Preferences for Information and Ambiguity*

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Abstract

This paper studies intrinsic preferences for how information is revealed. We enrich the standard dynamic choice model in two dimensions. First, we introduce a novel choice domain that allows preferences to depend on how information is revealed. Second, conditional on a given information partition, we allow preferences over state-contingent outcomes to depart from expected utility axioms. In particular, we accommodate ambiguity sensitive preferences. We establish that a dynamically consistent decision maker (DM) is averse to partial information if and only if her static preferences satisfy a property called Event Complementarity. We show that Event Complementarity is closely related to ambiguity aversion in popular families of ambiguity preferences.

Keywords: value of information, ambiguity aversion, dynamic choice.

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1. Introduction

This paper studies the link between intrinsic preferences for information and ambiguity preferences, in an environment with subjective uncertainty. To illustrate the problem that motivates these results, consider the situation two economics Ph.D. students (say Alice and Bob) on the job market face in December. Both have submitted many job applications, and are concerned about the possible job offers they might receive the following March. Their future job outcomes depend not only on their own quality and performance, but also on uncertain factors like the quality and performance of other candidates as well as funding and tastes of different employers. Starting in late December and early January, online forums like the Blu-Wiki and Economics Job Market Rumors post interview and fly-out schedules for different schools, which provide partial information about the uncertainty. We observe Alice and Bob exhibit very different attitudes regarding this partial information. Alice checks very frequently for updates, while Bob avoids (with an obvious effort) ever looking at this partial information.

Standard dynamic subjective expected utility (SEU) theory predicts that Alice and Bob should be indifferent to the rumor information, as their optimal actions are simply to maximize their own performance no matter what the rumors are. To accommodate Alice and Bob's choices, we enrich the standard dynamic choice model in two dimensions. First, we introduce a novel choice domain that allows for ex-ante preferences over state-contingent outcomes to be indexed by the intermediate information. Second, conditional on a given information partition, we allow preferences over state-contingent outcomes to depart from expected utility axioms. In particular we accommodate ambiguity sensitive preferences. Under recursivity and additional axioms, we show the equivalence of two properties: (i) aversion to partial information—that the decision maker (DM), just like Bob, prefers having uncertainties resolved in one shot to first getting partial information and then the remaining uncertainties resolved; and (ii) Event Complementarity—that the DM prefers evaluating the state-contingent outcomes as a whole rather than evaluating them conditional on separate events. We then show that Event Complementarity and aversion to partial information are closely related to ambiguity attitudes. In familiar classes of ambiguity preferences, we identify conditions that characterize aversion to partial information.

The connection we establish between ambiguity attitudes and intrinsic preferences for partial information is important for a number of reasons. From a theoretical perspective, when ambiguity aversion implies intrinsic preferences for information, then endogenous learning and information acquisition decisions can be different from those in a standard dynamic SEU model. In particular, one criticism regarding the importance of incorporating ambiguity in the long run steady state is that in a stationary environment, ambiguity could eventually be learnt away. If learning is endogenous and ambiguity aversion undermines the incentive to collect new information, however, then ambiguity can persist in the long-run steady state. Of more direct policy relevance, recent work illustrates the importance of ambiguity in finance and macroeconomics for providing more accurate and robust dynamic measures of risk in financial positions.[1] Our results suggest that the nature and timing of information could be an important additional component to include in the design of risk measures that account for ambiguity.

To illustrate the connection between ambiguity attitudes and information preferences more explicitly, consider the classical Ellsberg Urn. The urn has 90 balls. 30 balls are red, and 60 balls are either green or yellow, with the exact proportion unknown. The decision maker (DM) places bets on the color of a ball drawn from the urn. In the static setting, a typical Ellsbergian decision maker strictly prefers betting on red to betting on green, but strictly prefer betting on the event that the ball is either green or yellow ($\{G, Y\}$), to betting on the event that the ball is either red or yellow ($\{R, Y\}$).

$$\begin{pmatrix} 1 & R \\ 0 & G \\ 0 & Y \end{pmatrix} \succ_0 \begin{pmatrix} 0 & R \\ 1 & G \\ 0 & Y \end{pmatrix} \text{ and } \begin{pmatrix} 1 & R \\ 0 & G \\ 1 & Y \end{pmatrix} \prec_0 \begin{pmatrix} 0 & R \\ 1 & G \\ 1 & Y \end{pmatrix}$$

In the classical Ellsberg paradox, the relative attractiveness of betting on red to green is reversed when yellow is also included as a winning state. One intuition for this reversal is the complementarity between G and Y: while the probabilities of single events $\{G\}$ and $\{Y\}$ are imprecise (ranging from 0 to $\frac{2}{3}$), the joint event $\{G, Y\}$ has a precise probability $\frac{2}{3}$. This complementarity is considered indicative of ambiguity (see for example, [10]).

Information can erase this complementarity. To see this, suppose now there are two periods: at the end of period 1, the DM will learn whether the drawn ball is yellow or not, and at the end of period 2, the DM will learn the exact color of the drawn ball. The period-1 partial information can be described by the partition $\pi = \{\{R, G\}, \{Y\}\}$. The top event tree in Figure 1 illustrates the corresponding dynamic information structure. Suppose when expecting information π, the DM evaluates the dynamic bets by backward

[1] For applications of ambiguity in finance and macroeconomics, see [14], [24], [4], and [27]. In addition, [9] survey applications of ambiguity preferences in finance, and [2] survey applications of ambiguity preferences in macroeconomics. For work on dynamic risk measures under ambiguity, see [37] and [1] and references therein.

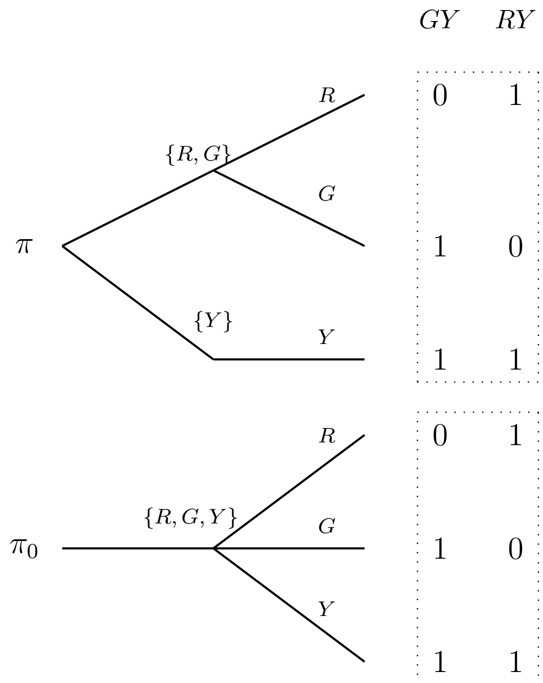

Figure 1: Event trees. GY is a bet that wins 1 if the drawn ball is either green or yellow and 0 otherwise. RY is a bet that wins 1 if the drawn ball is either red or yellow and 0 otherwise. In the top tree, the partition $\pi = \{\{R, G\}, \{Y\}\}$. In the bottom tree, the partition is the trivial no information partition $\pi_0 = \{\{R, G, Y\}\}$.

recursion: she first contemplates how she will rank acts at the end of stage 1, conditional on the realization of either event $\{R, G\}$ or event $\{Y\}$, and then aggregates these conditional preferences to form the ex-ante preferences expecting π. In this way, acts are evaluated separately for payoffs on events $\{R, G\}$ and payoffs on event $\{Y\}$, so the complementarity between G and Y is not taken into account. By partitioning the event $\{G, Y\}$ into the subevents $\{G\}$ and $\{Y\}$, information π breaks the complementarity between G and Y. On the other hand, if the DM is not told anything at the end of stage 1, an information structure illustrated by the bottom event tree π_0 in Figure 1, this complementarity is fully taken into account. So if a DM is ambiguity averse and values this complementarity, then she will prefer event tree π_0 to event tree π and exhibit an aversion to partial information in the interim stage.

Formally, we study a two-period model where state-dependent consequences are realized in the second period, and some partial information π, a partition of the state space S, is revealed in the first period. In particular, we relax *reduction*, an assumption that the

DM is indifferent to the temporal resolution of uncertainty. We do so by considering preferences on the product space of information partitions and Anscombe-Aumann acts ($\Pi \times \mathcal{F}$). So in ex-ante period 0, preferences are indexed by the expected period-1 partition π. In period 1, when an event in the partition π is revealed to contain the true state, preferences are updated conditional on this event. We consider as primitives the ex-ante and conditional preferences.

In Section 3, we give axioms under which ex-ante preferences and conditional preferences have a recursive representation. For a fixed partition π, ex-ante preferences are connected with conditional preferences through recursivity axioms. Across partitions, we characterize an updating rule that ensures all conditional preferences are derived from the same static unconditional preferences. In this way, ex-ante preferences across partitions are generated by the same static unconditional preferences and thus reflect consistent beliefs about events in S.

Under this recursive representation, we establish the equivalence between aversion to partial information in the ex-ante preferences and a property on the static unconditional preferences called Event Complementarity. We show that Event Complementarity captures the intuition of complementarity in the Ellsberg example and thus the concept of ambiguity aversion. In Section 4, we further explore the intersection between Event Complementarity (and thus preferences for partial information) and popular models of ambiguity preferences. We find that for maxmin expected utility (MEU) [16] and Choquet expected utility (CEU) [38], there is a tight connection between ambiguity aversion and aversion to partial information, and ambiguity loving and attraction to partial information. For the more general class of variational preferences [33], this connection is more delicate. For variational preferences, we identify a condition on the cost function that characterizes aversion to partial information. We also identify joint conditions on the cost function and acts that characterize local aversion to partial information at a particular act. Finally, we show that for multiplier preferences [24, 43], ex-ante preferences exhibit partial information neutrality.

In Section 5, we extend the model to allow for choices from menus after partial information is revealed, and study the value of information under ambiguity. The value of information is not monotonic, a natural implication of intrinsic aversion to partial information. We show that intrinsic aversion to partial information is equivalent to a preference for perfect information in the extended preferences.[2] We also show by a counterexample that the value of information is not monotone in the degree of ambiguity aversion.

[2]This is similar to Proposition 2 in [5].

This paper makes several novel contributions. First, we identify a connection between ambiguity attitudes and preferences for partial information, which is of both theoretical and applied interest. Second, this paper introduces a model of dynamic ambiguity preferences across different information structures, and reconciles the well-known tension between dynamic consistency and ambiguity preferences through relaxing reduction.[3] Third, this paper makes an independent contribution to the study of updating rules for ambiguity sensitive preferences. In particular, we provide a behavioral characterization for a simple updating rule for variational preferences.

One limitation of this work is that the behavioral characterization for updating is only well-defined for the class of translation invariant preferences. This rules out the second order belief models [29, 35, 40], another important family of ambiguity preferences. In section 6, we discuss information preferences for second order belief models.

1.1 Related Literature

This paper belongs to the literature on dynamic decision making under ambiguity. [13] axiomatize recursive preferences over adapted consumption processes where all conditional preferences are maxmin expected utility (MEU), and find that dynamic consistency (our π-Recursivity) implies that the prior belief set has to satisfy a "rectangularity" restriction. Later work axiomatizes recursive preferences for other static ambiguity preferences and finds similar restrictions [28, 34].

In fact, [41] shows that within a given filtration, dynamic consistency implies Savage's Sure-Thing Principle and Bayesian updating. Together with reduction, dynamic consistency rules out modal Ellsberg preferences and thus ambiguity.[4] To allow for ambiguity, Siniscalchi studies preferences over a richer domain of decision trees, and relaxes dynamic consistency by introducing a weaker axiom called Sophistication. Together with auxiliary axioms, he proposes a general approach where preferences can be dynamically inconsistent, and the DM addresses these inconsistencies through Strotz-type Consistent Planning.

In this paper, we start from the observation that the noted tension between dynamic consistency and ambiguity relies on reduction, that is, on the assumption that the DM is indifferent to the temporal resolution of uncertainties. However, experimental evidence

[3] This point is discussed in more detail in the related literature section.

[4] See also earlier work by [8].

suggests that reduction is often violated in environments with objective risk.[5] For example, [21] finds evidence for non-reduction of compound lotteries and ambiguity aversion, as well as a positive association between the two. In a dynamic portfolio choice experiment, [3] find that when a DM is committed to some ex-ante portfolio, higher frequency of information feedback leads to lower willingness to invest in risky assets. In this paper, we explore how dynamic consistency and unrestricted ambiguity preferences can be reconciled by relaxing reduction.

Thus this paper is also related to a rich literature that relaxes reduction and studies intrinsic preferences for early or late resolution of uncertainty. This was initially formalized by [32] by introducing a novel domain of objective temporal lotteries and subsequently extended by [11, 12] to study asset pricing. [18, 19] link time preferences to intrinsic preferences for information. In a purely subjective domain, [44] shows that even with standard discounting most models of ambiguity aversion display some preference with regard to the timing of resolution, with the notable exception of the MEU model. Motivated by experimental evidence,[6] recent work studies preferences for one-shot versus gradual resolution of (objective) uncertainty. In the domain of objective two-stage compound lotteries,[7] [5] identifies a link between preferences for one-shot resolution of uncertainty and Allais-type behaviors. In their reference-dependent utility model, [30] also find preferences for getting information "clumped together rather than apart." In contrast, here we identify a link between ambiguity attitudes and intrinsic preferences for partial information over subjective uncertainty.

Finally, our work is also related to the literature on consequentialist updating rules for preferences that violate Savage's Sure-Thing Principle.[8] [36] introduces a coherence property that characterizes the prior-by-prior Bayesian updating rule for MEU preferences. [6] then apply this coherency property to characterize full Bayesian updating for Choquet expected utility (CEU) preferences. Here we apply this property to general translation invariant preferences to connect unconditional and conditional preferences. We then show that this characterizes a simple updating rule for variational preferences, which nests previous results for Bayesian updating in the MEU and multiplier preferences cases.

[5] To my best knowledge, we don't have direct evidence on violation of reduction in environments with subjective uncertainty. One potential experimental design to test reduction is the Ellsberg example illustrated in the introduction.

[6] For example, [17], [20], and [3].

[7] [39] was the first to study two-stage compound lotteries without reduction.

[8] Alternatively, [22, 23] relax consequentialism, and characterize dynamically consistent updating rules for ambiguity preferences. They use a weaker notion of dynamic consistency than ours.

2. Set-up

2.1 Preliminaries

Subjective uncertainty is modeled by a finite set S of states of the world, with elements $s \in S$, describing all contingencies that could possibly happen. Let Σ be the power set of S. $\Delta(S)$ is the set of all probabilities on S. For any $E \subseteq S$, $\Delta(E)$ denotes the set of probabilities on (S, Σ) such that $p(E) = 1$.

Z is the set of deterministic consequences. We assume that Z is a separable metric space. Let $X = \Delta(Z)$, the set of all objective lotteries over Z, endowed with the weak topology. An act $f : S \to X$ is a mapping that associates to every state a lottery in X.

Let $\mathcal{F}$ be the set of all such acts, endowed with the product topology. An act f is *constant* if there is some $x \in X$ such that $f(s) = x, \forall s$; in this case f is identified with x. For all $f, g \in \mathcal{F}, E \in \Sigma$, fEg denotes the act such that $(fEg)(s) = f(s)$ if $s \in E$, and $(fEg)(s) = g(s)$ if $s \notin E$. For any $f, g \in \mathcal{F}, \alpha \in (0, 1)$, $\alpha f + (1 - \alpha)g$ denotes the pointwise mixture of f and g: $(\alpha f + (1 - \alpha)g)(s) = \alpha f(s) + (1 - \alpha)g(s)$.

Let $B(S)$ be the space of all real-valued functions on S, endowed with the sup-norm. For any interval $K \subseteq \mathbb{R}$, $B(S, K)$ denotes the subspace of functions that take values in K.

Partial information is a partition of S. A generic partition is denoted $\pi = \{E_1, \ldots, E_n\}$, where the sets E_i are nonempty and pairwise disjoint, $E_i \in \Sigma$ for each i, and $\cup_{i=1}^n E_i = S$. Let Π be the set of all such partitions. In particular, $\pi_0 = \{S\}$ denotes the coarsest partition, capturing the case when no information is learned in the intermediate stage, and $\pi^* = \{\{s_1\}, \ldots, \{s_{|S|}\}\}$ denotes the finest partition, capturing the case when all relevant uncertainties are resolved in the intermediate stage.

Finally, for all π, let $\mathcal{F}_\pi$ be the subset of π-measurable acts in $\mathcal{F}$.

2.2 Information Acquisition Problem

We consider a two stage information acquisition problem. The DM is endowed with some compact menu $F \subseteq \mathcal{F}$. At stage 1, the DM acquires some partial information π by paying a cost $c(\pi)$, where $c : \Pi \to \mathbb{R}$. At stage 2, she learns which event in π realizes, and chooses an action from the menu F contingent on that event. Finally, the state s realizes and the DM receives the consequence of her chosen action.

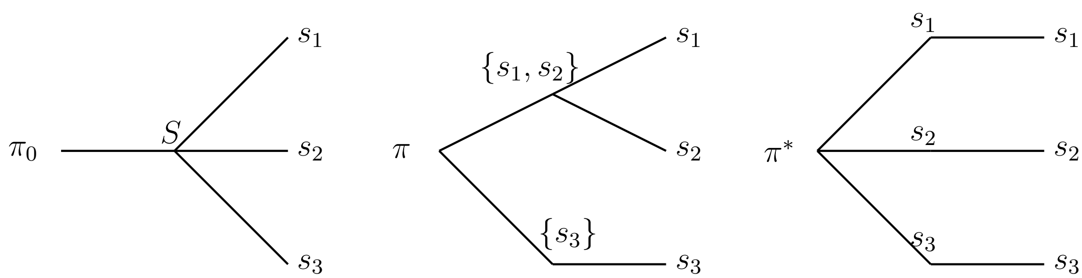

Figure 2: Information Partitions of $S = \{s_1, s_2, s_3\}$.

For any menu F, the information acquisition decision reflects the standard tradeoff between the cost and benefit of getting information π. The DM will choose $\pi \in \Pi$ to solve

$$\max_{\pi \in \Pi} V(\pi, F) - c(\pi)$$

where $V(\pi, F)$ is the value of the decision problem (π, F). Because the cost $c(\pi)$ is deterministic, we focus on how the value function is affected by ambiguity attitudes.

For any $\pi = \{E_1, \ldots, E_n\}$, $\forall E_i$, let f_i^* be the optimal act conditional on event E_i. Ex-ante, if information π is chosen, the DM can expect to get state contingent consequence of $f^* = f_1^* E_1 f_2^* E_2 \cdots f_{n-1}^* E_{n-1} f_n^*$, and the value of decision problem (π, F) is given by $V(\pi, F) = V(\pi, f^*)$. So the information acquisition problem can be reduced to the study of $V : \Pi \times \mathcal{F} \to \mathbb{R}$, the evaluation of singleton menus, expecting intermediate information π.

3. Intrinsic Preferences for Information

In this section, we show that ambiguity aversion is closely related to intrinsic information aversion. We first focus on the value of decision problems when menus are singletons, so the domain of preferences is $\Pi \times \mathcal{F}$. We develop a dynamic model of ambiguity averse preferences which retains recursivity but relaxes reduction, so information could potentially affect the evaluation of a single act. The extension to multi-action menus will be studied in the next section.

Formally, suppose the DM has ex-ante preferences $\succcurlyeq$ over $\Pi \times \mathcal{F}$.[9] Then $(\pi, f) \succcurlyeq (\pi', g)$

[9]We endow Π with the discrete topology, and put the product topology on $\Pi \times \mathcal{F}$.

means that the DM prefers act f (or equivalently, the singleton menu $\{f\}$) when anticipating information π, to act g when anticipating information π'. For given information π, upon learning that the state s lies in event E in the intermediate stage, the DM updates her prior preferences $\succcurlyeq$ to E-conditional preferences $\succcurlyeq_E$. We assume that the conditional preferences $\succcurlyeq_E$ depend only on the event E but not on π, so for each E, conditional preferences $\succcurlyeq_E$ are defined on $\mathcal{F}$.[10] We also denote by $\succcurlyeq_\pi$ the restriction of $\succcurlyeq$ to $\{\pi\} \times \mathcal{F}$, interpreted as the DM's ex-ante preferences over $\mathcal{F}$ when expecting information π. Thus $\succcurlyeq$ and $\{\succcurlyeq_E\}$ are the primitive preferences of our model.

We look for a dynamic model of preferences over $\Pi \times \mathcal{F}$ that satisfies two criteria. First, within a given partition $\pi = \{E_1, E_2, \cdots, E_n\}$, $\succcurlyeq_\pi$ and $\{\succcurlyeq_{E_i}\}_{i=1}^n$ satisfy a recursive relation, in the following sense. For any act f, construct another act f' by replacing f on each E_i by a constant act x_i, where $x_i \sim_{E_i} f$. So $f'(s) = x_i$ if $s \in E_i$, for all i. Recursivity requires that $f \sim_\pi f'$. Second, across two different information partitions π and π', $\succcurlyeq_\pi$ and $\succcurlyeq_{\pi'}$ are related by a unifying unconditional preference relation over $\mathcal{F}$. That is, there exists an unconditional preference relation $\succcurlyeq_0$ over $\mathcal{F}$ such that all conditional preferences $\{\succcurlyeq_E\}_{E\in\Sigma}$ are updated from $\succcurlyeq_0$. Thus if we observe any difference between $\succcurlyeq_\pi$ and $\succcurlyeq_{\pi'}$, it is due to differences in π and π' rather than ex-ante beliefs.

3.1 Recursive Model

In this section we impose axioms on $\{\succcurlyeq_\pi\}_{\pi\in\Pi}$ and $\{\succcurlyeq_E\}_{E\in\Sigma}$ that characterize the folding back evaluation procedure.

First we impose common basic technical axioms on $\succcurlyeq_\pi$ and $\succcurlyeq_E$, for each $\pi \in \Pi$ and $E \in \Sigma$. For convenience we group them together as Axiom 1.

Axiom 1.

1. (Continuity) For all π, E, f, $\{g \in \mathcal{F} : g \succcurlyeq_\pi f\}$, $\{g \in \mathcal{F} : f \succcurlyeq_\pi g\}$, $\{g \in \mathcal{F} : g \succcurlyeq_E f\}$, and $\{g \in \mathcal{F} : f \succcurlyeq_E g\}$ are closed.
2. (Monotonicity) For all $\pi, E \in \Sigma$, if $f(s) \succcurlyeq_\pi (\succcurlyeq_E) g(s)$, $\forall s$, then $f \succcurlyeq_\pi (\succcurlyeq_E) g$.
3. (Non-degeneracy) For all π, $f \succ_\pi g$ for some $f, g \in F$. Similarly, $\forall E \in \Sigma$, $f \succ_E g$ for some $f, g \in F$.

Axiom 2 (Stable Risk Preferences)**.** For all π, E, $\succcurlyeq_\pi$ and $\succcurlyeq_E$ agree on constant acts.

[10] In a two period model, there is no further information to expect after some event in π is realized, so it is reasonable to have conditional preferences defined only on $\mathcal{F}$.

Lemma 1. *Under Continuity and Stable Risk Preferences,* $\succcurlyeq$ *is a continuous preference relation on* $\Pi \times \mathcal{F}$.

Proof. See appendix. $\square$

Within a fixed partition $\pi = \{E_1, \cdots, E_n\}$, we impose π-recursivity to link prior preferences $\succcurlyeq_\pi$ and conditional preferences $\{\succcurlyeq_{E_i}\}_{i=1}^n$. This is similar to the Dynamic Consistency axiom in [13] and [34], simplified to two periods.

Axiom 3 (π-Recursivity)**.** For any π, $E \in \pi$, and $f, g, h \in \mathcal{F}$,

$$f \succcurlyeq_E g \Leftrightarrow fEh \succcurlyeq_\pi gEh$$

If all $\succcurlyeq_\pi$ satisfy π-Recursivity, then all conditional preferences $\{\succcurlyeq_E\}_{E\in\Sigma}$ satisfy *Consequentialism*, that is, $\forall f, g, h$, $\forall E$, $fEg \sim_E fEh$.[11] Intuitively, this says that outcomes in states outside E do not affect E-conditional preferences $\succcurlyeq_E$. We will return to this when discussing learning rules.

If an act f is π-measurable, then in both (π, f) and (π^*, f), all uncertainties about f are resolved in the first stage. So the additional information in π^* relative to that in π should not affect the evaluation of f. This idea is reflected in the following axiom.

Axiom 4 (Indifference to Redundant Information)**.** For all π, $f \in \mathcal{F}_\pi$, $(\pi, f) \sim (\pi^*, f)$.

The last axiom, Time Neutrality, abstracts information preferences from any effect due to preferences for early or late resolution of uncertainty, which is orthogonal to the information preferences of interest here.

Axiom 5 (Time Neutrality)**.** For all f, $(\pi^*, f) \sim (\pi_0, f)$.

Time Neutrality implies that $\succcurlyeq_{\pi^*} = \succcurlyeq_{\pi_0}$, and both can be viewed as the unconditional preferences over acts, denoted by $\succcurlyeq_0$ in the following text. In the next subsection, we specify how all conditional preferences are updated from a unifying unconditional $\succcurlyeq_0$, ensuring all $\succcurlyeq_\pi$ represent the same ex-ante belief.

[11] To see this, let $fEg = f'$ and $fEh = g'$. Then $f'Ef = g'Ef = f$. For $\pi = \{E, E^c\}$, $f'Ef \sim_\pi g'Ef$, and by π-Recursivity, $f' \sim_E g'$.

For a fixed $\pi = \{E_1, \cdots, E_n\}$, we define the conditional certainty equivalent mapping $c(\cdot|\pi) : \mathcal{F} \to \mathcal{F}_\pi$, as follows:

$$c(f|\pi) = \begin{pmatrix} c(f|E_1) & E_1 \\ c(f|E_2) & E_2 \\ \dots & \\ c(f|E_n) & E_n \end{pmatrix}$$

where for each i, $c(f|E_i) \in X$, and $c(f|E_i) \sim_{E_i} f$. That is, $c(f|E_i)$ is the certainty equivalent of f conditional on E_i. Existence is guaranteed by Continuity and Monotonicity of each $\succcurlyeq_{E_i}$, as proved in Lemma 4 in the appendix.

Recall that $\succcurlyeq$ is the ex-ante preference over $\Pi \times \mathcal{F}$, while for every π, $\succcurlyeq_\pi$ is the restriction of $\succcurlyeq$ to $\{\pi\} \times \mathcal{F}$. For an interval $K \subseteq \mathbb{R}$, $B(S, K)$ is the space of functions on S with range K. For any $k \in K$, denote by $\bar{k}$ the corresponding constant function in $B(S)$ taking value k. For any ξ and ϕ in $B(S, K)$, and any event $E \in \Sigma$, $\xi E \phi$ denotes the function such that $(\xi E \phi)(s) = \xi(s)$ if $s \in E$, and $(\xi E \phi)(s) = \phi(s)$ if $s \notin E$. For a functional $I : B(S, K) \to \mathbb{R}$, we say I is *monotone* if $\forall \xi, \phi \in B(S, K)$, $\xi \geq \phi$ implies $I_0(\xi) \geq I_0(\phi)$, and *strongly monotone* if in addition $\xi > \phi$ implies $I_0(\xi) > I_0(\phi)$. We say I is *normalized* if $I(\bar{k}) = k$ for all $k \in K$. Finally, we say I is *translation invariant* if $I(\xi + \bar{k}) = I(\xi) + k$ for all $\xi \in B(S, K)$ and $k \in K$ such that $\xi + \bar{k} \in B(S, K)$.

Lemma 2. *For preferences $\succcurlyeq$ and $\{\succcurlyeq_E\}_{E \in \Sigma}$ that are continuous and monotone, the following statements are equivalent:*

1. *$\{\succcurlyeq_\pi\}_{\pi \in \Pi}$ and $\{\succcurlyeq_E\}_{E \in \Sigma}$ satisfy π-Recursivity, Independence of Redundant Information, and Time Neutrality.*
2. *There exists a continuous function $u : X \to \mathbb{R}$, and a continuous, monotone, and normalized function $I_0 : B(\Sigma, u(X)) \to \mathbb{R}$ such that for each π, $\succcurlyeq_\pi$ can be represented by $V(\pi, \cdot) : \mathcal{F} \to \mathbb{R}$, where*

$$V(\pi, f) = I_0(u \circ c(f|\pi))$$

and $c(\cdot|\pi) : \mathcal{F} \to \mathcal{F}_\pi$ is the conditional certainty equivalent mapping.

Using Axioms 1-5, preferences $\succcurlyeq_\pi$ and $\{\succcurlyeq_E\}_{E \in \pi}$ satisfy π-Recursivity, under which the value of an act f expecting information π can be computed by a folding back procedure. For each event $E_i \in \pi$, replace f on E_i by its conditional certainty equivalent. The constructed act $c(f|\pi)$ is π-measurable, thus could be evaluated by the unconditional preferences $\succcurlyeq_0$, and

$$(\pi, f) \succcurlyeq (\pi', g) \Leftrightarrow c(f|\pi) \succcurlyeq_0 c(g|\pi')$$

Therefore, the ex-ante preferences $\succcurlyeq$ are dictated by the conditional preferences $\{\succcurlyeq_E\}_{E\in\Sigma}$ and unconditional preferences $\succcurlyeq_0$.

For any π, let $B(\pi, u(X))$ denote all the π-measurable functions in $B(S, u(X))$.

3.2 Updating Translation Invariant Preferences

In this subsection, we characterize an updating rule that specifies how the conditional preferences $\{\succcurlyeq_E\}_{E\in\Sigma}$ are derived from unconditional preferences $\succcurlyeq_0$. In this way, for two different information partitions π and π', $\succcurlyeq_\pi$ and $\succcurlyeq_{\pi'}$ are related by the same unconditional $\succcurlyeq_0$ and thus have the same underlying beliefs about events in S. Thus any difference between $\succcurlyeq_\pi$ and $\succcurlyeq_{\pi'}$ is due to differences in information partitions π and π' rather than ex-ante beliefs. In particular, to accommodate ambiguity sensitive $\succcurlyeq_0$, we look for an updating rule that (i) requires that each $\succcurlyeq_E$ satisfies Consequentialism, so outcomes on states outside E does not affect $\succcurlyeq_E$; (ii) does not exclude a preference for hedging in $\succcurlyeq_0$.

It does not make sense to discuss conditional preferences $\succcurlyeq_E$ if event E has "probability zero". We call an event E is Savage $\succcurlyeq_0$-non-null if it is not the case that $fEh \sim_0 gEh$ for all $f, g, h \in \mathcal{F}$. For simplicity, we require that for every event E in Σ is $\succcurlyeq_0$-non-null. For the purpose of updating ambiguity preferences, we need a stronger notion of non-null events.[12] Here we ensure every event is non-null for $\succcurlyeq_0$ by imposing a strong monotonicity axiom on $\succcurlyeq_0$.

Axiom 6 ($\succcurlyeq_0$-Strong Monotonicity)**.** For all $f, g \in \mathcal{F}$, if $f(s) \succcurlyeq_0 g(s)$ for all $s \in S$, then $f \succcurlyeq_0 g$. If in addition one of the preference rankings is strict, then $f \succ_0 g$.

Bayesian updating is the universal updating rule in Savage's SEU theory. The unconditional preference is represented by an expected utility functional with respect to some subjective belief p, and the conditional preference on E is represented by an expected utility functional with respect to the Bayesian posterior $p(\cdot|E)$. Behaviorally, $\succcurlyeq_E$ is derived from $\succcurlyeq_0$ by[13]

$$f \succcurlyeq_E g \Leftrightarrow fEh \succcurlyeq_0 gEh \text{ for some } h$$

[12] For a detailed discussion of the relationship between a Savage $\succcurlyeq_0$-non-null event and the stronger condition we need, see Appendix A.2.

[13] See, for example, [31, chap. 9].

We refer to this as Bayesian Updating in the rest of the paper. In Savage's theory, $\succcurlyeq_E$ is well-defined because $\succcurlyeq_0$ satisfies the Sure-Thing Principle (STP): for all f, g, h, h',

$$fEh \succcurlyeq_0 gEh \Leftrightarrow fEh' \succcurlyeq_0 gEh'$$

The Sure-Thing Principle requires that $\succcurlyeq_0$ is separable across events, which rules out a preference for hedging and Ellsberg-type preferences. This condition clearly is too strong for our purposes. Instead, we consider a weaker condition, called Conditional Certainty Equivalent Consistency. This condition requires that a constant act x is equivalent to an act f conditional on E if and only if x is also unconditionally equivalent to fEx, the act that gives f for states in E, and x for states outside E.

Axiom 7 (Conditional Certainty Equivalent Consistency)**.** $\forall f \in \mathcal{F}, x \in X$, $\forall E \in \Sigma$,

$$f \sim_E x \Leftrightarrow fEx \sim_0 x$$

Conditional Certainty Equivalent Consistency weakens Bayesisan Updating by restricting g and h to be a constant act x and considering only indifference relations. In particular, Bayesian Updating imposes two properties. First, $\succcurlyeq_0$ and $\succcurlyeq_E$ are dynamically consistent: if f and g agree outside event E, then f is preferred to g conditional on E if and only if f is preferred to g unconditionally. Second, $\succcurlyeq_E$ satisfies consequentialism: if f and g agree on event E, then f is equivalent to g conditional on E. It is straightforward to verify that under Conditional Certainty Equivalent Consistency, consequentialism is retained but not dynamic consistency.

Just as Savage's Bayesian Updating is not well-defined unless $\succcurlyeq_0$ satisfies the STP, we also need to impose some structural assumption on $\succcurlyeq_0$ to ensure that Conditional Certainty Equivalent Consistency is well-defined. The property needed is translation invariance of the corresponding aggregating functional I_0. The behavioral axiom that characterizes translation invariance is [33]'s Weak Certainty Independence.[14]

Axiom 8 (Weak Certainty Independence)**.** For all $f, g \in \mathcal{F}$, $x, y \in X$, and $\alpha \in (0,1)$,

$$\alpha f + (1-\alpha)x \succcurlyeq_0 \alpha g + (1-\alpha)x \Rightarrow \alpha f + (1-\alpha)y \succcurlyeq_0 \alpha g + (1-\alpha)y$$

Intuitively, Weak Certainty Independence of $\succcurlyeq_0$, and thus translation invariance of I_0, requires that the indifference curves in the space of utility profiles are parallel when moved

[14]By [33]'s Lemma 28,Weak Certainty Independence, Monotonicity, Continuity, and Non-degeneracy of $\succcurlyeq_0$ is equivalent to $\succcurlyeq_0$ can be represented by an affine risk utility u and normalized, monotone, and translation invariant functional aggregator I_0.

along the certainty line. Ambiguity preferences that satisfy translation invariance include MEU, CEU, and variational preferences. As mentioned in the discussion of related literature, Conditional Certainty Equivalence Consistency has been used by [36] to characterize prior-by-prior updating for MEU, and by [6] to characterize a generalized Bayes rule for CEU. In section 4, we characterize a simple update rule for variational preferences using this axiom.

We show that if $\succsim_0$ satisfies Weak Certainty Independence and Strong Monotonicity, then conditional preferences are well-defined. Thus only knowledge about $\succsim_0$ is needed to calculate the conditional certainty equivalent, and thus pin down the conditional preferences $\succsim_E$ for all E. Moreover, when combined with axioms characterizing recursiveness in the previous subsection, knowing $\succsim_0$ is sufficient to characterize $\succsim_\pi$ for all π. Below is a formal definition.

Definition 1. We say $\succsim$ on $\Pi \times \mathcal{F}$ and $\succsim_E$ on $\mathcal{F}$ *have a cross-partition recursive representation* (u, I_0) if

1. There exists a continuous, non-constant, and affine $u : X \to \mathbb{R}$, and a continuous, strongly monotone, normalized, and translation invariant $I_0 : u(X)^S \to \mathbb{R}$ such that

$$f \succsim_0 g \Leftrightarrow I_0(u \circ f) \geq I_0(u \circ g)$$

2. For all $E \in \Sigma$, $\succsim_E$ is represented by $V_E : \mathcal{F} \to \mathbb{R}$, where $V_E(f)$ is the unique solution to

$$k = I_0((u \circ f)E\bar{k})$$

3. $\succsim$ is represented by $V : \Pi \times \mathcal{F} \to \mathbb{R}$, where

$$V(\pi, f) = I_0(V_0(f|\pi))$$

and

$$V_0(f|\pi) = \begin{pmatrix} V_{E_1}(f) & E_1 \\ V_{E_2}(f) & E_2 \\ \dots & \\ V_{E_n}(f) & E_n \end{pmatrix}$$

In this case, we also say $\succsim$ *is recursively generated by* $\succsim_0$.

Theorem 1. *The following statements are equivalent:*

1. (i) $\{\succsim_\pi\}_{\pi\in\Pi}$ *and* $\{\succsim_E\}_{E\in\Sigma}$ *are continuous and monotone, satisfy* π*-Recursivity, Independence of Redundant Information, Time Neutrality, and Stable Risk Preferences;*

   (ii) $\succsim_0$ *satisfies Weak Certainty Independence and Strong Monotonicity;* $\succsim_0$ *and* $\{\succsim_E\}_{E\in\Sigma}$ *satisfy Conditional Certainty Equivalent Consistency.*

2. $\succsim$ *and* $\succsim_E$ *have a cross-partition recursive representation with* (u, I_0)*.*

Moreover, if both (u, I_0) *and* (u', I'_0) *represent* $\succsim_0$*, then there exists* $a > 0$ *and* $b \in \mathbb{R}$ *such that* $u' = au + b$ *and* $I'_0(\xi) = aI_0(\frac{\xi-b}{a}) + b$ *for all* $\xi \in (u'(X))^S$*.*

Proof. See appendix. □

3.3 Intrinsic Aversion to Partial Information

In this subsection, we define aversion to partial information as a property of the cross-partition preference $\succsim$. Then we show that under our recursive representation, aversion to partial information is equivalent to a property of $\succsim_0$ called Event Complementarity. We study the relationship between Event Complementarity and ambiguity aversion. In the next section, we consider familiar models of ambiguity preferences, and study the connection among ambiguity aversion, Event Complementarity, and aversion to partial information.

Definition 2. We say $\succsim$ exhibits *aversion to partial information at act* f if $(\pi_0, f) \succsim (\pi, f)$ for all π. We say $\succsim$ exhibits *aversion to partial information* if $\succsim$ exhibits aversion to partial information at all acts.

Attraction to partial information and information neutrality are defined analogously.

This definition of aversion to partial information is similar to Preferences for One-Shot Resolution of Uncertainty in [5], and preferences to get information "clumped together rather than apart" as in [30]. Our definition only requires that the DM prefers no information π_0 to any information π. This is weaker than the notion of information aversion defined in [18] and [42], which requires that coarser information is always preferred to finer information.[15] If the DM exhibits aversion to partial information at all acts and obeys Time Neutrality, then $(\pi_0, f) \sim (\pi^*, f) \succsim (\pi, f)$ for all f.

[15] In [18], finer information corresponds to higher Blackwell's informativeness ranking.

In the modal Ellsberg preferences, there is complementarity between the events $\{G\}$ and $\{Y\}$ in eliminating ambiguity. The DM knows that the joint event $\{G, Y\}$ has a precise probability $\frac{2}{3}$, while each subevent $\{G\}$ or $\{Y\}$ has an imprecise probability ranging from 0 to $\frac{2}{3}$. By partitioning the event $\{G, Y\}$ into the subevents $\{G\}$ and $\{Y\}$, the information regarding whether the ball drawn is yellow or not breaks this complementarity and creates ambiguity. A DM averse to ambiguity might naturally be averse to this information. We formalize this idea as a condition on $\succcurlyeq_0$ below.

Axiom 9 (Event Complementarity)**.** For all E and f, if $fEx \sim_0 x$ for some x, then $f \succcurlyeq_0 xEf$.

Intuitively, Event Complementarity captures the following thought experiment. For a given act f and event E, first calibrate the value of f conditional on E by finding its conditional certainty equivalent, that is, the constant act x such that $fEx \sim_0 x(= xEx)$. Then replace f on E by x, that is, consider the act xEf, and compare this to the original act f. By construction, xEf and f are equivalent conditional on E, and they are identical, and hence trivially equivalent, conditional on E^c. A DM who satisfies the Sure-Thing Principle would view f and xEf as equivalent. Replacing f by its conditional certainty equivalent x on E, however, breaks the potential complementarity between the events E and E^c with respect to the act f. A strict preference $f \succ_0 xEf$ reveals a DM who values such complementarity.

Proposition 1. *Suppose $\succcurlyeq_0$ is represented by (u, I_0) where I_0 is translation invariant. Then $\succcurlyeq_0$ satisfies Event Complementarity if and only if for any act f and constant act x such that $fEx \sim_0 x$,*

$$I_0(u \circ f) \geq I_0(u \circ (fEx)) + I_0(0E(u \circ f - u \circ x)) \tag{1}$$

Proof. Fix f, x, E such that $fEx \sim_0 x$. By translation invariance of I_0,

$$I_0(u \circ (xEf)) = I_0(0E(u \circ f - u \circ x)) + u(x).$$

Since $fEx \sim_0 x$, $I_0(u \circ (fEx)) = u(x)$, thus

$$I_0(u \circ (xEf)) = I_0(0E(u \circ f - u \circ x)) + I_0(u \circ (fEx))$$

Thus

$$I_0(u \circ f) \geq I_0(u \circ xEf)$$

if and only if

$$I_0(u \circ f) \geq I_0(u \circ fEx) + I_0(0E(u \circ f - u \circ x))$$

So $f \succcurlyeq_0 xEf$ if and only if (1) holds . $\square$

Inequality (1) describes Event Complementarity of $\succcurlyeq_0$ in terms of its utility representation (u, I_0). This gives us another way to understand this axiom. Given an act f and a constant act x such that $fEx \sim_0 x$, notice that the utility profile $u \circ f$ corresponding to f can be decomposed as follows:

$$u \circ f = u \circ (fEx) + 0E(u \circ f - u \circ x)$$

Since x is a constant act, $u \circ (fEx)$ varies only on E, and $0E(u \circ f - u \circ x)$ varies only on E^c by construction. Thus $u \circ f$ is decomposed into the sum of two utility profiles, one capturing the variation of $u \circ f$ on E and one capturing the variation of $u \circ f$ on E^c. Proposition 1 shows that Event Complementarity holds if and only if the value of utility profile $u \circ f$, $I_0(u \circ f)$, is greater than or equal to the sum of the values of these two pieces, $I_0(u \circ fEx) + I_0(0E(u \circ f - u \circ x))$. Notice that if I_0 is superadditive, then Event Complementarity holds. However, the converse is not generally true. This result will be useful in verifying that Event Complementarity holds in a number of classes of ambiguity preferences.

Finally, the following proposition shows that in our recursive model, aversion to partial information is equivalent to Event Complementarity.

Theorem 2. *Suppose $\succcurlyeq$ is recursively generated by $\succcurlyeq_0$. Then the following statements are equivalent:*

1. *$\succcurlyeq_0$ satisfy Event Complementarity.*
2. *$\succcurlyeq$ exhibits aversion to partial information.*

Proof. See appendix. $\square$

4. Ambiguity Preferences

In this section, we investigate further the link between ambiguity aversion and aversion to partial information. In particular, we examine whether partial information aversion is implied by ambiguity aversion for four familiar classes of translation invariant ambiguity preferences: MEU, multiplier preferences, variational preferences, and CEU. Another popular class of ambiguity preferences, the second order belief model, does not satisfy translation invariance and thus is not captured by our model. We defer discussion of second order belief models to Section 6.

We first introduce the ambiguity aversion axiom:[16]

Axiom 10 (Ambiguity Aversion)**.** For all $f, g \in \mathcal{F}$ and $\alpha \in (0,1)$,

$$f \sim_0 g \Rightarrow \alpha f + (1-\alpha) g \succcurlyeq_0 f$$

As argued by [16], Ambiguity Aversion captures a preference for state-by-state hedging. If $\succcurlyeq_0$ is represented by (u, I_0), and I_0 is continuous, monotone, normalized, and translation invariant, then $\succcurlyeq_0$ is ambiguity averse if and only if I_0 is concave.

4.1 Maxmin EU

MEU is the most popular model that captures ambiguity aversion. The static MEU model is axiomatized by [16], and a recursive MEU model is axiomatized by [13].[17]

We say $\succcurlyeq_0$ has an MEU representation $(u, \mathcal{P})$ if it can be represented by a function $V_0 : \mathcal{F} \to \mathbb{R}$ of the form

$$V_0(f) = \min_{p \in \mathcal{P}} \int_S u(f) dp$$

where $\mathcal{P}$ is a closed and convex subset of $\Delta(S)$.

For any convex and closed prior set $\mathcal{P}$ and any partition π, we define the *π-rectangular hull of $\mathcal{P}$* to be $rect_\pi(\mathcal{P}) = \{p = \sum_{i=1}^k p^i(\cdot|E_i) q(E_i) | \forall p^i, q \in \mathcal{P}\}$. The set $rect_\pi(\mathcal{P})$ is the largest set of probabilities that have the same marginal probabilities and conditional probabilities for events in π as elements of $\mathcal{P}$. By definition, $\mathcal{P} \subseteq rect_\pi(\mathcal{P})$ for any $\mathcal{P}$ and π. The set $\mathcal{P}$ is called *π-rectangular* if $rect_\pi(\mathcal{P}) = \mathcal{P}$. Whether $\mathcal{P}$ is π-rectangular is closely related to whether a DM with belief set $\mathcal{P}$ is strictly averse to partial information π. The next proposition summarizes the link between MEU preferences and aversion to partial information.

[16] In the literature, this axiom is usually called Uncertainty Aversion. Strictly speaking, it does not coincide with the definition of ambiguity aversion as in [15] or [7]. But for the four families of preferences we study, Axiom 10 implies ambiguity aversion.

[17] In contrast with our model, [13] assume reduction.

Proposition 2. *Suppose* $\succsim$ *is recursively generated by* $\succsim_0$*. Suppose* $\succsim_0$ *has a MEU representation* $(u, \mathcal{P})$*, and* $\succsim_E$ *has a MEU representation* $(u, \mathcal{P}_E)$*, for all* $E \in \Sigma$*. Then*

1. $\succsim$ *exhibits aversion to partial information at all acts.*
2. *For any partition* π*, there exists some act* f *such that* $\succsim$ *is strictly averse to* π *at* f*, i.e.,* $(\pi_0, f) \succ (\pi, f)$*, if and only if* $\mathcal{P}$ *is not* π*-rectangular.*

Proof. See appendix. □

Remark 1. MEU has an intuitive interpretation as a malevolent Nature playing a zero-sum game against the DM [34]. In this interpretation, Nature has a constraint set $\mathcal{P}$, and chooses a probability in order to minimize the DM's expected utility. In our recursive model without reduction, the information π turns this into a sequential game. In period 0, Nature chooses a probability from $\mathcal{P}$ for events in π. In period 1, Nature chooses a (possibly different) probability from $\mathcal{P}$ over states for every event in π, conditional on that event. In this way, information π expands Nature's constraint set from $\mathcal{P}$ to $rect_\pi(\mathcal{P})$. On the other hand, the DM has committed ex-ante to a fixed act f. So introducing information π helps Nature and hurts the DM. Part (2) of Proposition 2 shows that if information strictly expands Nature's constraint set, that is, if $\mathcal{P} \subsetneq rect_\pi(\mathcal{P})$, then Nature can make the DM strictly worse off at some act.

Remark 2. [13] develop a recursive MEU model in which they maintain reduction. They show that $\succsim$ is dynamically consistent with respect to π if and only if $\mathcal{P}$ is π-rectangular. Part (2) of Proposition 2 shows that if we instead maintain dynamic consistency but relax reduction, then information neutrality at π is equivalent to π-rectangularity of $\mathcal{P}$.

Remark 3. When the prior set $\mathcal{P}$ is a singleton (so the DM has SEU), or when $\mathcal{P} = \Delta(S)$, the DM is intrinsically information neutral.

4.2 Multiplier Preferences

Introduced by [24] to capture concerns about model misspecification, and later axiomatized by [43], multiplier preferences have found broad applications in macroeconomics.[18] We say $\succsim_0$ has a multiplier preferences representation (u, q, θ) if it can be represented by a function $V_0 : \mathcal{F} \to \mathbb{R}$ of the form

$$V_0(f) = \min_{p \in \Delta(S)} [\int u(f)dp + \theta R(p||q)]$$

[18] See [25] and references therein.

where $q \in \Delta(S)$ is the reference probability, $R(p||q) = \int \ln \frac{p}{q} dp$ is the relative entropy between p and reference probability q, and θ is a scalar measuring the intensity of ambiguity aversion.

Proposition 3. *Suppose $\succcurlyeq_0$ has a multiplier preferences representation (u, q, θ), and $\succcurlyeq$ is recursively generated by $\succcurlyeq_0$. Then $\succcurlyeq$ exhibits intrinsic information neutrality.*

Proof. See appendix. □

4.3 Variational Preferences

Variational preferences are introduced and axiomatized by [33, 34]. We say $\succcurlyeq_0$ has a variational representation (u, c) if it can be represented by a function $V_0 : \mathcal{F} \to \mathbb{R}$ of the form

$$V_0(f) = \min_{p \in \Delta(S)} \int u(f) dp + c(p)$$

where $c : \Delta(S) \to [0, +\infty]$ is a convex, lower semicontinuous and grounded (there exists p such that $c(p) = 0$) function. The function c is interpreted as the cost of choosing a probability. The MEU model and multiplier preferences model are special cases of variational preferences.[19] Variational preferences are the most general class of ambiguity averse preferences that satisfy translation invariance.

We let $dom(c) = \{p : c(p) < +\infty\}$ denote the domain of c. If $u(X)$ is unbounded, then for a given u, c is the unique minimum convex, lower semicontinuous, and grounded cost function that represents $\succcurlyeq_0$.

4.3.1 Updating Variational Preferences

For any non-empty $E \in \Sigma$, we say $\succcurlyeq_E$ has a variational representation (u_E, c_E) if it can be represented by a function $V_E : \mathcal{F} \to \mathbb{R}$ of the form

$$V_E(f) = \min_{p_E \in \Delta(S)} \int_S u_E(f) dp_E + c_E(p_E)$$

where $c_E : \Delta(S) \to [0, +\infty]$ is a convex, lower-semi-continuous, and grounded *conditional cost function.*

[19] Variational preferences have a MEU representation when c is 0 on a set $\mathcal{P}$ and $+\infty$ elsewhere, and a multiplier preferences representation when $c(p) = \theta R(p||q)$.

The next theorem shows that within the variational preferences family, Stable Risk Preferences and Conditional Certainty Equivalent Consistency characterize the following updating rule for conditional cost functions:

$$c_E(p_E) = \inf_{\{p \in \Delta(S): p(\cdot|E) = p_E\}} \frac{c(p)}{p(E)} \tag{2}$$

Taking the infimum over all probabilities with posterior p_E controls for any concern for model mis-specification outside event E, which is irrelevant to $\succsim_E$ due to consequentialism; normalization by $\frac{1}{p(E)}$ captures a maximum likelihood intuition: probabilities p assigning a higher probability on the event that occurred are more likely to be selected and determine c_E. Since we imposed Strong Monotonicity on $\succsim_0$, every event E is $\succsim_0$-non-null. In particular, $p(E) > 0$ for all $p \in dom(c)$. Then by Lemma 7 in the Appendix, the infimum in (5) attains at some p.

Theorem 3. *Suppose $\succsim_0$ has a variational representation (u, c) and satisfies Strong Monotonicity. Suppose for any non-empty $E \in \Sigma$, $\succsim_E$ has a variational representation (u_E, c_E). Then the following are equivalent:*

1. *$\succsim_E$ and $\succsim_0$ satisfy Stable Risk Preferences and Conditional Certainty Equivalent Consistency.*

2. *$\succsim_E$ has a variational representation (u, c_E) such that*

$$f \succsim_E g \Leftrightarrow \min_{p_E \in \Delta(E)} \int_E u(f)dp_E + c_E(p_E) \geq \min_{p_E \in \Delta(E)} \int_E u(g)dp_E + c_E(p_E)$$

   where

$$c_E(p_E) = \min_{\{p \in \Delta(S): p(\cdot|E) = p_E\}} \frac{c(p)}{p(E)}$$

Proof. See Appendix. □

This generalizes well-known updating rules for the two important subclasses of variational preferences: prior-by-prior updating in the MEU class, and Bayesian updating in the multiplier preferences class.

Corollary 1. *Suppose assumptions and Statement 1 in Theorem 3 hold.*

1. *If $\succsim_0$ also has a MEU representation $(u, \mathcal{P})$, then for any non-empty E, $\succsim_E$ has a MEU representation $(u, \mathcal{P}_E)$, where $\mathcal{P}_E$ is the set obtained from $\mathcal{P}$ by prior-by-prior updating, that is*

$$\mathcal{P}_E = \{p(\cdot|E) | p \in \mathcal{P}\}$$

2. *If $\succsim_0$ also has a multiplier preference representation (u, q, θ), then for any non-empty E, $\succsim_E$ has a multiplier preference representation (u, q_E, θ), where q_E is the Bayesian posterior of q.*

Proof. See Appendix. □

4.3.2 Variational Preferences and Preferences for Partial Information

In general, recursive variational preferences might not exhibit aversion to partial information at all acts. This can be explained by the following intuition. Similar to the MEU model, variational preferences also has the intuitive interpretation of a malevolent Nature playing a zero-sum game against the DM [34]. With variational preferences, Nature's constraint set is the domain of the cost function c, $dom(c)$. In addition, Nature has to pay a non-negative cost (or transfer) of $c(p)$ to the DM if it chooses a probability p in $dom(c)$. Nature seeks to minimize the DM's expected utility plus the transfer. In our recursive model without reduction, information π turns this into a sequential game, affecting both Nature's constraint set and how often Nature has to pay the DM a transfer. Similar to the MEU model, in period 0, Nature chooses a probability from $dom(c)$ for events in π. In period 1, Nature chooses a (possibly different) probability from $dom(c)$ over states for every event in π, conditional on that event. So information π expands Nature's constraint set from $dom(c)$ to $rect_\pi(dom(c))$. On the other hand, with information π, Nature also needs to pay a non-negative transfer to the DM at every node where it chooses a probability. The total transfer can be higher or lower than what Nature would have paid in the static game, depending on the cost function c. If the total transfer is higher, then this helps the DM. So the overall effect from information π is indeterminate. Below is an example in which when the transfer effect dominates and the DM strictly prefers information π at an act f.

Example 1 (Attraction to Partial Information in VP)**.** Suppose $S = \{s_1, s_2, s_3\}$. Let $u(x) = x$ (where $X = \mathbb{R}$). Consider the partition $\pi = \{\{s_1, s_2\}, \{s_3\}\}$. Let $E = \{s_1, s_2\}$. Let $\bar{p} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, and $\mathcal{P} = \{p \in \Delta(S) : p(s_i) \geq \delta, \forall i = 1, 2, 3\}$, for some $\delta \in (0, \frac{1}{5}]$.

Let $\alpha_{\bar{p}} = 0$. For all $p \in \mathcal{P} \backslash \bar{p}$, in the probability simplex illustrated by Figure 3, we connect $\bar{p}$ to p by a line segment and extend it to a point p' on the boundary of $\mathcal{P}$. Let α_p be the ratio of the length of line segment $\bar{p}p$ to the length of line segment $\bar{p}p'$. Consider the cost function

$$c(p) = \begin{cases} \alpha_p & \text{if } p \in P, \\ +\infty & \text{otherwise.} \end{cases}$$

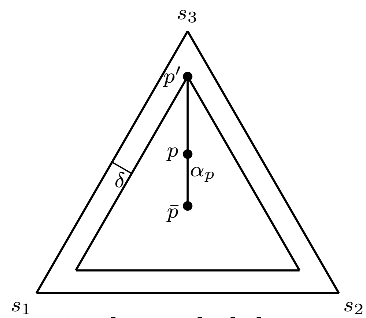

Figure 3: the probability simplex

Note that c is convex, lower semicontinuous, and grounded, so (u, c) characterizes some VP.

Consider the act $f = (0, 3K, 1K)$, where K is a large number in $\mathbb{R}_+$ and $K\delta > 10$. Without information, $V(\pi_0, f) = 4\delta K + 1$. Suppose the DM now gets partial information π. Then

$$V_E(f) = \min_{p_E \in \Delta(E)} 3K p_E(s_2) + \min_{p(\cdot|E)=p_E} \frac{c(p)}{p(E)} = \frac{1}{1-\delta}(3\delta K + 1)$$

$$V(\pi, f) = \min_p p(E) \frac{1}{1-\delta}(3\delta K + 1) + p(s_3)K + c(p) = 3\delta K + 1 + \delta K + 1 = 4\delta K + 2$$

Then $V(\pi, f) = 4\delta K + 2 > 4\delta K + 1 = V(\pi_0, f)$, so the DM has a strict preference for partial information π at f.

The following proposition identifies a necessary and sufficient condition on the unconditional cost function c under which aversion to partial information holds at all acts. In the zero-sum game against Nature interpretation, this condition ensures that the total transfer Nature pays under information π does not exceed that in the static game. To formalize this, we need some additional notation.

For all $p_E \in \Delta(E)$ and $p' \in \Delta(S)$, define $p_E \otimes_E p'$ by

$$(p_E \otimes_E p')(B) = p'(E)p_E(B) + p'(B \cap E^c), \forall B \in \Sigma$$

That is, in $p_E \otimes_E p'$, we substitute $p'(\cdot|E)$ by p_E for probability conditional on E, while measuring probabilities of events in E^c (including E^c) by p'.

Proposition 4. *Suppose $\succcurlyeq_0$ has a variational representation (u, c), and $\succcurlyeq$ is recursively generated by $\succcurlyeq_0$. Then $\succcurlyeq$ exhibits intrinsic aversion to partial information at all f if and only if for any non-empty $E \in \Sigma$,*

$$c(p) \geq \inf_{q_E \in \Delta(E)} c(q_E \otimes_E p) + p(E) \inf_{q \in \Delta(S)} \frac{c(p_E \otimes_E q)}{q(E)}, \quad \forall p, p(E) > 0$$

where p_E is the Bayesian posterior of p.

It is straightforward to verify that this condition holds for MEU and for multiplier preferences.

The above condition restricts the cost function c so that $\succcurlyeq$ exhibits partial information aversion at all acts. As shown in Example **??**, this can be violated by some variational preferences, where attraction to partial information at some act is possible. So this condition might be too strong for some purposes.

The next proposition characterizes a joint condition on the cost function c and an act f under which $\succcurlyeq$ exhibits aversion to partial information locally at f. This does not preclude the possibility that $\succcurlyeq$ exhibits attraction to partial information at some other act g. As we will explain later, this joint condition also has an intuitive interpretation.

Proposition 5. *Suppose $\succcurlyeq_0$ has a variational representation (u, c), and $\succcurlyeq$ is recursively generated by $\succcurlyeq_0$. Then for any act f such that*

$$c^{-1}(0) \cap \arg\min_{p \in \Delta}[\int_S u(f)dp + c(p)] \neq \emptyset \tag{3}$$

$\succcurlyeq$ exhibits aversion to partial information at f.

Proof. See appendix. □

If $\succcurlyeq_0$ has MEU representation $(u, \mathcal{P})$, then the cost function is an indicator function where

$$c(p) = \delta_{\mathcal{P}}(p) = \begin{cases} 0 & \forall p \in \mathcal{P} \\ +\infty & \text{otherwise} \end{cases}$$

In this case, for any act f, $\arg\min_{p \in \Delta}[\int_S u(f)dp + c(p)] \subseteq \mathcal{P} = c^{-1}(0)$. So the result that an MEU DM is averse to partial information at all acts follows as a natural corollary of Proposition 5.

Condition (3) has an intuitive interpretation in terms of comparative ambiguity. Following the notion of comparative ambiguity aversion in [15] and [7], given two static preferences $\succcurlyeq_1$ and $\succcurlyeq_2$ over $\mathcal{F}$, we say $\succcurlyeq_1$ *is more ambiguity averse than* $\succcurlyeq_2$ if for all $f \in \mathcal{F}$ and $x \in X$,

$$f \succcurlyeq_1 x \Rightarrow f \succcurlyeq_2 x$$

By [33] Proposition 8, if $\succcurlyeq_1$ has a variational representation (u_1, c_1) and $\succcurlyeq_2$ has a variational representation (u_2, c_2), then $\succcurlyeq_1$ is more ambiguity averse than $\succcurlyeq_2$ if and only if $u_1 \approx u_2$,[20] and $c_1 \leq c_2$ (provided $u_1 = u_2$). In the following when discussing comparative

[20] $u_1 \approx u_2$ if $u_1 = au_2 + b$, for some $a > 0$, $b \in \mathbb{R}$.

ambiguity aversion, we normalize risk utilities so that $u_1 = u_2$.[21]

We say an act f can be locally approximated by an SEU preference that is less ambiguity averse than $\succcurlyeq_0$ if there exists a preference relation $\geq'$ on $\mathcal{F}$ that admits an SEU representation

$$U'(f) = \int_S u'(f)dq$$

such that (i) $\geq'$ is less ambiguity averse than $\succcurlyeq_0$ and (ii) $V(f) = U'(f)$.

Proposition 6. *Suppose $\succcurlyeq_0$ has a variational representation (u, c). Condition (3) holds at some act f if and only if f can be locally approximated by an SEU preference that is less ambiguity averse than $\succcurlyeq_0$. In particular, if f can be locally approximated by an SEU preference that is less ambiguity averse than $\succcurlyeq_0$, then $\succcurlyeq$ exhibits aversion to partial information at f.*

Proof. Suppose f can be locally approximated by an SEU preference $\geq'$ that is less ambiguity averse than $\succcurlyeq_0$. Let $\geq'$ be represented by U' with risk utility u' and belief $q \in \Delta(S)$. Since $\geq'$ is less ambiguity averse than $\succcurlyeq_0$, we can normalize u' so that $u = u'$. In addition, $q \in c^{-1}(0)$ by [33] Lemma 32. Since $V(f) = U'(f)$,

$$V(f) = \min_{p\in\Delta}[\int_S u(f)dp + c(p)] = U'(f) = \int_S u(f)dq = \int_S u(f)dq + c(q)$$

The last equality follows from the fact that $q \in c^{-1}(0)$. So $q \in \arg\min_{p\in\Delta}[\int_S u(f)dp+c(p)]$ by definition. Together with $q \in c^{-1}(0)$, this implies that

$$c^{-1}(0) \cap \arg\min_{p\in\Delta}[\int_S u(f)dp + c(p)] \neq \emptyset$$

Thus condition (3) holds at f.

Now suppose there exists some $p^* \in c^{-1}(0) \cap \arg\min_{p\in\Delta}[\int_S u(f)dp + c(p)]$. Define U' by $U'(f) = \int_S u(f)dp^*$. Then by definition U' represents an SEU preference $\geq'$ that is less ambiguity averse than $\succcurlyeq_0$. Also

$$V(f) = \int_S u(f)dp^* + c(p^*) = \int_S u(f)dp^* = U'(f)$$

So f can be locally approximated by an SEU preference that is less ambiguity averse than $\succcurlyeq_0$. ∎

[21] In VP, u is unique up to positive affine transformation.

Proposition 7. *Suppose $\succcurlyeq_0^1$ has a variational representation (u^1, c^1) and f can be locally approximated by some SEU preference $\geq'$ that is less ambiguity averse than $\succcurlyeq_0^1$. Suppose $\succcurlyeq_0^2$ also has a variational representation (u^2, c^2), and let $\succcurlyeq^2$ be recursively generated by $\succcurlyeq_0^2$. If $\succcurlyeq_0^2$ is less ambiguity averse than $\succcurlyeq_0^1$ and more ambiguity averse than $\geq'$, then $\succcurlyeq^2$ exhibits partial information aversion at f.*

Proof. By Proposition 6, f can be locally approximated by an SEU preference $\geq'$ that is less ambiguity averse than $\succcurlyeq_0^1$ if and only if condition (3) holds. Then there exists $p^* \in c_1^{-1}(0) \cap argmin_{p\in\Delta}[\int_S u_1(f)dp + c_1(p)]$ such that $V_1(f) = \int_S u_1(f)dp^* + c_1(p^*)$, and $c_1(p^*) = 0$. By definition, $\succcurlyeq_0^2$ is less ambiguity averse than $\succcurlyeq_0^1$ if and only if $u_1 = u_2$ and $c_2 \geq c_1$. Since $\succcurlyeq_0^2$ is more ambiguity averse than $\geq'$, $u_2 = u'$ and $p^* \in c_2^{-1}(0)$. Let $u = u_1 = u_2 = u'$. Therefore:

$$\int_S u(f)dp^* + c_2(p^*) = \int_S u(f)dp^* + c_1(p^*) \leq \int_S u(f)dp + c_1(p) \leq \int_S u(f)dp + c_2(p), \forall p \in \Delta(S)$$

The first inequality follows from the fact that $p^* \in \arg\min_{p\in\Delta}[\int_S u_1(f)dp + c_1(p)]$, and the second from $c_1 \leq c_2$. Thus $p^* \in \arg\min_{p\in\Delta}[\int_S u(f)dp + c_2(p)]$. So

$$\arg\min_{p\in\Delta}[\int_S u(f)dp + c_2(p)] \cap c_2^{-1}(0) \neq \emptyset$$

and by Proposition 5, $\succcurlyeq^2$ exhibits aversion to partial information at f. □

4.4 Choquet EU

Finally, we look at the CEU model axiomatized by [38]. The CEU model is of particular interest because it allows for both ambiguity averse and ambiguity loving preferences, so this provides a framework for studying the relationship between information preferences and ambiguity attitudes more generally.

We say $\succcurlyeq_0$ has a CEU representation (u, ν) if it can be represented by a function $V_0 : \mathcal{F} \to \mathbb{R}$ of the form

$$V_0(f) = \int u(f)d\nu$$

where $\nu : \Sigma \to [0, 1]$ is a capacity, that is, $\nu(S) = 1$, $\nu(\emptyset) = 0$, and $\nu(E) \leq \nu(F)$ for all $E \subseteq F$.

If $\succcurlyeq_0$ satisfies Ambiguity Aversion, then ν is a convex capacity.[22] In this case, CEU preferences become a special case of MEU preferences, with the set of priors $\mathcal{P}$ being

[22] A capacity ν is convex if $\nu(E \cup F) + \nu(E \cap F) \geq \nu(E) + \nu(F)$ holds for all $E, F \in \Sigma$.

the core of the convex capacity ν.[23] So for CEU preferences, ambiguity aversion implies aversion to partial information.

For CEU preferences, we can say a bit more about the connection between ambiguity attitudes and information preferences. We can also define ambiguity loving.[24]

Axiom 11 (Ambiguity Loving)**.** For all $f, g \in \mathcal{F}$ and $\alpha \in [0, 1]$,

$$f \sim_0 g \Rightarrow f \succcurlyeq_0 \alpha f + (1 - \alpha) g$$

We show that within the CEU model, ambiguity aversion implies partial information aversion, and ambiguity loving implies partial information loving.

Proposition 8. *Suppose $\succcurlyeq_0$, $\{\succcurlyeq_E\}_{E \in \Sigma}$ have CEU representations, and $\succcurlyeq$ is recursively generated by $\succcurlyeq_0$.*

1. *If $\succcurlyeq_0$ satisfies Ambiguity Aversion, then $\succcurlyeq$ exhibits partial information aversion at all acts.*
2. *If $\succcurlyeq_0$ satisfies Ambiguity Loving, then $\succcurlyeq$ exhibits attraction to partial information at all acts.*

Proof. See appendix. □

5. Multi-action Menus and Information Acquisition

In this section we study decision problems with general menus. Recall the information acquisition problem is to choose $\pi \in \Pi$ to solve:

$$\max_{\pi} V(\pi, F) - c(\pi)$$

For a given menu F, the DM trades off the marginal cost and benefit of getting finer information to determine the optimal partition.

Let $\mathcal{M}$ be the collection of compact subsets of $\mathcal{F}$. We want to extend preferences over information and singleton menus, $\succcurlyeq$ on $\Pi \times \mathcal{F}$, to preferences over information and menus $\succcurlyeq^+$ over $\Pi \times \mathcal{M}$. This extension is straightforward since $\succcurlyeq$ is π-recursive for each π.

[23] For a convex capacity ν, its core is $\{p \in \Delta(S) | p(E) \geq \nu(E) \text{ for all } E \in \Sigma\}$.

[24] This is called "uncertainty appeal" in [38].

To that end, for every $F \in \mathcal{M}$ and $\pi = \{E_1, \cdots, E_n\}$, define

$$F^\pi = \{f_1 E_1 f_2 E_2 \cdots E_{n-1} f_n : f_i \in F, \forall i = 1, \cdots, n\}.$$

Note $F \subseteq F^\pi \subseteq \mathcal{F}$, and $F = F^\pi$ whenever F is a singleton.

Next, for a menu F and partition π, we define its conditional certainty equivalent as

$$c(F|\pi) = \begin{pmatrix} c(F|E_1) & E_1 \\ c(F|E_2) & E_2 \\ \cdots & \\ c(F|E_n) & E_n \end{pmatrix}$$

where $c(F|E_i) \in X$ and

$$u(c(F|E_i)) = \max_{f \in F} V_0(f|E_i)$$

We define the preferences $\succsim^+$ on $\Pi \times \mathcal{M}$ as follows:

$$(\pi, F) \succsim^+ (\pi', G) \text{ if and only if } \forall g \in G^{\pi'}, \exists f \in F^\pi, (\pi, f) \succsim (\pi', g)$$

In this case we say $\succsim^+$ *is extended from* $\succsim$.

Lemma 3. *Suppose $V : \Pi \times \mathcal{F} \to \mathbb{R}$ represents $\succsim$. If $\succsim^+$ is extended from $\succsim$, then $\succsim^+$ is represented by $\tilde{V} : \Pi \times \mathcal{M} \to \mathbb{R}$ where*

$$\tilde{V}(\pi, F) = \max_{f \in F^\pi} V(\pi, f) = V_0(c(F|\pi))$$

Since $\tilde{V}$ and V agree on $\Pi \times \mathcal{F}$, we abuse notation a bit by using V to denote the extended function $\tilde{V} : \Pi \times \mathcal{M} \to \mathbb{R}$. Here $V(\pi, F)$ is interpreted as the value of the decision problem (π, F).

The following proposition states some comparative statics of $V(\pi, F)$.

Proposition 9. *1. If $F \subseteq F'$, then $V(\pi, F) \leq V(\pi, F')$.*

2. Suppose $\succsim^1$ and $\succsim^2$ are recursively generated by variational preferences $\succsim_0^1$ and $\succsim_0^2$. If $\succsim_0^1$ is more ambiguity averse than $\succsim_0^2$, then $\forall \pi, F$, $V^1(\pi, F) \leq V^2(\pi, F)$.

The proof is straightforward and thus omitted.

Part (1) of Proposition 9 says that the DM always weakly prefers bigger menus. This distinguishes our model from that in [41]. In [41], the DM might prefer a smaller menu due to dynamic inconsistency and desire for commitment. This suggests one way to test the two models.

Part (2) of Proposition 9 says that the more ambiguity averse the DM is, the less she values any information and menu pair (π, F). However, this does not say that the value of information is decreasing in the degree of ambiguity aversion. Example 3 below illustrates this point.

Furthermore, $V(\pi, F)$ is not monotone in information π, so more information can be strictly worse. Formally, π_2 is (strictly) more informative than π_1, denoted $\pi_2 \geq (>) \pi_1$, if the partition π_2 is (strictly) finer than the partition π_1. If $\succsim_0$ displays non-trivial ambiguity aversion, then we can find a menu F and partitions $\pi_2 > \pi_1$ such that $V(\pi_2, F) < V(\pi_1, F)$. Below is an example.

Example 2. Suppose $S = \{s_1, s_2, s_3\}$, and $\succsim_0$ has a MEU representation $(u, \mathcal{P})$ where $\mathcal{P} = \{p \in \Delta^3 | p(s_1) = \frac{1}{3}, p(s_3) \in [\frac{1}{6}, \frac{1}{2}]\}$. For simplicity assume risk neutrality, so $u(x) = x$. Suppose the DM faces menu $F = \{(0, 1, 1), (0.49, 0.49, 0.49)\}$. Then $V(\pi_0, F) = \frac{2}{3}$. Let $\pi = \{\{s_1, s_2\}, \{s_3\}\} > \pi_0$. The informed DM will choose $(0.49, 0.49, 0.49)$ given $\{s_1, s_2\}$, and $(0, 1, 1)$ given $\{s_3\}$. Therefore $V(\pi, F) = 0.575 < \frac{2}{3} = V(\pi_0, F)$.[25] Information hurts.

This non-monotonicity is driven by intrinsic aversion to partial. [5] shows that a preference for one-shot resolution of uncertainty in two-stage compound lotteries is equivalent to a preference for perfect information in an extended model with intermediate choices. We show a similar result is also true in our model.

We say that $\succsim^+$ exhibits *a preference for perfect information* if $\forall F \in \mathcal{M}$ and $\pi \in \Pi$, $(\pi^*, F) \succsim^+ (\pi, F)$.

Proposition 10. *Suppose $\succsim$ is recursively generated by $\succsim_0$, and $\succsim^+$ is extended from $\succsim$. Then the following statements are equivalent:*

1. *$\succsim^+$ exhibits a preference for perfect information.*
2. *$\succsim$ exhibits partial information aversion at all acts $f \in F$.*
3. *$\succsim_0$ satisfies Event Complementarity.*

Proof. See appendix. □

[25] $\mathcal{P}(s_2 | \{s_1, s_2\}) = [\frac{1}{3}, \frac{3}{5}]$, so $(0.49, 0.49, 0.49) \succ_{\{s_1, s_2\}} (0, 1, 1)$.

In the rest of this section, we focus on the value of acquiring information π: $\Delta V(\pi, F) := V(\pi, F) - V(\pi_0, F)$. In appendix A.12, we analyze the marginal value of information, $V(\pi_2, F) - V(\pi_1, F)$ for any $\pi_2 \geq \pi_1$.

Acquiring information π affects the decision problem in two ways. First, information provides a way for the DM to fine-tune her strategy: expecting to get π, she conditions her choice of optimal action on the event realized in π, so her effective menu expands from F to F^π. This captures the instrumental value of information, and is always non-negative. Second, information directly affects the DM's utility from acts, thus also has intrinsic value. The value of information π in decision problem F admits the following decomposition:

$$\begin{aligned}\Delta V(\pi, F) &= V(\pi, F) - V(\pi_0, F) \\ &= [\max_{f \in F^\pi} V(\pi, F) - \max_{f \in F} V(\pi, f)] + [\max_{f \in F} V(\pi, f) - \max_{f \in F} V(\pi_0, f)]\end{aligned}$$

The first bracketed term captures the non-negative instrumental value of information. The second bracketed term captures the intrinsic value of information. It is zero if the DM is intrinsically neutral to information, so $V(\pi, f) = V(\pi_0, f)$ for all f, and non-positive if the DM is averse to partial information. So a DM's willingness to pay for information π is the resulting trade-off of these two components.

Next we look for conditions under which the value of information is non-negative, that is, the DM is still willing to acquire information π when it is free, regardless of ambiguity.

Let $F_0 = \arg\max_{f \in F} V(\pi_0, f)$ be the set of uninformed optimal acts.

Proposition 11. *For any menu F, if there exists an uninformed optimal act f_0 that is π-measurable, then $\Delta V(\pi, F) \geq 0$.*

Corollary 2. *Suppose $\succsim_0^1$ has a variational representation (u_1, c_1), and $x \in X$ is an uninformed optimal act from menu F for DM 1. If DM 2 has a variational representation (u_2, c_2) and is more ambiguity averse than DM 1, then $\Delta V^2(\pi, F) \geq 0$.*

Proposition 9 says that for variational preferences $\succsim_0$, $V(\pi, F)$ is decreasing in the degree of ambiguity aversion in $\succsim_0$ for all (π, F). Is the same comparative statics true for the value of information $\Delta V(\pi, F)$? The answer is no. The value of information is non-monotone in the degree of ambiguity aversion. Below is an example.

Example 3. Suppose DM 1 has SEU preferences with belief $p \in \Delta(S)$. DM 2 has MEU preferences with non-singleton prior set $\mathcal{P} \subsetneq \Delta(S)$, and $\mathcal{P}$ is not rectangular with respect to some partition π (therefore $\pi > \pi_0$). DM 3 has MEU preferences with prior set $\mathcal{Q} = rect_\pi(\mathcal{P})$. Assume further that these three DMs have the same risk preferences, so DM 3 is more ambiguity averse than DM 2, and DM 2 is more ambiguity averse than DM 1.

Since $\mathcal{P} \subsetneq \mathcal{Q}$, there exists $f \in \mathcal{F}$ such that $V^2(\pi_0, f) > V^3(\pi_0, f)$. Also $V^2(\pi, f) = V^3(\pi_0, f) = V^3(\pi, f)$.[26] Therefore

$$V^3(\pi, f) - V^3(\pi_0, f) > V^2(\pi, f) - V^2(\pi_0, f).$$

Increasing ambiguity aversion *increases* the value of information π in this case.

Alternatively, DM 1 is intrinsically neutral to information, so $V^1(\pi, f) = V^1(\pi_0, f)$. Therefore

$$V^1(\pi, f) - V^1(\pi_0, f) = 0 > V^2(\pi, f) - V^2(\pi_0, f).$$

Increasing ambiguity aversion *decreases* the value of information π in this case.

Finally, we end this section with an application to portfolio choice problems.

Example 4 (Portfolio Choice)**.** Consider the portfolio choice example in Dow and Werlang (1992). Suppose there is a risk-neutral DM with wealth W. There is a risky asset with unit price P and present value that is either high, H, or low, L. The DM has MEU preferences and believes the probability of H belongs to the interval $[\underline{p}, \bar{p}]$. For simplicity, we assume the DM could choose to buy a unit of the risky asset (B), short-sell a unit of the risky asset (S), or not do anything (N). So $F = \{B, S, N\}$. The DM's optimal portfolio choice is

$$f_0^*(P) = \begin{cases} B & \text{if } \underline{p}H + (1-\underline{p})L > P; \\ N & \text{if } \bar{p}H + (1-\bar{p})L \geq P \geq \underline{\pi}H + (1-\underline{p})L; \\ S & \text{if } P > \bar{p}H + (1-\bar{p})L. \end{cases}$$

We now add an information acquisition stage before the portfolio choice. The DM can acquire a binary signal, $\pi = \{h, l\}$, which is correlated with the state of the risky asset, with $p(h|H) = p(l|L) = q > \frac{1}{2}$. We want to know if the DM will collect information π if it is costless.

Suppose the DM's uninformed optimal choice is B. Then $V(\pi_0, B) = \underline{p}H + (1-\underline{p})L - P$, and $V(\pi, B) = [\underline{p}qH + (1-\underline{p})(1-q)L + \underline{p}(1-q)H + (1-\underline{p})qL - P] = V(\pi_0, B)$. By Lemma 10 in the appendix, π is valuable. The other two cases could be calculated similarly. Without the need to compute the informed optimal strategies and $V(\pi, F)$, we can conclude that in this portfolio choice problem the DM will want to collect information π if it is costless.

[26] The argument is similar to that in the proof of Proposition 2.

6. Discussion: Second Order Belief Models

Another important class of ambiguity preferences is the second order belief model [29, 35, 40]. We say $\succcurlyeq_0$ has a second order belief representation if

$$V(\pi_0, f) = \int_{\Delta(S)} \phi[\int_S u(f) dp_\theta] d\mu$$

where $\mu \in \Delta(\Delta(S))$ is a second order belief over the space of distributions $\Delta(S)$, and ϕ is a non-decreasing function capturing ambiguity attitude. When ϕ is smooth and concave (convex), the DM is ambiguity averse (loving).

For the second order belief models, translation invariance fails, and thus Conditional Certainty Equivalent Consistency cannot provide a well-defined update rule. Instead we adopt Bayes rule for the second order belief μ as our update rule.

Assumption 1. Suppose $\succcurlyeq_0$ has a second order belief representation $(u, \phi; \Theta, \mu)$. Then for any non-null event E, $\succcurlyeq_E$ has a second order belief representation $(u_E, \phi_E; \Theta_E, \mu_E)$ satisfying

1. Risk and ambiguity attitudes are not updated: $u_E = u$, $\phi_E = \phi$.
2. Prior by prior updating of first order belief: $\Theta_E = \{p_\theta(\cdot|E) | p_\theta \in \Theta\}$.
3. Bayes rule for second order belief:

$$\mu_E(\theta) = \frac{\mu(\theta) p_\theta(E)}{\int_\Theta p_{\theta'}(E) d\mu(\theta')} \tag{4}$$

In general, second order belief models exhibit no systematic relation between ambiguity aversion and information aversion, as the following example illustrates.

Example 5. Consider the standard three color Ellsberg urn. Let $S = \{R, G, Y\}$ and $\Theta = \{(\frac{1}{3}, \frac{2}{3}\theta, \frac{2}{3}(1-\theta)) | \theta = \frac{1}{3}, \frac{2}{3}\}$. Suppose the second order prior μ puts equal probability on $p_{\frac{1}{3}} = (\frac{1}{3}, \frac{2}{9}, \frac{4}{9})$, and $p_{\frac{2}{3}} = (\frac{1}{3}, \frac{4}{9}, \frac{2}{9})$. Assume the DM is risk neutral with $u(x) = x$, and ambiguity averse with $\phi(y) = log(y)$. Information is given by the partition $\pi = \{\{R, G\}, \{Y\}\}$. Let $E = \{R, G\}$. Suppose the above update rule captures conditional preferences, so $\mu_E(p_{\frac{1}{3}}) = \frac{5}{12}$, and $\mu_E(p_{\frac{2}{3}}) = \frac{7}{12}$. By computation we can show that the DM is strictly averse to π ($V(\pi, f) < V(\pi_0, f)$) at acts $f = (1, 0, 0)$ and $(0, 1, 1)$, and strictly loves π ($V(\pi, f) > V(\pi_0, f)$) at acts $f = (0, 1, 0)$ and $(1, 0, 1)$.

Observe that the partition $\pi' = \{\{R\}, \{G, Y\}\}$ contains only events with known probabilities. The two acts $(1, 0, 0)$ and $(0, 1, 1)$, at which the DM is strictly averse to partial information π, are measurable with respect to π' and thus unambiguous. This suggests that a DM with second order belief preferences will be averse to partial information at acts where she has local ambiguity neutrality. The next proposition formalizes this idea.

Following Definition 4 in [29], we say $\succcurlyeq_0$ displays (local) smooth ambiguity neutrality at act f if $V(\pi_0, f) = \phi[\int_{\Delta(S)} \int_S u(f) dp_\theta d\mu]$. In second order belief models, ambiguity aversion only implies partial information aversion at the subclass of locally ambiguity neutral acts.

Proposition 12. *Suppose $\succcurlyeq_0$ and $\{\succcurlyeq_E\}_{E\in\Sigma}$ are second order belief preferences, with update rule satisfying Assumption 1. If $\succcurlyeq_0$ is ambiguity averse (loving), then $\succcurlyeq$ exhibits partial information aversion (loving) at all acts where $\succcurlyeq_0$ displays (local) smooth ambiguity neutrality.*

Proof. See appendix. □

A Appendix: Proofs

A.1 Lemma 2

Proof of Lemma 1. Fix (π, f). We want to show that the sets $U = \{(\pi', g) : (\pi', g) \succcurlyeq (\pi, f)\}$ and $L = \{(\pi', g) : (\pi, f) \succcurlyeq (\pi', g)\}$ are closed.

Let $\{(\pi'_n, g_n)\}$ be a convergent sequence in the set U, with limit (π', g). We want to show (π', g) is also in U. Since $\pi'_n \to \pi'$ in the discrete topology on Π, there exists some N such that for all $n > N$, $\pi'_n = \pi'$. Continuity of $\succcurlyeq_\pi$ ensures there exists a constant act x_f, with $(\pi, f) \sim (\pi, x_f) \sim (\pi', x_f)$, where the last statement follows from Stable Risk Preferences. If $(\pi', x_f) \sim (\pi, f) \succ (\pi', g)$, then by continuity of $\succcurlyeq_{\pi'}$, there exists $M(> N)$ such that for all $n > M$, $(\pi', x_f) \succ (\pi', g_n)$. So $(\pi, f) \succ (\pi'_n, g_n)$ for sufficiently large n, a contradiction to the assumption $\{(\pi'_n, g_n)\} \subseteq U$. □

The next lemma verifies the existence of certainty equivalents as result of Continuity and Monotonicity.

Lemma 4. *For any nonempty $E \in \Sigma$, if $\succsim_E$ satisfies Continuity and Monotonicity, then for every act f we can find an E-conditional certainty equivalent $c(f|E) \in X$ such that $c(f|E) \sim_E f$.*

Proof. Let $f \in \mathcal{F}$. Since f is finitely ranged, by Monotonicity there exists $x^*, x_* \in X$ such that $x^* \succsim_E f \succsim_E x_*$. By continuity, $U = \{\alpha \in [0,1] : \alpha x^* + (1-\alpha)x_* \succsim_E f\}$ and $L = \{\alpha \in [0,1] : f \succsim_E \alpha x^* + (1-\alpha)x_*\}$ are closed subsets of $[0,1]$. Since $U \cup L = [0,1]$, by connectedness of $[0,1]$, $U \cap L \neq \emptyset$. Thus there exists $c(f|E) \in U \cap L$, and by definition $c(f|E) \sim_E f$. $\square$

Proof of Lemma 2. First we show equivalence of the two statements. (2) $\Rightarrow$ (1) is a straightforward verification. We show (1) $\Rightarrow$ (2).

Since $\succsim$ is a continuous and monotone preference relation, there exists a continuous function $V : \Pi \times \mathcal{F} \to \mathbb{R}$ that represents $\succsim$. By Time Neutrality, $(\pi_0, f) \sim (\pi^*, f), \forall f$, so $V(\pi_0, \cdot) = V(\pi^*, \cdot) : \mathcal{F} \to \mathbb{R}$ represents the restricted preference relations $\succsim_{\pi_0}$ and $\succsim_{\pi^*}$. Let $V_0(\cdot) := V(\pi_0, \cdot)$, and let $u : X \to \mathbb{R}$ be the restriction of V_0 to constant acts X, where $u(x) = V_0(x)$. Since V_0 is continuous, u is continuous. $u(X)$ is connected and thus an interval in $\mathbb{R}$, since $X = \Delta(Z)$ is connected. We define the functional $I_0 : B(\Sigma, u(X)) \to \mathbb{R}$ by $I_0(\xi) = V_0(f)$, where $\xi \in B(\Sigma, u(X))$, $f \in \mathcal{F}$ satisfy $u \circ f = \xi$. Then I_0 is well-defined and monotone by monotonicity of $\succsim_0$. For any $k \in u(X)$, choose the constant act x such that $u(x) = k$. Then by definition, $I_0(\bar{k}) = V_0(x) = u(x) = k$. So I_0 is normalized.

Similarly, for every π, the continuous function $V_\pi = V(\pi, \cdot) : \mathcal{F} \to \mathbb{R}$ represents $\succsim_\pi$. We show the connection between V_π and (u, I_0). Fix π, f, $E_i \in \pi$. By continuity and monotonicity of $\succsim_{E_i}$, we can find conditional certainty equivalent $c(f|E_i) \sim_{E_i} f$. By π-recursivity, $(\pi, f) \sim (\pi, c(f|\pi))$. Then $(\pi, c(f|\pi)) \sim (\pi^*, c(f|\pi)) \sim (\pi_0, c(f|\pi))$, where the first indifference is by Indifference to Redundant Information, and the second by Time Neutrality. By transitivity of $\succsim$, $(\pi, f) \sim (\pi_0, c(f|\pi))$, so $V_\pi(f) = V_0(c(f|\pi)) = I_0(u \circ c(f|\pi))$. $\square$

A.2 $\succsim_0$-non-null Events

This subsection clarifies the concept of a $\succsim_0$-non-null event for defining conditional preferences.

The literature normally adopts the condition of a non-null event from Savage. An event E is *Savage $\succsim_0$-non-null* if there exists f, g, h, such that $fEh \succ_0 gEh$.

We consider a stronger condition: an event E is $\succcurlyeq_0$*-non-null* if there exist constant acts x^*, x_* such that $x^* \succ_0 x_*$ and $x^* E x_* \succ_0 x_*$. An event E is Savage $\succcurlyeq_0$-non-null if it is $\succcurlyeq_0$-non-null, but not vice versa. The next lemma compares how these two definitions differ in the variational preference family.

Lemma 5. *Suppose $\succcurlyeq_0$ has a variational representation (u, c). An event E is $\succcurlyeq_0$-non-null if and only if $p(E) > 0$ for all $p \in c^{-1}(0)$. An event E is Savage $\succcurlyeq_0$-non-null if and only if there exists some act f and some $p \in \arg\min_{p' \in \Delta(S)} \int u(f)dp' + c(p')$ such that $p(E) > 0$.*

Proof. For the first claim, we prove E is $\succcurlyeq_0$-non-null iff $\exists p \in c^{-1}(0)$ such that $p(E) = 0$. Choose constant acts x^*, x_* such that $x^* \succ_0 x_*$. First, suppose $\exists p \in c^{-1}(0)$ such that $p(E) = 0$. Then

$$V_0(x^* E x_*) = u(x^*)p(E) + u(x_*)p(E^c) + c(p) = u(x_*) = V_0(x_*)$$

The first equality holds because $c(p) = 0$ and $p(E) = 0$. Next, suppose instead $p(E) > 0$ for all $p \in c^{-1}(0)$. Then let $p^* \in \arg\min_{p'} u(x^*)p'(E) + u(x_*)p'(E^c) + c(p')$. Either $p^* \in c^{-1}(0)$ and $p^*(E) > 0$, or $c(p^*) > 0$. In either case,

$$V_0(x^* E x_*) u(x^*)p^*(E) + u(x_*)p^*(E^c) + c(p^*) > u(x_*)$$

so $x^* E x_* \succ_0 x^*$.

For the second claim, suppose there exists some act f and some $p \in argmin_{p' \in \Delta(S)} \int u(f)dp' + c(p')$ such that $p(E) > 0$. Then we can construct an act f' such that $f'(s) = f(s)$ for all $s \in E^c$, and $u(f'_s) = u(f_s) - \epsilon$ for all $s \in E$, and some $\epsilon > 0$. Since $p(E) > 0$,

$$\begin{aligned} V_0(f) &= \int_E u(f)dp + \int_{E^c} u(f)dp + c(p) \\ &> \int_E u(f)dp - \epsilon p(E) + \int_{E^c} u(f)dp + c(p) \\ &= \int_S u(f')dp + c(p) \geq V_0(f') \end{aligned}$$

So $f \succ_0 f'$. For the converse, suppose there exists f, g, h such that $fEh \succ_0 gEh$. Let $p \in \arg\min_{p' \in \Delta(S)} \int u(gEh)dp' + c(p')$. We argue that $p(E) > 0$. If instead $p(E) = 0$, then

$$V_0(gEh) = \int_E u(g)dp + \int_{E^c} u(h)dp + c(p) = \int_E u(f)dp + \int_{E^c} u(h)dp + c(p) \geq V_0(fEh)$$

This contradicts $fEh \succ_0 gEh$. $\square$

Suppose $\succcurlyeq_0$ has an MEU representation $(u, \mathcal{P})$. As a corollary, E is $\succcurlyeq_0$-non-null if and only if $p(E) > 0$ for all $p \in \mathcal{P}$. In contrast, E is Savage $\succcurlyeq_0$-non-null if and only if there exists f and $p \in \arg\min_{p\in\mathcal{P}} \int u(f)dp$ such that $p(E) > 0$.

For the results about updating, the stronger $\succcurlyeq_0$-non-null condition is needed. [36] shows that if the unconditional preferences $\succcurlyeq_0$ have an MEU representation $(u, \mathcal{P})$ and all priors give positive probability to event E, then Conditional Certainty Equivalence Consistency is satisfied if and only if $\succcurlyeq_E$ has an MEU representation $(u, \mathcal{P}_E)$, where $\mathcal{P}_E$ is the prior-by-prior updated posteriors from $\mathcal{P}$. In section 4.3, we show that if the unconditional preferences $\succcurlyeq_0$ have a variational representation (u, c) and $p(E) > 0$ for all $p \in c^{-1}(0)$, then Conditional Certainty Equivalence Consistency is satisfied if and only if $\succcurlyeq_E$ has a variational representation (u, c_E), where c_E is obtained from c using update rule (5). In both cases, E has to be $\succcurlyeq_0$-non-null instead of Savage $\succcurlyeq_0$-non-null.

In the text, we impose Strong Monotonicity on $\succcurlyeq_0$ to ensure that updating is always well-defined. The following lemma follows directly by definition.

Lemma 6. *If $\succcurlyeq_0$ satisfies Strong Monotonicity, then every event E in Σ is $\succcurlyeq_0$-non-null.*

A.3 Theorem 1

We first recall a result from [33].

Lemma 28, [33] *A binary relation $\succcurlyeq_0$ on $\mathcal{F}$ satisfies Weak Order, Weak Certainty Independence, Continuity, Monotonicity, and Non-degeneracy if and only if there exists a nonconstant affine function $u : X \to \mathbb{R}$ and a normalized, monotone, and translation invariant $I_0 : B(S, u(X)) \to \mathbb{R}$ such that*

$$f \succcurlyeq_0 g \Leftrightarrow I_0(u(f)) \geq I_0(u(g))$$

Below we will apply this result to prove our representation Theorem 1.

Proof of Theorem 1. We verify only the direction (1) $\Rightarrow$ (2). The other direction is straightforward.

By Lemma 2, (i) implies there exists a continuous function $V_0 : \mathcal{F} \to \mathbb{R}$ such that that for each π, $\succcurlyeq_\pi$ can be represented by

$$V(\pi, f) = V_0(c(f|\pi))$$

where $c(\cdot|\pi) : \mathcal{F} \to \mathcal{F}_\pi$ is the conditional certainty equivalent mapping.

Define $u : X \to \mathbb{R}$ by $u(x) = V_0(x)$. Define $I_0 : B(\Sigma, u(X)) \to \mathbb{R}$ by $I_0(\xi) = V_0(f)$, for $\xi \in B(\Sigma, u(X))$, $f \in \mathcal{F}$ such that $u \circ f = \xi$. By Lemma 28 in [33], Weak Certainty Independence, Continuity, Monotonicity, and Non-degeneracy of $\succcurlyeq_0$ implies that u is continuous, nonconstant and affine, and I_0 is well-defined, continuous, normalized, and translation invariant. Moreover, for any $\xi, \xi' \in B(S, u(X))$ such that $\xi > \xi'$, there exists $f, g \in \mathcal{F}$ such that $u \circ f = \xi$ and $u \circ g = \xi'$, $f(s) \succcurlyeq_0 g(s)$ for all s, and $f(s) \succ_0 g(s)$ for some s. By Strong Monotonicity of $\succcurlyeq_0$, $f \succ_0 g$ and thus $I_0(\xi) > I_0(\xi')$. So I_0 is strongly monotone.

Next we show that for all f and nonempty $E \in \Sigma$, $k = I_0[(u \circ f)E\bar{k}]$ has a unique solution in $u(X)$.

Existence of solution. Fix f and nonempty E. Define $G(k) = I_0[(u \circ f)E\bar{k}] - k = I_0[(u \circ f - \bar{k})E0]$, for all $k \in u(X)$. Since f is finite-ranged, we can find x^*, x_* such that $x^* \succ_0 f(s) \succ_0 x_*$ for all s. Let $k^* = u(x^*)$, and $k_* = u(x_*)$. Then $G(k^*) \geq 0$, and $G(k_*) \leq 0$ by monotonicity of I_0. Since I_0 is continuous, G is a continuous function of k on $u(X)$. By the intermediate value theorem, there exists $k_0 \in [k_*, k^*]$ such that $G(k_0) = 0$.

Uniqueness of solution. Suppose k_1 and k_2 both solve $k = I_0[(u \circ f)E\bar{k}]$, and $k_1 \neq k_2$. Without loss of generality, let $k_1 > k_2$. By translation invariance of I_0,

$$I_0[(u \circ f - \bar{k}_1)E0] = I_0[u(fE\bar{k}_1)] - k_1 = 0 = I_0[u(fE\bar{k}_2)] - k_2 = I_0[(u \circ f - \bar{k}_2)E0]$$

Then $(u \circ f - \bar{k}_1)E0 < (u \circ f - \bar{k}_2)E0$, since E is non-empty. Since I_0 is strictly monotone, $I_0[(u \circ f - \bar{k}_1)E0] < I_0[(u \circ f - \bar{k}_2)E0]$. A contradiction.

For any $\pi = \{E_1, E_2, \cdots, E_n\}$, by Conditional Certainty Equivalent Consistency, x_i is the E_i-conditional Certainty Equivalent of f if and only if $x_i \sim_0 fE_ix_i$. This implies that $u(x_i)$ solves $k = I_0[(u \circ f)E_i\bar{k}]$ and $x_i \sim_0 c(f|E_i)$. So $u(x_i) = u(c(f|E_i))$, which implies $V_0(f|\pi) = u \circ c(f|\pi)$. As a result, $V(\pi, f) = V_0(c(f|\pi)) = I_0(V_0(f|\pi))$ by definition of I_0.

Finally, suppose both (u, I_0) and (u', I_0') represent $\succcurlyeq_0$. Since both u and u' are affine representations of $\succcurlyeq_0$ on X, by the Mixture Space Theorem [26], $u' = au + b$ for some $a > 0$ and $b \in \mathbb{R}$. For all f, let x_f be the constant act that $f \sim_0 x_f$. Then

$$I_0(u(f)) = u(x_f)$$

$$I_0'(u'(f)) = u'(x_f)$$

Substituting $u' = au + b$, we get

$$I_0'(u'(f)) = I_0'(au(f) + b) = au(x_f) + b$$

and thus $I_0'(au(f)+b) = aI_0(u(f))+b$. Since f is arbitrary, we have for all $\xi \in (u'(X))^S$, $I_0'(\xi) = aI_0(\frac{\xi-b}{a})+b$.

$\square$

A.4 Theorem 2

Proof. By Conditional Certainty Equivalent Consistency, $fEx \sim_0 x \Leftrightarrow x \sim_E f$, for all f, x, E. So it suffices to show that $\succcurlyeq$ exhibits partial information aversion if and only if $x \sim_E f \Rightarrow f \succcurlyeq_0 xEf$, for all f, x, E.

Suppose $\succcurlyeq_0$ satisfies Event Complementarity. Fix a finite partition $\pi = \{E_1, \cdots, E_n\}$, and an act f. For each $i = 1, \cdots, n$, let $x_i \in X$ be the E_i-conditional certainty equivalent of f, i.e., $x_i \sim_{E_i} f$. Let $f_0 := f$, $f_1 = x_1E_1f_0$, $f_2 = x_2E_2f_1$, $\cdots$, $f_n = x_nE_nf_{n-1} = (x_1E_1x_2E_2\cdots x_{n-1}E_{n-1}x_n)$. Note that f_n is π-measurable. Also $x_i \sim_{E_i} f_{i-1}, \forall i = 1, \cdots, n$, thus $(\pi_0, f_{i-1}) \succcurlyeq (\pi_0, f_i)$ by Event Complementarity, and $(\pi, f_0) \sim (\pi, f_1) \sim \cdots \sim (\pi, f_n)$ by π-Recursivity. Putting these results together yields:

$$\begin{aligned}(\pi, f) &\sim (\pi, f_n) \sim (\pi^*, f_n) \\ &\sim (\pi_0, f_n) \quad \text{(by Time Neutrality)} \\ &\preccurlyeq (\pi_0, f_{n-1}) \cdots \preccurlyeq (\pi_0, f)\end{aligned}$$

Since this is true for an arbitrary act f and partition π, $\succcurlyeq$ exhibits aversion to partial information.

We prove the converse by contradiction. Suppose not, so $\succcurlyeq$ exhibits aversion to partial information but there exists some π, $E \in \pi$, f, and x such that $f \sim_E x$, but $(\pi_0, xEf) \succ (\pi_0, f)$. Let $n_1, \cdots, n_m$ be labels for states in E^c, i.e., $E^c = \{s_{n_1}, \cdots, s_{n_m}\}$. Then consider the finer partition $\pi' = \{E, \{s_{n_1}\}, \cdots, \{s_{n_m}\}\}$. Thus xEf is π'-measurable, and by Axioms 4 and 5, $(\pi', xEf) \sim (\pi^*, xEf) \sim (\pi_0, xEf)$. By π'-Recursivity, $(\pi', f) \sim (\pi', xEf)$. By transitivity, $(\pi', f) \sim (\pi_0, xEf) \succ (\pi_0, f)$. This violates partial information aversion, a contradiction.

$\square$

A.5 Proposition 2

Proof. For part (1), by Theorem 2, it suffices to show that $\succcurlyeq_0$ satisfies Event Complementarity. Since $\succcurlyeq_0$ belongs to the MEU class, by Lemma 3.3 in [16], I_0 is superadditive.

Event Complementarity follows from that.

For part (2), if $\succcurlyeq_0$ has an MEU representation $(u, \mathcal{P})$ and $\succcurlyeq$ is recursively generated by $\succcurlyeq_0$, then $\succcurlyeq$ can be represented by

$$\begin{aligned} V(\pi, f) &= \min_{p \in \mathcal{P}} \sum_{i=1}^{n} [\min_{p^i \in \mathcal{P}} \int u(f) dp^i(\cdot|E_i)] p(E_i) \\ &= \min_{p \in \mathcal{P}} \min_{p^i \in \mathcal{P}} \sum_{i=1}^{n} [\int u(f) dp^i(\cdot|E_i)] p(E_i) \\ &= \min_{p' \in rect_\pi(\mathcal{P})} \int u(f) dp' \end{aligned}$$

Suppose $\mathcal{P}$ is not π-rectangular, so there exists $q \in rect_\pi(\mathcal{P}) \backslash \mathcal{P}$. Since $\mathcal{P}$ is convex and compact, by the strict separating hyperplane theorem, there exists a nonzero, bounded and measurable map $\xi \in B(\Sigma, \mathbb{R})$ such that

$$\int \xi dq < \int \xi dp, \forall p \in \mathcal{P}$$

Without loss of generality, let $0 \in int(u(X))$. There exists $f \in \mathcal{F}$ such that $u(f) = \alpha\xi$, for some $\alpha > 0$. Thus without loss of generality we can replace ξ by $u(f)$ in above inequality. By compactness of $\mathcal{P}$, $\min_{p \in \mathcal{P}} \int u(f) dp$ attains at some $p^* \in \mathcal{P}$, so using above

$$V(\pi, f) = \min_{q' \in rect_\pi(\mathcal{P})} \int u(f) dq' \le \int u(f) dq < \int u(f) dp^* = V(\pi_0, f)$$

Thus $\succcurlyeq$ is strictly averse to partition π at f.

For the converse, suppose $\mathcal{P}$ is π-rectangular, so $\mathcal{P} = rect_\pi(\mathcal{P})$. Then $V(\pi, f) = V(\pi_0, f), \forall f$, and $\succcurlyeq$ is intrinsically neutral to information π.

□

A.6 Proposition 3

Proof. By Theorem 1 in [43], if $\succcurlyeq_0$ has a multiplier representation, then Savage's Sure-Thing principle is satisfied. So $\forall f \in F$ and x such that $fEx \sim_0 x$, we have $f \sim_0 xEf$. By step 1 of our proof for Theorem 2, this yields information neutrality. □

A.7 Theorem 3 and Corollary 1

Lemma 7. *For the conditional cost function*

$$c_E(p_E) = \inf_{\{p\in\Delta(S):p(\cdot|E)=p_E\}} \frac{c(p)}{p(E)} \tag{5}$$

if $p(E) > 0$ for all $p \in c^{-1}(0)$, then the infimum attains at some $p \in \Delta(S)$, where $p(\cdot|E) = p_E$.

Proof. Let $Q(p_E) := \{p \in \Delta(S) : p(\cdot|E) = p_E\}$. Then $\overline{Q(p_E)} = Q(p_E) \cup \Delta(E^c)$ is compact in $\Delta(S)$. If $c(p) = +\infty$ for all $p \in Q(p_E)$, then $c_E(p_E) = +\infty$ and the infimum attains at any $p \in Q(p_E)$. Otherwise, $c_E(p_E) < +\infty$. By the definition of infimum, we can find a sequence $p^n \in Q(p_E)$, such that $\frac{c(p^n)}{p^n(E)}$ is decreasing and $\lim_n \frac{c(p^n)}{p^n(E)} = c_E(p_E)$. By compactness of $\overline{Q(p_E)}$, we can find a subsequence of $\{p^n\}$, say $\{p^k\}$, such that $p^k \to_k p^* \in \overline{Q(p_E)}$. It remains to show that if $p(E) > 0$ for all $p \in c^{-1}(0)$, then $p^* \notin \Delta(E^c)$.

Suppose not, so $p^* \in \Delta(E^c)$. By assumption, $c(p^*) > 0$, so $\frac{c(p^*)}{p^*(E)} = +\infty$. Yet by lower semicontinuity of c, $c_E(p_E) = \liminf_k \frac{c(p^k)}{p^k(E)} \geq \frac{c(p^*)}{p^*(E)} = +\infty$. A contradiction. □

From our discussion in Appendix 5, Strong Monotonicity of $\succcurlyeq_0$ ensures that all events are $\succcurlyeq_0$-non-null. As a result, the condition that $p(E) > 0$ for all $p \in c^{-1}(0)$ is satisfied for all E.

We then verify that c_E is convex, lower semicontinuous and grounded, so c_E can serve as a cost function.

Lemma 8. *The function $c_E : \Delta(E) \to [0, \infty]$ defined in (5) is (i) convex, (ii) lower semicontinuous, and (iii) grounded.*

Proof. Convexity. By the lower semicontinuity of c, $\forall p_E, q_E \in \Delta(E), \alpha \in [0, 1]$, we can find $p^*, q^* \in \Delta$ such that $p^*(\cdot|E) = p_E, q^*(\cdot|E) = q_E$, and $c_E(p_E) = \frac{c(p^*)}{p^*(E)}, c_E(q_E) = \frac{c(q^*)}{q^*(E)}$. Fix $\alpha \in [0, 1]$. Then there exists $\gamma \in [0, 1]$ such that $\frac{\gamma p^*(E)}{\gamma p^*(E)+(1-\gamma)q^*(E)} = \alpha$. Set $p' := \gamma p^* + (1 - \gamma)q^*$. Then $p'(\cdot|E) = \alpha p_E + (1 - \alpha)q_E$. Therefore,

$$\begin{aligned} c_E(\alpha p_E + (1-\alpha)q_E) &\leq \frac{c(p')}{p'(E)} \leq \frac{\gamma c(p^*) + (1-\gamma)c(q^*)}{\gamma p^*(E) + (1-\gamma)q^*(E)} \\ &= \frac{\gamma p^*(E)}{\gamma p^*(E) + (1-\gamma)q^*(E)} c_E(p_E) + \frac{(1-\gamma)q^*(E)}{\gamma p^*(E) + (1-\gamma)q^*(E)} c_E(q_E) \\ &= \alpha c_E(p_E) + (1-\alpha)c_E(q_E). \end{aligned}$$

Lower semicontinuity. We want to show the epigraph $epi(c_E)$ is closed. To that end, let $(p_E^n, r_n) \in epi(c_E)$, $(p_E^n, r_n) \to_n (p_E, r)$. We want to show $r \geq c_E(p_E)$. Since $p(E) > 0$ for all $p \in c^{-1}(0)$, by the previous lemma $c_E(p_E^n) = \frac{c(p^n)}{p^n(E)}$ for some p^n where $p^n(\cdot|E) = p_E^n$. Since $\Delta(S)$ is compact, there exists a subsequence $\{p^k\}$ of $\{p^n\}$ such that $p_k \to_k p^*$.

If $p^*(E) > 0$, then $p^*(\cdot|E) = \lim_k p^k(\cdot|E) = \lim_k p_E^k = p_E$. Then $\liminf_k \frac{c(p^k)}{p^k(E)} \geq \frac{c(p^*)}{p^*(E)}$ by lower semicontinuity of c. Since $r_k \to r$ and $r_k \geq c_E(p_E^k) = \frac{c(p^k)}{p^k(E)}$, $r \geq \liminf_k \frac{c(p^k)}{p^k(E)} \geq \frac{c(p^*)}{p^*(E)} \geq c_E(p_E)$. Then we are done.

If $p^*(E) = 0$, then there must be a subsequence $p^k(E) \to_k 0$. Since $r + \epsilon \geq \tilde{c}_E(p_E^k) = \frac{c(p^k)}{p^k(E)}$ for $\epsilon > 0$ and sufficiently large k, $\liminf_k c(p^k) = 0 \geq c(p^*) \geq 0$. Thus $p^*(E) = 0$ and $c(p^*) = 0$, a contradiction.

Groundedness. c is grounded, so there exists p^* such that $c(p^*) = 0$. By assumption, $p^*(E) > 0$, so $c_E(p^*(\cdot|E)) = 0$. $\square$

Lemma 9. *Consider two variational functionals $I(\phi) = \min_{p\in\Delta}\langle \phi, p\rangle + c(p)$, and $I'(\phi) = \min_{p\in\Delta}\langle \phi, p\rangle + c'(p)$. If $c(p_0) < c'(p_0)$ for some p_0, then there exists $\xi \in B(\Sigma)$ such that $I(\xi) < I'(\xi)$.*

Proof. Consider the epigraph of c':

$$epi(c') = \{(p, r) \in \Delta \times \mathbb{R} | r \geq c'(p)\}$$

Since c' is nonnegative, convex, lower semicontinous, and grounded, $epi(c')$ is nonempty, closed and convex. Let $r_0 = c(p_0)$. Since $c(p_0) < c'(p_0)$, $(p_0, r_0) \notin epi(c')$. By the strict separating hyperplane theorem there exists $(\xi_0, r^*) \in B(\Sigma) \times \mathbb{R}$, $(\xi_0, r^*) \neq 0$, that strictly separates (p_0, r_0) from the set $epi(c')$, such that, that is

$$\langle \xi_0, p_0\rangle + r_0 \cdot r^* < \inf_{r' \geq c'(p')} \langle \xi_0, p'\rangle + r' \cdot r^*$$

Note that we cannot have $r^* < 0$, otherwise we could take $r' = +\infty$ in the right hand side and the inequality fails. Also we cannot have $r^* = 0$, otherwise we get $\langle \xi_0, p_0\rangle < \inf_{p'}\langle \xi_0, p'\rangle \leq \langle \xi_0, p_0\rangle$, a contradiction. Thus $r^* > 0$, and we can rescale both sides by $\frac{1}{r^*}$ (take $\xi = \frac{1}{r^*}\xi_0$) to obtain

$$\langle \xi, p_0\rangle + r_0 < \inf_{r' \geq c'(p')} \langle \xi, p'\rangle + r'$$

Then

$$\langle \xi, p_0\rangle + r_0 = \langle \xi, p_0\rangle + c(p_0) \geq \min_{p\in\Delta}\langle \xi, p\rangle + c(p) = I(\xi)$$

and

$$\inf_{r' \geq c'(p')} \langle \xi, p' \rangle + r' = \min_{p' \in \Delta} \langle \xi, p' \rangle + c'(p') = I'(\xi)$$

Thus $I(\xi) \leq \langle \xi, p_0 \rangle + r_0 < \inf_{r' \geq c'(p')} \langle \xi, p' \rangle + r' = I'(\xi)$. □

Proof of Theorem 3. (2) ⇒ (1). Suppose (2) holds. It is straightforward to verify Stable Risk Preferences and Consequentialism. We prove Conditional Certainty Equivalent Consistency also holds.

Fix $f \in \mathcal{F}$ and $x \in X$ such that $x \sim_E f$. We must prove $fEx \sim_0 x$. Suppose c and c_E satisfy update rule (5). Then

$$\begin{aligned} x \sim_E f \Rightarrow u(x) &= \inf_{p_E \in \Delta(E)} \int_E u(f) dp_E + c_E(p_E) \\ &= \inf_{p_E \in \Delta(E)} \int_E u(f) dp_E + \inf_{p \in \Delta : p(\cdot|E) = p_E} \frac{c(p)}{p(E)} \end{aligned}$$

Let $p^* \in \Delta$ achieve the infimum above.[27]

$$\begin{aligned} u(x) &= p^*(E) \left[\int_E u(f) dp^*(\cdot|E) + \tfrac{c(p^*)}{p^*(E)} \right] + p^*(E^c) u(x) \\ &= \int_E u(f) dp^* + p^*(E^c) u(x) + c(p^*) \\ &\geq \min_{p \in \Delta} \int_E u(f) dp + p(E^c) u(x) + c(p) = V_0(fEx) \end{aligned}$$

It remains to show that the inequality cannot be strict. If not, then $u(x) > V_0(fEx)$. Let $\tilde{p} \in argmin_{p \in \Delta} \int_E u(fEx) dp + p(E^c) u(x) + c(p)$. Then

$$\begin{aligned} u(x) > V_0(fEx) &= \min_{p \in \Delta} \int_E u(f) dp + p(E^c) u(x) + c(p) \\ &= \int_E u(f) d\tilde{p} + \tilde{p}(E^c) u(x) + c(\tilde{p}) \end{aligned}$$

If $\tilde{p}(E) = 0$, then $u(x) > u(x) + c(\tilde{p})$, which contradicts the non-negativity of c. So

[27] Let $I_E : B(\Sigma_E, u(X)) \to \mathbb{R}$ be such that $I_E(\xi) = \inf_{p_E \in \Delta(E)} \int_E \xi dp_E + c_E(p_E)$. Then I_E is also a variational functional. Applying [33] Lemma 26, the infimum attains at some p_E^*. In addition, if $p(E) > 0$ for all $p \in c^{-1}(0)$, by the previous lemma there exists $p^* \in \Delta(S)$, $p^*(\cdot|E) = p_E^*$, at which the second infimum attains.

$\tilde{p}(E) > 0$. Then

$$\begin{aligned}
u(x) &> \frac{1}{\tilde{p}(E)}[\int_E u(f)d\tilde{p} + c(\tilde{p})] \\
&= \int_E u(f)d\tilde{p}(\cdot|E) + \frac{c(\tilde{p})}{\tilde{p}(E)} \\
&\geq \min_{p_E\in\Delta(E)} \int_E u(f)dp_E + \min_{p\in\Delta:p(\cdot|E)=p_E} \frac{c(p)}{p(E)} \\
&= V_E(f)
\end{aligned}$$

This contradicts the assumption that $x \sim_E f$. So $fEx \sim_0 x$.

For the converse, suppose $fEx \sim_0 x$. Then

$$\begin{aligned}
u(x) = V_0(fEx) &= \min_{p\in\Delta(S)} \int_E u(f)dp + u(x)p(E^c) + c(p) \\
&= \int_E u(f)dp^* + u(x)p^*(E^c) + c(p^*)
\end{aligned}$$

where $p^* \in argmin_p \int_E u(f)dp + u(x)p(E^c) + c(p)$. If $p^*(E) = 0$, then the equality above implies $c(p^*) = 0$, a contradiction to the assumption that $p(E) > 0, \forall p \in c^{-1}(0)$. So $p^*(E) > 0$, and

$$p^*(E)u(x) = \int_E u(f)dp^* + c(p^*)$$

Thus

$$\begin{aligned}
u(x) &= \int_E u(f)dp^*(\cdot|E) + \frac{c(p^*)}{p^*(E)} \\
&\geq \min_{p_E} \int_E u(f)dp_E + \inf_{p\in\Delta:p(\cdot|E)=p_E} \frac{c(p)}{p(E)} = V_E(f)
\end{aligned}$$

So $x \succsim_E f$.

Also, as argued before, we can find $q^* \in \Delta(S), q^*(E) > 0$, such that $V_E(f) = \int_E u(f)dq^*(\cdot|E) + \frac{c(q^*)}{q^*(E)}$. So

$$q^*(E)\left[\int_E u(f)dq^*(\cdot|E) + \frac{c(q^*)}{q^*(E)}\right] + q^*(E^c)u(x) \geq V_0(fEx) = u(x)$$

Thus $V_E(f) \geq u(x)$, or $f \succsim_E x$. So $x \sim_E f$.

(1) $\Rightarrow$ (2). By assumption, $\succsim_E$ has a representation of the form

$$V_E(f) = \min_{p\in\Delta(S)} \int_S u_E(f)dp + c_E(p)$$

By Stable Risk Preferences, $\succsim_0$ and $\succsim_E$ agree on constant acts X. We can normalize by setting $u_E = u$. Next we want to show only p with support on E can achieve the minimum defining V_E. For each $f \in \mathcal{F}$, choose $p^* \in \arg\min_{p \in \Delta(S)} \int_S u(f)dp + c_E(p)$. Without loss of generality, we can choose $x_* \in X$ such that $f(s) \succ_0 x_*$ for all s.[28] Since $(fEx_*)Ex = fEx$ for any x, by Conditional Certainty Equivalent Consistency, $fEx_* \sim_E f$. Then

$$V_E(f) = \int_S u(f)dp^* + c_E(p^*) = V_E(fEx_*) \leq \int_E u(f)dp^* + p^*(E^c)u(x_*) + c_E(p^*)$$

So $\int_{E^c}(u(f) - u(x_*))dp^* \leq 0$. Since $u(f) - u(x_*)$ is strictly positive on E^c, $\int_{E^c}(u(f) - u(x_*))dp^* \geq 0$, and this is an equality if and only if $p^*(E^c) = 0$. So $p^*(E) = 1$, and p^* has a natural imbedding in $\Delta(E)$. Therefore $\forall f$,

$$V_E(f) = \min_{p \in \Delta(E)} \int_E u(f)dp + c_E(p)$$

It remains to show that the (unique) conditional cost function c_E coincides with $\tilde{c}_E(p_E) := \inf_{p \in \Delta : p(\cdot|E) = p_E} \frac{c(p)}{p(E)}$. Suppose not, so $c_E \neq \tilde{c}_E$. Thus there exists p_E^* such that $c_E(p_E^*) \neq \tilde{c}_E(p_E^*)$. We prove a contradiction for the case $c_E(p_E^*) > \tilde{c}_E(p_E^*)$. The case $c_E(p_E^*) < \tilde{c}_E(p_E^*)$ can be proved by replicating the arguments. Applying Lemma 9, we can find $\xi_E \in B(\Sigma_E)$ such that $\min_{p_E} \int_E \xi_E dp_E + \tilde{c}_E(p_E) < \min_{p_E} \int_E \xi_E dp_E + c_E(p_E)$. Since $u(X)$ is unbounded, $B(\Sigma_E) \subseteq B(\Sigma_E, u(X)) + \mathbb{R}$. Thus there is an act $f \in \mathcal{F}$ such that $(u(f) + k)(s) = \xi_E(s)$ on E for some constant k. So $\min_{p_E} \int_E u(f)dp_E + \tilde{c}_E(p_E) < \min_{p_E} \int_E u(f)dp_E + c_E(p_E)$.

By Continuity, we can find $x \in X$ that is the E-conditional equivalent of f, $x \sim_E f$, and $u(x) = V_E(f) = \min_{p_E} \int_E u(f)dp_E + c_E(p_E)$.

Then

$$\begin{aligned}
u(x) &= \min_{p_E} \int_E u(f)dp_E + c_E(p_E) \\
&> \min_{p_E} \int_E u(f)dp_E + \tilde{c}_E(p_E) \\
&= \min_{p_E} \int_E u(f)dp_E + \inf_{p \in \Delta : p(\cdot|E) = p_E} \frac{c(p)}{p(E)} \\
&= \min_{p_E} \inf_{p \in \Delta : p(\cdot|E) = p_E} \int_E u(f)dp_E + \frac{c(p)}{p(E)} \\
&= \inf_{p \in \Delta, p(E) > 0} \frac{1}{p(E)} \left[\int_E u(f)dp + c(p) \right]
\end{aligned}$$

[28] If not, then $u(X)$ is bounded below and $\min_s u(f)(s)$ achieves the lower bound. By translation invariance p^* is also a minimizing probability for f' such that $u(f') = u(f) + \epsilon$. Then the whole argument works for f'.

As argued before, we can find $\underline{p} \in argmin_{p\in\Delta, p(E)>0}\frac{1}{p(E)}\left[\int_E u(f)dp + c(p)\right]$. Then multiplying both sides of the inequality by $\underline{p}(E)$ and adding $\underline{p}(E^c)u(x)$ to both sides yields

$$
\begin{aligned}
u(x) &> \underline{p}(E)\left(\frac{1}{\underline{p}(E)}[\int_E u(f)d\underline{p} + c(\underline{p})]\right) + \underline{p}(E^c)u(x) \\
&= \int_E u(f)d\underline{p} + \underline{p}(E^c)u(x) + c(\underline{p}) \\
&= \int u(fEx)d\underline{p} + c(\underline{p}) > V_0(fEx)
\end{aligned}
$$

So $x \succ_0 fEx$, violating Conditional Certainty Equivalent Consistency. □

Proof of Corollary 1. For part (1), suppose $\succcurlyeq_0$ has a MEU representation $(u, \mathcal{P})$. So it has a variational representation (u, c) with cost function c such that $c(p) = 0$ if $p \in \mathcal{P}$ and $c(p) = +\infty$ if $p \notin \mathcal{P}$. For any nonempty event E, Strong Monotonicity of $\succcurlyeq_0$ ensures that $p(E) > 0$ for all $p \in \mathcal{P}$. Applying updating rule 5,

$$
c_E(p_E) = \begin{cases} 0 \text{ if } p_E \in \mathcal{P}_E = \{p(\cdot|E)|p \in \mathcal{P}\} \\ +\infty \text{ otherwise} \end{cases}
$$

So $\succcurlyeq_E$ has MEU representation $(u, \mathcal{P}_E)$.

For part (2), suppose $\succcurlyeq_0$ also has a multiplier preference representation (u, q, θ). So it has a variational representation (u, c) with cost function $c(p) = \theta \int ln\frac{p}{q}dp$. For any nonempty event E, Strong Monotonicity of $\succcurlyeq_0$ ensures that $q(E) > 0$. Applying updating rule 5,

$$
\begin{aligned}
c_E(p_E) &= \min_{p\in\Delta(S):p(\cdot|E)=p_E} \frac{\theta}{p(E)}\int ln\frac{p}{q}dp \\
&= \min_{p\in\Delta(S):p(\cdot|E)=p_E} \frac{\theta}{p(E)}[(\int_E ln\frac{p_E}{q_E}dp_E)p(E) + (\int_{E^c} ln\frac{p_{E^c}}{q_{E^c}}dp_{E^c})p(E^c) \\
&\qquad + (p(E)ln\frac{p(E)}{q(E)} + p(E^c)\ln\frac{p(E^c)}{q(E^c)})] \\
&= \theta\int_E ln\frac{p_E}{q_E}dp_E
\end{aligned}
$$

In the last step, we choose p such that $p(E) = q(E)$ and $p(\cdot|E^c) = q(\cdot|E^c)$. So $\succcurlyeq_E$ has multiplier representation (u, q_E, θ). □

A.8 Proposition 4

Proof. By Theorem 2, $\succcurlyeq$ exhibits intrinsic information aversion at all acts if and only if $\forall f$ and x such that $fEx \sim_0 x$, $f \succcurlyeq_0 xEf$. By Conditional Certainty Equivalent Consistency, $fEx \sim_0$ if and only if $x \sim_E f$.

If $x \sim_E f$, then $u(x) = \min_{p_E \in \Delta(E)} \int_E u(f) dp_E + c_E(p_E)$. So

$$\begin{aligned}
V_0(xEf) &= \min_{p\in\Delta} p(E)u(x) + \int_{E^c} u(f)dp + c(p) \\
&= \min_{p\in\Delta} p(E)[\min_{p_E\in\Delta(E)} \int_E u(f)dp_E + \hat{c}_E(p_E)] + \int_{E^c} u(f)dp + c(p) \\
&= \min_{p\in\Delta} \min_{p_E\in\Delta(E)} p(E)[\int_E u(f)dp_E + \hat{c}_E(p_E)] + + \int_{E^c} u(f)dp + c(p) \\
&= \min_{q\in\Delta} \min_{q_E\in\Delta(E)} \int u(f)dq + q(E)\hat{c}_E(q_E) + c(q_E \otimes_E q) \\
&\text{(change of variable: } q = p_E \otimes_E p, \text{ and } q_E = p(\cdot|E)) \\
&= \min_{q\in\Delta} \int u(f)dq + q(E)\hat{c}_E(q_E) + \min_{q_E\in\Delta(E)} c(q_E \otimes_E q)
\end{aligned}$$

Also

$$V_0(f) = \min_{q\in\Delta} \int u(f)dq + c(q)$$

"If" direction. Suppose $\inf_{q_E\in\Delta(E)} c(q_E \otimes_E p) + p(E) \inf_{q\in\Delta(S)} \frac{c(p_E\otimes q)}{q(E)} \leq c(p), \forall p$. Then for all f, q,

$$\int u(a)dq + q(E)\hat{c}_E(q_E) + \min_{q_E\in\Delta(E)} c(q_E \otimes_E q) \leq \int u(f)dq + c(q),$$

so $V_0(xEf) \leq V_0(f)$. Thus the DM is averse to partial information at all f.

"Only if" direction. For each $E \in \Sigma$, define

$$\tilde{c}(p) = \begin{cases} \inf_{q_E\in\Delta(E)} c(q_E \otimes_E p) + p(E)c_E(p(\cdot|E)) & \text{if } p(E) > 0 \\ +\infty & \text{otherwise} \end{cases}$$

Define $\tilde{I} : B(S, \mathbb{R}) \to \mathbb{R}$ by $\tilde{I}(\xi) = \inf_{p\in\Delta(S)} \int \xi dp + \tilde{c}(p)$. By the calculation above, we have $\forall f \in \mathcal{F}$, $x \sim_E f$, $V_0(xEf) = \tilde{I}(u(f))$.

If statement (2) fails, then there exists p such that $\tilde{c}(p) > c(p)$. By Lemma 9, we can find $\xi \in B(S, \mathbb{R})$ such that $\tilde{I}(\xi) > I(\xi)$. By unboundedness, $B(S, \mathbb{R}) \subseteq B(S, u(X)) + \mathbb{R}$, so there exists $f \in F$ such that $u(f) + k = \xi$ for some constant k. So we can find $f \in \mathcal{F}$ such that $V_0(xEf) = \tilde{I}(u(f)) > I(u(f)) = V_0(f)$. This contradicts aversion to partial information. $\square$

A.9 Proposition 5

Proof. Let $p^* \in c^{-1}(0) \cap \arg\min_{p\in\Delta}[\int_S u(f)dp + c(p)]$. Then $\forall \pi = \{E_1, \cdots, E_n\}$,

$$\begin{aligned}
V(\pi, f) &= \min_{p\in\Delta} \sum p(E_i) \left[\min_{p_i\in\Delta(E_i)} \int u(f)dp_i + c_{E_i}(p_i)\right] + \min_{\{q\in\Delta(S):q=p \text{ on } \pi\}} c(q) \\
&\leq \sum p^*(E_i) \left[\int u(f)dp^*(\cdot|E_i) + c_{E_i}(p^*(\cdot|E_i))\right] + \min_{\{q\in\Delta(S):q=p^* \text{ on } \pi\}} c(q^*) \\
&= \int_S u(f)dp^* \\
&= \int_S u(f)dp^* + c(p^*) = V(\pi_0, f)
\end{aligned}$$

The second equality follows from

$$c_{E_i}(p^*(\cdot|E_i)) = \min_{p(\cdot|E_i)=p^*(\cdot|E_i)} \frac{c(p)}{p(E_i)} = 0$$

and

$$\min_{\{q\in\Delta(S):q=p^* \text{ on } \pi\}} c(q^*) = 0.$$

□

A.10 Proposition 8

Proof. (1) Suppose $\succcurlyeq_0$ has a CEU representation (u, ν) and satisfies Uncertainty Aversion. By the Proposition in [38] the corresponding functional I_0 is concave and superadditive. By Proposition 1, this implies that Event Complementarity holds. By Theorem 2, $\succcurlyeq$ exhibits aversion to partial information.

(2) Suppose $\succcurlyeq_0$ has a CEU representation (u, ν) and satisfies Uncertainty Loving. By [38] Remark 6 the corresponding functional I_0 is convex and subadditive. By Proposition 1 and Theorem 2, $\succcurlyeq$ exhibits attraction to partial information. □

A.11 Lemma 3

Proof. We first show $\succcurlyeq^+$ is represented by $\tilde{V}(\pi, F) = \max_{f\in F^\pi} V(\pi, f)$. For all (π, F) and (π', G), $(\pi, F) \succcurlyeq^+ (\pi', G)$ if and only if

$$\forall g \in G^{\pi'}, \exists f \in F^\pi, (\pi, f) \succcurlyeq (\pi', g)$$

Since $V : \Pi \times \mathcal{F}$ represents $\succsim$, this is equivalent to

$$\max_{f \in F^{\pi}} V(\pi, f) \geq \max_{g \in G^{\pi'}} V(\pi', g)$$

Thus $(\pi, F) \succsim^{+} (\pi', G)$ if and only if $\tilde{V}(\pi, F) \geq \tilde{V}(\pi', G)$.

Then we show $\max_{f \in F^{\pi}} V(\pi, f) = V_0(c(F|\pi))$. By definition, $F^{\pi} = \{f_1 E_1 f_2 E_2 \cdots E_{n-1} f_n : f_i \in F, \forall i = 1, \cdots, n\}$. So

$$\begin{aligned}
\max_{f \in F^{\pi}} V(\pi, f) &= \max_{f_1 \in F} \ldots \max_{f_n \in F} V(\pi, f_1 E_1 \cdots E_{n-1} f_n) \\
&= \max_{f_1 \in F} \ldots \max_{f_n \in F} V(\pi, [c(f_i|E_i), E_i]_1^n) \quad \text{by } \pi\text{-Recursivity} \\
&= \max_{f_1 \in F} \ldots \max_{f_n \in F} V_0([c(f_i|E_i), E_i]_1^n) \\
&= V_0([c(F|E_i), E_i]_1^n) \quad \text{by } \pi_0\text{-monotonicity}
\end{aligned}$$

where the second to last equality is due to Independence from Redundant Information and Time Neutrality. ☐

A.12 Proposition 10 and 11

Proof of Proposition 10. (2) ⇔ (3) is due to Theorem 2.

To show (1) ⇒ (2), take any singleton menu $F = \{f\}$. A preference for perfect information implies $(\pi^*, f) \succsim (\pi, f), \forall \pi$. By Time Neutrality, $(\pi^*, f) \sim (\pi_0, f)$, so $(\pi_0, f) \succsim (\pi, f)$.

To show (2) ⇒ (1). Let $\pi \in \Pi$ and $F \in \mathcal{M}$. Then

$$V(\pi^*, F) - V(\pi, F) = [\max_{f \in F^{\pi^*}} V(\pi^*, f) - \max_{f \in F^{\pi}} V(\pi^*, f)] + [\max_{f \in F^{\pi}} V(\pi^*, f) - \max_{f \in F^{\pi}} V(\pi, f)]$$

The first term is non-negative since $F^{\pi} \subseteq F^{\pi^*}$. By (2) and Time Neutrality, $V(\pi^*, f) = V(\pi_0, f) \geq V(\pi, f)$, for all π, f. So

$$max_{f \in F^{\pi}} V(\pi, f) = V(\pi, f^*) \leq V(\pi^*, f^*) \leq \max_{f \in F^{\pi}} V(\pi^*, f)$$

where $f^* \in F^{\pi}$ is the act that maximizes $V(\pi, \cdot)$. So the second term is also non-negative. Thus $V(\pi^*, F) \geq V(\pi, F)$ and the DM has preferences for perfect information. ☐

Next we prove Proposition 11. We first prove a lemma. Let $F_0 = \arg\max_{f\in F} V(\pi_0, f)$ be the set of uninformed optimal acts. By our decomposition, as long as the DM is not strictly averse to information π at some $f_0 \in F_0$, then information is valuable.

Let $F_i^* = \arg\max_{f\in F} V_{E_i}(f)$ be the set of optimal acts in F conditional on learning about E_i. Consider $F^* = \{f_1^* E_1 f_2^* E_2 \cdots E_{n-1} f_n^* : f_i^* \in F_i^*, \forall i\} \subseteq F^\pi$. The instrumental value of information is zero if and only if $F^* \cap F \neq \emptyset$. We collect these observations below.

Lemma 10. *1. If there exists an unconditional optimal act $f_0 \in F_0$ such that $V(\pi, f_0) \geq V(\pi_0, f_0)$ at f_0, then $V(\pi, F) - V(\pi_0, F) \geq 0$.*

2. If there exists a conditional optimal strategy $f^* \in F^*$ such that $f^* \in F$ and $V(\pi, f^*) \leq (<) V(\pi_0, f^*)$, then $V(\pi, F) - V(\pi_0, F) \leq (<) 0$.

Proof. By definition $V(\pi_0, f_0) = \max_{f\in F} V(\pi_0, f)$. If $V(\pi, f_0) \geq V(\pi_0, f_0)$, then the intrinsic value of information π at menu F is non-negative:

$$\max_{f\in F} V(\pi, f) - \max_{f\in F} V(\pi_0, f) \geq V(\pi, f_0) - V(\pi_0, f_0) \geq 0.$$

As the instrumental value of information is always non-negative, $V(\pi, F) - V(\pi_0, F) \geq 0$ and π is valuable.

If there exists $f^* \in F \cap F^*$, then the instrumental value of π, $V(\pi, f^*) - \max_{f\in F} V(\pi, f) = 0$. In addition $\max_{f\in F} V(\pi, f) = V(\pi, f^*) \leq V(\pi_0, f^*) \leq \max_{f\in F} V(\pi_0, f)$, so the intrinsic value of π is non-positive. ☐

Remark 4. The first condition is helpful, as it requires only calculation of an optimal act in the uninformed case. This could simplify checking whether ambiguity aversion generates information aversion or not. In MEU models, this is equivalent to $V(\pi, f_0) = V(\pi_0, f_0)$, when the intrinsic value of information π for menu F vanishes.

Proof of Proposition 11. If there exists an uninformed optimal act f_0 that is π-measurable, then $V(\pi, f_0) = V(\pi^*, f_0) = V(\pi_0, f_0)$. By the above lemma, $\Delta V(\pi, F) \geq 0$. ☐

Proof of Corollary 2. Let x be the uninformed optimal act for DM 1. So $V^1(\pi_0, x) \geq V^1(\pi_0, f)$, for all f in menu F. Since DM 2 is more ambiguity averse than DM 1, $u_2 = u_1$ and $c_2 \leq c_1$. So for all $f \in \mathcal{F}$,

$$V^2(\pi_0, f) = \min_{p\in\Delta(S)} \int_S u(f)dp + c_2(p) \leq \min_{p\in\Delta(S)} \int_S u(f)dp + c_1(p) = V^1(\pi_0, f)$$

and $V^1(\pi_0, x) = u(x) = V^2(\pi_0, x)$. Thus $V^2(\pi_0, x) \geq V^2(\pi_0, f)$ for all $f \in F$. Since x is π-measurable, by Proposition 11 we have $\Delta V^2(\pi, F) \geq 0$. ☐

Marginal Value of Information

For any menu F, consider two partitions $\pi_2 \geq \pi_1$. The marginal value of getting the finer information π_2 is:

$$V(\pi_2, F) - V(\pi_1, F) = [\max_{f \in F^{\pi_2}} V(\pi_2, f) - \max_{f \in F^{\pi_1}} V(\pi_2, f)] + [\max_{f \in F^{\pi_1}} V(\pi_2, f) - \max_{f \in F^{\pi_1}} V(\pi_1, f)]$$

The first term captures the instrumental value of getting finer information π_2 relative to π_1, and since $F^{\pi_1} \subseteq F^{\pi_2}$ this term is non-negative. The second part captures the intrinsic value of information π_2 relative to π_1.

Lemma 10 can be generalized as follows.

Lemma 11. *1. If there exists an optimal strategy f^{*1} for decision problem (π_1, F) such that $V(\pi_1, f^{*1}) \leq V(\pi_2, f^{*1})$, then $V(\pi_2, F) - V(\pi_1, F) \geq 0$.*

2. If there exists an optimal strategy f^{*2} for decision problem (π_2, F) such that $f^{*2} \in F^{\pi_1}$ and $V(\pi_1, f^{*2}) \geq V(\pi_2, f^{*2})$, then $V(\pi_2, F) - V(\pi_1, F) \leq 0$.

The proof is similar to the proof of Lemma 10 and hence omitted.

A.13 Proposition 12

Proof. Fix $\pi = \{E_1, \cdots, E_n\}$. Suppose $\succcurlyeq_0$ has second order belief representation $(u, \phi; \Theta, \mu)$ and $\succcurlyeq_0$ is ambiguity averse. Then by [29] Proposition 1, ϕ is concave. Let f be an act where $\succsim_0$ displays local ambiguity neutrality. Then

$$\begin{aligned}
V(\pi, f) &= \int_\Theta \phi[\sum_{i=1}^n p_{\theta'}(E_i)\phi^{-1}[\int_\Theta \phi(\int u(f)dp_{\theta_i}(\cdot|E_i))d\mu_{E_i}(\theta_i)]]d\mu(\theta') \\
&\leq \int_\Theta \phi[\sum_{i=1}^n p_{\theta'}(E_i)[\int_\Theta \int u(f)dp_{\theta_i}(\cdot|E_i)d\mu_{E_i}(\theta_i)]]d\mu(\theta') \\
&= \int_\Theta \phi[\sum_{i=1}^n p_{\theta'}(E_i)(\int_\Theta \int_{E_i} u(f)dp_{\theta_i}\frac{d\mu(\theta_i)}{\int p_{\theta''}(E_i)d\mu(\theta'')}]d\mu(\theta') \\
&= \int_\Theta \phi[\sum_{i=1}^n (\int_\Theta \int_{E_i} u(f)dp_{\theta_i}d\mu(\theta_i)\frac{p_{\theta'}(E_i)}{\int p_{\theta''}(E_i)d\mu(\theta'')}d\mu(\theta')] \\
&\leq \phi\int_\Theta [\sum_{i=1}^n (\int_\Theta \int_{E_i} u(f)dp_{\theta_i}d\mu(\theta_i)\frac{p_{\theta'}(E_i)}{\int p_{\theta''}(E_i)d\mu(\theta'')}d\mu(\theta')] \\
&= \phi[(\sum_{i=1}^n \int_\Theta \int_{E_i} u(f)dp_{\theta_i}d\mu(\theta_i)(\int_\Theta \frac{p_{\theta'}(E_i)}{\int p_{\theta''}(E_i)d\mu(\theta'')}d\mu(\theta'))] \\
&= \phi(\int_\Theta \int_S u(f)dp_\theta d\mu) = V(\pi_0, f)
\end{aligned}$$

The two inequalities follow from the concavity of ϕ. The last equality holds because $\succsim_0$ displays local ambiguity neutrality at f.

The case for ambiguity loving $\succsim_0$ can be proved analogously. □

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