A Network Theory of Military Alliances *

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Abstract

This paper introduces network game theory into the study of international relations and specifically, military alliances. Using concepts from graph theory, I formally define defensive alliance, offensive alliance and powerful alliance, and on the basis of which, develop a novel network game that takes these forms of alliances as steady states any given collectivity of countries might evolve into. For the complex variations of the game, I propose a solution algorithm and show the robustness of the model in affirming many historic facts including those from World War I and World War II.

1 Introduction

In political economy, scholars have reached the consensus on the lack of an accepted, theoretically compelling and operational definition of military alliance, the lack of which has limited the theorizing about alliance behavior. This paper makes a step towards providing an operational definition (or theory) of military alliances, by addressing a set of critical questions: How are alliances different? How can alliances help their members and thereby impact the broader cooperation and conflict between members and outsiders?

The current literature on military alliances has proposed three main theories of alliance behavior: balance of power, balance of threat and balance of interest. The gist of the bal-

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ance of power theory is the specific idea that if one state gains excessive power, the power might be transformed into offensive capabilities to attack weaker neighbors, which provides an incentive for the threatened to unite for survival (Waltz, 1979). Instead, Walt argues that when confronted by a significant external threat, states may choose between the strategies of balancing and bandwagoning and develops balance of power theory into balance of threats theory. (Walt, 1987) While balancing is alignment against the prevailing threat, bandwagoning is alignment with the source of danger. States choose to bandwagon because it may be a form of appeasement. Along similar lines, past scholarship notes many other tactics states can choose when facing threats, such as buck-passing, chain-ganging, tethering or hedging, and so forth (Snyder, 1990; Mearsheimer, 2001; Weitsman, 2004; Pape, 2005). However, a criticism can be made that without a carefully developed and commonly understood foundation, it could be hard to distinguish and mediate different claims because any one of them only speaks to one facet of reality. Regarding this, Schweller distinguishes between bandwagoning and balancing on the basis of the respective motivation. (Schweller, 1994) The distribution of capabilities, by itself, does not determine the stability of the system; an equally important factor is the interests of countries to which those capabilities are applied, which entails the basic analysis of costs and benefits. An example is that a hegemony can coexist in harmony with multiple other great powers because their well-beings are inextricably linked together. (Schweller, 2010) He thus proposes a balance of interest theory to address the concerns for the previous two theories: while bandwagoning is commonly done in the expectation of making gains, balancing is done for security and always entails costs.

Empirically, many investigations on alliances often make use of certain datasets, which have facilitated the analysis of alliance behavior but whose coding is usually based on the specific pacts the countries signed, such as the pacts of defense, neutrality, nonaggression and entente.(Small and Singer, 1969; Gibler and Sarkees, 2004) This typology is straightforward and useful; however, it adds to rather than reduces the complexity in defining alliances. First, by applying a de jure definition of alliances, many alliances remain under-defined. For example, the Triple Alliance in World War I, which had signed defense pacts and would never have signed "offense" pacts in the first place, still had offensive motives and should have been counted as an offensive alliance. So simply using the datasets leaves an intractable issue in the literature, offense-defense indistinguishability, unaddressed; second, each of the pacts provides for a certain behavior in the case of a conflict but the kind of behavior provided for is very different in each case, which creates problems for theoretical purposes. To address the problems above, a microfoundation is of fundamental importance, showing the need for a rigorous and repeatable methodology. (Dolan, 2007) Strategic-interaction models or any kind of formalism would be useful for this purpose. Previous models in the literature, for instance, Smith's two-stage games in alliance formation (Smith, 1995) and Morrow's model of arms trade-off (Morrow, 1991), can work in many scenarios but network game can be a much more ideal alternative. From an empirical perspective, alliances are essentially networks and the structural characteristics do influence the strategies of both signatories and outsiders; second, from a theoretical perspective, modeling agents' "network behavior" can incorporate many forms of extra-dyadic relationships, which can broaden the analysis to a good extent.

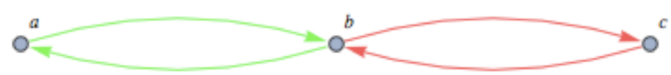


Figure 1: a **and** b **are allies and** b **and** c **are foes**

Consider the collection of countries with different interests and power in Figure 1 above. To represent interests, we may assume country a and b are allies and country b and c are having a war. We go on to assume each has some material power. We then examine how countries optimally allocate resources to their different interests or relations. For example, country a has to decide on the proportion of its total resources to spend on supporting b's war with c; country b and c decide respectively on the amount of resources to spend on the war with each other. Though the basic formulation takes the relations as exogenous and leaves out "network formation", understanding countries' resource allocation to different interests or "network behavior" in a given relational structure is already a fruitful attempt.

So in this paper, I construct a theory of military alliances using network analysis that has integrated systemic features, the alliance structure as well as state characteristics in one single framework. In particular, I take alliances' structures as they already are in reality and explain countries' resource allocation on networks. The basic idea of the theory is that aggregating individual countries' micro-level optimizing behavior gives different macro-level military alliances.

First, I formally define multiple types of alliances using a combination of graph theoretic concepts and a resource-allocation framework, which provides basic and essential theorizing of alliances. The formulation helps to mediate among the three aforementioned existing theories of alliance behavior, and to pave the way for a network game that eventually produces these defined alliances as equilibrium. Though still underexploited, network games suit the analysis of military alliances especially well.

Second, I construct a network game as well as an algorithm that solves any variation of the game. With the relations (ally, foe or no relation) for any given dyad as given, countries in the network allocate their total capacity into self-defense, support for allies and threat towards foes. The allocation should be optimal for these countries such that they obtain the highest possible utility associated with the *state* attained. Here I define an Alliance Network Nash Equilibrium as the solution concept, meaning that given the strategies of all the other countries, any country would have no incentives to deviate from the current strategy. Aggregating the optimizing behavior of countries give different forms of alliances as steady states, such as defensive alliance and powerful alliance.

The model is related to the current scholarship on networks and specifically on network games. (Bala and Goyal, 2000; Jackson and Wolinsky, 1996; Konrad and Kovenock, 2009; Bloch and Jackson, 2006; Hiller, 2011; Jackson and Zenou, 2012; Papadimitriou, 2003; Menache and Ozdaglar, 2011) It even furthers the current studies by embedding a resource allocation framework into graphs. The model also borrows insights into modeling alliance behavior from some statistical analyses of political networks in political science literature. For instance, Warren has shown stochastic actor-oriented models combined with Markov simulations of network evolution to be a productive alternative method of modeling interstate alliances, which allows the incorporation of extra-dyadic interdependence; Maoz's analysis of alliance and trade networks over the 18702003 period reveals strong evidence that alliance networks are affected by the "homophily" processes.(Maoz, 2012)

Third and lastly, I test the model with two conflicts involving the most complex alliance dynamics we ever know of – World War I and World War II. The empirical testing consists of two parts. First, I draw up the network structures of the two wars in consecutive years and solve each game with the algorithm, using real data on states' military expenditure and relation patterns. I then match the game solution with historic facts. Second, I conduct simulations which predict the likelihood for any country in the given game to be in any state (aggregating the probabilities for individual countries, I derive the probabilities for different alliances to occur as equilibrium). The network games in World War I and World War II yield empirical regularities that are consistent with historic facts. Notably, the model and the algorithm work even better with interstate conflicts of more complex relation patterns.

In all, this paper makes two main contributions: first, it imports and adapts concepts in graph theory into use for political economy, providing an operational concept (or theory) of military alliances. It also has laid a solid micro-foundation for military alliances with a novel network game; third, the model is highly useful for corroborating many historic facts. I have tested the game and the solution algorithm with examples from World War I and World War II. [1] To my knowledge, this paper is the first attempt to operationalize military alliances. It is also the first one in international relations to use network games for theory building and for a systematic testing of the two great wars.

The paper is organized as follows: I propose a formal definition of alliances as well as a theoretical mechanism to explain how alliances are different from each other. On this basis, I work through the game that predicts alliance behavior and alliance patterns in equilibrium. Lastly, I present results from empirical testing.

[1] Games for smaller-scaled conflicts involving military alliances are simple and can be solved without running the optimization algorithm.

2 A Network Game of Alliances

The new definition of military alliances is adapted from a graph-theoretic definition. This graph-theoretic definition builds on a political logic: an alliance is a collection of entities such that the union is stronger than the individual, which can either function to protect against attack, or to assert collective will against others. This definition was first articulated by Kristiansen, Hedetniemi and Hedetniemi.(Kristiansen, Hedetniemi and Hedetniemi, 2004) They argue that in graphs any given collectivity of nodes is an alliance and the node connectivity determines its type — defensive, offensive or powerful. Informally, given a graph $G = (V; E)$, a set V is an offensive alliance if every other vertex that is adjacent to V is outgunned by V. In recent years, researchers in graph theory and theoretical computer science have furthered the studies of alliances in graphs along similar lines(Bermudo et al., 2010; Brigham, Dutton and Hedetniemi, 2007; Dutton, 2009; Rodríguez-Velazquez and Sigarreta, 2006; Sigarreta, Bermudo and Fernau, 2009) .

Formally speaking, the original definition of alliances assumes all the edges are of equal weight and models the number of all connecting nodes as the overall defense support. However, for this specific application, I refine it further to incorporate the resource-allocation structure: the nodes are countries; the node connectivity denotes bilateral relations (or state interests), such as between allies or foes; the edge weights denote state investments on those relations. So for instance, the defense support for a given country should be modeled by the sum of its connected edges' weights rather than the number of its connected nodes.

With this refinement, the graphic-theoretic definition of alliances becomes political-economic — alliances can be defined on the basis of the type and amount of the capability investment. Alliances are *defensive* if their defensive capabilities are sufficient to ward off any attacks towards them. Alliances are *offensive* if their foes are vulnerable. This can be due to the offensive capabilities of the alliances that are sufficient to crush the defense of their foes. If an alliance is both offensive and defensive, it is *powerful*. From the perspective of a network game, they can be viewed as equilibria or steady states a given collectivity of states converges to.

2.1 The Network Mechanics

Consider a directed graph $G = (V, E)$, where V denotes the set of all countries and E denotes the set of relations between countries.

Definition 1. Node A node in the graph represent a country. I assume that one country can support another country, attack it or do nothing. These three actions represent three types of relation: "ally", "foe" and "none".

Definition 2. Connectivity Directed edges between nodes represents the relation they have. Any two connected nodes are either "allies" or "foes". A and Φ are two disjoint sets of E that $A \cup \Phi = E$. A refers to the collection of *all* ally relations. If $(i, j) \in A$, i will support j in this game. Similarly, Φ refers the collections of *all* foe relations.

If two countries have no relations, they can not affect each other's strategy, so they are not connected. So if country i has no relations with country j, (i, j) will not be in the set E.

Definition 3. Succeeding Node and Preceding Node In (i, j), j is i's **succeeding node**, which means country i initiates certain behavior towards j. In other words, i is j's **preceding node**. $Succ(i) = \{j|(i, j) \in E\}$ denotes i's *all* succeeding nodes, and $Pred(i) = \{j|(i, j) \in E\}$ denotes i's *all* preceding nodes.

Specifically, $Succ(i)$ can be a country i attacks or defends; similarly, $Pred(i)$ can be a country attacking i or defending i. A more convenient notion $N(i) = Pred(i) \cup i$ represents the closed in-neighborhood[2] of i. $\partial V = \bigcup_{i \in V} Succ(i) \setminus V$ represents the out-neighborhood of V.

Definition 4. Node Capacity c_i is the **capacity** of country i, which denotes the overall national capabilities. C is the collection of countries' national capabilities.

Definition 5. Edge Weight $w_{i,j}$ is the **weight** of i's relation to j. $w_{i,j}$ denotes the amount of capabilities i invests towards j. If $(i, j) \notin E$, $w_{i,j} = 0$ and $\sum_{(i,j) \in E} w_{i,j} + w_{i,i} \leq c_i$.

[2] "Closed" means $N(i)$ contains i itself.

If $(i,j) \in A$, $w_{i,j}$ measures the defense support i gives to j; if $(i,j) \in \Phi$, $w_{i,j}$ measures the offense u have for j. $w_{i,i}$ is i's self-defense. The total efforts i invests for self defense, support for its allies and threat for its foes must not exceed its total capacity.

Before proceeding to the game, some specific definitions should be made. By assumption, any node i (or country i) in the graph is either ally or foe with its connected nodes. It can be further defined as below that there are four sets of relations between i and its connected nodes.

Definition 6. Ally Set and Foe Set A_i and Φ_i are two disjoint sets of $Succ(i)$. $A_i = \{j|(i,j) \in A\}$ refers to the set such that country i defends as allies and is therefore called the **ally set** of i. Similarly, $\Phi_i = \{j|(i,j) \in \Phi\}$ will be i's **foe set**, the set of countries that i threatens.

Definition 7. Support Set and Threat Set Ξ_i and Θ_i are two disjoint sets of $Pred(i)$. Ξ_i refers to the set that defends country i as allies and is therefore called the **support set** for i. Similarly, Θ_i will be i's **threat set**, the collection of countries that threaten i.

On the basis of the four sets, a behavioral assumption can be made that for any country in an alliance, it has to **support** its allies *and* **be supported** by them[3]; additionally, for any country with foes, it **threatens** them and **is threatened** by them. We can formally denote the total support and threat facing any country.

Definition 8. Total Support and Total Threat $\xi_i = \sum_{j \in \Xi_i} w_{j,i} + w_{i,i}$ represents total support for country i; and $\theta_i = \sum_{j \in \Theta_i} \max\{w_{j,i} - w_{i,j}, 0\}$ represents total threat for country u. [4]

2.2 The Game

On the basis of the network mechanics, consider the following game: I model a multiplayer game with complete information, which incorporates the decision structure of each country

[3] A country under the other's protection is not considered as ally, because it solely receives support.

[4] Note that with the assumption of directed graph, mostly we have that $w_{j,i} \neq w_{i,j}$ if the links between i and j are bidirectional, which means the mutual defense contributions in an alliance are not equal.

as investing the total capacity towards self-defense, support for allies and threats towards foes. Formally, the game is specified by the collection $\Gamma = \{V, \mathcal{R}, \mathbf{R}, \mathbf{\Lambda}, \mathcal{C}, \mathbf{C}, \mathbf{W}, s, u\}$.

The mathematical representation of the game is as follows:

- A collection of countries $V = \{1, 2, \ldots, N\}$.
- A collection of relations $\mathcal{R} = \{\texttt{self}, \texttt{ally}, \texttt{foe}, \texttt{neutral}\}$ and a matrix $\mathbf{R} = [r_{i,j}] \in \mathcal{R}^{N \times N}$ assigning relations between countries. We require that $\forall i \in V, r_{i,i} = \texttt{self}$; $\forall i, j \in V$, if $r_{i,j} = \texttt{ally}$ or $\texttt{foe}$, then $r_{j,i} = \texttt{ally}$ or $\texttt{foe}$.
- A matrix $\mathbf{\Lambda} = [\lambda_{i,j}] \in (0, 1]^{N \times N}$ assigning willingness of investment to countries. Country i's **willingness of investment** for j is $\lambda_{i,j}$. It is the ratio that denotes the maximum support i is willing to give to j relative to i's total capacity.
- A compact set of possible capacities $\mathcal{C} \subset \mathbb{R}_+$ and a vector $\mathbf{C} = [c_1, \ldots, c_N]^T \in \mathcal{C}^N$ assigning c_i to country i as its capacity.
- A compact set of strategy $\mathcal{C}^N$ for each country. Denote strategy of country i as $\mathbf{W}_i = [w_{i,1}, \ldots, w_{i,N}] \in \mathcal{C}^N$ where $w_{i,j}$ is the investment of country i to country j. The adjacent matrix $\mathbf{W} = [\mathbf{W}_1^T, \ldots, \mathbf{W}_N^T]^T$ describes the investment distribution. We require that $\forall i, j \in V, w_{i,j} \le \lambda_{i,j} c_i$, and $\|\mathbf{W}_i\|_1 = c_i$.
- A state function $\sigma : \mathcal{C}^N \times \mathcal{R}^N \to [0, 1]^N$ assigning N *pairwise* states to any country based on its interactions with the other countries.
- A utility function or characteristic function $u : [0, 1]^N \to \mathbb{R}$ assigns the final state to a country based on its pairwise states.

Some basic concepts can now be formalized as follows:

- $E = \{(i, j) | i, j \in V, r_{i,j} \in \{\texttt{ally}, \texttt{foe}\}\}$
- $A = \{(i, j) | i, j \in V, r_{i,j} = \texttt{ally}\}$
- $\Phi = \{(i, j) | i, j \in V, r_{i,j} = \texttt{foe}\}$

Three auxiliary relation matrices $\mathbf{R}^A = [r^A_{i,j}] \in \{0, 1\}^{N \times N}$, $\mathbf{R}^S = [r^S_{i,j}] \in \{0, 1\}^{N \times N}$ and

$\mathbf{R}^{\Phi} = [r^{\Phi}_{i,j}] \in \{0,1\}^{N \times N}$ are defined as

$$r^{A}_{i,j} = \begin{cases} 1 & r_{i,j} = \texttt{ally} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$r^{S}_{i,j} = \begin{cases} 1 & r_{i,j} = \texttt{self} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$r^{\Phi}_{i,j} = \begin{cases} 1 & r_{i,j} = \texttt{foe} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$\mathbf{R}^{A}$ is the adjacent matrix of A, and $\mathbf{R}^{\Phi}$ is the adjacent matrix of Φ. $\mathbf{R}^{S}$ is an identity matrix.

Assumption 1. Complete Information Each country in the game knows all the total capacities and all the relations for those involved. Formally, relation function r, willingness function ι, capacity function c, state function σ, and utility function u are known to everyone in the game.

Assumption 2. Allocation of Investment Each country allocates the total capacity into the usages of self-defense, support for allies (if there are any) and threat towards foes (if there are any).

Formal definitions of the state and the utility can be represented as below.

Definition 9. State The state matrix $\mathbf{S} = [s_{i,j}] \in [0,1]^{N \times N}$ denotes pairwise states. $s_{i,j}$ describes the effectiveness of the influence i exerts on j. $\mathbf{S}_i = [s_{i,1}, \ldots, s_{i,N}]$, $\mathbf{S}'_i = [s_{1,i}, \ldots, s_{N,i}]^T$.

Definition 10. State Function Country i calculates its states using a state function σ, a function of $\mathbf{W}'_i = [w_{1,i}, \ldots, w_{N,i}]^T$ and $\mathbf{R}'_i = [r_{1,i}, \ldots, r_{N,i}]^T$. $\mathbf{S}'_i = \sigma(\mathbf{W}'_i, \mathbf{W}_i, \mathbf{R}'_i)$. Specifically, $\mathbf{R}^{S\prime}{}_i = [r^{S}_{1,i}, \ldots, r^{S}_{N,i}]^T$, $\mathbf{R}^{A\prime}{}_i = [r^{A}_{1,i}, \ldots, r^{A}_{N,i}]^T$ and $\mathbf{R}^{\Phi\prime}{}_i = [r^{\Phi}_{1,i}, \ldots, r^{\Phi}_{N,i}]^T$. The state function is identical in form for each country. Formally, we have,

$$\begin{aligned}\mathbf{S}'_i &= \sigma(\mathbf{W}'_i, \mathbf{W}_i, \mathbf{R}'_i) \\ &= \min\{1, \frac{\mathbf{W}'_i \cdot (\mathbf{R}^{A'}{}_i + \mathbf{R}^{S'}{}_i)}{\max\{\mathbf{0}, \mathbf{W}'_i - \mathbf{W}^T_i\} \cdot \mathbf{R}^{\Phi'}{}_i}\}\mathbf{R}^{A'}{}_i + \min\{1, \frac{\max\{\mathbf{0}, \mathbf{W}'_i - \mathbf{W}^T_i\} \cdot \mathbf{R}^{\Phi'}{}_i}{\mathbf{W}'_i \cdot (\mathbf{R}^{A'}{}_i + \mathbf{R}^{S'}{}_i)}\}\mathbf{R}^{\Phi'}{}_i\end{aligned} \tag{4}$$

In this equation, $(\mathbf{W}'_i \cdot (\mathbf{R}^{A'}{}_i + \mathbf{R}^{S'}{}_i))$ is actually ξ_i, and $(\max\{\mathbf{0}, \mathbf{W}'_i - \mathbf{W}^T_i\} \cdot \mathbf{R}^{\Phi'}{}_i)$ is θ_i. For instance, if $r_{j,i} = \texttt{ally}$, $s_{j,i} = \min\{1, \frac{\mathbf{W}'_i \cdot (\mathbf{R}^{A'}{}_i + \mathbf{R}^{S'}{}_i)}{\max\{\mathbf{0}, \mathbf{W}'_i - \mathbf{W}^T_i\} \cdot \mathbf{R}^{\Phi'}{}_i}\}$. $s_{j,i}$ is capped to be 1 if $\xi_i > \theta_i$. However, for $r_{j,i} = \texttt{foe}$, $s_{j,i}$ is 1 if $\xi_i < \theta_i$. After obtaining $\mathbf{S}'_i$ for N countries, $\mathbf{S}$ and $\mathbf{S}_i$ can be automatically derived.

Definition 11. Utility Function Country i calculates its utility using utility function u with $\mathbf{S}_i = [s_{i,1}, \ldots, s_{i,N}]$ and $\mathbf{R}_i = [r_{i,1}, \ldots, r_{i,N}]$. $U_i = u(\mathbf{S}_i, \mathbf{R}_i)$. The utility function for each country is identical. It takes a complex form and as will soon be discussed, is piecewise continuous.

Formally, we have

$$U_i = u(\mathbf{S}_i, \mathbf{R}_i) = \begin{cases} \mathbf{S}_i \cdot \mathbf{R}^S_i & \mathbf{S}_i \cdot \mathbf{R}^S_i < 1 \\ 1 + \mathbf{S}_i \cdot \mathbf{R}^A_i & \mathbf{S}_i \cdot \mathbf{R}^S_i = 1 \text{ and } \mathbf{S}_i \cdot \mathbf{R}^A_i < \|\mathbf{R}^A\|_1 \\ 1 + \|\mathbf{R}^A\|_1 + \mathbf{S}_i \cdot \mathbf{R}^\Phi_i & \text{otherwise} \end{cases} \tag{5}$$

For simplicity, the utility function is a complete characterization of four situations or realistic states any country could be in:

1. `vulnerable` country i's support does not add up to its threats. In other words, $s_{i,i} = \mathbf{S}_i \cdot \mathbf{R}^S_i < 1$;
2. `self-defensive` country i's support exceeds its threats, but the support for at least one of its allies does not exceed the threat. In other words, $\sum_{j \in A_i} s_{i,j} = \mathbf{S}_i \cdot \mathbf{R}^A_i < \|\mathbf{R}^A_i\|_1 = |A_i|$;
3. `defensive` for country i and all its allies', the support exceeds threats, but at least one of i's foes is not vulnerable. In other words, $\sum_{j \in \Phi_i} s_{i,j} = \mathbf{S}_i \cdot \mathbf{R}^\Phi_i < \|\mathbf{R}^\Phi_i\|_1 = |\Phi_i|$;
4. `powerful` for country i and all its allies', the support exceeds threats, and all i's foe

are vulnerable. The utility reaches its maximum, $1 + \|\mathbf{R}_i^A\|_1 + \|\mathbf{R}_i^\Phi\|_1 = |Succ(i)| + 1$.

The utility function and the four situations only constitute one interpretation of the state matrix. The state matrix contains much more information that can be used for decision making. Nevertheless, this special form of utility function conforms to two important realistic assumptions about countries' preferences in resource allocation.

Therefore, the maximization problem for each country is:

$$\begin{aligned} \underset{\mathbf{W}_i}{\text{maximize}} \quad & U_i(\mathbf{W}) \\ \text{subject to} \quad & \forall j \in V, w_{i,j} \leq \lambda_{i,j} c_i, \text{ and } \|\mathbf{W}_i\|_1 = c_i \end{aligned} \tag{6}$$

By making such investments, countries can reach certain utility as specified previously. The four situations in the utility function are actually ranked by order and thereby operationalizes a *preference structure* for countries' investment.

Assumption 3. Each country weakly prefers investing in self-defense to investing in defense support for allies.

Assumption 4. Each country weakly prefers investing in defense (self-defense and defense support allies) to investing in offense.

Given the preferences, when the total support for i, ξ_i, including its self-defense efforts and assistance from allies, exceeds the total threat, θ_i. I assume that the country in question will invest all the resources in self-defense rather than allocating some to support allies if there are any. Its state s_i will be smaller than 1 as previously defined. By the same logic, only if it has sufficient resources for self-defense, it will invest additional resources on supporting its allies; and only when it is able to self-defend and defend *all* allies, it will invest additional resources on attacking its foes. Trivially, it can be seen that country i's utility increases with its total capacity c_i.

Obviously the utility U_i is a function of the power distribution and is piecewise continuous in terms of total capacity c_i. For example, suppose a and b are foes of c. $c_a = 2$, $c_b = 2$, $c_c = 4$. We have that $U_c = \frac{4}{4} = 1$. However, a small increment in c_c will make U_c 3 because now $s_{c,a} + s_{c,b} + 1 = 2 + 1 = 3$. So it is piecewise continuous.

If all of country i's foes are vulnerable (not necessarily because of attacks from i[5]), we can call i `offensive`. Since I have assumed countries would prioritize self-defense over support for allies, and defense over offense. `offensive` only exists theoretically but it will not happen as a steady state. The countries, by investing in the three kinds of capacities — self-defense, support for allies and threat towards foes, try to attain the highest possible state. For countries with both allies and foes, the highest state it could obtain would be `powerful`; for countries with only allies, it would be `defensive`; for countries without allies, it would be `self-defensive`. So the highest possible state for any country to attain would be `powerful`.

Assumption 5. In terms of relation intensities, `ally` and `foe` both describe continuums of relations.

Importantly, solving the game needs the realistic assumption on countries' interest, their willingness to invest, which is previously captured by $\omega_{i,j}$. This assumption extends the understanding of allies and foes from two extremes to a spectrum. It measures every country's **relation intensity** with each ally and foe at any point of them.

How much efforts countries are willing to invest towards certain relation is a factor not much emphasized in the current literature; in addition to countries' capacities and relations, "issue salience" is crucial to almost every event. The differences in relation intensities can be attributed to various factors, such as contiguity/geography (in old time, it would be harder for a distant country to move its troops overseas to help some allies in conflict), and certain idiosyncrasies(the allies might need to cope with geopolitical threats or unintended events with discretionary resources).

Definition 12. Deviation Let country i's deviation from strategy profile $\mathbf{W} = [\mathbf{W}_1^T, \ldots, \mathbf{W}_N^T]^T$ be $\boldsymbol{\Delta}_i = [\delta_{j,k}] \in \mathbb{R}^{N \times N}$ that $\forall j \neq i, \delta_{j,k} = 0$ and $\tilde{\mathbf{W}} = \mathbf{W} + \boldsymbol{\Delta}_i$ is a valid strategy profile.

Theorem 1. Alliance Network Nash Equilibrium (ANNE)

[5]This point will be elaborated in the next section.

Let $\Gamma = \{V, \mathcal{R}, \mathbf{R}, \mathbf{\Lambda}, \mathcal{C}, \mathbf{C}, \mathbf{W}, s, u\}$. be a game with N players, where $\mathbf{W}_i$ is the strategy set for player i, and U_i is the payoff for player i. When each player $i \in V$ chooses strategy $\mathbf{W}_i$ resulting in strategy profile $\mathbf{W} = [\mathbf{W}_1^T, \ldots, \mathbf{W}_N^T]^T$ then player i obtains utility U_i. Note that the state depends on the strategies chosen by all the players. A strategy profile $\mathbf{W}^*$ is an Alliance Network Nash Equilibrium if no unilateral deviation in strategy by any single player is profitable for that player, that is $\forall \mathbf{\Delta}_i$, $U_i(\mathbf{W}^*) \geq U_i(\mathbf{W}^* + \mathbf{\Delta}_i)$.

While a formal proof of equilibrium existence is hard to derive, many examples in which the equilibrium exists can be given.

Example 1. A Three Player Case There are nine relation structures for a three-player case of countries a, b and c as below in Figure 2.[6] The seven relations are of two categories: 1. the three countries in the first four subfigures represent a connected graph; in other words, any two of the three has a relation; 2. the latter five subfigures mean two of the three have no bilateral relation.

Take the first graph as example. Assuming the willingness parameter to be 100%, the equilibrium still requires a discussion of the power distribution. Then there are three cases to consider:

Case 1: $c_a + c_b > c_c$. First, it is definite that a and b are $\texttt{self} - \texttt{defensive}$ and c is `vulnerable`. Given that a and b are allies, a and b are defensive.

With the assumption on countries' priority in defense, c will not use most or all resources on defeating one foe even if it might be able to. Rather, it will use all the resources for defense against a and b because in this case its threats are larger than the support.

Furthermore, if at least one of a and b has more individual capacity than that of c, the country (countries) will be powerful; otherwise, both of them will be defensive and c is still vulnerable.

In any of the possibilities, none of them has incentives to deviate.

Case 2: $c_a + c_b = c_c$. This case only has a type of equilibrium where each country is defensive. Further, a and b are `defensive` and c is $\texttt{self} - \texttt{defensive}$.

[6] Note that here I exclude the cases of unilateral relations and the case where the three have no relation with any other.

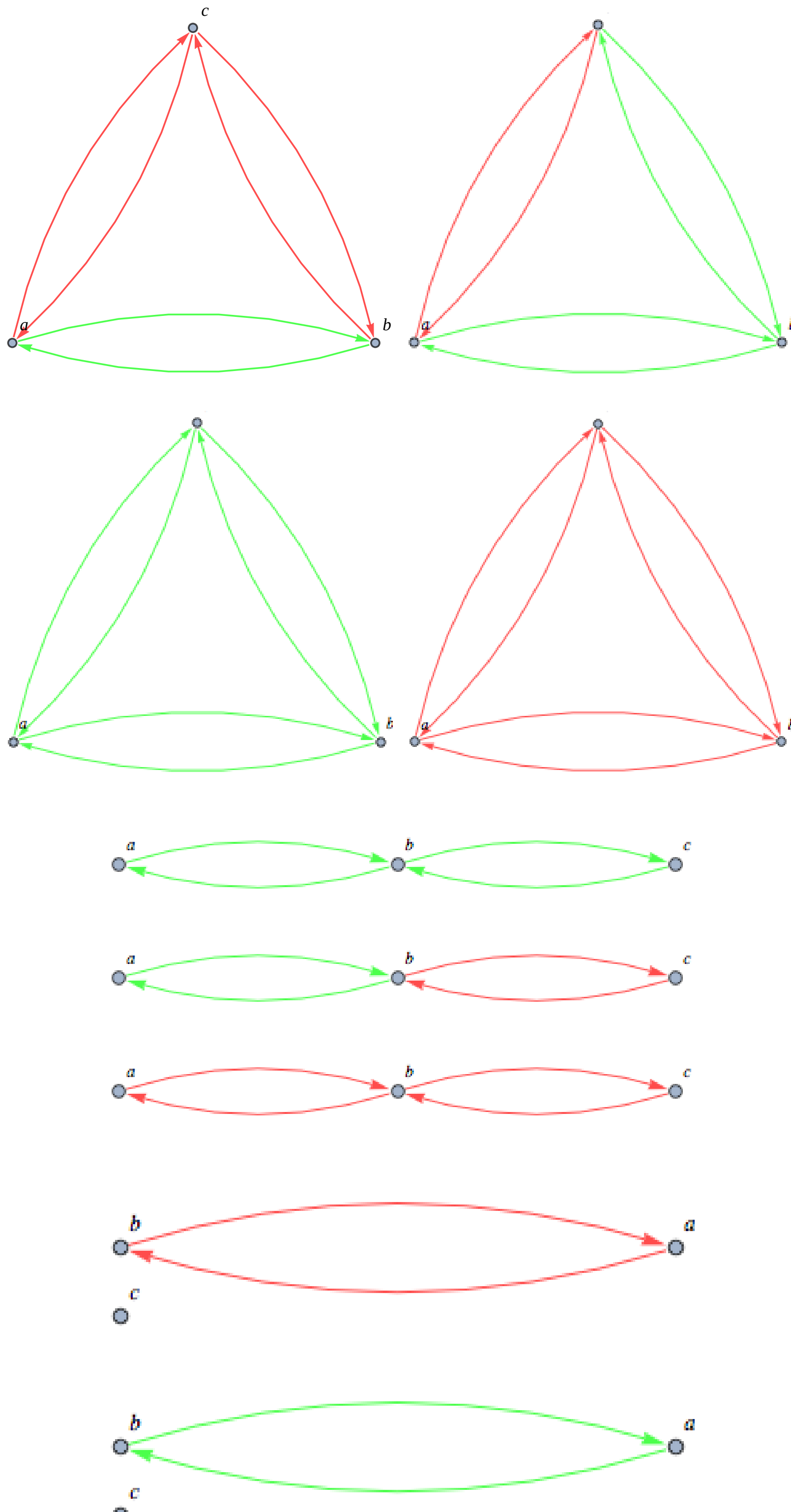
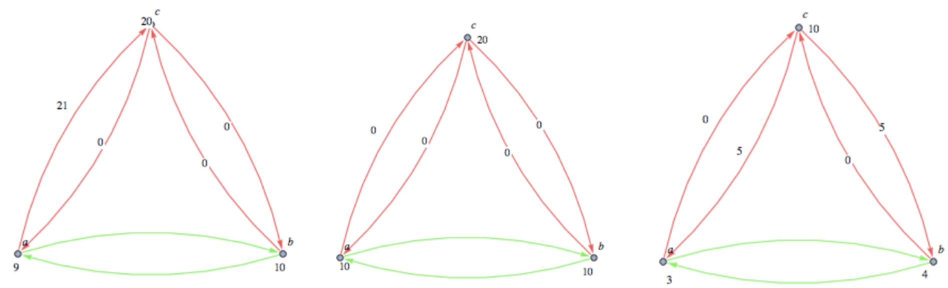


Figure 2: Red lines mean “foe” and green lines mean “ally”



(a) $c(a)+c(b)>c(c)$ (b) $c(a)+c(b)=c(c)$ (c) $c(a)+c(b)<c(c)$

Case 3: $c_a + c_b < c_c$. First, a and b are `vulnerable`. It is obvious that c is powerful. None of them has incentives to deviate. □

I can proceed to derive the equilibrium similarly for the rest of the relation structures. The first to notice is that holding the relations constant, the situation or state each would possibly be in is a function of their power and the willingness to invest in any type of relation; and holding their power and interest constant, their states in equilibrium turns into a function of their relations, i.e. node connectivity. Furthermore, comparing to a and b having a mutual enemy c in the first figure, b's two allies, a and c, are foes in the second figure. Such distinction would be explored further in the definition of *complete alliance* in the next section, where I will show how this distinction can make different alliances in equilibrium.

Example 2. Multiple States Consider the example: a and b are allies and both of them are in conflict with c. Assuming the total capacities $c_a = 30$, $c_b = 10$ and $c_c = 20$. Figure 4 illustrates an equilibrium in which each country represents a different state and has no incentives to deviate to other states given the state of the others: a is powerful (it invests 21 in offense towards c and keeps 9 as self-defense), b is defensive (its ally a and itself are both self-defensive) and c is vulnerable. [7]

Example 3. Multiple Types of Equilibria Consider the following example that illustrates the case of multiple equilibria: c is in conflict with a and b respectively. We assume

[7] Red pointed edges represent relation `foe` and green pointed edges represent relation `ally`. Investments on the red pointed edges denote threat towards the foes and investments on the green pointed edges denote support for allies. For simplicity, $\iota = 1$ for all relations in the examples of Section 2.

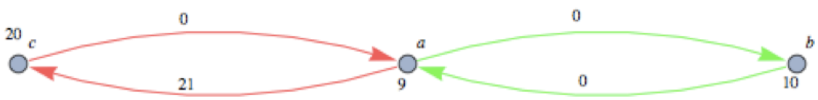


Figure 4: **Multiple States**
$S_a =$ `powerful`, $S_b =$ `defensive` and $S_c =$ `vulnerable`

$c_a = 5$, $c_b = 9$ and $c_c = 5$. There will be two equilibria illustrated in Figure 5 and 6: the first with a, b and c all being defensive, and the second with a and c being powerful and b vulnerable.

For the first case, with c's strategies staying fixed, however a changes its investment will not change its state. b does not have enough capacity to overwhelm a and c and cannot promote itself to a higher state, either. In the second case, a and c have reached their highest possible state, while b on the other hand does not have enough capacity to be self-defensive.

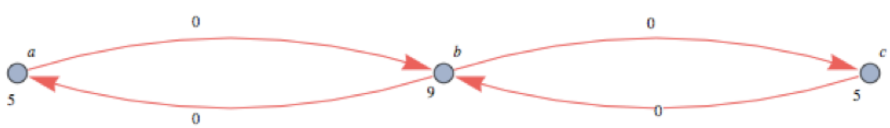


Figure 5: **Type 1 Equilibrium**
$S_a =$ `defensive`, $S_b =$ `defensive` and $S_c =$ `defensive`

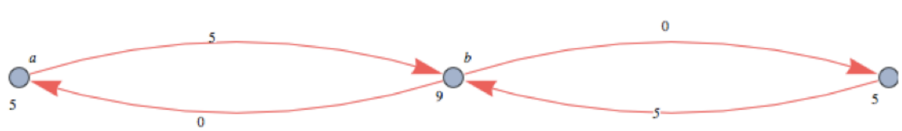


Figure 6: **Type 2 Equilibrium**
$S_a =$ `powerful`, $S_b =$ `vulnerable` and $S_c =$ `powerful`

2.2.1 Equilibrium Analysis: A Characterization of Different Alliances

Aggregating all the countries' maximizing behavior produces the aforementioned equilibrium. Different types of alliances in equilibrium can be the end products to which collectivities of countries converge, which can be defined formally as below. [8]

Definition 13. Alliance. For a set of countries V, $\forall i, j \in V$, $r(i,j) \neq \texttt{foe}$ and $\forall i \in V$, $\exists j \in V$ s.t. $r(i,j) = \texttt{ally}$ or $r(j,i) = \texttt{ally}$.

Definition 14. Complete Alliance. An alliance V is complete if $\forall i \in V$ and $\forall j \notin V$, $r(i,j) \neq \texttt{ally}$.

Using a concept of *defensiveness*, I define types of alliances to represent some possible end products of countries' behavior.

Definition 15. Defensive Alliance. An alliance V is a defensive alliance against a set of countries Z if *all* countries in V are self-defensive against Z, $\forall i \in V, \xi_i \geq \sum_{j \in Z \cap \Theta_i} \max\{0, w_{j,i} - w_{i,j}\}$.

ξ_i represents all the defense capabilities any country i in alliance can have, which include its own defensive investment and allies' support, even from allies in other alliances. $\sum_{j \in Z \cap \Theta_i} w_{j,i}$ represents the the threats facing i. So long as all countries' defense capabilities exceed enemies' offense efforts, they can ward off threats and form a defensive alliance against the foes. Consider the example in Figure 7, a and b form an alliance, and a and d, d and c are in conflicts. $c_a = 2$, $c_b = 0.5$, $c_c = 5$ and $c_d = 4$. We can see the alliance formed by a and b is defensive.

Note that an alliance can be defensive only because of certain strong outsiders who have mutual enemies with it. In other words, if not for the outsiders' offense, some alliances could not have been defensive. Though the alliance in Figure 7 would not have been defensive if the outsider c had not helped to overcome the threats, it became defensive without the defense support of c. Accordingly, we have the definition of strictly defensive alliance to allow for

[8] A country can be part of several different alliances. Although we do not require mutual support in an alliance, we do require there is no conflict in an alliance; in other words, there's no "foe edge" in the graph of an alliance.

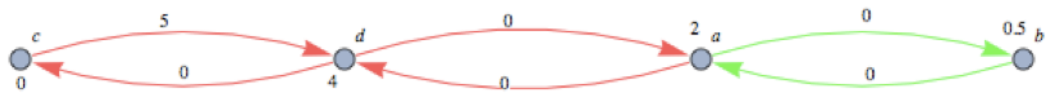


Figure 7: **Defensive Alliance**
a and b form a defensive alliances

the case that an alliance is defensive only because of the capacities and support *among* its member countries. By this definition, the alliance in Figure 7 is both defensive and strictly defensive.

Definition 16. Strictly Defensive Alliance. An alliance V is a strictly defensive alliance against a set of countries Z if *all* countries in V are self-defensive against Z, $\forall i \in V$ and $\forall j \in V$, $\sum_{j \in V \cap \Xi_i} w_{j,i} + w_{i,i} \geq \sum_{j \in Z \cap \Theta_i} \max\{0, w_{j,i} - w_{i,j}\}$.

Figure 8 shows a strictly defensive alliance formed by a and b, where $c_a = 5$, $c_b = 5$, $c_c = 6$, because both a and b are self-defensive.

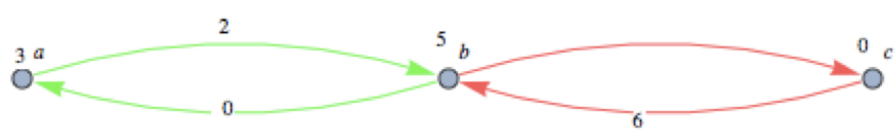


Figure 8: **Strictly Defensive Alliance**
a and b form a defensive alliance

Proposition 1. If an alliance is complete, defensive alliance is equivalent to strictly defensive alliance.

Proof: If an alliance is complete and defensive, each country in the alliance does not have outsider allies and is self-defensive against the set of foes the alliance might have.

By the definition of strictly defensive alliance, we know it must be strictly defensive.

The two concepts are equivalent for a complete alliance. □

We can have both a weak and a strong version of defensive alliance.

Definition 17. Weakly Defensive Alliance. V is a defensive alliance against country j if all countries in V are self-defensive against $\{j\}$, $\forall i \in V, \xi_i \geq \max\{0, w_{j,i} - w_{i,j}\}$.

A weakly defensive alliance can also be called *regional defensive alliance* because it can still defend itself in a regional conflict against a single country. Figure 9 shows a weakly defensive alliance formed by a and b, where $c_a = 6$, $c_b = 4$, $c_c = 12$, $c_d = 6$. Though a is self-defensive against d, b is vulnerable and a cannot help b to overcome c. So a and b are only self-defensive against one foe, d.

Figure 9: **Weakly Defensive Alliance**
a and b form a weakly defensive alliance against d

Definition 18. Strongly Defensive Alliance. V is a strongly defensive alliance if all countries in V are self-defensive, $\forall i \in V, \xi_i \geq \theta_i$.

A strongly defensive alliance can also be called *global defensive alliance* because even if every country outside the alliance becomes a foe, the alliance can still defend itself. Figure 10 shows a strongly defensive alliance formed by a and b, where $c_a = 5$, $c_b = 6$, $c_c = 1$, $c_d = 6$. Obviously, a and b are self-defensive against both c and d.

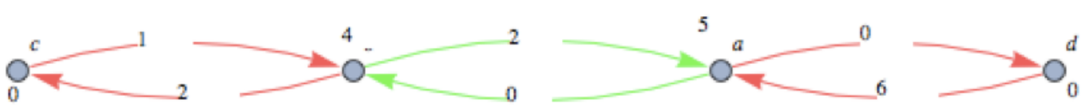


Figure 10: **Strongly Defensive Alliance**
a and b form a weakly defensive alliance against c and d

Proposition 2. Countries in Complete Strongly Defensive Alliance must be defensive.

Proof: By the definition of complete alliance, every country must only be ally with countries also in the alliance. By the definition of strongly defensive alliance, every country in the alliance must be self-defensive.

This is equivalent to saying every country as well its allies is self-defensive, which is exactly the definition of "defensive". So countries in complete strongly defensive alliance must be defensive.

Note that in non-complete alliance, some countries can be allies with outsiders, who might not be self-defensive. So countries in non-complete alliance are not necessarily defensive.

An example is given in Figure 11. We assume $c_a = 1$, $c_b = 1$ and $c_c = 1$; a is ally with both b and c, and b and c are foes. The alliance formed by a and c is therefore non-complete because a is also ally with an outsider b. Since b has conflict with c, we say a, b and c cannot form an alliance by the assumption of "no-conflict" in alliances. The equilibrium shows that a is not defensive because one of its ally b is made vulnerable by c. □

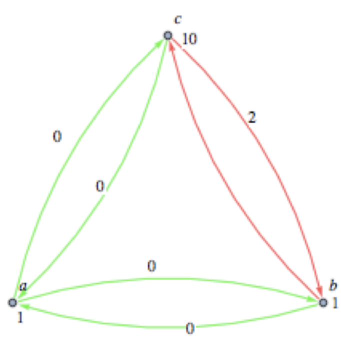


Figure 11: **Non-Complete Strongly Defensive Alliance**
$S_a \neq$ `defensive`

Definition 19. Offensive Alliance. V is an offensive alliance *for* Z if no country in Z being V's neighbor is self-defensive, $\forall j \in \partial V \cap Z, \xi_j < \theta_j$.

As long as the threat is larger than the support, we say Z is vulnerable, regardless of the origins of the threats. The threats can be from V itself or from elsewhere. To illustrate the case that Z is vulnerable *because of* V, I propose a more strict definition of offensive alliance below.

Definition 20. Strictly Offensive Alliance. V is a strictly offensive alliance *towards* Z if no country in Z being V's neighbor is self-defensive against V, $\forall j \in \partial V \cap Z, \xi_j < \sum_{i \in V \cap \Theta_j} \max\{0, w_{j,i} - w_{i,j}\}$.

Strictly offensive alliance is the opposite of defensive alliance. If V is a defensive alliance against Z, Z will never be a strictly offensive alliance towards V and vice versa. However, Z can still be an offensive alliance. However, the other threats V might have to make precautions for could have prevented V from becoming a defense alliance.

Figure 12 represents an offensive alliance and a strictly offensive alliance. We assume a and c, a and b, d and e, are allies, while c and b, b and d are foes. In addition, $c_a = 0.5$, $c_b = 8$, $c_c = 12$, $c_d = 5$ and $c_e = 5$. The solution shows the alliance formed by d and e is offensive alliance, because even though b is vulnerable, it is vulnerable because of c. The alliance formed by a and c is strictly offensive alliance because it makes the only foe b vulnerable.

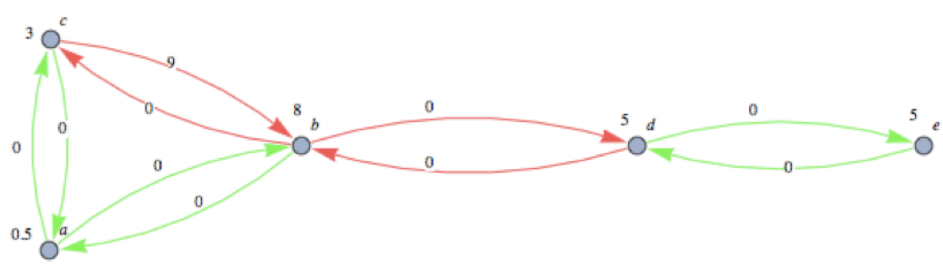


Figure 12: **Offensive Alliance and Strictly Offensive Alliance**
a and c form a strictly offensive alliance; d and e form an offensive alliance

Proposition 3. Strictly offensive alliance must be offensive alliance.

Proof: By definition, if $\forall j \in \partial V \cap Z, \xi_j < \sum_{i \in V \cap \Theta_j} \max\{0, w_{j,i} - w_{i,j}\}$,

it must be that $\forall j \in \partial V \cap Z, \xi_j < \theta_j$.

So strictly offensive alliance must be offensive alliance. □

Note that the definition of offensive alliance refers to alliances attacking a set of countries. Then we can further refine this concept into a weak and a strong version.

Definition 21. Weakly Offensive Alliances. V is a weakly offensive alliance towards j if j is not self-defensive.

Definition 22. Strongly Offensive Alliances. V is a strongly offensive alliance if no country being V's neighbor is self-defensive.

Proposition 4. If an alliance Z is the complement of a complete alliance V, $Z = \overline{V}$, and if V is strongly offensive alliance, V is also strictly offensive.

Proof: By the definition of strongly offensive alliance, no country as V's neighbor is self-defensive. Given that $Z = \overline{V}$, no country in Z as V's neighbor is self-defensive, which is exactly the definition of strictly offensive alliance.

So if Z is the complement of V, $Z = \overline{V}$, strongly offensive alliance is equivalent to strictly offensive alliance. □

On the basis of the definitions of defensive and offensive alliances, we come to the definition of powerful alliance, by which we refer to such a kind of alliance both strongly offensive and strongly defensive for *its neighbors*.

Proposition 5. Powerful Alliance. If V is strongly offensive in equilibrium, V must be strongly defensive. We call U a powerful alliance.

Proof: "Strongly offensive" and "strongly defensive" are equilibrium concepts.

Given the priority assumption, we know the countries always prioritize defense over offense. So for strongly offensive alliance in equilibrium, it must have already been strongly defense.

We call this kind of alliance "powerful" alliance. □

Figure 13 illustrates a powerful alliance. a, b and c are allies with one another while d has conflict with both a and b. We have that $c_a = 10$, $c_b = 10$, $c_c = 1$ and $c_d = 9$. The only foe for the alliance formed by a, b and c is vulnerable. The alliance is both strongly offensive and strongly defensive and thus powerful by definition.

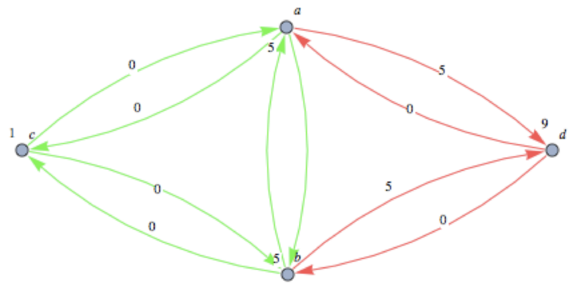


Figure 13: **Powerful Alliance**
a, b and c form a powerful alliance

Table 1: Notations

Symbol	Expression	Explanation
V		set of nodes (vertices)
A		set of "ally" edges
Φ		set of "foe" edges
E	$A \cup \Phi$	set of edges
$w_{i,j}$		edge weight of (i,j)
$\mathbf{W}$	$\bigcup_{(i,j)\in E}\{w_{i,j}\}$	set of edge weights
c_j		node j's node capacity
$\mathcal{C}$	$\bigcup_{j\in V}\{c_j\}$	set of capacities
$\lambda_{u,v}$		node i's willingness of investment for j
Λ	$\bigcup_{(i,j)\in E}\{\iota_{i,j}\}$	set of willingness parameters
A_j	$\{i \| (j,i) \in A\}$	"ally" nodes of node j
Φ_j	$\{i \| (j,i) \in \Phi\}$	"foe" nodes of node j
Ξ_j	$\{i \| (i,j) \in A\}$	"support" nodes for node j
Θ_j	$\{j \| (i,j) \in \Phi\}$	"threat" nodes for node j
ξ_j	$\sum_{i\in\Xi_j} w_{i,j} + w_{j,j}$	total support for node j
θ_i	$\max\{w_{j,i} - w_{i,j}, 0\}$	total threat for node j
$\pi_{i,j}$	$\frac{\theta_j}{\xi_j} w_{i,j}$	Node i's PDR for node j
S_i		Country i's pairwise state vector
R_i		Country i's relation vector
U_i		Country i's utility

3 A Solution Algorithm

The model operationalizes a basic idea for alliance behavior: with survivalism as the utmost motive, countries engage in investment in offense and defense capabilities. Country relations (node connectivity), capacity distribution (node capacity) and investments (edge weight) combine to determine the type of the alliance. Given these, network theory is not only important for a operational definition of military alliances but also for a testable theory for alliance behavior.

Given the complexity of the alliance structures and the possibility of multiple types of equilibria of the game, it would be efficient to solve the game by computation with an optimizing algorithm because some variations of the game can be extremely complex. The game involves the basic idea of evolution, taking countries as entities of learning and imitation and letting their behavior evolve into certain equilibrium. Obviously many equilibria exist in most cases. However, I do not pay much attention to the details of those equilibria as long as they belong to the same *type*. Instead I pay attention to their properties and distribution.

Starting from any point in solution space, we can always find an equilibrium using a proper evolutionary algorithm. The probability for each type of the equilibria to occur depends on the proportion of initial points that finally converge to the equilibrium. We can randomly sample the solution space and estimate the distributional property. And a proper evolutionary algorithm like *simulated annealing* is used for finding an equilibrium to which an initial point converges.

We have designed and tested an algorithm to compute the optimal allocation between edge weights (resources invested on attacking or assisting other countries) and node weights (resources retained for self-defense) in the network games for any type of alliance structure.

We first simulate networks on the basis of node capacity and the connectivities based on data of military alliances and national capabilities. The algorithm will work in such a way that we first generate the initial population of edge weights randomly, and then evaluate the fitness of the edge weights for attaining certain goal, for example, defensive alliance. Such process will repeat on until termination when a sufficient fitness is achieved.

Thus, the flow of decisions is described in detail below:

First, all countries randomly assign weights to their out-edges. The assignment does not have to be optimal (in fact it should not be optimal), but have to be completely random because we are randomly sampling solution space.

The node weights and all the edge weights will evolve until an equilibrium is reached. As mentioned previously, we should consider the previous preference issue for countries in assigning capacities towards various kinds of investments: first, assuming survivalism is of utmost importance, countries have to fulfill their self-defense requirements; then they may also have to support their allies; finally if they have extra capacities, they can invest in offense. A country is impossible to be powerful if it does not have enough capacity to attack its foes while maintaining its self-defense and its allies' defense.

Basically, the criterion is that for each country[9],

1. it is powerful, or
2. it is defensive and incapable of being powerful, or
3. it is self-defensive and incapable of supporting all its allies, or
4. it is incapable of being self-defensive.

Note that although the algorithm works by dynamic updating of edge weights and node weights until an equilibrium is obtained, the model itself is static per se. Therefore, to efficiently execute the algorithm, we use an variable named "Proportional Defense Responsibility" to proxy for countries' investment update in each round.

Definition 23. Country i's **Proportional Defense Responsibility** for j is $\pi_{i,j}^t = \frac{w_{i,j}^t}{\xi_j^t} \cdot \theta_j^t$ (which denotes at round t the proportion of i's support for j to j's overall defense support, multiplied by the sums of threat for node v). And $\pi_{i,i}^t$ is called self-defense requirement and recognized as a form of PDR.

The idea is that in each round of evolution, all countries firstly calculate the proportional defense responsibilities (PDRs) for itself and its allies. If, holding fixed the external threats,

[9]Given that the alliance always prioritizes defense, it would be realistic to further assume that countries' aim would be to first sustain a defensive alliance and second, if possible, a powerful one.

all countries fulfill their PDRs, they will just become defensive at the end of this round. If any of them has not fulfilled the PDRs, the updating processes will repeat until an equilibrium is reached. It is possible that they would all be defensive or some of them fail to be even self-defensive at the end of the updating process.

Formally, for all countries, an updating process will repeat until an equilibrium is reached:

1. Collecting Total Defense Investments

   Country v "collects" its defense investments in the previous round including defense support for allies and its own self-defense, so that they can be reallocated in the new round.

$$\kappa = w_{j,j}^{t-1} + \sum_{i \in A_j} w_{j,i}^{t-1} \tag{7}$$

   κ is the total amount of capacity the country can reallocate on defense.

2. Prioritizing Defense to Offense

   Given that the PDRs for v is obviously,

$$PDR_j^t = \pi_{j,j}^t + \sum_{i \in A_j} \pi_{j,i}^t \tag{8}$$

   If total defense investments κ is smaller than the sum of the PDRs, country j ought to "retract" some threats made towards foes (if there are any), in other words, some offense investments, in order to defend. And if total capacity c_j (which includes both the defense and offense investments) is larger than the sum of the PDRs, country j is also able to retract some of the previously made threats. Note that this behavior is, in particular, due to the priority-of-defense assumption we made at the beginning.

   Let the amount of offense investments retracted be just sufficient to satisfy the requirements of PDRs. The gaps in resources country j has to fill would be given by (Note that if the total capacity is not even enough for the PDRs, country j should retract all the offenses investments):

$$(\frac{\pi_{j,j}^t + \sum_{i \in A_j} \pi_{j,i}^t - \kappa}{\sum_{i \in \Phi_j} w_{j,i}^{t-1}}) w_{j,i}^{t-1} \qquad if c_j \geq \pi_{j,j}^t + \sum_{i \in A_j} \pi_{j,i}^t \tag{9}$$

So the offense investment by j towards i, $w_{j,i}^t$, will be updated into:

$$w_{j,i}^t = \begin{cases} (1 - \frac{\pi_{j,j}^t + \sum_{i \in A_j} \pi_{j,i}^t - \kappa}{\sum_{i \in \Phi_j} w_{j,i}^{t-1}}) w_{j,i}^{t-1} & c_j \geq \pi_{j,j}^t + \sum_{i \in A_j} \pi_{j,i}^t \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Now adding the retracted offense investment to the total defense investments κ, which becomes,

$$\kappa = \begin{cases} \pi_{j,j}^t + \sum_{i \in A_j} \pi_{j,i}^t & c_j^t \geq \pi_{j,j}^t + \sum_{i \in A_j} \pi_{j,i}^t \\ c_j^t & \text{otherwise} \end{cases} \tag{11}$$

3. Prioritizing Self-Defense to Defense Support

The priority assumption also states that countries have to fulfill its self-defense requirement first. In the case that the total capacity is not even sufficient for self-defense, we assume it should retract all the other investments, use all its capacity just for self defense and end this round, hoping some allies would assist.

$$w_{j,j}^t = \begin{cases} \pi_{j,j}^t & c_j^t \geq \pi_{j,j}^t \\ c_j^t & \text{otherwise} \end{cases} \tag{12}$$

So if the total capacity is sufficient for self-defense, the remaining capacity can be retained for other purposes such as defense support for allies.

$$c_j^t = c_j^t - \pi_{j,j}^t \tag{13}$$

4. Supporting Allies

   After fulfilling the self-defense requirement, countries would try to meet PDRs for allies with what remains of c_v (the c_v^t on the left-handside of Equation 7). If the available capacity is not sufficient, we let it rescale PDRs so that it can afford them and end this round, hoping its allies can receive additional support from the other sources. So for ally i of country j,

$$w_{j,i}^t = \begin{cases} \pi_{j,i}^t & c_j^t \geq \sum_{s \in A_j} \pi_{j,s}^t \\ \dfrac{c_j^t}{\sum_{s \in A_j} \pi_{j,s}^t} \pi_{j,i}^t & \text{otherwise} \end{cases} \tag{14}$$

   Once again, we go on to update the remaining total capacity c_j^t.

$$c_j^t = c_j^t - \sum_{s \in A_j} \pi_{j,s}^t \tag{15}$$

5. Waging Offense

   Finally after meeting both self-defense and PDRs, we assume the country can arbitrarily spend the extra capacity, such as on offense.

Example 4. 1910 Europe Using the algorithm, we examine the case of the Triple Alliance and its foes in 1910. Figure 14 represents the investments within the alliance and towards a foe, Turkey. Note that though having been rescaled, their military spendings reflect the real capacity distribution. Take Germany as an example: 31.7667 is its self-defense effort; and the "60" within the bracket denotes the level of total capacity; 52.0047:0 denotes the defense-offense balance: 52.0047 includes its self-defense and the allies' support [10]; since Germany did not have foes that year, it would optimally make 0 offense investment. Just as predicted by the model, with the support from Germany and Austria-Hungary, Italy became powerful and defeated Turkey with a large advantage. Germany, Austria-Hungary and Romania were defensive, Italy was powerful and Turkey was vulnerable. (See Figure 14)

[10] This graphical presentation of investments will also be used for all solution graphs in the section of empirical testing.

Algorithm 1: Finding $w_{j,i}$ and $w_{j,j}$

Data: V, A, Φ, C
Result: W

```
begin
    foreach j ∈ V do RandomSeed (j, c_j);
    repeat
        t ← t + 1;
        foreach j ∈ V do                          /* calculate supports and threats */
            ξ_j^t ← ∑_{i∈Ξ_v} w_{i,j}^{t-1} + w_{j,j}^{t-1};
            θ_j^t ← ∑_{i∈Θ_j} max{0, w_{i,j} − w_{j,i}}^{t-1};
        foreach j ∈ V do                                       /* update weights */
            foreach i ∈ A_j ∪ {j} do                           /* calculate PDR */
                π_{j,i}^t ← (θ_i^t / ξ_i^t) w_{j,i}^{t-1};
            κ ← w_{j,j}^{t-1} + ∑_{i∈A_j} w_{j,i}^{t-1};       /* retract support */
            foreach i ∈ A_j do
                w_{j,i}^t ← 0;
            if κ < π_{j,j}^t + ∑_{i∈A_j} π_{j,i}^t then
                foreach i ∈ Φ_j do          /* retract threat if cannot defend */
                    w_{j,i}^t ← (1 − (π_{j,j}^t + ∑_{i∈A_j} π_{j,i}^t − κ) / (∑_{i∈Φ_j} w_{j,i}^{t-1})) w_{j,i}^{t-1};
                κ ← π_{j,j}^t + ∑_{i∈A_j} π_{j,i}^t;
            if κ < π_{j,j}^t then
                w_{j,j}^t ← κ;                 /* use all weights for self defense */
            else
                w_{j,j}^t ← π_{j,j}^t;                                /* self defense */
                κ ← κ − π_{j,j}^t;
                if κ < ∑_{i∈A_j} π_{j,i}^t then
                    foreach i ∈ A_j do   /* use all weights for alliance defense */
                        w_{j,i}^t ← (κ / ∑_{s∈A_v} π_{j,s}^t) π_{j,i}^t;
                else
                    foreach i ∈ A_j do                          /* alliance defense */
                        w_{j,i}^t ← π_{j,i}^t;
                    κ ← κ − ∑_{i∈A_j} π_{j,i}^t;
                    RandomSeed (j, κ);
        (cnt_o^t, cnt_d^t) ← CheckSteady (V, W^t);
    until cnt_o^t = 0 and cnt_d^t = 0;
```

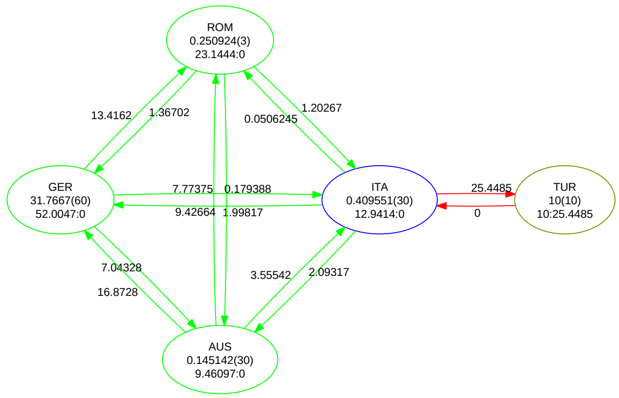


Figure 14: **1910 Europe**
Green circles are countries that are self-defensive; blue circles are countries that are both self-defensive and with whom the foes are vulnerable (not necessarily powerful); though not shown in this graph, yellow circles are countries that are vulnerable. The Triple Alliance was powerful because it was defensive and its foes was vulnerable. This is an equilibrium because no country will deviate from the current state.[11]

4 Empirical Testing

The model can in principal work with interstate conflicts involving military alliances of any context and geographic scale. The previous example of 1910 Europe is an application of the model to a geographically small-scaled conflict between an alliance, the Triple Alliance, and a single country, Turkey. While many more similar cases such as the Gulf War and the Sino-Japanese war could be studied under the current framework and presented, an empirical testing with World War I and World War II, the two largest conflicts in human history that embody the most complex alliance dynamics, adds more credibility to the model.

While the network mechanics in the model can comprise any interstate conflict in history, I acknowledge certain conditions have to be specified for model testing. First, it has to be an interstate conflict involving military alliances; otherwise, the specified decision structure for agents would be of little use. Moreover, as will be shown in the below sections, the effectiveness of the model in restoring historic facts increases with the complexity of the network structures or the number of countries involved. I hypothesize that noises or idiosyncratic factors could more easily impact interstate conflicts that are structurally simpler. On the contrary, those involving complex alliance dynamics are more likely to reflect the influences of what are fundamental to the outcomes of wars – material factors like relative power and country interest. Second, the conflict is preferably contemporary. Otherwise the study would be greatly limited by data constraints.

The empirical testing consists of two parts: first, I take the military spendings, alliance structures and conflict occurrences for the countries involved in the given year as data inputs of the game and illustrate graphically one solution obtained with the aforementioned heuristic; second, given the existence of multiple types of equilibria, I predict the likelihood for each type of equilibria to occur by simulations.

I mainly make use of datasets from the Correlates of War project, which include the National Material Capability dataset, the Militarized Interstate Dispute dataset as well as the Alliance Treaty Obligations and Provisions (ATOP) dataset. [12]Overall, the data indicates two patterns in the alliance and conflict dynamics: first, there are more militarized conflicts than wars; second, as more countries engaged in wars, more countries joined alliances with each other.[13]

Note that solving the game relies heavily on the willingness parameter[14], which serves as an upper bound on the resources any country could make use of in pursuing certain behavior. For the empirical testing, certain criteria are applied to input the parameter: first, for a dyad with great contiguity, the willingness parameter for both would be high; second, for dyads that are confirmed to have been important allies or in intensive wars, the willingness parameter would also be high. For the rest, I use historic cases to enter the parameters.

[12]The country-year variable "military spending" in the National Military Capacity dataset is used to proxy the total capacity of countries in a given year. For 1816-1913, military spending was coded in thousands of current year British Pounds; and for 1914 onwards, it was in thousands of current year US Dollars.

[13]Please refer to Table 2 - 6 to see these patterns.

[14]The year-by-year willingness parameters for any country in any given relation will be given in supplementary materials.

4.1 A Visualization of Europe in World War I

I first examine the period from 1914 to 1918 in Europe. In the decade before the start of the World War I, two alliances gradually formed. One was by Germany in alignment with Austria-Hungary and Italy, forming what was known as the "Triple Alliance". The other started with France cultivating friendship with Russia after the Franco-Prussian war in the 1870s, then with Britain seeking out an alliance with these two continental powers, which became the "Triple Entente".

There are four main results from the empirical testing, which are:

Result 1: Germany had faced an increasing probability of failure as the war dragged on.

Result 2: Without the entry of the US, Germany could have been defensive and reversed the disadvantage because it had accumulated tremendous power in 1918

Result 3: As Russia's resources were being depleted and even much of it had to be diverted to deal with the domestic revolution, this greatly impaired the prospect for retaining a powerful state

Result 4: The entry of the US helped the UK and France greatly. Obviously, these regularities are consistent with historic facts.

4.1.1 1914-1915

The map in Figure 15 shows the geopolitical situation of Europe as of the year of 1914: the country dyads in wars, in militarized conflicts (not wars) and in bilateral defense pacts. The first two sets of relations are denoted by red lines while the third relation is captured by green lines. More specifically, I listed all the relations below in Table 2.

The relation patterns show that Germany was at war with some major powers such as the UK, France and Russia. The results of the game show that given the capacity distribution, country relations and their willingness to invest, neither Germany nor Austria-Hungary was successful in these two years. To empirically account for these results, initially Germany opened the war on the Western Front, which was intended to quickly conquer France through Belgium. However, Belgium fought back and sabotaged the German rail system,

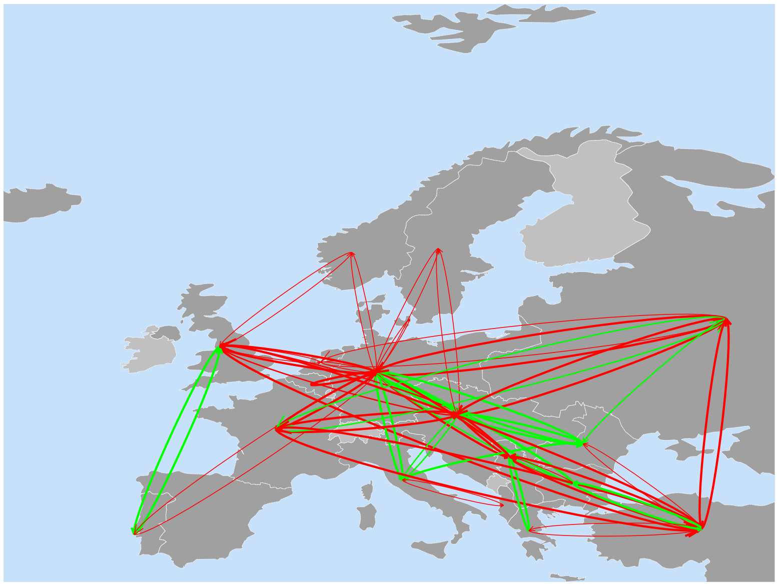


Figure 15: **Geopolitics of Europe in 1914**

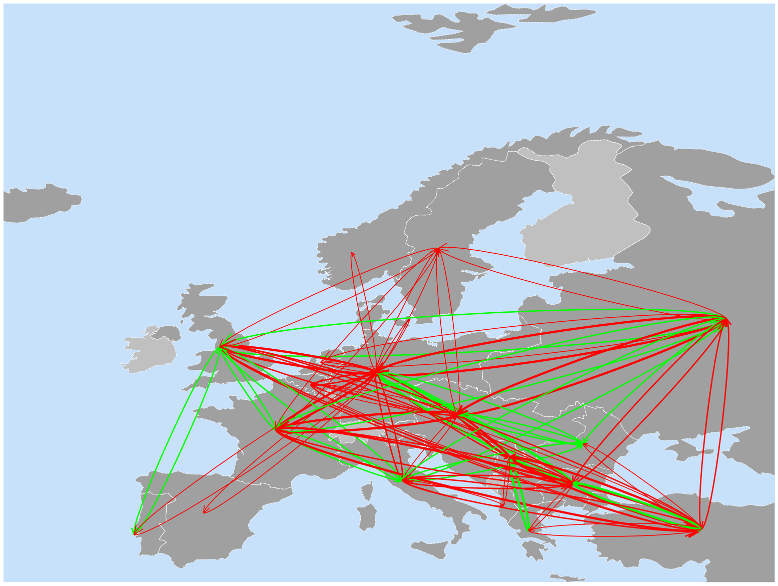


Figure 16: **Geopolitics of Europe in 1915**

Table 2: 1914 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
UK - Germany	UK - The Netherlands	UK - Portugal
UK - Austria-Hungary	UK - Norway	Germany - Austria-Hungary
UK - Turkey	The Netherlands - Germany	Germany - Italy
Belgium - Germany	The Netherlands - Russia	Austria-Hungary - Italy
France - Germany	Switzerland - Germany	Germany - Romania
France - Austria-Hungary	Switzerland - Austria-Hungary	Romania - Austria-Hungary
France - Turkey	Portugal - Germany	Italy - Romania
Germany - Yugoslavia	Germany - Romania	France - Russia
Germany - Russia	Germany - Sweden	Yugoslavia - Greece
Austria-Hungary - Russia	Germany - Norway	Bulgaria - Turkey
Austria-Hungary - Yugoslavia	Germany - Denmark	Austria-Hungary - Bulgaria
Yugoslavia - Turkey	Italy - Albania	Germany - Bulgaria
Russia - Turkey	Yugoslavia - Bulgaria	Russia - Romania
	Austria-Hungary - Italy	
	Greece - Bulgaria	
	Greece - Turkey	
	Romania - Turkey	

which greatly delayed Germany. Though Germany was very successful in earlier battles in 1914, France with assistance from the British forces halted the German advance at the First Battle of the Marne. Importantly, German armies intended for the Western front were also diverted to cope with Russia's attacks on the Eastern front.(Foley, 2006; Keegan, 1999) Aside from the cooperation between France and Britain, Germany also suffered from the problems of communications and questionable command. Additionally, in 1915, Austria-Hungary soon entered war with Russia, which greatly limited its coordination with Germany.(Keegan, 1999; Strachan, 2001)

Also, through the relation patterns in 1915, we can see that Italy revoked the Triple Alliance and joined the Entente against Germany and Austria-Hungary. So despite the quick victories in the early phases of the war, these unfavorable factors landed Germany in besiege, making it extremely difficult to be “defensive” or “powerful”, as has been shown in Figure 9 and 10.

Table 3: 1915 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
UK - Germany	Bulgaria - Turkey	UK - Portugal
UK - Austria-Hungary	UK - Sweden	Germany - Austria-Hungary
UK - Turkey	The Netherlands - Germany	Italy - Russia
Belgium - Germany	The Netherlands - Russia	France - Italy
France - Germany	Belgium - Austria-Hungary	Germany - Romania
France - Austria-Hungary	Belgium - Bulgaria	Romania - Austria-Hungary
France - Turkey	Belgium - Turkey	Italy - Romania
Germany - Yugoslavia	Germany - Romania	France - Russia
Germany - Russia	Germany - Sweden	Yugoslavia - Greece
Austria-Hungary - Russia	Germany - Norway	Bulgaria - Turkey
Austria-Hungary - Yugoslavia	Germany - Denmark	Austria-Hungary - Bulgaria
Yugoslavia - Turkey	Italy - Albania	Germany - Bulgaria
Russia - Turkey	Italy - Bulgaria	Russia - Romania
UK - Bulgaria	Greece - Turkey	UK - France
France - Bulgaria	Romania - Turkey	UK - Italy
France - Turkey	France - Sweden	UK - Russia
Germany - Italy	Spain - Germany	
Italy - Bulgaria	Portugal - Germany	
Italy - Turkey	Austria-Hungary - Italy	
Yugoslavia - Bulgaria	Austria-Hungary - Sweden	
Bulgaria - Russia	Albania - Yugoslavia	
	Greece - Bulgaria	
	Russia - Sweden	

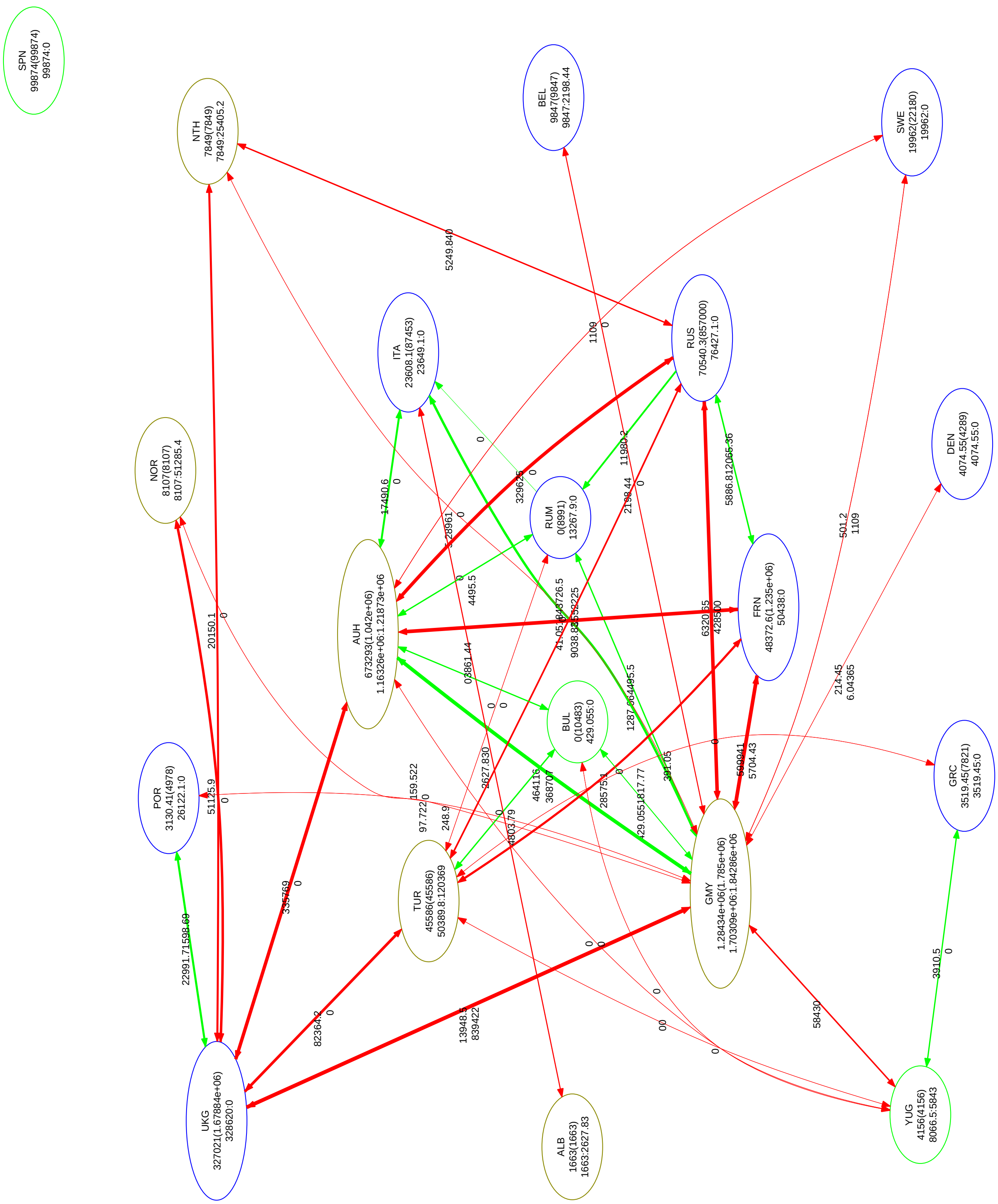


Figure 17: **1914 Europe**

As previously represented, Green circles are countries that are self-defensive, blue circles are countries that are both self-defensive and with whom the foes are vulnerable, and yellow circles are countries that are vulnerable.

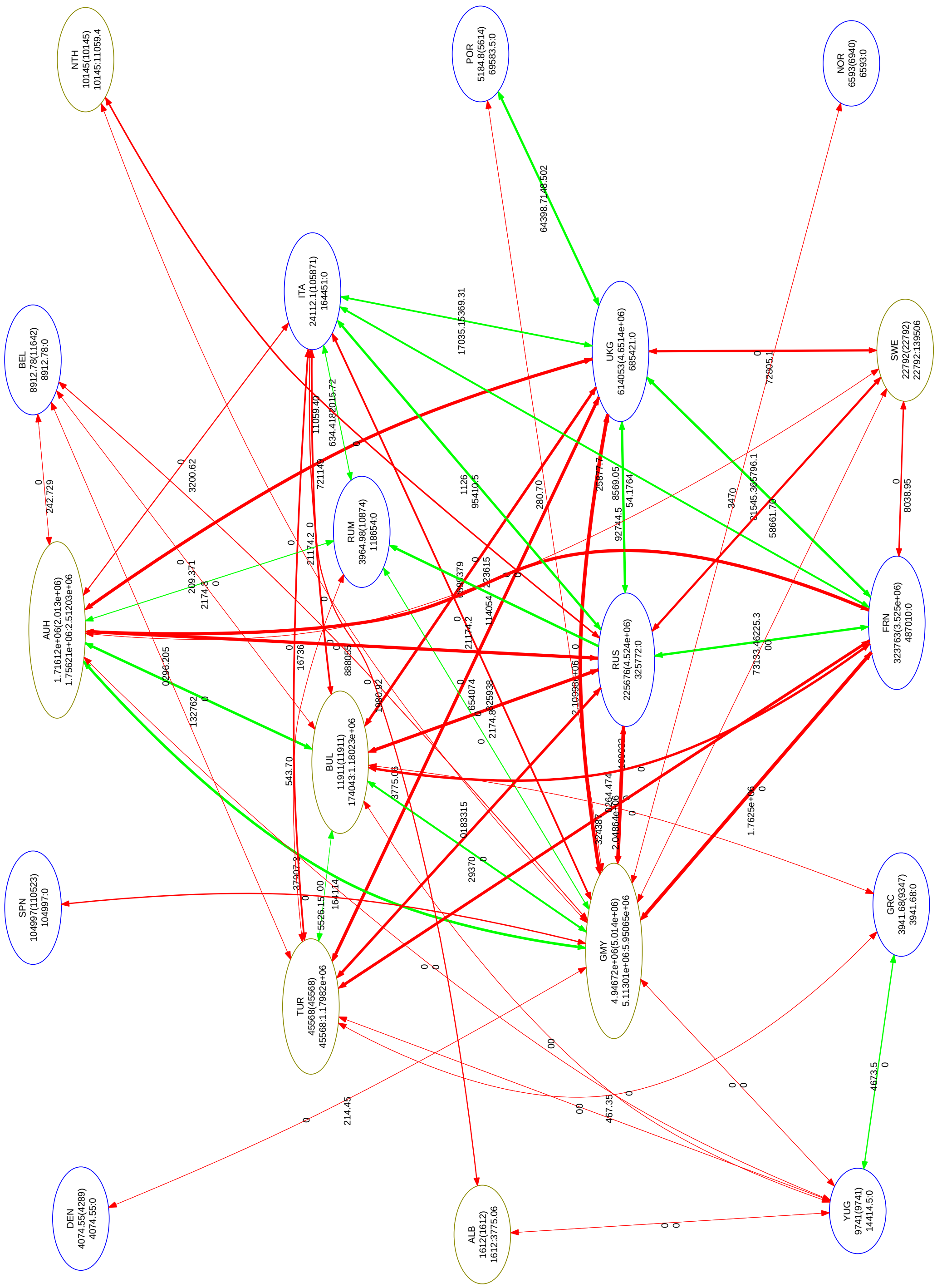


Figure 18: **1915 Europe**

4.1.2 1916

In 1916 the alliance structure had remained largely stable but had undergone some changes. For instance, between 1914 and 1915, despite allied with Russia, Romania was also having defense pacts with Germany and Italy. From 1916 onwards, it relinquished the partnership with Germany and Italy and joined on the UK and France against the Triple Alliance. Additionally, Turkey also became aligned with Germany against the Entente.

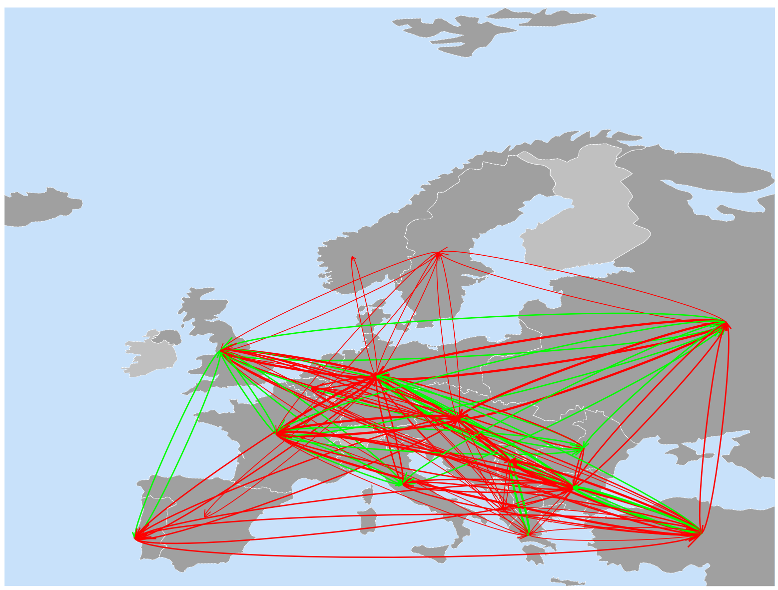


Figure 19: **Geopolitics of Europe in 1916**

The solution of the game reflects the situation in 1916, characterized by two great battles on the Western front, at Verdun and Somme, between Germany and the joint armies of France and Britain. The casualties from Verdun pushed Britain to start the Battle of the Somme in July 1916, which was part of a multinational plan of the Entente to attack Germany on different fronts simultaneously. (Prete, 2009; Strachan, 1998)Britain won over Germany in the battle, which marked the point at which German morale began a permanent decline and the strategic initiative was lost.

Table 4: 1916 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
UK - Germany	UK - The Netherlands	UK - Portugal
UK - Austria-Hungary	UK - Greece	Germany - Austria-Hungary
UK - Turkey	UK - Sweden	France - Russia
Belgium - Germany	The Netherlands - Germany	Yugoslavia - Greece
France - Germany	Belgium - Austria-Hungary	Bulgaria - Turkey
France - Austria-Hungary	Belgium - Bulgaria	Austria-Hungary - Bulgaria
France - Turkey	Belgium - Turkey	Germany - Bulgaria
Germany - Yugoslavia	Germany - Sweden	Russia - Romania
Germany - Russia	Germany - Norway	UK - France
Austria-Hungary - Russia	Germany - Albania	UK - Italy
Austria-Hungary - Yugoslavia	Germany - Greece	UK - Russia
Yugoslavia - Turkey	Albania - Bulgaria	France - Italy
Russia - Turkey	Albania - Turkey	Italy - Russia
UK - Bulgaria	France - Greece	Germany - Turkey
France - Bulgaria	France - Sweden	UK - Romania
Germany - Romania	Spain - Germany	France - Romania
Germany - Italy	Austria-Hungary - Albania	
Italy - Bulgaria	Austria-Hungary - Sweden	
Italy - Turkey	Albania - Yugoslavia	
Yugoslavia - Bulgaria	Albania - Bulgaria	
Bulgaria - Russia	Albania - Turkey	
Bulgaria - Romania	Greece - Bulgaria	
Portugal - Germany	Greece - Turkey	
Portugal - Austria-Hungary	Russia - Sweden	
Portugal - Bulgaria		
Portugal - Turkey		

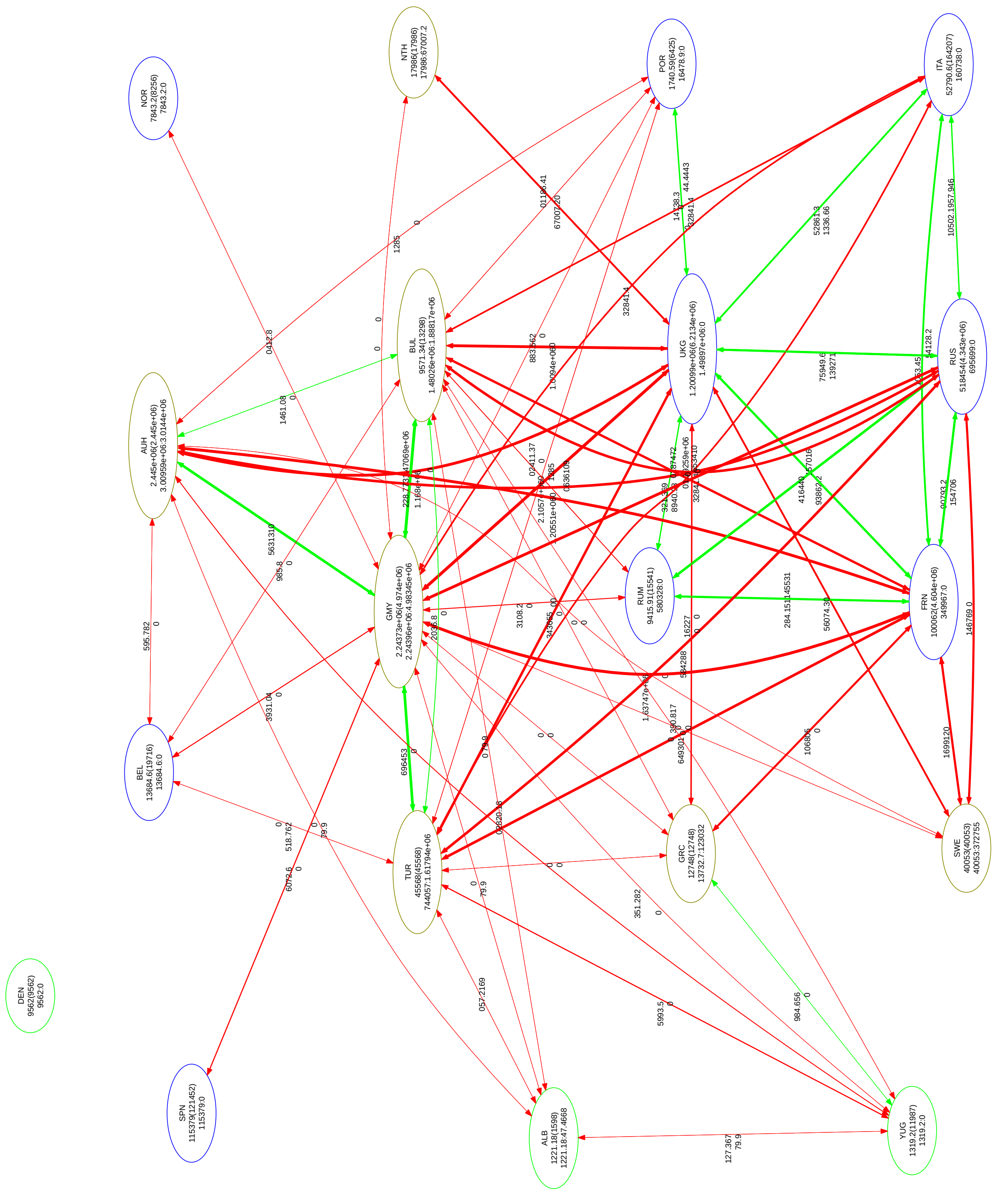


Figure 20: **1916 Europe**

4.1.3 1917-1918

In 1917, Germany's military spending peaked within the course of war and surpassed all of its rivals. Their morale was helped by a series of victories against countries including Greece, Italy, and Russia and had been at its greatest since 1914 at the end of 1917 and beginning of 1918 with Russia lapsed into revolution. (Cruttwell, 1934; Herwig, 2014)

1917-1918 saw a major structural change, the US entry, which eventually put an end to the war. The solution of the game in 1918 would have been entirely different had this factor not been considered. Actually it was not that Germany did not foresee this to happen. So Germany also offered a military alliance to Mexico, which outraged the US just as Germany started defeating the US in submarine warfare. Wilson asked Congress for "a war to end all wars" and the US declared war on Germany on April 6, 1917.(Link and Wilson, 1954)

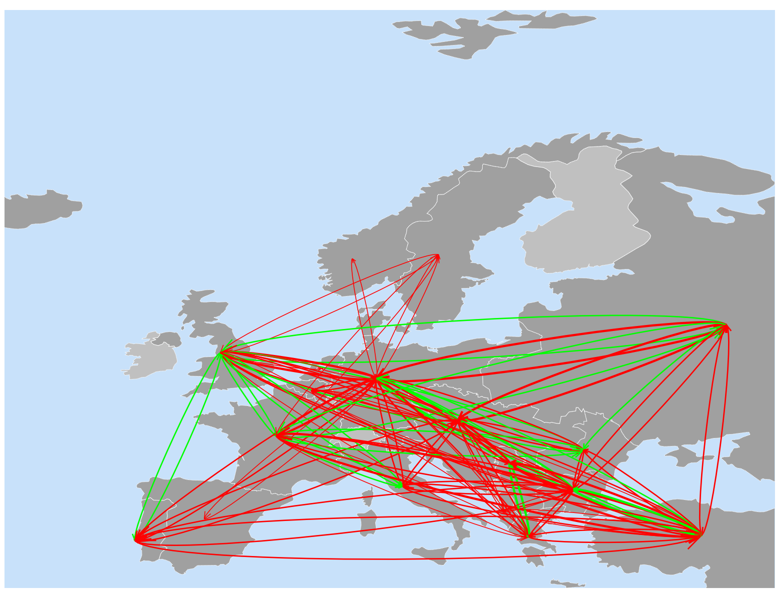


Figure 21: **Geopolitics of Europe in 1917**

Comparing Figure 25 to 24, obviously if it had not been the assistance from the US, Germany would have been "defensive", which means that it could have overcome the joint attacks from the Entente. Paul Kennedy in *The Rise and Fall of the Great Powers* also noted

Table 5: 1917 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
UK - Germany	UK - The Netherlands	UK - Portugal
UK - Austria-Hungary	UK - Greece	Germany - Austria-Hungary
UK - Turkey	UK - Sweden	Germany - Bulgaria
Belgium - Germany	The Netherlands - Germany	Germany - Turkey
France - Germany	Belgium - Austria-Hungary	Austria-Hungary - Bulgaria
France - Turkey	Belgium - Bulgaria	Bulgaria - Turkey
Austria-Hungary - Italy	Belgium - Turkey	France - Russia
Germany - Yugoslavia	Austria-Hungary - Albania	Yugoslavia - Greece
Germany - Russia	Germany - Sweden	UK - France
Austria-Hungary - Russia	Germany - Norway	UK - Italy
Austria-Hungary - Yugoslavia	Germany - Albania	UK - Russia
Yugoslavia - Turkey	Albania - Bulgaria	France - Italy
Russia - Turkey	Albania - Turkey	France - Italy
UK - Bulgaria	France - Greece	UK - Romania
Romania - Turkey	Spain - Germany	France - Romania
France - Bulgaria	Albania - Bulgaria	Russia - Romania
Germany - Romania	Albania - Turkey	
Germany - Italy	Greece - Bulgaria	
Italy - Bulgaria	Greece - Turkey	
Italy - Turkey		
Yugoslavia - Bulgaria		
Bulgaria - Russia		
Bulgaria - Romania		
Portugal - Germany		
Portugal - Austria-Hungary		
Portugal - Bulgaria		
Portugal - Turkey		
Germany - Greece		
Austria-Hungary - Greece		
Austria-Hungary - Romania		
Greece - Bulgaria		
Greece - Turkey		

Table 6: 1918 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
UK - Germany	UK - Russia	UK - Portugal
UK - Austria-Hungary	The Netherlands - Germany	Germany - Austria-Hungary
UK - Turkey	Belgium - Austria-Hungary	Germany - Bulgaria
Belgium - Germany	Belgium - Bulgaria	Germany - Turkey
France - Germany	Belgium - Turkey	Austria-Hungary - Bulgaria
France - Turkey	France - Albania	Bulgaria - Turkey
Austria-Hungary - Italy	France - Russia	Italy - Romania
Germany - Yugoslavia	Spain - Germany	Yugoslavia - Greece
Austria-Hungary - Yugoslavia	Germany - Sweden	UK - France
Yugoslavia - Turkey	Germany - Norway	UK - Italy
Russia - Turkey	Germany - Albania	France - Romania
UK - Bulgaria	Germany - Russia	France - Italy
Romania - Turkey	Albania - Italy	UK - Romania
France - Bulgaria	Italy - Russia	
Germany - Italy	Albania - Bulgaria	
Italy - Bulgaria	Austria-Hungary - Albania	
Italy - Turkey	Albania - Yugoslavia	
Yugoslavia - Bulgaria	Albania - Turkey	
Bulgaria - Russia	Yugoslavia - Romania	
Bulgaria - Romania	Yugoslavia - Russia	
Portugal - Germany	Greece - Russia	
Portugal - Austria-Hungary	Romania - Russia	
Portugal - Bulgaria	Russia - Estonia	
Portugal - Turkey	Russia - Lithuania	
Germany - Greece		
Austria-Hungary - Greece		
Greece - Bulgaria		
Greece - Turkey		
France - Austria-Hungary		

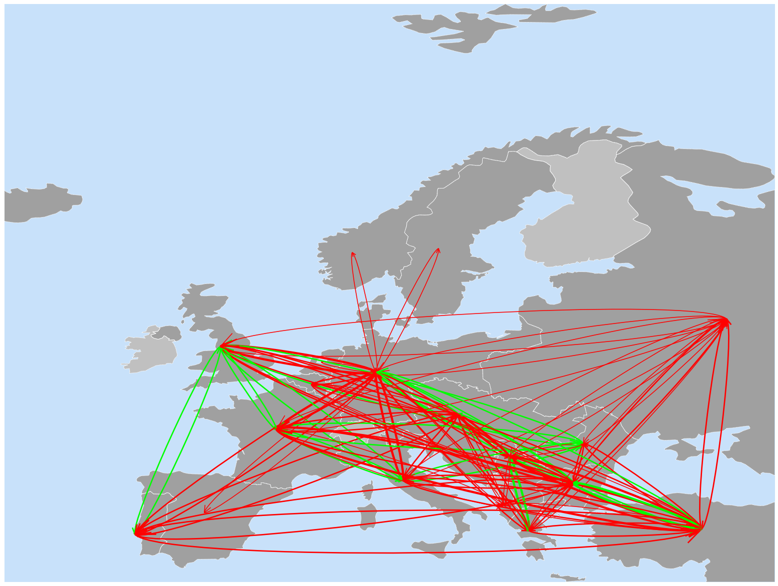


Figure 22: **Geopolitics of Europe in 1918**

the advanced railway system of Germany, which facilitated the transportation of the troops between the two fronts.(Kennedy, 2010) However, as the morale waned, it would be hard to predict whether it would have been realistic for Germany to reverse the failing situation and defeat the Entente. Figure 25 shows the final outcome of the war: the Entente countries were either powerful or defensive, while Austria-Hungary and Germany were vulnerable.

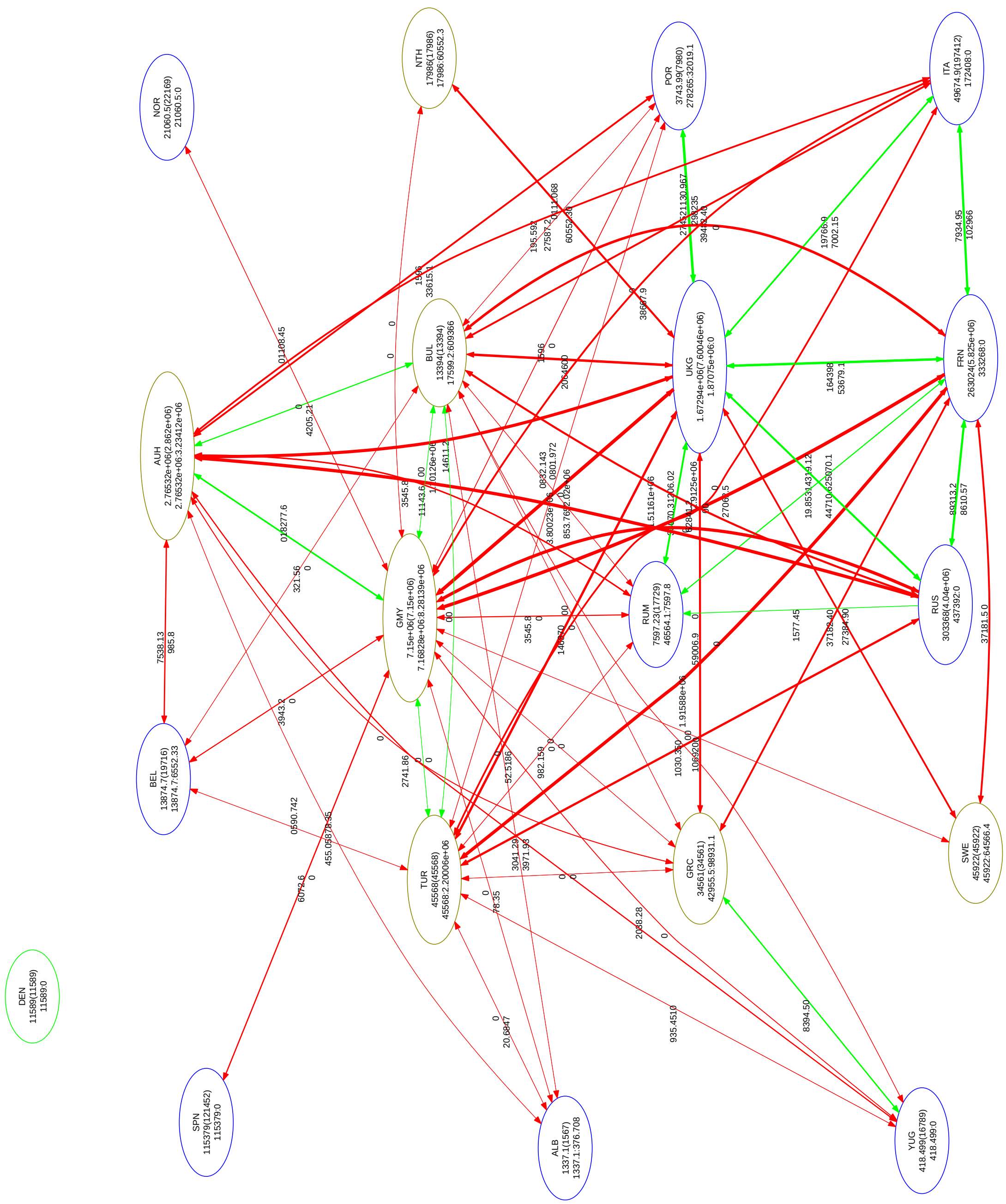


Figure 23: **1917 Europe**

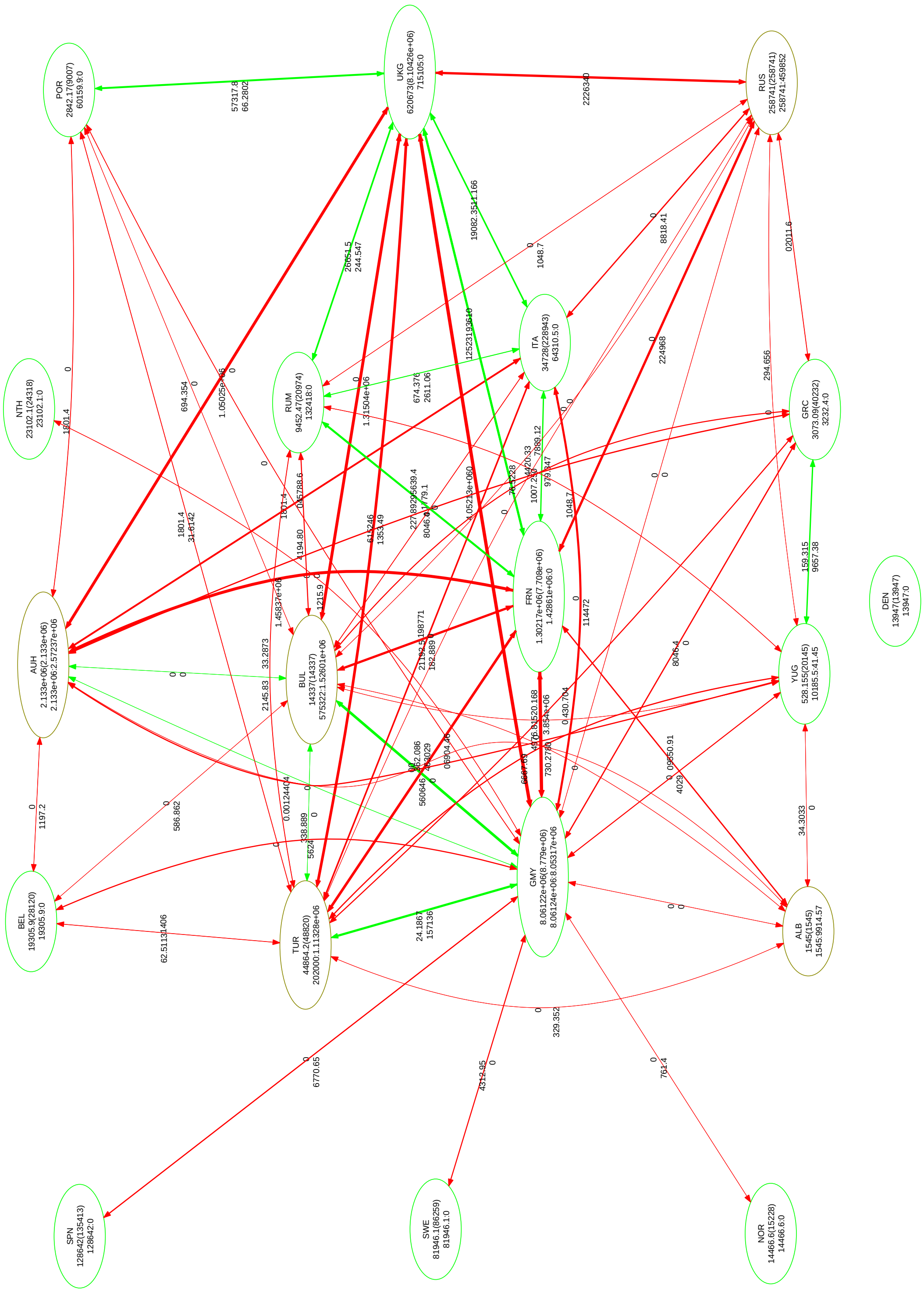


Figure 24: **1918 Europe (Without US Entry)**

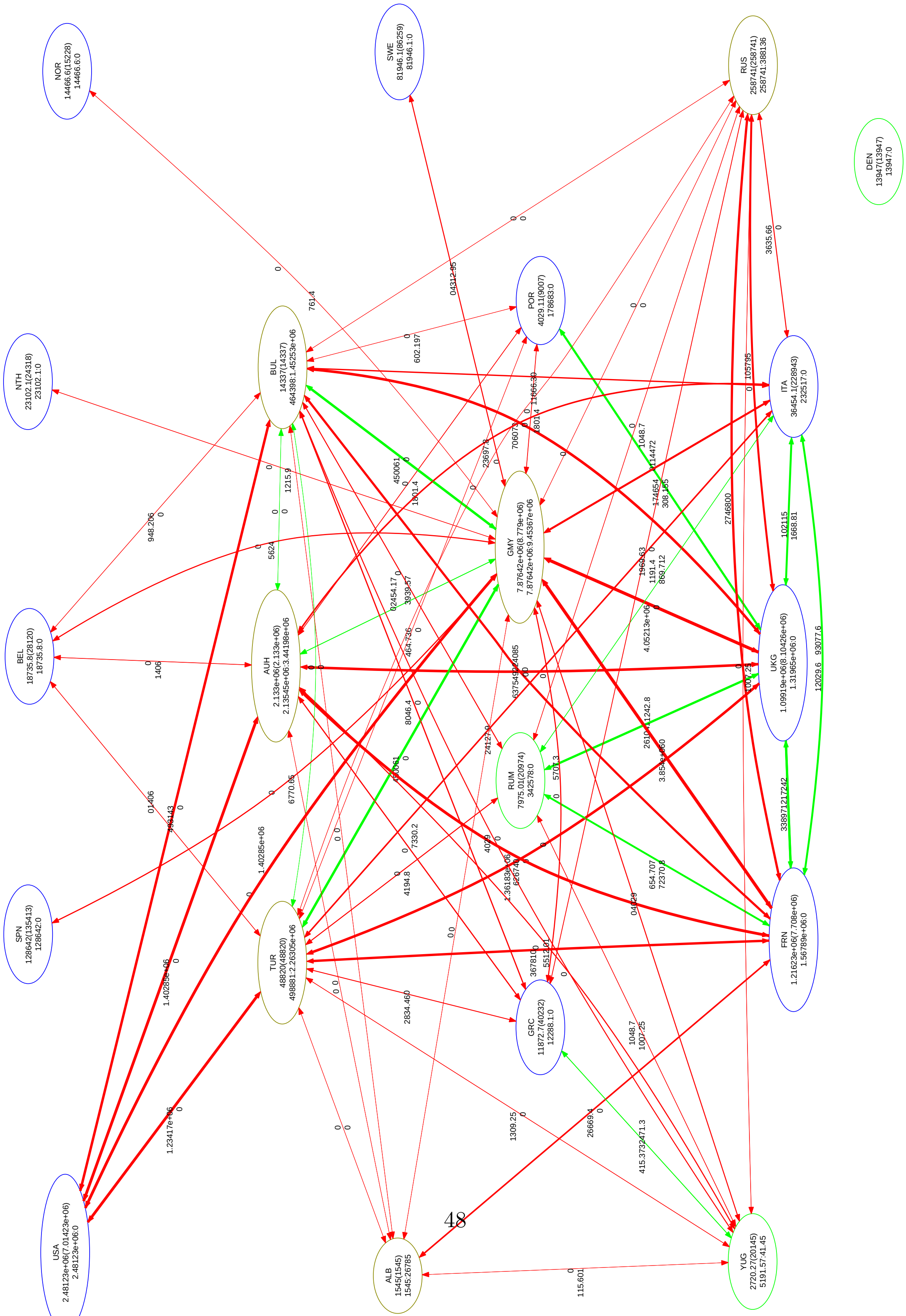


Figure 25: **1918 Europe (With US Entry)**

4.1.4 Simulation Results

Having obtained 10,000 possible solutions of the game from simulation, I present Table 7, which shows the probability of being defensive, powerful and vulnerable for selected countries in World War I.

Table 7: Probability of Being in Different States for Selected Countries from 1914-1918

Country-Year	Pr(Vulnerable)	Pr(Defensive)	Pr(Powerful)
Germany-1914	91.2%	8.76%	0%
Germany-1915	94.53%	5.47%	0%
Germany-1916	97.56%	2.44%	0%
Germany-1917	98.23%	1.77%	0%
Germany-1918 (N/A USA Entry)	0%	100%	0%
Germany-1918 (With USA Entry)	97.92%	2.08%	0%
Austria-Hungary-1914	98.2%	1.8%	0%
Austria-Hungary-1915	99.8%	0.2%	0%
Austria-Hungary-1916	99.69%	0.31%	0%
Austria-Hungary-1917	98.18%	1.82%	0%
Austria-Hungary-1918 (N/A USA Entry)	100%	0%	0%
Austria-Hungary-1918 (With USA Entry)	99.99%	0.01%	0%
UK-1914	0%	10.51%	89.49%
UK-1915	0%	6.05%	93.95%
UK-1916	0%	2.98%	97.02%
UK-1917	0%	3.59%	96.41%
UK-1918 (N/A USA Entry)	0%	100%	0%
UK-1918 (With USA Entry)	0%	2.09%	97.91%
France-1914	0%	10.51%	89.49%
France-1915	0%	6.05%	93.95%
France-1916	0%	2.98%	97.02%
France-1917	0%	1.77%	98.23%
France-1918 (N/A USA Entry)	0%	100%	0%
France-1918 (With USA Entry)	0%	2.09%	97.91%
Russia-1914	0%	10.51%	89.49%
Russia-1915	0%	6.05%	93.95%
Russia-1916	0%	2.98%	97.02%
Russia-1917	0%	3.59%	96.41%
Russia-1918 (N/A USA Entry)	100%	0%	0%
Russia-1918 (With USA Entry)	100%	0%	0%

Several regularities can be observed: (1) the likelihood of failure persistently increases for Germany; (2) without the entry of the US, Germany could have been defensive; (3) Russia's domestic revolution and later withdrawal from the war greatly impaired its prospect for retaining a powerful state; (4) The entry of the US helped the Entente to end the war. Obviously, these regularities are consistent with historic facts.

4.2 A Visualization of the World in World War II

The model and algorithm work even better with the case of World War II that manifests even more complex relation patterns and encompasses much greater geographical scale than World War I. The alliance dynamics in the period from 1939 to 1945 are mainly between the Axis (Germany, Italy, Japan, Hungary, Romania, Bulgaria) and the Allies (U.S., Britain, France, USSR, Australia, Belgium, Brazil, Canada, China, Denmark, Greece, Netherlands, New Zealand, Norway, Poland, South Africa, Yugoslavia).(Amt, 1948)

The model confirms the major historic events from 1939 to 1945, especially:

Fact 1: The 1939 Appeasement. With the nonaggression pacts signed by the Soviet Union and Germany, Germany quickly conquered Poland, then defeated Britain and France with large advantages.

Fact 2: Massive Successes of Germany in Western Europe in 1940. 1940 saw Germany invade and defeat nearly all of the Western Europe.

Fact 3: Turning Point at the Eastern Front in 1941. The Battle of Moscow between the Soviet Union and Germany marks the beginning of the loss of advantages for Germany.

Fact 4: Turning Point in the Asia-Pacific in 1942. The US greater involvement in the war after the Pearl Harbor attack marks the turning point of the war.

Fact 5: The Allied Victory. From 1943 to 1945, the joint efforts by the Allies overwhelmed that of the Axis.

4.2.1 1939

Though the Soviet Union joined the Allies after being attacked by Nazi Germany in 1941, it began World War II with non-aggression pacts with Nazi Germany. The nonaggression pacts, along with the other secret protocols, divided the whole of Eastern Europe into German and the Soviets spheres of influence. (Amt, 1948).

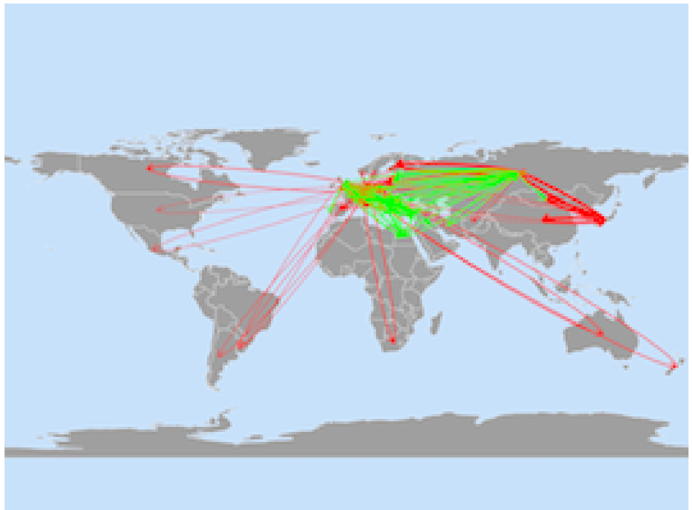


Figure 26: **1939 Europe**

Figure 27 shows that under the attacks by Nazi Germany and the Soviet Union, Poland was vulnerable. Though was requested by Poland for military assistance, France and Britain did not immediately declare war on Germany. Warsaw surrendered soon after and Germany gained a swift victory. Figure 27 also shows the outcome of the Sino-Japanese war on the Asia-Pacific battlefield. Due to great advantages over China, Japan held a winning position in 1939. Comparatively speaking, the Axis powers won over the Allies in 1939.

Table 8: 1939 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
China - Japan	Spain - Poland	United Kingdom - Egypt
Russia - Finland	Spain - Italy	United Kingdom - France
Russia - Japan	United Kingdom - Netherlands	United Kingdom - Yugoslavia
Russia - France	United Kingdom - Japan	United Kingdom - Greece
Russia - United Kingdom	United Kingdom - Italy	United Kingdom - Iraq
Germany - United Kingdom	United Kingdom - Uruguay	United Kingdom - Bulgaria
Germany - Belgium	Russia - Estonia	United Kingdom - Romania
Germany - France	Russia - Poland	United Kingdom - Turkey
Germany - Poland	Russia - Latvia	United Kingdom - Portugal
Germany - Australia	Russia - Afghanistan	United Kingdom - Poland
Germany - New Zealand	Italy - Albania	France - Yugoslavi
Germany - Canada	France - Italy	France - Greece
Germany - South Africa	France - Japan	France - Bulgaria
Mongolia - Japan	Hungary - Romania	France - Romania
	Poland - Estonia	France - Turkey
	United States - Germany	France - Egypt
	United States - United Kingdom	France - Poland
	Germany - The Netherlands	Russia - France
	Germany - Belgium	Russia - Turkey
	Germany - Luxemburg	Russia - Egypt
	Germany - Finland	Russia - Iran
	Germany - Latvia	Russia - Estonia
	Germany - Sweden	Russia - Latvia
	Germany - Estonia	Russia - Lithuania
	Germany - Russia	Russia - Czechoslo
	Germany - Mexico	Russia - Mongolia
	Germany - Uruguay	Bulgaria - Romaniav
	Germany - Argentina	Bulgaria - Russia
	Germany - Norway	Bulgaria - Turkey
	Germany - Lithuania	Bulgaria - Egypt
	Germany - Italy	Romania - Russia
	Germany - Spain	Romania - Egypt
	Germany - Switzerland	Romania - Poland
	Germany - Czechoslovakia	Romania - Czechoslo
		Romania - Yugoslavia
		Germany - Italy
		Italy - Albania
		Greece - Romania
		Greece - Turkey
		Greece - Bulgaria
		Greece - Russia
		Greece - Egypt
		Yugoslavi - Greece
		Yugoslavi - Turkey
		Yugoslavi - Bulgaria
		Yugoslavi - Russia
		Yugoslavi - Egypt
		Latvia - Estonia
		Turkey - Romania
		Turkey - Egypt

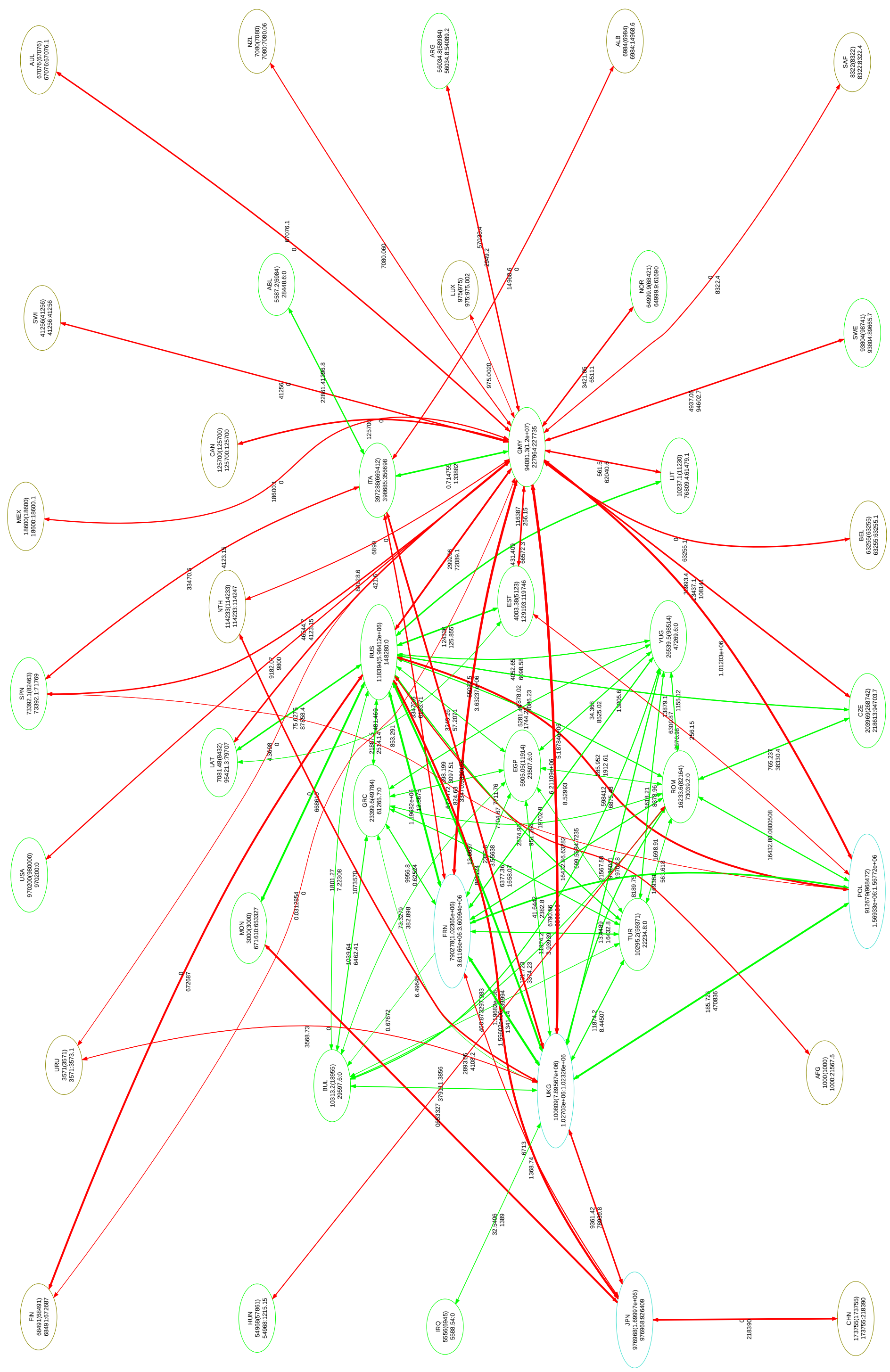


Figure 27: **1939 Europe**

4.2.2 1940

Table 9 indicates wars between Germany and Western Europe including France and Britain as well as Scandinavian countries like Norway. Originally Hitler had intended to respect Norways neutrality but still invaded Norway in 1940. In Western Europe, Germany's sweeping victory led to an occupation of the Netherlands, Belgium and France and to a British evacuation from Dunkirk(Klemann and Kudryashov, 2012).

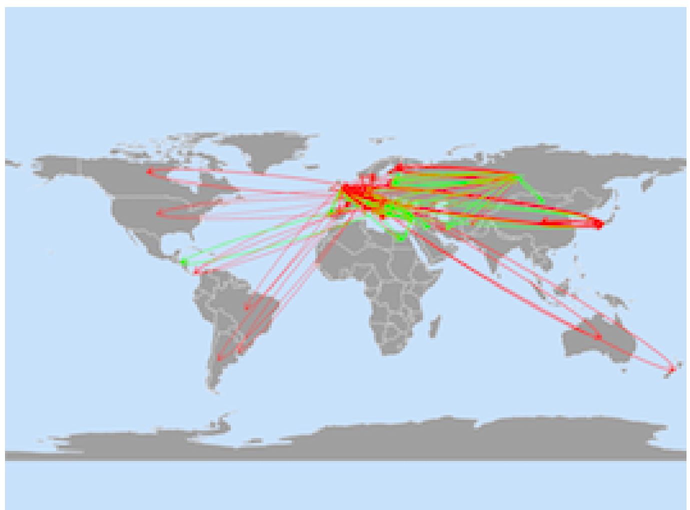


Figure 28: **1940 Europe**

Despite being shown on the graph at the same time, the war between Germany and Britain happened after France was conquered. Germany lay down a plan of attack to destroy British air power and so open the way for the amphibious invasion. However, the victory by the Royal Air Force (RAF) Fighter Command blocked this possibility (Weinberg, 1994).

In this year, the collapse of France convinced Mussolini that the time to implement his Pact of Steel with Hitler had come, and on June 10, 1940, Italy declared war against France and Britain. (Knox, 1986) Soon France was conquered and Germany entered Paris on June 14th, 1940 (Paxton, 2001).

Results of the game show the follows: first, France was vulnerable mainly because of the joint attacks from Germany and Italy; Britain was simultaneously engaged in several conflicts

Table 9: 1940 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
China - Japan	Spain - France	United Kingdom - Egypt
Russia - Finland	Spain - United Kingdom	United Kingdom - France
United Kingdom - Japan	United Kingdom - France	United Kingdom - Iraq
United Kingdom - Italy	United Kingdom - Japan	United Kingdom - Turkey
United Kingdom - France	United Kingdom - Norway	United Kingdom - Portugal
Germany - United Kingdom	United Kingdom - Denmark	France - Turkey
Germany - Belgium	United Kingdom - Italy	Russia - Iran
Germany - France	United Kingdom - Switzerland	Russia - Czechoslovakia
Germany - Norway	United Kingdom - Sweden	Russia - Mongolia
Germany - Australia	United Kingdom - Romania	Russia - Estonia
Germany - New Zealand	United Kingdom - Yugoslavia	Russia - Latvia
Germany - Canada	United Kingdom - Portugal	Russia - Lithuania
Germany - Greece	United Kingdom - Russia	Germany - Japan
Germany - The Netherlands	United Kingdom - Brazil	Germany - Hungary
Italy - Greece	United States - Romania	Germany - Romania
	United States - Germany	Germany - Italy
	United States - United Kingdom	Romania - Turkey
	United States - Romania	Romania - Italy
	Germany - Ireland	Romania - Hungary
	Germany - The Netherlands	Romania - Japan
	Germany - Belgium	Italy - Hungary
	Germany - Luxemburg	Italy - Japan
	Germany - Finland	Greece - Romania
	Germany - Latvia	Greece - Turkey
	Germany - Sweden	Turkey - Egypt
	Germany - Estonia	Yugoslavi - Greece
	Germany - Norway	Yugoslavi - Romania
	Germany - Yugoslavia	Yugoslavi - Turkey
	Germany - Uruguay	Yugoslavi - Russia
	Germany - Argentina	Latvia - Estonia
	Germany - Russia	
	Germany - Romania	
	Germany - Italy	
	Germany - Turkey	
	Germany - Bulgaria	
	Germany - Panama	
	Germany - Hungary	
	Germany - Portugal	
	Russia - Romania	
	Russia - Turkey	
	Russia - Latvia	
	Russia - Lithuania	
	Russia - Estonia	
	Russia - Afghanistan	
	Russia - Bulgaria	
	Italy - Yugoslavia	
	Italy - France	
	Italy - Spain	
	Italy - Turkey	
	Italy - Iran	
	Italy - Egypt	
	Italy - Panama	
	Italy - Sweden	
	Italy - Greece	

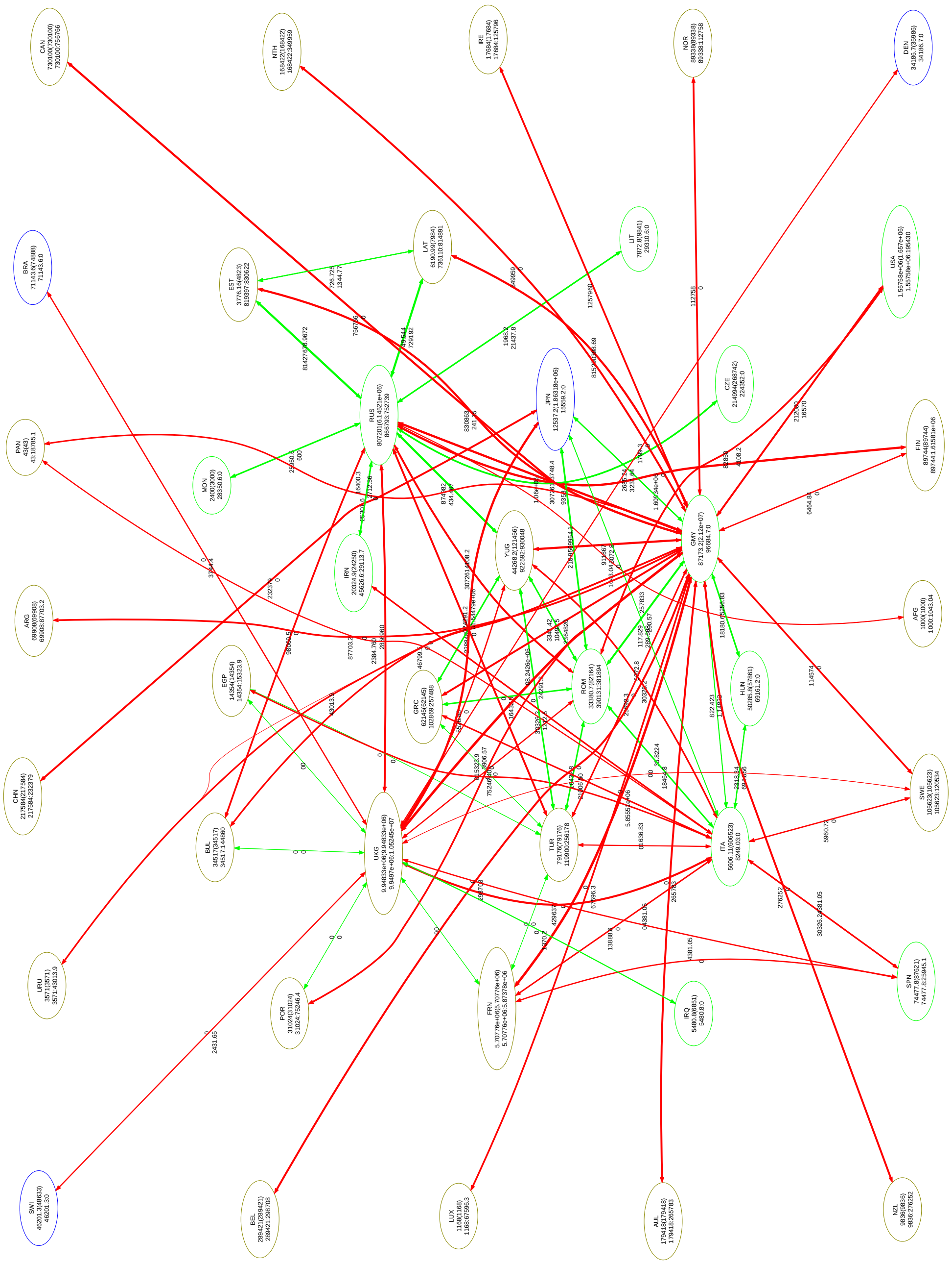


Figure 29: **1940 Europe**

with countries including Germany, Russia and Brazil. Facing the attacks on multiple fronts, Britain was also predicted to be "vulnerable"; accordingly, both Germany and Italy were defensive, though not yet powerful in this year. Generally speaking, the game predictions largely support the key historic facts in this period.

4.2.3 1941

In 1941, Germany launched attacks in the Balkans and Yugoslavia and Greece surrendered. In addition, this year has the following events: the Germany-Soviet Union alignment broke down and Germany attacked the Soviet Union, Japan attacked Pearl Harbor, the US and Britain declared war on Japan, and Germany and Italy entered into conflict with the US. With continental Europe under Nazi's control, the war took on a more global dimension in this year.

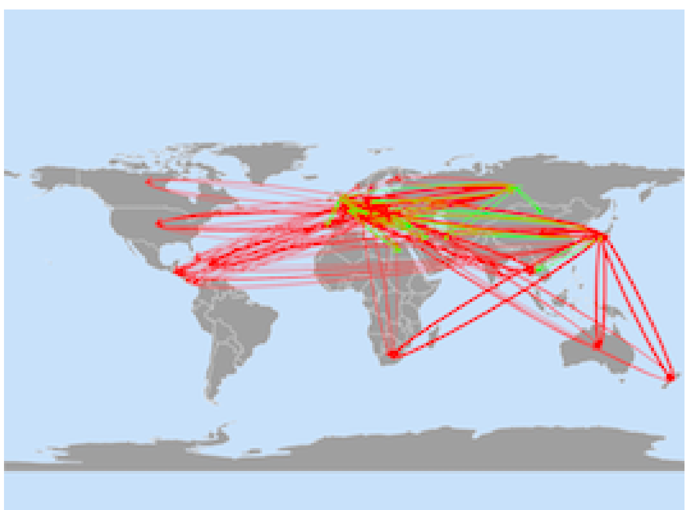


Figure 30: **1941 Europe**

While the conflict with Britain continued, Germany invaded Russia executing what was known as "Operation Barbarossa" on June 22. The initial advance was swift and Moscow coming under attack at the end of the year. The bitter Russian winter, however, crippled the German advance. The Soviets counterattacked in December successfully and the So-

Table 10: 1941 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
China - Japan	Spain - Russia	United Kingdom - Egypt
United Kingdom - France	United Kingdom - Hungary	United Kingdom - Russia
United Kingdom - Japan	United Kingdom - France	United Kingdom - Turkey
United Kingdom - Italy	United Kingdom - Finland	
United Kingdom - Bulgaria	United Kingdom - Norway	United Kingdom - Iraq
United Kingdom - Romania	United Kingdom - Germany	United Kingdom - Portugal
Germany - Canada	United Kingdom - Iraq	Hungary - Italy
Germany - Yugoslavia	United Kingdom - Bulgaria	Hungary - Bulgaria
Germany - Greece	United Kingdom - Greece	Hungary - Romania
Germany - Australia	United States - Hungary	Hungary - Japan
Germany - New Zealand	United States - Bulgaria	France - Yugoslavia
Germany - Russia	United States - Japan	France - Greece
Germany - South Africa	United States - Germany	France - Bulgaria
Germany - United Kingdom	United States - Romania	France - Romania
Hungary - Russia	Germany - Haiti	France - Turkey
United States - Germany	Germany - Nicaragua	France - Egypt
United States - Italy	Germany - Costa Rica	France - Poland
United States - Japan	Germany - Cuba	Russia - Czechoslovakia
Italy - Yugoslavia	Germany - Dominica	Russia - Mongolia
Italy - Greece	Germany - Salvador	Russia - Iran
Greece - Bulgaria	Germany - Guatemala	Germany - Hungary
Russia - Romania	Germany - Honduras	Germany - Bulgaria
Russia - Finland	Germany - Panama	Germany - Romania
Russia - Japan	Germany - Yugoslavia	Germany - Japan
Japan - South Africa	Germany - Sweden	Germany - Italy
Japan - Australia	Germany - Russia	Italy - Bulgaria
Japan - New Zealand	Germany - Romania	Italy - Romania
France - Thailand	Germany - Turkey	Italy - Japan
	Germany - Bulgaria	Bulgaria - Japan
	Germany - Egypt	Romania - Bulgaria
	Germany - Portugal	Romania - Japan
	Yugoslavia - Bulgaria	Japan - Thailand
	Yugoslavia - Romania	
	Romania - South Africa	
	Romania - Australia	
	Romania - New Zealand	
	Romania - Nicaragua	
	Romania - Haiti	
	Romania - Hungary	
	Russia - Romania	
	Russia - Iran	
	Russia - Bulgaria	
	Bulgaria - Nicaragua	
	Bulgaria - Hungary	
	Bulgaria - Haiti	
	Bulgaria - Turkey	
	Bulgaria - Canada	
	Italy - Haiti	
	Italy - Nicaragua	
	Italy - Costa Rica	
	Italy - Cuba	
	Italy - Dominica	
	Italy - Salvador	
	Italy - Guatemala	
	Italy - Honduras	
	Italy - Panama	
	Japan - Thailand	
	Japan - Nicaragua	
	Japan - Haiti	
	Japan - Costa Rica	
	Japan - Cuba	
	Japan - Dominica	
	Japan - Salvador	
	Japan - Guatemala	
	Japan - Honduras	
	Japan - Panama	
	Hungary - Haiti	
	Hungary - Nicaragua	
	Hungary - Yugoslavia	

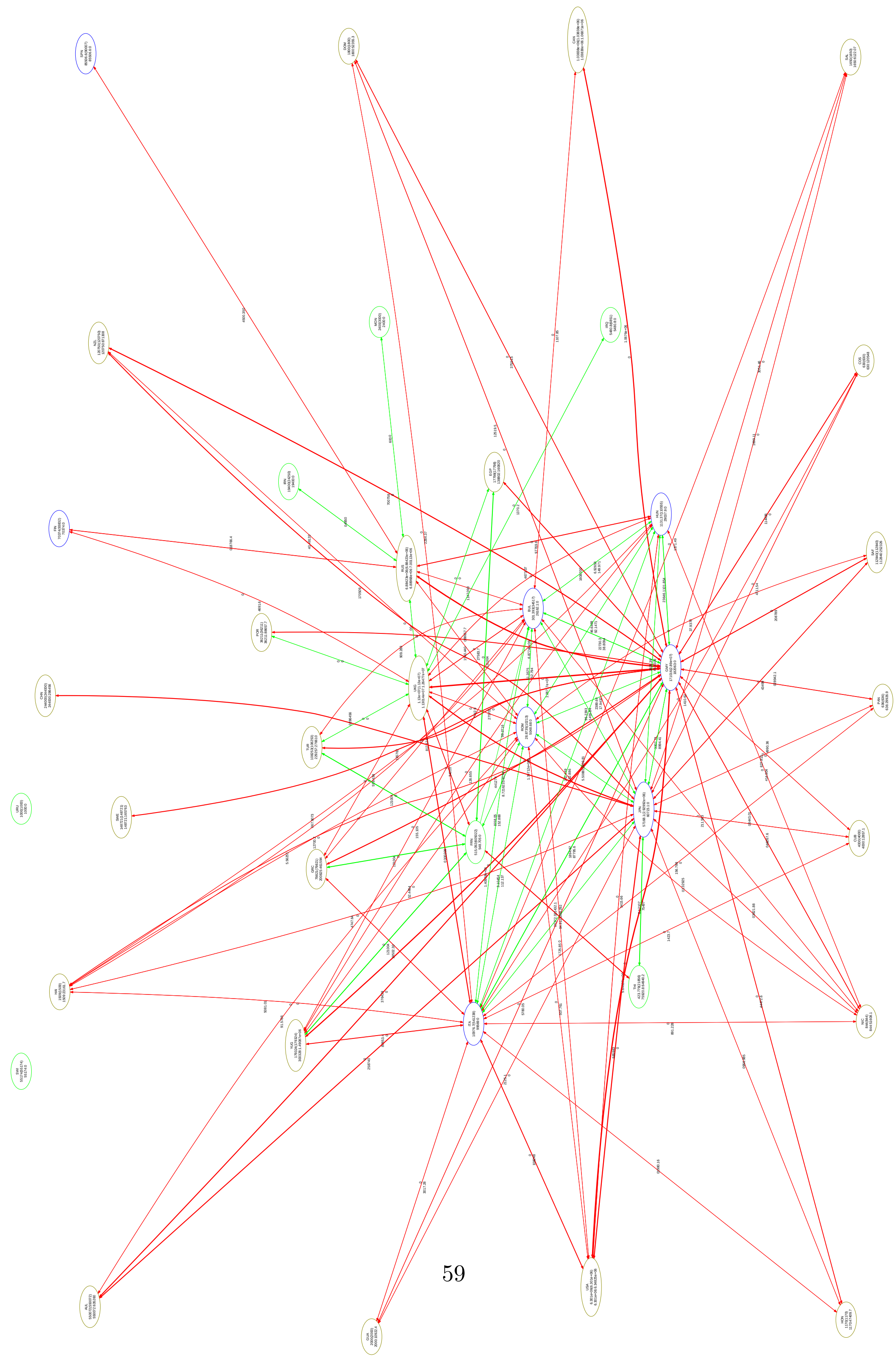


Figure 31: **1941 Europe**

viet counteroffensives were even counted as one of the greatest turning points in the entire war.(Fugate, 1984)

In the Asia-Pacific, Japan mounted a surprise attack on the US Navy base of Pearl Harbor in Hawaii on December 7. (Weinberg, 1994) With Germany declaring war on the US, more conflicts ensued and led to the Pacific war.

The solution of the game in Figure 31 illustrates the defeat of the US under the attacks by Japan and Germany (low willingness parameter caused by little involvement in the war), the massive victory of Germany in Europe and also Japan's success in other Asia-Pacific areas. Though the battle of Moscow had crushed the German offensive against the Soviet Union, its effects did not become pronounced in this year. In other words, the Soviet Union was still vulnerable in the Soviet-German conflict.

4.2.4 1942 - 1943

The US declared war on Japan and assisted Britain in the air battles with Germany in 1942. In the Pacific, June saw the peak and swift decline of Japanese expansion, because of the Battle of Midway, in which US sea-based aircraft destroyed four Japanese carriers and a cruiser and which marked the turning point in the Pacific War. (Weinberg, 1994)

The year of 1942 also saw a reversal of German fortunes. British forces under Montgomery gained the initiative in North Africa at El Alamein, and Russian forces counterattacked at Stalingrad. (Weinberg, 1994) Each of the two battles proved itself to be the respective turning point in the North African and the European fields.

The solutions in Figure 34 and Figure 35 shows the vulnerability of Germany, Italy and Japan and the powerful states of the Allies including the US, Britain and the Soviet Union in these two years. German surrendered to the Soviet armies in February 1943 at Stalingrad. In mid-May German and Italian forces in North Africa surrendered to the Allies, who invaded Sicily in July. In the Asia-Pacific, the US forces defeated the Japanese army at Guadalcanal.

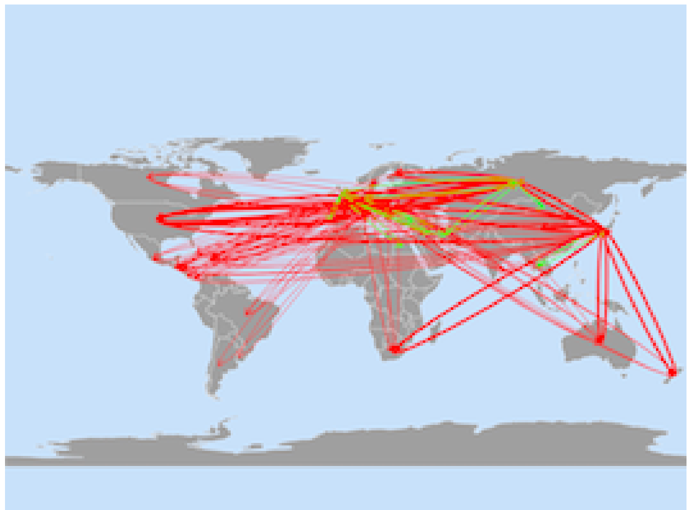


Figure 32: **1942 Europe**

Figure 33: **1943 Europe**

Table 11: 1942 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
China - Japan	Spain - Russia	United Kingdom - Egypt
United Kingdom - France	United Kingdom - Hungary	United Kingdom - Russia
United Kingdom - Japan	United Kingdom - Egypt	United Kingdom - Turkey
United Kingdom - Italy	United Kingdom - Finland	United Kingdom - Iraq
United Kingdom - Bulgaria	United Kingdom - Thailand	United Kingdom - Portugal
United Kingdom - Romania	United Kingdom - Germany	United Kingdom - Iran
Germany - Canada	Canada - Japan	Russia - Czechoslovakia
Germany - Russia	Ecuador - Peru	Russia - Mongolia
Germany - Australia	Thailand - Australia	Russia - Iran
Germany - New Zealand	United States - Hungary	Germany - Italy
Germany - South Africa	United States - Bulgaria	Japan - Thailand
Germany - United Kingdom	United States - Romania	
Hungary - Russia	Germany - Brazil	
United States - France	Germany - Argentina	
United States - Germany	Germany - Haiti	
United States - Italy	Germany - Nicaragua	
United States - Japan	Germany - Costa Rica	
Russia - Romania	Germany - Cuba	
Russia - Finland	Germany - Dominica	
Russia - Japan	Germany - Salvador	
Japan - South Africa	Germany - Guatemala	
Japan - Australia	Germany - Honduras	
Japan - New Zealand	Germany - Panama	
	Germany - Ethiopia	
	Germany - Sweden	
	Germany - Mexico	
	Germany - Chile	
	Germany - Uruguay	
	Romania - South Africa	
	Romania - Australia	
	Romania - New Zealand	
	Romania - Nicaragua	
	Romania - Haiti	
	Bulgaria - Nicaragua	
	Bulgaria - Haiti	
	Bulgaria - Canada	
	Italy - Brazil	
	Italy - Mexico	
	Italy - Ethiopia	
	Italy - Haiti	
	Italy - Nicaragua	
	Italy - Costa Rica	
	Italy - Cuba	
	Italy - Dominica	
	Italy - Salvador	
	Italy - Guatemala	
	Italy - Honduras	
	Italy - Panama	
	Italy - Sweden	
	Japan - Mexico	
	Japan - Ethiopia	
	Japan - Nicaragua	
	Japan - Haiti	
	Japan - Costa Rica	
	Japan - Cuba	
	Japan - Dominica	
	Japan - Salvador	
	Japan - Guatemala	
	Japan - Honduras	
	Japan - Panama	

Table 12: 1943 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
China - Japan	Spain - Russia	United Kingdom - Egypt
United Kingdom - France	United Kingdom - Hungary	United Kingdom - Russia
United Kingdom - Japan	United Kingdom - Egypt	United Kingdom - Turkey
United Kingdom - Italy	United Kingdom - Finland	United Kingdom - Iraq
United Kingdom - Bulgaria	United Kingdom - Thailand	United Kingdom - Portugal
United Kingdom - Romania	United Kingdom - Germany	United Kingdom - Iran
Germany - Italy	Canada - Japan	Russia - Czechoslovakia
Germany - Russia	Canada - Bulgaria	Russia - Mongolia
Germany - Australia	Thailand - Australia	Russia - Iran
Germany - New Zealand	United States - Hungary	Germany - Italy
Germany - South Africa	United States - Bulgaria	Japan - Thailand
Germany - United Kingdom	United States - Romania	
Hungary - Russia	Germany - Iraq	
United States - France	Germany - Saudi Arabia	
United States - Germany	Germany - Switzerland	
United States - Italy	Germany - Nicaragua	
United States - Japan	Germany - Colombia	
Russia - Romania	Germany - Brazil	
Russia - Finland	Germany - Iran	
Russia - Japan	Germany - Bolivia	
Japan - South Africa	Germany - Ethiopia	
Japan - Australia	Germany - Italy	
Japan - New Zealand	Germany - Sweden	
Italy - Bulgaria	Germany - Spain	
	Germany - Panama	
	Germany - Egypt	
	China - Mongolia	
	China - Russia	
	Romania - South Africa	
	Romania - Australia	
	Romania - New Zealand	
	Russia - Sweden	
	Italy - Iraq	
	Italy - Thailand	
	Italy - Brazil	
	Italy - Bolivia	
	Italy - Ethiopia	
	Italy - United States	
	Italy - United Kingdom	
	Japan - Iraq	
	Japan - Saudi Arabia	
	Japan - Bolivia	
	Japan - Ethiopia	

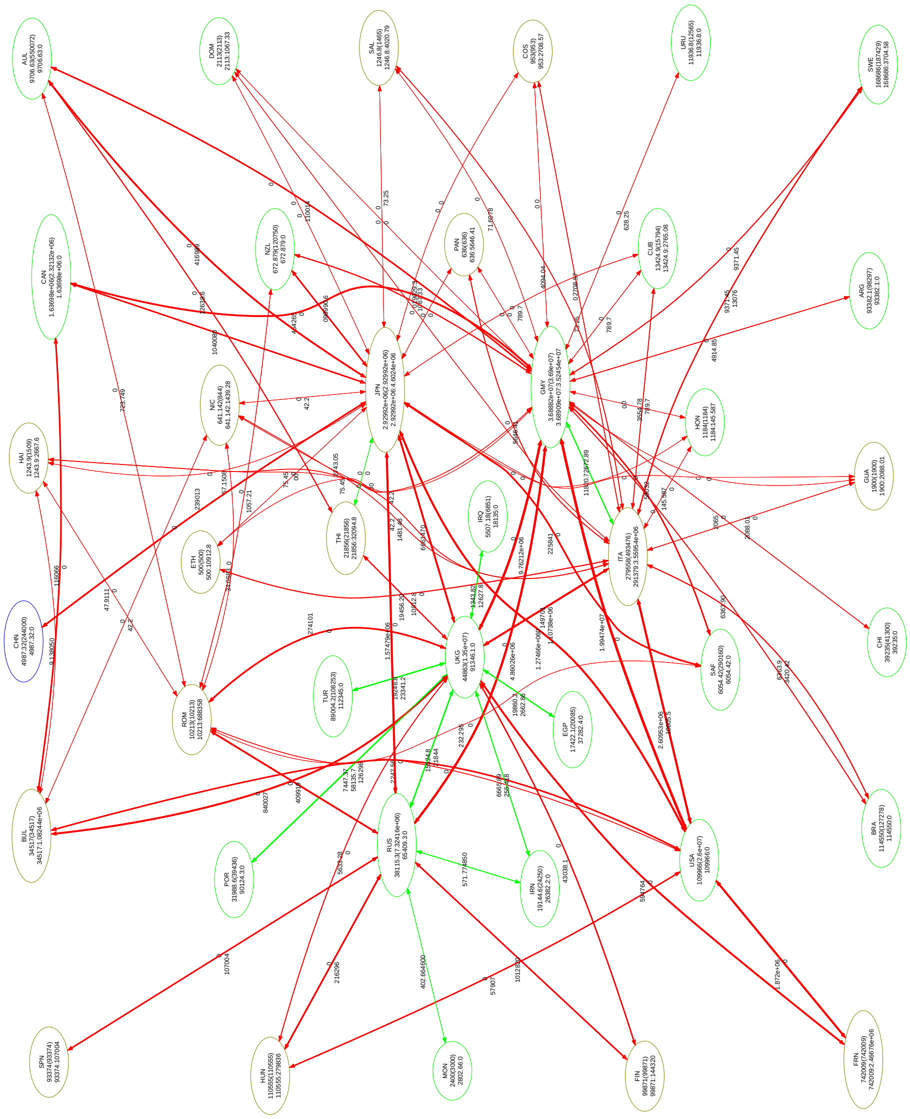
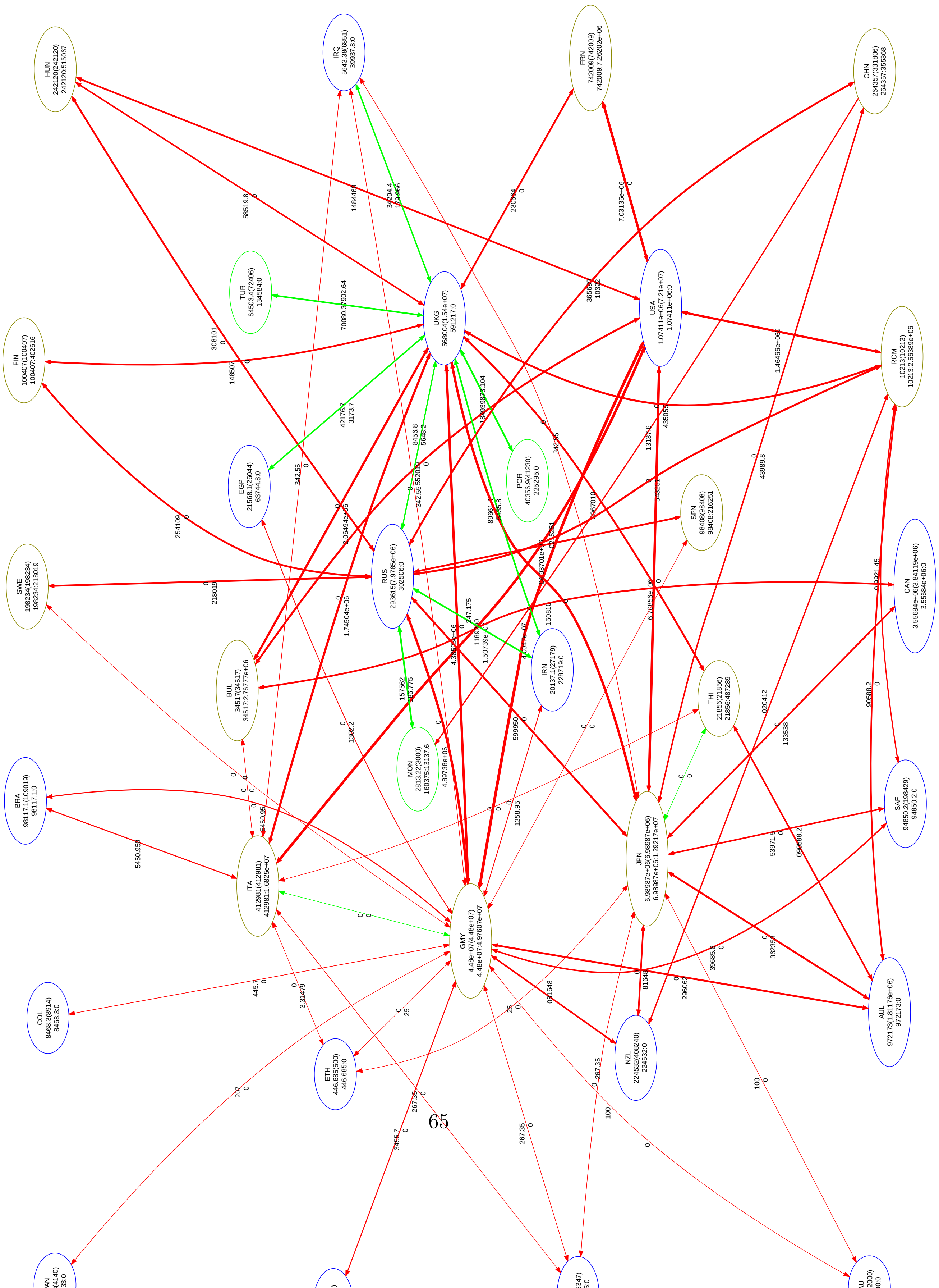


Figure 34: **1942 Europe**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IRQ
5643.38(6851)
39937.8:0
FRN
742009(742009)
742009:7.26202e+06
CHN
264357(331806)
264357:355368
TUR
64503.4(72406)
134584:0
UKG
568004(1.54e+07)
591217:0
USA
1.07411e+06(7.21e+07)
1.07411e+06:0
FIN
100407(100407)
100407:402616
ROM
10213(10213)
10213:2.56389e+06
EGP
21568.1(26044)
63744.8:0
POR
40356.9(41230)
225295:0
SPN
98408(98408)
98408:216251
SWE
198234(198234)
198234:218019
RUS
293615(7.9785e+06)
302506:0
CAN
3.55684e+06(3.84119e+06)
3.55684e+06:0
BUL
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IRN
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228719:0
THI
21856(21856)
21856:487289
BRA
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MON
2813.22(3000)
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SAF
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ITA
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JPN
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4.2.5 1944 - 1945

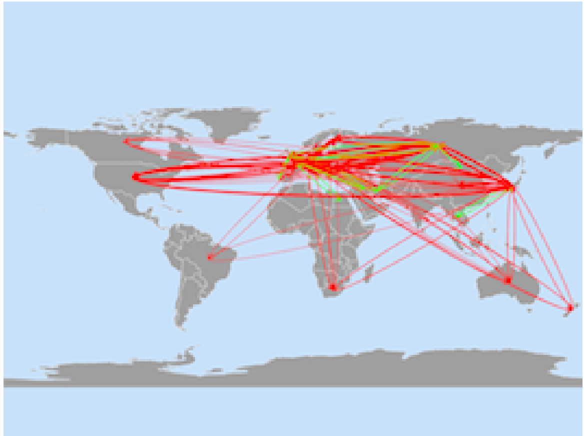


Figure 36: **1944 Europe**

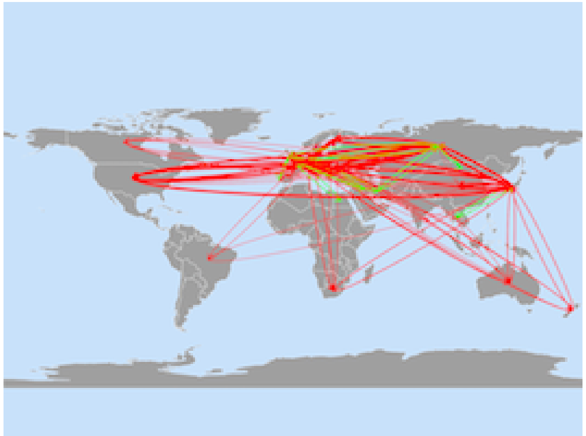


Figure 37: **1945 Europe**

1944 - 1945 was the end of the war. Japan began its last offensive in China, capturing further territory in the south. In Europe, the Normandy landings make Germany unable to counter-attack with the necessary speed and strength (Weinberg, 1994).

Table 13: 1944 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
China - Japan	Spain - Russia	United Kingdom - Egypt
United Kingdom - France	United Kingdom - Hungary	United Kingdom - Russia
United Kingdom - Japan	United Kingdom - Finland	United Kingdom - Turkey
United Kingdom - Bulgaria	United Kingdom - Thailand	United Kingdom - Iraq
United Kingdom - Romania	France - Hungary	United Kingdom - Portugal
Germany - Canada	France - Japan	United Kingdom - Iran
Germany - Russia	Canada - Japan	Russia - Czechoslovakia
Germany - Australia	Canada - Bulgaria	Russia - Mongolia
Germany - New Zealand	Thailand - Australia	Russia - Iran
Germany - South Africa	United States - Hungary	Russia - France
Germany - United Kingdom	United States - Bulgaria	Japan - Thailand
Germany - Italy	United States - Romania	Australia - New Zealand
Germany - Finland	Germany - Iraq	
Germany - Bulgaria	Germany - Turkey	
Hungary - Russia	Germany - Portugal	
United States - France	Germany - Nicaragua	
United States - Germany	Germany - Hungary	
United States - Japan	Germany - Brazil	
Russia - Hungary	Germany - Iran	
Russia - Romania	Germany - Colombia	
Russia - Finland	Germany - Liberia	
Russia - Japan	Germany - Romania	
Japan - South Africa	Germany - Sweden	
Japan - Australia	Germany - Bulgaria	
Japan - New Zealand	China - Russia	
Italy - Bulgaria	Romania - South Africa	
	Romania - Australia	
	Romania - New Zealand	
	Romania - Hungary	
	Russia - Bulgaria	
	Italy - Iraq	
	Italy - Bolivia	
	Bulgaria - Turkey	
	Japan - France	
	Japan - Brazil	
	Japan - Iraq	
	Japan - Liberia	
	Japan - Bolivia	

In the New Year the Soviet army liberated the Auschwitz. They also continued its battles from the east, while from the west the Allies raced with them to be the first to enter into Berlin. The Russians reached Berlin first and Germany surrendered unconditionally on 7

May (Weinberg, 1994). The war in Europe was over.

In the Pacific, however, conflicts had continued to rage throughout this time. Plans were being prepared for an Allied invasion of Japan, but fearful of fierce resistance and massive casualties, the atomic bombs were dropped in Hiroshima and Nagasaki. The Japanese surrendered on 14 August. (Weinberg, 1994) The biggest conflict in history had lasted almost six years and ended.

Table 14: 1945 Relation

Dyads in War	Dyads in Militarized Conflict	Dyads in Alliance
China - Japan	Spain - Russia	United Kingdom - Egypt
United Kingdom - France	United Kingdom - Hungary	United Kingdom - Russia
United Kingdom - Japan	United Kingdom - Thailand	United Kingdom - Turkey
Germany - Canada	France - Hungary	United Kingdom - Iraq
Germany - Russia	France - Japan	United Kingdom - Portugal
Germany - Australia	Canada - Japan	United Kingdom - Iran
Germany - New Zealand	Thailand - Australia	Russia - Czechoslovakia
Germany - South Africa	United States - Hungary	Russia - Mongolia
Germany - United Kingdom	Germany - Brazil	Russia - Iran
Germany - Italy	Germany - Iraq	Russia - France
Germany - Finland	Germany - Portugal	Russia - China
Germany - Bulgaria	Germany - Hungary	Russia - Poland
Germany - France	Germany - Peru	Russia - Yugoslavia
Hungary - Russia	Germany - Uruguay	Japan - Thailand
United States - Germany	Germany - Argentina	Australia - New Zealand
United States - Japan	Germany - Egypt	
Russia - Japan	Germany - Venezuela	
Japan - South Africa	Germany - Paraguay	
Japan - Australia	Germany - Romania	
Japan - New Zealand	Russia - Iran	
	Russia - Turkey	
	Russia - Japan	
	China - Russia	
	Italy - Iraq	
	Japan- Venezuela	
	Japan - Paraguay	
	Japan - Iran	
	Japan - Egypt	
	Japan - Chile	
	Japan - Argentina	
	Japan - Uruguay	
	Japan - Peru	
	Japan - Mongolia	
	Japan - Brazil	
	Japan - Iraq	
	Japan - Liberia	
	Japan - Bolivia	

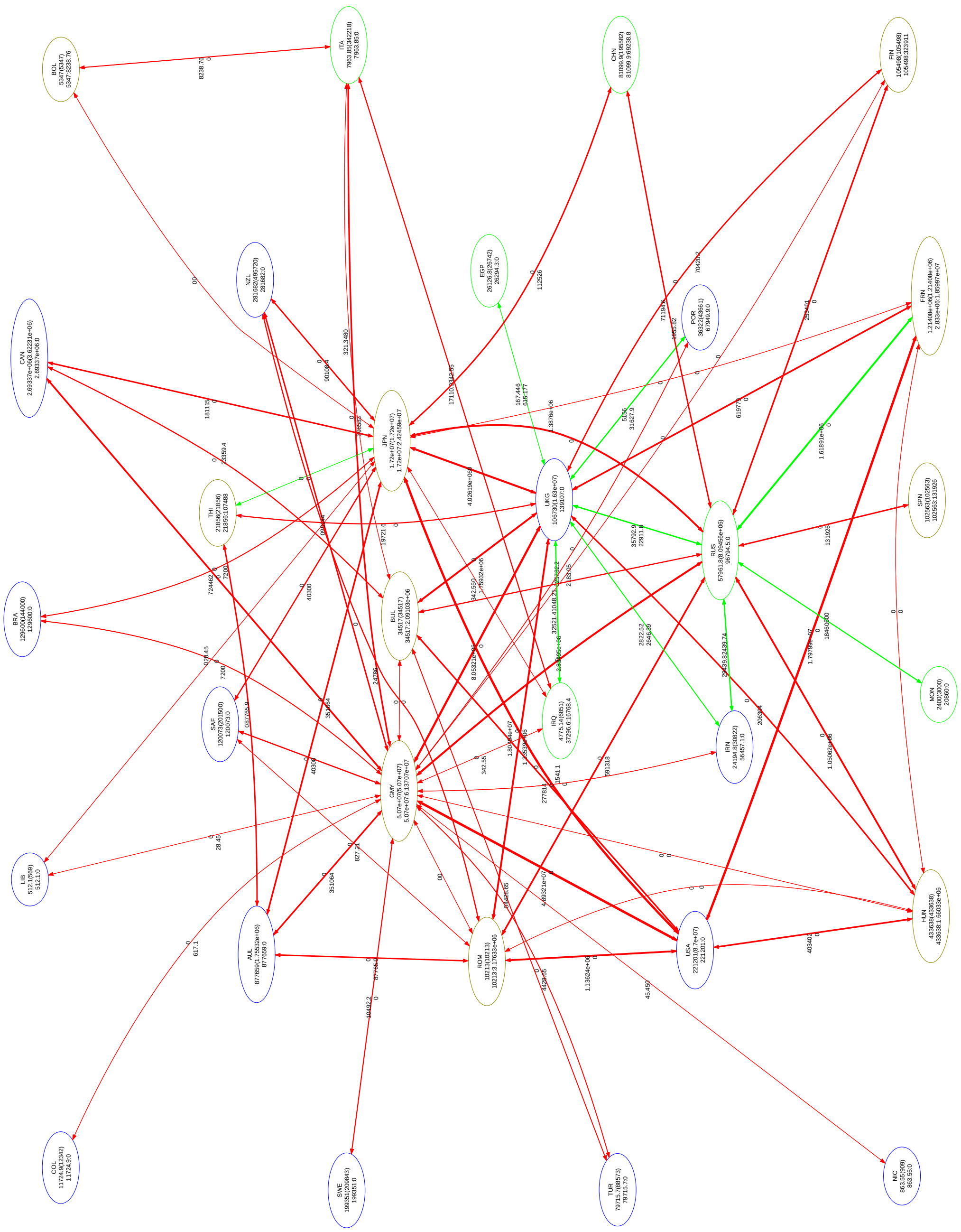
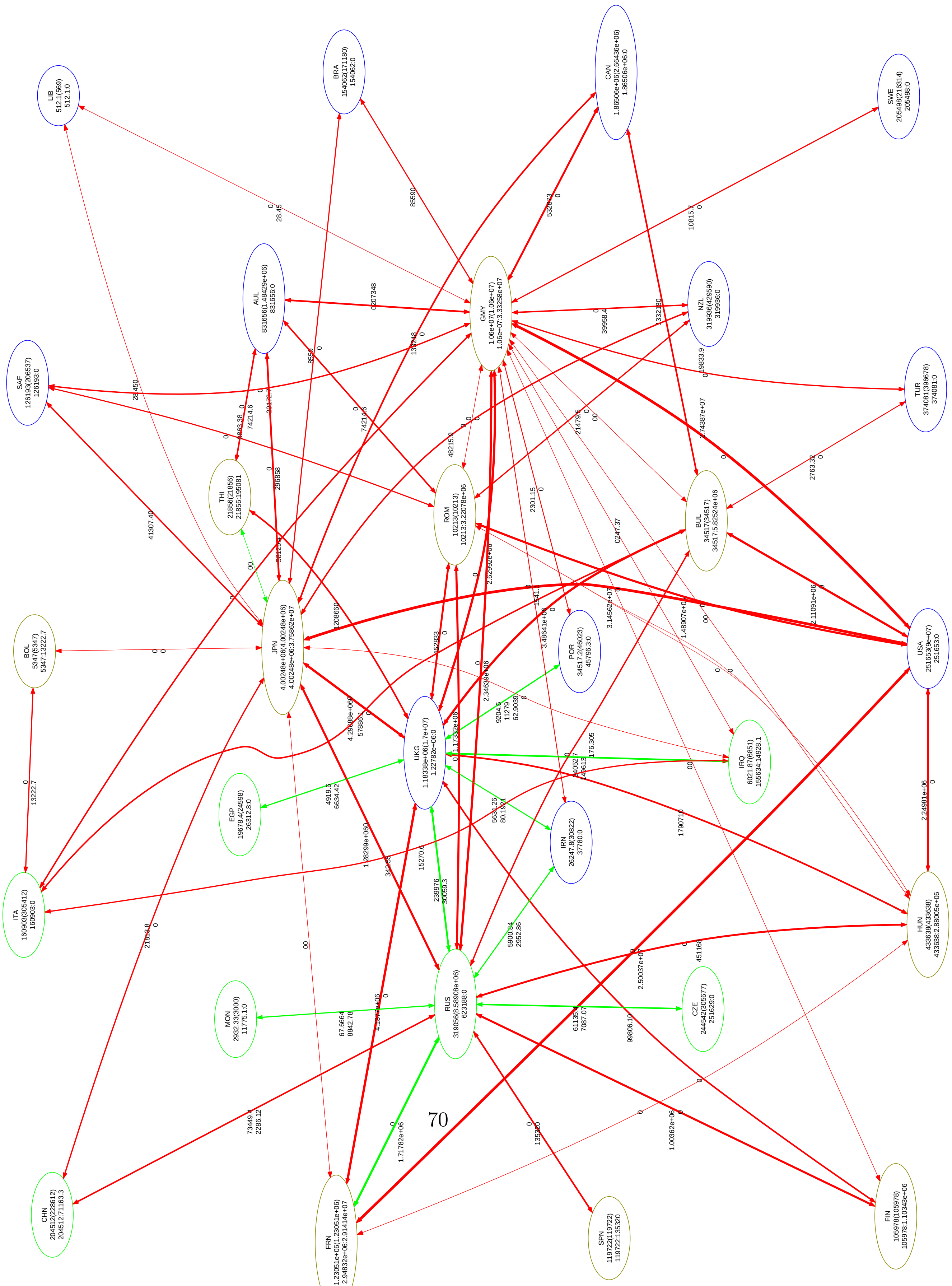


Figure 38: **1944 Europe**



4.2.6 Simulation Results

Table 8 shows the probability of being in different states for selected countries in World War II. Italy and France were not included because the former withdrew from the war in 1943 and the latter was conquered and turned into Vichy France from 1942 onwards.

Table 15: Probability of Being in Different States for Selected Countries from 1939-1945

Country-Year	Pr(Vulnerable)	Pr(Defensive)	Pr(Powerful)
Germany-1939	0.08%	99.92%	0%
Germany-1940	0%	100%	0%
Germany-1941	0%	2.52%	97.48%
Germany-1942	72.15%	27.85%	0%
Germany-1943	99.94%	0.06%	0%
Germany-1944	100%	0%	0%
Germany-1945	100%	0%	0%
Japan-1939	0%	100%	0%
Japan-1940	0%	0.07%	99.93%
Japan-1941	0%	2.52%	97.48%
Japan-1942	100%	0%	0%
Japan-1943	100%	100%	0%
Japan-1944	100%	0%	0%
Japan-1945	100%	0%	0%
UK-1939	1.17%	98.83%	0%
UK-1940	99.96%	0.04%	0%
UK-1941	99.68%	0.32%	0%
UK-1942	0.04%	29.38%	70.58%
UK-1943	0%	0.08%	99.92%
UK-1944	0%	0%	100%
UK-1945	0%	0%	100%
Soviet Union-1939	0.05%	99.95%	0%
Soviet Union-1940	0%	100%	0%
Soviet Union-1941	100%	0%	0%
Soviet Union-1942	0.7%	27.15%	72.15%
Soviet Union-1943	0.01%	0.06%	99.93%
Soviet Union-1944	0.02%	99.49%	0.49%
Soviet Union-1945	0%	99.81%	0.09%

The simulation results indicate the year of 1942 as the watershed of the war, during which the greater involvement of the US profoundly reversed the fortunes of the Axis powers. They also show the processes within which the Axis powers turned into a powerful alliance from a defensive alliance, and eventually became vulnerable, and the the Allies lifted itself from vulnerability to a powerful alliance

5 Extending the Model

In all, the paper makes two main contributions: first, for the first time, it has provided basic and necessary theorizing of military alliances with graphic-theoretic concepts and network games, having laid a solid microfoundation for understanding the workings of military alliances. Its theoretical implications go beyond military alliances and can be extended to alliances in general; second, the theoretical predictions of the game rely heavily on empirical evidence from 1816 to 2012. The model can be applied to any interstate conflict involving military alliances. I discussed specifically the cases of World War I and World War II and have proven that the model is even robust to cases of such complexity. The simplicity of the model has enabled the introduction of the basic ideas and intuitions and yielded clear results.

The model presented here is a simple one, leaving room for further extensions, in particular for the sake of applications in studying more complex situations. Most obviously, a major extension would be to incorporate alliance formation and stability into the current model. It is straightforward and natural to model agents' establishment of new relations as well as deviations from current relations under the current framework. Some previous work on the evolution of social and economic networks(Jackson and Watts, 2002; Jackson and Zenou, 2012) can be partly combined with the current baseline model for this aim.

There are several directions in which formation and stability of military alliances could be studied, which I intend to follow up. One possibility would be to introduce a novel but intuitive relation formation and deviation protocol into the current model, which is based on countries' trade-off between the costs and benefits of doing so. Since my model uses directed graph, it would be necessary to assume that relation formation should be bilateral and relation deviation be unilateral in terms of `ally` and the reverse in terms of `foe`. Another possible direction would be to study different initial states of relation structures and their effects on the final structures in equilibrium as a result of countries' optimizing behavior. This might overcome another simplification of the model.

References

Amt, Allemagne (1871-1945). Auswärtiges. 1948. *Nazi-Soviet Relations, 1939-1941. Documents from the Archives of the German Foreign Office. Edited by Raymond James Sontag and James Stuart Beddie.* US Government Printing Office.

Bala, Venkatesh and Sanjeev Goyal. 2000. "A noncooperative model of network formation." *Econometrica* 68(5):1181–1229.

Bermudo, Sergio, Juan A Rodríguez-Velázquez, José M Sigarreta and Ismael G Yero. 2010. "On global offensive k-alliances in graphs." *Applied Mathematics Letters* 23(12):1454–1458.

Bloch, Francis and Matthew O Jackson. 2006. "Definitions of equilibrium in network formation games." *International Journal of Game Theory* 34(3):305–318.

Brigham, Robert C, Ronald D Dutton and Stephen T Hedetniemi. 2007. "Security in graphs." *Discrete applied mathematics* 155(13):1708–1714.

Cruttwell, CRMF. 1934. *A history of the great war, 1914-1918.* Clarendon Press Oxford.

Dutton, Ronald D. 2009. "On a graph's security number." *Discrete Mathematics* 309(13):4443–4447.

Foley, Robert T. 2006. "The Real Schlieffen Plan." *War in History* 13(1):91–115.

Fugate, Bryan I. 1984. *Operation Barbarossa: Strategy and Tactics on the Eastern Front, 1941.* Presidio Press.

Gibler, Douglas M and Meredith Reid Sarkees. 2004. "Measuring alliances: The correlates of war formal interstate alliance dataset, 1816–2000." *Journal of Peace Research* 41(2):211–222.

Herwig, Holger H. 2014. *The First World War: Germany and Austria-Hungary 1914-1918.* Bloomsbury Publishing.

Hiller, Timo. 2011. "Alliance formation and coercion in networks.".

Jackson, Matthew O and Alison Watts. 2002. "The evolution of social and economic networks." *Journal of Economic Theory* 106(2):265–295.

Jackson, Matthew O and Asher Wolinsky. 1996. "A strategic model of social and economic networks." *Journal of economic theory* 71(1):44–74.

Jackson, Matthew and Yves Zenou. 2012. "Games on networks.".

Keegan, John. 1999. "The First World War.".

Kennedy, Paul. 2010. *The rise and fall of the great powers.* Random House LLC.

Klemann, Hein AM and Sergei Kudryashov. 2012. *Occupied Economies: An Economic History of Nazi-occupied Europe, 1939-1945.* Vol. 5 Berg.

Knox, MacGregor. 1986. *Mussolini unleashed, 1939-1941: politics and strategy in fascist Italy's last war.* Cambridge University Press.

Konrad, Kai A and Dan Kovenock. 2009. "The alliance formation puzzle and capacity constraints." *Economics Letters* 103(2):84–86.

Kristiansen, Petter, Sandra M Hedetniemi and Stephen T Hedetniemi. 2004. "Alliances in graphs." *Journal of Combinatorial Mathematics and Combinatorial Computing* 48:157–178.

Link, Arthur Stanley and Woodrow Wilson. 1954. "Woodrow Wilson and the progressive era, 1910-1917.".

Maoz, Zeev. 2012. "Preferential Attachment, Homophily, and the Structure of International Networks, 1816–2003." *Conflict Management and Peace Science* 29(3):341–369.

Mearsheimer, John J. 2001. *The tragedy of great power politics.* WW Norton & Company.

Menache, Ishai and Asuman Ozdaglar. 2011. "Network Games: Theory, Models, and Dynamics." *Synthesis Lectures on Communication Networks* 4(1):1–159.

Morrow, James D. 1991. "Alliances and asymmetry: An alternative to the capability aggregation model of alliances." *American Journal of Political Science* pp. 904–933.

Papadimitriou, Christos. 2003. Games and Networks. In *Fundamentals of Computation Theory*. Springer pp. 157–157.

Pape, Robert A. 2005. "Soft balancing against the United States." *International Security* 30(1):7–45.

Paxton, Robert O. 2001. *Vichy France: old guard and new order 1940-1944*. Columbia University Press.

Prete, Roy A. 2009. "Joffre and the Origins of the Somme: A Study in Allied Military Planning." *The Journal of Military History* 73(2):417–448.

Rodríguez-Velazquez, Juan Alberto and Jose Maria Sigarreta. 2006. "Global offensive alliances in graphs." *Electronic Notes in Discrete Mathematics* 25:157–164.

Schweller, Randall L. 1994. "Bandwagoning for profit: Bringing the revisionist state back in." *International Security* 19(1):72–107.

Schweller, Randall L. 2010. *Unanswered threats: political constraints on the balance of power*. Princeton University Press.

Sigarreta, Jose Maria, Sergio Bermudo and Henning Fernau. 2009. "On the complement graph and defensive k-alliances." *Discrete Applied Mathematics* 157(8):1687–1695.

Small, Melvin and J David Singer. 1969. "Formal alliances, 1816-1965: An extension of the basic data." *Journal of Peace Research* 6(3):257–282.

Smith, Alastair. 1995. "Alliance formation and war." *International Studies Quarterly* pp. 405–425.

Snyder, Jack. 1990. "Chain gangs and passed bucks: predicting alliance patterns in." *International Organization* 44(2):137–168.

Strachan, Hew. 1998. "The battle of the Somme and British strategy." *The Journal of Strategic Studies* 21(1):79–95.

Strachan, Hew. 2001. *The First World War: Volume I: To Arms.* Oxford University Press.

Walt, Stephen M. 1987. *The origins of alliances.* Cornell University Press.

Waltz, Kenneth Neal. 1979. *Theory of international politics.* Vol. 5 McGraw-Hill New York.

Weinberg, Gerhard L. 1994. *A world at arms: a global history of World War II.* Cambridge University Press.

Weitsman, Patricia A. 2004. *Dangerous alliances: proponents of peace, weapons of war.* Stanford University Press.

A Appendix

List of Figures and Tables

Figures

Tables

Data Appendix

Sources

1. Country Total Capacity: National Material Capabilities Dataset (v4.0) in Correlates Of War Project
2. War and Militarized Conflict: Militarized Interstate Dispute Dataset in Correlates Of War Project
3. Alliance Membership: The Alliance Treaty Obligations and Provisions (ATOP) Dataset

Coverage

Table 16: Countries Included in Empirical Analysis

Country	Abbre.	Country	Abbre.
Afghanistan	AFG	Iran	IRN
Argentina	ARG	Iraq	IRA
Austria-Hungary	AUH	Ireland	IRE
Austria	AUS	Japan	JPN
Australia	AUL	Latvia	LAT
Albania	ALB	Liberia	LIB
Belgium	BEL	Lithuania	LIT
Bolivia	BOL	Luxemburg	LUX
Brazil	BRA	Mongolia	MON
Bulgaria	BUL	New Zealand	NEZ
Canada	CAN	Nicaragua	NIC
Chile	CHI	Norway	NOR
China	CHN	Panama	PAN
Colombia	COL	Paraguay	PAR
Costa Rica	COS	Peru	PER
Cuba	CUB	Poland	POL
Czechslovakia	CZE	Portugal	POR

Table 16: (continued)

Country	Abbre.	Country	Abbre.
Denmark	DEN	Russia or the Soviet Union	RUS
Dominica	DOM	Romania	ROM
Ecuador	ECU	Saudi Arabia	SAU
Egypt	EGY	South Africa	SAF
El Salvador	SAL	Spain	SPN
Estonia	EST	Sweden	SWE
Ethiopia	ETH	Switzerland	SWI
France	FRN	Thailand	THI
Finland	FIN	Turkey or the Ottoman Empire	TUR
Germany	GMY	The Netherlands	NTH
Greece	GRC	United Kingdom	UKG
Guatemala	GUA	United States	USA
Haiti	HAI	Uruguay	URU
Hungary	HUN	Venezuela	VEN
Honduras	HON	Yugoslavia	YUG
Italy	ITA		