Interviewing in Many-to-One Matching Market

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April 11, 2014

Abstract

I propose a new model of interviewing for a many-to-one matching market. This model with a finite number of firms and a continuum of students, introduces a stage of costly information acquisition before the matching process. The strategic decisions of firms to interview the optimal set of students is the focus of this discussion. I present this first interviewing model in a many-to-one setting. It predicts the following anecdotally observed phenomena. A firm targets its interview offers instead of extending them only to the best students. It strategically extends its interview offers to a few stars, a few medium ranked students and a few safe bets. The strategic choice by firms causes some students to fall through the cracks.

1 Introduction

Market Design as a field, over the past couple of decades, has grown and has had an immense influence in various practical markets–the redesign of National Residency Match Program (NRMP), school allocation, kidney exchange, and course allocations in business schools and colleges–to name a few. The bulk of the research in this field has focused on the offer extension process, the final stage of a matching market which takes the preferences of all the market participants as given. However, a matching market has various stages which start from the application submission stage by one side of the market, say

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†I would like to thank Drew Fudenberg, Divya Kirti, Scott Kominers, Maciej Kotowski, Eric Maskin, Assaf Romm, and Al Roth for extremely helpful discussions. I would also like to thank Clayton Featherstone, Jacob Leshno, and especially Sankarsan Basu, and Saral Mukherjee for conversations that started this project.

graduating students, to the other side, say firms.[1] It also has some intermediate stages like creation of shortlists or interview offers, and scheduling of interviews.

Interviewing processes are organized in many ways. At one end of the spectrum there is completely centralized interviewing where the firms and students are matched for interviews through a centralized mechanism, e.g. the placement process at Indian colleges and business schools (IITs, IIMs, etc.). At the other end of the spectrum there is completely decentralized process where no central institution co-ordinates any aspect of the interviewing process, e.g. the interview process for residency positions in US hospitals, placement process of college seniors in universities like Peking University, etc. There are also some hybrid mechanisms where there is some centralized co-ordination early on but there are fly-outs and final interview stages that happen in a decentralized manner, e.g. senior placements from US colleges, Economics PhD placements, etc. A guideline for the choice of a mechanism under different circumstances will emerge after a sound understanding of the preference formation process.

There is a lot of anecdotal evidence and wordly wisdom about application processes and interviewing stages. E.g. a high school senior targets which colleges to apply to rather than applying just to the very top or to the very low ranked colleges. When deciding on which candidates to extend the flyout offers a medium-tier university strategically chooses some top students likely to be overlooked by the very top universities. A second-best firm usually does not interview only the best students in the best colleges. However, the existing small but growing theoretical literature does not capture any of these insights.

This paper will take a step in this direction and formally establish some of the above intuitions which are new to the theory.[2]

Model Overview

The starting point of my analysis is a many-to-one matching market under complete information about the common value of students for all firms. I assume that each student has a 'firm fitness indicator,' which is either 1 or 0, i.e. a student is a "fit" or a "misfit" with respect to a particular firm. This fit might correspond to intangible factors related to firm culture or idiosyncratic characteristics. This indicator can only be discovered by interviewing a candidate and a firm never wants to hire a "misfit" candidate. Each firm has a

[1] Similarly, applications are submitted by high school seniors to colleges, doctors to hospitals, law graduates to law firms, etc.

[2] There is existing literature on preference formation processes reviewed in **Related Literature**. On application decisions in the context of college admissions see Chade and Smith (2006) and Chade, Lewis, and Smith (2014). For interviewing models see Lee and Schwarz (2012), Ely and Siegel (2013), Josephson and Shapiro (2013), and Rastegari, Condon, Immorlica, and Leyton-Brown (2013).

specific number of open positions called a recruitment quota. Interviewing is costly for firms and this is modeled as a quota of interview slots.

This setup although very stylized, can be motivated in many ways. A firm might want to find out if a student would be a good fit for the firm's competitive or creative culture. It might also look for some firm-specific skills in the student which can only be discovered through an interview. A firm, typically, invests some pre-decided number of employee-hours on recruiting based on its expectations about the students.

I apply this model in various settings to generate a few key insights. Even with complete information and common value about the student, firms will interview more (mass of) students than (the mass corresponding to the) open positions due to the absence of information on the fitness factors. If there is a clear ranking over all the firms among the students, under independent fitness factor, a second ranked firm could expend only a few of its interview slots for the top students who are also being interviewed by the first ranked firm. The second ranked firm, thus, startegically targets its interview offers. The intuition for this is straightforward. It will find a better use of its scarce interview slots in extending them to those students for whom it faces no competition rather than competing with the best firm. Similarly, a lower ranked firm will typically interview a diverse set of students–a few students at the top, a few in the middle and a few where it does not face any competition from the higher ranked firms.

The above setup is extended to compare markets where there is no clear ranking among firms and to those markets where the fitness factor is correlated with the common value of the students or across firms. In all the above situations, there would be students who would have 'justified envy' for not getting an interview offer from a firm although students ranked lower than him/her are interviewed by such firms. In case such a student does not get an offer from the top firms, we would see the phenomenon of falling through the cracks in the interview process. This setup can be easily extended for multi-dimensional indices for students with different firms caring more or less about one dimension versus the other.

The results about targetting by firms and justified envy, and falling through the cracks of students are robust to the relaxation of common-value assumption as long as the firms do not care about different factors which are completely negatively correlated and the students are not very homogenous.

Related Literature

There have been a few important contributions to the literature on preference formation processes. Chade and Smith (2006) focus on the problem of portfolio choice for applications for a single student. They present a greedy algorithm which solves the combinatorial optimization problem optimally. The finite options facing a student are characterized by their value upon success, probability of success, and cost associated with applying. A student wants to choose the best portfolio of colleges to apply for. Our model can be interpretted as a generalization of their model where the role of a student is replaced by finite firms making choices for their optimal portfolio of (interview) offers to candidates. The probability of success at each candidate is found at equilibrium and is thus endogenized. Chade, Lewis, and Smith (2014) (henceforth CLS) talk about the equilibrium model of college admissions in a setup with two ranked colleges. With incomplete information about the student quality as seen by the colleges and incomplete information about the portfolio of students by the colleges, CLS generate interesting results about 'stretch' and 'safety' application portfolios. If everything that needs to be found out about students can not be inferred from the application file and there is room for actual interviews, as is in the case of MBA school applications or job applications, our model offers a tractable alternative.

Lee and Schwarz (2012) focus on the network aspect of an interview schedule for multiple firms and multiple agents in a one-to-one matching market. They find that interviewing schedules with maximum overlap are welfare improving as compared to the ones with less overlap. The intuition being if a firm loses out a candidate to a rival firm due to the preference of that candidate, the chance of getting somebody else are maximum if (at best) the firms are interviewing the exact same pool of candidates although each firm does not interview the whole market but only a subset of it. Ely and Siegel (2013) analyze the implications of revelation of intermediate interviewing decisions by firms in a common-value labor market. They restrict their attention to the setup where firms compete for a single worker. The common ranking of the firms, common-value of the worker and no discriminatory information revelation to firms during the interview process are key elements of their setup. They show that severe adverse selections shuts off all the firms except the top firm(s) from participation in recruiting.

Josephson and Shapiro (2013) look at information-based unemployment resulting from a schedule of interviews presented to the participants (by a central coordinating organization). Their key result is that with two ordered firms and three types–high, medium, and low–of two agents, the lower ranked firm will never want to interview an agent not hired by the first firm in the first time period. The reason for this inefficient matching is that the cost of interviewing is higher than the expected gain from interviewing a candidate who could at best only be a medium type. Rastegari, Condon, Immorlica, and Leyton-Brown (2013) solve the problem of centralized interview schedule for partially informed agents with the objective of stability and minimum number of interviews. Their model matches agents and firms at every round of interview and the results are communicated back to the central coordinating body. Using one-to-one model they establish a computationally efficient interview minimizing policy.

The interviewing literature has focused on markets with finite agents on both sides in a one-to-one matching setting. The innovation that we bring in is using the continuum setup to get around the combinatorial optimization problem and extend the problem to its more natural context of many-to-one matching. The rest of the paper is organized as follows. The next section sets up the model, preferences, and the role of interviews. Section 3 focuses on the simplest setup of common-value of agents and ordered firms with independent fitness factors. Section 4 extends the basic model to two telling examples and presents some key insights. Section 5 concludes with some minor extensions and directions for future work. All the proofs are relegated to the appendix.

2 General Model

There are $F \geq 2$ firms and a continuum of students. We use F to denote both the number of firms and the set of firms $F = \{1, 2, \cdots, F\}$. The set $-i$ is the set of firms except a particular firm i. A student of type θ has a characteristic-rank vector $\mathrm{x} \in \mathbb{X} = [0, 1]^r$ where r is the number of characteristic-ranks. Each student has a complete strict preference ordering $\succ^\theta$ over the set of all firms and being unmatched. Let $\mathcal{P}$ be the set of all such strict preference relations. The distribution of students over types in $\Theta = \mathbb{X} \times \mathcal{P}$ is given by a measure η.

Each firm i wants to recruit q_i mass of students. The vector of quotas for positions is given by q where its ith element is q_i. A market is a tuple $\{F, \mathrm{q}, \Theta, \eta\}$. Each student with characteristic-rank x is associated with a value function $V_i(\mathrm{x})$ for firm i. The vector function $\mathbb{V}$ stands for the value functions of all the F firms. Each student has a fitness factor with respect to a firm which is either 1 or 0, i.e. a "fit" or a "misfit" respectively. The probability that a student of type θ with characteristic-rank vector x is found 'fit' by firm i is given by probability $p(\theta, i)$. The market is characterized by the function tuple $\{\mathbb{V}, p(\cdot, \cdot)\}$.

I will maintain the following assumptions about the setup throughout.

Assumption 2.1 $V_i(\mathrm{x})$ *is a continuous and strictly decreasing function along each of the characteristic-rank dimensions.*

Assumption 2.2 *Interviews are costly for the firms.*

Assumption 2.3 *All firms simultaneously extend their interview offers.*

Assumption 2.4 *All firms prefer to leave a position empty rather than employ a "misfit" candidate.*

Assumption 2.5 *The "fitness factor" is revealed perfectly to the firm after the interview.*

Assumption 2.1 implies that the lower values of the characteristic-ranks are better, e.g. a student with 0.1 value is better than one with 0.2 on a particular characteristic-rank, ceteris paribus. Assumption 2.2 is captured by assuming that each firm i can interview up to $k_i q_i$ mass of students. The vector of interview quotas is given by k where its ith element is $k_i q_i$. The other assumptions are partly motivated by the real world observations and partly to keep the model tractable.

This formulation provides us various levers to set up a model. These levers can be adjusted to generate implications of the interviewing setup. For instance, the characteristic-rank vector can be just a scalar or a pair of characteristics or even a vector of individual ranking by the firms. The fitness factor can represent completely idiosyncratic performance related factors which are not correlated with the characteristic-rank, firm or the preference of the student, i.e. $p(\theta, i) = p$ for all θ and i. Or it could be correlated with the student preferences, i.e. a firm could find a student fit with a higher probability if the student prefers that firm over some other firm. The other interpretations of the fitness factor being correlated with a particular characteristic-rank are equally possible.

The timing of the model is as follows.

1. Firms consider all students. Firms shortlist and send interview offers to as many students as they want.

2. Students interview with (all) the firms they received interview offers from.

3. Firms learn the fitness factor for the students interviewed.

4. Firms and students match either centrally or in a decentralized manner where the firms extend offers.

Consider two characteristic-rank vectors x_1 and x_2. We say that $\mathrm{x}_1 < \mathrm{x}_2$ if the inequality holds element by element weakly and strictly at least for one characteristic-rank. The following concepts are important for the properties of the equilibria that arise.

Definition 2.1 *A student* 1 *with characteristic-rank vector* x_1 *is said to be* ***more capable*** *than student* 2 *with vector* x_2 *if* $\mathrm{x}_1 < \mathrm{x}_2$. *Also student* 2 *is said to be* ***less capable*** *than* 1.

Definition 2.2 *A student 1 has* ***justified envy*** *for a particular firm i with some other less capable student 2 if (s)he does not get an interview offer from i and student 2 has an interview offer from i.*

Definition 2.3 *A firm i has* ***diversity on student ranks*** *in its interview offers if the set of students with interview offers from i is not connected.*

Definition 2.4 *If there are no job offers for a non-zero measure of students following a justified envy for a particular firm with some other less capable student then it is said that a non-zero mass of students has* ***fallen through the cracks****.*

Definition 2.5 *An equilibrium of interview offers will be termed* ***essentially unique*** *if all possible equilibria differ only in zero measure students.*

The characteristic-rank vector of the students can be interpretted as resume evaluations by the firm along different characteristics. Diversity in any firms' interview offers necessarily implies justified envy but not vice versa.

3 Market with ranked firms, independent fitness, and single index

The F firms are ranked and there is agreement about the ranking in the market, i.e. all students have the same preference $\succ^\theta = 1 \succ 2 \succ 3 \succ \cdots \succ F$. There is a single index x of characteristic rank, i.e. $r = 1$ and the students are uniformly distributed over $[0, 1]$.[3] Let the probability of fitness for any student with any firm be constant, i.e. $p(\theta, i) = p$, and the value function be identical across firms, i.e. for all i, $V_i(x) = V(x)$.[4]

Assume that the final matching, for the ease of analysis, happens in a centralized manner. A decentralized market with firms extending the offers can also be modeled as a centralized market as long as the offers are not exploding offers or the offers could be exploding but the students can renege on them.[5] We call the above market as one with ranked firms, independent fitness, and single index. We begin by a simple two firm example which provides most of the intuition for the general result with more than two firms.

[3]The uniform distribution assumption is relatively innocuous in this setting of a single index. All the results are robust to this assumption. Consider the more general set up with a general distribution with the density function as $G(x)$ and let $\tilde{V}(x)$ be the continuous non-increasing value function. This can be transformed in to a uniform distribution setting with a related continuous non-increasing value function $V(x)$.

[4]This assumption is just for ease of exposition and complete information about $\mathbb{V}$ is sufficient for the results to hold.

[5]See Niederle and Roth (2009) for a justification about how market cultures dictate the offer extension process and its implications.

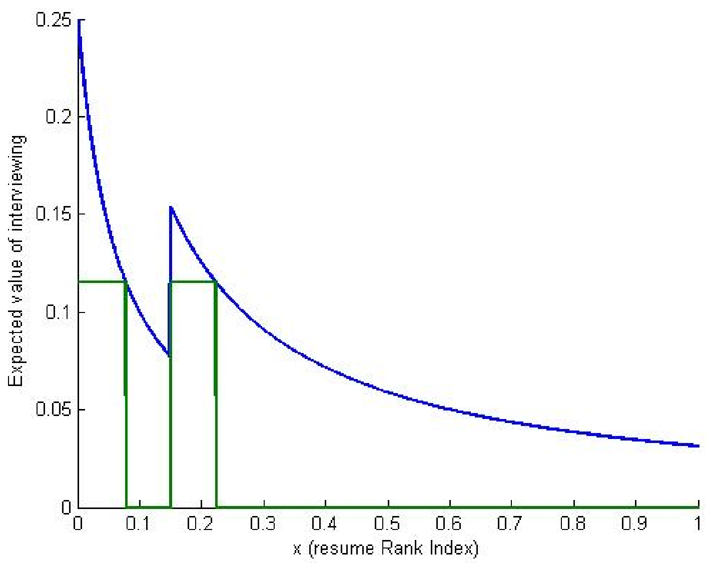
3.1 Two firms and independent fitness factors

Consider two firms 1 and 2 such that all students prefer $1 \succ 2$. There is a continuum of students indexed by x uniformly distributed over $[0, 1]$. The value function is given by $V(x) = \frac{1}{1+15x}$. Each firm wants to interview up to 0.15 mass of students and wants to recruit up to 0.075 mass. The probability of fitness is 0.5, independent of the index for both the firms. There exists an essentially unique equilibrium in this market which can be established by iterated elimination of strictly dominated strategies. The strategy of the best ranked firm can be established first as it does not depend on the strategies of any of the other players. The second ranked firm best responds to the strategy of the best ranked firm.

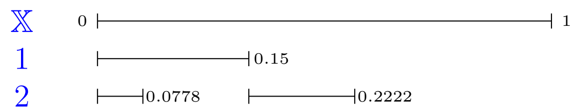
In equilibrium, 1 interviews the top 0.15 mass of students. There is a 50% probability that the students will be found fit by firm 2. However in the region [0.0.15], of those found fit, half will be found fit by firm 1 as well and 2 will lose those students to it. Thus, 2 is looking at the following function representing the effective value of interviewing a student at index x.

$$\text{Eff\_}V(x) = \begin{cases} 0.25V(x) & \text{if } x \leq 0.3, \\ 0.5V(x) & \text{if } x > 0.3 \end{cases}$$

The following graph shows this and also summarizes the (unique) choice 2 actually makes.



The green region of $[0, 0.0778] \cup [0.15, 0.2222]$ is the essentially unique optimal region for 2 to interview. This diversity results in justified envy for students in the region $(0.0778, 0.15)$ with students in $[0.15, 0.2222]$. The essentially unique equilibrium can be summarize by the following diagram. $\mathbb{X}$ is the continuum of students and the line segments in front of i represent the interview offers from firm i.



The two firm example provides the intuition behind the following proposition. The offer extension process has an essentially unique equlbrium in a continuum setting. The strategies for interviewing can be established by iterated elimination of strictly dominated strategies for the firms considered in the order of their ranking.

Proposition 3.1 *For a market with ranked firms, independent fitness, and single index, there is an essentially unique equilibrium of interview offers by the firms.*

3.2 Justified Envy and Diversity in student ranks

We will make the following four technical assumptions to make some interesting predictions about the essentially unique equilibrium outcome.

Assumption 3.1
- *Limited top firm quota:* $k_1 q_1 \leq \Sigma_{-1} \min(k_i q_i, \frac{q_i}{p})$

This assumption is to ensure that the best firm, i.e. firm 1 does not have an interview quota greater than the sum of quotas for all the other firms.

- *Comparable students:* $(1-p)^{F-1} V(0) < V\big(\max_{i \in -F}(k_i q_i)\big)$

Comparable students assumption restricts the analysis to the markets where the students at the very top with index 0 are not significantly better than the students just outside the top few students. This is reasonable in markets where a few 'superstar' students significantly better than most of the other students do not exist.

- *Sufficiently thick students' market:* $(1-p)V(0) > V(1)$ *and* $\forall\, i \;\; k_i q_i << 1$

The student with index 1 is sufficiently different from the one with index 0 and there are enough students that any firm i can not interview all the students.

- *Sufficiently thick firms' market:* $\Sigma_i \min(k_i q_i, \frac{q_i}{p}) \geq 1$

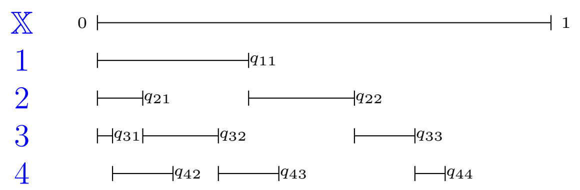
This assumption ensures that there are enough firms that together they would interview all the students if there were no overlap in the interview regions.

Proposition 3.2 *For a market with ranked firms, independent fitness, and single index, satisfying the above assumptions, the essentially unique equilibrium involves* ***justified envy*** *for some non-zero mass of students and there is* ***diversity in student ranks*** *in the choice of at least one firm.*

The intuition behind the proof of proposition 3.2 is broadly that interviewing at the top, i.e. with index 0, is preferred rather than going for the students all the way at the bottom. The continuity and monotonicity of the value function will create the diversity result where a firm interviews some students also being interviewed by better firms and some without any competition from such firms.

We interpret the above proposition as providing the sufficient conditions under which we get **diversity in student ranks** in the choice of at least one firm and existence of **justified envy**. However, the main take-away from this proposition is the possibility of these phenomena (even) in broader set of markets (which may fail these conditions). The two firm example above in subsection 3.1 did not meet the technical conditions but we still saw diversity and justified envy.

The following four firm example will further elucidate the results in the proposition. Analysis similar to subsection 3.1 can be done with four firms and $p = 0.5$. The equilibrium can be described by the following diagram.



Without any assumptions about the parameters of the problem like interview quota or recruiting quota, we establish a unique equilibrium by iterated elimination of dominated strategies.

The equilibrium is described as follows if the value function satisfies the assumptions listed.

1. The region $[0, q_{11}]$ represents the interview offers from firm 1.

2. Firm 2 interviews the region $[0, q_{21}] \cup [q_{11}, q_{22}]$ such that $0.5 \times V(q_{21}) = V(q_{22})$.

3. Firm 3 interviews the region $[0, q_{31}] \cup [q_{21}, q_{32}] \cup [q_{22}, q_{33}]$ and it ensures that $0.25 \times V(q_{31}) = 0.5 \times V(q_{32}) = V(q_{33})$.

4. Firm 4 interviews only $[q_{31}, q_{42}] \cup [q_{32}, q_{43}] \cup [q_{22}, q_{44}]$ as the value function $V(\cdot)$ is such that there does not exist a $0.125V(q_{41}) = V(q_{44})$ in fact $0.125V(0) < 0.25 \times V(q_{42}) = 0.5 \times V(q_{43}) = V(q_{44})$.

Firm 3's choice provides example for a firm extending interview offers to some stars, some medium tier students, and some safe bets. The analysis becomes interesting for 4 as it does not interview any students around index $x = 0$. Thus, a sufficiently low ranked firm does not interview the top mass of the students but starts lower down the order. This example brings forth a lot of intuitions in line with the anecdotal evidence.

4 A tale of Two Markets: Similar Firms Market and Correlated Fitness Market

This section discusses more results in two markets closely related to the simple setup in subsection 3.1.

4.1 Two similar firms with independent fitness

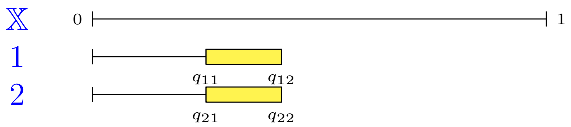
We consider a market with two firms, 1 and 2. There is a continuum of students indexed by x uniformly distributed over $[0, 1]$. Half the population of students has preferences $1 \succ 2$ and the other half has preferences $2 \succ 1$. The two firms are exactly identical in terms of quotas and interview quotas. The fitness probability is independent of the type, i.e. $p(\theta, i) = p$. A market is considered significantly heterogeneous if the students at the top are better than the ones in the interview region, i.e. $[1 - \frac{p}{2}]V(0) > V(kq)$.

The strategy of each firm will depend on the strategy of the other. If firm 1 interviews the top mass of students, firm 2 will best respond by interviewing a few at the top and a few in a region where it does not face any competition. Intuitively, this is because if firm 2 finds a student 'fit,' there is a 0.5 chance that the firm 1 also finds him/her 'fit' and furthermore there is a 0.5 chance that the student prefers firm 1. Thus a fraction 0.25 of the mass is lost in the region of overlap. If there is no overlap in the interview offers then firm 2 gets all the students it finds 'fit.' Thus with continuity of value function, we get a region of interview offers where both firms compete with each other and there is a region where they anti-coordinate. E.g. two similar geographically separated firms would compete for the best students by visiting the top-tier college campuses but only hire locally for the lower ranked students and thus anti-coordinate geographically.

Proposition 4.1 *In a market with two exactly similar firms with same quotas, single index and constant independent fitness probability, there is a single class of infinite equilibria where both the firms compete at the top and anti-co-ordinate the interview offers in any way for the lower index students.*

Moreover, for a sufficiently heterogenous market, there is a non-zero mass in both the competition region and the anti-co-ordination region for both the firms.

The following diagram represents the equilibria that arise. In every equilibrium the line segments indicate the interview offers where both firms compete and the yellow box indicates the region where the firms anti-co-ordinate and do not overlap with each other. The mass in the yellow box can be split arbitrarily up to each firm's interview quota. At least half the students in the yellow box will have justified envy.



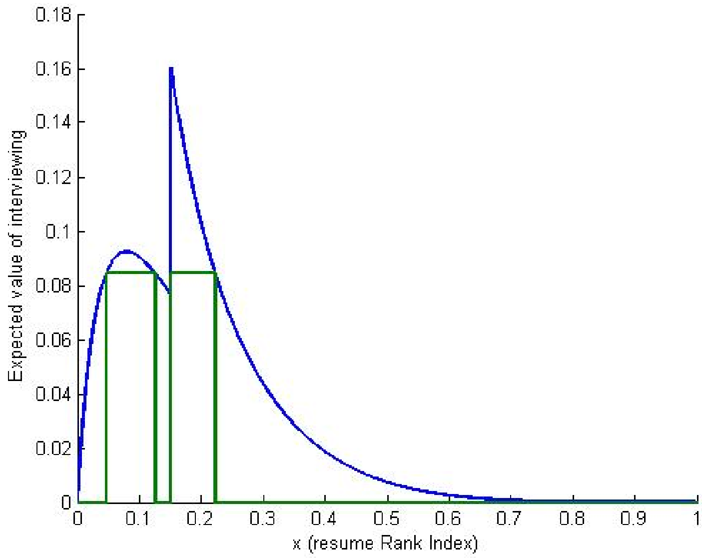
4.2 Two ordered firms and Index-correlated fitness factor

We look at a market where a student who is very good is expected to be found fit with very high probability to a firm and suppose that the fitness probability decreases with the index. We focus on ordered firms where all students have preferences of the type $\succ^\theta$: $1 \succ 2$. The strategic choices facing firm 2 change as compared to the constant independent fitness probability case in subsection 3.1. It will not interview all the way at the top as these candidates will be interviewed and found fit by 1 with high probability. So instead 2 finds it advantageous to find the sweet spot in the middle of firm 1's interviewees that are good but also have a good chance of being rejected by the top firm. This story resonates well with a medium tier department or firm trying to find the 'stars' possibly overlooked or incorrectly judged by the top tier departments or firms.

Consider the same setup as above in subsection 3.1 with two firms. Instead of a constant probability of fitness, suppose that $p(x) = (1-x)^4$. Now the effective value function for 2 is

$$\text{Eff\_}V(x) = \begin{cases} (1-x)^8 V(x) & \text{if } x \leq 0.3, \\ (1-x)^4 V(x) & \text{if } x > 0.3 \end{cases}$$

The following graph plots this and also summarizes the choice 2 actually makes.



Firm 2 finds that interviewing students in the region $[0.0468, 0.1148] \cup [0.15, 0.222]$ is optimal as the effective value function depicts above. Although the students with index closer to 0 are highly attractive, there is high probability that the top firm will find them acceptable and extend offers. Hence starting slightly lower than the absolute top is optimal for the second tier firm. These students in $[0, 0.0468)$ have justified envy with the lower index students with interview offer from 2 and in fact some of them will fall through the cracks as the probability of fitness with firm 1 is less than 1 (except for the zero measure students at $x = 0$).

5 Conclusion

In this paper, we present the first model for a many-to-one matching market with a stage of costly interviewing. This setup with a continuum of students and a finite number of firms is not only tractable but also offers great insights. This model delivers the real world phenomenon about diversity in student ranks in a firm's interview offers, i.e. strategic targeting by second and lower tier firms. It also resonates with the anecdotal evidence that some students do fall through the cracks and end up with too few or no interview or job offers. The result about competition for the best students and anti-co-ordination for medium tier students is an interesting result in the setting with comparable firms.

In these concluding remarks, we briefly discuss three extensions of the model. In the discussion so far, the fitness factor was assumed to be independent across firms. First, suppose in a market the fitness factor is correlated between firms although it is constant across the index of the student. A student found fit by firm i will be found fit with a higher probability by some other firm j. It is easy to infer that, due to adverse selection, the lower ranked firms compete less with the higher ranked firms and have smaller overlapping regions if there is more correlation. In this setup justified envy increases, ceteris paribus.

Secondly, consider a two similar firms setup where the students who prefer, say, firm 1 would be found fit with higher probability by 1 instead of having independent fitness across firms. The probability of fitness for 1 is say, p_1 and for firm 2 is say $p_2 < p_1$. It is clear that as the probability p_1 increases the competition region increases and smaller mass of students would have justified envy, for a given aggregate fitness probability.

Third, a market has students indexed on two different characteristic-ranks or abilities, e.g. numerical ability and creative mindset. In this setup, the index is a two dimensional vector. The fitness factor can be independent for ease of exposition. In the presence of different value functions for different firms, with complete information, we will still get a unique equilibrium for ordered firms in the spirit of proposition3.1. The firms which care for a particular dimension more compared to firms slightly above will extend more interview offers for students with higher index along that dimension. This says that comparable firms can endogenously choose their niche abilities as a focal point equilibrium.

An important dimension to take this forward would be to compare the coordinated (or centralized) versus non-coordinated (or decentralized) interviewing setup. That along with the ability to analyze contracts (salaries) is left as an avenue for future research.

Appendix

Proof of Proposition 3.1

The equilibrium response can be found by solving the game backwards. For a given interview schedule and final preferences, the final stage of the matching market always has a unique stable matching which follows from the facts that the outcome of DAA coincides with that of the Serial Dictatorship (SD) when all agents on one side of the market agree about the ranking of the other side (say firms). Moreover, this result can also be established from some natural extensions of the conditions under which Theorem 1 of Azevedo and Leshno (2011) holds. If the matching market has a unique stable match then truth telling is an equilibrium which follows again from DAA coinciding with SD in this setting. More importantly, the outcome of all other equilibria, if any, are exactly the same as the outcome from truth-telling of all the participants.

The interviewing stage problem can be solved for all the firms one-by-one.

We can define the best ranked firm's strategy first and then the second best and so on. The following algorithm establishes that and proves the uniqueness due to strictly decreasing value function. The algorithm starts by defining the strategy for the best firm as interviewing the top students (Step 1). For firm 2, it starts by assuming that it interviews the non-overlapping region and hence interviews at k_1q_1 and beyond ensuring that the interviewing capacity is not violated and the firm does not interview more than it needs to (Step 2). It sets up flags for $at\_capacity$ and $full\_recruitment$ to decide appropriate actions (Step 3 and 4). If the interview offers in non-overlapping region exhaust all students and still the firm has capacity to interview and needs to interview to recruit the desired number of students, the algorithm enters Step 5. Firm 2 extends interview offers to some students in the region where firm 1 is also interviewing.

If however the firm is able to recruit the desired mass of students through the non-overlapping region, Step 6 investigates whether some interviewing should happen at the top as the effective value is higher at those spots. If the firm is at its interviewing capacity Step 7 finds out if some mass of students should be substituted in to the overlapping region.

Step 1 $s_{11} = \min(k_1q_1, \frac{q_1}{p}, 1)$ and $S_1 = [0, s_{11}]$. We can restrict our attention to $1 > k_1q_1 \leq \frac{q_1}{p}$. Hence $s_{11} = k_1q_1$.

Step 2 $s_{22} = \min(s_{11} + k_2q_2, s_{11} + \frac{q_2}{p}, 1)$, $s_{21} = 0$ and $S_2 = [s_{11}, s_{22}]$.

Step 3 If $|S_2| = k_2q_2$ $at\_capacity = true.$

Step 4 If $|S_2| = \frac{q_2}{p}$ $full\_recruitment = true.$

Step 5 If $\Big[\neg at\_capacity \;\&\&\; \neg full\_recruitment\Big]$ $s_{21} = \frac{1}{1-p}\big[\frac{q_2}{p} - |S_2|\big]$ and go to step 5.1

Else go to step 6.

Step 5.1 If $|S_2| + s_{21} > k_2q_2$ then $s_{21} = k_2q_2 - |S_2|$ $at\_capacity = true$ and go to step 7.

Else go to step 5.2

Step 5.2 If $|S_2| + s_{21} = k_2q_2$ $at\_capacity = true.$

Step 5.3 $full\_recruitment = true$ and go to step 6.

Step 6 If $\Big[\neg at\_capacity \;\&\&\; full\_recruitment\Big]$ then go to step 6.1

Else go to step 7.

Step 6.1 If $(1-p)V(s_{21}) \leq V(s_{22})$ then EXIT the algorithm

Else go to step 6.2.

Step 6.2 Find Δ_1 such that

$(1-p)V(s_{21} + \frac{\Delta_1}{1-p}) = V(s_{22} - \Delta_1)$ and go to step 6.3

Step 6.3 If $s_{21} + \frac{\Delta_1}{1-p} + |S_2| - \Delta_1 > k_2q_2$ then go to step 6.4

Else go to step 6.5

Step 6.4 Find Δ_2 such that $s_{21} + \frac{\Delta_2}{1-p} + |S_2| - \Delta_2 = k_2 q_2$
Define $s_{21} = s_{21} + \frac{\Delta_2}{1-p}$ and $s_{22} = s_{22} - \Delta_2$
$at\_capacity = true$ and $full\_recruitment = false$. Go to step 7.
Step 6.5 $s_{21} = s_{21} + \frac{\Delta_1}{1-p}$ and $s_{22} = s_{22} - \Delta_2$.
$S_2 = [0, s_{21}] \cup [s_{11}, s_{22}]$ and $full\_recruitment = true$. Go to step 6.6.
Step 6.6 If $|S_2| = k_2 q_2$ then $at\_capacity = true$ and EXIT the algorithm.
Else EXIT the algorithm.

Step 7 If $(1-p)V(s_{21}) \leq V(s_{22})$ EXIT the algorithm
else go to step 7.1.
Step 7.1 Find Δ_1 such that $(1-p)V(s_{21} + \Delta_1) = V(s_{22} - \Delta_1)$
Define $s_{21} = s_{21} + \Delta_1$, $s_{22} = s_{22} - \Delta_1$ and $S_2 = [0, s_{21}] \cup [s_{11}, s_{22}]$ and EXIT the algorithm.

Firm 1 offers its interviews to $[0, s_{11}]$. Firm 2 offers its interviews to $[0, s_{21}] \cup [s_{11}, s_{22}]$.

The algorithm above establishes the strategy for the first two firms. It can be extended to define the strategies for all the remaining firms one-by-one. Intuitively, the algorithm finds the maximum support of the effective value function for a given firm i ensuring that the support is not greater than its capacity to interview $k_i q_i$ and that it will not recruit more than q_i candidates, should they be found acceptable. The effective value function is defined as follows.

$$\begin{aligned} \text{Eff\_}V_i(x) &= \text{Probability that the candidate is found unfit by all firms better than } i \\ &\quad \times p \times V(x) \end{aligned}$$

Proof of Proposition 3.2

From Proposition 3.1 we know that there is a unique equilibrium. Consider the strategies of the firms one by one in the order of their ranking. We will prove that at least one firm has **diversity** in its choice of interviews. **Justified envy** necessarily results with **diversity**. Recall the technical assumptions 3.1 which are required to prove the result.

- *Limited top firm quota:* $k_1 q_1 \leq \Sigma_{-1} \min(k_i q_i, \frac{q_i}{p})$
- *Comparable students:* $(1-p)^{F-1} V(0) < V\big(\max_{i \in -F}(k_i q_i)\big)$
- *Sufficiently thick students' market:* $(1-p)V(0) > V(1)$ *and* $\forall\, i \;\; k_i q_i << 1$
- *Sufficiently thick firms' market:* $\Sigma_i \min(k_i q_i, \frac{q_i}{p}) \geq 1$

In equilibrium, one of the following will necessarily occur.

1. The first $F-1$ firms extend their interview offers to completely non-overlapping regions of students.
2. There is some overlap in the interview offers of the first $F-1$ firms.

If (1) occurs then we know that the $F-1$th firm finds it profitable to interview the non-overlapping region rather than compete with 1. Hence the effective value of interviewing at the end of its interview region is higher than $(1-p)V(0)$, which is the effective value of interviewing at $x=0$. Due to continuity of the value function Firm F will find it optimal to interview in the non-overlapping region. However, with sufficiently thick markets we know that $\Sigma_i \min(k_i q_i, \frac{q_i}{p}) \geq 1$ and hence F extends interview offers for students with index $x=1$. Under Sufficiently heterogeneous students assumption $(1-p)V(0) > V(1)$, firm F will find expending it interview slots at $x=0$ more valuable than interviewing students at $x=1$. Due to continuity there will be also a non-zero mass of students at the top who will get interview offers. Thus, firm F has interview offers at the top and a few in the non-overlapping region at the bottom and thus **diversity** results in firm F's interview offers.

If (2) occurs then at least two firms, say i and j with $i<j$, of the first $F-1$ firms have some overlap in the interview offers. Without loss of generality let us assume that i and j are the lowest ranked such firms with overlap. With independence of the fitness probability, the strategy to interview the same mass of students at the top, i.e. $[0,\bar{q}]$ being interviewed by j dominates any other strategy with other regions of overlap. If $j \neq 2$ then firm 2 has a profitable deviation by interviewing the same region as firm j at the top. There is diversity if 2 interviews a few students at the top and a few without any competition with a hole in the middle then the result holds. Suppose there is overlap but 2's interview offers are continuous regions. Let $[0,q_{11}]$ be the region of firm 1's interview offers and $[0,q_{22}]$ be the region of firm 2's interview offers. The region will be continuous only if $p(1-p)V(q_{11}) > pV(q_{22})$. For firm 3 if there is no overlap then6 the strategy to interview at least some students in $(q_{11},\cdot]$ rather than interviewing at q_{22} and beyond because by continuity $p(1-p)V(q_{11}^{+}) > pV(q_{22}^{+})$. If firm 3 has holes in its interview offers then the result holds. Suppose not, then 3 interviews the entire region overlapping with 2 alone (but not 1) and hence firm 3 will find it profitable to interview at $x=0$ because $p(1-p)^2V(0) > p(1-p)^2V(q_{11}) > p(1-p)V(q_{22})$. This could result in holes in firm 3's offer. However, if still there is no hole in 3's offer region, we can continue the above analysis for firm 4 and then one by one for each firm. We know that from comparable students assumption, i.e. $(1-p)^{F-1}V(0) < V\big(\max_{i\in -F}(k_i q_i)\big)$, at least firm F will find it better to extend interview offers with some holes. Thus diversity results.

Thus, in either case we get diversity and hence also justified envy for a non-zero mass of students.

Proof of Proposition 4.1

The argument about truth-telling as a unique equilibrium outcome which held for proposition 3.1 also holds in this context as firms have an agreement about the indices of the student however they might disagree about the fitness factor. We can focus on the equilibrium description for interviewing strategies.

In any equilibrium, at least one of the firms extends interview offers to the top kq mass of students. If not, there is a profitable deviation for a firm to substitute the lower ranked student mass with the mass at the top. If there is no overlap at all between the interview regions for the two firms, each firm will extend offers to kq and thus $2kq$ mass of students will have interview offers from one of the two firms. Without loss of generality consider that firm 2 is not interviewing at 0^+. This can not be an equilibrium response because extending interview offers to that region and competing with 1 instead of the region beyond kq (which is non-zero) is a profitable deviation. 2 will lose $\frac{p}{2}$ mass of students to 1 in the region of overlap as half the population prefers 1 over 2 and it is found fit by firm 1 with probability p. By assumption, $[1-\frac{p}{2}]V(0) > V(kq)$. Thus by continuity both the firms extend interview offers to non-zero mass of students around 0.

The existence of anti-co-ordination region follows immediately by continuity. If both firm extend interview offers to the exact same region, i.e. kq. Then either of the firms, say firm 2 has a profitable deviation by interviewing at kq^+ instead of kq^-. In the region of competition the effective value of interviewing a student with index kq^- is $p[1-\frac{p}{2}]V(kq^-)$ whereas the same for a student without competition is $pV(kq^+)$.

Thus, in all equilibria there is a competition and an anti-co-ordination region. Furthermore, both regions are non-zero if sufficiently heterogenous students exist.

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