THE CONTINUOUS APPROXIMATION METHOD FOR BAYESIAN GAMES WITH GENERICALLY CONTINUOUS PAYOFFS [1]

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This paper proposes a new method, *the continuous approximation method,* to study *Bayesian games with generically continuous payoff functions* where discontinuities of payoff functions take place only at a set of first category. Generically continuous functions include continuous function, step functions, and monotone functions in $\mathbf{R}^n$ and this class of games generalizes the canonical model of Milgrom and Weber (1985), encompasses a large class of auctions and mechanism design problems, and synthesizes the previous models by Dasgupta and Maskin (1996), Jackson, Simon, Swinkels, and Zame (2002), and Barelli, Govindan, and Wilson (2014a,b). The continuous approximation method analyzes the game via its continuous approximations and their first order conditions, and can yield economic insights beyond existence of equilibrium. First, a Bayesian game with generically continuous payoffs has a Bayesian Nash equilibrium via continuous approximations when the better reply security condition of Reny (1999) holds. Second, we generalizes the existence result of Jackson and Swinkels (2006) existence theorem of a Bayesian Nash equilibrium of auction games to the model that involves the primary dealership as in Kastl and Hortacsu (2012). Third, we present the unified analysis of the optimal auction mechanism of Myerson (1981) and the nonlinear pricing mechanisms of Mussa and Rosen (1978). Fourth, we present a new characterization of the optimal auction mechanism with heterogeneous objects and multidimensional private values from continuous distributions by applying the sweeping method of Rochet and Chone (1998) that first becomes possible with the continuous approximation method.

KEYWORDS: Games with Discontinuous Payoffs, Bayesian Nash Equilibrium, Optimal Auction Design, Nonlinear Pricing.

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