Low Risk-free Rates, Competition, and Bank Lending Booms

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Macro Workshop

April 2, 2014

Motivation

- AN ON-GOING DEBATE
  Was the low risk-free rate environment during 2002–2006 an important reason for the credit boom leading up to the subprime crisis?
  Generally, is the risk-taking channel of monetary policy important?
  - Recent empirical work show the importance of such a channel.
    - Maddaloni & Peydro (2011), Ioannidou et al. (2009), Altunbas et al. (2010), Jimenez et al. (2014).
- One particular mechanism of the risk-taking channel:
  Lower risk-free rate $\Rightarrow$ banks compete more $\Rightarrow$ take more risk.
- QUESTION
  How to understand such a competition mechanism in theory?

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Overview of this paper

- Based on my work (Liu, 2014), I study the implications of time-varying risk-free rate on the dynamics of bank lending and competition.
- I model bank's risk-taking behavior through endogenously chosen lending standards.
    - In most works on risk-taking, banks choose risk *level* directly.
- MAIN RESULTS
    - An inverse U relationship between risk-taking and risk-free rate $\Rightarrow$ very low risk-free rate leads to high risk via low lending standards.
    - Commitment to low risk-free rate over an extended period leads to more risk-taking over time.

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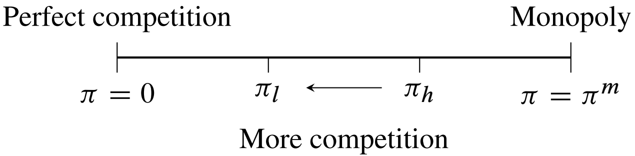
My interpretation of changes in competition

Let π be the realized profit rate of a bank in a market where a (small) number of banks compete with each other



Basic model setup

- TIME: discrete, infinite horizon.
- AGENTS: unit-mass borrowers; N identical banks, $i = 1, \ldots, N$.
- TECHNOLOGY: borrowers invest by borrowing from banks; banks screen "bad" borrowers from "good" ones.
- PREFERENCES: both agents are risk-neutral, maximizing discount value of payoffs in each period.
- AGGREGATE SHOCK: (gross) risk-free rate shock $r_{f,t} \sim$ Markovian.

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Borrower's investment opportunity

- In every period, each borrower draws an idiosyncratic shock $\theta \in \{\theta^g, \theta^b\} \equiv \{g, b\}$ with $\theta^g > \theta^b$, iid over time and borrowers.
- θ is a borrower's private information.
- Each θ borrower has a 1-period investment opportunity:

with $x > 1 > c$.

- Per unit $\text{NPV}^\theta \equiv \frac{\theta x + (1-\theta)c - r_f}{r_f} = \frac{\theta x + (1-\theta)c}{r_f} - 1$.

  Assume $\text{NPV}^g > 0 > \text{NPV}^b, \forall r_f$.

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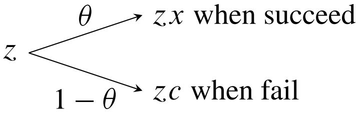

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$$z \begin{cases} \xrightarrow{\theta} zx \text{ when succeed} \\ \xrightarrow{1-\theta} zc \text{ when fail} \end{cases} \quad \text{with } x > 1 > c.$$

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Bank financing

- Borrowers have limited liability, but no resource for investment.
- Each bank publicly offers a loan contract $\ell = (r, q)$.
  r: gross interest rate; q: screening intensity.
- Each borrower applies for a loan of a given size z from one bank.
- If lending to a θ borrower at $r \geq 1$, bank's unit (expected) payoff:

$$\eta^{\theta}(r, r_f) = \frac{\theta r + (1-\theta)c}{r_f} - 1.$$

  And borrower's unit (expected) payoff: $\theta(x - r)/r_f$.

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Costly screening

- As $\text{NPV}^b < 0$, lending to bad borrowers is never profitable.
- Banks distinguish good borrowers from bad ones by screening.
- At intensity q, screening generates a signal $\phi \in \{G, B\}$ satisfying

$$\Pr(\phi = G|\theta = g) = \Pr(\phi = B|\theta = b) = q \in \left[\tfrac{1}{2}, 1\right].$$

  q determines posterior probability $\Pr(g|\phi, q)$ via Bayes law.
- Banks pay a cost $zC(q)$ for screening a borrower at intensity q. $C(q)$ is convex in q.

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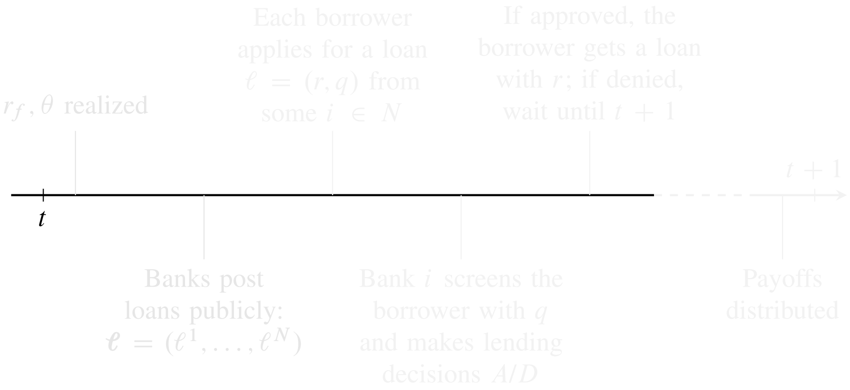

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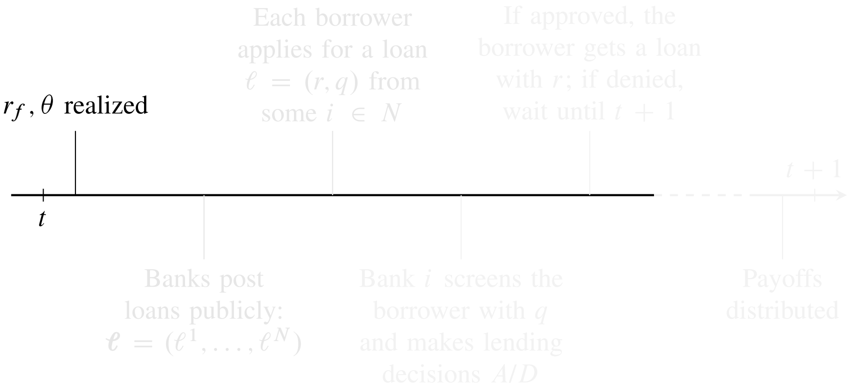

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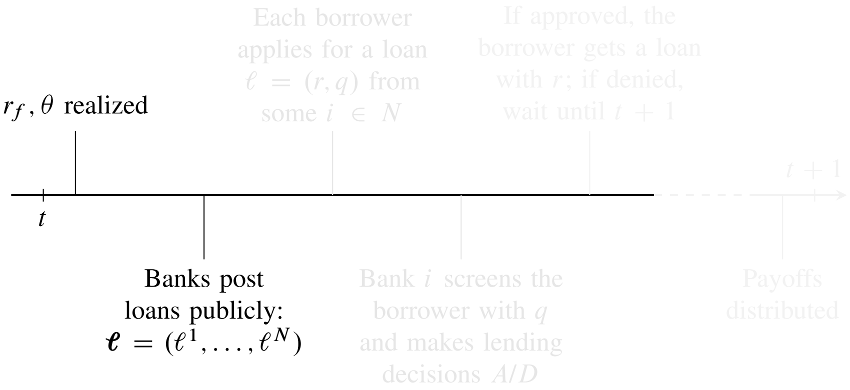

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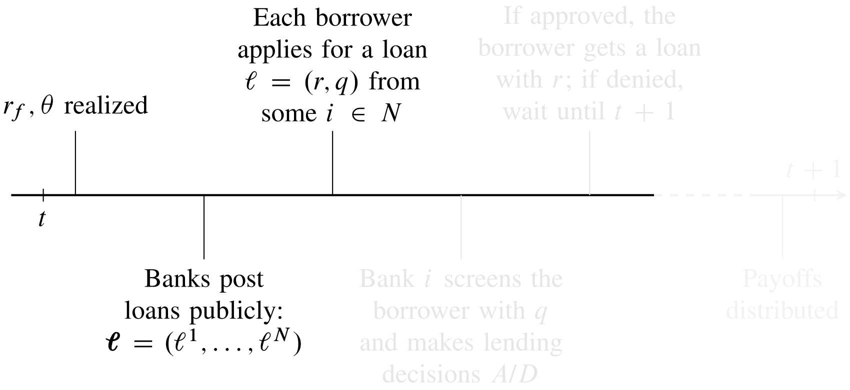

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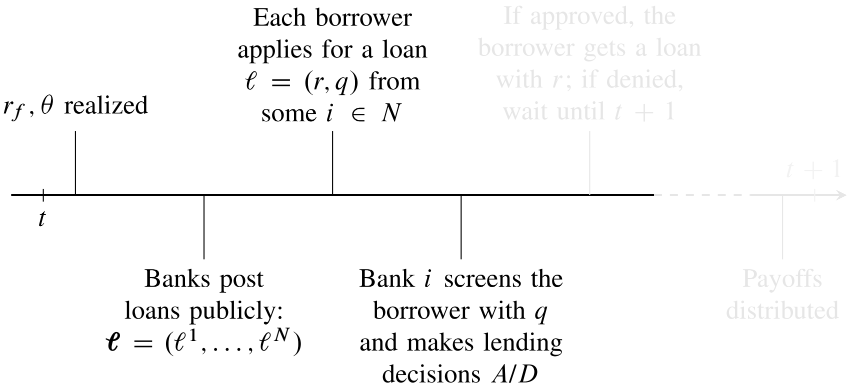

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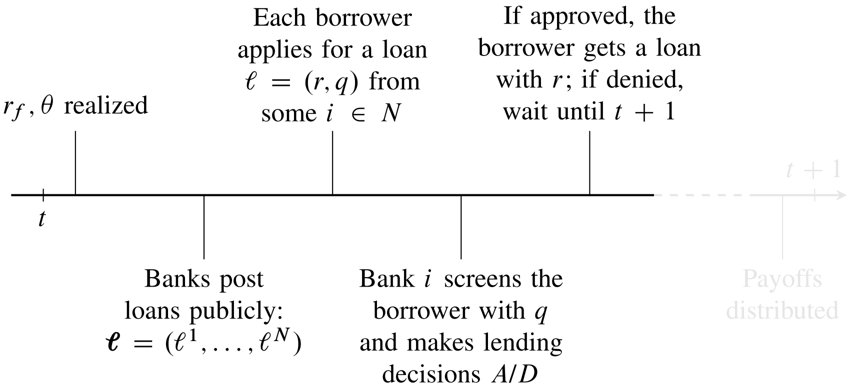

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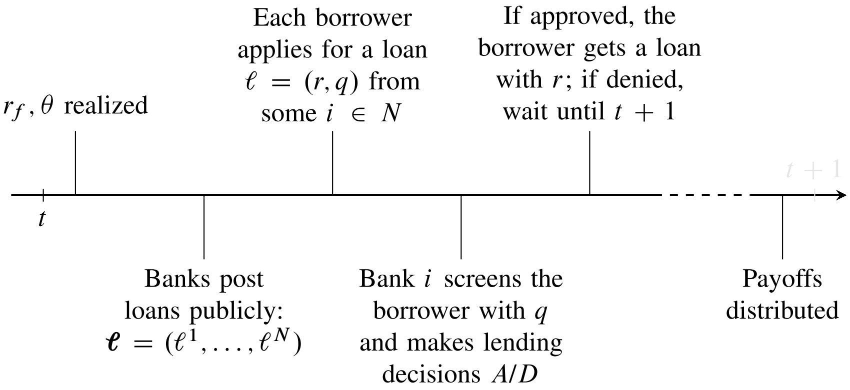

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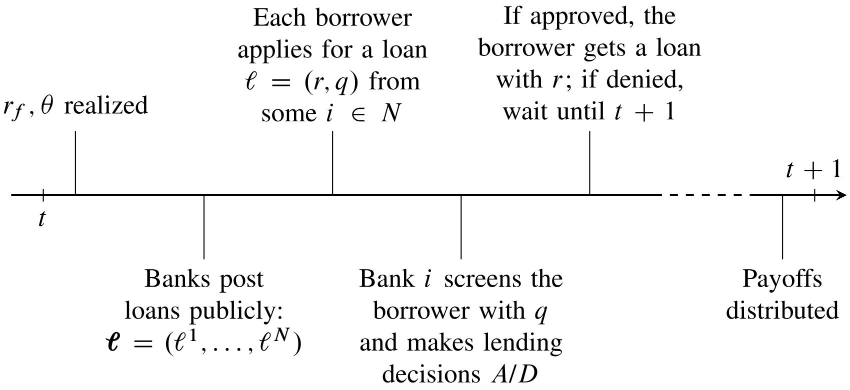

Formulating the lending game

- A θ borrower's strategy: choose which ℓ^i to apply for given $\boldsymbol{\ell}$.
- A θ borrower's unit payoff of choosing $\ell^i = (r^i, q^i)$:

$$\frac{\theta(x - r^i)}{r_f} p^\theta(\ell^i), \quad p^\theta(\ell) : \text{ approval probability.}$$

- Bank i's strategy: first choose what ℓ^i to post, then choose whether to approve or deny the loan when a borrower applies for ℓ^i.

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Formulating the lending game (contd.)

- Recall that $\eta^g(r, r_f) = [\theta^g r + (1-\theta)c]/r_f - 1$; also $\eta^b(r, r_f)$.
- Bank's unit payoff from lending to a ϕ borrower at $\ell = (r, q)$:

$$\eta^\phi(\ell, r_f) = \Pr(g|\phi, q)\eta^g(r, r_f) + \Pr(b|\phi, q)\eta^b(r, r_f).$$

- Bank i's net unit payoff if borrowers apply for ℓ^i:

$$\eta(\ell^i, r_f) - C(q^i) \quad \text{where}$$

$$\eta(\ell^i, r_f) = \Pr(G)\max\{\eta^G(\ell^i, r_f), 0\} + \Pr(B)\max\{\eta^B(\ell^i, r_f), 0\}.$$

- This lending game is repeated over time.

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Determination of lending standard

- Under mild parametric assumptions, banks always choose an efficient q^e such that all B borrowers are denied credit.
- Moreover, given r, such q^e solves

$$\max_q \eta(\ell, r_f) - C(q) = \eta(r, q, r_f) - C(q),$$

and $q^e(r, r_f)$ is increasing in r and decreasing in r_f.

- If borrowers choose $\ell = (r, q^e(r, r_f))$, the profit rate function is

$$\pi(\ell, r_f) = \eta(r, q^e(r, r_f), r_f) - C(q^e(r, r_f)),$$

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Bank's profit function

- Given $\ell = (r, q^e(r, r_f))$, payoffs to good and bad borrowers are

$$[\theta^g(x-r)/r_f]q^e(r, r_f) \quad \text{and} \quad [\theta^b(x-r)/r_f](1-q^e(r)).$$

  Moreover, both types of borrowers prefer a loan with a lower r.

- Let $\boldsymbol{\ell} = (\ell^1, \ldots, \ell^N)$ where $\ell^i = (r^i, q^e(r^i, r_f))$, and let $\ell_{\min}$ be the contract with minimum r.
- Bank's i's profit function:

$$\Pi^i(\boldsymbol{\ell}, r_f) = \begin{cases} \frac{1}{N_{\min}}\pi(\ell_{\min}, r_f), & \text{if } \ell^i = \ell_{\min}; \\ 0, & \text{otherwise.} \end{cases}$$

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Repeated game: a quick view details

- The repeated lending game can be solved as a repeated game among N banks with profit function $\Pi^i(\boldsymbol{\ell}, r_f)$.
- I focus on the optimal symmetric subgame-perfect equilibrium (SSPE).
- I use standard results in the repeated game literature to solve the model. In particular, it is sufficient to consider grim trigger strategy.
- Let $0 < \delta < 1$ denote the common discount factor for all banks.

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Equilibrium dynamics: iid $\{r_{f,t}\}$

Suppose $r_f \in [1, R] \sim F(r_f)$

Proposition

There exists $\bar{\delta}$ *such that if* $1 - \frac{1}{N} \leq \delta < \bar{\delta}$, *such that in the optimal SSPE*

- $r^*(r_f)$ *is decreasing in* r_f;
- $q^*(r_f) = q^e(r^*(r_f), r_f)$ *is first increasing then decreasing in* r_f;
- $\pi^*(r_f)/\pi^m(r_f)$ *is increasing in* r_f, *where* $\pi^*(r_f) = \pi(r^*(r_f), q^*(r_f), r_f)$ *and* $\pi^m(r_f)$ *is the monopoly profit rate.*

Intuition

- With iid shock, the equilibrium incentive constraints are

$$N\pi(\ell, r_f) \leq \pi(\ell, r_f) + \frac{\delta}{1-\delta}\mathbb{E}\pi(\ell, r_f), \quad \forall r_f$$

- Low r_f causes deviation incentive to be high
$\Rightarrow$ monopoly profit rate can no be sustained for small r_f.
- Banks compete away this extra profit (relative to π^m) by lowering r.
- However, as $\partial_1 q^e(r, r_f) > 0 > \partial_2 q^e(r, r_f)$, only for very small r_f

$$\frac{\mathrm{d}q^*(r_f)}{\mathrm{d}r_f} = \frac{\mathrm{d}}{\mathrm{d}r_f} q^e(r^*(r_f), r_f) > 0.$$

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An extension to persistent shock

Suppose $r_{f,t+1} = (1-\rho)\bar{r}_f + \rho r_{f,t} + \epsilon_{t+1}$.

Proposition

There exists $\bar{\delta}$ such that if $1 - \frac{1}{N} \leq \delta < \bar{\delta}$, such that in the optimal SSPE

- $r^*(r_f)$ *is decreasing in* r_f;
- $q^*(r_f) = q^e(r^*(r_f), r_f)$ *is first increasing then decreasing in* r_f;
- $\pi^*(r_f)/\pi^m(r_f)$ *is increasing in* r_f.

Commitment to low r_f

Suppose the central bank commits to low risk-free rate over a period:

$$r_{f,t} \begin{cases} = 1, & t = 0, \dots, T \\ \sim F(\cdot), & t = T+1, \dots, \infty \end{cases}$$

Proposition

For δ such that $1 - \frac{1}{N} \leq \delta < \bar{\delta}$, the optimal SSPE is characterized by

$$r_0^{**} > r_1^{**} > \cdots > r_T^{**} = r^*(1), \quad q_0^{**} > q_1^{**} > \cdots > q_T^{**} = q^*(1)$$

in the first $T+1$ periods, and $r^*(r_f)$, $q^*(r_f)$ since $T+1$.

To sum up

- An inverse U relationship between risk-taking and risk-free rate $\Rightarrow$ very low risk-free rate leads to high risk via low lending standards.
- Commitment to low risk-free rate over an extended period leads to more risk-taking over time within the commitment period.

Repeated game: strategy and equilibrium back

- History — $h^t = (s_t, \boldsymbol{\ell}_{t-1}, s_{t-1}, \ldots, \boldsymbol{\ell}_0, s_0)$, $\{s_t\}$ Markov.
- Strategy — $\sigma_t^i : h^t \mapsto \ell^i \in \mathscr{C}^e(s_t)$, $\sigma_t = \left(\sigma_t^i\right)_{i \in N}$, $\sigma = \{\sigma_t\}_{t=0}^{\infty}$.
- Common discount factor $0 < \delta < 1$, discount value under $\sigma | h^t$

$$V^i\left(\sigma \middle| h^t\right) = \mathbb{E}^\sigma \left[\sum_{\tau=0}^{\infty} \delta^\tau \Pi^i\left(\boldsymbol{\ell}_{t+\tau}; s_{t+\tau}\right) \middle| h^t \right].$$

- Equilibrium: optimal symmetric subgame-perfect equilibrium (SSPE).
  — collusive equilibrium.

Optimal SSPE and grim trigger strategy

- Following standard results in the repeated game literature.
- Suffice to use symmetric grim trigger strategy:
  — $\ell^*(s) = (R^*(s), q^*(s))$ along the optimal equilibrium path.
  — revert to $\ell^0(s)$ from t onwards if any deviation occurs at $t-1$,
  $\Rightarrow$ 0 continuation value, i.e., optimal punishment.
- $\Pi^*(s) \equiv \Pi(\ell^*(s); s) = z(s)\pi(\ell^*(s); s)/N$ and the optimal value $\mathbb{E}V^*(s) = \mathbb{E}\Pi^*(s)/(1-\delta) = \mathbb{E}z(s)\pi(\ell^*(s); s)/[N(1-\delta)]$.

Associated maximization problem

- Optimal SSPE $\{\ell^*(s)\}$ solves $\max_{\ell^e(s) \in \mathscr{C}^e(s)} \mathbb{E}\Pi(s)$

  subject to the intertemporal incentive constraint (IIC):

$$N\Pi(s) \le \Pi(s) + \delta \mathbb{E}_s V(\ell(\cdot)|s'), \quad \forall s.$$

- Deviation incentive: $N\Pi(s) = z\pi(\ell(s))$ — infinitesimal undercutting.
- After deviation, 0 continuation value.
- Resemble to Rotemberg & Saloner '86 when s_t iid.