Information Acquisition and Strategic Sequencing in Bilateral Trading: Is Ignorance Bliss?*

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Abstract

This paper examines the optimal sequencing of complementary deals based on their privately known values. It finds that an informed buyer sequences deals from low to high value – the opposite of the (anticipated) price offers; in response, the sellers adjust their offers. Together, the positive sequencing and negative pricing effects determine the value of information to the buyer: it is negative for moderate complements and positive for strong complements. That is, for moderate complements the buyer would optimally choose to sequence uninformed even with no information cost, while for strong complements she would seek unlikely deals. The optimal information acquisition is, therefore, inefficient: too little for moderate complements and too much for strong complements. It is shown that when its acquisition is unobservable, the buyer has an added incentive to be informed, which may improve social welfare. Related settings with "exploding" offers and substitutes are also examined and our main conclusions are demonstrated to hold.

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1 Introduction

Acquiring complementary goods and services often entails dealing with independent sellers. Examples include: a real estate developer buying adjacent parcels from different landowners; a lobbyist securing a bipartisan support; an employer recruiting a team of employees; a

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vaccine manufacturer obtaining various antigens from patent holders; and a hedge fund manager enticing key investors into a new fund. In many cases, the buyer needs to deal with the sellers one-by-one – perhaps convening multiple sellers is infeasible, or the sellers fear leaking proprietary business plans. Given complementarity between them, a careful sequencing of the sellers should be an important bargaining tool for the buyer.[1] Complicating the buyer's strategy, however, is her potential uncertainty about each deal's worth. In this paper, we explore optimal informed sequencing and its value to the buyer. Our main observation is that when sequencing the sellers, ignorance may be bliss for the buyer even though it may reduce trade.

Our base model comprises three risk-neutral players: two sellers with complementary goods and one buyer with unit demands. The buyer's valuation for the bundle is commonly known while her stand-alone valuations are uncertain.[2] The buyer can privately resolve this uncertainty at a cost prior to meeting with the sellers. In each bilateral meeting, the seller posts a public price,[3] and if previously uninformed (or ignorant), the buyer privately learns her valuation – perhaps, through free consultation. Having observed all the prices and valuations, the buyer makes her purchases. Our solution concept is perfect Bayesian equilibrium throughout.

Our investigation reveals that equilibrium prices trend downward: the leading seller prices aggressively to claim the extra surplus from complementarity while ensuring the follower's coordination at the joint valuation. This implies that the buyer will receive a payoff if and only if she has a high value for the second good and obtains that good alone. Thus, in our model the buyer seeks information to get the (value) sequence right. We show that an informed buyer indeed sequences goods from low to high value – the opposite of the (anticipated) price trend; in response, the second seller raises his price, diminishing the buyer's payoff. Together, the (positive) sequencing and the (negative) pricing effects determine the value of information for the buyer. For moderate complements, the value of information is *negative*; hence, even with *costless* information, the buyer would optimally commit to being

[1]For an interesting discussion and further applications of sequencing in bilateral trading, see Sebenius (1996) and Wheeler (2005).

[2]In particular, the buyer's stand-alone valuations are assumed to be more uncertain than her joint valuation. For instance, a developer may be less sure about market demand for a smaller subdivision built on a single parcel; a lobbyist may be more worried about passage of the legislation through only one-party endorsement; and a vaccine manufacturer may be more uncertain about the effectiveness of the vaccine that uses only a subset of the antigens.

[3]Below we show that the buyer has an incentive to make prices public.

uninformed so the sellers would not "read" into her sequence. She can achieve such commitment perhaps by: overloading herself with many (unrelated) tasks (Aghion and Tirole, 1997); delegating the sequencing to an uninformed third party; or letting the sellers sequence themselves.[4] For strong complements, the value of information is positive; hence, for a low enough cost, the buyer would optimally become informed even though stand-alone deals are unlikely.[5] Note that as being her only source of surplus, the buyer's optimal strategy is aimed at diluting the adverse pricing of the last seller. Given complementarity, it is therefore (socially) inefficient. Specifically, the buyer acquires information too little for moderate complements and too much for strong complements.

In many applications, the buyer may fail to follow her optimal strategy because her information acquisition is unobservable to the sellers.[6] Under such unobservability, the buyer seeks information beyond the optimal level: unable to control the adverse pricing effect, she overweighs the advantage of informed sequencing. This added incentive for being informed may, however, improve welfare for moderate complements.

For comparison and robustness, we also examine a setting in which the buyer receives "exploding" price offers that require a rapid purchasing decision. Such offers are prevalent in labor and real estate markets (Niederle and Roth, 2009; Armstrong and Zhou, 2010). With exploding offers, past payments are ignored by the sellers (a form of holdup) and thus equilibrium prices trend upward. This induces informed sequencing from high to low value – again the opposite of the price trend. The value of information is also qualitatively similar to nonexploding offers: it is negative for moderate complements and positive for strong complements. Nevertheless, exploding offers differ from the nonexploding in one important respect: for strong complements, they create a *positive* pricing effect. To increase demand, the leading seller offers a discount to alleviate the buyer's holdup but does so more for an informed buyer. This implies that unlike in the base model, (1) the buyer who faces exploding offers may actually prefer strategic sellers who read into her sequencing to those who do not; and (2) the unobservability may discourage information acquisition.

[4]For instance, an employer can assign the scheduling of job interviews to an (uninformed) administrative assistant or ask job candidates to pick an interview slot from available ones. In some applications, the buyer's sheer concern for "fairness" may also commit her to random (or uninformed) sequencing, as is the case for judicial recruitments (Greenstein and Sampson 2004, ch.7).

[5]The value of information is trivially zero for weak complements (as would be the case for unrelated goods) in our model and thus not the focus of our discussion.

[6]It is conceivable that an employer can secretly study job candidates' CVs before setting up their interviews or a lobbyist can privately research the long-term political significance of a democratic or republican endorsement.

Information acquisition and sequencing issues can also be relevant for substitutes; e.g., job candidates with comparable skills or land parcels at rival locations. We, however, show that with substitutes, the buyer's incentive to learn her valuations is limited since the sellers' primary concern is competition – not coordination,– which engenders equal price offers regardless of the buyer's information.

Related Literature. Our paper relates to a growing literature on optimal negotiation sequence. With two exceptions discussed below, this literature assumes commonly-known valuations, so informed sequencing or information acquisition is not an issue. Marx and Shaffer (2007) show that with contingent price contracts, the buyer strictly prefers to negotiate first with the weaker seller in order to extract rents from the stronger seller. Xiao (2010) finds the same ordering in a complementary-goods setting with noncontingent cash offers. Li (2010) studies an infinite-horizon bargaining model of complementary goods and establish that any ordering is sustainable in equilibrium.[7] A similar indeterminacy is proved by Moresi et al. (2010) in a fairly general model of bilateral negotiations.[8] Our paper is also related to Noe and Wang (2004) and Krasteva and Yildirim (2012a) who note that the buyer is (weakly) better off conducting negotiations confidential. In contrast, we note that with private information, the buyer is often better off making price offers and the sequence public.

Our paper is closest to Chatterjee and Kim (2005) and Krasteva and Yildirim (2012b). Chatterjee and Kim examine a bargaining model in which the buyer values one item twice as much as the other, but the exact valuations are her private information. These authors do not study the value of information, which is at the heart of our investigation. Krasteva and Yildirim explore a similar setting to this paper except that they rule out ex ante information acquisition. Here we consider complementary settings where information cost is not too high and sequencing is purely informational.[9] Nevertheless, we find that even with costless information, the buyer might choose to stay uninformed.

The strategic value of being uninformed has also been indicated in other contexts. For instance, Carrillo and Mariotti (2000) argue that a decision-maker with time-inconsistent preferences may choose to remain ignorant of the state to control future consumption. In

[7]See also Horn and Wolinsky (1988) and Cai (2000) who assume a fixed order of negotiations.

[8]In a labor union-multiple firms framework, Marshall and Merlo (2004) examine "pattern bargaining" where the buyer offers in the second negotiation the contract that is agreed upon in the first negotiation. In their case with non-pattern sequential negotiations, the buyer does not, however, care about the order. See also Banerji (2002).

[9]The sequencing in Krasteva and Yildirim (2012b) is driven by the *ex ante* heterogeneity in the sellers' bargaining powers.

a principal-agent framework, Riordan (1990), Cremer (1995), Dewatripont and Maskin (1995) and Taylor and Yildirim (2011), among others show that an uninformed principal may better motivate an agent while Kessler (1998) makes a similar point for the agent who may stay ignorant to obtain a more favorable contract. Perhaps, in this vein, papers closest in spirit to ours are those that incorporate signaling. Among them, Kaya (2010) examines a repeated contracting model without commitment and finds that the principal may delay information acquisition to avoid costly signaling through contracts. In a duopoly setting with role choice, Mailath (1993) and Daughety and Reinganum (1994) show that the choice of production period (as well as production level) may have signaling value and dampen incentives to acquire information. The issue of signaling in our setting is very different from these models, and the value of information critically depends on the prior belief in a non-monotonic way.

The rest of the paper is organized as follows. The next section sets up the base model, followed by the equilibrium characterization with exogenous information in Section 3. Section 4 endogenizes information. We extend the analysis to exploding offers in Section 5 and substitutes in Section 6. Section 7 concludes. The proofs of formal results are relegated to an appendix.

2 Base model

A risk-neutral buyer (b) wants to purchase two complementary goods from two risk-neutral sellers (s_i, $i = 1, 2$). At the outset, it is commonly known that the buyer attaches a normalized value of 1 to the bundle, while her stand-alone value for good i, v_i, is independently drawn from a nondegenerate Bernoulli distribution where $\Pr\{v_i = 0\} = q \in (0, 1)$ and $\Pr\{v_i = \frac{1}{2}\} = 1 - q$. We say that as q increases, goods become stronger complements for the buyer.[10] In particular, with probability q^2 goods are believed to be perfect complements.

The buyer meets with the sellers only once and in sequence: $s_1 \to s_2$ or $s_2 \to s_1$. Refer to Figure 1. Prior to the meetings, the buyer can privately learn both v_1 and v_2 by paying a fixed cost $c > 0$.[11] The decision to acquire information is also private to the buyer. We assume that the sellers are on the short side of the market and make the price offers.[12] In her

[10] Goods are assumed *ex ante* identical in order to isolate *ex post* heterogeneity borne by informed sequencing. In addition, our qualitative results would not change by a more general support $0 \le \underline{v} < \overline{v} \le \frac{1}{2}$ (see Appendix B).

[11] We rule out $c = 0$ in the analysis to avoid a trivial equilibrium multiplicity when the value of information is exactly zero, though some of our results will hold even for $c = 0$.

[12] For instance, there might be many realtors competing to acquire the adjacent land parcels; or there might be many employers trying to recruit among scarce talents. The assumption of price-setting sellers also isolates

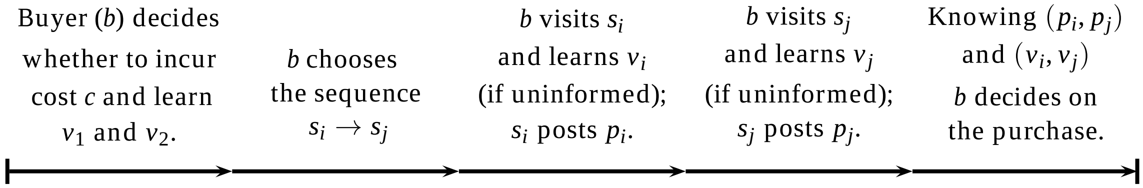

Figure 1: Timing and Information Structure

meeting with seller i, the buyer receives a price offer p_i, which becomes public. Moreover, if previously uninformed, the buyer privately learns her value v_i in this meeting – perhaps, through free consultation. Upon securing price offers, p_1 and p_2, and discovering all her valuations, v_1 and v_2, the buyer chooses which goods to purchase, if any. Our solution concept is perfect Bayesian equilibrium throughout. Note that under complements, a joint sale is (socially) efficient. We break indifferences in favor of efficiency unless they are pinned down in equilibrium.

Discussion of the model. The assumption of public price offers can be justified on two grounds. First, if the buyer uses public funds, then in many countries she will be subject to "sunshine" laws that, with few exemptions, require business meetings and transactions be available for public observation, participation and/or inspection (e.g., Berg et al. 2005). Second, we show in Section 4.3 that the buyer would be worse off keeping prices confidential. The anticipation of price disclosure can also explain why the sequence might be observable to the sellers; though, even without such disclosure, the sequence might be discerned by the calendar time or by the publicity surrounding the buyer-seller meetings. The assumption that the buyer learns all the prices prior to a purchase is mild. This may be due to extended deadlines and return policies adopted as industry standards or enforced by consumer protection laws.[13] Nevertheless, in Section 5 we examine exploding price offers with very short deadlines and demonstrate the robustness of our main conclusions. Finally, we restrict attention to one-time bilateral interactions. This greatly simplifies the analysis and is reasonable if the buyer has a limited time to undertake the project or an employer is in urgent need of filling

sequencing as the only signaling device for the buyer. Nonetheless, we briefly discuss the case of a powerful buyer in the conclusion.

[13]For instance, the Federal Trade Commission's (FTC) cooling-off rule allows consumers three days to cancel a contract or return a good for a full refund if the transaction is more than $25 and made outside the vendor's permanent workplace, e.g., at the buyer's home. (http://www.consumer.ftc.gov/articles/0176-protections-home-purchases-cooling-rule).

vacancies.

3 Equilibrium with exogenous information

We begin our analysis by assuming that with an exogenous and commonly known probability ϕ, the buyer is privately informed of her valuations prior to visiting the sellers while with probability $1-\phi$, she approaches them uninformed. Without loss of generality, we re-label the sellers according to the sequence $s_1 \to s_2$. Let $p_k(\phi)$ and $q_k(\phi)$ denote the kth seller's price and posterior belief of having the low value, respectively. Our first result characterizes the equilibrium prices.

Lemma 1 ***(Prices)*** *Fix the posterior beliefs. Then, equilibrium prices are given by*

$$(p_1^*(\phi), p_2^*(\phi)) = \begin{cases} \left(\frac{1+q_2(\phi)}{2}, \frac{1-q_2(\phi)}{2}\right) & \text{if} \quad q_2(\phi) > \underline{q} \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if} \quad q_2(\phi) \leq \underline{q}, \end{cases}$$

where $\underline{q} = \frac{\sqrt{5}-1}{2} \approx .62$.

Lemma 1 says that in equilibrium, the sellers coordinate their prices to the buyer's joint value, 1. Keeping the follower's incentive to accommodate, the leading seller targets the buyer's extra surplus from complementarity and posts a price (weakly) greater than her stand-alone value, thereby relying solely on a joint sale. The leading seller raises his price, and in response the follower lowers his, to the extent of the belief that the second good alone is unlikely to be valuable to the buyer. Note that only under such unequal prices can the buyer obtain a positive payoff – by having a high stand-alone value from the second good and purchasing *only* that.[14] This observation is key to understanding the buyer's behavior. In particular, it suggests that (1) an informed buyer will sequence from low to high value – the opposite of the (expected) price trend; in turn, (2) the buyer will be motivated to seek information; but (3) because an informed buyer is likely to purchase only one good – not the bundle of complements, – information acquisition is unlikely to be efficient. To formalize these intuitions succinctly, we observe that information acquisition is trivially precluded *if* equilibrium prices do not respond to the buyer's information. Thus, in what follows we focus on *responsive* equilibrium in which they do.

[14] It is worth pointing out that the buyer always purchases the second good – either alone or as part of the bundle.

Lemma 2 ***(Existence)*** *There exists a (responsive) equilibrium if and only if* $q > \underline{q}$.

That is, a responsive equilibrium exists for sufficiently strong complements. The reason is that in a responsive equilibrium, informed sequencing must (partially) reveal a high value for the low-price second good, decreasing the posterior, i.e., $q_2^*(1) \leq q$. Thus, if $q \leq \underline{q}$ so that goods are weak complements, Lemma 1 implies that equilibrium prices remain at their monopoly levels, $\left(\frac{1}{2}, \frac{1}{2}\right)$, for both informed and uninformed buyers.[15]

To characterize the (responsive) equilibrium, note that the posterior belief of having the low value can be written:

$$q_k(\phi) = \phi q_k(1) + (1-\phi)q, \tag{1}$$

where $q_k(1)$ denotes the posterior conditional on an informed buyer. This posterior clearly depends on sequencing. Let $\theta_k(v_i, v_{-i})$ be the probability that an informed buyer visits the seller with valuation v_i the kth if the other seller yields v_{-i}. In particular, with heterogenous valuations, $\theta_1(0, \frac{1}{2})$ and $\theta_2(\frac{1}{2}, 0)$ refer to the probabilities of placing the low value seller first and the high value seller second, respectively. To ease the analysis, we require sequencing to be symmetric:[16]

$$\theta_k(v, v) = \frac{1}{2} \text{ and } \theta_1(0, \frac{1}{2}) = \theta_2(\frac{1}{2}, 0). \tag{2}$$

Eq. (2) implies that *ex ante* each seller is equally likely to be approached first or second. It also reduces the posterior to:

$$\begin{aligned} q_k(1) &= \frac{q^2\frac{1}{2} + q(1-q)\theta_k(0, \frac{1}{2})}{\frac{1}{2}} \\ &= q^2 + 2q(1-q)\theta_k(0, \frac{1}{2}). \end{aligned} \tag{3}$$

Eq. (3) is intuitive. The seller in kth place will have the low value if the buyer has low values on both goods, occurring with probability q^2. He may add to this probability depending on how an informed buyer sequences heterogenous values, realized with probability $2q(1-q)$. Proposition 1 pins down equilibrium sequencing.

[15] As with most signalling games, an unresponsive equilibrium always exists in ours too. The following is one: regardless of her information, an informed buyer picks a "favorite" seller to visit first and the sellers offer their uninformed prices, with an off-equilibrium belief that switching the sequence would mean a high-value favorite with certainty, i.e., $\tilde{q}_2 = 0$. Intuitively, the buyer is discouraged from placing the favorite second because, by the off-equilibrium belief, doing so would engender the price pair $(\frac{1}{2}, \frac{1}{2})$ and leave no surplus to her.

[16] We conjecture that an asymmetric sequencing equilibrium may exist but will not be payoff-relevant since posterior beliefs, and thus prices, are often uniquely determined.

Proposition 1 ***(Informed sequencing)*** *In equilibrium, an informed buyer is more likely to sequence heterogenous goods from low to high value. This sequencing becomes strict if goods are strong complements. Formally,* $\theta_2^*(\frac{1}{2}, 0) > \frac{1}{2}$ *if* $\underline{q} < q \leq \overline{q}$*; and* $\theta_2^*(\frac{1}{2}, 0) = 1$ *if* $q > \overline{q}$*, where* $\overline{q} = \sqrt{\frac{\sqrt{5}-1}{2}} \approx .79$.

Proposition 1 confirms our intuition that the buyer places the high value good second in order to take advantage of a low price by that seller. This is the *sequencing effect* of being informed. This effect is positive: if the sellers behaved nonstrategically and did not update beliefs based on the sequence, then the buyer would strictly benefit from being informed (see Eq.(7) below). The strategic sellers, however, do update beliefs and in turn are likely to adjust their prices as Corollary 1 shows.

Corollary 1 ***(Informed prices).*** *In equilibrium with an informed buyer,*

- *if* $q > \overline{q}$*, then* $q_2^* = q^2 > \underline{q}$ *and* $(p_1^*(1), p_2^*(1)) = (\frac{1+q^2}{2}, \frac{1-q^2}{2})$*;*
- *if* $\underline{q} < q \leq \overline{q}$*, then* $q_2^* \leq \underline{q}$ *and* $(p_1^*(1), p_2^*(1)) = (\frac{1}{2}, \frac{1}{2})$.

Since informed sequencing implies a higher demand for the second (-place) good, the second seller posts a higher price than he would for an uninformed buyer, namely $\frac{1-q}{2}$. This is the *pricing effect* of being informed. This effect is negative because a price increase by the second seller erodes the only source of surplus for the buyer. Together, the sequencing and pricing effects determine the value of information to the buyer, as we study next.

4 Information acquisition

Before characterizing information acquisition when it is unobservable, we establish two benchmarks, one in which information acquisition is observable so that the buyer can optimally commit to visiting the sellers informed or uninformed, and the other in which a social planner dictates such commitment.

4.1 Optimal vs. efficient information acquisition

Optimal information acquisition. As discussed above, the buyer obtains a positive payoff if and only if she has a high value for the second good *and* purchases only that good. When the buyer is uninformed, $\phi = 0$, the sellers are visited at random and thus do not update priors, $q_k(0) = q$. From Lemma 1, the equilibrium pair of prices is $(\frac{1+q}{2}, \frac{1-q}{2})$ for $q > \underline{q}$, which implies that the buyer's expected payoff is the net surplus from a high-value second good, $\frac{q}{2}$, discounted by its probability, $1-q$; that is, the expected uninformed payoff is

$$B^U(q) = (1-q)\frac{q}{2} \text{ if } q > \underline{q}. \tag{4}$$

When the buyer is informed, $\phi = 1$, Corollary 1 reveals that for $q > \overline{q}$, the buyer's net surplus from a high-value second good is $\frac{q^2}{2}$, which she grabs by realizing at least one high value, occurring with probability $1-q^2$. The buyer's expected informed payoff is therefore

$$B^I(q) = \begin{cases} (1-q^2)\frac{q^2}{2} & \text{if} \quad q > \overline{q} \\ 0 & \text{if} \quad \underline{q} < q \leq \overline{q}. \end{cases} \tag{5}$$

The buyer's value of information is then $\Delta(q) \equiv B^I(q) - B^U(q)$ or

$$\Delta(q) = \begin{cases} \frac{q(1-q)(q^2+q-1)}{2} & \text{if} \quad q > \overline{q} \\ -\frac{q(1-q)}{2} & \text{if} \quad \underline{q} < q \leq \overline{q}. \end{cases} \tag{6}$$

Eq. (6) reveals that for moderate complements, $q \in (\underline{q}, \overline{q}]$, the buyer is strictly worse off being informed! This is because informed sequencing causes the second seller to set a significantly higher price, leaving no surplus to the buyer. Put differently, for moderate complements, the negative pricing effect of being informed dominates its positive sequencing effect. For strong complements, $q > \overline{q}$, it is the positive sequencing effect that dominates because the second seller's posterior of having a high value does not increase as much to justify a substantial price increase. The buyer weighs the value of being informed against its direct cost c. From (6), the following result is immediate.

Proposition 2 *If goods are strong complements, $q > \overline{q}$, and the information cost is low enough, $c < \Delta(q)$, then the buyer optimally acquires information, $\phi^o = 1$. If, on the other hand, goods are moderate complements, $\underline{q} < q \leq \overline{q}$, she optimally stays uninformed, $\phi^o = 0$.*

According to Proposition 2, the buyer prefers informed sequencing if and only if goods are strong complements and the information cost is low. Otherwise, the buyer prefers to visit the sellers uninformed even with no information cost. The buyer can credibly remain uninformed by: (1) significantly raising her information cost, perhaps through overloading herself with multiple tasks (Aghion and Tirole, 1997); (2) delegating her sequencing decision to an uninformed third party; or (3) letting the sellers sequence themselves (see Footnote 4).

It is worth noting that if the sellers were nonstrategic, the buyer's informed payoff would be $\overline{B}^I(q) = (1-q^2)\frac{q}{2}$ for $q > \underline{q}$. Subtracting $B^U(q)$ from $\overline{B}^I(q)$ would produce the nonstrategic value of information:

$$\overline{\Delta}(q) = \frac{(1-q)q^2}{2} \text{ if } q > \underline{q}. \tag{7}$$

Evidently, $\overline{\Delta}(q) > 0$, highlighting the negative effect of strategic pricing. In addition, $\overline{\Delta}(q) > \Delta(q)$, which means that the buyer would value information more if sellers were nonstrategic. Since it is aimed at purchasing a single unit of complements, the buyer's optimal strategy is unlikely to be (socially) efficient. We explore this comparison next.

Efficient information acquisition. Suppose that a social planner who maximizes the (expected) welfare can publicly instruct the buyer whether or not to acquire information. By definition, the welfare is the maximum total surplus realized from a joint purchase minus the lost total surplus from the purchase of the second good alone. For strong complements, $q > \overline{q}$, Eq.(4) implies that with probability $1-q$, an uninformed buyer obtains the second good and generates a total surplus of $\frac{1}{2}$, which means a lost surplus of $\frac{1}{2}$. Thus, for $q > \overline{q}$, the uninformed welfare is $W^U(q) = 1-(1-q)\frac{1}{2} = \frac{1+q}{2}$. By the same token, Eq.(5) implies that the welfare under an informed buyer is $W^I(q) = 1-(1-q^2)\frac{1}{2} = \frac{1+q^2}{2}$. Together, the social value of information is $\Delta_W(q) \equiv W^I(q) - W^U(q) = -\frac{q(1-q)}{2}$. Similarly, for $q \in (\underline{q}, \overline{q}]$, we find that $W^U(q) = \frac{1+q}{2}$ and $W^I(q) = 1$, giving rise to $\Delta_W(q) = \frac{1-q}{2}$. In sum, the social value of information is

$$\Delta_W(q) = \begin{cases} -\frac{q(1-q)}{2} & \text{if} \quad q > \overline{q} \\ \frac{1-q}{2} & \text{if} \quad \underline{q} < q \leq \overline{q}. \end{cases} \tag{8}$$

Efficient information acquisition directly follows from Eq.(8).

Proposition 3 *If goods are moderate complements, $\underline{q} < q \leq \overline{q}$, and the information cost is low enough, $c < \Delta_W(q)$, then it is efficient for the buyer to acquire information, $\phi^w = 1$. If, on the other hand, goods are strong complements, $q > \overline{q}$, then it is efficient for her to stay ignorant, $\phi^w = 0$.*

Comparing with Proposition 2, it is clear that the optimal information acquisition is inefficient. The reason is that maximizing welfare requires a joint sale which in turn requires that the negative pricing effect of being informed dominate the positive sequencing effect so that the buyer is discouraged from purchasing a single unit. As explained in Proposition 2, this dominance occurs for moderate complements, implying that $0 = \phi^o \leq \phi^w$ while the opposite is true for strong complements, implying that $\phi^o \geq \phi^w = 0$. That is, from social standpoint, the buyer's optimal information acquisition is too little for moderate complements and too much for strong complements.

Armed with these (commitment) benchmarks, we now turn to the base model in which information acquisition is unobservable to the sellers.

4.2 Equilibrium information acquisition

To fix ideas, consider the case of moderate complements for which the buyer would commit to sequencing sellers uninformed. But, if the sellers believed this, they would offer their uninformed prices, yielding a positive value of information, $\overline{\Delta}(q)$, to the buyer. In the case of strong complements, the commitment value of information, $\Delta(q)$, is positive; so the buyer is likely to acquire information when the acquisition is unobservable, too. Hence, letting ϕ^* be the buyer's equilibrium probability of being informed, we reach

Proposition 4 *When unobservable, the buyer acquires information more frequently than optimal. Formally, $\phi^o \leq \phi^*$, with strict inequality if $\underline{q} < q \leq \overline{q}$ and $c < \overline{\Delta}(q)$; or if $q > \overline{q}$ and $c \in \big(\Delta(q), \overline{\Delta}(q)\big)$.*

Proposition 4 follows because when information acquisition is unobservable, the buyer cannot soften the second seller's pricing by influencing his posterior; in turn, she relies on informed sequencing more than she would under commitment. In other words, the buyer attaches a higher value to information (up to $\overline{\Delta}(q)$) when its acquisition is private. To illustrate, we record ϕ^* for a small c here.[17]

$$\phi^* = \begin{cases} 1 & \text{if} \quad q > \overline{q} \\ \frac{q-\underline{q}}{q(1-q)} & \text{if} \quad \underline{q} < q \leq \overline{q}. \end{cases} \tag{9}$$

For moderate complements, even though no information acquisition is optimal, the buyer acquires some in equilibrium. As mentioned above, the buyer would want to sequence uninformed in this region, but this is not credible. The buyer would not sequence informed either because the (commitment) value of information is negative, establishing strict mixing in equilibrium. The strategic uncertainty about the buyer's level of information induces the leading

[17] Formally, $c \leq \frac{q}{\overline{q}}\overline{\Delta}(q)$. See Proposition A1 for a full characterization.

seller to also mix:

$$p_1^* = \begin{cases} \frac{1+q}{2} & \text{w. p.} \quad \frac{c}{\frac{q}{\underline{q}}\overline{\Delta}(q)} \\ \frac{1}{2} & \text{w. p.} \quad 1 - \frac{c}{\frac{q}{\underline{q}}\overline{\Delta}(q)}. \end{cases} \tag{10}$$

The low price of $\frac{1}{2}$ is aimed at guaranteeing a sale whereas the high price $\frac{1+q}{2}$ lies strictly between the informed and uninformed levels, as expected.[18] For strong complements, given the low cost, the buyer acquires information in equilibrium and engenders the informed prices in Corollary 1. Eq.(9) implies that as with the optimal benchmark, ϕ^* is increasing in q; that is, the buyer is more likely to be informed when goods are stronger complements and thus less likely to have individual values.

It is intuitive that by restricting her ability to commit, the unobservability of information acquisition cannot make the buyer better off; but by providing an added incentive to be informed, it may strictly raise the welfare. To see this, recall that for moderate complements, while inefficient, the buyer would optimally sequence uninformed, yielding welfare $W^U(q) = \frac{1+q}{2}$. For a negligible information cost, Eq.(10) implies that both sellers post equilibrium prices of $\frac{1}{2}$, inducing a joint purchase and a greater welfare, $W^*(q) \approx 1$. The unobservability may also lower welfare. Note that although uninformed sequencing is both efficient and optimal for strong complements and an intermediate cost, $c \in \left(\Delta(q), \overline{\Delta}(q)\right)$, informed sequencing may emerge in equilibrium.

4.3 Confidential prices

Up to now, price offers are assumed public. This is reasonable if, as with government procurements, certain "sunshine laws" oblige the buyer to open her business dealings to the public, or if the buyer voluntarily discloses such information. In many applications, though, the buyer keeps the details of business meetings confidential. For instance, private companies often adopt strict confidentiality policies for employee records containing salary and benefits information, but they may find it difficult to conceal the interview schedule of job candidates – because interview slots can be inferred from the calendar time or because interviews are highly visible. To examine the role of such "partial" confidentiality on information acquisition, we modify our base model by assuming that the sellers cannot observe each other's price while they continue to observe the sequence. This means that prices can be conditioned on the

[18] Again, the follower coordinates by setting $p_2^* = 1 - p_1^*$.

sequence only. As the next result indicates, this is likely to generate less unequal equilibrium prices.

Lemma 3 *Suppose the buyer is informed. For all q, there is an equal-price equilibrium, $(\frac{1}{2}, \frac{1}{2})$. This equilibrium is unique for $q < \overline{q}$. For $q \geq \overline{q}$, there are two more sets of coordination equilibria: I) $p_1^c \in (\frac{1}{2q^2}, \frac{1+q^2}{2}]$ and $p_2^c = 1 - p_1^c$; and II) $p_1^c = 1 - p_2^c$ and $p_2^c \in (\frac{1}{2q^2}, \frac{1+q^2}{2}]$.*

Lemma 3 is best understood in conjunction with Corollary 1. Unable to ensure coordination by the follower, the leading seller can no longer assume price leadership under confidential prices. This explains the equilibrium multiplicity. Compared to public prices, we see that the confidentiality softens the leader's pricing and intensifies the follower's. In particular, under confidentiality, it is possible that the leader posts the lower price, which induces the buyer to sequence from high to low value – as opposed to sequencing from low to high value. More importantly, by creating less unequal prices, the confidentiality limits the value of being informed. For instance, under the equal-price equilibrium (which now exists for all q), an informed buyer obtains zero payoff, leaving her no incentive to seek information. This may, however, improve welfare. The following result generalizes this observation for all equilibria found in Lemma 3.

Proposition 5 *Under confidential prices, (1) information acquisition is less likely, $\phi^c \leq \phi^*$; (2) the buyer is worse off, $B^c(q) \leq B^*(q)$; and (3) if $q > \overline{q}$ or $c \geq \frac{q}{q}\overline{\Delta}(q)$, the welfare is higher, $W^c(q) \geq W^*(q)$.*

Hence, the buyer has an incentive to make price offers public. This benefits her by engendering a lower second price through a better price coordination between the sellers. This finding further supports our assumption of public prices in the base model and implies that the buyer is hurt by a privacy policy even though it may be welfare-improving to adopt one. The latter is the case for strong complements: according to Proposition 3, no information acquisition is socially optimal and a privacy policy helps curb the buyer's incentive to be informed. For moderate complements, however, a privacy policy may diminish welfare since information acquisition is socially desirable in this case.[19]

Proposition 5 may seem puzzling in light of Noe and Wang (2004) and Krasteva and Yildirim (2012a). These authors establish that with commonly known payoffs, the buyer often

[19]The ambiguous welfare comparison is due also to equilibrium multiplicity. Otherwise, under equal-pricing equilibrium, Proposition 5 holds for all q.

favors confidentiality in order to create strategic uncertainty between the sellers. Our result says that with private information, the buyer need not rely on such endogenous uncertainty.

For comparison and robustness, we extend our analysis to "exploding" price offers and then to substitutes in the following two sections.

5 Exploding offers

In the base model, price offers are assumed not to expire until the buyer secures both. As discussed in the model setup, this may be due to extended deadlines and return policies adopted as industry standards or imposed by certain consumer protection laws such as the FTC cooling-off rule. In this section, we consider "exploding" price offers that carry short deadlines, requiring the buyer to make a quick purchasing decision without visiting the next seller. Exploding offers are ubiquitous in labor and real estate markets (Niederle and Roth, 2009; Armstrong and Zhou, 2010).

Under exploding offers, a key strategic concern for the buyer is being held up by the last seller. Hence, contrary to the base model, we expect that equilibrium prices with short deadlines will trend upward; in response, an informed buyer will sequence heterogenous goods from high to low value. Despite this sequencing difference from the base model, we conjecture that our predictions about information acquisition will remain largely unchanged. To formalize, consider the model in Section 2 and let $h \in \{0, 1\}$ indicate the buyer's purchase history of the first good. Assume that the sequence as well as the history are public.[20] Proposition 6 summarizes the equilibrium prices and the informed sequencing in this section.

Proposition 6 *Under exploding offers, there exists a (responsive) equilibrium if and only if $q > \frac{1}{2}$. Equilibrium prices are given by: $p_2^X(h=0) = \frac{1}{2}$, and*

(a) uninformed buyer:

$$p_1^X = \begin{cases} \frac{1-q}{2} & w.\,p. \quad \frac{1-q}{q} \\ \\ \frac{1}{2} & w.\,p. \quad \frac{2q-1}{q} \end{cases} \quad and\ p_2^X(h=1) = \begin{cases} \frac{1}{2} & w.\,p. \quad 1-q \\ \\ 1 & w.\,p. \quad q; \end{cases}$$

[20]The actual price offer by the first seller is assumed unobservable to the second seller.

(b) informed buyer: $p_1^X = p_2^X(h=1) = \frac{1}{2}$ *and* $\theta_2^X(0, \frac{1}{2}) > \frac{1}{2}$ *for* $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$; *and*

$$p_1^X = \begin{cases} \frac{1-q^2}{2} & w.\ p. \quad \frac{1-q^2}{q^2} \\ \frac{1}{2} & w.\ p. \quad \frac{2q^2-1}{q^2} \end{cases} \quad and\ p_2^X(h=1) = \begin{cases} \frac{1}{2} & w.\ p. \quad 1-q^2 \\ 1 & w.\ p. \quad q^2, \end{cases}$$

and $\theta_2^X(0, \frac{1}{2}) = 1$ *for* $q > \frac{1}{\sqrt{2}}$.

(c) Demand: *A buyer with* $v_1 = 0$ *accepts only the low* p_1^X *but all* $p_2^X(h=1)$ *whereas a buyer with* $v_1 = \frac{1}{2}$ *accepts all* p_1^X *but only the low* $p_2^X(h=1)$.

Consider an uninformed buyer. Upon observing the purchase of the first good, the second seller optimally charges the buyer's marginal value from the bundle, which is $\frac{1}{2}$ or 1. He must strictly mix between these two prices: otherwise, a sure price of 1 would strictly discourage a low valuation buyer from acquiring the first good and lead the second seller to reduce his price to $\frac{1}{2}$; on the other hand, a sure price of $\frac{1}{2}$ would guarantee the sale of the first good and encourage the second seller to raise his price to 1 given that in a (responsive) equilibrium, the prior strictly favors a low value buyer, $q > \frac{1}{2}$. Not surprisingly, seller 2's price stochastically increases with q. Note that a low value buyer demands the first good in the hope of paying less than the full price for the second. In particular, in equilibrium, such a buyer expects to pay $\frac{1+q}{2}$ for the second good and is therefore willing to pay $\frac{1-q}{2}$ for the first, which is exactly what seller 1 might offer. Seller 1 might, however, also offer a high price of $\frac{1}{2}$ to target a high value buyer. Seller 1's mixing between these two prices accommodates that of 2's. As q increases, seller 1 drops his discount price, $\frac{1-q}{2}$, to (partially) subsidize a low value buyer for a subsequent holdup, but interestingly he also drops the frequency, $\frac{1-q}{q}$, of this enticing offer so that his subsidy is not captured by seller 2.[21] The uninformed prices in part (a) also explain equilibrium demand in part (c): a low value buyer purchases the first good only at the discount price, upon which she proceeds to purchase the second with certainty, while the opposite is true for a high value buyer.

Further inspecting the uninformed prices, it is evident that the buyer enjoys a surplus if and only if she has a high value and a low price for the first good. Thus, much like in the base model with "nonexploding" offers, the buyer seeks information to get the (value) sequence

[21]Indeed, with probability $\frac{1-q}{q}q = (1-q)$, a low valuation buyer acquires both goods but ends up with a loss of $\frac{1-q}{2}$, illustrating the holdup problem. Such a holdup is absent in the base model since the buyer purchases fully informed of all prices and valuations.

right. The strategic difference is that under exploding offers, an informed buyer is more likely to sequence from high to low value, as indicated in part (b).[22] This again points to a positive sequencing effect; that is, if the sellers were nonstrategic, the buyer would strictly gain from being informed. To quantify this, note that the expected payoff of an uninformed buyer is

$$\begin{aligned} B^{X,U}(q) &= (1-q)\frac{1-q}{q}\left(\frac{1}{2}-\frac{1-q}{2}\right) \\ &= \frac{(1-q)^2}{2}. \end{aligned} \tag{11}$$

Analogously, facing nonstrategic sellers, the expected payoff of an informed buyer would be

$$\overline{B}^{X,I}(q) = \frac{(1-q^2)(1-q)}{2}. \tag{12}$$

As before, the gap between (12) and (11) measures the nonstrategic value of information:

$$\overline{\Delta}^{X}(q) = \frac{(1-q)^2 q}{2}. \tag{13}$$

Unlike in the base model, the effect of strategic pricing is ambiguous for exploding offers. Part (b) reveals that anticipating informed sequencing, seller 1 raises his price to $\frac{1}{2}$ for moderate complements, here $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$, which fully extracts the buyer's surplus. For strong complements, $q > \frac{1}{\sqrt{2}}$, informed sequencing is strict, inducing seller 1 to reduce his posterior of facing a low value buyer from q to q^2; in turn, seller 1 increases his discount price above the uninformed while simultaneously increasing the probability of offering it. Together, the expected informed payoff of the buyer when the sellers are strategic is given by:

$$B^{X,I}(q) = \begin{cases} \frac{(1-q^2)^2}{2} & \text{if} \quad q > \frac{1}{\sqrt{2}} \\ 0 & \text{if} \quad \frac{1}{2} < q \leq \frac{1}{\sqrt{2}}. \end{cases} \tag{14}$$

From Eqs.(12) and (14), it follows that strategic pricing makes an informed buyer worse off for moderate complements, signifying a negative pricing effect, and better off for strong complements, signifying a positive pricing effect. The latter contrasts with nonexploding offers under which strategic pricing always hurts the buyer. It implies that an informed buyer strictly prefers the sellers who read into the sequence to those who do not.[23] This observation also

[22]Recall that such sequencing is also possible under nonexploding, confidential offers but here it is the unique prediction.

[23]This can also be seen by noting that under exploding offers, seller 1's average informed price is $\frac{q^2}{2}$, which is less than his average uninformed price, $\frac{q}{2}$.

helps explain information acquisition under exploding offers. Subtracting (11) from (14), the value of information for the buyer is

$$\Delta^X(q) = \begin{cases} \frac{(1-q)^2 q(q+2)}{2} & \text{if} \quad q > \frac{1}{\sqrt{2}} \\ -\frac{(1-q)^2}{2} & \text{if} \quad \frac{1}{2} < q \leq \frac{1}{\sqrt{2}}. \end{cases} \tag{15}$$

Comparing Eqs.(6) and (15), we see that the buyer's value of information under exploding offers follows the same sign pattern as the nonexploding. Hence, the optimal information acquisition found in Proposition 2 should remain qualitatively intact. The optimal strategy is also unlikely to be efficient since it is tailored to maximize the buyer's surplus. When information acquisition is unobservable to the sellers, we predict that the buyer should be more informed than optimal if, similar to nonexploding offers, the pricing effect is negative, which is the case for moderate complements. For strong complements, however, we predict that the buyer will be less informed in equilibrium than optimal owing to the positive pricing effect. We confirm these predictions in,

Proposition 7 *Suppose price offers are exploding and* $q > \frac{1}{2}$.

(a) (**Optimal information acquisition**) *If goods are strong complements,* $q > \frac{1}{\sqrt{2}}$*, and the information cost is low enough,* $c < \Delta^X(q)$*, then the buyer optimally acquires information,* $\phi^{x,o} = 1$. *If, on the other hand, goods are moderate complements,* $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$*, she optimally stays uninformed,* $\phi^{x,o} = 0$.

(b) (**Efficient information acquisition**) *The social value of information is positive and exceeds its private value to buyer; i.e.,* $\Delta^X_W(q) > 0$ *and* $\Delta^X_W(q) > \Delta^X(q)$. *Hence, the optimal information acquisition is less than efficient.*

(c) (**Equilibrium information acquisition**) *If* $q > \frac{1}{\sqrt{2}}$*, then* $\phi^{x,*} \leq \phi^{x,o}$*, with strict inequality for* $c \in \big((1+q)\bar{\Delta}^X(q), \Delta^X(q)\big)$. *If, on the other hand,* $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$*, then* $\phi^{x,*} \geq \phi^{x,o}$*, with strict inequality for* $c < \bar{\Delta}^X(q)$.

Part (a) parallels Proposition 2. Parts (b) and (c) slightly differ from Propositions 3 and 4, respectively: for strong complements, since strategic pricing benefits the buyer, informed sequencing improves trade, leading to a greater social value of information than the private value. Indeed, it is readily verified from Proposition 6 that for $q > \frac{1}{\sqrt{2}}$, an informed buyer purchases the bundle with probability $(1-q^2)(2-q^2)$, which exceeds the corresponding probability, $(1-q)(2-q)$, for an uninformed buyer. Moreover, for strong complements, the buyer acquires information less frequently in equilibrium than optimal. Intuitively, contrary to nonexploding offers, the buyer would now want the sellers to believe that she is informed; but, given this belief, she would also want to save on the information cost when it is incurred privately.

It is edifying to further compare efficiency of exploding and nonexploding offers. Recall that under nonexploding offers, the buyer purchases the bundle whenever she has a low value for the second good. Thus, if the buyer is uninformed, the probability of an efficient sale is q. For responsive equilibria, i.e., $q > \underline{q}$, observe that $(1-q)(2-q) < q$; that is, an efficient trade is less likely under exploding offers if the buyer is uninformed. From Corollary 1 and Proposition 6, the same is also true if the buyer is informed. The intuition is that under exploding offers, a joint purchase means a greater exposure to holdup, which the buyer is trying to avoid, but the same concern is absent under nonexploding offers.

6 Substitutes

The issues of sequencing and information acquisition can also be pertinent to substitute goods, e.g., employees of similar skills and land parcels at rival locations. Our result, however, suggests that with substitutes, the buyer would have little incentive to acquire information since the sellers are likely to set competing – not coordinating – prices, resulting in equal offers. Formally, let the buyer's joint value be 1 and stand-alone values be independently distributed such that $\Pr\{v_i = 1\} = q_s \in (0,1)$ and $\Pr\{v_i = \frac{1}{2}\} = 1 - q_s$. Goods are said to become closer substitutes as q_s increases. Keeping with the base model, we assume public prices and ex post purchasing decisions. Proposition 8 is our finding in this section.

Proposition 8 *Consider substitute goods. For any q_s and ϕ, equilibrium prices must be equal, i.e., $p_1^*(\phi) = p_2^*(\phi)$. Hence, when information acquisition is unobservable, the buyer stays uninformed, $\phi^* = 0$.*

To understand Proposition 8, note first that much like weak complements, $q \leq \frac{1}{2}$, each seller sets his monopoly price of $\frac{1}{2}$ for weak substitutes, $q_s \leq \frac{1}{2}$, too. For close substitutes, $q_s > \frac{1}{2}$, each seller posts a price strictly above the unlikely realization of a low valuation, $\frac{1}{2}$, with the highest price pair $(1,1)$ always emerging as an equilibrium.[24] Given substitution, this implies that the buyer acquires at most one unit, creating competition between the sellers. If the buyer is uninformed, the sellers are ex ante identical, so it is not surprising that they will end up setting equal prices in equilibrium. In particular, the following seller matches the leader's price.[25] If the buyer is informed, the sellers may choose unequal prices depending on the sequence; but, as with the complements, the optimal sequencing would move in the opposite direction to the price trend and engender less unequal prices than those for an uninformed buyer, explaining equal equilibrium prices in general. Anticipating equal surplus from each seller, the buyer would then have no incentive to be informed.

Together, Propositions 4 and 8 imply that the buyer is less likely to pre-invest in finding out her demands for substitutes than for complements. As such, the buyer's meeting with each seller is likely to involve both free consultation and price solicitation for substitutes and only price solicitation for complements.

7 Conclusion

In this paper, we have explored information-based sequencing of complementary deals and identified a clear conflict between private and social values of being informed. Our analysis has produced three main results. First, an informed buyer sequences deals the opposite of the (anticipated) price offers. With nonexploding offers, there is a first-mover advantage for the sellers, implying declining equilibrium prices and an optimal sequence from low to high value deal. With exploding offers, there is a second-mover advantage due to a familiar holdup problem, implying rising prices and an optimal sequence from high to low value deal. Second, the buyer may be strictly worse off with informed sequencing because of strategic pricing. And third, the buyer's information acquisition is inefficient: too little for moderate complements and too much for strong complements. That is, the buyer is likely to invest in information when she is unlikely to discover any valuable deal.

[24] The fact that the equilibrium prices grow more collusive for closer substitutes is consistent with a standard Stackelberg duopoly. Specifically consider demand functions: $q_i = 1 - p_i + \alpha p_j$, where a greater $\alpha \in (0,1)$ corresponds to closer substitutes. It is readily verified that the Stackelberg prices are:

$$(p_1^S, p_2^S) = \left(\frac{2+\alpha}{2(2-\alpha^2)}, \frac{4+2\alpha-\alpha^2}{4(2-\alpha^2)}\right),$$

and each price is *increasing* in α.

[25] Underlying the exact price matching is the unique equilibrium belief that the buyer breaks ties in favor of the follower.

In closing, we note that our model is special in that the buyer cannot make or counter the sellers' offers in our model. We conjecture that sequencing, and thus being informed, would be less valuable for a buyer who can directly negotiate prices, with the extreme case being a price-setting buyer. Our model is also special in that the buyer learns all valuations by paying a fixed cost. If the marginal cost of information is, however, significant, the buyer may choose to learn only one valuation. We believe that such "partial" information will be more easily inferred by the sellers, making an all-or-nothing information acquisition optimal.

Appendix A

Proof of Lemma 1. Recall that without loss of generality, we have re-labeled the sellers by the sequence $s_1 \rightarrow s_2$. Fixing the posterior beliefs, $q_1(\phi)$ and $q_2(\phi)$, let $P_2(p_1)$ denote s_2's best response. Note that if $p_1 < \frac{1}{2}$, then $p_2 = 1 - p_1$ generates a sale for s_2 only if $v_1 = 0$, while $p_2 = \frac{1}{2}$ guarantees a sale for him. Comparing s_2's resulting payoffs, $(1 - p_1)q_1(\phi)$ and $\frac{1}{2}$, it follows that $P_2(p_1) = 1 - p_1$ if $p_1 < 1 - \frac{1}{2q_1(\phi)}$, and $P_2(p_1) = \frac{1}{2}$ if $1 - \frac{1}{2q_1(\phi)} \leq p_1 \leq \frac{1}{2}$ (where $p_1 = \frac{1}{2}$ is trivially included in this interval). If, on the other hand, $p_1 > \frac{1}{2}$, then since $v_1 \leq \frac{1}{2}$, good 1 is purchased only if the buyer acquires the bundle. Given this, the price $p_2 = 1 - p_1$ ensures a sale for s_2 whereas $p_2 = \frac{1}{2}$ is accepted only if $v_2 = \frac{1}{2}$, leading to the respective payoffs: $1 - p_1$ and $(1 - q_2(\phi))\frac{1}{2}$. From here, $P_2(p_1) = 1 - p_1$ if $\frac{1}{2} < p_1 \leq \frac{1+q_2(\phi)}{2}$, and $P_2(p_1) = \frac{1}{2}$ if $p_1 > \frac{1+q_2(\phi)}{2}$. To sum up,

$$P_2(p_1) = \begin{cases} 1 - p_1 & \text{if} \quad 0 \leq p_1 < 1 - \frac{1}{2q_1(\phi)} \\ \frac{1}{2} & \text{if} \quad 1 - \frac{1}{2q_1(\phi)} \leq p_1 \leq \frac{1}{2} \\ 1 - p_1 & \text{if} \quad \frac{1}{2} < p_1 \leq \frac{1+q_2(\phi)}{2} \\ \frac{1}{2} & \text{if} \quad p_1 > \frac{1+q_2(\phi)}{2}. \end{cases} \tag{A-1}$$

Turning to s_1, note that his optimal price must satisfy $p_1 \geq \frac{1}{2}$; otherwise, for $p_1 < \frac{1}{2}$, he could slightly increase p_1 without risking a sale. Furthermore, $p_1 = \frac{1+q_2(\phi)}{2}$ must be optimal in $(\frac{1}{2}, \frac{1+q_2(\phi)}{2}]$ since the response $P_2(p_1) = 1 - p_1 < \frac{1}{2}$ in this region implies a constant trade probability, $q_2(\phi)$, for s_1. No price in $(\frac{1+q_2(\phi)}{2}, 1]$ can be optimal for s_1, however, as it would induce no purchase of good 1. Thus s_1 must choose between the prices $\frac{1}{2}$ and $\frac{1+q_2(\phi)}{2}$, yielding the expected payoffs $\frac{1}{2}$ and $\frac{1+q_2(\phi)}{2}q_2(\phi)$, respectively. From here, equilibrium prices, $p_1^*(\phi)$ and $p_2^*(\phi) = P_2(p_1^*(\phi))$, are found to be

$$(p_1^*(\phi), p_2^*(\phi)) = \begin{cases} \left(\frac{1+q_2(\phi)}{2}, \frac{1-q_2(\phi)}{2}\right) & \text{if} \quad q_2(\phi) > \underline{q} \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if} \quad q_2(\phi) \leq \underline{q}, \end{cases} \tag{A-2}$$

where $\underline{q} \equiv \frac{\sqrt{5}-1}{2}$, as recorded in the text. ■

Proofs of Lemma 2, Proposition 1 and Corollary 1. As defined in the text, equilibrium is responsive (to the buyer's information) if and only if $p_k^*(1) \neq p_k^*(0)$ for some seller k. We consider three regions for q.

$q \le \underline{q}$ (weak complements): No responsive equilibrium exists in this region, i.e., $p_k^*(1) = p_k^*(0)$ for all k. To see this, note that since $q_k(0) = q$ for an uninformed buyer, $p_1^*(0) = p_2^*(0) = \frac{1}{2}$ from Lemma 1. Thus, equilibrium is responsive if and only if $q_2^*(1) > \underline{q}$, implying $p_2^*(1) = \frac{1-q_2^*(1)}{2}$. To determine $q_2^*(1)$, we next write an informed buyer's expected payoff conditional on realizing (v_i, v_{-i}):

$$B^I(v_i, v_{-i}) = \theta_2(v_i, v_{-i}) \max\{0, v_i - p_2^*(1)\} + \theta_2(v_{-i}, v_i) \max\{0, v_{-i} - p_2^*(1)\}, \qquad \text{(A-3)}$$

where we use the fact that the buyer can receive a positive surplus only by purchasing from the (lower price) second seller. By our symmetry assumption in (2), clearly $B^I(0, \frac{1}{2}) = B^I(\frac{1}{2}, 0)$. From (A-3), $B^I(0, \frac{1}{2}) = \theta_2(\frac{1}{2}, 0)\frac{q_2(1)}{2}$, which implies that setting $\theta_2^*(\frac{1}{2}, 0) = 1$ is optimal for the buyer, i.e., strictly sequencing from low to high value. This means $q_2^*(1) = q^2 \le \underline{q}$ by Eq.(3) – a contradiction.

$q > \overline{q} \equiv \sqrt{\frac{\sqrt{5}-1}{2}}$ (strong complements): We characterize the responsive equilibrium stated in Proposition 1 and Corollary 1. Observe that $q_2^*(1) \ge q^2 (> \underline{q})$ by Eq.(3). Then, by Lemma 1, $p_1^*(1) = \frac{1+q_2^*(1)}{2}$ and $p_2^*(1) = \frac{1-q_2^*(1)}{2}$. Next, observe that $B^I(0, \frac{1}{2}) = \theta_2(\frac{1}{2}, 0)(\frac{1}{2} - p_2^*(1))$, which implies the strict sequencing $\theta_2^*(\frac{1}{2}, 0) = 1$, confirming Proposition 1 for this region. Finally, $\theta_2^*(\frac{1}{2}, 0) = 1$ induces the posterior $q_2^*(1) = q^2$, and thus $p_1^*(1) = \frac{1+q^2}{2}$ and $p_2^*(1) = \frac{1-q^2}{2}$, confirming the prices in Corollary 1, too.

$\underline{q} < q \le \overline{q}$ (moderate complements): To characterize the responsive equilibrium, note that $q_2^*(1) \le \underline{q}$: otherwise, $q_2^*(1) > \underline{q}$ would engender prices $p_1^*(1) = \frac{1+q_2^*(1)}{2}$ and $p_2^*(1) = \frac{1-q_2^*(1)}{2}$ by Lemma 1, and imply $\theta_2^*(\frac{1}{2}, 0) = 1$ (as for strong complements), resulting in $q_2^*(1) = q^2 \le \underline{q}$ – a contradiction. Hence, equilibrium prices are $p_1^*(1) = p_2^*(1) = \frac{1}{2}$, as recorded in Corollary 1. Under these prices, the buyer receives zero payoff, making her indifferent to the order. However, to satisfy $q_2^*(1) \le \underline{q}$, Eq.(3) requires that $\theta_2^*(0, \frac{1}{2}) \le \frac{\underline{q}-q^2}{2q(1-q)}$, whose r.h.s. is strictly less than $\frac{1}{2}$ for $q < \overline{q}$. That is, $\theta_2^*(0, \frac{1}{2}) < \frac{1}{2}$ or equivalently $\theta_2^*(\frac{1}{2}, 0) > \frac{1}{2}$, proving Proposition 1. ■

Proofs of Proposition 2 and 3. Immediate from the text. ■

Before proving Proposition 4, we first fully characterize equilibrium under unobservable information acquisition.

Proposition A1 (Unobservable information acquisition).

- *For* $\underline{q} < q \le \overline{q}$,

– *information acquisition:*

$$\phi^* = \begin{cases} \frac{q-\underline{q}}{q(1-q)} & \textit{if} \quad \frac{c}{\overline{\Delta}(q)} \leq \frac{\underline{q}}{q} \\ \frac{1}{1-q} - \frac{2c}{q^2(1-q)^2} & \textit{if} \quad \frac{\underline{q}}{q} < \frac{c}{\overline{\Delta}(q)} < 1 \\ 0 & \textit{if} \quad \frac{c}{\overline{\Delta}(q)} \geq 1; \end{cases}$$

– *prices:* $p_2^*(\phi^*) = 1 - p_1^*(\phi^*)$ *and*

$$p_1^*(\phi^*) = \begin{cases} \begin{cases} \frac{1+\underline{q}}{2} & w.\ p. \quad \frac{c}{\frac{\underline{q}}{q}\overline{\Delta}(q)} \\ \frac{1}{2} & w.\ p. \quad 1 - \frac{c}{\frac{\underline{q}}{q}\overline{\Delta}(q)} \end{cases} & \textit{if} \quad \frac{c}{\overline{\Delta}(q)} \leq \frac{\underline{q}}{q} \\ \frac{1+\frac{2c}{q(1-q)}}{2} & \textit{if} \quad \frac{\underline{q}}{q} < \frac{c}{\overline{\Delta}(q)} < 1 \\ \frac{1+q}{2} & \textit{if} \quad \frac{c}{\overline{\Delta}(q)} \geq 1. \end{cases}$$

- *For* $q > \overline{q}$,

– *information acquisition:*

$$\phi^* = \begin{cases} 1 & \textit{if} \quad \frac{c}{\overline{\Delta}(q)} \leq q \\ \frac{1}{1-q} - \frac{2c}{q^2(1-q)^2} & \textit{if} \quad q < \frac{c}{\overline{\Delta}(q)} < 1 \\ 0 & \textit{if} \quad \frac{c}{\overline{\Delta}(q)} \geq 1; \end{cases}$$

– *prices:* $p_2^*(\phi^*) = 1 - p_1^*(\phi^*)$ *and*

$$p_1^*(\phi^*) = \begin{cases} \frac{1+q^2}{2} & \textit{if} \quad \frac{c}{\overline{\Delta}(q)} \leq q \\ \frac{1+\frac{2c}{q(1-q)}}{2} & \textit{if} \quad q < \frac{c}{\overline{\Delta}(q)} < 1 \\ \frac{1+q}{2} & \textit{if} \quad \frac{c}{\overline{\Delta}(q)} \geq 1. \end{cases}$$

Proof. Recall that the buyer receives a surplus if and only if she has a high value for the (low-price) second good and purchases only that. Therefore, under endogenous information acquisition, the buyer's ex-ante payoffs from being informed and uninformed for a fixed belief ϕ by the sellers are, respectively, $\hat{B}^I(\phi) = (1-q^2)(\frac{1}{2} - p_2^*(\phi))$ and $\hat{B}^U(\phi) = (1-q)(\frac{1}{2} - p_2^*(\phi))$. Taking the difference, the value of information for the buyer is:

$$\hat{\Delta}(\phi) = q(1-q)(\frac{1}{2} - p_2^*(\phi)). \tag{A-4}$$

Clearly, $\hat{\Delta}(\phi) > 0$ if and only if $p_2^*(\phi) < \frac{1}{2}$. Such equilibrium pricing requires that $q_2(\phi^*) \geq \underline{q}$; $p_2^*(\phi) = \frac{1-q_2(\phi)}{2}$, and the sequencing be strictly from low to high value, i.e., $\theta_2^*(0, \frac{1}{2}) = 1$, inducing the posterior $q_2^*(1) = q^2$ and

$$q_2(\phi) = \phi q^2 + (1-\phi)q. \tag{A-5}$$

We analyze two regions for q.

$\underline{\underline{q} < q \leq \overline{q}}$: From (A-4) and (A-5), $\phi^* = 0$ if $\hat{\Delta}(0) = \frac{(1-q)q^2}{2} \equiv \overline{\Delta}(q) \leq c$ where $\overline{\Delta}(q)$ is as defined in (7). For $c < \overline{\Delta}(q)$, we must have $\phi^* > 0$, implying $\hat{\Delta}(\phi^*) > 0$. Then $q_2(\phi^*) \geq \underline{q}$, which requires $\phi^* \leq \frac{q-\underline{q}}{q(1-q)} (< 1)$. Therefore $c < \overline{\Delta}(q)$ implies $\phi^* \in (0,1)$ and in turn $\hat{\Delta}(\phi^*) = c$. Suppose $\phi^* < \frac{q-\underline{q}}{q(1-q)}$. By (A-4), $\hat{\Delta}(\phi^*) = q(1-q)\frac{\phi^* q^2 + (1-\phi^*)q}{2}$, resulting in $\phi^* = \frac{1}{1-q} - \frac{2c}{q^2(1-q)^2}$. Clearly, $\phi^* \in (0, \frac{q-\underline{q}}{q(1-q)})$ if and only if $\frac{c}{\overline{\Delta}(q)} \in (\frac{\underline{q}}{q}, 1)$. In this interval, $q_2^*(\phi^*) = \frac{2c}{q(1-q)}$ and by Lemma 1, $p_1^*(\phi^*) = \frac{1+\frac{2c}{q(1-q)}}{2}$. For $\frac{c}{\overline{\Delta}(q)} \leq \frac{\underline{q}}{q}$, the only equilibrium candidate is $\phi^* = \frac{q-\underline{q}}{q(1-q)}$, which engenders $q_2(\phi^*) = \underline{q}$. The resulting value of information is $\hat{\Delta}(\phi^*) = q(1-q)\frac{\underline{q}}{2}$ or equivalently $\hat{\Delta}(\phi^*) = \frac{\underline{q}}{q}\overline{\Delta}(q)$, which evidently exceeds c since $\frac{c}{\overline{\Delta}(q)} \leq \frac{\underline{q}}{q}$. Thus, the equilibrium cannot support pure strategy pricing $p_1^* = (1+\underline{q})/2$. Instead, given $q_2(\phi^*) = \underline{q}$, the leading seller must mix between the prices $\frac{1}{2}$ and $(1+\underline{q})/2$. Let μ be s_1's probability of posting the price $\frac{1}{2}$. The value of information under this mixing is $\hat{\Delta}(\phi^*) = (1-\mu)\frac{\underline{q}}{q}\overline{\Delta}(q)$. Then $\hat{\Delta}(\phi^*) = c$ yields $\mu^* = 1 - \frac{c}{\frac{\underline{q}}{q}\overline{\Delta}(q)}$, which also supports the buyer's mixing $\phi^* = \frac{q-\underline{q}}{q(1-q)}$.

$\underline{q > \overline{q}}$: In this case, it must be that $q_2(\phi) > \overline{q}$ for any ϕ. As previously derived, $\phi^* = 0$ for $c \geq \overline{\Delta}(q)$ with the corresponding prices implied by Lemma 1. $\phi^* = 1$ is supported as an equilibrium for $\hat{\Delta}(1) = q(1-q)\frac{q^2}{2} = q\overline{\Delta}(q) \geq c$ with $q_2^*(1) = q^2$ and the corresponding prices implied by Lemma 1. For $q < \frac{c}{\overline{\Delta}(q)} < 1$, the previous case also reveals $\phi^* = \frac{1}{1-q} - \frac{2c}{q^2(1-q)^2}$ and $p_1^*(\phi^*) = \frac{1+\frac{2c}{q(1-q)}}{2}$. ■

Proof of Proposition 4. The proof follows immediately by comparing ϕ^o from Proposition 2 and ϕ^* from Proposition A1.■

Proof of Lemma 3. Under confidential prices, the sellers play a simultaneous-move pricing game and thus the equilibrium occurs at the intersection of their best responses. The best response of the follower is as recorded in (A-1). Switching the labels, the best response for the leading seller is analogous:

$$P_1(p_2) = \begin{cases} 1 - p_2 & \text{if} \quad 0 \le p_2 < 1 - \frac{1}{2q_2(\phi)} \\ \frac{1}{2} & \text{if} \quad 1 - \frac{1}{2q_2(\phi)} \le p_2 \le \frac{1}{2} \\ 1 - p_2 & \text{if} \quad \frac{1}{2} < p_2 \le \frac{1+q_1(\phi)}{2} \\ \frac{1}{2} & \text{if} \quad p_2 > \frac{1+q_1(\phi)}{2}. \end{cases} \tag{A-6}$$

Note that $p_1^c(\phi) = p_2^c(\phi) = \frac{1}{2}$ constitute an equilibrium for all posteriors $q_1(\phi)$ and $q_2(\phi)$. Moreover, the only prices that satisfy the equilibrium condition $P_1(P_2(p_1^c(\phi))) = p_1^c(\phi)$ are $p_1^c(\phi) \in [\frac{1-q_1(\phi)}{2}, 1 - \frac{1}{2q_1(\phi)}) \cup (\frac{1}{2q_2(\phi)}, \frac{1+q_2(\phi)}{2}]$ and $p_2^c(\phi) = 1 - p_1^c(\phi)$. The interval is non-empty if and only if $q_1(\phi) > \underline{q}$ or $q_2(\phi) > \underline{q}$.

In order to complete the equilibrium characterization under an informed buyer, i.e. $\phi^c = 1$, we need to determine $q_1(1)$ and $q_2(1)$ in equilibrium. Consider first $p_1^c(1) > \frac{1}{2}$ and $p_2^c(1) < \frac{1}{2}$. Then the buyer will strictly approach the high-value seller second, i.e., $\theta_2^c(0, \frac{1}{2}) = 0$. As a result, by (3), $q_2(1) = q^2$ and the equilibrium set is characterized by $p_1^c(1) \in (\frac{1}{2q^2}, \frac{1+q^2}{2}]$ and $p_2^c(1) = 1 - p_1^c(1)$. The price interval is non-empty if and only if $q > \overline{q}$. This establishes the type I equilibrium in Lemma 3. To establish the type II equilibrium, let $p_1^c(1) < \frac{1}{2}$ and $p_2^c(1) > \frac{1}{2}$. Then the buyer will approach the high-value seller first, i.e., $\theta_1^c(0, \frac{1}{2}) = 0$, which induces the posterior $q_1(1) = q^2$ and equilibrium prices $p_2^c(1) \in (\frac{1}{2q^2}, \frac{1+q^2}{2}]$ and $p_1^c(1) = 1 - p_2^c(1)$. Similar to type I, the price interval is non-empty if and only if $q > \overline{q}$. ■

Proof of Proposition 5. From the proof of Lemma 3, $p_1^c(\phi) = p_2^c(\phi) = \frac{1}{2}$ is an equilibrium for all beliefs $q_1(\phi)$ and $q_2(\phi)$. Since such equal-pricing leaves no surplus to the buyer independent of her information, an equilibrium with $\phi^c = 0$ always exists. Hence, $\phi^c \le \phi^*$ and $B^c(q) = 0 \le B^*(q)$ hold trivially. The welfare comparison also holds since, under equal-pricing, $W^c(q) = 1 \ge W^*(q)$.

Next we show that every equilibrium involving unequal pricing, namely $p_1^c(\phi^c) \neq p_2^c(\phi^c)$, also satisfies $\phi^c \le \phi^*$ and $B^c(q) \le B^*(q)$, and that $W^c(q) \ge W^*(q)$ if $q > \overline{q}$ or $c \ge \frac{q}{\underline{q}}\overline{\Delta}(q)$. First, observe that for $q \le \underline{q}$, there is no unequal pricing equilibrium: if there were, then from the proof of Lemma 3, it would require $p_k^c(\phi^c) < \frac{1}{2}$ and $q_k(\phi^c) > \underline{q}$ for some k, which would in turn induce an informed buyer to visit the high-value seller in the kth place and generate the posterior $q_k(\phi^c) = \phi^c q^2 + (1 - \phi^c)q$. Clearly this would imply $q_k(\phi^c) \le \underline{q}$ for all ϕ – a contradiction.

Next consider $q > \underline{q}$ and without loss of generality, the set of equilibria with $p_1^c(\phi^c) > p_2^c(\phi^c)$ where $p_2^c \in [\frac{1-q_2(\phi^c)}{2}, 1 - \frac{1}{2q_2(\phi^c)})$. Then, analogous to the case of public prices, the buyer would strictly sequence from low to high value, engendering the posterior $q_2(\phi^c) = \phi^c q^2 + (1-\phi^c)q$ and the value of information $\hat{\Delta}(\phi^c) = q(1-q)(\frac{1}{2} - p_2^c(\phi^c))$ (refer to the proof of Proposition A1). Suppose $q > \overline{q}$ or $c \geq \frac{q}{\underline{q}}\overline{\Delta}(q)$. From the proof of Proposition A1, this implies $\hat{\Delta}(\phi^*) = q(1-q)(\frac{1}{2} - p_2^*(\phi^*))$ where $p_2^*(\phi) = \frac{1-q_2(\phi^*)}{2}$. Since $p_2^c(\phi) \geq p_2^*(\phi)$, we have $\hat{\Delta}(\phi) \leq \hat{\Delta}(\phi)$, revealing $\phi^c \leq \phi^*$. Now, note that for a fixed ϕ, the buyer's expected equilibrium payoff and social welfare can be written as:

$$\begin{aligned} E[B(\phi)] &= (1-q_2(\phi))(\frac{1}{2} - p_2(\phi)) \\ &= (1-q)(1+\phi q)(\frac{1}{2} - p_2(\phi)) \end{aligned} \tag{A-7}$$

and

$$\begin{aligned} E[W(\phi)] &= \frac{1-q_2(\phi)}{2} + q_2(\phi) \\ &= (1-q)(1+\phi q)\frac{1}{2} + (1-(1-q)(1+\phi q)). \end{aligned} \tag{A-8}$$

Clearly, the fact that $p_2^c(\phi) \geq p_2^*(\phi)$ and $\phi^c \leq \phi^*$ implies $B^c(q) = E[B(\phi^c)] \leq B^*(q) = E[B(\phi^*)]$. Since $E[W(\phi)]$ is decreasing in ϕ, it also implies $E[W(\phi^c)] = W^c(q) \geq E[W(\phi^*)] = W^*(q)$, as claimed.

To complete the proof, now suppose $q \leq \overline{q}$ and $c < \frac{q}{\underline{q}}\overline{\Delta}(q)$ (again in the region $q > \underline{q}$). From Proposition A1, we know that $\phi^* = \frac{q-\underline{q}}{q(1-q)}$ in this case. Moreover, from the proof of Lemma 3, the unequal pricing requires $q_2(\phi^c) > \underline{q}$, which implies $\phi^c \leq \frac{q-\underline{q}}{q(1-q)} = \phi^*$. Moreover, since $\phi^c < 1$, we have $\hat{\Delta}(\phi^c) = q(1-q)(\frac{1}{2} - p_2^c(\phi^c)) \leq c$, revealing $p_2(\phi^c) \geq \frac{1}{2} - \frac{cq}{2\overline{\Delta}(q)}$. As a result, by (A-7) $B^c(q) \leq (1-q)(1+q\phi^c)\frac{cq}{2\overline{\Delta}(q)}$. From Proposition A1, the equilibrium payoff under public prices is $B^*(q) = (1-q)(1+q\phi^*)\frac{cq}{2\overline{\Delta}(q)}$, which establishes $B^*(q) \geq B^c(q)$, as desired. ■

Proposition A2 (Equilibrium prices under exploding offers). *Given $q_1^X(\phi)$, $p_2^X(h=0) = \frac{1}{2}$ and*

- *if $q_1^X(\phi) < \frac{1}{2}$, then $p_1^X = p_2^X(h=1) = \frac{1}{2}$ and the buyer purchases both goods;*
- *if $q_1^X(\phi) = \frac{1}{2}$, then $p_1^X = \frac{\alpha^*}{2}$ and $p_2^X(h=1) = \begin{cases} \frac{1}{2} & w.\ p. \quad \alpha^* \\ 1 & w.\ p. \quad 1-\alpha^* \end{cases}$, where $\alpha^* \in [\frac{1}{2}, 1]$. The buyer purchases the first good for sure and the second only if $v_1 = 0$ or $\alpha^* = 1$;*

- *if $q_1^X(\phi) > \frac{1}{2}$, then*

$$p_1^X = \begin{cases} \frac{1-q_1^X(\phi)}{2} & w.\ p. \quad \frac{1-q_1^X(\phi)}{q_1^X(\phi)} \\ \\ \frac{1}{2} & w.\ p. \quad \frac{2q_1^X(\phi)-1}{q_1^X(\phi)} \end{cases} \quad \text{and } p_2^X(h=1) = \begin{cases} \frac{1}{2} & w.\ p. \quad 1-q_1^X(\phi) \\ \\ 1 & w.\ p. \quad q_1^X(\phi). \end{cases}$$

A buyer with $v_1 = 0$ accepts only low p_1^X and all $p_2^X(h=1)$ whereas a buyer with $v_1 = \frac{1}{2}$ accepts all p_1^X but only the low $p_2^X(h=1)$.

Proof. We first consider the pricing decision by s_2. Note that while the equilibrium posterior $q_k^X(\phi)$ uses only information regarding the buyer's equilibrium information acquisition and sequencing, s_2 could further update this posterior on the basis of the buyer's equilibrium purchasing history h. Thus, let $q_k^X(\phi|h)$ denote s_2's equilibrium belief upon observing $h \in \{0,1\}$ in the first period. If $h = 0$, the buyer's purchase of the second good depends on $q_2^X(\phi|0)$. Since in this case, s_2 can realize a positive payoff only when $v_2 = \frac{1}{2}$, $p_2^X(h=0) = \frac{1}{2}$ is an equilibrium for all $q_2^X(\phi|0)$. Next consider the history $h = 1$. Conditional on purchasing the first good, a buyer with $v_1 = 0$ will accept any offer $p_2^X(h=1) \leq 1$ whereas a buyer with $v_1 = \frac{1}{2}$ will accept only the prices $p_2^X(h=1) \leq \frac{1}{2}$ for the second good. Thus s_2's optimal price is:

$$p_2^X(h=1) = \begin{cases} \frac{1}{2} & \text{if} \quad q_1^X(\phi|1) \leq \frac{1}{2} \\ \\ 1 & \text{if} \quad q_1^X(\phi|1) \geq \frac{1}{2}. \end{cases} \tag{A-9}$$

In general, anticipating $p_2^X(h=1)$, a buyer with valuation v_1 is willing to pay $\overline{p}_1(v_1)$ for good 1 such that

$$\max\{1 - \overline{p}_1(v_1) - p_2^X(h=1), v_1 - \overline{p}_1(v_1)\} = 0,$$

or equivalently,

$$\overline{p}_1(v_1) = \max\{1 - p_2^X(h=1), v_1\}. \tag{A-10}$$

Next we argue that $q_1^X(\phi|1) \leq \frac{1}{2}$ in equilibrium. To the contrary, suppose $q_1^X(\phi|1) > \frac{1}{2}$. Then $p_2^X(h=1) = 1$ and $\overline{p}_1(v_1 = 0) = 0$ and $\overline{p}_1(v_1 = \frac{1}{2}) = \frac{1}{2}$. But this would imply $p_1^X = \frac{1}{2}$ and in turn reduce the posterior to $q_1^X(\phi|1) = 0$ – a contradiction. Hence, $q_1^X(\phi|1) \leq \frac{1}{2}$ in equilibrium. We now consider two possibilities.

$\underline{q_1^X(\phi|1) < \frac{1}{2}}$: Then $p_2^X(h=1) = \frac{1}{2}$ from (A-9), and $\overline{p}_1(v_1 = 0) = \overline{p}_1(v_1 = \frac{1}{2}) = \frac{1}{2}$ from (A-10). This means $q_1^X(\phi|1) = q_1^X(\phi)$. Thus such an equilibrium exists if and only if $q_1^X(\phi) < \frac{1}{2}$, which reveals that the buyer purchases the bundle independent of her valuations.

$\underline{q_1^X(\phi|1) = \frac{1}{2}}$: Then by (A-9), s_2 is indifferent between the prices $\frac{1}{2}$ and 1. Suppose s_2 offers $\frac{1}{2}$ with probability α. Then, by (A-10), $\overline{p}_1(v_1 = \frac{1}{2}) = \frac{1}{2}$ and $\overline{p}_1(v_1 = 0) = \frac{\alpha}{2}$. Suppose that s_1 mixes between the prices $\frac{1}{2}$ and $\frac{\alpha}{2}$ by offering the latter with probability γ. Evidently, the buyer always accepts $\frac{\alpha}{2}$ whereas only the buyer with $v_1 = \frac{1}{2}$ accepts $\frac{1}{2}$. By Bayes' rule, $q_1^X(\phi|1) = \frac{\gamma q_1^X(\phi)}{\gamma q_1^X(\phi) + 1 - q_1^X(\phi)}$, which, together with the hypothesis $q_1^X(\phi|1) = \frac{1}{2}$, implies $\gamma = \frac{1-q_1^X(\phi)}{q_1^X(\phi)}$. Note that $\gamma \leq 1$ requires $q_1^X(\phi) \geq \frac{1}{2}$. For $q_1^X(\phi) > \frac{1}{2}$, $\gamma \in (0,1)$. The strict mixing by s_1 requires $\frac{\alpha}{2} = \frac{1-q_1^X(\phi)}{2}$, revealing $\alpha^* = 1 - q_1^X(\phi)$. For $q_1^X(\phi) = \frac{1}{2}$, $\gamma^* = 1$. This gives rise to the multiplicity of equilibria that satisfy $\frac{\alpha^*}{2} \geq \frac{1-q_1^X(\phi)}{2}$ or equivalently $\alpha^* \geq \frac{1}{2}$. ■

Proof of Proposition 6. Under the equilibrium prices found in Proposition A2, we determine equilibrium $q_1^X(\phi)$. Since $q_1^X(0) = q$, part (a) of Proposition 6 is immediate. Consider $\phi = 1$. If $q_1^X(1) > \frac{1}{2}$, then given the equilibrium prices in Proposition A2, the buyer strictly sequences from high to low value, i.e. $\theta_2^X(0, \frac{1}{2}) = 1$. As a result, $q_1^X(1) = q^2 > \frac{1}{2}$, which holds if and only if $q > \frac{1}{\sqrt{2}}$. Then $q \leq \frac{1}{\sqrt{2}}$ implies $q_1^X(1) \leq \frac{1}{2}$, resulting in equilibrium prices $p_1^X = p_2^X(h=1) = \frac{1}{2}$ (the multiplicity of equilibria at $q = \frac{1}{\sqrt{2}}$ is resolved in favor of efficiency here). In this case, the buyer is indifferent in the order. To guarantee $q_1^X(1) \leq \frac{1}{2}$, we need $q_1^X(1) = q^2 + 2q(1-q)\theta_1^X(0, \frac{1}{2}) \leq \frac{1}{2}$, or $\theta_1^X(0, \frac{1}{2}) \leq \frac{\frac{1}{2} - q^2}{2q(1-q)}$. The r.h.s. expression is in $(0, \frac{1}{2})$ for $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$, which implies that $\theta_2^X(0, \frac{1}{2}) = 1 - \theta_1^X(0, \frac{1}{2}) > \frac{1}{2}$. Finally, comparing the equilibrium prices under $\phi = 0$ and $\phi = 1$, it is observed that equilibrium is responsive if and only if $q > \frac{1}{2}$. ■

Proof of Proposition 7. Part (a) follows by comparing c and $\Delta^X(q)$ from Eq.(15). To prove part (b), we derive $\Delta_W^X(q) = W^{X,I}(q) - W^{X,U}(q)$. Consider first an uninformed buyer, $\phi = 0$. From Proposition 6(a) and (c),

$$\begin{aligned} W^{X,U}(q) &= (1-q)[\frac{1}{2} + (1-q)\frac{1}{2}] + q[\frac{1-q}{q} + \frac{2q-1}{q}(1-q)\frac{1}{2}] \\ &= \frac{1}{2}(1-q)(3+q). \end{aligned}$$

Next consider an informed buyer, $\phi = 1$. By Proposition 6(b), if $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$, the sellers' offers are accepted with certainty. Thus $W^{X,I}(q) = 1$. For $q > \frac{1}{\sqrt{2}}$, using Proposition 6(b) and (c), the equilibrium welfare is found to be

$$\begin{aligned} W^{X,I}(q) &= (1-q^2)[\frac{1}{2}+(1-q^2)\frac{1}{2}]+q^2\frac{1-q^2}{q^2} \\ &= \frac{1}{2}(1-q^2)(4-q^2). \end{aligned}$$

Given $W^{X,U}(q)$ and $W^{X,I}(q)$,

$$\Delta_W^X(q) = \begin{cases} 1-\frac{1}{2}(1-q)(3+q) & \text{if} \quad q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}] \\ \\ \frac{1}{2}(1-q)(1+3q-q^2-q^3) & \text{if} \quad q > \frac{1}{\sqrt{2}}. \end{cases}$$

Straightforward algebra shows $\Delta_W^X(q) > 0$ and $\Delta_W^X(q) > \Delta^X(q)$ for all $q > \frac{1}{2}$.

To prove part (c), suppose $q > \frac{1}{\sqrt{2}}$. Then $q_1^X(\phi) = \phi q_1^X(1) + (1-\phi)q \geq \phi q^2 + (1-\phi)q > \frac{1}{2}$. Armed with the equilibrium prices in Proposition A2, it is readily verified that an informed buyer strictly sequences the high-value seller first, inducing $q_1^X(1) = q^2$. Moreover the value of information under endogenous information acquisition is $\hat{\Delta}^X(\phi) = q(1-q)\frac{1-q_1^X(\phi)}{2}$. Clearly $\hat{\Delta}^X(\phi) \leq \Delta^X(q)$ for all ϕ and thus $\phi^{x,*} \leq \phi^{x,o}$. To determine when this inequality is strict, note first that $\phi^{x,*} = \phi^{x,o} = 0$ for $c > \Delta^X(q)$. For $c < \Delta^X(q)$, $\phi^{x,o} = 1$. Having $\phi^{x,*} = 1$ requires $\hat{\Delta}^X(\phi = 1) = q(1-q)\frac{1-q^2}{2} = (1+q)\overline{\Delta}^X(q) \geq c$. Therefore, if $(1+q)\overline{\Delta}^X(q) < c < \Delta^X(q)$, then it must be that $\phi^{x,*} < \phi^{x,o}$. Finally, suppose $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$. Then $\phi^{x,o} = 0 \leq \phi^{x,*}$. To see when this inequality is strict, note that $\phi^{x,*} = 0$ implies $q_1^X(0) = q$ and in turn the equilibrium prices described in Proposition 6(a). Then the value of information satisfies $\hat{\Delta}(\phi = 0) = q(1-q)\frac{1-q}{2} = \overline{\Delta}^X(q)$. For $c < \overline{\Delta}^X(q)$, $\phi^{x,*} = 0$ cannot be supported as an equilibrium and thus $\phi^{x,o} < \phi^{x,*}$. ■

Proof of Proposition 8. As with complements, without loss of generality, we label sellers according to the sequence $s_1 \to s_2$. Let q_i denote the posterior probability of realizing a high valuation, 1, from s_i. To determine equilibrium prices, we again begin with s_2's best response $P_2(p_1)$. We exhaust several regions.

<u>$p_1 \in [0, \frac{1}{2})$</u>: Then $v_1 - p_1 > 0$ for all v_1. In order for s_2 to sell, he needs to set p_2 such that $\max\{1-p_1-p_2, v_2-p_2\} \geq v_1 - p_1$. The following table lists s_2's best response candidates and the sellers' implied probabilities of sales.

p_2	s_2's prob. of sale	s_1's prob. of sale
$p_1+\frac{1}{2}$	$q_2(1-q_1)$	$1-q_2(1-q_1)$
$\frac{1}{2}$	$1-q_1$	$1-q_2(1-q_1)$
p_1	$1-q_1(1-q_2)$	$1-q_2$
0	1	$1-q_2$.

From here,

$$P_2(p_1) = \begin{cases} \frac{1}{2} & \text{if} \quad p_1 \leq \min\{\frac{1-q_1}{2(1-q_1(1-q_2))}, \frac{1-q_2}{2q_2}\} \\ p_1 + \frac{1}{2} & \text{if} \quad p_1 \in \left[\frac{1-q_2}{2q_2}, \frac{q_2(1-q_1)}{2(1-q_2(1-q_1)-q_1(1-q_2))}\right] \\ p_1 & \text{if} \quad p_1 \in \left[\max\{\frac{1-q_1}{2(1-q_1(1-q_2))}, \frac{q_2(1-q_1)}{2(1-q_2(1-q_1)-q_1(1-q_2))}\}, \frac{1}{2}\right). \end{cases} \tag{A-11}$$

$\underline{p_1 = \frac{1}{2}}$: Similar to the previous case, in order to realize a sale, s_2 sets p_2 such that $\max\{1 - p_1 - p_2, v_2 - p_2\} \geq v_1 - p_1$. The table below lists the candidates for s_2's best response.

p_2	s_2's prob. of sale	s_1's prob. of sale
$\frac{1}{2}$	$1 - q_1(1 - q_2)$	$1 - q_2$
1	$q_2(1 - q_1)$	$1 - q_2(1 - q_1)$
0	1	$1 - q_2$

Hence,

$$P_2(\frac{1}{2}) = \begin{cases} \frac{1}{2} & \text{if} \quad 2q_2(1-q_1) + q_1(1-q_2) \leq 1 \\ 1 & \text{if} \quad 2q_2(1-q_1) + q_1(1-q_2) \geq 1. \end{cases} \tag{A-12}$$

$\underline{p_1 \in \left(\frac{1}{2}, 1\right)}$: Then $1 - p_1 - p_2 < \max\{v_1 - p_1, v_2 - p_2\}$, implying that the buyer will never purchase the bundle. For her to purchase from s_2, it must be that $\max\{v_2 - p_2, 0\} \geq \max\{v_1 - p_1, 0\}$, which yields

p_2	s_2's prob. of sale	s_1's prob. of sale
1	$q_2(1 - q_1)$	q_1
p_1	q_2	$q_1(1 - q_2)$
$\frac{1}{2}$	$1 - q_1(1 - q_2)$	$q_1(1 - q_2)$
$p_1 - \frac{1}{2}$	1	$0.$

Suppose $2q_2(1-q_1) + q_1(1-q_2) \geq 1$. Then $q_1 < \frac{1}{2} < q_2$. This implies that s_2's payoff from $p_2 = 1$ exceeds his payoff from $p_2 = \frac{1}{2}$, and that his payoff from $p_2 = p_1$ exceeds his payoff from $p_2 = p_1 - \frac{1}{2}$ since $p_1 < 1 < \frac{1}{2(1-q_2)}$. As a result,

$$P_2(p_1) = \begin{cases} 1 & \text{if} \quad p_1 \leq 1 - q_1 \\ p_1 & \text{if} \quad p_1 \geq 1 - q_1. \end{cases} \tag{A-13}$$

For $2q_2(1-q_1) + q_1(1-q_2) \leq 1$, s_2's payoff from $p_2 = \frac{1}{2}$ exceeds his payoff from $p_2 = 1$. In sum,

$$P_2(p_1) = \begin{cases} \frac{1}{2} & \text{if} \quad p_1 \leq \min\{1 - \frac{q_1(1-q_2)}{2}, \frac{1-q_1(1-q_2)}{2q_2}\} \\ p_1 & \text{if} \quad p_1 \in \left[\frac{1-q_1(1-q_2)}{2q_2}, \frac{1}{2(1-q_2)}\right] \\ p_1 - \frac{1}{2} & \text{if} \quad p_1 \geq \max\{1 - \frac{q_1(1-q_2)}{2}, \frac{1}{2(1-q_2)}\}. \end{cases} \tag{A-14}$$

$\underline{p_1 = 1}$: Then, it is straightforward to verify that the only candidates for best response are 1 and $\frac{1}{2}$, implying

p_2	s_2's prob. of sale	s_1's prob. of sale
1	q_2	$q_1(1-q_2)$
$\frac{1}{2}$	1	0.

Hence,

$$P_2(1) = \begin{cases} \frac{1}{2} & \text{if} \quad q_2 \leq \frac{1}{2} \\ 1 & \text{if} \quad q_2 > \frac{1}{2}. \end{cases} \tag{A-15}$$

Turning to s_1, note that for $p_1 < \frac{1}{2}$, Eq.(A-11) reveals that the highest possible price that is accepted with probability $1 - q_2(1-q_1)$ is $p_1^a \equiv \max\{\frac{1-q_1}{2(1-q_1(1-q_2))}, \frac{q_2(1-q_1)}{2(1-q_2(1-q_1)-q_1(1-q_2))}\}$. A price higher than p_1^a would be accepted with probability $1-q_2$ and result in a lower payoff for the buyer than $p_1 = \frac{1}{2}$. A price lower than p_1^a would be accepted with the same probability as p_1^a. Therefore, the only equilibrium candidate for $p_1 < \frac{1}{2}$ is p_1^a. It can be similarly established that the only other equilibrium candidates are $p_1^b = \frac{1}{2}$, $p_1^c = 1$ and

$$p_1^d = \begin{cases} 1 - q_1 & \text{for} \quad 2q_2(1-q_1) + q_1(1-q_2) \geq 1 \\ \max\{1 - \frac{q_1(1-q_2)}{2}, \frac{1}{2(1-q_2)}\} & \text{for} \quad 2q_2(1-q_1) + q_1(1-q_2) \leq 1 \end{cases}$$

Next, we show that in equilibrium $p_1^* = p_2^* \geq \frac{1}{2}$.

First we rule out the possibility that $p_1^* = p_1^a < \frac{1}{2}$. From (A-11), $P_2(p_1^a) > p_1^a$. This, in turn, implies that if $p_1^* = p_1^a$, an informed buyer would strictly prefer to approach the high value seller first, inducing $q_1 \geq q_2$ in equilibrium. However, for $q_1 \geq q_2$, straightforward algebra shows $\pi_1(\frac{1}{2}) > \pi_1(p_1^a)$, implying that p_1^a cannot be supported as an equilibrium price.

Second we consider an equilibrium with $p_1^* = \frac{1}{2}$. We show that the only possible equilibrium response by s_2 is $p_2^* = \frac{1}{2}$. Suppose, instead, that $p_2^* = 1$, which by (A-12) requires $2q_2(1-q_1) + q_1(1-q_2) \geq 1$ and thus $q_1 < \frac{1}{2} < q_2$. However, given the unequal prices $\frac{1}{2}$ and 1, an informed buyer would sequence the sellers from high to low value, engendering $q_1 > q_2$ – a contradiction.

Next suppose that $p_1^* = p_1^d \in (\frac{1}{2}, 1)$. Similar to the previous case, we show that the only possible equilibrium response is $p_2^* = p_1^d$. If $2q_2(1-q_1) + q_1(1-q_2) \geq 1$, then we must have $q_1 < \frac{1}{2} < q_2$. Moreover, by (A-13), $p_2^* = 1$, which implies that an informed buyer would optimally sequence the sellers from high to low value, yielding $q_1 \geq q_2$ – a contradiction. If, on the other hand, $2q_2(1-q_1) + q_1(1-q_2) \leq 1$, we must have $p_1^d \geq 1$ for $q_2 \geq \frac{1}{2}$, violating our hypothesis $p_1^d \in (\frac{1}{2}, 1)$. For $q_2 < \frac{1}{2}$, suppose that $p_2^* \neq p_1^*$. Then, by (A-14), $p_2^* < p_1^*$, inducing an informed buyer to sequence the sellers from low to high value and thus $q_1 < q_2 < \frac{1}{2}$.

Given these equilibrium beliefs, it follows that $\pi_1(\frac{1}{2}) = \frac{1-q_2}{2} > \pi_1(p_1^d) = q_1(1-q_2)p_1^d$ for s_1, implying a profitable deviation to $p_1 = \frac{1}{2}$. Hence, $p_1^* = p_1^d \neq p_2^*$ cannot be supported as an equilibrium, either.

Finally, consider $p_1^* = 1$. From (A-15), it follows that s_1 sells with a positive probability if and only if $p_2^* = 1$. Thus, $p_1^* = 1 \neq p_2^*$ cannot be supported as an equilibrium. This completes the proof of the claim that $p_1^* = p_2^* \geq \frac{1}{2}$ for *all* q_s and ϕ. Given the equal pricing by the sellers, under unobservable information acquisition $\hat{\Delta}(\phi) = 0$, implying that $\phi^* = 0$.

We complete the proof of Proposition 8 by showing that $p_1^* = p_2^* = \frac{1}{2}$ for $q_s \leq \frac{1}{2}$ and $p_1^* = p_2^* > \frac{1}{2}$ for $q_s > \frac{1}{2}$. First, for $q_s \leq \frac{1}{2}$, the price pair $p_1^* = p_2^* = \frac{1}{2}$ can be supported as an equilibrium by a complete randomization over the sequence by an uninformed buyer, giving rise to $q_1 = q_2 = q_s$. To establish its uniqueness, we easily rule out $p_1^* = 1$ and $p_1^* = p_1^d \in (\frac{1}{2}, 1)$ for $q_s < \frac{1}{2}$.[26] To see that $p_1^* = p_2^* = 1$ cannot be supported as an equilibrium, note from (A-15) that the highest possible payoff for s_1 is $\pi_1(1) = q_1(1-q_2)$ for $q_2 > \frac{1}{2}$. Since $q_s \leq \frac{1}{2}$, we have $q_2 > \frac{1}{2} > q_1$ and thus $\pi_1(\frac{1}{2}) = \frac{1-q_2}{2} > \pi_1(1)$. To show $p_1^* = p_2^* = p_1^d \in (\frac{1}{2}, 1)$ cannot be supported as an equilibrium either, note from (A-14) that $p_1^d = \frac{1}{2(1-q_2)}$ is the only possible equilibrium price that satisfies equal pricing in the region $p_1 \in (\frac{1}{2}, 1)$. Moreover, no deviation by s_1 requires $\pi_1(p_1^d) = \frac{q_1}{2} \geq \pi_1(\frac{1}{2}) = \frac{1-q_2}{2}$, which implies $q_1 + q_2 \geq 1$. Analogous to the complements case, $q_1 + q_2 = 2q_s$ should hold for all posteriors. Therefore, $q_s \geq \frac{1}{2}$ is necessary for s_1 not to deviate from $p_1^* = p_1^d$. Last, by detecting a profitable deviation for s_1, we show that $p_1^* = p_2^* = \frac{1}{2}$ is not an equilibrium for $q_s > \frac{1}{2}$. If $q_1 > q_2 \geq \frac{1}{2}$, then $\pi_1(1) = q_1(1-q_2) > \pi_1(\frac{1}{2}) = \frac{1-q_2}{2}$. If $q_1 > \frac{1}{2} > q_2$, then $\pi_1(p_1^d) \geq \frac{q_1}{2} > \pi_1(\frac{1}{2})$ since $q_1 + q_2 = 2q_s > 1$. Finally, if $q_2 > \frac{1}{2} > q_1$, then $\pi_1(p_1^a) > \pi_1(\frac{1}{2})$. Hence, $p_1^* = p_2^* > \frac{1}{2}$ for $q_s > \frac{1}{2}$. ■

Appendix B

In this appendix, we extend our base model with non-exploding offers to a Bernoulli distribution with a general support $v_i \in \{\underline{v}, \overline{v}\}$ where $0 \leq \underline{v} < \overline{v} \leq \frac{1}{2}$ and $\Pr\{v_i = \underline{v}\} = q \in (0,1)$. Define

$$\underline{q}^g \equiv \max\{\frac{\sqrt{1+4\frac{\overline{v}}{1-\overline{v}}}-1}{2\frac{\overline{v}}{1-\overline{v}}}, \frac{1-\overline{v}}{1-\underline{v}}\}.$$

The following Lemma characterizes the equilibrium pricing for fixed posterior beliefs.

[26] For $q_s = \frac{1}{2}$, a price pair $p_1^* = p_2^* = 1$ is an equilibrium as well. Since the two sellers are indifferent between the two price pairs, we break the indifference in favor of the efficient pricing.

Lemma B1. *Fix the posterior beliefs.*

- *If* $\frac{1-\overline{v}}{1-\underline{v}} < 1 - \underline{v}/\overline{v}$, *then*

$$(p_1^*(\phi), p_2^*(\phi)) = \begin{cases} (1-\overline{v}, \overline{v}) & if \quad q_2(\phi) \leq \underline{q}^g \\ (1-(1-q_2(\phi))\overline{v}, (1-q_2(\phi))\overline{v}) & if \quad q_2(\phi) \in \left(\underline{q}^g, 1-\underline{v}/\overline{v}\right) \\ (1-\underline{v}, \underline{v}) & if \quad q_2(\phi) \geq 1-\underline{v}/\overline{v}; \end{cases}$$

- *if* $\frac{1-\overline{v}}{1-\underline{v}} \geq 1 - \underline{v}/\overline{v}$, *then*

$$(p_1^*(\phi), p_2^*(\phi)) = \begin{cases} (1-\overline{v}, \overline{v}) & if \quad q_2(\phi) \leq \underline{q}^g \\ (1-\underline{v}, \underline{v}) & if \quad q_2(\phi) > \underline{q}^g. \end{cases}$$

Proof. Let p_i^m denote the optimal monopoly price of a seller who focuses on selling only his product. Then,

$$p_i^m(q_i) = \begin{cases} \overline{v} & \text{if} \quad q_i < 1-\underline{v}/\overline{v} \\ \underline{v} & \text{if} \quad q_i \geq 1-\underline{v}/\overline{v}. \end{cases}$$

The corresponding monopoly profit is given by

$$\pi_i^m(q_i) = \begin{cases} (1-q_i)\overline{v} & \text{if} \quad q_i < 1-\underline{v}/\overline{v} \\ \underline{v} & \text{if} \quad q_i \geq 1-\underline{v}/\overline{v}. \end{cases} \tag{B-1}$$

Analogous to the base model, it is straightforward to establish that any price $p_1 \leq 1-\overline{v}$ would be accepted by the buyer since s_2 would optimally respond by $P_2(p_1) = 1-p_1 \geq \overline{v} > \pi_2^m(q_2)$, inducing the purchase of the bundle. Therefore, $p_1^*(\phi) \geq 1-\overline{v}$. For $p_1 > 1-\overline{v}$, s_2's best response is found to be:

$$P_2(p_1) = \begin{cases} 1-p_1 & \text{if} \quad p_1 \in (1-\overline{v}, 1-\pi_2^m(q_2)] \\ \pi_2^m & \text{if} \quad p_1 > 1-\pi_2^m(q_2). \end{cases}$$

Given $P_2(p_1)$, s_1 chooses between the price $1-\overline{v}$, which is accepted for sure and the price $1-\pi_2^m(q_2)$, which is accepted with probability q_2. Comparing the implied payoffs, we determine

$$(p_1^*, p_2^*) = \begin{cases} (1-\overline{v}, \overline{v}) & \text{if} \quad q_2 \le \frac{1-\overline{v}}{1-\pi_2^m} \\ (1-\pi_2^m, \pi_2^m) & \text{if} \quad q_2 > \frac{1-\overline{v}}{1-\pi_2^m}. \end{cases} \tag{B-2}$$

For $q_2 \ge 1 - \underline{v}/\overline{v}$, by eq.(B-1) $\pi_2^m = \underline{v}$. Then, by (B-2),

$$(p_1^*, p_2^*) = \begin{cases} (1-\overline{v}, \overline{v}) & \text{if} \quad 1-\underline{v}/\overline{v} \le q_2 \le \frac{1-\overline{v}}{1-\underline{v}} \\ (1-\underline{v}, \underline{v}) & \text{if} \quad q_2 > \frac{1-\overline{v}}{1-\underline{v}}. \end{cases} \tag{B-3}$$

For $q_2 < 1 - \underline{v}/\overline{v}$, by eq. (B-1) $\pi_2^m = (1-q_2)\overline{v}$. Then, by (B-2)

$$(p_1^*, p_2^*) = \begin{cases} (1-\overline{v}, \overline{v}) & \text{if} \quad q_2 \le \tilde{q} \\ (1-(1-q_2)\overline{v}, (1-q_2)\overline{v}) & \text{if} \quad \tilde{q} < q_2 < 1-\underline{v}/\overline{v}. \end{cases} \tag{B-4}$$

where $\tilde{q} \equiv \frac{\sqrt{1+4\frac{\overline{v}}{1-\overline{v}}}-1}{2\frac{\overline{v}}{1-\overline{v}}}$ solves $\tilde{q} = \frac{1-\overline{v}}{1-(1-\tilde{q})\overline{v}}$. Straightforward algebra reveals that $\underline{q}^g = \tilde{q}$ for $\frac{1-\overline{v}}{1-\underline{v}} < 1 - \underline{v}/\overline{v}$ and $\underline{q}^g = \frac{1-\underline{v}}{1-\overline{v}}$ otherwise. Using this fact, and rearranging (B-3) and (B-4), we obtain the equilibrium prices, as desired. ■

As in the base model, the equilibrium pricing reveals that the buyer will be motivated to seek information for sufficiently strong complements and that the optimal sequencing should be more likely to be from low to high value. The following Proposition formalizes this generalization.

Proposition B1. *The equilibrium is responsive if and only if $q > \underline{q}^g$. Moreover, in a responsive equilibrium, an informed buyer is more likely to sequence heterogeneous goods from low to high value. Formally, $\theta_2^*(\frac{1}{2}, 0) > \frac{1}{2}$ for $\underline{q}^g < q \le \overline{q}^g$; and $\theta_2^*(\frac{1}{2}, 0) = 1$ for $q > \overline{q}^g$ where $\overline{q}^g = \sqrt{\underline{q}^g}$.*

Proof. To show that the equilibrium is unresponsive for $q \le \underline{q}^g$, note that a responsive equilibrium in this region requires price $p_2^* < \overline{v}$. By Lemma B1, this implies that $q_2(1) > \underline{q}^g$. Such pricing, however, results in a strict sequencing preference from low to high value and thus $q_2(1) = q^2 < \underline{q}^g$, contradicting $p_2^* < \overline{v}$. Therefore, for $q \le \underline{q}^g$, an informed buyer must be indifferent in the order and $p_2^* = \overline{v}$. The above argument also implies that $q_2(1) \le \underline{q}^g$ for $q \le \sqrt{\underline{q}^g}$, which by Lemma B1 implies that $p_2 = \overline{v}$. As a result, the buyer is indifferent in the order. Then, $\theta_2^*(\frac{1}{2}, 0) = 1 - \theta_2^*(0, \frac{1}{2})$ is determined by the condition $q_2(1) = q^2 + 2q(1-q)\theta_2^*(0, \frac{1}{2}) \le \underline{q}^g$. It immediately follows that $\theta_2^*(\frac{1}{2}, 0) > \frac{1}{2}$. Finally, for $q > \sqrt{\underline{q}^g}$, $q_2(1) > \underline{q}^g$ for any possible mixing by the buyer. By Lemma B1, this implies that $p_2^* < \overline{v}$. As a result, the buyer has a strict sequencing preference from low to high value, i.e. $\theta_2^*(\frac{1}{2}, 0) = 1$. ■

Corollary B1. (**Informed Prices**) *In equilibrium with an informed buyer,*

- *if* $\frac{1-\overline{v}}{1-\underline{v}} < 1 - \underline{v}/\overline{v}$,

$$(p_1^*(1), p_2^*(1)) = \begin{cases} (1-\overline{v}, \overline{v}) & if \quad q \leq \overline{q}^g \\ (1-(1-q^2)\overline{v}, (1-q^2)\overline{v}) & if \quad q \in \left(\overline{q}^g, \sqrt{1-\underline{v}/\overline{v}}\right) \\ (1-\underline{v}, \underline{v}) & if \quad q \geq \sqrt{1-\underline{v}/\overline{v}}; \end{cases}$$

- *if* $\frac{1-\overline{v}}{1-\underline{v}} \geq 1 - \underline{v}/\overline{v}$,

$$(p_1^*(1), p_2^*(1)) = \begin{cases} (1-\overline{v}, \overline{v}) & if \quad q \leq \overline{q}^g \\ (1-\underline{v}, \underline{v}) & if \quad q > \overline{q}^g. \end{cases}$$

Proof. It is immediate from Lemma B1 and Proposition B1. ■

Using prices and sequencing in a responsive equilibrium, we determine the buyer's expected uninformed and informed payoffs. For $\frac{1-\overline{v}}{1-\underline{v}} < 1 - \underline{v}/\overline{v}$,

$$B^U(q) = \begin{cases} q(1-q)\overline{v} & \text{if} \quad q \in \left(\underline{q}^g, 1-\underline{v}/\overline{v}\right) \\ (1-q)(\overline{v}-\underline{v}) & \text{if} \quad q \geq 1-\underline{v}/\overline{v} \end{cases}$$

and

$$B^I(q) = \begin{cases} 0 & \text{if} \quad q \leq \overline{q}^g \\ q^2(1-q^2)\overline{v} & \text{if} \quad q \in \left(\overline{q}^g, \sqrt{1-\underline{v}/\overline{v}}\right) \\ (1-q^2)(\overline{v}-\underline{v}) & \text{if} \quad q \geq \sqrt{1-\underline{v}/\overline{v}}. \end{cases} \tag{B-5}$$

For $\frac{1-\overline{v}}{1-\underline{v}} \geq 1 - \underline{v}/\overline{v}$,

$$B^U(q) = (1-q)(\overline{v}-\underline{v})$$

and

$$B^I(q) = \begin{cases} 0 & if \quad q \leq \overline{q}^g \\ (1-q^2)(\overline{v}-\underline{v}) & if \quad q > \overline{q}^g. \end{cases}$$

Corollary B2 shows that the buyer's value of information $\Delta(q) = B^I(q) - B^U(q)$ follows the same sign pattern as in the base model.

Corollary B2. *The buyer's value of information satisfies:* $\Delta(q) < 0$ *for* $q \leq \overline{q}^g$ *and* $\Delta(q) > 0$ *for* $q > \overline{q}^g$.

Proof. For $q \leq \overline{q}^g$, the result follows immediately since $B^I(q) = 0$. For $q > \overline{q}^q$, if $\frac{1-\overline{v}}{1-\underline{v}} \geq 1 - \underline{v}/\overline{v}$, then $\Delta(q) = q(1-q)(\overline{v} - \underline{v}) > 0$. For $\frac{1-\overline{v}}{1-\underline{v}} < 1 - \underline{v}/\overline{v}$, if $q \geq \sqrt{1 - \underline{v}/\overline{v}}$, $\Delta(q) = q(1-q)(\overline{v} - \underline{v}) > 0$. For $q \in \left(\overline{q}^g, \sqrt{1 - \underline{v}/\overline{v}}\right)$, by (B-5), $B^I(q) = q^2(1-q^2)\overline{v}$. Moreover, it is straightforward to verify that $B^U(q)$ is continuous and decreasing in q for $q > \overline{q}^g$. Therefore, $\Delta(q) \geq q^2(1-q^2)\overline{v} - q(1-q)\overline{v} = q(1-q)\overline{v}(q(1+q)-1) > 0$ for $q > \frac{\sqrt{5}-1}{2}$. Since $\overline{q}^g = \sqrt{\underline{q}^g} > \sqrt{\frac{1}{2}} > \frac{\sqrt{5}-1}{2}$, we have $\Delta(q) > 0$ for $q > \overline{q}^g$. ■

Consistent with the base model, the above corollary shows that the buyer is strictly worse off being informed for moderate complements, $q \in (\underline{q}^g, \overline{q}^g]$, since the negative pricing effect dominates the positive sequencing effect in this region. For strong complements, $q > \overline{q}^g$, the sequencing effect dominates the negative pricing effect and the buyer benefits from being informed.

Next, we derive the social value of information. Analogous to the base model, it is straightforward to establish that the uninformed welfare is $W^U(q) = q + (1-q)\overline{v}$ for $q > \underline{q}^g$. From Corollary B1 and Proposition B1, the informed welfare is

$$W^I(q) = \begin{cases} 1 & \text{if} \quad q \leq \overline{q}^g \\ q^2 + (1-q^2)\overline{v} & if \quad q > \overline{q}^g. \end{cases}$$

Therefore, the social value of information is given by

$$\Delta_W(q) = \begin{cases} (1-q)(1-\overline{v}) & \text{if} \quad \underline{q}^g < q \leq \overline{q}^g \\ -q(1-q)(1-\overline{v}) & \text{if} \quad q > \overline{q}^g. \end{cases}$$

The comparison between the social and the private value of information is also consistent with the base model. While the social value of information exceeds the private value of information for moderate complements, $q \in (\underline{q}^g, \overline{q}^g]$, the reverse is true for strong complements, $q > \overline{q}^g$. As a result, as found in the base model, the optimal information acquisition is inefficient: too little for moderate complements and too much for strong complements.

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