Mediated Coordination with Restricted Private Communication

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1 Introduction

It has been shown that preplay communication with a trust worthy mediator can make players substantially better off in games of both complete and incomplete information. The correlated equilibrium (Aumann(1974)) and communication equilibrium (see Myerson(1986) and Forges(1986)) concepts allow players to expand the set of equilibrium outcomes in such settings well beyond that of independent play (*i.e.* Nash equilibria) whenever such mediated communication is available. While there has been an extensive line of literature focusing on how to achieve such outcomes by removing or replacing the mediator with some subset of players of the game (e.g. see Girardi(2004)), the assumption that has been maintained throughout is that the mediator, or its equivalent subset of players, can communicate privately and directly with the other players of the game. This paper looks to relax this assumption by considering a private communication network $\mathcal{N}$ where the mediator and the players of the game represent the vertices and the (possibly directed) edges represent the private communication channels. I then characterize the necessary and sufficient conditions on the network $\mathcal{N}$ such that any correlated equilibrium can be implemented as a sub game perfect equilibrium of the game in question augmented by a finite preplay cheap talk communication phase.

2 Results

To formally state the problem, consider a finite normal form game $\Gamma = (I, (A_i)_{i\in I}, (u_i)_{i\in I})$ with a trustworthy mediator M, and a communication network $\mathcal{N} = (V, E)$ with the set of vertices $V = I \cup M$ and a set $E \subset \{ij : i \in V, j \in V\}$ of directed edges. Throughout we will only consider private communication and assume that player $i \in V$ can send messages to player $j \in V$ if and only if $ij \in E$. Now, let Q be a correlated equilibrium (CE) of Γ and $\mathcal{P}(\mathcal{N}) = (T, \rho, \hat{\sigma})$ a finite cheap talk communication protocol over the network $\mathcal{N}$ consisting of T periods of communication, a history dependent *communication strategy* profile $\rho = (\rho_i)_{i\in I}$, and a time T history dependent *action strategy* profile $\hat{\sigma} = (\hat{\sigma}_i)_{i\in I}$ of Γ. Then, we say that $\mathcal{P}(\mathcal{N})$ *implements* Q *on* $\mathcal{N}$ if it satisfies the conditions that $i.)$ $(\rho, \hat{\sigma})$ is a sub game perfect equilibrium of the game Γ augmented by T periods of preplay cheap talk communication and $ii.)$ $\mathbb{P}_\rho(\hat{\sigma} = a) = Q(a)$. The question we are interested in is what are the necessary and sufficient conditions on the network $\mathcal{N}$ such that for any game Γ and correlated equilibrium Q of Γ, there exists a protocol $\mathcal{P}(\mathcal{N})$ that implements Q on $\mathcal{N}$.

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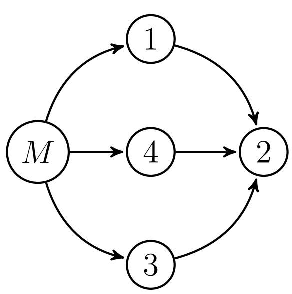
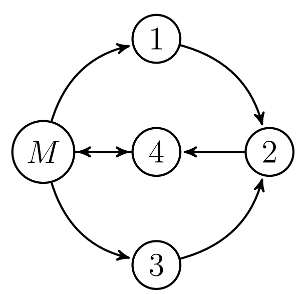
Before stating the main results we need to introduce a definition on the connectivity of the network $\mathcal{N}$. Namely, we say that $\mathcal{N}$ is *k-connected from $i \in V$ to $j \in V$* if there exist k vertex disjoint directed paths from i to j. Clearly then a necessary condition for the existence of a protocol $\mathcal{P}(\mathcal{N})$ that implements any CE on $\mathcal{N}$ is that the network $\mathcal{N}$ be 1-connected from M to i for all $i \in I$ so that M can feasibly (albeit indirectly) communicate the strategy suggestion $\tilde{a}_i$ to each player $i \in I$. Further, if we denote by $C(M) = \{i \in I | Mi \in E\}$ the set of direct successors to M, then whenever $\mathcal{N}$ satisfies the condition that $C(M) = I$ we are in the trivial case where all CE can be implemented on $\mathcal{N}$ via direct private suggestions from M. Finally, for the moment we assume that the set of feasible messages, $\mathcal{M}$, that players can send in the communication phase is some arbitrary but infinite set. We are now ready to state our main results.

Theorem 1 Let Γ be a finite game, Q a CE of Γ, and $\mathcal{N} = (I \cup M, E)$ a directed communication network. Then there exists a CT protocol $\mathcal{P}(\mathcal{N})$ that implements Q on $\mathcal{N}$ if and only if for all $i \in I \backslash C(M)$, $\mathcal{N}$ satisfies one of the following two conditions:
(1) $\mathcal{N}$ is 3-connected from M to i.
(2) $\mathcal{N}$ is 2-connected from M to i and 1-connected from i to M with all three connecting paths disjoint.

Two networks satisfying conditions (1) and (2) respectively are illustrated in figure 1. This theorem states the conditions under which it is possible to send the suggested strategy to every player $i \in I$ that is not a direct successor to the mediator without provoking any unilateral deviations from neither the communication strategy ρ, nor the action strategy $\hat{\sigma}$. As we will see, these conditions guarantee *perfect resiliance* and *perfect security*. Namely, for any unilateral deviation in the communication phase, player $i \in I \backslash C(M)$ receives their correct suggested strategy with probability 1, and the probability that any player $j \in I \backslash \{i\}$ learns the suggested strategy of player i is zero. I will now provide a short sketch of the proof of theorem 1.



Network satisfying condition (1).



Network satisfying condition (2).

Figure 1: A pair of networks of a 4-player game with $I \backslash C(M) = \{2\}$.

Sketch of proof. ($\Rightarrow$) The proof of sufficiency uses a preliminary result regarding modular arithmetic. Namely if $x \sim U[0, 1)$ and $y \in [0, 1)$ then $x \oplus y \sim U[0, 1)$ where $U[0, 1)$ is the uniform distribution over $[0, 1)$ and $\oplus$ represents addition modulus 1.[1] Using this result, I construct for each of the two cases of theorem 1 a protocol $\mathcal{P}(\mathcal{N})$ that uses an encryption/decryption scheme such that any message that player $j \in I$ is required to forward is uniformly distributed on $[0, 1)$ given j's information at that time. Hence, player j does not learn anything about the strategy suggestion

[1]Here we write $x \oplus y$ to mean;

$$x \oplus y = \begin{cases} x + y & \text{if } x + y < 1 \\ x + y - 1 & \text{otherwise.} \end{cases}$$

of any player $i \in I\backslash C(M)$ given the messages they are required to forward. Further, I construct the protocols such that given any unilateral deviation in the communication phase made by some player $j \in I\backslash\{i\}$, player $i \in I\backslash C(M)$ still receives the correct suggested strategy from the players on the other path(s) connecting M to i, not containing j. Further, the probability that i receives a *different* suggested strategy from the path containing player j is zero. Hence, player $i \in I\backslash C(M)$ plays the only strategy that is suggested to her, which is the correct suggested strategy with probability 1, and therefore such a deviation cannot be profitable for any player $j \in I\backslash\{i\}$. $\square$

Sketch of proof. ($\Leftarrow$) The proof of necessity relies on a preliminary *reduction* result that states that whenever there exists a protocol $\mathcal{P}(\mathcal{N})$ that implements some CE Q on $\mathcal{N}$, then one can construct another protocol $\hat{\mathcal{P}}(\mathcal{N})$ that also implements Q on $\mathcal{N}$ and (i) only requires players in $C(M)$ to forward messages between M and the players $I\backslash C(M)$ $(i.e.\ j \in C(M)$ does not generate any messages), and (ii) does not require any two players in $C(M)$ to communicate with each other. Then, I construct a game Γ and CE Q that cannot be implemented on a particular network $\mathcal{N}^*$ such that any strengthening of the connectivity of $\mathcal{N}^*$ results in a network satisfying one of the conditions of theorem 1[2].

Now I will give a rough intuition as to why there exists a CE Q that cannot be implemented on the aforementioned network $\mathcal{N}^*$. Namely, I construct $\mathcal{N}^*$ such that $I\backslash C(M) = \{i\}$ and then I find a game Γ and CE Q with two key properties. First, there exists $j \in I\backslash\{i\}$ such that if with any positive probability (as determined by the protocol when implementing Q) player j ever learns the strategy suggested to player i, then player j has a profitable deviation in the play phase. Second, there exists $k \in I\backslash\{i, j\}$ such that if with any positive probability player i learns the incorrect suggested strategy via some deviation from ρ_k, then this is a profitable deviation for player k. I then show that based on the network $\mathcal{N}^*$ and the construction of the game Γ and CE Q, that the only way to prevent both of the aforementioned deviations is for the protocol to allow player i to *report* any deviation made by player k, resulting in a strategy profile that punishes player k. Finally, whenever this is the case, I show that player i has a profitable deviation to report that a deviation has been made by player k in every realization of the protocol, as there exists no strategy profile that punishes player k without making this deviation profitable for player i. $\square$

3 Conclusion

This paper characterizes the necessary and sufficient conditions under which we can implement any correlated equilibrium on a private communication network with a mediator who has restricted private communication possibilities. If one thinks of the mediator as the principle decision maker of an organization and the private communication channels as costly flows of information, then this paper shows that when facing conflicts of interest among its privately informed agents, the principle must face a tradeoff between the set of equilibrium outcomes they can achieve and the cost of the flows of information they send and receive. In fact, the results above state that if we are interested in a 3-player game Γ, and we consider two-way communication channels (i.e. undirected edges), then restricting private communication between the mediator M and any player $i \in I$ results in a network $\mathcal{N}$ where we cannot guarantee implementation of *any* correlated equilibrium on $\mathcal{N}$. Hence, the choice of which costly communication channels to maintain could be of considerable strategic interest in even the simplest of organizations. Finally, we note that when we restrict our attention to cheap talk protocols with finite message spaces, while the above conditions are still necessary, they are no longer sufficient. Hence, the problem of implementation with a mediator becomes even more difficult when we make this simple but realistic restriction.

[2]In particular, I construct a CE Q that cannot be implemented on a network $\mathcal{N}^*$ that is strongly 2-connected from M to $i \in I\backslash C(M)$, strongly 2-connected from $i \in I\backslash C(M)$ to M, and such that only 2 of the 4 connecting paths are disjoint. Then, using a result that 2-connectivity from M to i is a necessary condition for implementation, we can see that strengthening the connectivity of $\mathcal{N}^*$ requires adding a disjoint directed path either from M to i or from i to M. Adding either type of path to $\mathcal{N}^*$ results in a network satisfying conditions (1) and (2) respectively.

References

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