

The Option-Implied Foster-Hart Riskiness

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February 4, 2015

Foster and Hart (2009) introduce an objective measure of the riskiness of an asset that implies a bound on how much of one's wealth is 'safe' to invest in the asset while (a.s.) guaranteeing no-bankruptcy in the long run. In this work, we translate the Foster-Hart bound from abstract repeated one-player games to applied finance using risk-neutral densities that are nonparametrically estimated from S&P 500 call and put option prices covering 2003 to 2013. The option-implied Foster-Hart bound is analyzed and assessed in light of well-known risk measures including value at risk, expected shortfall and risk-neutral volatility.

Keywords: risk measure, risk-neutral densities, value at risk, expected shortfall

JEL: D81, D84, G01, G32

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The price which a man whose available fund is n pounds may prudently pay for a share in a speculation... (Whitworth 1870, p.217)

1 Introduction

Foster and Hart (2009) introduce a concept that relates the riskiness of a given gamble to a critical wealth level above which it is ‘safe’ to enter that gamble. Entering gambles below these critical wealth levels is not safe in the sense that this risks bankruptcy in finite time. Conversely, the power of the Foster-Hart measure of riskiness is that safe gambles (a.s.) guarantee no-bankruptcy. An important feature of the Foster-Hart measure of riskiness as a wealth level is that it is a law-invariant; i.e. it depends only on the underlying distribution and not on the risk attitude of the investor. It is in this sense that Foster and Hart (2009) refer to it as ‘operational’.

When the gamble is scalable, then the Foster-Hart measure can be re-interpreted as a bound on the fraction/ ‘share’ of one’s wealth that is safe to invest. The (a.s.) no-bankruptcy guarantee carries over to this setting. The importance of the non-bankruptcy criterion in this sense is illustrated by the fact that Whitworth (1870) proves an analogous result for buying a ‘share’ of a ‘speculation’ as early as 1870 (pp. 216-219).¹ In this note, we focus on the ‘shares’ interpretation of Foster and Hart (2009)’s risk measure. In fact, where appropriate, we use Riedel and Hellmann (2014)’s generalization of Foster and Hart (2009) to continuous random variables.

To know what an investment in the spirit of the Foster-Hart bound means in practice seems of high practical importance as it promises answers to questions regarding how much of one’s wealth one can invest in the stock market without risking bankruptcy. Obviously, the answer is not straightforward, because the pig in the poke regarding such real-world investment decisions is the underlying probability distribution, which typically is unknown not only to the decision-maker but also to us as scientists. One may use the word ‘speculation’ instead of ‘gamble’ to stress this feature (as in the above citation by Whitworth 1870). In the rare real-world situations where the return distribution is indeed known to the decision-maker and to the scientist, such as with roulette in the casino, the Foster-Hart bound would often commend not to invest at all.

The aim of our work is to translate the Foster-Hart bound (FH bound) from abstract gambles to applied finance. In turn, this means we have to translate the underlying financial ‘speculation’ back into a ‘gamble’ with a well-defined distribution. The question we shall seek out to address is what investment behavior is considered safe in financial investment practice. The main challenge for this approach is to get an estimate of the underlying return distribution. We approach this by analysis of risk-neutral densities (RNDs) that are nonparametrically estimated from S&P 500 call and put options prices. We favor this approach over other alternatives such as historical return distributions as this information

¹See the above citation.

is in fact available to the decision-maker at the time of decision. The motivating assumption is that the investor considers how much of his (unleveraged) wealth to invest in one single index stock, in our case one that is tied to the S&P 500, based on information about the associated options market. We analyze and assess the option-implied FH bound in light of well-known risk measures including value at risk, expected shortfall and risk-neutral volatility.

Our work builds on risk-neutral density estimation techniques that we shall discuss in detail in subsequent sections where we introduce them. Our work is also related to that of Bali et al. (2011) whose generalized measure of riskiness nests those of Aumann and Serrano (2008) and Foster and Hart (2009). However, quite differently to theirs, ours is an attempt to return to the operationality feature of the FH bound in terms of its independence from risk aversion. We achieve this by making a somewhat ‘brutal’ (because direct) move from the physical probability measure (\mathbb{P}) to the option-implied risk-neutral measure (\mathbb{Q}). While this is inappropriate for most financial analyses where alternative approaches are preferred (e.g. Bliss and Panigirtzoglou 2004), the direct \mathbb{P} - \mathbb{Q} move is both theoretically and empirically validated in our setting as it results in a ‘bound for the FH bound’; i.e. more sophisticated option-implied risk-neutral density estimations that take into account agents’ risk-aversion would only lead to higher and thus riskier FH bounds. It is in this sense that our approach is closer in spirit to the original ‘satisficing’ approach of Foster and Hart (2009) (as opposed to an ‘optimizing’ approach as in, for example, Kelly 1956), and in that way also independent of risk aversion/ ‘operational’ (as opposed to, for example, the closely related measures by Aumann and Serrano 2008 or Bali et al. 2011).

The main contribution of this paper is the translation of the operational measure proposed by Foster and Hart (2009) to applied finance. In particular, we analyze FH bounds on real-world financial investments (in our case into the S&P 500 stock index) using as the underlying probability distribution, which is unknown to us and to the investor, an estimator based on options data. Moreover, we analyze and assess the option-implied FH bounds in light of three well-known risk measures; value at risk, expected shortfall and risk-neutral volatility. The two main findings are as follows. First, the FH bound correlates less strongly with the other three risk measures than they all correlate with each other. Supposing that the FH bound has informative content by itself, an investor therefore potentially gains additional information from looking also at the FH bound than from looking only at the known risk measures. Second, the informativeness of the FH bound is empirically validated as the FH bound is shown to be a significant predictor of large return draw-downs.

The remainder of this document is structured as follows. Next, we formally introduce and discuss the Foster-Hart measure of riskiness in section two, and turn to the estimation of risk-neutral densities in section three. Section four contains the analysis. Section five concludes.

2 The Foster-Hart measure of riskiness

2.1 No-bankruptcy in the “shares” setup

When applying the Foster-Hart measure of riskiness to finance, it will prove useful to work within the setup where the decision maker is allowed to take any proportion of the offered gamble (Foster and Hart, 2009). In our case the gamble g consists of buying some multiple of the risky asset at price S_0 , holding it over a period $T > 0$ and finally selling it at price S_T . Including dividends, we may define g as the absolute return $g := S_T + Y - S_0$, where Y is the monetary amount of dividends being paid over the period. This allows us to define the Foster & Hart bound (FH bound) $\alpha_{\text{FH}} \in (0, 1)$ for a gamble with positive expectation as the zero of the equation

$$\text{E} [\log (1 + \alpha_{\text{FH}} r)] = 0, \quad (1)$$

with $r := g/S_0 = (S_T + Y - S_0)/S_0$ being the relative return. Since in reality any risky asset exhibits a positive probability of default, α_{FH} is bounded from above by 1. Riedel and Hellmann (2014) show that there exist gambles for which equation (1) has no solution $\alpha_{\text{FH}} \in (0, 1)$, even if the expected return is positive. In this case we may consistently set the FH bound to one, $\alpha_{\text{FH}} = 1$.

The FH bound connects to the original definition of Foster and Hart’s operational measure of riskiness R simply as $\alpha_{\text{FH}} = S_0/R$ (Foster and Hart, 2009, p. 791). Varying between 0 and 1, one may interpret it as the fraction of wealth at which it becomes risky to invest in the asset. Formally this may be expressed via a no-bankruptcy criterion. Following Foster and Hart (2009), we define no-bankruptcy as a vanishing probability for ending up with zero wealth when confronted with a sequence of gambles

$$\text{P} \left[\lim_{t \rightarrow \infty} W_t = 0 \right] = 0. \quad (2)$$

Foster and Hart (2009) (theorem 2) show that no-bankruptcy is guaranteed if and only if the fraction of wealth invested in the risky asset is always smaller than the FH bound, that is,

$$\alpha = S_0/W < \alpha_{\text{FH}}. \quad (3)$$

In this case the wealth even diverges; $\lim_{t \rightarrow \infty} W_t \rightarrow \infty$ (a.s.).

2.2 Positive and maximal growth rate

The FH bound can be interpreted as the limit between the positive and negative geometric means of the gamble outcomes. A simple example may provide some intuition. Assume that a risky asset at price $S_0 = \$300$ will, with equal probability, increase to $S_T = \$420$ or decrease to $S_T = \$200$. Solving equation (1) reads as $(1 + \alpha \frac{2}{3})(1 - \alpha \frac{1}{3}) = 1$. The solution

$\alpha_{\text{FH}} = 0.5$ is exactly the quantity that balances the potential gain and loss to an expected geometric mean of 1. By contrast, investing a higher (lower) fraction of wealth will result in a negative (positive) expected geometric mean. Thus α_{FH} separates the regimes of expected negative and positive growth rates of wealth. For an infinite sequence of gambles only investments in the latter avoid bankruptcy.

A natural question is why the FH bound (equation 1) sets the expected growth rate to zero instead of maximizing it. Indeed, there is an extensive literature on a corresponding maximal growth rate criterion, often referred to as the ‘Kelly criterion’ (Kelly, 1956; Samuelson, 1979). Foster and Hart succinctly comment on this relation as follows Foster and Hart (2009):

“While the log function appears there too, our approach is different. We do not ask who will win and get more than everyone else [...], but rather who will not go bankrupt and will get good returns. It is like the difference between ‘optimizing’ and ‘satisficing’.”

In our eyes, and more importantly for our purposes, the main difference between the Kelly criterion and FH bound lies in their respective applications. While the first is an investment strategy explicitly stating how to allocate one’s portfolio, the latter is a risk measure indicating the set of mathematically problematic portfolio allocations in the sense of bankruptcy. For us, the goal is to identify risky investment decisions, which is why we prefer the latter interpretation.

2.3 A more conservative bound

While the FH bound (equation 1) is defined under the physical probability measure \mathbb{P} , we will evaluate it under the option-implied risk-neutral measure \mathbb{Q} . Although Cox et al. (1985) argue from a theoretical model that the risk-neutral density will converge to the physical probability density as the aggregate wealth of an economy rises, more recent econometric work questions this hypothesis (e.g. Brown and Jackwerth 2001). Since Bliss and Panigirtzoglou (2004) find remarkable consistency in the deviation of the two measures across markets, utility functions and time horizons, we shall address in this section what our direct move between these measures means for the validity of the option-implied FH bound in our analysis.

Intuitively, given a risk-averse representative investor, the FH bound will be lower under \mathbb{Q} than under \mathbb{P} . Hence, our thus derived FH bound as a ‘bound on the bound’ is justified. To see this, we follow Bliss and Panigirtzoglou (2004) to reconstruct the subjective density function p from the risk-neutral density q assuming, as an example, a power utility function;

$$p(S_T) = \frac{q(S_T)/U'(S_T)}{\int q(x)/U'(x)dx} = \frac{q(S_T)S_T^\gamma}{\int q(x)x^\gamma dx}, \quad (4)$$

where S_T is the price of the underlying at maturity and $U(S_T) = (S_T^{1-\gamma} - 1)/(1 - \gamma)$. For a positive relative risk aversion coefficient $\gamma > 0$ it is clear that this transformation shifts probability mass from lower towards higher prices.² Be $S_1 > S_0 > 0$, then

$$\frac{p(S_1)/p(S_0)}{q(S_1)/q(S_0)} = \left(\frac{S_1}{S_0}\right)^\gamma > 1. \quad (5)$$

Technically, p first-order stochastically dominates q and the FH bound increases as the gamble becomes more attractive (Foster and Hart, 2009). This means that the option-implied FH bound will be a more conservative risk measure and importantly the no-bankruptcy property persists given direct move between physical and option-implied measure. Throughout the literature one finds positive coefficients of relative risk aversion, albeit of various magnitude (e.g. Arrow 1971; Friend and Blume 1975; Hansen and Singleton 1982, 1984; Epstein and Zin 1991; Normandin and St-Amour 1998). In the spirit of Foster and Hart (2009) we restrain from making assumptions on the utility of a representative agent and pursue with option-implied quantities.

3 Estimating risk-neutral densities

3.1 Theory

The fundamental theorem of asset pricing, stating that in a complete market the current price of a derivative may be determined as the discounted expected value of the future payoff under the unique risk-neutral measure (e.g. Delbaen and Schachermayer, 1994), guides the way of inferring information from financial options. The price C_0 of a standard European call option with exercise price K on a stock with price S is thus given as

$$C_0(K) = e^{-r_f T} \mathbb{E}_0^{\mathbb{Q}} [\max(S_T - K, 0)] = e^{-r_f T} \int_K^\infty (S_T - K) f(S_T) dS_T, \quad (6)$$

where \mathbb{Q} and f are the risk-neutral measure and corresponding risk-neutral density, respectively. Since option prices as well as the risk-free rate r_f and time to maturity T are observable, we may hope to invert equation (6) for the risk-neutral density.³

Several methods for inverting have been proposed, of which Jackwerth (2004) provides an excellent review. Besides parametric approaches where one assumes a specific form for the risk-neutral density with parameters that minimize the pricing error, a ‘trick’ by Breeden and Litzenberger (1978) opens another route: if strikes are distributed continuously on the

²Note that the same argument applies to exponential utilities with $U(S_T) = -(e^{-\gamma S_T})/\gamma$, i.e. to the other type of utility function discussed by Bliss and Panigirtzoglou (2004).

³One may at least proxy the *true* risk-free rate with, say, yields on 13-week Treasury bills or the rate of interbank lending Libor.

positive real line, we can simply differentiate equation (6) with respect to K to obtain the risk-neutral distribution F and density f as

$$F(S_T) = e^{r_f T} \frac{\partial}{\partial K} C_0(K) + 1, \quad f(S_T) = e^{r_f T} \frac{\partial^2}{\partial K^2} C_0(K). \quad (7)$$

Again various methods exist to overcome the numerical problems associated with the fact that options are only traded at discrete and unevenly spaced strikes.

3.2 A nonparametric approach

For our purposes the relatively new approach by Figlewski (2010) is most suited. It combines a 4th-order polynomial interpolation of data points in implied volatility space with appended generalized extreme value (GEV) tails beyond the range of observed strikes. We shall briefly present this method here.

We start from bid and ask quotes for puts and calls with a given maturity and transform the mid-prices to implied-volatility space via the Black-Scholes equation (Black and Scholes, 1973). Note that we do not assume the Black-Scholes model to price options correctly, but only use the equation as a mathematical tool. The implied volatilities of puts and calls are blended together such that only the more liquid and thus informative out of the money and at the money data points are considered while ensuring a smooth transition from puts to calls. The resulting famous ‘volatility smirk’ is interpolated with a 4th-order polynomial weighted by open interest, thus, giving higher importance to data points which contain more market information.⁴ After a retransformation of the fit values to price space, we numerically evaluate the empirical part of the risk neutral distribution and density according to equation (7).

As the range of strikes is finite, we have to choose a functional form of the tails. Instead of the often-used log-normal function, Figlewski (2010) employs the family of generalized extreme value (GEV) distributions (Embrechts et al., 2005, p. 265). The Fisher-Tippett theorem (Embrechts et al., 2005, p. 266) supports this choice, stating that under weak regularity conditions and after rescaling, the maximum of any i.i.d. random variable sample converges in distribution to a GEV distribution. The GEV family contains many relevant distributions, in particular also those with heavy tails.⁵ A distribution of GEV type is

⁴Since our data set admits open interest weighting, we deviate in this point from the original approach by Figlewski, who weights such that fits outside the bid-ask spread are penalized.

⁵The conceptually correct choice of extreme value family is the generalized Pareto distribution (GPD), since the risk-neutral tails correspond rather to the peaks-over-threshold method than the block-maxima method (Embrechts et al., 2005, pp. 264-291). Mathematically this translates into applying the Pickands-Balkema-de Haan theorem instead of Fisher-Tippett. However, because of their asymptotic equivalence and quantitative similarity we may use either and refer to Embrechts et al. (2005, 1997); Birru and Figlewski (2012) for a more detailed discussion.

characterized by three parameters: location, scale and shape. We determine them by imposing the following three connection conditions for the left and right tail separately: the GEV density should match the empirical one at two specified quantile points and conserve the probability mass in the tail.

Joining the empirical part with the tails eventually gives the full option-implied risk neutral density. While there exist many approaches to estimate risk-neutral densities, we argue that Figlewski’s method, as a combination of a model-free empirical part and flexible extreme value tails, belongs to the most unbiased ones. Allowing for non-standard features such as bimodality and fat tails will be of advantage for analyzing the highly different regimes around the Global Financial Crisis of 2008. We refer the interested reader to a supplementary paper to ours (Leiss et al., 2014) in which we discuss in detail the properties of the RNDs during and around the Financial Crisis (see also Figlewski 2010; Birru and Figlewski 2012).

3.3 Data

In this study we employ end-of-day data for standard European call and put options on the Standard & Poor’s 500 stock market index. Our data is provided by Stricknet and covers the period from January 1st, 2003, to October 23rd, 2013. The data consists of bid and ask quotes as well as open interest across various maturities, but we focus only on the extremely liquid quarterly options with expiration in March, June, September and December, respectively. Daily values for index level, its dividend yield and the yield of the Three-Month Treasury bill as a proxy of the risk-free rate are taken from the Thomson Reuters Datastream.

We follow Figlewski (2010) in filtering the raw data, ignoring quotes with bids below \$0.50 and those that are more than \$20.00 in the money, as such bids come with high ambiguity due to large spreads. Moreover, we also discard data points with midprices violating static no-arbitrage conditions. Finally, to ensure well-behaved densities, we restrict our analysis to dates with time to expiration of at least two weeks.⁶

4 Empirical results

4.1 Relation to value at risk, expected shortfall and risk-neutral volatility

A pioneering work on option-implied risk measures is Aït-Sahalia and Lo (2000), who suggest that value at risk (VaR) evaluated under the risk-neutral measure may capture aspects

⁶(i) As the range of relevant strikes shrinks on the way towards maturity, the densities show a strong peaking. (ii) Figlewski (2010) also notes that another reason may be price effects from rollovers of hedge positions into later maturities around contract expirations.

of market risk that VaR under the physical measure does not. Ait-Sahalia and Lo (2000) argue that “risk management is a complex process that is unlikely to be driven by any single risk measure”, and conclude that the option-implied measure should rather be seen as a compliment than substitute. In a similar fashion, Bali et al. (2011) set out to assess the added value of their ‘generalized risk measure’, a measure that is related to ours (as discussed previously), against traditional ones such as VaR and expected shortfall (ES), and also against the risk-neutral measure of skewness QSKEW (Xing et al., 2010). (Fama and MacBeth, 1973) use a Fama-MacBeth regression (Fama and MacBeth, 1973) to show that their option-implied ‘generalized risk measure’ successfully explains the cross section of 1-, 3-, 6- and 12-month-ahead risk-adjusted stock returns, indicating that, controlling for the other measures, the option-implied measure adds value.⁷

So let us take a step back and ask what the FH bound may add when evaluating all measures under the risk-neutral measure?⁸ In this we depart crucially from previous work, who compared option-implied risk measures against physical measures. To make that comparison, we calculate VaR and ES for option-implied log-returns at the 5% level,

$$\text{VaR}_\alpha = \inf\{x \in \mathbb{R} : F_r(x) \leq \alpha\}, \quad \text{ES}_\alpha = \mathbb{E}^{\mathbb{Q}}[x \in \mathbb{R} : x \leq \text{VaR}_\alpha], \quad (8)$$

where F_r is the implied distribution of log-returns.⁹ The risk-neutral volatility is defined as the second moment of the (rescaled) risk-neutral density $f(S_T/S_0)$. Figure 1 displays and compares the resulting quantities. All measures exhibit signatures of the Global Financial Crisis of 2008 as well as the Greek and European sovereign debt crises in 2010 and late 2011, respectively. Yet, it appears from figure 1 that the FH bound’s behavior is distinctly different. A correlation table provides some first quantification of the relation between the various measures (see table 1). While the tail measures VaR and ES, as well as risk-neutral volatility, are respectively highly correlated amongst each other (with 98% and 87%), the FH bound only exhibits a linear correlation of 38% to 48% to the others. Indeed, it seems that the FH bound captures different information than VaR, ES or risk-neutral volatility.

4.2 Drops of ahead-returns

Bali et al. (2011) systematically assess the performance of their generalized risk measure vis-a-vis other measure through regression of ahead-returns on the various risk measures. We shall not follow this approach here. The reason is that we cannot expect a satisfying

⁷Note, however, that the asset allocation implications are limited: across all investment horizons the time-varying investment choice of an investor with a relative risk aversion of three over the whole sample period of 1996–2008 ranges only over a few percentage points (Bali et al., 2011).

⁸Indeed, Bali et al. (2011) compare the option-implied Bali measure to historical VaR and ES. Evaluating all risk measures under the same information set represents a somewhat more level playing field.

⁹One can easily go from annualized log-returns to prices as $r = \log(S_T/S_0)/T$. The risk-neutral density expressed in log-returns is $f_r(r) = TS_T f_S(S)$.

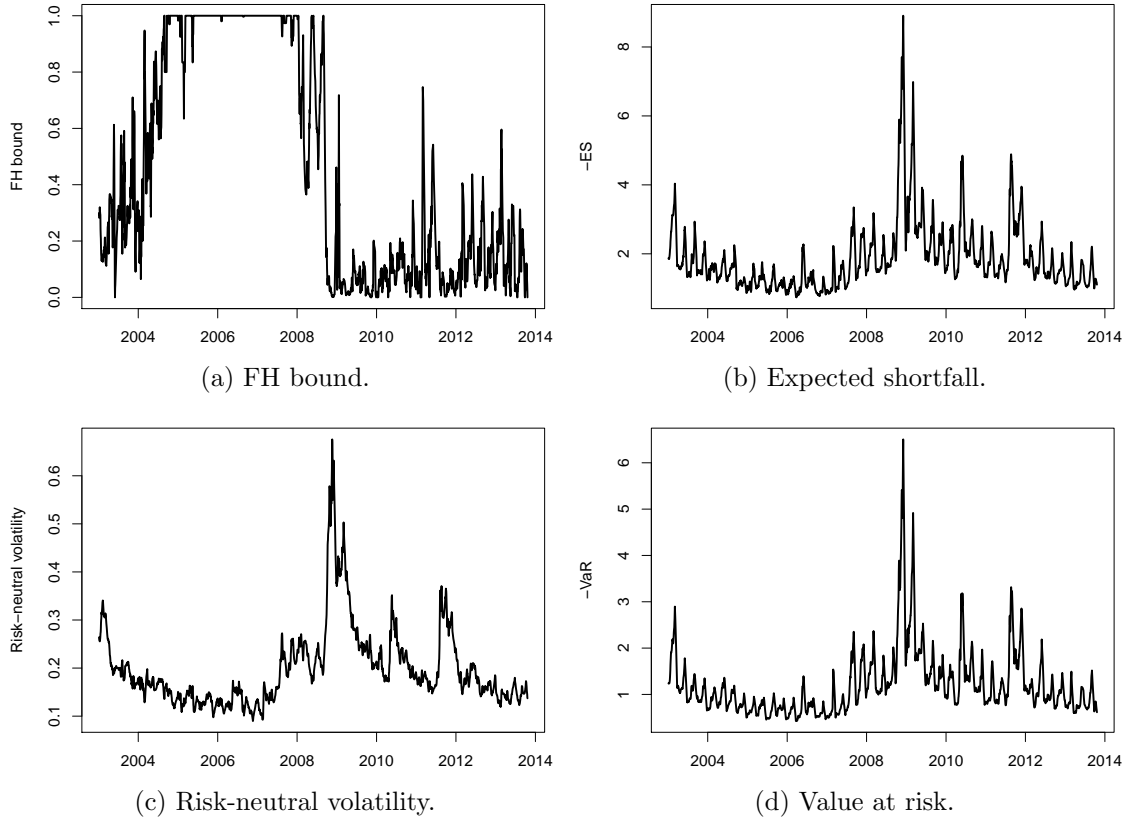


Figure 1: Various risk measures evaluated under the option-implied risk-neutral measure (weekly rolling mean). Although determined on the same information set, the FH bound (a) clearly shows a different behavior from expected shortfall (b), risk-neutral volatility (c) and value at risk (d). Expected shortfall and value at risk are evaluated on the log-return density and for readability flipped to positive values (i.e. expressed in losses).

	FH.bound	VaR	ES
FH.bound			
VaR	0.38***		
ES	0.42***	0.98***	
RN.Vol	-0.48***	-0.87***	-0.87***

Table 1: Linear correlation coefficients between FH bound, value at risk, expected shortfall and risk-neutral volatility, all evaluated under the risk-neutral measure.

performance of the FH bound, because the FH bound is defined as the quantity with an expected return of zero. Indeed, over the whole period of 2003 to 2013, a portfolio allocation implied by the FH bound results in a Sharpe ratio that does not significantly differ from zero, which is exactly expressive of this fact.

More intuition concerning the FH bound’s added value can be gained from inspection of its definition in equation (1). Due to the logarithm as a highly concave function, the FH bound may be particularly sensitive to left-tail risks, i.e. extreme losses. We test this hypothesis using a dummy variable Δr_t^ρ that is one whenever the S&P 500 ahead-return until maturity of the option, $r_{t \rightarrow T} := \log(S_T/S_0)$, significantly reduces by more than a value ρ as compared to the day before and zero otherwise. Value at risk VaR_t and expected shortfall ES_t as left-tail measures are the natural quantities to compare the FH bound FH_t with, yet to avoid a multicollinearity problem do to their high correlation we only control for one of them at once. That means we run the following regressions:

$$\Delta r_t^\rho = a_{0,t} + a_{1,t} \text{FH}_t + a_{2,t} \text{VaR}_t, \quad \Delta r_t^\rho = a_{0,t} + a_{1,t} \text{FH}_t + a_{2,t} \text{ES}_t. \quad (9)$$

Results for $\rho = -100\%$ and $\rho = -200\%$ are reported in Table 2 and are robust with respect to the choice of the threshold ρ over a wide range of numerical values. Note, that we are dealing with logarithmic returns where a $\rho = -100\%$ and $\rho = -200\%$, respectively, mean that the asset value at maturity S_T drops to $e^{-100\%} = 37\%$ and $e^{-200\%} = 14\%$ of its previous value S_0 . All estimated slope coefficients are negative, most of them significantly, indicating that a drop of the ahead-return may be explained by both FH bound, VaR and ES. When controlling for a left-tail measure the FH bound loses explanatory power, however remains significant in half of the cases. One reason for the better performance of traditional left-tail measures might be due to the no-bankruptcy property: figure 1a shows that even during the seemingly safe period of 2005 to 2008 the FH bound cannot exceed the upper limit of one, the signal becomes essentially flat and loses information relative to the unbounded VaR and ES. For drawdowns of intermediate size, however, the FH bound keeps explanatory power even when controlling for left-tail measures.

5 Conclusion

The main contribution of this paper has been the translation of the objective risk measure by Foster and Hart (2009) to applied finance. This was done by extracting the underlying risk-neutral densities from option prices and deriving the option-implied FH bound. Rather than optimal estimates, we chose an approach which could be described as deriving a conservative bound on these. This bound has been shown to have additional information compared to known risk measures. It had interesting macroscopic patterns in that it indicates a rather extreme regime shift in the dawn of the financial crisis. The bound also has been shown to have microscopic interest in that it is a significant predictor of large return draw-downs. It is a challenge for future work in similar spirit to consider investment strategies that are not restricted to one asset and / or allow leverage.

Regression of $\Delta r_t^{-100\%}$ on FH bound, VaR and ES					
(Intercept)	0.17*** (0.01)	0.04*** (0.01)	0.06*** (0.02)	0.03** (0.01)	0.05*** (0.02)
FH bound	-0.08*** (0.02)		-0.03* (0.02)		-0.03* (0.02)
VaR		-0.08*** (0.01)	-0.08*** (0.01)		
ES				-0.06*** (0.01)	-0.05*** (0.01)
Regression of $\Delta r_t^{-200\%}$ on FH bound, VaR and ES					
(Intercept)	0.06*** (0.01)	-0.04*** (0.01)	-0.03*** (0.01)	-0.04*** (0.01)	-0.03*** (0.01)
FH bound	-0.05*** (0.01)		-0.01 (0.01)		-0.01 (0.01)
VaR		-0.06*** (0.01)	-0.06*** (0.01)		
ES				-0.04*** (0.00)	-0.04*** (0.00)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 2: This table reports the intercept and slope coefficients of the regression of drops in logarithmic return between successive days of more than 100% (upper part) or 200% (lower part) on FH bound, value at risk and expected shortfall. Logarithmic returns of -100% and -200% mean that the asset value at maturity drops to $e^{-100\%} = 37\%$ and $e^{-200\%} = 14\%$ of its base value, respectively. Standard deviations are given in brackets, significance according to p -values is indicated by stars. All estimated slope coefficients are negative, most of them significantly.

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