Preference Uncertainty and Conflict of Interest in Committees*

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– *Preliminary* –

Abstract

A committee of agents with interdependent values votes on whether to accept an alternative or stick to the status quo. Agents hold two-dimensional private information: about a quality criterion of the alternative, and about their individual preference type. In equilibrium committee members adopt cutoff strategies, and an agent's preference type is reflected in his acceptance standard: More extreme types adopt more stringent acceptance standards and act less strategic. Agents lower their acceptance standard if they believe to face a more partisan type. By contrast, more preference uncertainty will encourage an agent to raise his acceptance standard.

Keywords: voting, interdependent values, multi-dimensional private information, preference uncertainty

JEL Classifications: C72, D72, D82

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1 Introduction

In most organizations it has become the prevailing practice that complex decisions are not made by individuals but by committees. Typically, this is done by a more or less elaborate voting procedure. Corporate boards decide how to invest, whom to hire and whether or not to adopt a new technology. Examples include decisions about the allocation of research grants, the approval of new drugs by the FDA for the market and academic hiring. By the complexity of matters which are put to vote committee members usually cannot assess all information about the alternatives. Rather, committee members receive private noisy signals. For example, jurors obtain this signal from listening to the evidence presented in a trial whereas in the hiring process signals arise from evaluating application materials and interviews.

It is often the case that committee members have two types of private information; they possess information not only about the state of the world, which is payoff relevant to all agents, but also about their *preference type*, which determines how they aggregate available information into preferences. This means that, even if all private signals were publicly revealed, it would still remain private information to the agents how they translate signals about the state of the world into preferences – there is *preference uncertainty*. The preference types of jurors could, for example, reflect their confidence level about their abilities to assess the evidence presented in a trial. In a committee of specialist who each have private information about the quality of the proposal in the dimension of their own specialty, the preference types reflect the agents' levels of partisanship.

In this paper we study a committee setting in which agents have interdependent preferences and hold two-dimensional private information: They have differential information about the payoff relevant state of the world and, moreover, there is preference uncertainty. We present a model which captures these features. Our goal is to understand the interaction between the two types of private information and how preference uncertainty, individual preferences types and beliefs about them affect committee decisions, in particular equilibrium acceptance sets.

A group of agents faces a binary decision, whether to implement an alternative or stick to the status quo. The decision is made by generalized majority voting. That is, each agent can indicate whether he accepts the proposal, and the majority rule determines the minimal number of votes required to adopt the alternative. Prior to voting each agent obtains a private signal about the quality of the alternative.[1] Committee members have interdependent values. To be precise, the payoff of the status quo is zero for all agents, and the payoff obtained from adopting an alternative is a convex combination over the private signals of all agents, in which agents put most weight on their own signal. The private preference type of an agent reflects the extent to which he favors his own private signal.

In this setting, we analyze voting behavior of agents and the resulting acceptance sets. Our main contributions include establishing the existence of a pure strategy Nash equilibrium in undominated strategies, and characterizing properties of equilibrium strategies. As is typical for voting models with continuous signals, in equilibrium agents adopt cutoff strategies.[2] That is, an agent accepts an alternative whenever his private signal is above a certain threshold. We find that an agent's private type is reflected in the cutoff he adopts: more extreme types act less strategic. For example, under unanimity voting, agents with higher preference types adopt more stringent acceptance standards. We conclude the equilibrium characterization by establishing, under some common distributional assumption, equilibrium uniqueness for the unanimity rule.

Next, we exploit the flexibility of the new committee voting model introduced in this paper, to address the question how private preference types and beliefs about them affect committee decision. The results are illustrated for a two-member committee. We study comparative statics with respect to shifts in the distribution of agents' types.[3] If consensus is required to adopt the alternative, we find that, if an agent believes to face a more confident or partisan committee member[4] he will lower his acceptance standard while the other committee member reacts by increasing his. A further contribution of

[1] A random draw from a continuous signal space.

[2] See Feddersen and Pesendorfer (1997), Duggan and Martinelli (2001), Li and Suen (2009).

[3] To be precise, shifts in the distribution of types in terms of first and second order stochastic dominance.

[4] In the sense of a first order stochastic dominance shift in the distribution of agents' types.

this paper is to characterize the effect of more preference uncertainty on the equilibrium strategies of committee members. One might think that more preference uncertainty would lead an agent to be more cautious and adopt a more lenient acceptance standard. It may be somewhat surprising that the opposite occurs: more preference uncertainty causes an agent to increase his acceptance standard, essentially focusing more on his own private signal. This result illustrates a special feature of the private information about the preference type. For more preference uncertainty, defined as a mean-preserving spread of the distribution of preference types of the other committee member, all agents adjust their equilibrium strategies. This is true even though for the decision of an agent only the expectation over all preference types of the other committee member is relevant. By contrast, for a risk-neutral agent for whom only the expectation over all payoff-relevant signals is relevant for his decision, a mean-preserving spread of the distribution of states does not affect the agent's best response.

Some further comments on our payoff structure: It captures the natural assumption that agents pay attention to the aspect of the alternative which is most important for them, but are aware that the signals held by the other committee members also contain relevant information. This implies that agents have interdependent, but typically not purely common, values. Moreover, even if all signals about the proposal were publicly observable, it is often private information to the agents how they aggregate this information. In our model this is captured by the preference type, which indicates how an agent weighs his own signal compared to the signals held by other agents, that is, how biased the agent is towards his own information. This brings more precision to the interpretation presented above that preference types reflect, for example, the confidence of jurors or the partisanship levels of specialists. The extent of the confidence or partisanship level is intrinsic in nature, can be regarded as part of the personality of an individual and is thus his private information. Consequently, there is conflict of interest among committee members, but uncertainty about the extent of the conflict.

Related Literature An early observation made by Condorcet (1785) is that, by pooling the information of their members, groups may take better decisions than individuals. This statement, known as the *Condorcet Jury Theorem*, was initially formulated for non-strategic voters and thus a purely statistical result. Starting from this insight, there is an extensive literature on collective decision making, now typically focusing on strategic voters who update their beliefs about the information held by other agents conditional on the event of being pivotal. Li and Suen (2009) provide a good survey.

Most of the theoretical voting models study settings in which individuals share a common interest. That is, committee members would agree on the best outcome if they knew the state of the world. Often an even stronger assumption is made, namely, that agents have perfectly aligned preferences. This assumption implies that there is an underlying consensus: agents would agree on the best action if there were no asymmetric information, that is, if all private information were publicly available. Li et al. (2001) relax the second assumption, but still assume that agents share a common objective. If there is uncertainty about the state of the world there may be conflict of interest, but disagreement vanishes if all uncertainty is resolved. The authors discuss how the level of conflict among committee members affects their incentives to strategically misrepresent their information and thus may hinder information aggregation. In the model by Li et al. (2001) and related papers,[5] agents are heterogeneous in the sense that they require different levels of evidence to prefer the alternative over the status quo. This specification of heterogeneity implies that between any pair of agents there is only one direction of disagreement, which is determined by the different evidence levels they require to favor the alternative. By contrast, in the interdependent values model we consider, the direction of conflict is not given and there exist different types of disagreement. A more detailed discussion of the relation between these models is provided at the end of Section 2.

Two recent related papers that also study settings in which committee members have interdependent values are Moldovanu and Shi (2013) and Yildirim (2012).[6] Yildirim (2012) identifies time-consistent majority rules, that is, majority rules which a designer can implement if he cannot commit to a rule prior to observing the votes. Moldovanu and Shi (2013) consider

[5]e.g. Austen-Smith and Feddersen (2006) and Li and Suen (2009).

[6]Further interdependent values voting models are Grüner and Kiel (2004) and Rosar (2012). They consider a different (quadratic) functional form of utilities.

an infinite horizon search model where the decision to stop is made by a committee by unanimity voting. They characterize a stationary equilibrium in cutoff-strategies, and discuss how the level of conflict among committee members' preferences affects acceptance sets and welfare. The preference structure adopted in both papers, Yildirim (2012) and Moldovanu and Shi (2013), is similar to ours in that agents have interdependent, additively separable utilities. To be precise, agents' utilities are a convex combination over private signals of all agents, and all committee members possess the same bias-level or preference type.[7] Our model departs from these assumptions in two aspects: First, we allow agents to have individual preference types, and second these are private information to the agents, that is, there is preference uncertainty.[8] Our model is therefore more flexible than the existing models in the literature. This allows to address new questions, for example how individual preference types and the composition of a committee may affect group decisions. In this paper, we focus on the effects of beliefs about preference types on equilibrium acceptance sets.

The rest of the paper is structured as follows. The model is introduced in Section 2 and a more detailed discussion of its special features is provided. Section 3 and Section 4 contain our main results. In Section 3 we establish equilibrium existence and characterize fundamental properties of equilibrium strategies. This is followed by a discussion of equilibrium uniqueness under unanimity voting. Our results of the effects of preference uncertainty on equilibrium strategies and acceptance sets are presented in Section 4. In Section 5 we discuss the benchmark case in which preference types are common knowledge. Section 6 concludes. All proofs are relegated to the appendix.

2 The Model

Consider a committee of n agents, $\mathcal{I} = \{1, \dots, n\}$, who take a binary decision, for example, whether to stick to the status quo or accept an alternative. The payoff of the status quo is 0 for all members. An alternative (proposal)

[7]Meyer and Strulovici (2013) extend some of the results of Moldovanu and Shi (2013) to more general preference structures.

[8]The benchmark in which agents have individual preference types which are common knowledge is discussed in Section 5.

is characterized by an n-dimensional vector $x = (x_1, \ldots, x_n) \in \mathcal{X} \subseteq \mathbb{R}^n$ where $\mathcal{X} = \times_{i=1}^n \mathcal{X}_i$ is a closed compact set in $\mathbb{R}^n$. Let $\mathcal{X}_{-i} := \underset{j \neq i}{\times} \mathcal{X}_j$ and $x_{-i} := (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$. We refer to $x_1, \ldots, x_n$ as the *attribute values* of x. Attribute values x_i, $i \in \mathcal{I}$ are determined by independent random draws from $\mathcal{X}_i$ with distribution function F_i which are twice continuously differentiable. The realization x_i is private information to agent i, the distributions F_i of the respective random variables X_i are common knowledge.

For agent i, the payoff of alternative x is

$$v_i(\theta_i, x) = \theta_i x_i + (1 - \theta_i) \underbrace{\frac{1}{n-1} \sum_{j \neq i} x_j}_{=: \bar{x}_{-i}}, \tag{1}$$

where $\theta_i \in \Theta_i \subseteq [0, 1]$ reflects agent i's *preference type*. We assume that $E(X_i) = 0$ for every $i \in \mathcal{I}$. This implies that before agents observe their private signal x_i about the proposal, they neither favor the status quo nor the alternative. It is common knowledge that the payoff structure has the form of Equation 1. Agents' types are independently distributed on Θ_i with distributions G_i and densities $g_i > 0$. Preference type θ_i is private information to agent i, the distribution of types is common knowledge.

We assume that agents' valuations satisfy the single-crossing property with respect to agents' private signals x_i:

Assumption 1 ((SC))**.** *For all $i, j \in \mathcal{I}$, $j \neq i$:*

$$\frac{\partial v_i}{\partial x_i}(\theta_i, x) \geq \frac{\partial v_j}{\partial x_i}(\theta_j, x) \quad \forall x \in \mathcal{X}.$$

For the parametric form of Equation 1 this equivalent to $\theta_i \geq \frac{1}{n-1}(1 - \theta_j)$, for all $(\theta_i, \theta_j) \in \Theta_i \times \Theta_j$, $j \neq i$. It follows that $\Theta_i \subseteq \left[\frac{1}{n}, 1\right]$ for every $i \in \mathcal{I}$. Note that, if $\theta_i = 1$, agent i has private values, whereas $\theta_i = \frac{1}{n}$ for all $i = 1, \ldots, n$ corresponds to the pure common values case.

Given our model specifications, agents hold two-dimensional private information; (x_i, θ_i) is private information to agent i.

To interpret the model, one can think of the attribute values representing the quality of the proposal x in different dimensions, or simply as the private signals observed by the agents. For example, they could represent the private signals committee members obtain from evaluating the application material of a job-candidate or the private information jury members acquire by following a trial. An agent's preference type could be interpreted to reflect his level of partisanship, altruism or confidence.[9] Higher type agents are more confident or partisan (less altruistic) than lower type agents.

Consider for example a committee of specialists. In this case, the payoff structure can be interpreted as follows: Agents are specialists in different fields and can only assess the quality of the proposal in their own area of expertise. Specialists are biased towards their own specialization but acknowledge that there may be spill-over effects. This results in agents having interdependent values. The realization of the levels of partisanship θ_i are private information.

Under Assumption 1 the specialization of each agent is common knowledge and hence the direction of his partisanship. In the jury model, Assumption 1 implies that an agent is more confident about his own signal than about any other signal.[10]

Decision Rule and Equilibrium Concept.
We assume that the committee decision is rendered by generalized majority voting where the majority rule is characterized by a integer $k \in \{1, \ldots, n\}$. The majority rule k is publicly announced, agents indicate whether they want to accept or reject the alternative and the alternative is adopted if and only if there are at least k affirmative votes.

To avoid trivial equilibria we employ the concept of undominated Nash equilibrium, that is, we restrict attention to Nash equilibria in which no agent uses a weakly dominated strategy.[11] This is standard in the voting literature and as Feddersen and Pesendorfer (1997) we refer to it as a *voting equilibrium.*

Important Features of the Model. In this paper we present a very flexible model which relaxes some of the common assumptions made in the voting

[9] E.g. how confident an agent is about his ability to evaluate the evidence presented in a trial, or the level of partisanship of a specialist towards his own area of expertise.

[10] Similarly, agents may be altruistic but not to an extent of being selfless.

[11] This eliminates trivial equilibria where all agents play extreme strategies, i.e., always accept respectively, always reject the project.

literature. Our framework allows us to study new questions, and in particular to better understand the role of private preference types and preference uncertainty in voting models. The model we suggest has some important and distinctive features, which we will now discuss in some more detail. We assume neither that agents have identical preferences, and differences in opinion arise only from differential information, nor that there is an underlying consensus. In the latter case, agents would agree on the best outcome if the state of the world were known. Consequently, our model differs from the models adopted in most of the voting literature, even if there is no preference uncertainty, that is, if the preference types are common knowledge.[12] Li et al. (2001) and related papers keep the assumption of an underlying consensus among agents, but allow for preference heterogeneity. They assume that, if there is uncertainty about the state of the world, conflict among committee members arises from different preferences of agents for type-I and type-II errors, which results in different evidence levels required to accept the alternative. In this model of preference heterogeneity, the direction of conflict is always the same: If a pair of agents disagrees, it is always the same agent who supports the alternative whilst the other favors staying with the status quo. By contrast, in our model, if agents disagree it is not always the same agent who votes for the alternative. For any two agents A and B, there exist states of the world such that if the state were known agent A would want to accept the alternative, while agent B would prefer to stick to the status quo, and states in which this relation is reversed.[13] The the two types of conflict are illustrated in Figure 1.

3 Equilibrium Characterization

In this section we provide a discussion of equilibria in out model. We first establish existence of a pure strategy equilibrium, to then characterize fundamental properties of the equilibrium strategies.

We consider a binary decision problem in which agents indicate whether they want to accept an alternative (proposal) or stay with the status quo. A

[12] The models in Moldovanu and Shi (2013) and Yildirim (2012) are notable exceptions and if preference types are common knowledge special cases of the model we study.

[13] To be precise this is not true if $\theta_A = \theta_B = \frac{1}{n}$ in which case, agent A and B have perfectly aligned preferences. Not however, that this is a non-generic case.

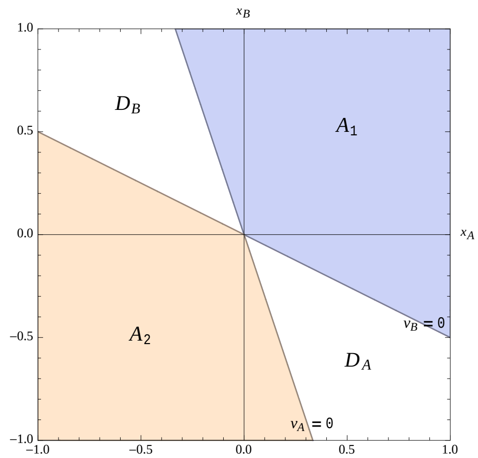

Figure 1: Agreement and disagreement sets in our model for $(\theta_A,\, \theta_B) = \left(\frac{1}{2},\, \frac{1}{3}\right)$, if the state is known.
Agreement sets: A_1 – agents prefer the alternative; A_2 – agents prefer the status quo.
Disagreement sets: D_A – agent A prefers the alternative, agent B favors status quo;
D_B – agent B prefers the alternative, agent A favors the status quo.

mixed strategy for agent i is a measurable function:

$$\begin{aligned} \sigma_i :\ \Theta_i \times \mathcal{X}_i &\to [0,\, 1] \\ (\theta_i,\, x_i) &\mapsto \sigma_i\,(\theta_i,\, x_i)\,, \end{aligned}$$

where $\sigma_i(\theta_i,\, x_i)$ is the probability that agent i votes affirmatively (i.e. in favor of the alternative) if his type is θ_i and his private signal is x_i. Strategy σ_i is pure if $\sigma_i(\theta_i,\, x_i) \in \{0,\, 1\}$, for every $(\theta_i,\, x_i) \in \Theta_i \times \mathcal{X}_i$. It will sometimes be convenient to consider the strategy of a given type, θ_i, of agent i. With a slight abuse of notation we will denote the strategy of agent i with type θ_i by σ_{θ_i}, where $\sigma_{\theta_i}(x_i) := \sigma_i(\theta_i,\, x_i)$.

In the binary decision problem we consider, a pure strategy for an agent characterizes for each of his preference types θ_i a corresponding *acceptance set* $A_i^+(\theta_i) := \sigma_{\theta_i}^{-1}(1) \subseteq \mathcal{X}_i$, which is the set of signals $x_i \in \mathcal{X}_i$ that will induce the agent to vote affirmatively. A strategy of agent i thus characterizes a set of acceptance sets $\{A_i^+(\theta_i)\}_{\theta_i \in \Theta_i}$.

Let $\overline{\mathcal{X}}_i = \mathcal{X}_i \cup \{\widetilde{x}\}$ be the space obtained by adjoining a point $\widetilde{x}$ to $\mathcal{X}_i$ at the upper boundary of $\mathcal{X}_i$.[14] A strategy σ_i of agent i is a *cutoff-strategy* if for

[14]Formally, consider $\widetilde{x}$ as a duplicate of $\overline{x}$ and $\overline{\mathcal{X}}_i = \mathcal{X}_i \cup \{\widetilde{x}\}$ as the space equipped with the following topology: Let the set of open sets $\mathcal{O}$ consist of all subsets $O \subseteq \overline{\mathcal{X}}_i$ such

every θ_i there exists some $\chi_i(\theta_i) \in \overline{\mathcal{X}}_i$ such that type θ_i votes affirmatively if and only if he observes a signal $x_i \geq \chi_i(\theta_i)$. Here $\chi_i(\theta_i) = \widetilde{x}$ represents the case in which agent i always rejects the proposal, irrespective of his private signal. If agents adopt cutoff strategies, the acceptance sets $A_i^+(\theta_i)$ are intervals of the form $[\chi_i(\theta_i),\, \overline{x}]$.[15]

In a voting game, rational agents condition their decision on the event of being pivotal. That is, they take into account the information they can extract from the event of being pivotal. Consider agent i and suppose all other agents adopt strategy profile σ_{-i}. Then, for agent i with private information $(\theta_i,\, x_i)$, the expected payoff from implementing the alternative, conditional on being pivotal is:

$$V_i\left((\theta_i,\, x_i); \sigma_{-i}\right) = \theta_i x_i + (1-\theta_i) \cdot \mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i} \,|\, piv\right], \tag{2}$$

where $\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i} \,|\, piv\right]$ is the expected value agent i attaches to the average signal of the other agents, conditional on being pivotal.[16]

Suppose $\left\{A_{-i}^+(\theta_{-i})\right\}$ is the set of acceptance sets corresponding to strategy profile σ_{-i}. For any majority rule k and type-profile θ_{-i}, the *pivotal set* $A_i^{piv}(\theta_{-i}) := \{x_{-i} : |\{j \in \mathcal{I}\backslash\{i\} : \pi_j(x_{-i}) \in A^+(\theta_j)\}| = k-1\}$ is the set of signal profiles x_{-i} for which agent i is pivotal.[17] This implies:

$$\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i} \,|\, piv\right] = \mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i} \,|\; x_{-i} \in A_i^{piv}(\theta_{-i})\right].$$

We use the Tychonoff-Schauder fixed-point theorem to establish existence of a pure-strategy equilibrium. In equilibrium, agents adopt cutoff-strategies.

Theorem 1. *Consider an n-person committee with interdependent values and private types θ_i. Then for any quorum rule $k \in \{1, \ldots, n\}$, there exists a voting equilibrium. In every voting equilibrium agents adopt cutoff strategies*

that for each $x \in O$ there is an interval $I_x \in \{(a,b)\,,\,(a,\overline{x}]\}$, with $x \in I_x \subseteq O$. And let $int\,[a,\widetilde{x}] := (a,\overline{x}]$, where *int* denotes the interior.

[15]with $[\widetilde{x},\, \overline{x}] := \emptyset$.

[16]Reminder: $\overline{x}_{-i} = \sum_{j\neq i} x_j$.

[17]Here, π_j denotes the j^{th} projection map which maps vector x_{-i} to coordinate x_j.

which, for all agent $i \in \mathcal{I}$, are characterized by:

$$\chi_i^*(\theta_i) = \begin{cases} \underline{x} & \text{if } V_i\left((\theta_i,\, x_i); \sigma_{-i}\right) \geq 0 \;\forall\, x_i \in [\underline{x}, \overline{x}] \\ \widetilde{x} & \text{if } V_i\left((\theta_i,\, x_i); \sigma_{-i}\right) < 0 \;\forall\, x_i \in [\underline{x}, \overline{x}] \\ -\frac{1-\theta_i}{\theta_i}\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i} \mid piv\right] & \text{otherwise,} \end{cases} \tag{3}$$

where

$$\begin{aligned} E_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right] =& \mathbb{E}_{\Theta_{-i},\mathcal{X}_{-i}}\left[\overline{x}_{-i} \mid x_{-i} \in A_i^{piv}(\theta_{-i})\right] \\ =& \frac{1}{\mathbb{P}(\sigma_{-i})}\int_{\Theta_{-i}}\int_{\mathcal{X}_{-i}} \overline{x}_{-i} \cdot \mathbb{1}_{A_i^{piv}(\theta_{-i})}\, dF_{-i}(x_{-i})\, dG_{-i}(\theta_{-i}), \end{aligned}$$

and

$$\mathbb{P}(\sigma_{-i}) := \int_{\Theta_{-i}}\int_{\mathcal{X}_{-i}} \mathbb{1}_{A_i^{piv}(\theta_{-i})}\, dF_{-i}(x_{-i})\, dG_{-i}(\theta_{-i}),$$

is the probability that agent i is pivotal.

The rest of the section is devoted to characterize and provide a better understanding of some fundamental properties of equilibrium strategies.

Lemma 1. *In any voting equilibrium, for all $i \in \mathcal{I}$,*

(i) $\chi_i^*(1) = 0$ *and agent i's cutoffs $\chi_i^*(\theta_i)$ have the same sign for all types $\theta_i \in \Theta_i \backslash \{1\}$.*

(ii) the cutoff functions χ_i^* are continuous on Θ_i and twice continuously differentiable a.e.,

(iii) $|\chi_i^*(\theta_i)|$ *is non-increasing in θ_i.*

Let us provide some intuition for this result. If agents have interdependent values, every agent – in his decision of choosing an optimal cutoff – takes into account the expected value he attaches to the information held by other committee members, where he updates his beliefs conditional on the event of being pivotal. In particular, being pivotal is either good news (if $E_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right] > 0$) or bad news ($E_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right] < 0$) for an agent. In the first case, conditional on the event of being pivotal, agent i attaches a positive expectation to the information held by other agents. He will therefore require weaker evidence himself to accept an alternative, that is, adjust his own acceptance standard and adopt a negative cutoff.

The expected information derived from the event of being pivotal is the same for all types of agent i. Consequently, all types $\theta_i \neq 1$ will strategically adjust their acceptance standards in the same direction. An agent with type $\theta_i = 1$ has private values, that is, signals received by the other agents are not payoff relevant for him, and the information he derives from the event of being pivotal does not affect his decision. A private values type will therefore vote *sincerely*, that is, solely based on his own private signal, and adopt cutoff 0. Since equilibrium cutoff strategies are continuous (and even smooth a.e.), similar types will adopt similar cutoffs. In particular, under unanimity voting, all cutoffs are non-positive.

Corollary 1 (Unanimity Voting)**.** *For the unanimity rule $k = n$, equilibrium cutoffs are non-positive:*

$$\chi_i^*(\theta_i) \in [\underline{x},\, 0] \quad \forall \theta_i \in \Theta_i,\; i \in \mathcal{I},$$

and $\chi_i^*(\theta_i)$ is increasing in θ_i.

We now discuss if agents' equilibrium strategies are *responsive*. That is, whether agents condition their voting decision on the private signal they observe, or not.

Definition 1. We say that type θ_i of agent $i \in \mathcal{I}$ is *responsive*, if he conditions his decision whether or not to vote affirmatively on his observed signal. For cutoff-strategies this is equivalent to adopting an (interior) cutoff, $\chi_i(\theta_i) \in (\underline{x}, \overline{x}]$.
We say that agent i's strategy is *responsive* if there exists a set of types $\Theta_i^R \subseteq \Theta_i$ with non-empty interior, such that all types $\theta_i \in \Theta_i^R$ are responsive.

The question if agents' equilibrium strategies are responsive is of fundamental importance since it is a necessary condition for information aggregation.

Lemma 2. *In any voting equilibrium, agents' cutoff strategies are responsive. In particular, for every agent i, either all types are responsive, or there exists some type $\hat{\theta}_i$ such that all types $\theta_i > \hat{\theta}_i$ are responsive, whereas χ_i^* is constant* *on* $\left[0, \hat{\theta}_i\right]$ *and all of these types adopt the same extreme cutoff, either* $\underline{x}$ *or* $\widetilde{x}$*. That is,*

$$\chi_i^*(\theta_i) \in (\underline{x}, \overline{x}], \forall\, \theta_i \in \left(\hat{\theta}_i, \overline{\theta}\right], \ \textit{and}$$
$$\chi_i^*(\theta_i) \equiv \textit{constant} \in \{\underline{x},\, \widetilde{x}\}, \forall\, \theta_i \in \left[0, \hat{\theta}_i\right].$$

This result shows that there is always a set of responsive preference types with non-empty interior. That is, in any voting equilibrium a positive measure of preference types (and profiles) condition their decision on the signal they observe – some information aggregation occurs.

The following lemma characterizes the properties of the equilibrium cutoff-functions in some more detail. It establishes that if for some agent equilibrium cutoffs are non-positive (non-negative), the corresponding cutoff-function is concave (convex) on the set of responsive types.

Lemma 3. *In any voting equilibrium, equilibrium cutoff functions satisfy:*

$$(\chi^*)' \cdot (\chi^*)'' \leq 0. \tag{4}$$

In particular, equilibrium cutoff functions are concave (convex) on the set of responsive types $\left[\hat{\theta}_i, \overline{\theta}\right]$ *if cutoffs are non-positive,* $\chi^*(\theta_i) \leq 0$ *(non-negative,* $\chi^*(\theta_i) \geq 0$*).*

To illustrate the typical equilibrium strategies, we provide an example for which we explicitly compute equilibrium cutoff-strategies and further illustrate them in Figure 2.

Example 1. Consider a two-member committee and unanimity voting, that is $n = k = 2$. Suppose attribute values and types are uniformly distributed: $X_i \overset{iid}{\sim} U[-1, 1]$ and $\theta_i \overset{iid}{\sim} U[1/2, 1]$.

Equilibrium cutoff-strategies are characterized by:

$$\chi_i^*(\theta_i) = \begin{cases} \underline{x} & \text{if } V_i\left((\theta_i,\, x_i); \sigma_j\right) \geq 0 \; \forall x_i \in [\underline{x}, \overline{x}] \\ \widetilde{x} & \text{if } V_i\left((\theta_i,\, x_i); \sigma_j\right) < 0 \; \forall x_i \in [\underline{x}, \overline{x}] \\ -\frac{1-\theta_i}{\theta_i} \mathbb{E}_{\chi_j^*}\left[x_j \mid piv\right] & \text{otherwise.} \end{cases} \tag{5}$$

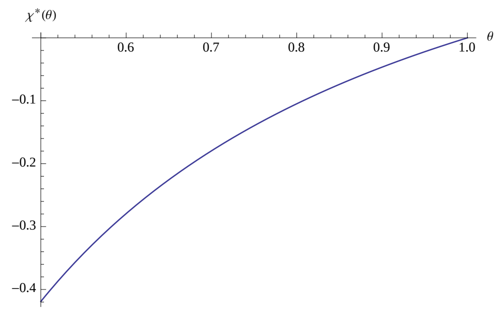

Figure 2: Equilibrium cutoffs in a 2-member committee under unanimity voting with uniformly distributed signals and types: $X_i \overset{iid}{\sim} U[-1,1]$ and $\theta_i \overset{iid}{\sim} U[1/2,1]$.

In the current example we obtain:

$$\mathbb{E}_{\chi_i^*}[x_i|piv] = \frac{1}{1+\log 4}$$
$$\mathbb{P}(\chi_i^*) = \frac{\log(4)}{1+\log(4)} \approx 0.58$$
$$\chi_i^*(\theta_i) = -\frac{1-\theta_i}{\theta_i} \cdot \frac{1}{1+\log 4}$$

The resulting cutoff-function is displayed in Figure 2.

3.1 Equilibrium Uniqueness

It is well known that for general majority rules there are typically multiple equilibria. We now provide conditions under which there exists a unique voting equilibrium in undominated strategies, as characterized in Theorem 1. We show that, if all committee members have sufficiently strong preference types, then, under some standard distributional assumptions on the attribute values, for the unanimity rule there exist a unique voting equilibrium. We need to introduce the following definition:

Definition 2. A random variable X with support $\mathcal{X} \subseteq \mathbb{R}$ has the strict *diminishing mean residual life property* (DMRL), if for $x \in \mathcal{X}$

$$\mathbb{E}[X|X \geq x] - x$$

is strictly decreasing in x.[18]

[18]To be precise, this is the natural generalization of the standard definition for non-

Definition 2 is equivalent to

$$\mathbb{E}\left[X|X \geq x\right] - \mathbb{E}\left[X|X \geq x'\right] < x - x' \quad \forall x > x'. \tag{6}$$

Formally, this is a Lipschitz-continuity assumption for the truncated mean $E\left[X|X \geq x\right]$, with Lipschitz constant 1.[19] The DMRL-property also has an intuitive interpretation in our committee-voting setting. The left hand side of Equation 6 represents the effect of a change in cutoffs from x to x', on the informational value of an approval vote for the other committee members. It requires that the effect of a change of an agent's cutoff on the estimate of his information conditional on him voting affirmatively is limited.

Theorem 2. *If, for all committee members $i \in \mathcal{I}$, the distributions of attribute values F_i satisfy the monotone hazard rate property, and $\theta_i \in \left[\frac{1}{2}, 1\right]$, then in any unanimity voting game among n agents with private preference types, there exists a unique voting equilibrium.*

Notice that $\theta_i \in \left[\frac{1}{2}, 1\right]$ implies $\frac{\partial v_i}{\partial x_i} \geq \sum_{j \neq i} \frac{\partial v_i}{\partial x_j}$. That is, the effect of a marginal increase in agent i's signal on his own valuation is at least as strong as the sum of marginal changes in the signals of other agents. In the two-agent case, the conditions on preference types for equilibrium existence ($\theta_i \in \left[\frac{1}{n}, 1\right]$) and equilibrium uniqueness ($\theta_i \in \left[\frac{1}{2}, 1\right]$) coincide. The condition that the distributions of attribute values satisfy the monotone hazard rate property implies that for any subset $J \subseteq I$, the random variable $Y_J := \sum_{i \in J} X_i$ has the DMRL-property. The combination of the two conditions then implies that the effect of a marginal change of agents' cutoffs on the expected value an agent attaches to the average signal of the other agents, conditional on being pivotal, is limited. In the proof of Theorem 2, this property is used to contradict the existence of multiple equilibria.

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negative random variables to the case of real-valued random variables.

The diminishing mean residual life property is a slightly weaker condition than the familiar monotone hazard rate assumption which is standard in the mechanism design literature. It requires that $\frac{f(x)}{1-F(x)}$ be increasing in x on the support of X.

[19]If $E\left[X|X \geq x\right]$ is differentiable in x, the DMRL property is equivalent to

$$\frac{\partial E\left[X|X \geq x\right]}{\partial x} \leq 1.$$

4 Effects of Preference Uncertainty

The model presented in this paper establishes a framework to study how the belief of an agent about the distribution of types of the other committee members affects the acceptance standards and acceptance sets in equilibrium. In this section, we address the following questions: How do agents' acceptance standards change according to their beliefs about the preference types they face – particularly when they believe to find themselves among committee members with extreme preference types? And what are the effects of more preference uncertainty? That is, how do agents adjust their cutoffs if they have in a sense less information about the preference types of their fellow committee members?

Our analysis focuses on the two-agent case, referring to them as agent A and agent B. We discuss comparative static effects with respect to the distribution of agents' types. More precisely, we analyze how shifts in the distribution of agents' types in terms of first and second-order stochastic dominance affect the equilibrium outcome, that is, cutoffs and acceptance sets.

To study these effects, we keep the distribution, G_A, of agent A's types fixed, whereas the distribution of agent B's types is either G_B or H_B. We restrict attention to the case in which attribute values have the DMRL-property and hence for any distribution of types with support $\Theta_i = [1/2, 1]$, there exists a unique voting equilibrium. For distribution profiles (G_A, G_B) and (G_A, H_B), the corresponding equilibrium cutoff-function profiles are denoted by $(\chi_A^{*,G}, \chi_B^{*,G})$ and $(\chi_A^{*,H}, \chi_B^{*,H})$, respectively.

4.1 Facing Higher Preference Types

We start off our discussion by considering the case in which agent A faces "on average" a higher preference type. That is, we study the effect of a shift in the distribution of agent B's types in terms of first-order stochastic dominance. In the applications we discussed, this could be interpreted as agent B being on average more confident or partisan.

Which effects are to be expected for a shift of the distribution of agents' preference types? Consider a first-order shift of agent B's types. It is easily seen that this has no direct effect on the best response function of each individual type of agent B. Indeed, for any strategy agent A may choose, for agent B the best response for each of his types remains the same.[20] Now, consider the situation for agent A. For a first-order shift in the distribution of agent B's types, agent A is more likely to face an agent with a high preference type. From Lemma 3 we know that those types adopt more stringent cutoffs than low types. This implies that by keeping a strategy of agent B fixed, a first-order shift in agent B's types increases agent A's expected value of agent B's information conditional on being pivotal. The best response for any type of agent A is to adopt a lower cutoff than before. This results in agent B being less optimistic in the event of being pivotal which induces him to increase his cutoff. This reasoning shows that agents' cutoffs are strategic complements.

It still remains an open question in which direction agents adjust their equilibrium cutoffs. It could be that for a first-order shift in agent B's distribution of types, all types of this agent choose to lower their equilibrium cutoffs, which would counteract the direct effect of the first-order shift on agent A' estimate of agent B's information conditional on being pivotal. Alternatively agent B may increase his acceptance standard, which would enforce the direct effect of the shift in the distribution of types. It is not obvious which of these effects prevails and one may think that both cases may occur depending on the distribution of types or attributes. The next result shows that the reaction of agents' equilibrium strategies on first-order shifts of the distribution of types is unambiguous. It is always the case that all preference types of agent A will lower their cutoffs whereas all types of agent B raise their acceptance standards.

Theorem 3. *Suppose the distribution of attribute values F satisfies the DMRL-property. Keeping the distribution G_A of agent A's types fixed, then,*

[20]Keeping agent A's strategy σ_A fixed, the best response function of agent B only depends on θ_B and $\mathbb{E}_{\sigma_A}\left[X_A|piv\right]$ and not on the distribution of his own type.

if the distribution H_B first-order stochastically dominates G_B, ($H_B \geq_{st} G_B$):

$$\chi_A^{*,G}(\theta_A) \geq \chi_A^{*,H}(\theta_A) \quad \textit{and}$$
$$\chi_B^{*,G}(\theta_B) \leq \chi_B^{*,H}(\theta_B) \quad \forall \theta_A, \theta_B \in \left[\frac{1}{2}, 1\right].$$

Interpreting this result in a setting of a committee of specialists indicates that agents lower their acceptance standards if they face a more partisan group. At the same time an agent from a more partisan population will adopt a more stringent cutoff than an agent with the same level of partisanship from a less partisan group.

4.2 More Preference Uncertainty

The next result provides insights into how more preference uncertainty affect equilibrium strategies. More preference uncertainty corresponds to a mean preserving spread of the distribution of types, denoted $G_B \geq_{MPS} H_B$. If agent B's types are distributed according to G_B, agent A is "more uncertain" (so in a sense has less information) about the preference type of the other agent. That is, agent A faces more preference uncertainty. One could think that in this case agent A will adopt a less aggressive strategy and reduce his acceptance standard. As we see in the next theorem the opposite is the case.

Theorem 4. *Suppose the distribution of attribute values F satisfies the DMRL property and is such that $E[X|X \geq x]$ is strictly increasing and concave in x.*[21] *Keeping the distribution of agent A's types, G_A, fixed, then, if G_B is a mean preserving spread of H_B ($G_B \geq_{MSP} H_B$):*

$$\chi_A^{*,G}(\theta_A) \geq \chi_A^{*,H}(\theta_A) \quad \textit{and}$$
$$\chi_B^{*,G}(\theta_B) \leq \chi_B^{*,H}(\theta_B) \quad \forall \theta_A, \theta_B \in \left[\frac{1}{2}, 1\right].$$

We again interpret this result for a specialist committee. Suppose that, in a committee of specialists, agent A faces more preference uncertainty about the level of partisanship of the other committee member. This results in the event of being pivotal being "less good news" than before and thus yields a

[21] This is for example the case if attribute values are uniformly distributed.

reduction of agent A's estimate of the other committee member's information, conditional on being pivotal. Consequently, agent A will "play it safe" and focuses more on his own private signal. He adopts a more stringent acceptance standard. Agent B reacts by lowering his acceptance standard.

5 Benchmark: No Preference Uncertainty

In this section we analyze the benchmark case of a standing committee in which the level of partisanship of committee members is common knowledge.[22] That is, θ_A, $\theta_B \in \left[\frac{1}{2}, 1\right]$ are common knowledge whereas attribute values x_A and x_B are private information to agent A, respectively B. We restrict attention to the case when unanimity is required to accept the alternative. This is without loss of generality in two-member committees.

We assume that attribute values are determined by random i.i.d. draws from the interval $[\underline{x}, \overline{x}]$, $\underline{x} < 0 < \overline{x}$, with $\underline{x} = -\overline{x}$.

To simplify the exposition, we use the following notation for the left- and right-truncated mean in our analysis:

$$E^+(\hat{x}) := E\left[x|x \geq \hat{x}\right] = \frac{1}{1 - F(\hat{x})} \int_{\hat{x}}^{\overline{x}} x f(x)\,\mathrm{d}x,$$
$$\text{and} \quad E^-(\hat{x}) := E\left[x|x \leq \hat{x}\right] = \frac{1}{F(\hat{x})} \int_{\underline{x}}^{\hat{x}} x f(x)\,\mathrm{d}x.$$

A (mixed) strategy for agent A with partisanship type θ_A is:

$$\sigma_A : \ X_A \to [0,\, 1]$$
$$x_A \mapsto \sigma_A(x_A),$$

where $\sigma_A(x_A)$ is the probability that agent A votes affirmatively (i.e. in favor of the alternative) if his private signal is x_A. Strategy σ_A is pure if $\sigma_A(x_A) \in \{0,\, 1\}$, for every $x_A \in X_A$. Strategies for agent B are characterized analogously.

We consider a binary decision problem in which agents indicate whether

[22] This is an extension of the discussion in Yildirim (2012), allowing for individual heterogeneity levels, but restricting attention to 2-member-committees. Yildirim assumes that all agents share the same level of partisanship, that is, $\theta_A = \theta_B$, the non-generic case in our setting.

they want to accept a reform or stay to the status quo. Consequently, a pure strategy for an agent is characterized by the corresponding *acceptance set*, that is the set of private signals $x_i \in \mathcal{X}_i$ which will induce the agent to vote affirmatively. Agents' acceptance sets are denoted by $A^+ \subseteq \mathcal{X}_A$ for agent A and $B^+ \subseteq \mathcal{X}_B$ for agent B. A strategy σ_i is a *cutoff-strategy* if there exists some $\hat{x}_i \in \mathcal{X}_i$ such that agent i votes affirmatively if and only if $x_i \geq \hat{x}_i$. In this case the acceptance set A^+ is an interval of the form $[\hat{x}_i, \overline{x}]$.
In a voting game agents choose their optimal action conditional on being pivotal, that is, in their decision they take into account the information they can extract from the event of being pivotal. In our unanimity voting expert model, given agent B's strategy and corresponding acceptance set B^+, agent A's expected payoff from the alternative is

$$V_A = \theta_A x_A + (1 - \theta_A) E\left[x_B | x_B \in B^+\right].$$

It is straightforward to establish equilibrium existence using Brouwer's fixed-point theorem and show that in equilibrium agents adopt cutoff-strategies.

Proposition 1. *In a unanimity voting game of two agents with individual preference types θ_A, θ_B, there exists a voting equilibrium. In a voting equilibrium agents adopt cutoff strategies which are characterized by the following equations:*

$$x_A^* = -\frac{1-\theta_A}{\theta_A} \cdot E^+(x_B^*), \tag{7}$$

$$x_B^* = -\frac{1-\theta_B}{\theta_B} \cdot E^+(x_A^*). \tag{8}$$

In equilibrium agents will adopt informative strategies, that is, they base their decision whether to support the reform or not on the signal they observe. This means in particular that they will adopt interior cutoffs.

Lemma 4. *In the 2-member unanimity voting game with individual preference types, in equilibrium agents adopt interior cutoffs, that is, $x_A^*, x_B^* \in (\underline{x}, \overline{x})$. Moreover, cutoffs are non-positive, $x_A^*, x_B^* \leq 0$.*

Intuition: In their decision of choosing an optimal cutoff, agents take into account the expected information of the other conditional on him voting affirmatively. This expected information is always positive which yields negative equilibrium cutoffs.

Equilibrium Uniqueness

To establish equilibrium uniqueness, as before we assume that attribute values satisfy the DMRL property (cf. Definition 2).

Proposition 2. *Consider unanimity voting among two agents with individual preference types. If the distributions of attribute values F_A, F_B satisfy the strict DMRL-property, the voting game has a unique equilibrium in cutoff strategies.*

5.1 Comparative Statics: Changes in the Level of Partisanship

In the given setting it is interesting to understand how the preference type of agents affects equilibrium cutoffs. That is, do agents with higher preference types adopt higher or lower acceptance standards than their less biased counterparts? How do agents adjust to the preference type of other committee member?

5.1.1 How the Level of Partisanship affects Acceptance Standards

We start by studying how acceptance standards, represented by equilibrium cutoffs x_A^* and x_B^* change with the profile of partisanship levels (θ_A, θ_B). Keeping agent B's bias, θ_B, fixed, we analyze the effect of a (marginal) change of θ_A on equilibrium cutoffs. Agents' cutoffs are strategic substitutes. If agent A lowers his cutoff, in the event of being pivotal agent B's has a lower estimate of agent A's information about the candidate. Agent B wants to compensate for this effect and thus raises his acceptance standard. Now, if agent A gets more partisan there are two possible scenarios: Either agent A increases his equilibrium cutoffs whereas agent B adopts a less stringent cutoff or the other way around. We show that, whenever Definition 2 holds, as an agent gets more partisan, he will increase his acceptances standard. In this sense, the partisanship of an agent's preferences in reflected in the partisanship of his vote.

Proposition 3. *Suppose the distribution of attribute values F satisfies the DMRL-property. Then, keeping θ_B fixed, in the voting equilibrium, x_A^* is increasing in θ_A and x_B^* is decreasing in θ_A. That is,*

$$x^*_{A,\theta_A} > 0 \ \ and \ \ x^*_{B,\theta_A} < 0.$$

Sketch of proof (The formal proof is relegated to the appendix).
If agent A gets more partisan he puts less weight on the dimension of the alternative about which agent B is holds private information. Thus, for the same cutoff $\hat{x}_B$ of agent B, conditional on being pivotal, agent A puts less weight on the expected information of agent B. Consequently, agent A requires stronger evidence of the alternative to be of high quality and thus adopts a higher acceptance standard. Agent A's best response to $\hat{x}_B$ increases. Given that agent B's partisanship-level does not change, neither does his best response function. If agent A raises his cutoff, agent B will react by lowering his cutoff. This behavior even enforces the direct effect of an increase in θ_A. □

That is, lower type agents will be more lenient and accept more candidates. Furthermore, for any fixed preference type of agent A, his cutoff is increasing in agent B's type θ_B. That is, if agent B gets less biased, agent A will use this and react by increasing his acceptance standard.

6 Conclusion

We have studied a flexible committee voting model in which agents have two dimensional private information: about the payoff-relevant state of the world and a private preference parameter. An important distinction between the two kinds of private information is that the private information about the state directly affects the utilities of all committee members. By contrast, the private information about an agents preference type only has an indirect effect on the utilities of other agents through equilibrium voting. We have established equilibrium existence, characterized fundamental properties of equilibrium strategies and discussed how private preference types and preference uncertainty affect equilibrium outcomes. We find that, a shift in the distribution of preference types in terms of second order stochastic dominance, induces all agents to adjust their equilibrium strategies, even though for the decision of an agent only the expectation over all preference types of the other committee member is relevant. This result therefore illustrates a distinction between the two kinds of private information.

We discuss implication of our results for various applications, among them juries which consist of jurors with different confidence levels and committees of specialists with private partisanship levels.

Appendix

A Proofs

Proof of Theorem 1. Consider some agent $i \in \mathcal{I}$. Suppose σ_{-i} denotes the strategies adopted by all agents but i, and let $\{A^{+}_{-i}(\theta_{-i})\}_{\theta_{-i}\in\Theta_{-i}}$ be the corresponding set of acceptance sets. For agent i with type θ_i, the expected payoff of the alternative, conditional on being pivotal, and observing signal x_i is:[23]

$$V_i\left((\theta_i,\, x_i);\sigma_{-i}\right) = \theta_i x_i + (1-\theta_i)\cdot \mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}\,|\, piv\right] \quad \text{with} \qquad (9)$$
$$\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right] := \frac{1}{\mathbb{P}(\sigma_{-i})}\int_{\Theta_{-i}}\int_{\mathcal{X}_{-i}} \overline{x}_{-i}\cdot \mathbb{1}_{A^{piv}_{-i}(\theta_{-i})}\,\mathrm{d}F_{-i}(x_{-i})\,\mathrm{d}G_{-i}(\theta_{-i}).$$

It is agent i's best response to vote in favor of the alternative if and only if the expected payoff of the alternative, conditional on him being pivotal, is greater than the payoff of the status quo, which is 0. Consequently, it is agent i's best response to vote affirmatively if and only if $V_i\left((\theta_i,\, x_i);\sigma_{-i}\right) \geq 0$.

It is easy to see from Equation 9 that $V_i\left((\theta_i,\, x_i);\sigma_{-i}\right)$ is continuous and

[23] Suppose agent i updates his beliefs, assuming that he is pivotal and all other agents play according to σ_{-i}. Then his expected payoff if he votes affirmatively is:

$$\begin{aligned}\widetilde{V}_i\left((\theta_i,\, x_i);\sigma_{-i}\right) &= \int_{\Theta_{-i}}\int_{\mathcal{X}_{-i}}\left[\theta_i x_i + \frac{1-\theta_i}{n-1}\sum_{j\neq i} x_j\right]\cdot \mathbb{1}_{A^{piv}_{-i}(\theta_{-i})}\,\mathrm{d}F_{-i}(x_{-i})\,\mathrm{d}G_{-i}(\theta_{-i})\\ &= \mathbb{P}(\sigma_{-i})\theta_i x_i + \frac{1-\theta_i}{n-1}\int_{\Theta_{-i}}\int_{\mathcal{X}_{-i}}\left(\sum_{j\neq i} x_j\right)\cdot \mathbb{1}_{A^{piv}_{-i}(\theta_{-i})}\,\mathrm{d}F_{-i}(x_{-i})\,\mathrm{d}G_{-i}(\theta_{-i}),\end{aligned}$$

where $\mathbb{P}(\sigma_{-i}) = \int_{\Theta_{-i}}\int_{\mathcal{X}_{-i}} \mathbb{1}_{A^{piv}_{-i}(\theta_{-i})}\,\mathrm{d}F_{-i}(x_{-i})\,\mathrm{d}G_{-i}(\theta_{-i})$ is the probability that agent i is pivotal. Using that $\overline{x}_i := \frac{1}{n-1}\sum_{j\neq i} x_j$ and conditioning on the event of being pivotal yields Equation 9.

strictly increasing in x_i. This readily establishes that agent i's best response is to follow a cutoff-strategy. That is, for every preference type θ_i, there exists some $\chi_i(\theta_i) \in \overline{\mathcal{X}}_i$ such that type θ_i votes affirmatively if and only if he observes a signal $x_i \geq \chi_i(\theta_i)$.[24] In particular, suppose all other agents adopt strategy profile σ_{-i}, then agent i's best response is to adopt cutoff-function ϕ_i^{BR} characterized by:

$$\phi_i^{BR}(\theta_i) = \begin{cases} \underline{x} & \text{if } V_i\left((\theta_i,\, x_i); \sigma_{-i}\right) \geq 0 \;\forall x_i \in [\underline{x}, \overline{x}] \\ \widetilde{x} & \text{if } V_i\left((\theta_i,\, x_i); \sigma_{-i}\right) < 0 \;\forall x_i \in [\underline{x}, \overline{x}] \\ -\frac{1-\theta_i}{\theta_i}\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right] & \text{otherwise.} \end{cases} \tag{10}$$

From now on we assume that agents adopt cutoff-strategies. By a slight abuse of notation, we denote agent i's cutoff strategy by a function $\chi_i : \Theta_i \to \overline{\mathcal{X}}_i$ where $\chi_i(\theta_i)$ is the cutoff agent i adopts if his type is θ_i.

Let $\overline{\mathcal{X}}_i^{\Theta_i}$ be the space of functions $f : \Theta_i \to \overline{\mathcal{X}}_i$ endowed with the product topology (here: the topology of pointwise convergence). We denote agent i's best response function by $\phi_i^{BR} : \overline{\mathcal{X}}^{\Theta} \to \overline{\mathcal{X}}_i^{\Theta_i}$.[25] This is well-defined since we have shown that for any strategy profile σ_{-i} agent i's best response is a cutoff function.

The discussion shows that, for every strategy profile σ_{-i} a unique best response for agent i exists, and, best responses take the form of cutoff functions. It follows that the best response correspondence is a function, characterized by:

$$\begin{aligned} \Phi :\; \overline{\mathcal{X}}_1^{\Theta_1} \times \cdots \times \overline{\mathcal{X}}_n^{\Theta_n} &\longrightarrow \overline{\mathcal{X}}_1^{\Theta_1} \times \cdots \times \overline{\mathcal{X}}_n^{\Theta_n} \\ \boldsymbol{\chi} = (\chi_1, \ldots, \chi_n) &\longmapsto \left(\phi_i^{BR}(\boldsymbol{\chi}), \ldots, \phi_n^{BR}(\boldsymbol{\chi})\right). \end{aligned}$$

We use the *Tychonoff's Fixed Point Theorem*[26] to establish equilibrium existence.

First, notice that for every $i \in \mathcal{I}$, Θ_i is compact. Moreover, $\overline{\mathcal{X}}_i$ is com-

[24] Reminder: $\overline{\mathcal{X}}_i := \mathcal{X}_i \cup \{\widetilde{x}\}$, and cutoff $\widetilde{x}$ represents the case in which the agent rejects all proposals.

[25] ϕ^{BR} identifies for every σ_{-i} a corresponding cutoff-function $\phi^{BR}(\sigma_i, \sigma_{-i}) = \chi_i \in \overline{\mathcal{X}}_i^{\Theta_i}$. Notice that ϕ^{BR} is constant in σ_i.

[26] cf. Aliprantis and Border (2006) p. 583

pact for the given topology we have chosen. It is possible to interpret $\overline{\mathcal{X}}_i^{\Theta_i}$ as an infinite product of $\overline{\mathcal{X}}_i$. It therefore follows from Tychonoff's theorem that $\overline{\mathcal{X}}_i^{\Theta_i}$ is compact. Applying Tychonoff's theorem again, yields that $K := \overline{\mathcal{X}}_1^{\Theta_1} \times \cdots \times \overline{\mathcal{X}}_n^{\Theta_n}$ is compact. It is easily verified that K is non-empty and convex.

We also have to verify that the best response function Φ is continuous for which it suffices to show continuity for each of the coordinate functions. Consider the coordinate function

$$\Phi_i : \overline{\mathcal{X}}_i^{\Theta_i} \times \overline{\mathcal{X}}_{-i}^{\Theta_{-i}} \to \overline{\mathcal{X}}_i^{\Theta_i}$$
$$\boldsymbol{\chi} = (\chi_i, \chi_{-i}) \mapsto \phi^{BR}(\boldsymbol{\chi})$$

It is easily seen that Φ_i is constant in χ_i. Moreover, since the expectation operator is linear, and in the given setting bounded, it follows that Φ_i is continuous in χ_{-i} (cf. Equation 10). This shows that every coordinate function Φ_i is continuous, and so is Φ.

We can finally apply Tychonoff's Fixed Point Theorem to establishes the existence of a fixed point of ϕ. This completes the proof of equilibrium existence. □

Proof of Lemma 1.
(i): An agent with preference type $\theta_i = 1$ has private values. In particular, $V_i\left((1, x_i); \sigma^*_{-i}\right) = x_i$, $\forall x_i \in X_i$. Given that the payoff of the status quo is 0 it follows directly that $\chi^*_i(1) = 0$. That is, these types always vote sincerely and adopt cutoff 0 in equilibrium.

Consider any equilibrium strategy profile σ^* with corresponding acceptance sets $\{A^*_i(\theta_i)\}_{i \in \mathcal{I}, \theta_i \in \Theta_i}$. Notice that, for every agent i, $\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right]$ is constant in θ_i. Moreover, $-\frac{1-\theta_i}{\theta_i} < 0$ for all $\theta_i \in (0, 1)$. It follows that equilibrium cutoffs $\chi^*_i(\theta_i)$ have the same sign for all types $\theta_i \in \Theta_i \backslash \{0\}$:

$$sign\left[\chi^*_i(\theta_i)\right] = -sign\, \mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right].$$

(ii): *Continuity of equilibrium cutoff-functions.*
Consider an agent with type θ_i who adopts and interior equilibrium cutoff, that is, $\chi^*_i(\theta_i) = -\frac{1-\theta_i}{\theta_i}\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right] \in (\underline{x}, \overline{x}]$. Given the equilibrium charac-

terizing conditions in Theorem 1 it must hold that $V_i\left((\theta_i,\, x_i);\sigma^*_{-i}\right) \underset{(<)}{>} 0$ for $x_i \underset{(<)}{>} \chi^*_i(\theta_i)$. Since $V_i\left((\theta_i,\, x_i);\sigma^*_{-i}\right)$ is continuous in θ_i and x_i this implies that there exist an $\epsilon > 0$ s.t. $V_i\left((\theta_i,\, \overline{x});\sigma^*_{-i}\right) > 0$ and $V_i\left((\theta_i,\, \underline{x});\sigma^*_{-i}\right) < 0$ for all $\theta_i' \in \mathcal{B}_\epsilon(\theta_i)$, where $\mathcal{B}_\epsilon(\theta_i)$ is the open $\epsilon-$ball about θ_i. It then follows from the equilibrium characterizing conditions of Theorem 1, that all preference types $\theta_i' \in \mathcal{B}_\epsilon$ adopt interior cutoffs. In this case equilibrium cutoffs are characterized by $\chi^*_i(\theta_i) = -\frac{1-\theta_i}{\theta_i}\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right]$. Since $\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right]$ is constant and $-\frac{1-\theta_i}{\theta_i}$ is twice continuously differentiable in θ_i, it follows that $\chi^*_i(\theta_i)$ is continuously differentiable in θ_i.

Now, consider a preference type $\theta_i \in \Theta_i$ who adopts a boundary cutoff, $\chi^*_i(\theta_i) \in \{\underline{x},\, \widetilde{x}\}$, that is for type θ_i either $V_i\left((\theta_i,\, x_i);\sigma^*_{-i}\right) \geq 0$ for all $x_i \in X_i$, or $V_i\left((\theta_i,\, x_i);\sigma^*_{-i}\right) < 0$ for all $x_i \in X_i$. Consider the first case, that is, $\chi^*_i(\theta_i) = \underline{x}$ and $V_i\left((\theta_i,\, x_i);\sigma^*_{-i}\right) \geq 0,\ \forall\, x_i \in X_i$.[27] Given that $V_i\left((\theta_i,\, x_i);\sigma^*_{-i}\right)$ is monotone increasing in x_i a necessary and sufficient condition for $V\left((\theta_i,\, x_i);\sigma^*_{-i}\right) \geq 0$ is $V\left((\theta_i,\, \underline{x}_i);\sigma^*_{-i}\right) \geq 0$. Since $V_i\left((\theta_i,\, x_i);\sigma^*_{-i}\right)$ is continuous in θ_i, the set $\{\theta_i \in \Theta_i | V_i\left((\theta_i, x_i),\sigma^*_{-i}\right) \geq 0\}$, which is the inverse image of $\left[0, V_i\left((\theta_i,\, \overline{x});\sigma^*_{-i}\right)\right]$, is closed. Since $\frac{\partial V_i}{\partial \theta_i}\big|_{x_i=\underline{x}_i} \leq 0$, it follows that there exists some $\hat{\theta} \in \Theta_i$ such that $\{\theta_i \in \Theta_i | V_i\left((\theta_i, x_i),\sigma^*_{-i}\right) \geq 0\} = \left[\underline{\theta}, \hat{\theta}\right]$. By Theorem 1, $\chi^*(\theta_i) = \underline{x}$ for $\theta_i \in \left[\underline{\theta}, \hat{\theta}\right]$. That is, the equilibrium cutoff function is constant on this set and thus twice continuously differentiable in θ_i. It is easy to check that $\lim\limits_{\theta_i \downarrow \hat{\theta}} \chi^*(\theta_i) = 0$, which establishes continuity of the equilibrium cutoff function. However, $\chi^*(\theta_i)$ is not differentiable at $\hat{\theta}$, hence equilibrium cutoff-functions are only differentiable almost everywhere and the same holds true for higher order differentiability.

(iii): To prove the last statement we use again that $\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right]$ is constant in θ_i. For all θ_i such that $\chi^*_i(\theta) \in (\underline{x}, \overline{x}]$, we have shown that the cutoff-function is continuously differentiable. We obtain:

$$\left|\frac{\partial}{\partial \theta_i}\chi^*_i\right| = \frac{1}{\theta_i^2} \cdot \left|\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right]\right|,$$

from which it follows directly that $|\chi^*_i(\theta_i)|$ is non-increasing in θ_i whenever

[27] The second case can be easily verified using analogous arguments.

cutoffs are interior. But, given that for all $\theta_i \in \left[\underline{\theta}, \hat{\theta}_i\right]$ these types of agent i adopt extreme cutoffs in $\{\underline{x}, \widetilde{x}\}$. Using that cutoff-functions are continuous, we can conclude that $|\chi_i^*(\theta_i)|$ is non-increasing in θ_i for all types $\theta_i \in \Theta_i$. □

Proof of Corollary 1. For unanimity voting, agent i is pivotal if and only if all other committee members vote affirmatively. This implies

$$\begin{aligned}\mathbb{E}_{\sigma_{-i}}\left[\overline{x}_{-i}|piv\right] =&\mathbb{E}_{\Theta_{-i},\mathcal{X}_{-i}}\left[\frac{1}{n-1}\sum_{j\neq i} X_j \mid X_j \geq \chi_j^*(\theta_j)\right]\\ &> \mathbb{E}_{\Theta_{-i},\mathcal{X}_{-i}}\left[\frac{1}{n-1}\sum_{j\neq i} X_j \mid X_j \geq \underline{x}\right] = 0\end{aligned}$$

We obtain the last equality because $\mathbb{E}\left[X_j\right] = 0$ for every $j \in \mathcal{I}$. The inequality is strict because $\chi_j^*(1) = 0$ and $\chi_j^*(\theta_j)$ is continuous in θ_j, which implies that in a neighborhood of $\boldsymbol{\theta_{-i}} = \mathbf{1}$, $\chi_j^*(\theta_j) \neq \underline{x}$ for all $j \in \mathcal{I}\backslash\{i\}$.

It follows that

$$\chi_i^*(\theta_i) = -\frac{1-\theta_i}{\theta_i}\mathbb{E}_{\Theta_{-i},\mathcal{X}_{-i}}\left[\frac{1}{n-1}\sum_{j\neq i} X_j \mid X_j \geq \chi_j^*(\theta_j)\right] \leq 0.$$

That $\chi_i^*(\theta_i)$ is increasing in θ_i for every $i \in \mathcal{I}$ then follows directly from Lemma 1 (iii). □

Proof of Lemma 2. In Lemma 1 it was shown that χ_i^* is continuous in θ_i, $\chi_i^*(1) = 0$ and $|\chi_i^*(\theta_i)|$ is non-increasing in θ_i. Moreover, if $\chi_i^*(\theta_i) \in \{\underline{x}, \widetilde{x}\}$, then $\chi_i^*(\theta_i') = \chi_i^*(\theta_i) \in \{\underline{x}, \widetilde{x}\}$ for all $\theta_i' \leq \theta_i$. Continuity of χ_i^* for interior cutoffs and $\chi_i^*(1) = 0$ yield that if there are types which adopt extreme cutoffs in $\{\underline{x}, \widetilde{x}\}$, then there exists some type $\hat{\theta}_i$ such that all types $\left[0, \hat{\theta}_i\right]$ adopt extreme cutoffs in $\{\underline{x}, \widetilde{x}\}$ whereas all types $\left(\hat{\theta}_i, 1\right]$ adopt interior cutoffs. □

Proof of Lemma 3. By Lemma 1 $\chi_i^*(\theta_i)$ is constant on the set of non-responsive types. That is, $(\chi_i^*)'(\theta_i) = 0$ for all $\theta_i \in \left[0, \hat{\theta}_i\right)$ and Equation 4 is trivially satisfied.

Now consider any responsive type $\theta_i \in \left(\hat{\theta}, \overline{\theta}\right]$. By Lemma 1 we know that χ_i^* is twice continuously differentiable at θ_i. It is easily verified that for

$\theta_i \in \left(\hat{\theta}, \overline{\theta}\right]$ we obtain

$$(\chi_i^*)' \cdot (\chi_i^*)'' (\theta_i) = -\frac{2}{\theta_i^5} \mathbb{E}_{\sigma^*_{-i}} \left[\overline{x}_{-i} | piv\right]^2 \leq 0.$$

The result about concavity/convexity of equilibrium cutoff functions follows by combining this with the result of Lemma 1 (iii). □

Proof of Theorem 2. We prove the result for $n = 2$, and then discuss how the result extends to unanimity voting in an n-member committee for $n \geq 2$. We refer to the two agents as agent A and B.

The proof consists of two steps. We first show that a voting equilibrium is *essentially unique* to then derive the stronger statement of equilibrium uniqueness. Formally, a voting equilibrium is *essentially unique*, if there exist no voting equilibria which differ for a non zero-measure set of types, $\widetilde{\Theta} \subseteq \Theta_A \times \Theta_B$.[28]

Claim 1: A voting equilibrium is essentially unique.

Suppose multiple equilibria exist; two of them be characterized by the distinct cutoff profiles (χ_A^*, χ_B^*) and (ξ_A^*, ξ_B^*), and suppose they differ on a non zero-measure set of types. Both of these cutoff profiles have to satisfy the equilibrium characterizing Equation 3. Moreover, for the two agent case we know that all equilibrium cutoffs are interior (cf. Lemma 3). Combining these observations we obtain that the cutoff profiles have to satisfy the following equations:

$$\xi_A^*(\theta_A) - \chi_A^*(\theta_A) = -\frac{1-\theta_A}{\theta_A} \mathbb{E}_{\Theta_B} \left[\mathbb{E}^+_{\mathcal{X}_B} \left[\xi_B^*(\theta_B)\right] - \mathbb{E}^+_{\mathcal{X}_B} \left[\chi_B^*(\theta_B)\right]\right]$$
$$\xi_B^*(\theta_B) - \chi_B^*(\theta_B) = -\frac{1-\theta_B}{\theta_B} \mathbb{E}_{\Theta_A} \left[\mathbb{E}^+_{\mathcal{X}_A} \left[\xi_A^*(\theta_A)\right] - \mathbb{E}^+_{\mathcal{X}_A} \left[\chi_A^*(\theta_A)\right]\right]$$

where

$$\mathbb{E}^+_{\mathcal{X}_A} \left[\chi_A^*(\theta_A)\right] := \mathbb{E}_{\mathcal{X}_A} \left[X_A | X_A \geq \chi_A^*(\theta_A)\right]$$

We use this compactified notation in following discussion.

[28]That is, $\widetilde{\Theta}$ has non-empty interior, and $\lambda(\widetilde{\Theta}) > 0$, where λ is the Lebesgue measure.

For any preference type $\theta_A \in \Theta_A$ it must hold that:

$$\begin{aligned}
|\xi_A^*(\theta_A) - \chi_A^*(\theta_A)| &= \left| -\frac{1-\theta_A}{\theta_A} \mathbb{E}_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\xi_B^*(\theta_B)\right] - \mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^*(\theta_B)\right]\right]\right| \\
&\leq \frac{1-\theta_A}{\theta_A} \mathbb{E}_{\Theta_B}\left[\left|\mathbb{E}^+_{\mathcal{X}_B}\left[\xi_B^*(\theta_B)\right] - \mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^*(\theta_B)\right]\right|\right] \\
&< \frac{1-\theta_A}{\theta_A} \cdot \mathbb{E}_{\Theta_B}\left[|\xi_B^*(\theta_B) - \chi_B^*(\theta_B)|\right].
\end{aligned}$$

We obtain the last inequality by using that the distribution functions F_i satisfy the strict DMRL-property (cf. Equation 6). Applying an analogous argument to $|\xi_B^*(\theta_B) - \chi_B^*(\theta_B)|$ yields:

$$|\xi_B^*(\theta_B) - \chi_B^*(\theta_B)| < \mathbb{E}_{\Theta_A}\left[\left|\xi_A^*(\widehat{\theta}_A) - \chi_A^*(\widehat{\theta}_A)\right|\right].$$

Combining these two inequalities we obtain:

$$\begin{aligned}
|\xi_A^*(\theta_A) - \chi_A^*(\theta_A)| &< \mathbb{E}_{\Theta_B}\left[\mathbb{E}_{\Theta_A}\left[|\,\xi_A^*(\theta_A) - \chi_A^*(\theta_A)|\right]\right] \\
&\leq \mathbb{E}_{\Theta_B}\left[\mathbb{E}_{\Theta_A}\left[\left|\,\xi_A^*(\widehat{\theta}_A) - \chi_A^*(\widehat{\theta}_A)\right|\right]\right] \\
&< \left|\,\xi_A^*(\widehat{\theta}_A) - \chi_A^*(\widehat{\theta}_A)\right|,
\end{aligned}$$

where $\widehat{\theta}_A := \arg\max_{\theta_A \in \Theta_A} \{|\xi_i^*(\theta_A) - \chi_i^*(\theta_A)|\}$. The above inequality has to be satisfied for all $\theta_A \in \Theta_A$ including $\widehat{\theta}_A$, for which $|\xi_A^*(\hat{\theta}_A) - \chi_A^*(\hat{\theta}_A)| > 0$. This yields a contradiction which proves *Claim 1.*

Claim 2: If the equilibrium of Theorem 1 is essentially unique, it is unique.

If the voting equilibrium is essentially unique, equilibrium cutoff-functions may differ on a null set. However, two equilibrium cutoff-functions which only differ on a null set yields the same expectations $\mathbb{E}^+_{\mathcal{X}_i}\left[\chi_i^*(\theta_i)\right]$ for $i \in \{A, B\}$. By Theorem 1 this fully characterizes the equilibrium cutoff for each preference type θ_i which completes the proof of equilibrium uniqueness for the 2-agent case.

To extend this result to $n > 2$ it is important to notice that the DMRL-property is not closed under convolution. This means, that if all random variables X_i, $i \in \mathcal{I}$ satisfy the DMRL-property, this does not necessarily imply that $\sum_{j \neq i} X_j$ has the DMRL-property. However, if all random variables X_i have a monotone hazard rate[29], then $\sum_{j \neq i} X_j$ has the DMRL-property.[30] The result, that the voting equilibrium is essentially unique under unanimity voting can then be established by following the same line of reasoning as for the case $n = 2$. □

Proof of Theorem 3. We use an indirect argument to prove this. Consider any continuous, monotone increasing cutoff function $\widehat{\chi}_B : \Theta_B \to \mathcal{X}_B$.[31] In this case $\mathbb{E}^+(\widehat{\chi}_B(t_B)) = \mathbb{E}\left[X_B | X_B \geq \widehat{\chi}_B(t_B)\right]$ is monotone increasing in t_B, which implies that, if H_B first-order stochastically dominates G_B, $H_B \geq_{st} G_B$, then

$$\int_{\underline{\theta}_B}^{\overline{\theta}_B} \mathbb{E}^+(\widehat{\chi}_B(t_B))\,\mathrm{d}G_B(t_B) \leq \int_{\underline{\theta}_2}^{\overline{\theta}_2} \mathbb{E}^+(\widehat{\chi}_B(t_B))\,\mathrm{d}H_B(t_B). \tag{11}$$

It follows that

$$\chi_A^{BR,G}(\theta_A, \widehat{\chi}_B) \geq \chi_A^{BR,H}(\theta_A, \widehat{\chi}_B) \quad \forall \theta_A \in \Theta_A \tag{12}$$

Claim 1. The equilibrium cutoff shifts in the same direction for all types.

Let $\mathbb{E}_H(\widehat{\chi}_B) := \int_{\underline{\theta}_2}^{\overline{\theta}_2} E^+(\widehat{\chi}_B(t_B))\,\mathrm{d}H_B(t_B)$ and similarly for $\mathbb{E}_G(\widehat{\chi}_B)$. Then, if $\theta_A \sim G_A$ and $\theta_B \sim G_B$, equilibrium conditions are

$$\chi_A^{*,G}(\theta_A) = -\frac{1-\theta_A}{\theta_A}\mathbb{E}_G(\chi_B^{*,G}) \;\text{ and }\; \chi_B^{*,G}(\theta_B) = -\frac{1-\theta_B}{\theta_B}\mathbb{E}_G(\chi_A^{*,G}),$$

and similarly for $\theta_B \sim H_B$. Since, $\mathbb{E}_G(\chi_B^{*,G})$ and $\mathbb{E}_H(\chi_2^{*,H})$ are constant in θ_A and θ_B, we obtain

$$\chi_A^{*,G}(\theta_A) \geq \chi_A^{*,H}(\theta_A) \;\Leftrightarrow\; \mathbb{E}_G(\chi_B^{*,G}) \leq \mathbb{E}_H(\chi_B^{*,H}),$$

and similarly for the equilibrium cutoff functions of agent B.
Thus, if $\chi_A^{*,G}(\theta_A) \geq \chi_A^{*,H}(\theta_A)$ for some θ_A, then $\chi_A^{*,G}(\theta_A) \geq \chi_A^{*,H}(\theta_A)$ for all

[29] In the statistics literature this is known as the *increasing failure rate* property.

[30] cf. Shaked and Shanthikumar (2007), theorem 2.A.23 and corollary 2.A.24

[31] This is w.l.o.g. since equilibrium cutoff functions are continuous and monotone increasing in θ_B (cf. Lemma 3)

$\theta_A \in \left[\frac{1}{2}, 1\right]$ which verifies Claim 1.

Claim 2. If $H_B \geq_{st} G_B$, equilibrium cutoffs of agent A and B move in different directions.[32]

Suppose $\widehat{\chi}_B(\theta_B) \leq \widetilde{\chi}_B(\theta_B)\ \forall \theta_B \in \Theta_B$, then

$$\mathbb{E}_H(\widehat{\chi}_2) = \int_{\underline{\theta}_2}^{\bar{\theta}_2} \mathbb{E}^+(\widehat{\chi}_B(t_B))\, \mathrm{d}H(t_B) \leq \int_{\underline{\theta}_2}^{\bar{\theta}_2} \mathbb{E}^+(\widetilde{\chi}_B(t_B))\, \mathrm{d}H(t_B) = \mathbb{E}_H(\widetilde{\chi}_B) \tag{13}$$

$$\Rightarrow \chi_A^{BR,H}(\theta_A, \widehat{\chi}_B) \geq \chi_A^{BR,H}(\theta_A, \widetilde{\chi}_B)$$

Now, suppose $\chi_B^{*,G}(\theta_B) \leq \chi_B^{*,H}(\theta_B)\ \forall \theta_B \in \left[\frac{1}{2}, 1\right]$, then

$$\chi_A^{*,G}(\theta_A) = \chi_A^{BR,G}(\theta_A, \chi_B^{*,G}) \geq \chi_A^{BR,H}(\theta_A, \chi_B^{*,G}) \geq \chi_A^{BR,H}(\theta_A, \chi_B^{*,H}) = \chi_A^{*,H}(\theta_A)$$

where the first inequality follows from Equation 12 and the second inequality follows by Equation 13.
With similar arguments we can show that if $\chi_A^{*,G}(\theta_A) \leq \chi_A^{*,H}(\theta_A)$, $\forall \theta_A \in \Theta_A$, then $\chi_B^{*,G}(\theta_B) \geq \chi_B^{*,H}(\theta_B)$, using that $\chi_B^{BR,G}(\theta_B, \widehat{\chi}_A) = \chi_B^{BR,H}(\theta_B, \widehat{\chi}_A)$ given that the distribution G_A does not change. This proves Claim 2.

To prove the theorem, we still have to contradict the case

$$\chi_A^{*,G}(\theta_A) \leq \chi_A^{*,H}(\theta_A) \quad \text{and}$$
$$\chi_B^{*,G}(\theta_B) \geq \chi_B^{*,H}(\theta_B).$$

[32]Agents cutoffs are strategic complements wrt first order shifts in the distribution function of types.

Assume those inequalities were true. We obtain:

$$\begin{aligned}
0 \leq & \chi_A^{*,H}(\theta_A) - \chi_A^{*,G}(\theta_A) \\
= & -\frac{1-\theta_A}{\theta_A}\left[\mathbb{E}_{\Theta_B}^{H}\left[\mathbb{E}_{\mathcal{X}_B}^{+}\left[\chi_B^{*,H}(\theta_B)\right]\right] - \mathbb{E}_{\Theta_B}^{G}\left[\mathbb{E}_{\mathcal{X}_B}^{+}\left[\chi_B^{*,G}(\theta_B)\right]\right]\right] \\
= & \frac{1-\theta_A}{\theta_A}\mathbb{E}_{\Theta_B}^{H}\left[E_{\mathcal{X}_B}^{+}\left[\chi_B^{*,G}(\theta_B)\right] - E_{\mathcal{X}_B}^{+}\left[\chi_B^{*,H}(\theta_B)\right]\right] \\
& + \frac{1-\theta_A}{\theta_A}\left[\mathbb{E}_{\Theta_B}^{G}\left[\mathbb{E}_{\mathcal{X}_B}^{+}\left[\chi_B^{*,G}(\theta_B)\right]\right] - \mathbb{E}_{\Theta_B}^{H}\left[\mathbb{E}_{\mathcal{X}_B}^{+}\left[\chi_B^{*,G}(\theta_B)\right]\right]\right] \\
\leq & \frac{1-\theta_A}{\theta_A}\mathbb{E}_{\Theta_B}^{H}\left[\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B)\right] \\
& + \frac{1-\theta_A}{\theta_A}\left[\mathbb{E}_{\Theta_B}^{G}\left[\mathbb{E}_{\mathcal{X}_B}^{+}\left[\chi_B^{*,G}(\theta_B)\right]\right] - \mathbb{E}_{\Theta_B}^{H}\left[\mathbb{E}_{\mathcal{X}_B}^{+}\left[\chi_B^{*,G}(\theta_B)\right]\right]\right] \\
\leq & \frac{1-\theta_A}{\theta_A}\mathbb{E}_{\Theta_B}^{H}\left[\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B)\right]
\end{aligned}$$

The last inequality follows since $H \geq_{st} G$ implies

$$\mathbb{E}_{\Theta_B}^{H}\left[\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B)\right] = \int_{\underline{\theta}_B}^{\bar{\theta}_B} E^{+}(\chi_B^{*,H}(t_B))\left[\,\mathrm{d}G_B(t_B) - \,\mathrm{d}H_B(t_B)\right] \leq 0.$$

Analogous arguments show:

$$0 \leq \chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B) < \frac{1-\theta_B}{\theta_B}\mathbb{E}_{\Theta_A}^{H}\left[\chi_A^{*,H}(\theta_A) - \chi_A^{*,G}(\theta_A)\right]$$

Combining these inequalities yields:

$$\begin{aligned}
\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B) < & \overbrace{\frac{1-\theta_B}{\theta_B}}^{\in[0,1]}\mathbb{E}_{\Theta_A}^{H}\left[\overbrace{\frac{1-\theta_A}{\theta_A}}^{\in[0,1]}\mathbb{E}_{\Theta_B}^{H}\left[\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B)\right]\right] \\
\leq & \mathbb{E}_{\Theta_A}^{H}\left[\mathbb{E}_{\Theta_B}^{H}\left[\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B)\right]\right] \\
= & \mathbb{E}_{\Theta_B}^{H}\left[\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B)\right]
\end{aligned}$$

Integrating both sides, to be precise, taking expectations using distribution

G_B, we obtain:

$$\begin{aligned}
&\mathbb{E}^G_{\Theta_B}\left[\chi^{*,G}_B(\theta_B)-\chi^{*,H}_B(\theta_B)\right]\le \mathbb{E}^G_{\Theta_B}\left[\mathbb{E}^H_{\Theta_B}\left[\chi^{*,G}_B(\theta_B)-\chi^{*,H}_B(\theta_B)\right]\right]\\
\Rightarrow\quad &\mathbb{E}^G_{\Theta_B}\left[\chi^{*,G}_B(\theta_B)-\chi^{*,H}_B(\theta_B)\right]\le \mathbb{E}^H_{\Theta_B}\left[\chi^{*,G}_B(\theta_B)-\chi^{*,H}_B(\theta_B)\right]
\end{aligned}$$

a contradiction.[33] □

Proof of Theorem 4. As characterized in Theorem 1, equilibrium cutoff functions are of the form $\chi^*_i(\theta_i)=-\frac{1-\theta_i}{\theta_i}\cdot C_i$ with $\theta_i\in\Theta_i$ and some constant $C_i\in\mathcal{X}_i$. In particular, they are non-positive, strictly increasing and concave in θ (cf. Lemma 3 and Corollary 1). By assumption, the distribution of attribute values is such that $\mathbb{E}[X|X\ge x]$ is strictly increasing and concave in x. It follows that for every equilibrium cutoff function, $\mathbb{E}[X|X\ge\chi^*(\theta)]$ is non-negative, strictly increasing and concave in θ.[34]

Consider any fixed distribution G_A, and distributions G_B and H_B such that G_B is a mean-preserving spread of H_B, $G_B\ge_{MSP}H_B$. The corresponding equilibrium cutoff-strategy profiles are $\chi^{*,G}=\left(\chi^{*,G}_A,\chi^{*,G}_B\right)$ and $\chi^{*,H}=\left(\chi^{*,H}_A,\chi^{*,H}_B\right)$.

Consider any strictly increasing and concave function $\widehat{\chi}_B:\Theta_B\to\mathcal{X}_B$. In this case, $\mathbb{E}[X|X\ge\widehat{\chi}_B(\theta)]$ is strictly increasing and concave, and $G_B\ge_{MSP}H_B$ implies:

$$\begin{aligned}
&\underbrace{\int_{\underline{\theta}_B}^{\overline{\theta}_B}\mathbb{E}^+_{\mathcal{X}_B}\left[\widehat{\chi}_B(\theta_B)\right]\,\mathrm{d}H_B(\theta_B)}_{=\mathbb{E}^H_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}[\widehat{\chi}(\theta_B)]\right]}\ge\underbrace{\int_{\underline{\theta}_B}^{\overline{\theta}_B}\mathbb{E}^+_{\mathcal{X}_B}\left[\widehat{\chi}_B(\theta_B)\right]\,\mathrm{d}G_B(\theta_B)}_{=\mathbb{E}^G_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}[\widehat{\chi}(\theta_B)]\right]}\\
&\qquad\Rightarrow\quad\chi^{BR,H}_A(\theta_A,\widehat{\chi}_B)\le\chi^{BR,G}_A(\theta_A,\widehat{\chi}_B)\quad\forall\,\theta_A\in\Theta_A.
\end{aligned}\tag{14}$$

Moreover, consider any two cutoff-functions $\widehat{\chi}_B,\widetilde{\chi}_B:\Theta_B\to\mathcal{X}_B$ such that

[33] By assumption $\chi^{*,G}_B(\theta_B)-\chi^{*,H}_B(\theta_B)\ge 0$. Moreover, $\chi^{*,G}_B(\theta_B)-\chi^{*,H}_B(\theta_B)$ is monotone decreasing in θ_B:

$$\chi^{*,G}_B(\theta_B)-\chi^{*,H}_B(\theta_B)=\underbrace{-\frac{1-\theta_B}{\theta_B}}_{\text{incr. in }\theta_B}\underbrace{\left[\mathbb{E}^G_{\Theta_A}\left[\chi^{*,G}_A(\theta_A)\right]-\mathbb{E}^H_{\Theta_A}\left[\chi^{*,H}_A(\theta_A)\right]\right]}_{\le 0}.$$

Thus $H\ge_{st}G$ would imply $lhs\ge rhs$.

[34] Indeed, if functions g and f are strictly increasing and concave, so is the composition $g\circ f$. Here, $g(x):=\mathbb{E}[X|X\ge x]$ and $f(\theta):=\chi^*(\theta)$.

$\widehat{\chi}_B(\theta_B) \leq \widetilde{\chi}_B(\theta_B)$ for all $\theta_B \in \Theta_B$. Since $\mathbb{E}\left[X|X \geq x\right]$ is increasing in x, it follows that, for any distribution of preference types H_B:

$$\mathbb{E}^H_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\widehat{\chi}(\theta_B)\right]\right] \leq \mathbb{E}^H_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\widetilde{\chi}(\theta_B)\right]\right] \tag{15}$$

$$\Rightarrow \quad \chi_A^{BR,H}(\theta_A, \widehat{\chi}_B) \geq x_A^{BR,H}(\theta_A, \widetilde{\chi}_B) \quad \forall\, \theta_A \in \Theta_A. \tag{16}$$

Claim 1: For every agent i, the equilibrium cutoff function shifts in the same direction for all preference-types $\theta_i \in \Theta_i$.

We use an indirect proof to establish the result. Suppose $\chi_B^{*,G}(\theta_B) \leq \chi_B^{*,H}(\theta_B)$ for all $\theta_B \in \Theta_B$. Then

$$\chi_A^{*,H}(\theta_A) = \chi_A^{BR,H}(\theta_A, \chi_B^{*,H}) \leq \chi_A^{BR,H}(\theta_A, \chi_B^{*,G}) \leq \chi_A^{BR,G}(\theta_A, \chi_B^{*,G}) = \chi_A^{*,G}(\theta_A),$$

where the first inequality follows from Equation 16 and the second inequality follows by Equation 14. This contradicts the case:

$$\chi_A^{*,G}(\theta_A) \leq \chi_A^{*,H}(\theta_A) \text{ and } \chi_B^{*,G}(\theta_B) \leq \chi_B^{*,H}(\theta_B)\; \forall \theta_A, \theta_B.$$

Analogous arguments can be used to contradict the case:

$$\chi_A^{*,G}(\theta_A) \geq \chi_A^{*,H}(\theta_A) \text{ and } \chi_B^{*,G}(\theta_B) \geq \chi_B^{*,H}(\theta_B)\; \forall \theta_A, \theta_B.$$

This shows that if $G_B \geq_{MSP} H_B$, equilibrium cutoffs move in different directions, that is, cutoff-functions are strategic substitutes.

Claim 2: If $G_B \geq_{MSP} H_B$, then:

$$\chi_A^{*,G}(\theta_A) \geq \chi_A^{*,H}(\theta_A) \text{ and } \chi_B^{*,G}(\theta_B) \leq \chi_B^{*,H}(\theta_B)\; \forall\, (\theta_A, \theta_B) \in \Theta_A \times \Theta_B.$$

Again, we use an indirect argument to prove this statement. To be precise, we contradict the case:

$$(*) \quad \chi_A^{*,G}(\theta_A) \leq \chi_A^{*,H}(\theta_A) \text{ and } \chi_B^{*,G}(\theta_B) \geq \chi_B^{*,H}(\theta_B)\; \forall\, (\theta_A, \theta_B) \in \Theta_A \times \Theta_B.$$

Suppose these inequalities were true. This would imply:

$$\begin{aligned}
0 \leq & \chi_A^{*,H}(\theta_A) - \chi_A^{*,G}(\theta_A) \\
= & -\frac{1-\theta_A}{\theta_A}\left[\mathbb{E}^H_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^{*,H}(\theta_B)\right]\right] - \mathbb{E}^G_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^{*,G}(\theta_B)\right]\right]\right] \\
= & \frac{1-\theta_A}{\theta_A}\mathbb{E}^H_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^{*,G}(\theta_B)\right] - \mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^{*,H}(\theta_B)\right]\right] \\
& + \frac{1-\theta_A}{\theta_A}\left[\mathbb{E}^G_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^{*,G}(\theta_B)\right]\right] - \mathbb{E}^H_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^{*,G}(\theta_B)\right]\right]\right] \\
\leq & \frac{1-\theta_A}{\theta_A}\mathbb{E}^H_{\Theta_B}\left[\chi_B^{*,G}(t_B) - \chi_B^{*,H}(t_B)\right] \\
& + \frac{1-\theta_A}{\theta_A}\left[\mathbb{E}^G_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^{*,G}(\theta_B)\right]\right] - \mathbb{E}^H_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^{*,G}(\theta_B)\right]\right]\right] \\
\leq & \frac{1-\theta_A}{\theta_A}\mathbb{E}^H_{\Theta_B}\left[\chi_B^{*,G}(t_B) - \chi_B^{*,H}(t_B)\right] \qquad (17)
\end{aligned}$$

The last inequality follows, from the fact that, since $E^+_{\mathcal{X}_B}\left[\chi_B^{*,G}(\theta_B)\right]$ is non-decreasing and concave in θ_B, $G_B \geq_{MSP} H_B$ implies

$$\mathbb{E}^G_{\Theta_B}\left[E^+_{\mathcal{X}_B}\left[\chi_B^{*,G}(\theta_B)\right]\right] - \mathbb{E}^H_{\Theta_B}\left[\mathbb{E}^+_{\mathcal{X}_B}\left[\chi_B^{*,G}(\theta_B)\right]\right] \leq 0.$$

Using analogous arguments, if $(*)$ holds true, it follows that:

$$0 \leq \chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B) \leq \frac{1-\theta_B}{\theta_B}\mathbb{E}^G_{\Theta_A}\left[\chi_A^{*,H}(\theta_A) - \chi_A^{*,G}(\theta_A)\right]. \qquad (18)$$

Notice that

$$\chi_B^{*,H}(\theta_B) - \chi_B^{*,G}(\theta_B) = -\frac{1-\theta_B}{\theta_B}\mathbb{E}^G_{\Theta_A}\left[E^+_{\mathcal{X}_A}\left[\chi_A^{*,H}(\theta_A)\right] - E^+_{\mathcal{X}_A}\left[\chi_A^{*,G}(\theta_A)\right]\right].$$

The expectation on the rhs is constant in θ_B and, under assumption $(*)$, positive. This implies that, since $-\frac{1-\theta_B}{\theta_B}$ is non-decreasing and concave in θ_B, so is $\chi_B^{*,H}(\theta_B) - \chi_B^{*,G}(\theta_B)$.

Combining Equation 18 with Equation 17 yields

$$\begin{aligned}
\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B) \leq & \mathbb{E}^G_{\Theta_A}\left[\mathbb{E}^H_{\Theta_B}\left[\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B)\right]\right] \qquad (19) \\
= & \mathbb{E}^H_{\Theta_B}\left[\chi_B^{*,G}(\theta_B) - \chi_B^{*,H}(\theta_B)\right].
\end{aligned}$$

This inequality states that the expectation of $\chi_B^{*,G}(\theta_B)-\chi_B^{*,H}(\theta_B)$ given distribution H_B is greater than $\chi_B^{*,G}(\theta_B)-\chi_B^{*,H}(\theta_B)$ for every θ_B. This statement obviously cannot be satisfied, unless agents' cutoff-functions are constant across preference types. Moreover, if any of the inequalities in $(*)$ are strict on a non-zero measure set of types, the inequality in Equation 19 is strict. We obtain a contradiction to $(*)$, which completes the proof. □

Proof of Proposition 1. Consider a committee of experts with partisanship levels $\theta_A,\ \theta_B \in \left[\frac{1}{2}, 1\right]$.

Suppose agent B adopts strategy σ_B with corresponding (measurable) acceptance set B^+. The expected payoff of the alternative for agent A, conditional on being pivotal, as a function of his private signal x_A is:

$$V_A(x_A, \sigma_B) = \theta_A x_A + (1-\theta_A)E\left[x_B | x_B \in B^+\right]. \tag{20}$$

It is agent A's best response to vote in favor of the reform if and only if the expected payoff of the reform, conditional on him being pivotal, is greater than the payoff of the status quo, which is 0. Consequently, agent A votes affirmatively if and only if $V_A(x_A, \sigma_B) \geq 0$. It is easy to see from Equation 20 that $V_A(x_A, \sigma_B)$ is continuous and strictly increasing in x_A. This readily establishes that if agents do not adopt weakly dominated strategies, agent A's best response is to follow a cutoff-strategy. The same reasoning applies to agent B's best response.

From now on we restrict attention to agents' adopting cutoff strategies. By a slight abuse of notation we denote an agents' cutoff strategy by the relevant threshold. That is, $\hat{x}_B$ is agent B's cutoff strategy where he votes affirmatively if and only if $x_B \geq \hat{x}_B$.

Note that, $E\left[X|X \geq \underline{x}\right] = E(X) = 0$ and $E\left[X|X \geq \overline{x}\right] = \overline{x}$. Moreover, on $(\underline{x}, \overline{x})$, $E\left[X|X \geq \hat{x}\right]$ is (strictly) increasing in $\hat{x}$.[35] Thus, $E\left[X|X \geq \hat{x}\right] \in [0, \overline{x}]$. When combined with Equation 20 and the observation that it is agent A's best response to vote affirmatively if and only if $V_A(x_A, \hat{x}_B) \geq 0$, we obtain

[35]Indeed,

$$\frac{\partial}{\partial x}E\left[X|X \geq x\right] = \frac{f(x)}{1-F(x)}\left(E\left[X|X \geq x\right] - x\right) \geq 0 \tag{21}$$

that agent A's best-response function is characterized by:

$$x_A^{BR}(\hat{x}_B) = -\frac{1-\theta_A}{\theta_A} E\left[X_B | X_B \geq \hat{x}_B\right]. \tag{22}$$

Since $\frac{1-\theta_A}{\theta_A} \in [0,1]$ and $E\left[X | X \geq \hat{x}\right] \in [0, \overline{x}]$ it follows that $x_A^{BR}(\hat{x}_B) \in X_A$, for every $\hat{x}_B \in X_B$.

The discussion shows that, for any cutoff-strategy of agent B, there exists a unique best response of agent A and vice versa. It follows that the best response correspondence, $B(\cdot)$, is a function:

$$\begin{aligned} B:\ X_A \times X_B &\longrightarrow X_A \times X_B \\ (\hat{x}_A,\, \hat{x}_B) &\longmapsto \left(x_A^{BR}(\hat{x}_B),\, x_B^{BR}(\hat{x}_A)\right) \end{aligned}$$

We use a simple fixed-point argument to show equilibrium existence:

Note that $X = [\underline{x},\, \overline{x}]$ with $\underline{x} < 0 < \overline{x}$. Hence, X non-empty and compact, and so is $X \times X$ (by Tychonoff's theorem). Moreover, $X \times X$ is a regular polygon in $\mathbb{R}^2$ and thus convex. The best response function $B(\cdot, \cdot)$ is continuous for which it suffices to show continuity for each of the coordinate functions. Consider

$$\begin{aligned} B_A(\cdot, \cdot) : X \times X &\longrightarrow X \\ (\hat{x}_A, \hat{x}_B) &\longmapsto x_A^{BR}(\hat{x}_B). \end{aligned}$$

It is easy to see that this function is constant in $\hat{x}_A$ and monotone decreasing in $\hat{x}_B$ (cf. Equation 22). This implies that $B_A(\cdot, \cdot)$ is continuous and analogous arguments show continuity of $B_B(\cdot, \cdot)$.

We can finally apply Brouwer's fixed point theorem to establishes the existence of a fixed point of $B(\cdot, \cdot)$. This completes the proof equilibrium existence. □

Proof of Lemma 4. As seen above, agents best responses are characterized by Equation 22. Note that, if $\theta_A \neq 0 \vee \hat{x}_B \neq \overline{x}$, then $x_A^{BR}(\hat{x}_B) \in (\underline{x}, \overline{x})$, and the same holds true for agent B.
If $\theta_A = 0$, $x_A^{BR}(\overline{x}) = -\overline{x} = \underline{x}$. But $x_B^{BR}(\underline{x}) = 0$ for every $\theta_B \in [0,1]$. Since, in equilibrium, agents' strategies have to be mutually best responses to each other this shows that even for $\theta_A = 0$ equilibrium cutoffs have to be interior

which proves the first statement of the lemma.

Non-positive cutoffs: Equilibrium existence was already established in Proposition 1. Moreover, since equilibrium cutoffs are interior and $E[X|X \geq \hat{x}]$ is increasing in $\hat{x}$, $E[X|X \geq x_B^*] \in (0, \overline{x})$. That is, under unanimity voting, conditional on being pivotal, agent A's posterior estimate of agent B's signal is positive. From the equilibrium characterizing equations (Equation 7 and Equation 8) we obtain: $x_A^*, x_B^* \leq 0$. That is, under unanimity voting, agents adopt non-positive cutoffs in equilibrium. □

Proof of Proposition 2. Equilibrium existence was already established in Proposition 1. We only have to show that under Definition 2 the equilibrium is unique. By Lemma 4 equilibrium cutoffs are non-positive, that is, $x_A^*, x_B^* \leq 0$.

Now, assume there exist multiple equilibria and assume two of them are characterized by the distinct cutoff profiles (x_i^*, x_j^*) and $(x_i^* + \delta_i, x_j^* + \delta_j)$. Both of these cutoff profiles have to satisfy the equilibrium conditions Equation 7 and Equation 8. Combining them (i.e. subtracting them from each other) we obtain:

$$(1 - \frac{\theta_i}{2})\delta_i + \frac{\theta_i}{2}\left[E^+(x_j^* + \delta_j) - E^+(x_j^*)\right] = 0 \tag{23}$$

$$(1 - \frac{\theta_j}{2})\delta_j + \frac{\theta_j}{2}\left[E^+(x_i^* + \delta_i) - E^+(x_i^*)\right] = 0 \tag{24}$$

There exist multiple equilibria if and only if this system of equations has a nontrivial solution (i.e. a solution $(\delta_i, \delta_j) \neq (0, 0)$).

We can re-write Equation 23 and Equation 24 as:

$$\delta_i = -\frac{\theta_i}{2 - \theta_i}\left[E^+(x_j^* + \delta_j) - E^+(x_j^*)\right] \quad \text{and}$$
$$\delta_j = -\frac{\theta_j}{2 - \theta_j}\left[E^+(x_i^* + \delta_i) - E^+(x_i^*)\right]$$

Now, since $c = \min\{\frac{2-\theta_1}{\theta_1}, \frac{2-\theta_2}{\theta_2}\}$ it follows that $\max\{c \cdot \frac{\theta_i}{2-\theta_i}, c \cdot \frac{\theta_j}{2-\theta_j}\} = 1$. Assumption (A1$^{\text{s}}$) implies that $|E^+(x) - E^+(y)| < c|x - y|$ for all $x, y \in$

$\left[\underline{x}^{BR}, 0\right]$ which yields:

$$\begin{aligned}|\delta_i| &= | - \frac{\theta_i}{2-\theta_i}\left[E^+(x_j^* + \delta_j) - E^+(x_j^*)\right]| \\ &< |\delta_j| \\ &= | - \frac{\theta_j}{2-\theta_j}\left[E^+(x_i^* + \delta_i) - E^+(x_i^*)\right]| \\ &< |\delta_i|\end{aligned}$$

a contradiction. Thus, no multiple equilibria can exist. □

Proof of Proposition 3. From Equation 7 and Equation 8 which characterize the equilibrium cutoffs, taking the derivative with respect to θ_i on both sides, we obtain

$$\begin{aligned}\frac{\mathrm{d}V_i(x_i^*, x_j^*; (\theta_i, \theta_j))}{\mathrm{d}\theta_i} &= -\frac{1}{2}x_i^* + \frac{1}{2}E^+(x_j^*) + (1 - \frac{\theta_i}{2})x_{i,\theta_i}^* + \frac{\theta_i}{2}E_{x_j^*}^+(x_j^*)x_{j,\theta_i}^* = 0 \\ \frac{\mathrm{d}V_j(x_i^*, x_j^*; (\theta_i, \theta_j))}{\mathrm{d}\theta_i} &= (1 - \frac{\theta_j}{2})x_{j,\theta_i}^* + \frac{\theta_j}{2}E_{x_i^*}^+(x_i^*)x_{i,\theta_i}^* = 0 \\ \Leftrightarrow \quad x_{j,\theta_i}^* &= -\frac{\theta_j}{2-\theta_j}E_{x_i^*}^+(x_i^*)x_{i,\theta_i}^* \end{aligned} \tag{25}$$

Combining these equations we obtain

$$\underbrace{\frac{1}{2}\left(\underbrace{-x_i^*}_{\geq 0} + \underbrace{E^+(x_j^*)}_{>0}\right)}_{:=G(x^*)>0} + \underbrace{\left[(1 - \frac{\theta_i}{2}) - \frac{\theta_i\theta_j}{2(2-\theta_j)}E_{x_j^*}^+(x_j^*)E_{x_i^*}^+(x_i^*)\right]}_{:=H} x_{i,\theta_i}^* = 0 \tag{26}$$

Given assumption (A1$^{\mathrm{s}}$), $E^{+\prime}(\hat{x}) < c = \min\{\frac{2-\theta_1}{\theta_1}, \frac{2-\theta_2}{\theta_2}\}$, $\forall \hat{x} \in [\underline{x}, 0]$, we obtain

$$\begin{aligned}H(x^*) &= 1 - \frac{\theta_i}{2}\left[1 + \overbrace{\frac{\theta_j}{2-\theta_j}E_{x_j^*}^+(x_j^*)}^{\in[0,1]}\overbrace{E_{x_i^*}^+(x_i^*)}^{\in[0,c)}\right] \\ &> 1 - \frac{\theta_i}{2}\left[1 + \frac{2-\theta_i}{\theta_i}\right] = 0\end{aligned}$$

Finally, Equation 26 yields

$$x^*_{i,\theta_i} = -\frac{G(x^*)}{H(x^*)} < 0$$

which shows that agent i's equilibrium cutoff x^*_i is decreasing in θ_i. Moreover, from Equation 25 we obtain:

$$x^*_{j,\theta_i} = -\frac{\theta_j}{2-\theta_j} \underbrace{E^+_{x^*_i}(x^*_i)}_{>0} \underbrace{x^*_{i,\theta_i}}_{<0} > 0$$

That is, agent j's equilibrium cutoff is increasing in θ_i. □

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