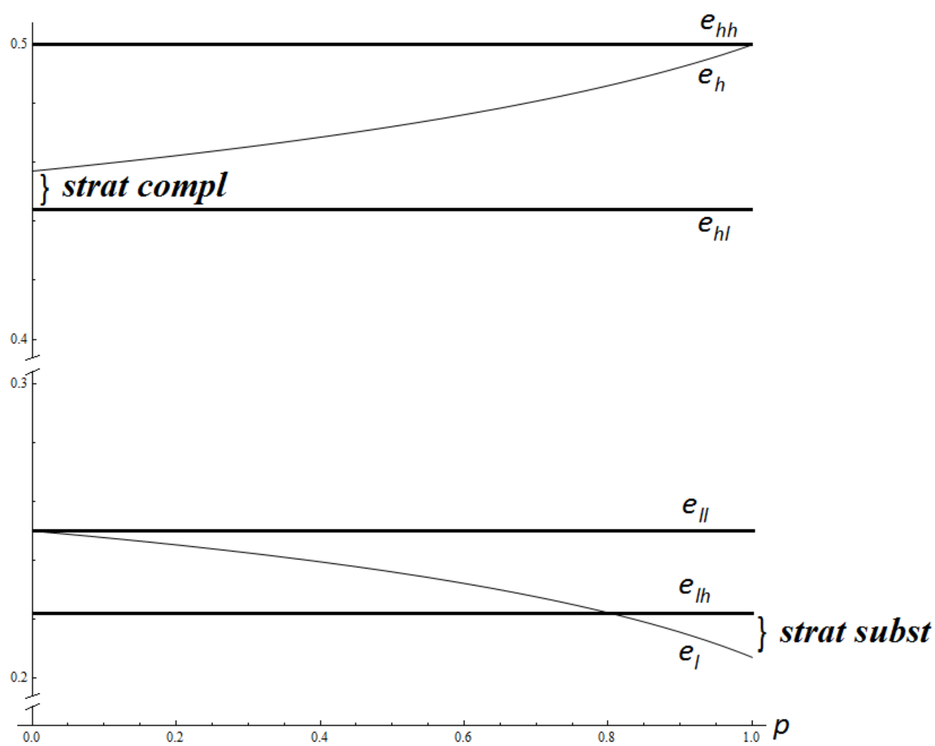

Figure 2: Equilibrium efforts as functions of p assuming $r = 1$, $l = 1$ and $h = 2$. The two segments named "strat compl" and "strat subst" embody the *upward bias* and *downward bias* explained in the text.

where the first (resp. second and third) element corresponds to the information regime in case of contingency $\{h, h\}$ (resp. $\{h, l\}$ and $\{l, l\}$). If, for instance, the information regime is $\mathcal{R} = \{Pri, Pri, Pub\}$, then the only contingency in which contestants are told their rival's type is when two low-types show up.[18] As before, $\mathcal{R}$ is common-knowledge. Upon observing Pub, contestants get to know the types. Upon observing Pri, contestants update their beliefs on the rival's type because they are aware of $\mathcal{R}$. We define these regime-induced beliefs using p_h to indicate the belief of a high-type of being against another high-type under Pri, and p_l for the belief of a low-type of being against another low-type in Pri. Residual beliefs are the residual probabilities. Hence, if $\mathcal{R} = \{Pri, Pri, Pub\}$ as in the above-mentioned example, then $p_h = p$ and $p_l = 0$. Although the strong commitment power analyzed here formally nests the analysis of Section 3, it might not be an option in some circumstances, and hence we keep the two sections separate.

Note that the existence of an equilibrium in pure strategy for any $\mathcal{R}$ is again special case of Theorem 1 in Einy et al. (2013). We first prove the following.

Proposition 3 *In equilibrium, i)* $\frac{\partial e_h}{\partial p_h} > 0$*, ii)* $\frac{\partial e_l}{\partial p_l} > 0$*, iii)* $\frac{\partial e_h}{\partial p_l} > 0$*, and iv)* $\frac{\partial e_l}{\partial p_h} < 0$*.*

Parts i) and ii) of Proposition 3 are trivial: in even contests the effort exertion is greater than in uneven contests. Part iii) is the by-product of the strategic complementarity of efforts for the high-type: the more the low-type believes she is likely to be against another low-type, the more effort she exerts, the more effort the high-type exerts as a best response. Part iv) is the by-product of the strategic substitutability of efforts for the low-type: the more the high-type believes she is likely to be against another high-type, the more effort she exerts, the less effort the low-type exerts as a best response.

We use the results of Proposition 3 together with the generalization of Proposition 1 (see (25) in the Appendix) to prove the main result of this section.

Theorem 4 *If contingent information regimes are possible, expected aggregate effort is maximum if only realization* $\{h, h\}$ *is made public (i.e.,* $\mathcal{R} = \{Pub, Pri, Pri\}$*).*

The idea behind Theorem 4 is as follows. A contest is a game in which for some players (high-types) actions are Strategic Complements - i.e., SC-players -, and for others (low-types) actions are Strategic Substitutes - i.e., SS-players. The optimal information regime of Theorem 4 leads an SC-player to be aware of her rival's type, and an SS-player to be unaware.[19] Therefore, an SS-player knows that she would never face an SC-player unaware of the disparity of the match, and hence the latter player would not choose a disproportionately high action that would severely drag down the SS-player's action ("avoid the losses of strategic substitutability"). On top of this, an SS-player does not know when she is facing an SC- or an SS-player, and hence she chooses an intermediate action. The SC-player knows when she faces an SS-player, and knows that the SS-player's intermediate action is greater than the one she would make if she knew she is against an SC-player. This fact leads the SC-player to increase her action ("benefits from strategic complementarity").

[18] Information regimes of the form "high-types are aware and low-types are not" are nested in the information regimes considered in this section.

[19] The SC-player is either told of being against another SC-player or is not told, in which case she perfectly infers that she is against an SS-player.

5 Discussion

We analyzed the expected aggregate effort exerted by contestants of dichotomous stochastic type (high or low) who simultaneously compete for a fixed prize. We find the following: i) public information extracts more (less) expected aggregate effort than private information if the distribution of types is skewed towards high-types (low-types), and ii) if contingent information regime is possible, expected aggregate effort is maximum in case of public information only for the most favorable information (two high-types).

We voluntarily ignored contestants' selection and participation issues. For the former, it may be effort improving to endow the model with an entry fee capable of sifting out weak applicants. For the latter, contestants' participation in the contest may depend on the declared information regime itself. With regard to this extension, we conjecture that private information deters participation, especially for the low-types, and this might be beneficial in terms of expected aggregate effort.

We assumed that the objective function is the expected sum of contestants' efforts. Although this assumption fits several applications of contests and it is by far the most common assumption in the literature, we acknowledge that there are at least two sensible alternatives: 1) maximization of the probability of a high-type winner if the contest's goal is contestant selection, such as in an assessment center for hiring new employees or in the economic job market, and 2) maximization of the expected winning effort if the contest aims at quality of the winning submission disregarding overall competition, such as in an architectural contest or in a private procurement. Optimal contest design in the latter case is introduced and studied in detail by Serena (2014).

Hurley and Shogren (1998) claim that "the more tractable one-sided asymmetric information contest might be sufficient to capture contest behavior under uncertainty", and this appears to be the current direction of the literature on private information in contests. The naturalness of full (two-sided) private information contests is quite indisputable. We overcome its apparent lack of tractability.

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A Appendix

A.1 Proof of Proposition 1

The existence of an equilibrium is a special case of Theorem 1 in Einy et al. (2013). We show that (5) holds in every equilibrium.

The expected utility of player i is given by (3), whose FOC depends on i's type:

$$p\frac{re_h^r e_{\theta_i}^{r-1}}{(e_h^r + e_{\theta_i}^r)^2} + (1-p)\frac{re_l^r e_{\theta_i}^{r-1}}{(e_{\theta_i} + e_l)^2} = \frac{1}{\theta_i} \text{ for } \theta_i = h, l$$

Therefore, the system of equations characterizing the equilibrium is

$$\begin{cases} p\frac{r}{4e_h} + (1-p)\frac{re_l^r e_h^{r-1}}{(e_h^r+e_l^r)^2} = \frac{1}{h} \\ (1-p)\frac{r}{4e_l} + p\frac{re_h^r e_l^{r-1}}{(e_h^r+e_l^r)^2} = \frac{1}{l} \end{cases} \tag{6}$$

We divide the second term of the first equation by that of the second equation.

$$\frac{(1-p)e_l}{pe_h} = \frac{\frac{4e_h - prh}{4e_h h}}{\frac{4e_l-(1-p)rl}{4e_l l}}$$

We simplify and obtain

$$4(1-p)he_l - (1-p)^2 rhl = 4ple_h - p^2 rhl$$

and, dividing by rhl

$$\frac{4(1-p)e_l}{rl} - (1-p)^2 = \frac{4pe_h}{rh} - p^2$$

and using the expressions for e_{hh} and e_{ll} in (4) the result follows.

A.2 Proof of Theorem 2

We check the sign of $\pi^{Pub-Pri}$ defined as the difference in the expected sum of efforts under Pub and Pri. We prove the claim in three steps, and we suggest that the reader sees Figure 1 to follow the upcoming proof. **Step 1:** we show that the function $\pi^{Pub-Pri}$ takes the value 0 for exactly three values of p (0, $\frac{1}{2}$, and 1). **Step 2:** we show that the derivative of $\pi^{Pub-Pri}$ with respect to p in $p = \frac{1}{2}$ is strictly positive. **Step 3:** we show that $\pi^{Pub-Pri}$ is continuous in p. These three results together necessarily lead to the sign of $\pi^{Pub-Pri}$ as in Figure 1, and hence Theorem 2 follows.

Step 1. First, we analyze when $\pi^{Pub-Pri}$ takes the value 0, i.e.,

$$\pi^{Pub-Pri} = p^2[2e_{hh} - 2e_h] + 2p(1-p)[e_{hl} + e_{lh} - e_h - e_l] + (1-p)^2[2e_{ll} - 2e_l] = 0$$

which simplifies to

$$p^2 e_{hh} + (1-p)^2 e_{ll} + p(1-p)[e_{hl} + e_{lh}] - pe_h - (1-p)e_l = 0$$

We now substitute (4) and (5) and obtain

$$p^2\frac{rh}{4} + (1-p)^2\frac{rl}{4} + p(1-p)\frac{r(h+l)h^r l^r}{(h^r + l^r)^2} - p\frac{h+l}{h}e_h - \frac{1-2p}{4}rl = 0$$

or, equivalently,

$$p^2\frac{rh}{4} + p^2\frac{rl}{4} + p(1-p)\frac{r(h+l)h^r l^r}{(h^r + l^r)^2} - p\frac{h+l}{h}e_h = 0$$

A solution is $p = 0$, when the information regime does not play any role because the prior (2) collapses to certainty of high-type. We rearrange terms and obtain

$$pr\frac{h+l}{4} + (1-p)\frac{r(h+l)h^r l^r}{(h^r+l^r)^2} = \frac{h+l}{h}e_h$$

which is equivalent to

$$pe_{hh} + (1-p)e_{hl} = e_h \tag{7}$$

Hence, (7) is a condition for the indifference between Pub and Pri written in terms of the efforts exerted by the high-type only. When $p = 1$, (6) leads to $e_h = e_{hh}$, which solves (7). Hence, $p = 1$ is a second solution of $\pi^{Pub-Pri} = 0$. With a similar procedure used to find (7), we can obtain the value of e_l for which the administrator is indifferent between Pub and Pri, which symmetrically to (7) is

$$(1-p)e_{ll} + pe_{lh} = e_l \tag{8}$$

To see if there are other values of p asides from 0 and 1 leading to indifference between Pub and Pri, we plug (7) and (8) into the top equation of (6) and see if any $p \in (0, 1)$ solves the resulting equation.

First, we use (4) to rewrite the expressions (7) and (8) for e_l and e_h as

$$e_h = rh\frac{p(h^r+l^r)^2 + 4(1-p)h^r l^r}{4(h^r+l^r)^2} \tag{9}$$

$$e_l = rl\frac{(1-p)(h^r+l^r)^2 + 4ph^r l^r}{4(h^r+l^r)^2} \tag{10}$$

These efforts are those that, if exerted in Pri, lead to indifference between Pri and Pub. Now, using (6), we check whether these effort levels are reached for some parameter values. Hence, we rewrite the top equation of (6) as

$$pr\frac{h}{4} + (1-p)r\frac{he_h^r e_l^r}{(e_h^r+e_l^r)^2} = e_h \tag{11}$$

Plugging (9) into (11), we obtain the following simplified expression

$$\frac{e_h^r e_l^r}{(e_h^r+e_l^r)^2} = \frac{h^r l^r}{(h^r+l^r)^2} \tag{12}$$

Finally, we plug (9) and (10) where we defined $a = p(h^r+l^r)^2 + 4(1-p)h^r l^r$ and $b = (1-p)(h^r+l^r)^2 + 4ph^r l^r$ into (12), and obtain

$$\begin{aligned}
\frac{h^r l^r a^r b^r}{(h^r a^r + l^r b^r)^2} &= \frac{h^r l^r}{(h^r+l^r)^2} \\
h^{2r}a^{2r} + l^{2r}b^{2r} + 2h^r l^r a^r b^r &= h^{2r}a^r b^r + l^{2r}a^r b^r + 2h^r l^r a^r b^r \\
h^{2r}a^r(a^r - b^r) &= l^{2r}b^r(a^r - b^r) \qquad (13)
\end{aligned}$$

and the unique solution of (13) is $a = b$, which is equivalent to

$$\begin{aligned}
p(h^r+l^r)^2 + 4(1-p)h^r l^r &= (1-p)(h^r+l^r)^2 + 4ph^r l^r \\
4(1-2p)h^r l^r &= (1-2p)(h^r+l^r)^2
\end{aligned}$$

which leads to a third and last solution in p of $\pi^{Pub-Pri}=0$: $p=\frac{1}{2}$. Similar algebra shows that $p=\frac{1}{2}$, (9) and (10) satisfy also the second equation of (6). Hence, we proved that there are three values of p for which $\pi^{Pub-Pri}=0$: 0, $\frac{1}{2}$, and 1.

Step 2. To show that $\pi^{Pub-Pri}<0$ if $p\in(0,\frac{1}{2})$ and $\pi^{Pub-Pri}>0$ if $p\in(\frac{1}{2},1)$, we write the system (6) as a unique equation in terms of e_h and parameters only, and then we make use of the implicit function theorem to evaluate the derivative of $\pi^{Pub-Pri}$ in $p=\frac{1}{2}$, and prove that it is positive. Remember that efforts under Pub are not a function of p, unlike the efforts under Pri. We omitted this detail so far in the notation, and we now write it for clarity as we need to differentiate with respect to p. We want to show that

$$\left.\frac{\partial \pi^{Pub-Pri}}{\partial p}\right|_{p=\frac{1}{2}}>0$$

from (7) - where we simplified a p -

$$\begin{aligned}
\left.\frac{\partial}{\partial p}\left[p^2 e_{hh}+p(1-p)e_{hl}-pe_h(p)\right]\right|_{p=\frac{1}{2}} &> 0 \\
\left.\left[2pe_{hh}+e_{hl}-2pe_{hl}-e_h(p)-p\frac{\partial e_h(p)}{\partial p}\right]\right|_{p=\frac{1}{2}} &> 0 \qquad (14)\\
2e_{hh}-2e_h\left(\frac{1}{2}\right) &> \left.\frac{\partial e_h(p)}{\partial p}\right|_{p=\frac{1}{2}} \qquad (15)
\end{aligned}$$

When $p=\frac{1}{2}$, we know that $\pi^{Pub-Pri}=0$, and hence from (7), we know that $e_h\left(\frac{1}{2}\right)=\frac{e_{hh}+e_{hl}}{2}$. Therefore, (15) is equivalent to

$$e_{hh}-e_{hl}>\left.\frac{\partial e_h(p)}{\partial p}\right|_{p=\frac{1}{2}} \qquad (16)$$

The left-hand side of (16) is known by (4). The right-hand side is trickier. First, we isolate e_l in (5):

$$\begin{aligned}
(1-p)\frac{e_l}{e_{ll}} &= p\frac{e_h}{e_{hh}}+(1-2p) \\
(1-p)e_l &= p\frac{l}{h}e_h+(1-2p)\frac{rl}{4} \\
4(1-p)he_l &= 4ple_h+(1-2p)rlh \\
e_l &= \frac{4ple_h+(1-2p)rlh}{4(1-p)h} \qquad (17)
\end{aligned}$$

We now use (17) into the top equation of (6), and we obtain

$$f(e_h,p)\equiv p\frac{r}{4e_h}+4^r h^r l^r r\frac{(1-p)^{r+1}e_h^{r-1}[h(1-2p)+4pe_h]^r}{[4^r h^r(1-p)^r e_h^r+(4ple_h+hl(1-2p))^r]^2}-\frac{1}{h}=0$$

Notice that the defined $f(e_h,p)$ is an equation in p and e_h only, and hence by the implicit function theorem

$$\left.\frac{\partial e_h(p)}{\partial p}\right|_{p=\frac{1}{2}}=-\frac{\left.\frac{\partial f(e_h,p)}{\partial p}\right|_{p=\frac{1}{2}}}{\left.\frac{\partial f(e_h,p)}{\partial e_h}\right|_{p=\frac{1}{2}}} \qquad (18)$$

We evaluate the numerator and denominator of (18). We start with the denominator.

$$
\begin{aligned}
\left.\frac{\partial f(e_h,p)}{\partial e_h}\right|_{p=\frac{1}{2}} &= \frac{\partial f(e_h,\frac{1}{2})}{\partial e_h} \\
&= \frac{\partial}{\partial e_h}\left[\frac{r}{8e_h} + 2^{2r}h^r l^r r \frac{2^{-(r+1)}e_h^{r-1}(2e_h)^r}{[2^r h^r e_h^r + 2^r l^r e_h^r]^2}\right]\Bigg| \\
&= \frac{\partial}{\partial e_h}\left[\frac{r}{8e_h} + \frac{h^r l^r r}{2\left(h^r+l^r\right)^2 e_h}\right]\Bigg| \\
&= -\frac{r}{8e_h^2} - \frac{h^r l^r r}{2(h^r+l^r)^2 e_h^2} \\
&= -\frac{r}{e_h^2}\left[\frac{h^{2r}+l^{2r}+6h^r l^r}{8(h^r+l^r)^2}\right] \qquad (19)
\end{aligned}
$$

Note that when $p=\frac{1}{2}$, the solution to (6) is $e_h = rh\frac{h^{2r}+l^{2r}+6h^r l^r}{8(h^r+l^r)^2}$ (see also Fey (2008)). We use this fact into (19) and obtain

$$
\left.\frac{\partial f(e_h,p)}{\partial e_h}\right|_{p=\frac{1}{2}} = -\frac{1}{he_h}
$$

Hence, expression (18) reads

$$
\begin{aligned}
\left.\frac{\partial e_h(p)}{\partial p}\right|_{p=\frac{1}{2}} &= -\frac{\left.\frac{\partial f(e_h,p)}{\partial p}\right|_{p=\frac{1}{2}}}{\left.\frac{\partial f(e_h,p)}{\partial e_h}\right|_{p=\frac{1}{2}}} \\
&= he_h\left(\left.\frac{\partial f(e_h,p)}{\partial p}\right|_{p=\frac{1}{2}}\right) \\
&= \frac{rh}{4} + 4^r h^{r+1} l^r r e_h^r\left[\frac{\partial}{\partial p}\left[\frac{(1-p)^{r+1}[h(1-2p)+4pe_h]^r}{[4^r h^r (1-p)^r e_h^r + (4ple_h + hl(1-2p))^r]^2}\right]\Bigg|_{p=\frac{1}{2}}\right] \\
&= \frac{rh}{4} + 4^r h^{r+1} l^r r e_h^r\left[\frac{\partial}{\partial p}\left[\frac{a(p)b(p)}{[c(p)]^2}\right]\Bigg|_{p=\frac{1}{2}}\right] \qquad (20)
\end{aligned}
$$

where we defined

$$
\begin{aligned}
a(p) &= (1-p)^{r+1} \\
b(p) &= [h(1-2p)+4pe_h]^r \\
c(p) &= 4^r h^r (1-p)^r e_h^r + (4ple_h + hl(1-2p))^r
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\frac{\partial}{\partial p}\left[\frac{a(p)b(p)}{[c(p)]^2}\right]\Bigg|_{p=\frac{1}{2}} &= \left.\frac{[a'(p)b(p)+a(p)b'(p)]\,[c(p)]^2 - 2a(p)b(p)c(p)c'(p)}{[c(p)]^4}\right|_{p=\frac{1}{2}} \\
&= \left.\frac{a'(p)b(p)+a(p)b'(p)}{[c(p)]^2}\right|_{p=\frac{1}{2}} - \left.\frac{2a(p)b(p)c'(p)}{[c(p)]^3}\right|_{p=\frac{1}{2}} \qquad (21)
\end{aligned}
$$

We obtain

$$
\begin{aligned}
a\left(\frac{1}{2}\right) &= \frac{1}{2^{r+1}} \\
b\left(\frac{1}{2}\right) &= 2^r e_h^r \\
c\left(\frac{1}{2}\right) &= 2^r e_h^r (h^r + l^r)
\end{aligned}
$$

and

$$
\begin{aligned}
a'\left(\frac{1}{2}\right) &= -\frac{r+1}{2^r} \\
b'\left(\frac{1}{2}\right) &= 2^{r+1} r e_h^{r-1} [e_h - h/2] \\
c'\left(\frac{1}{2}\right) &= -2^{r+1} h^r r e_h^r + 2^{r+1} l^r r e_h^{r-1} (e_h - h/2)
\end{aligned}
$$

We plug these results into (21) to write (20) in the following way

$$
\begin{aligned}
\left.\frac{\partial e_h(p)}{\partial p}\right|_{p=\frac{1}{2}} &= \frac{rh}{4} - 4^r h^{r+1} l^r r e_h^r \left[\frac{e_h^r (r+1) - r e_h^{r-1}(e_h - h/2)}{\left[2^r e_h^r (h^r + l^r)\right]^2} + \frac{r e_h^r 2^{r+1}\left[-h^r e_h^r + l^r e_h^{r-1}(e_h - h/2)\right]}{\left[2^r e_h^r (h^r + l^r)\right]^3}\right] \\
&= \frac{rh}{4} - h^{r+1} l^r r \frac{1 + r e_h^{-1}(h/2)}{\left[(h^r + l^r)\right]^2} - 2h^{r+1} l^r r^2 \frac{-h^r + l^r - l^r e_h^{-1}(h/2)}{(h^r + l^r)^3} \\
&= \frac{rh}{4} + h^{r+2} l^r r^2 \frac{2l^r - (h^r + l^r)}{2e_h (h^r + l^r)^3} - h^{r+1} l^r r \frac{l^r(2r+1) + h^r(1-2r)}{(h^r + l^r)^3} \\
&= \frac{rh}{4} + 4h^{r+1} l^r r \frac{l^r - h^r}{(h^r + l^r)\left[(h^r + l^r)^2 + 4h^r l^r\right]} - h^{r+1} l^r r \frac{l^r(2r+1) + h^r(1-2r)}{(h^r + l^r)^3}
\end{aligned}
$$

in which we used $e_h = rh\frac{h^{2r} + l^{2r} + 6h^r l^r}{8(h^r + l^r)^2}$ in the last step.
Therefore, we can finally evaluate expression (16):

$$
\begin{aligned}
\frac{rh}{4} - rh\frac{h^r l^r}{(h^r + l^r)^2} &> \frac{rh}{4} + 4h^{r+1} l^r r \frac{l^r - h^r}{(h^r + l^r)\left[(h^r + l^r)^2 + 4h^r l^r\right]} + \\
&\quad - h^{r+1} l^r r \frac{l^r(2r+1) + h^r(1-2r)}{(h^r + l^r)^3} \\
4\frac{l^r - h^r}{\left[(h^r + l^r)^2 + 4h^r l^r\right]} &< \frac{l^r(2r+1) + h^r(1-2r) - h^r - l^r}{(h^r + l^r)^2} \\
4\frac{l^r - h^r}{\left[(h^r + l^r)^2 + 4h^r l^r\right]} &< 2r\frac{l^r - h^r}{(h^r + l^r)^2} \\
2(h^r + l^r)^2 &> r\left[(h^r + l^r)^2 + 4h^r l^r\right]
\end{aligned}
$$

By $r \leq 1$, it suffices to show that

$$
\begin{aligned}
2(h^r + l^r)^2 &> (h^r + l^r)^2 + 4h^r l^r \\
(h^r - l^r)^2 &> 0
\end{aligned}
$$

and the result follows.

Step 3. The continuity of e_h and e_l in p directly follows from the Maximum Theorem applied to (3), noting that (3) is strictly concave and continuous in e_i. The continuity of $\pi^{Pub-Pri}$ follows from the continuity of e_h and e_l.

A.3 Proof of Proposition 3

For Proposition 3, we provide the following Lemmata. The primary difference here with respect to the framework of Section 3 is that instead of a unique p, the information regime induces a p_h and a p_l, where the first (second) is the belief of a high (low) type that she is against another high (low) type.

Lemma 5 *For any quintuple $\{h, l, r, p_h, p_l\}$ with $h > l > 0$, $r \in [0, 1]$, and $p_h, p_l \in [0, 1]$, it holds in equilibrium that $e_h > e_l$.*

Proof of Lemma 5. Instead of (6), the system of necessary and sufficient FOCs is now

$$\begin{cases} p_h A + (1 - p_h)B = \frac{1}{h} \\ p_l C + (1 - p_l)D = \frac{1}{l} \end{cases} \tag{22}$$

where we define

$$A \equiv \frac{r}{4e_h},\ B \equiv \frac{r e_l^r e_h^{r-1}}{(e_h^r + e_l^r)^2},\ C \equiv \frac{r}{4e_l},\ D \equiv \frac{r e_h^r e_l^{r-1}}{(e_h^r + e_l^r)^2}$$

Note that (22) generalizes (6) for $p_l = 1 - p$ and $p_h = p$.

Now, assume by contradiction that $e_h \leq e_l$. Then, it is routine to show that $A \geq B$, $A \geq C$, $B \geq C$ and $B \geq D$. Since $\frac{1}{h} < \frac{1}{l}$, (22) implies $p_h A + (1 - p_h)B < p_l C + (1 - p_l)D$, which is impossible if $B \geq C$. Hence, $B < C$ or, equivalently,

$$\begin{aligned} \frac{e_l^r e_h^{r-1}}{(e_h^r + e_l^r)^2} &< \frac{1}{4e_l} \\ 4e_l^{r+1} e_h^{r-1} &< e_h^{2r} + e_l^{2r} + 2e_h^r e_l^r \end{aligned} \tag{23}$$

By $e_h \leq e_l$, we know that $e_l^{r-1} e_h^{1-r} \leq 1$, and hence a necessary condition for (23) is that

$$\begin{aligned} 4e_l^{r+1} e_h^{r-1} e_l^{r-1} e_h^{1-r} &< e_h^{2r} + e_l^{2r} + 2e_h^r e_l^r \\ 4e_l^{2r} &< e_h^{2r} + e_l^{2r} + 2e_h^r e_l^r \\ 3e_l^{2r} - 2e_h^r e_l^r - e_h^{2r} &< 0 \end{aligned} \tag{24}$$

(24) is a convex parabola whose roots are $e_l^r = e_h^r$ and $e_l^r = -\frac{e_h^r}{3}$, and hence it holds true iff $e_h > e_l$. This condition contradicts the assumption. The result follows. ■

Lemma 6 *For any quintuple $\{h, l, r, p_h, p_l\}$ with $h > l > 0$, $r \in [0, 1]$, and $p_h, p_l \in [0, 1]$, it holds in equilibrium that $e_h \leq \frac{rh}{4}$ and $e_l \leq \frac{rl}{4}$.*

Proof of Lemma 6. Consider the first equation of (22): a convex combination between A and B with weights p_h and $1 - p_h$ equals a constant. If $p_h = 1$, $e_h = \frac{rh}{4}$. Consider now a lower $p_h (< 1)$. Then, by $A > B$, the convex combination decreases. To keep it equal to the constant, e_h must decrease because both A and B are decreasing functions of e_h. Hence, $e_h \leq \frac{rh}{4}$. The proof of $e_l \leq \frac{rl}{4}$ is symmetric. ■

We are now ready to prove Proposition 3.

Proof of Part iv) $\partial e_l/\partial p_h < 0$. Similar steps to those used in the proof of Proposition 1 to get (5) from (6) can be used in (22) to obtain

$$\frac{e_l \frac{4}{rl} - p_l}{1 - p_l} = \frac{e_h \frac{4}{rh} - p_h}{1 - p_h} \tag{25}$$

Note that by Lemma 6 the rhs of (25) decreases in p_h if we ignore the dependency of e_h on p_h. Assume by contradiction that $\partial e_l/\partial p_h \geq 0$. Then, lhs increases in p_h, and hence so does the rhs. Therefore, it has to be that $\partial e_h/\partial p_h \geq 0$. The lhs of the second equation in (22) decreases in both e_h and e_l by Lemma 5, hence $\partial e_l/\partial p_h \geq 0$ and $\partial e_h/\partial p_h \geq 0$ lead to a contradiction. Therefore, $\partial e_l/\partial p_h < 0$. ■

Proof of Part i) $\partial e_h/\partial p_h > 0$. Using again the fact that the lhs of the second equation in (22) decreases in both e_h and e_l, an increase in p_h decreases e_l (as just proved), and hence to keep the lhs constant, we need e_h to increase. ■

Proof of Part ii) $\partial e_l/\partial p_l > 0$. We use e_h' to denote $\partial e_h/\partial p_l$, and e_l' to denote $\partial e_l/\partial p_l$. The lhs of the first equation of (22) increases in e_l and decreases in e_h, hence $e_l' > 0$ iff $e_h' > 0$. We now prove that $e_l' \leq 0$ and $e_h' \leq 0$ lead to a contradiction. We do so by differentiating the second equation in (22) with respect to p_l. (for the differential of $(1-p_h)B$ we apply the same formula of (21))

$$\begin{aligned} &\frac{e_l - p_l e_l'}{4e_l^2} + \frac{-e_h^r e_l^{r-1} + (1-p_l)\left(r e_h^{r-1} e_l^{r-1} e_h' + (r-1) e_h^r e_l^{r-2} e_l'\right)}{\left(e_h^r + e_l^r\right)^2} - \frac{2(1-p_l) r e_h^r e_l^{r-1}\left[e_h^{r-1} e_h' + e_l^{r-1} e_l'\right]}{\left(e_h^r + e_l^r\right)^3} = 0 \\ &\frac{1}{4e_l} - \frac{p_l e_l'}{4e_l^2} - \frac{e_h^r e_l^{r-1}}{\left(e_h^r + e_l^r\right)^2} + (1-p_l)\frac{\left[r e_h^{r-1} e_l^{r-1} e_h' + (r-1) e_h^r e_l^{r-2} e_l'\right]\left(e_h^r + e_l^r\right) - 2r e_h^r e_l^{r-1}\left[e_h^{r-1} e_h' + e_l^{r-1} e_l'\right]}{\left(e_h^r + e_l^r\right)^3} = 0 \end{aligned} \tag{26}$$

In (26), the second term is positive by $e_l' \leq 0$, and the sum of the first and third terms is strictly positive; hence, proving that the numerator of the fourth term is positive suffices to obtain a contradiction. We do so by factoring out e_h' and e_l'.

$$\begin{aligned} e_h'\left[r e_h^{2r-1} e_l^{r-1} + r e_h^{r-1} e_l^{2r-1} - 2r e_h^{2r-1} e_l^{r-1}\right] + e_l'\left[(r-1) e_h^{2r} e_l^{r-2} + (r-1) e_h^r e_l^{2r-2} - 2r e_h^r e_l^{2r-2}\right] \geq 0 \\ e_h'\left[r e_l^r - r e_h^r\right] + e_l'\left[(r-1) e_h^{2r} e_l^{r-2} + (r-1) e_h^r e_l^{2r-2} - 2r e_h^r e_l^{2r-2}\right] \geq 0 \end{aligned} \tag{27}$$

where the first square brackets of (27) is negative by Lemma 5, and the second square-brackets is negative because each term is negative. By $e_h' \leq 0$ and $e_l' \leq 0$, (27) holds, and hence the contradiction in (26) is reached. The result follows. ■

Proof of Part iii) $\partial e_h/\partial p_l > 0$. It follows by $\partial e_l/\partial p_l > 0$ and the fact that $\partial e_l/\partial p_l > 0$ iff $\partial e_l/\partial p_l > 0$ as proved in the proof of Part ii) above. ■

A.4 Proof of Theorem 4

First, note that $\mathcal{R} \in \{\{Pri, Pub, Pub\}, \{Pub, Pri, Pub\}, \{Pub, Pub, Pri\}, \{Pri, Pub, Pri\}\}$ are outcome-equivalent to $\mathcal{R} = \{Pub, Pub, Pub\}$, because contestants perfectly infer types despite Pri. Hence, it suffices to prove that the expected sum of efforts under $\{Pub, Pub, Pub\}$ is greater than under $\{Pri, Pri, Pub\}$ (**Lemma 7**), and then that under $\{Pub, Pri, Pri\}$ it is greater than under $\{Pub, Pub, Pub\}$ (**Lemma 8**) or under $\{Pri, Pri, Pri\}$ (**Lemma 9**). The proof of each lemma in this section works similarly: first, we simplify the difference in the expected sum of efforts under the two information regimes using (25) with the appropriate p_l and p_h; second, we use the results in Proposition 3 to conclude the proof. For completeness, we present here all of these proofs. We denote by $\pi^{\mathcal{R}}$ the expected sum of efforts under regime $\mathcal{R}$.

Lemma 7 $\pi^{\{Pub,Pub,Pub\}} - \pi^{\{Pri,Pri,Pub\}} > 0$

Proof. We denote by e_h and e_l the efforts under Pri and regime $\{Pri, Pri, Pub\}$. The claim is equivalent to

$$\begin{aligned} p^2(2e_{hh} - 2e_h) + 2p(1-p)(e_{hl} + e_{lh} - e_h - e_l) &> 0 \\ pe_{hh} + (1-p)(e_{hl} + e_{lh}) - (1-p)e_l - e_h &> 0 \end{aligned}$$

We use (4) for e_{hh}, e_{hl} and e_{lh}, and we obtain

$$p\frac{rh}{4} + (1-p)r\frac{h^r l^r (h+l)}{(h^r + l^r)^2} - (1-p)e_l - e_h > 0$$

We use (25) with $p_l = 0$ and $p_h = p$ to eliminate e_h, and we obtain

$$\begin{aligned} p\frac{rh}{4} + (1-p)r\frac{h^r l^r (h+l)}{(h^r + l^r)^2} - (1-p)\frac{h+l}{l}e_l - p\frac{rh}{4} &> 0 \\ r\frac{h^r l^r}{(h^r + l^r)^2} - \frac{e_l}{l} &> 0 \\ e_{lh} &> e_l \end{aligned} \tag{28}$$

Now, (28) follows from $e_{lh} = e_l^{\{Pub,Pri,Pub\}} > e_l^{\{Pri,Pri,Pub\}}$ by Proposition 3 (moving from $\{Pub, Pri, Pub\}$ to $\{Pri, Pri, Pub\}$ leads p_h to increase and p_l to remain constant). ■

Lemma 8 $\pi^{\{Pub,Pri,Pri\}} - \pi^{\{Pub,Pub,Pub\}} > 0$

Proof. We denote by e_h and e_l the efforts under Pri and regime $\{Pub, Pri, Pri\}$. The claim is equivalent to

$$\begin{aligned} 2p(1-p)(e_h + e_l - e_{hl} - e_{lh}) + (1-p)^2(2e_l - 2e_{ll}) &> 0 \\ e_l + pe_h - p(e_{hl} + e_{lh}) - (1-p)e_{ll} &> 0 \end{aligned}$$

We use (4) for e_{ll}, e_{hl} and e_{lh}, and we obtain

$$e_l + pe_h - pr\frac{h^r l^r (h+l)}{(h^r + l^r)^2} - (1-p)\frac{rl}{4} > 0$$

We use (25) with $p_l = 1-p$ and $p_h = 0$ to eliminate e_l, and we obtain

$$\begin{aligned} p\frac{h+l}{h}e_h + (1-p)\frac{rl}{4} - pr\frac{h^r l^r (h+l)}{(h^r + l^r)^2} - (1-p)\frac{rl}{4} &> 0 \\ \frac{e_h}{h} - r\frac{h^r l^r}{(h^r + l^r)^2} &> 0 \\ e_h &> e_{hl} \end{aligned} \tag{29}$$

Now, (29) follows from $e_{hl} = e_h^{\{Pub,Pri,Pub\}} < e_h^{\{Pub,Pri,Pri\}}$ by Proposition 3 (moving from $\{Pub, Pri, Pub\}$ to $\{Pub, Pri, Pri\}$ leads p_h to remain constant and p_l to increase). ■

Lemma 9 $\pi^{\{Pub,Pri,Pri\}} - \pi^{\{Pri,Pri,Pri\}} > 0$

Proof. Now both regimes include some Pri's, and hence there are two efforts under Pri for each type according to the regime. We denote by $\hat{e}_h$ and $\hat{e}_l$ the efforts under Pri and regime $\{Pub, Pri, Pri\}$ and by $\bar{e}_h$ and $\bar{e}_l$ the efforts under Pri and regime $\{Pri, Pri, Pri\}$. The claim is equivalent to

$$\begin{aligned} p^2(2e_{hh} - 2\bar{e}_h) + 2p(1-p)(\hat{e}_h + \hat{e}_l - \bar{e}_h - \bar{e}_l) + (1-p)^2(2\hat{e}_l - 2\bar{e}_l) &> 0 \\ p^2 e_{hh} + p(1-p)\hat{e}_h - p\bar{e}_h + (1-p)\hat{e}_l - (1-p)\bar{e}_l &> 0 \end{aligned}$$

We use (4) for e_{hh}, and we obtain

$$p^2\frac{rh}{4} + p(1-p)\hat{e}_h - p\bar{e}_h + (1-p)\hat{e}_l - (1-p)\bar{e}_l > 0$$

We use (25) twice with $p_l = 1-p$ and $p_h = 0$ to eliminate $\hat{e}_h$ and with $p_l = 1-p$ and $p_h = p$ to eliminate $\bar{e}_h$, and we obtain

$$\begin{aligned} p^2\frac{rh}{4} + (1-p)\frac{h+l}{l}\hat{e}_l - (1-p)^2\frac{rh}{4} - (1-p)\frac{h+l}{l}\bar{e}_l + (1-2p)\frac{rh}{4} &> 0 \\ \hat{e}_l &> \bar{e}_l \end{aligned} \tag{30}$$

Now, (30) follows directly from Proposition 3 (moving from $\{Pub, Pri, Pri\}$ to $\{Pri, Pri, Pri\}$ leads p_h to increase and p_l to remain constant). ■

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