Job Market Signaling with Imperfect Competition among Employers

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Abstract

Spence (1973) assumes perfect competition among receivers (or employers) in a job market signaling model. In this paper, by adopting the Hotelling model, we investigate job market signaling characterized by imperfect competition among employers. In our model, workers are differentiated in the vertical and the horizontal dimensions: productivity and location (or preference), respectively. We identify both separating and pooling equilibrium. We conclude that if competition is sufficiently strong, there exists a separating equilibrium, whereas if competition is sufficiently weak, there only exists a pooling equilibrium. By comparing two different information structures with respect to workers preference, we show that, if a market is sufficiently competitive, a worker prefer the structure where her preference is publicly known to the structure where it is privately known. Moreover, we show that, with a large portion of high productivity workers, a perfectly competitive market is worse than the least competitive market in terms of social welfare.

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1 Introduction

Many realized job markets are characterized by imperfect competition among employers. For instance, in the job market for Ph.D. candidates, each candidate who has private information about her own productivity has a "(locational) preference" over a finite number of potential employers. Such private information can be signaled through a CV, writing samples, and so on.

Spence (1973) investigates the job market signaling model, in which workers (senders) signal their productivity to employers (receivers) through education which is costly but unproductive. One crucial assumption in his paper is that the market is perfectly competitive among employers. Under this assumption, employers cannot offer any wage other than those which are exactly the same as a worker's expected productivity, and earn zero profits. This makes the problem significantly simple to solve, because it allows one to focus on only workers' signaling decision. However, the limitation of the model is that it is impossible to investigate the employers' strategic behaviors, which may depend on the degree of competition among them.

In this paper, we mainly investigate employers' optimal wage offering strategies and workers signaling strategies in a job market characterized by imperfect competition among employers. Our basic model is a mixture of the job market signaling model, which has the vertical dimension of a worker's productivity, and the Hotelling model, which includes the horizontal dimension of a worker's preference. This feature widens the scope of the analysis significantly.

First, by adopting the horizontal dimension, we cannot only examine the signaling behaviors of workers in a job market, but also analyze the employers' equilibrium wage offers, which cannot be accomplished by Spence's basic model. Second, we investigate the impact of changes in competitiveness on agents' equilibrium behaviors and the social welfare. Third, we examine the changes of the agents' equilibrium strategies according to a worker's preference. In this regard, we compare signaling strategies and payoffs of workers with different preferences, and also compare agents' equilibrium behaviors under two different informational structures regarding the horizontal dimension; publicly and privately known preference. Finally, we also study a market with n firms and a case involving the continuous productivity and education level.

We investigate both separating and pooling equilibrium. Furthermore, by using the intuitive criterion, we specify the condition for pooling equilibria to survive this refinement of beliefs.

1.1 Related Literature

After Spence (1973) suggested a signaling model with incomplete information, a number of studies have examined and developed it. First of all, to resolve the problem of the plethora of equilibrium in Spence (1973), various refinements of equilibrium have been proposed (Wilson (2008), Riley (1979), and Cho and Kreps (1987)). Cho and Kreps (1987) suggest the concept of the intuitive criterion which allows elimination of many of the pooling equilibria which have counter-intuitive or unrealistic off-equilibrium beliefs.

Previous studies also have expanded or generalized the job market signaling model in the sense of the timing of games or the informational structure. McCormick (1990) studies signaling during job search. He concludes that too few unskilled workers accept interim (unskilled) jobs because a search strategy may be used as a signal of their productivity. In Noldeke and van Damme (1990) and Swinkels (1999), it is assumed that firms make offers before workers complete their education. In Noldeke and van Damme (1990), offers are public, while they are private in Swinkels (1999). Beaudry and Poitevin (1993) examine how the possibility of renegotiation affects the contractual outcomes in environments in which adverse selection is a problem. Finally, Alós-Ferrer and Prat (2012) analyze job market signaling where a employer learns the information of workers' productivity from hiring a part of them, and conclude that, when learning is efficient enough, pooling equilibria can survive the intuitive criterion.

Other studies examine job market signaling models in different frameworks, such as matching theory and experimental study. Under the matching framework, Hopkins (2012) models a labor market where workers signal their types to firms with various levels of (observable) quality. He concludes that the highest quality workers earned the highest education and are hired by the firms with the highest quality. Qingmin Liu and Samuelson (2012) try to identify stable matching with incomplete information.

Kübler et al. (2008) compare a signaling and a screening of a job market signaling model in a laboratory experiment. They conclude that there are more separating equilibrium in a signaling model than in a screening. They also observe that wages in signaling is increasing in the number of firms, but wage in screening is not.

While much has been studied on the refinements of equilibrium or the variations of the setting in a job market signaling model, there is yet no study which focuses mainly on the effect of competitiveness among firms. As mentioned above, the current paper investigates the effect of imperfect competition among firms on the equilibrium behaviors of agents in a job market signaling model with incomplete information. In this regard, Daughety and Reinganum (2008) is a good reference. They examine price competition among firms which are producing substitutes differentiated in both horizontal and vertical dimension. In particular, they focus on the effect of imperfect competition among firms and incomplete information between firms and a representative consumer. They conclude that competition among firms is weakened by incomplete information about quality. They also show that competition moderates the higher prices that are induced by incomplete information. The main difference between Daughety and Reinganum (2008) and the current paper is that, in Daughety and Reinganum (2008), there is competition among senders, while in ours, there is competition among receivers.

We also adopt some of the basic procedures of the competitive nonlinear pricing literature (Armstrong and Vickers (2001), Rochet and Stole (1997), and Yang and Ye (2008)).

The remainder of the paper is organized as follows. The next section illustrates the basic model. In Section 3, we conduct the equilibrium analysis of the duopoly model, and discuss the existence and features of certain types of the equilibrium.[1] In Section 4, we extend our model to the n-firm case, and compare the results to those from the duopoly case. Section 5 investigates an extension with continuum of types of workers and shows that the resulting equilibrium strategies are consistent with the basic model. Section 6 concludes the paper. The Appendices contain proofs and some more technical materials not presented in the main body of the paper.

2 Model

Consider a market in which there are two firms, $i = 1$ or 2, and a continuum of workers uniformly distributed in $[0, 1]$. As shown in Figure 1, each firm $i \in \{1, 2\} \equiv N$, is located at either end of the unit interval $[0, 1]$. Each worker is identified with two characteristics in vertical and horizontal dimensions: a type, θ, and a location, x. A worker (θ, x) has a productivity of $\theta \in \Theta = \{1, 2\} \equiv \{\theta_l, \theta_h\}$, and is located at $x \in [0, 1]$. In our basic model, (θ, x) is privately known. Firms share a common prior $q = Pr(\theta = \theta_l) \in (0, 1)$.

[1] We abuse the concepts "duopoly" or "monopoly" in this paper. In a job market situation, firms are buyers and workers are sellers. So, it would be more precise to use "duopsony" or "monopsony." However, those words are not commonly used in the literature. So, to avoid unfamiliarity, we use "duopoly" and "monopoly."

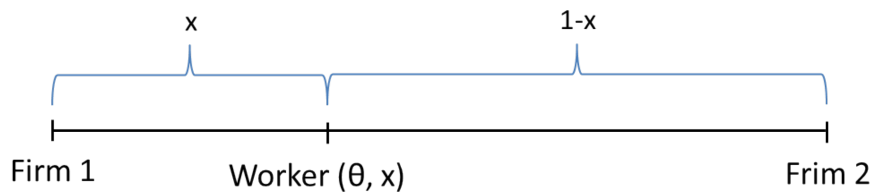

Figure 1: Basic duopoly model

A worker chooses her education level $e \in E = \mathbb{R}_+$ to signal her type to the firms. Education is costly and unproductive, but observable. Let $c(e;\theta)$ represent the cost function of acquiring education level e for a worker with productivity θ, where $c_e(e;\theta) > 0$, $c_\theta(e;\theta) < 0$ and $c_{e\theta}(e;\theta) < 0$.[2] Following Spence (1973), we assume that $c(e;\theta) = \frac{e}{\theta}$. A wage schedule, (e, w), offered by a firm is a contract to pay a wage w to a worker with education level e.

The timing of a game is as follows.

1. Nature determines each worker's characteristic, which is private information of the worker.
2. Each worker chooses an education level $e \in E = \mathbb{R}_+$.
3. Each firm, $i \in N$ offers a wage schedule (e, w) for workers who signal.
4. Each worker accepts/rejects a wage offer from a firm $i \in N$.

The utility of a worker, (θ, x), who acquires education level e, and accepts a wage w, is parametrized by the utility function:

$$u(w, e; \theta, x) = w - kx - c(e;\theta) = w - kx - \frac{e}{\theta}, \tag{1}$$

where k, $k > 0$, is a per unit "transportation" cost.[3] Notice that the utility function is increasing in the wage and decreasing in the level of education. A worker who rejects all wage offers receives a payoff of $-\frac{e}{\theta}$.

Firm i has a per-consumer profit function given by

$$\pi^i(w^i;\theta) = \theta - w^i, \tag{2}$$

[2] The last condition is required to satisfy the single-crossing property.

[3] This k can be interpreted as "locational preference" or "(relative) preference" parameter. At the same time, we can interpret it as the inverse of the degree of competition. We will discuss this further at the end of this section.

when a worker with θ accepts the firm's wage offer w^i.[4]

We define Firm i's interim belief about the type of the worker as $\mu^i(\theta|e)$ such that the firm which observes the education level of a worker, e, believes that, with probability $\mu^i(\theta|e)$, the worker has a productivity θ. Firm i's strategy is a wage function $w^i : \mathbb{R}_+ \to \mathbb{R}_+$ such that Firm i offers a wage w^i to a worker with $e \in \mathbb{R}_+$, $w^i(e) = w^i$.

We use the concept of the perfect Bayesian equilibrium. We focus on the symmetric solution in which each firm offers the identical wage schedule and has identical prior and interim beliefs on the types of the workers, and the resulting market shares are symmetric. In equilibrium, we can thus drop the superscripts to write $w^i(e) = w(e)$ and $\mu^i(\theta|e) = \mu(\theta|e)$, for both firms, $i = 1$ or 2.

We first identify a separating equilibrium in which different type-θ workers choose a different level of education.[5] We start by setting

$$\mu(\theta = 1|e < e_h) = 1 \text{ and } \mu(\theta = 2|e \geq e_h) = 1 \tag{3}$$

for a separating equilibrium to be in place with some positive value of $e_h > 0$. We proceed by backward induction.

At the last stage of the game, a worker at x would accept Firm i's wage offer, instead of Firm j's offer, only when

$$w^i - kx \geq \max\left\{w^j - k(1-x), 0\right\}. \tag{4}$$

When a worker makes the "take it or leave it" decision, they do not take the cost of signaling into account. (It is a sunk cost.) Since firms are aware of this, they also consider the signaling cost as "sunk" when they offer wage schedules.[6] Let $x \leq \frac{1}{2}$ without loss of generality. Then, in any symmetric equilibrium, where $w_i = w_j$, workers simply accept Firm 1's offer if and only if $w - kx \geq 0$. Hence, we omit the workers' strategies at the last stage from the specification of any equilibrium.

[4] From now on, subscripts are for types and locations of workers, (θ, x), and superscripts are for firms. To make the notations simple, we are going to abuse the subscripts for the types, θ_l or θ_h. For some variable T, we use T_l or T_h for T_{θ_l} or T_{θ_h}, respectively.

[5] The kind of a separating equilibrium we are going to use in the our duopoly model is with respect to the type-θ, not to the location x. However, in the complete information benchmark, the signaling strategy could vary not only in θ, but also in x.

[6] The impact of this sunk cost is similar to that of vendor lock-in or holdup problem. As the vendor lock-in and holdup problem could deter the investment or the relational specific contract, the fact that the signaling cost will be sunk after the first stage could discourage the signaling. Unlike the vendor lock-in or holdup problem, however, this signaling cost is not firm or relationship specific. This cost is sunk and deters signaling only because of the timing of the game, which gives all the bargaining powers to firms after signaling.

Given the beliefs, expressed in (3), and the workers' rational strategies at the last stage, expressed in (4), firms solve their profit maximization problems by choosing sequentially rational wage schedules. By solving the problem,

$$\max_{0 \le x_\theta\left(w_\theta^i, w_\theta^j\right) \le \frac{1}{2},\ 0 \le w_\theta^i} x_\theta\left(w_\theta^i, w_\theta^j\right)\left(\theta - w_\theta^i\right) \tag{5}$$

where $x_\theta\left(w_\theta^i, w_\theta^j\right)$ is the market share of Firm i, we identify a firm's sequentially rational strategy for each type.

After identifying sequentially rational strategies for the firms, we investigate workers' sequentially rational signaling strategies given the firms' wage schedules. Lastly, we check if the beliefs specified in (3) are consistent with the workers' separating signaling strategies.

In a similar way, we also examine pooling equilibria. We show that, for some parameter values, certain types of pooling equilibria fail the intuitive criterion of Cho and Kreps (1987).

3 Duopoly

In this section, we investigate the duopoly model. As a benchmark, we first investigate the situation that the location of each worker, x, is publicly known. Then, we move on to the main analysis under the assumption of privately known location and productivity. At the end of this section, we compare those two information structures in the sense of agents' payoffs and surpluses, and provide comparative statics with respect to the parameter k.

3.1 Complete Information Benchmark

In this subsection, we assume that the location of each worker x is publicly known. We are still focusing on the separating equilibrium in the sense that a different type of worker sends different signal in equilibrium, $e_l = 0$ and $e_h > 0$. This means that, in any separating equilibrium, firms can correctly infer a worker's type θ. Moreover, in this subsection, because the worker's location x is observable to the firms, they can differentiate wage offers based on the location x as well as signals from the workers.

To identify the sequentially rational wage schedules, we compare the firms' willingness to pay, θ, to a worker's willingness to sell, by inspecting the worker's (I.R.) conditions for both firms. Without loss of generality, we assume $x \le \frac{1}{2}$.

$$w_\theta^1 - kx \geq 0 \Leftrightarrow w_\theta^1 \geq kx \tag{6}$$

$$w_\theta^2 - k(1-x) \geq 0 \Leftrightarrow w_\theta^2 \geq k(1-x) \tag{7}$$

(6) and (7) are a worker (θ, x)'s I.R. condition for Firm 1 and Firm 2. A worker (θ, x) are willing to accept the wage offers only when Firm 1 and Firm 2's offers are greater or equal to kx and $k(1-x)$ respectively. With these, we examine three different scenarios.

First, if $kx > \theta$, or equivalently $x > \frac{\theta}{k}$, the worker's willingness to sell for either Firm 1 or Firm 2, kx or $k(1-x)$, is greater than the firms' willingness to pay, θ. Hence, the worker will not be hired by any firm. Therefore, when $k > 2\theta$, $x \in \left(\frac{\theta}{k}, 1 - \frac{\theta}{k}\right)$ is excluded from the market.

Second, if $kx \leq \theta$ and $k(1-x) > \theta$, the firms' willingness to pay is greater than or equal to the worker's willingness to sell for Firm 1, but is not for Firm 2. It means that, there is no competition between firms for the worker. Therefore, for the worker (θ, x), Firm 1 acts like a local monopolist, and offer the wage $w_\theta^1 = kx$.

Finally, if $kx \leq \theta$ and $k(1-x) \leq \theta$, the firms' willingness to pay is greater than or equal to the worker's willingness to sell for both firms. Hence, a worker (θ, x) would accept Firm 1's offer if $w_\theta^1 - kx - \frac{e_\theta}{\theta} \geq w_\theta^2 - k(1-x) - \frac{e_\theta}{\theta}$. As mentioned before, Firm 2 is willing to pay up to $w_\theta^2 = \theta$. Combine this with I.R. condition, we get $w_\theta^1 = kx$ if $\theta - k(1-2x) \leq kx$, and $w_\theta^1 = \theta - k(1-2x)$ if $\theta - k(1-2x) > kx$.

From the analysis above, we identify the sequentially rational wage schedules, when $k \leq 2$.[7]

$$w(x, e < e_h) = \begin{cases} kx, & x \leq 1 - \frac{1}{k} \\ 1 - k(1-2x), & 1 - \frac{1}{k} < x \leq \frac{1}{2} \end{cases}$$

$$w(x, e \geq e_h) = 2 - k(1-2x).$$

Corresponding sequentially rational and Pareto dominant signaling for both types of workers, when $k \leq 2$, are[8]

$$e(x, \theta = 1) = 0 \tag{8}$$

$$e(x, \theta = 2) = \begin{cases} 2 - k(1-x), & x \leq 1 - \frac{1}{k} \\ 1, & 1 - \frac{1}{k} < x \leq \frac{1}{2} \end{cases}. \tag{9}$$

[7] If $k > 2$, a high type worker $x \in [0, k-2)$ is better off by exiting from the market than by choosing the sequentially rational signaling. Hence, there is no separating equilibrium.

[8] For the high type workers, there can be other signaling strategy which is dominated by (9) in the sense of the Pareto optimality. In this paper, we just identify the Pareto dominant signaling strategy.

We summurize the above analysis as follows.

Definition 1 (Worker's payoff)**.**

$$
\begin{aligned}
u(e;\theta_h,x) =& 2-k(1-2x)-kx-\tfrac{e}{2}\\
=& \begin{cases} 1-(1-x)\frac{k}{2}, & x\le 1-\frac{1}{k}\\ \frac{3}{2}-(1-x)k, & 1-\frac{1}{k}<x\le\frac{1}{2}\end{cases}\\
u(e;\theta_l,x) =& \begin{cases} 0, & x\le 1-\frac{1}{k}\\ 1-(1-x)k, & 1-\frac{1}{k}<x\le\frac{1}{2}\end{cases}
\end{aligned}
$$

Definition 2 (Workers' surplus: Producer surplus)**.**

$$
WS(k)=\begin{cases} \frac{3}{2}-\frac{1}{2}q-\frac{3}{4}k, & k\le 1\\ \frac{1}{4}k-\frac{1}{2}-\frac{1}{2}q+\frac{1}{k}, & 1<k\le 2\end{cases}
$$

Definition 3 (Firms' surplus: Consumer surplus)**.**

$$
FS(k)=\begin{cases} \frac{k}{2}, & k\le 1\\ 2q-\frac{q}{k}+\frac{k}{2}-kq, & 1<k\le 2\end{cases}
$$

Definition 4 (Total surplus: Social welfare)**.**

$$
SW(k)=\begin{cases} \frac{3}{2}-\frac{1}{2}q-\frac{1}{4}k, & k\le 1\\ \frac{3}{2}q-\frac{q}{k}-kq+\frac{3}{4}k+\frac{1}{k}-\frac{1}{2}, & 1<k\le 2\end{cases}
$$

3.2 Separating equilibrium

From now on, we focus on the incomplete information scenario. Now we assume that a worker's location x is also privately known to the worker. Investigating procedure is as follows. First, under assumption that a different type of worker chooses a different education level, we identify the firms' sequentially rational wage schedules. After that, we examine which education levels are incentive compatible. For every k, there is a continuum of equilibria with different levels of education. We identify all the equilibria in the following propositions. As a refinement, we also specify the Pareto dominant separating equilibrium, which is also optimal for the workers. After identifying the separating equilibria, we also examine the existence of pooling equilibria for every possible value of k. Then, by using the intuitive criterion, we refine the pooling equilibria we found. Finally, we conduct welfare analysis along with comparative statics results.

3.2.1 Firms' wage offers

To identify the firms' sequentially rational wage offers in a separating equilibrium, we study three different ranges of k separately, $k \leq \frac{2}{3}\theta$, $k > \theta$, and $\frac{2}{3}\theta < k \leq \theta$.

Full-coverage under competition For $k \leq \frac{2}{3}\theta$, we assume that, in a symmetric equilibrium, the two firms will cover all workers. As we identify a separating equilibrium, we assume $\mu(\theta = 1|e < e_h) = 1$ and $\mu(\theta = 1|e \geq e_h) = 0$ for some positive value of $e_h > 0$. In this full-coverage regime, it is assumed that every worker is willing to accept at least one offer from either firm. A worker (θ, x) would prefer to accept Firm 1's wage offer w_θ^1 if

$$w_\theta^1 - kx \geq w_\theta^2 - k(1-x).$$

From this, we get the market share of Firm 1, x_θ^1.

$$x_\theta^1 = \frac{w_\theta^1 - w_\theta^2}{2k} + \frac{1}{2}$$

Then, the maximization problem for Firm 1 turns out to be

$$\max_{w_\theta^1 < \theta} \left(\frac{w_\theta^1 - w_\theta^2}{2k} + \frac{1}{2} \right) (\theta - w_\theta^1).$$

The first order condition with respect to w_θ^1 is

$$\text{(F.O.C.)} \qquad \frac{1}{2k}(\theta - w_\theta^1) - \left(\frac{w_\theta^1 - w_\theta^2}{2k} + \frac{1}{2} \right) \leq 0 \text{ with equality if } w_\theta^1 < \theta.$$

Therefore, in a symmetric solution where $w_\theta^1 = w_\theta^2$, each firm's wage offer is $w_\theta^D = \theta - k$. To be consistent with the full coverage assumptions, the wage offer should satisfy the following constraint.

$$\text{(10)} \qquad w_\theta^1 - kx_\theta^1 = w_\theta^1 - k\left(\frac{w_\theta^1 - w_\theta^2}{2k} + \frac{1}{2} \right) \geq 0$$

This constraint requires that the I.R. conditions for all workers are satisfied. In symmetric equilibrium, $w_\theta^1 = w_\theta^2 = w_\theta^D = \theta - k$, this condition holds for any $k \leq \frac{2}{3}\theta$.

$$w_\theta^1 - k\left(\frac{w_\theta^1 - w_\theta^2}{2k} + \frac{1}{2} \right) = \theta - k - \frac{1}{2}k = \theta - \frac{3}{2}k \geq 0$$

Therefore, for $k \leq \frac{2}{3}\theta$, the wage schedules for the both types of workers are

$$w_l^D = 1 - k, \quad k \leq \frac{2}{3}$$

$$w_h^D = 2 - k, \quad k \leq \frac{4}{3}$$

, for the low and the high type worker, respectively.

Partial-coverage For $k > \theta$, we assume that some workers in the middle will not be hired by any of the two firms. Hence, we need to redefine the market share, x_θ^1. Unlike the full-coverage case, in the partial-coverage regime, it is assumed that every worker is willing to accept at most one offer from one of the two firms which is closer to the worker. Hence, a workers $(\theta, x) \in \Theta \times \left[0, \frac{1}{2}\right]$ would accept Firm 1's wage offer if

$$w_\theta^1 - kx \geq 0.$$

This is satisfied with equality at x_θ^1 .

$$x_\theta^1 = \frac{w_\theta}{k}$$

Therefore, when $k > \theta$, Firm 1's objective function is as follows.

$$\max_{w_\theta < \theta} x_\theta^1(\theta - w_\theta^1) = \frac{w_\theta}{k}\left(\theta - w_\theta^1\right).$$

The first order condition is,

$$\theta - 2w_\theta^1 \leq 0 \text{ with equality if } w_\theta^1 < \theta. \tag{11}$$

In a symmetric solution, $w_\theta^1 = w_\theta^2 = w_\theta^D = \frac{\theta}{2}$. Because the corresponding market share is $x_\theta^1 = \frac{\theta}{2k} < \frac{1}{2}$, the solution is consistent with the partial coverage assumption. Therefore, when $k > \theta$, the firm's wage offers are

$$w_l^D = \tfrac{1}{2}, 1 < k,$$

$$w_h^D = 1, 2 < k.$$

Full-coverage without competition So far, we find sequentially rational wage schedules for both types of workers when $k > \theta$, and $k \leq \frac{2}{3}\theta$. Now, we need to identify the wage offers for the intermediate value of k where $\frac{2}{3}\theta < k \leq \theta$.

First, we argue that, for $\frac{2}{3}\theta < k \leq \theta$, the partial-coverage wage offer is not sequentially rational. In a symmetric equilibrium that we are focusing on, to be in the partial-coverage regime, a firm's market share should be less than $\frac{1}{2}$. Let $x_\theta^1 = \frac{w_\theta^1}{k} < \frac{1}{2}$, which implies $w_\theta^1 < \frac{k}{2}$. By substituting this into the first order condition of the profit maximization in the partial-coverage regime, (11), we get

$$\theta - 2w_\theta^1 > \theta - k \geq 0.$$

This implies that, at any value of the wage offer $w_\theta^1 < \frac{k}{2}$, the firm has incentive to raise the wage level. Hence, the partial-coverage wage offer is not sequentially rational when $\frac{2}{3}\theta < k \leq \theta$.

Second, to cover all workers in a symmetric equilbrium, the market share for each firm should be $\frac{1}{2}$. From the full-coverage condition (10), we can conclude that, the wage offer which covers all workers (and makes the entire wage schedule continuous in k), is $\frac{k}{2}$. Let $w_\theta^2 = \frac{k}{2}$, and check Firm 1's deviation incentive from $\frac{k}{2}$ to some other value of $w_\theta^1 > \frac{k}{2}$.

$$\max_{w_\theta \geq \frac{k}{2}} \left(\frac{w_\theta^1 - w_\theta^2}{2k} + \frac{1}{2} \right) (\theta - w_\theta^1) = \left(\frac{w_\theta^1}{2k} + \frac{1}{4} \right) \left(\theta - w_\theta^1\right).$$

First order condition for this problem is as follows.

$$\frac{\theta - w_\theta^1}{2k} - (\frac{w_\theta^1}{2k} + \frac{1}{4}) \leq 0 \Leftrightarrow \theta - 2w_\theta^1 - \frac{k}{2} \leq 0 \text{ with equality if } w_\theta > \tfrac{k}{2}$$

From this condition, we can conclude that at any value of $w_\theta > \frac{k}{2}$, the firm has incentive to decrease the level of the wage offer.

$$\theta - 2w_\theta^1 - \frac{k}{2} < \frac{3k}{2} - 2w_\theta^1 - \frac{k}{2} = k - 2w_\theta^1 < 0$$

It implies that, for $\frac{2}{3}\theta < k \leq \theta$, the sequentially rational wage offer is $w_\theta = \frac{k}{2}$.

Therefore, the entire sequentially rational wage schedules for the low type worker and the high type worker are as follows.

$$w_l^D = \begin{cases} 1 - k, & k \leq \frac{2}{3} \\ \frac{k}{2}, & \frac{2}{3} < k \leq 1 \\ \frac{1}{2}, & 1 < k \end{cases} \tag{12}$$

$$w_h^D = \begin{cases} 2-k, & k \leq \frac{4}{3} \\ \frac{k}{2}, & \frac{4}{3} < k \leq 2 \\ 1, & 2 < k \end{cases} \tag{13}$$

3.2.2 Workers' signaling strategies

For the last step, given the firms' wage schedules, (12) and (13), and beliefs, (3), we identify the workers' sequentially rational signaling strategies. By investigating the incentive compatibility of each type of worker, for a given value of k, we define the separating signaling strategy, where $e_l = 0$ and $e_h > 0$.[9] Without loss of generality, we still assume $x \leq \frac{1}{2}$

By substituting (12) or (13) into (1), we define the expected utility of each type of worker who take the education of e.

$$u(w,e;\theta,x,k) = \begin{cases} \theta - k - kx - \frac{e}{\theta}, & k \leq \frac{2\theta}{3} \\ \frac{k}{2} - kx - \frac{e}{\theta}, & \frac{2\theta}{3} < k \leq \theta \\ \frac{1}{2} - kx - \frac{e}{\theta}, & \theta < k \end{cases}$$

To be incentive compatible, there should be no deviation incentive for both types of worker. Hence, for any given value of k, we need to identify the level of education which satisfies

$$(IC_l) \qquad u(w, e = e_l; \theta_l, x, k) \geq u(w, e = e_h; \theta_l, x, k)$$
$$(IC_h) \qquad u(w, e = e_h; \theta_h, x, k) \geq u(w, e = e_l; \theta_h, x, k).$$

Each type's corresponding signaling strategy which does not violate the above incentive compatibility conditions are as follows. (See Appendix A.2.1 for the detail analysis. We show that there is no sequentially rational separating signaling strategy if $k > \frac{5}{4}$.)

$$e(\theta = 1, x; k) = 0, \quad k \leq \frac{5}{4}$$
$$e(\theta = 2, x; k) = \begin{cases} [1,2] & k \leq \frac{2}{3} \\ \left[2 - \frac{3}{2}k, 4 - 3k\right], & \frac{2}{3} < k \leq 1 \\ \left[\frac{3}{2} - k, 4 - 3k\right], & 1 < k \leq \frac{5}{4} \end{cases}$$

[9] In any separating equilibrium, e_l should always be zero. If $e_h > e_l > 0$ and $\mu(\theta = 1|e < e_h) = 1$, there is alway an incentive to deviate from $e = e_l$ to $e = 0$.

3.2.3 Propositions

We can summarize above analysis in the next proposition.

Proposition 1 (Separating Equilibrium in the Duopoly Model)**.**
In duopoly case with any value of $q \in (0,1)$, for every possible $k \leq \frac{5}{4}$ and corresponding value of education $e_h \in E(k)$, there is a separating equilibrium consisting of a strategy profile $\langle w(e;k), e(\theta,x;k)\rangle$ and supporting beliefs $\mu(\theta|e)$.
Specifically, for a given $k \leq \frac{5}{4}$ and $e_h \in E(k)$, a separating equilibrium is a triple, $\langle w(e;k), e(\theta,x;k), \mu(\theta|e)\rangle$, which satisfies

$$w(e<e_h;k)=\begin{cases}1-k, & k \leq \frac{2}{3}\\ \frac{k}{2}, & \frac{2}{3}<k\leq 1\\ \frac{1}{2}, & 1<k\leq\frac{5}{4}\end{cases},\ w(e\geq e_h;k)=2-k,\ \ k\leq\frac{5}{4}$$

$$e(\theta,x;k)=\begin{cases}0, & \theta=1\\ e_h, & \theta=2\end{cases}$$

$$\textit{where } e_h\in E(k)=\begin{cases}[1,2] & k\leq\frac{2}{3}\\ \left[2-\frac{3}{2}k, 4-3k\right], & \frac{2}{3}<k\leq 1\\ \left[\frac{3}{2}-k, 4-3k\right], & 1<k\leq\frac{5}{4}\end{cases},$$

$$\mu(\theta=1;e<e_h)=1,\ \ \mu(\theta=1;e\geq e_h)=0.$$

Proposition 2 (Pareto Dominant Separating Equilibrium in Duopoly)**.**
In duopoly case with any value of $q \in (0,1)$, for every possible $k \leq \frac{5}{4}$, there is a separating equilibrium consisting of a strategy profile $\langle w(e;k), e(\theta,x;k)\rangle$ and supporting beliefs $\mu(\theta|e)$. Specifically, for a given $k \leq \frac{5}{4}$, a separating equilibrium is a triple, $\langle w(e;k), e(\theta,x;k), \mu(\theta|e)\rangle$, which satisfies

$$w(e<e_h;k)=\begin{cases}1-k, & k \leq \frac{2}{3}\\ \frac{k}{2}, & \frac{2}{3}<k\leq 1\\ \frac{1}{2}, & 1<k\leq\frac{5}{4}\end{cases},\ w(e\geq e_h;k)=2-k,\ \ k\leq\frac{5}{4}$$

$$e(\theta,x;k)=\begin{cases}0, & \theta=1\\ e_h(k), & \theta=2\end{cases}$$

$$\textit{where } e_h(k)=\begin{cases}1, & k\leq\frac{2}{3}\\ 2-\frac{3}{2}k, & \frac{2}{3}<k\leq 1\\ \frac{3}{2}-k, & 1<k\leq\frac{5}{4}\end{cases},$$

$$\mu(\theta=1;e<e_h)=1,\ \ \mu(\theta=1;e\geq e_h)=0.$$

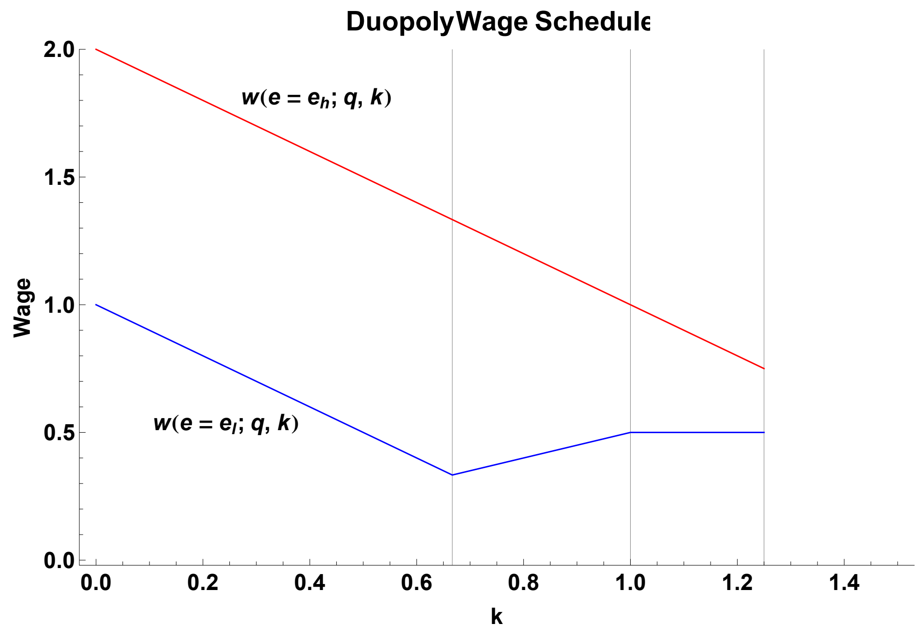

Figure 2: the Separating Equilibrium Wage Schedule.

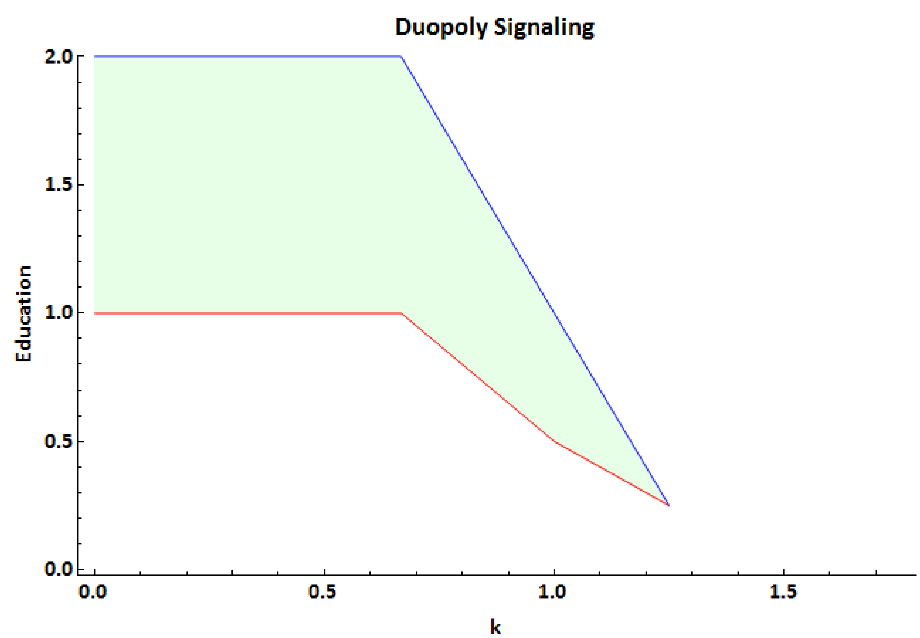

Figure 3: Signaling in Duopoly.

Figure 3 illustrates the sequentially rational signaling for the duopoly model. The red line represents the sequentially rational signaling in the Pareto dominant equilibrium.

We summarize the main economic implications of the above results in the following remarks. The first remark is about relationship between the competition and the existence of the separating equilibrium. Only when there exists a separating equilibrium, signaling could be meaningful.

Remark 1 (The Degree of Competition and Signaling)**.**
When $k \leq \frac{5}{4}$, there exists a symmetric separating equilibrium. In words, when there is enough competition in the market, signaling works. Therefore, when $k > \frac{5}{4}$, there is no separating equilibrium. If the market is similar to a monopoly market, then signaling does not work.

One of the main goals of this paper is to examine the role of the horizontal dimension in signaling behavior, i.e., location and transportation costs. Therefore, it is important to investigate the parameter in that dimension, k. There are two different interpretations of the parameter k. The first one is "the degree of the competition". Our main conjecture is that, when competition softens, firms which may have higher market powers would lower the level of the wage offers. Moreover, if the degree of the competition is sufficiently small, then the incentive for workers to take costly educations would disappear because the lowered wage would decrease the (expected) marginal benefit of the signaling.

The second interpretation of the parameter k is that "the intensity of the preferences." When k increases, the I.R. condition of a worker is harder to be satisfied. In other words, the worker's (locational) preference is stronger in the market with higher k. So, to hire a worker, a firm may offer relatively higher wage. As a result, the intensity of preferences may affect the features or even existence of certain types of equilibria in our model.

Remark 2 (Conflict between two different effects related to k)**.**
*There are two opposed effects of increasing in k. When k increases, the competition would be lessened (**the competition effect**), but at the same time, the preference would be more stronger (**the preference effect**). The former may decrease firms' wage offers, and the later may increase firms' wage offers. The conflict between the aforementioned effects results in a non-monotonic wage schedule in k.*

- $k \leq \frac{2}{3}$: *For both types of workers, the competition effect dominates the preference effect. The levels of the equilibrium wages which are competitive wages are decreasing in k. Higher competition implies higher* *wage. If $k = 0$, or equivalently if the market is perfectly competitive, the competitive wage for each type-θ converges to θ, which is exactly the same as in Spence (1973).*

- $\frac{2}{3} < k \leq \frac{5}{4}$: *For the high type worker, the competition effect still dominates. So, the level of the equilibrium wage offer for the high type worker is decreasing in k. For the low type of worker, the preference effect dominates. Ergo, the level of the equilibrium wage offer for the low type worker is (weakly) increasing in k.*

From the worker's point of view, the (expected) marginal benefit of the signaling decreases in k, while the marginal cost of taking education $\frac{1}{\theta}$ is constant. This discourages the worker from taking education. Therefore, when k increases, a worker would have less incentive to take costly education. It implies that, if k is too high, there cannot exist any separating equilibrium because no worker wants to signal. (See Appendix A.2.2 for the further analysis.)

3.3 Pooling equilibrium

In the pooling equilibrium, both types take the same education level. Intuitively, the education level they would take in equilibrium should be the lowest possible level of education, which is 0 in our model. So, all firms will have the same beliefs for the low level of education.

$$\mu(\theta = \theta_l | e = 0) = q \tag{14}$$

From this belief, we calculate the firms' (ex-ante) expected marginal benefit from hiring a worker on the equilibrium path.

$$E(\theta) = q\theta_l + (1-q)\theta_h = 2 - q$$

The analysis procedure to find a pooling equilibrium is similar to that for a separating equilibrium, but even simpler. For $k \leq \frac{2}{3}(2-q)$, we assume all workers are hired by one of the two firms. The firms maximize

$$\max_{w_P^1 < 2-q} \left(\frac{w_P^1 - w_P^2}{2k} + \frac{1}{2} \right) ((2-q) - w_P^1). \tag{15}$$

By solving (15), we identify the wage schedule when $k \leq \frac{2}{3}(2-q)$, which is consistent with the full coverage assumption.

$$w_P = (2-q) - k, \;\; k \leq \frac{2}{3}(2-q)$$

For $(2-q) < k$, we assume some workers will not be hired by either firm. From the maximization problem,

$$\max_{w_P} x_P \left((2-q) - w_P^1\right) = \frac{w_P^1}{k}\left((2-q) - w_D^1\right),$$

we can specify the sequentially rational wage schedule, which is consistent with the partial coverage assumption,

$$w_P = \frac{(2-q)}{2}, \;\; (2-q) < k.$$

For $\frac{2}{3}(2-q) < k \leq 2(2-q)$, as in the separating equilibrium, the candidate wage schedule is $\frac{k}{2}$. From the maximization problem as follows,

$$\max_{w_P \geq \frac{k}{2}} \left(\frac{w_P^1 - w_P^2}{2k} + \frac{1}{2}\right)((2-q) - w_P^1) = \left(\frac{w_P^1}{2k} + \frac{1}{4}\right)\left((2-q) - w_P^1\right),$$

we confirm that the sequentially rational wage schedule is $\frac{k}{2}$ when $\frac{2}{3}(2-q) < k \leq 2(2-q)$.

These wage schedules have exactly same functional forms as those for the separating equilibrium. Hence, we can follow the similar procedure as above to identify the sequentially rational wage schedule given the belief, (14).

$$w_P^D = \begin{cases} (2-q) - k, & k \leq \frac{2}{3}(2-q) \\ \frac{k}{2}, & \frac{2}{3}(2-q) < k \leq (2-q) \\ \frac{(2-q)}{2}, & (2-q) < k \end{cases}.$$

Unlike the separating equilibrium case, for the rationality of the workers' pooling strategy, we don't have to check all possible deviation incentives. In fact, because a possible deviation is from the equilibrium strategy to the action off the equilibrium path, and there could be infinitely many consistent beliefs and corresponding sequentially rational wage schedules, it is impossible and unnecessary to check every possible case. Only by specifying appropriate off the equilibrium belief, which does not allow any profitable deviation, we can fairly identify pooling equilibria, which are as follows.

Proposition 3 (Pooling Equilibrium in the Duopoly Model)**.**
In duopoly case, for any $q \in (0,1)$ and $k > 0$, there exists a pooling equilibrium consisting of a strategy profile $\langle w_p(e;k), e_p(\theta,x;k)\rangle$ and supporting beliefs $\mu_p(\theta|e)$.
Specifically, for given $q \in (0,1)$ and $k > 0$, a separating equilibrium is a triple, $\langle w_P(e;k), e_P(\theta,x;k), \mu_P(\theta|e)\rangle$, which satisfies

$$w_p(e=0;k) = \begin{cases} (2-q)-k, & k \leq \frac{2}{3}(2-q) \\ \frac{k}{2}, & \frac{2}{3}(2-q) < k \leq (2-q) \\ \frac{(2-q)}{2}, & (2-q) < k \end{cases},$$

$$w_p(e=1;k) = \begin{cases} (2-r)-k, & k \leq \frac{2}{3}(2-r) \\ \frac{k}{2}, & \frac{2}{3}(2-r) < k \leq (2-r) \\ \frac{(2-r)}{2}, & (2-r) < k \end{cases}.$$

$$e_p(\theta,x;k) = 0$$

$$\mu_p(\theta=1|e=0) = q, \;\; \mu_p(\theta=1|e=1) = r > q.$$

We refine these multiple pooling equilibria by using the intuitive criterion. We defer the formal proof to A.3 and state the propositions here.

Proposition 4 (Failing the Intuitive Criterion)**.**
If $k < \frac{4}{3}$, all pooling equilibria fail the Intuitive Criterion.

Proposition 5 (Survival of pooling equilibrium)**.**
If $k \geq \frac{4}{3}$, all pooling equilibria survive the Intuitive Criterion.

The fact that, when $k > \frac{4}{3}$, all pooling equilibria survive the intuitive criterion is related to the degree of competition. If the market is not competitive and more like monopoly market, then each local monopoly firm has almost all the market power in either one of two locally separated markets. So, the workers, who realize they will only get the wages which just cover the transportation costs, do not have any incentive to acquire costly education. Therefore, there is no (potentially) profitable deviation from the pooling equilibrium strategy for any type of workers.

Remark 3 (Existence of and Types of Equilibria)**.**

1. *Separating Equilibrium: The existence depends on the value of k.*
   - *If $k \leq \frac{5}{4}$, then there exists a separating equilibrium.*
   - *If $k > \frac{5}{4}$, then there is no separating equilibrium.*

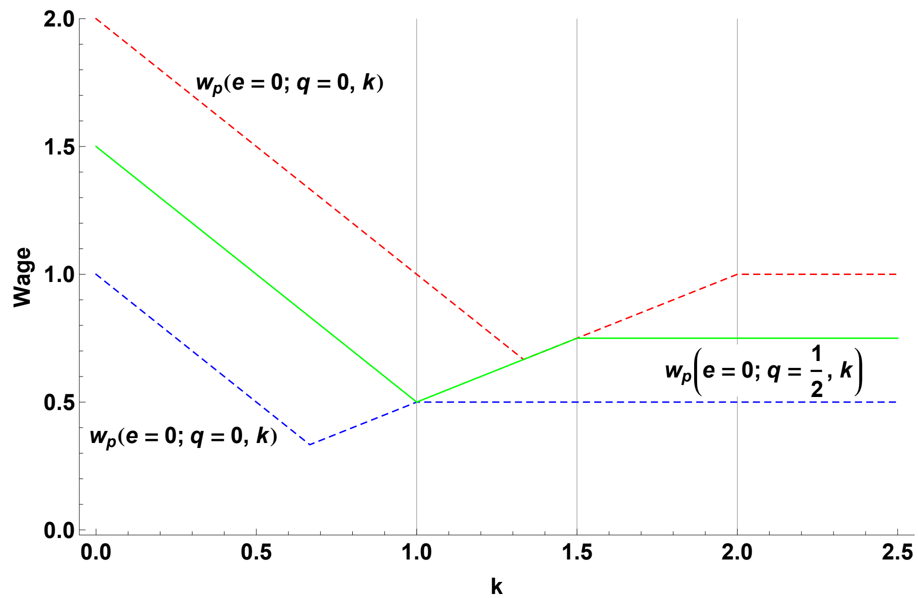

Figure 4: the Pooling Equilibrium Wage Schedule.

2. *Pooling Equilibrium: There exists a pooling equilibrium for any k.*
   - *If $k \leq \frac{4}{3}$, then all pooling equilibria fail the intuitive criterion.*
   - *If $k > \frac{4}{3}$, then all pooling equilibria survive the intuitive criterion.*
3. *These results do not depend on the vale of q.*

3.4 Comparative statics and welfare analysis

Figure 5 compares the Pareto optimal levels of the signaling of the high type workers under the incomplete information and that under the complete information. Under both structures, the level of education is weakly decreasing in k. As we discussed above, as k increases, then the competition decreases. This would (weakly) lessen the incentive for the high type workers to take costly education. The second observation we make is that the education is weakly higher in the complete information than in the incomplete information. In the sense of incentives, we conclude that there are (weakly) higher incentive for the high type workers to signal under the complete information.

We also compare the workers' levels of utilities under two different information structures.

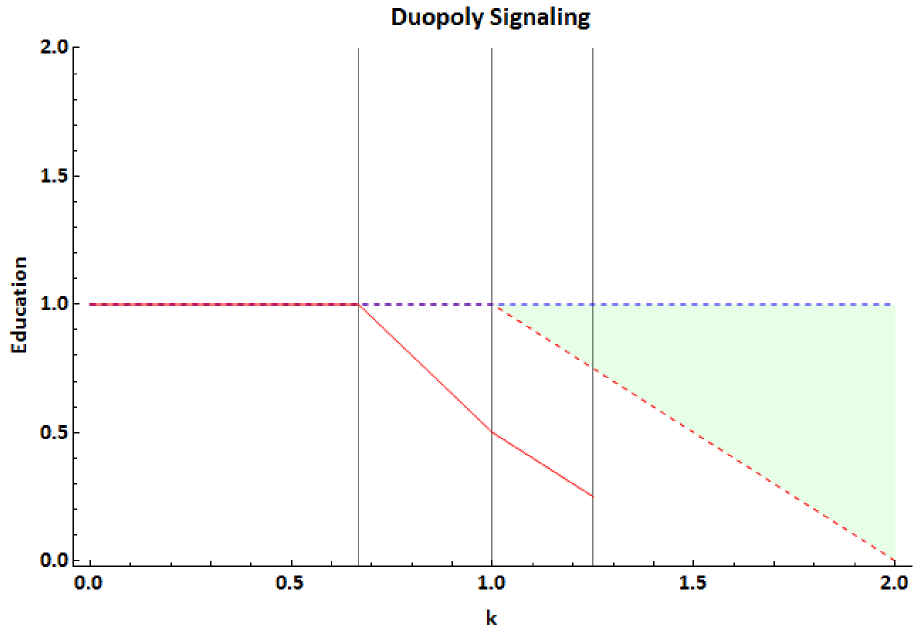

Figure 5: Signaling in Duopoly by comparing with incomplete information.

Definition 5 (Worker's payoff)**.**

$$u(e^D;\theta_h,x)=\begin{cases}2-k-kx-\frac{1}{2}=\frac{3}{2}-(1+x)k, & k\leq\frac{2}{3}\\ 2-k-kx-\frac{2-\frac{3}{2}k}{2}=1-(\frac{1}{4}+x)k, & \frac{2}{3}<k\leq 1\\ 2-k-kx-\frac{\frac{3}{2}-k}{2}=\frac{5}{4}-(\frac{1}{2}+x)k, & 1<k\leq\frac{5}{4}\end{cases}$$

$$u(e^D;\theta_l,x)=\begin{cases}1-k-kx, & k\leq\frac{2}{3}\\ \frac{k}{2}-kx, & \frac{2}{3}<k\leq 1\\ \frac{1}{2}-kx, & 1<k\leq\frac{5}{4}\ and\ x\leq\frac{1}{2k}\end{cases}$$

Figure 6 and Figure 7 depict the utilities of the high type workers and the low type workers respectively under two information structures. Under the complete information, a worker near the middle of the interval $x=\frac{1}{2}$ gets higher utility than one near the end $x=0$, because the degree of competition varies in the location x, and more specifically it increases in the location x. On the contrary, under the incomplete information, a worker near the end gets higher utility than one near the middle. Under the incomplete information, the equilibrium wage and the signaling level do not vary in the location x. Hence, in equilibrium, a workers utility with lower x, whose

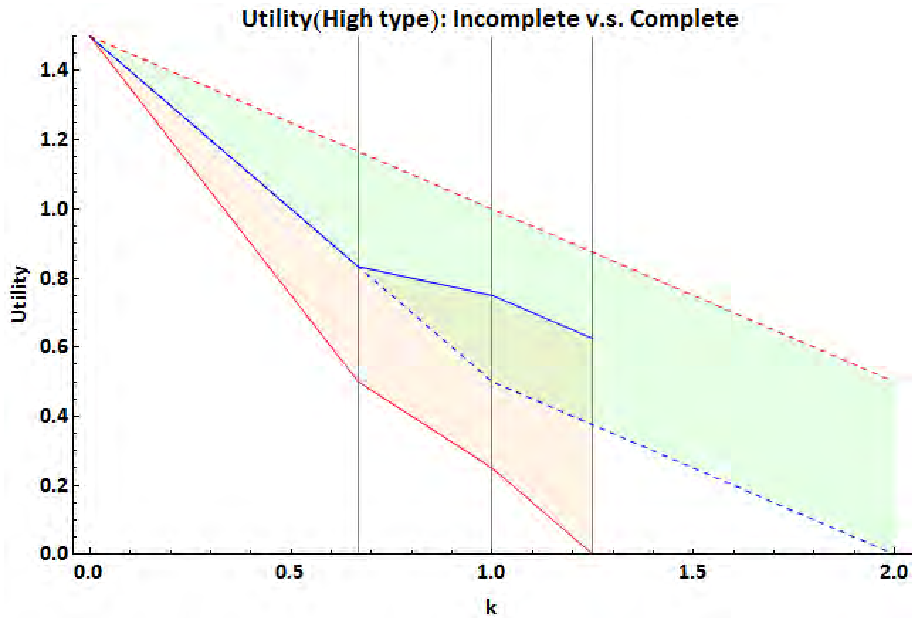

Figure 6: Utility of a high type worker.

gross transportation cost kx is relatively low, gets higher utility than one with higher x.

When $k < \frac{2}{3}$, all workers better off under the complete information except a worker with $x = 0$, who is indifferent. When $\frac{2}{3} < k \leq 1$, under the incomplete information, the competition disappears for low type workers, so their utilities increase in k, whereas there is still enough competition on low type workers under the complete information. Hence, under the incomplete information, the Pareto optimal signaling level, $e = 2 - \frac{3}{2}k$, at which the I.C. of the low type worker binds, decreases in k, while that under the complete information stays high at $e = 1$. Therefore, workers who are located near the middle of the unit interval are better off under the complete information, while workers who are located near the end are better off under the incomplete information. If $1 < k \leq \frac{5}{4}$, the low type workers who are located near the middle are even excluded from the market under the incomplete information. Under the complete information, the competition disappears the low type workers who are located near the ends. Hence, they get the wages only covering the transportation costs. Therefore, those who are located near the ends prefer the incomplete information, and vice versa.

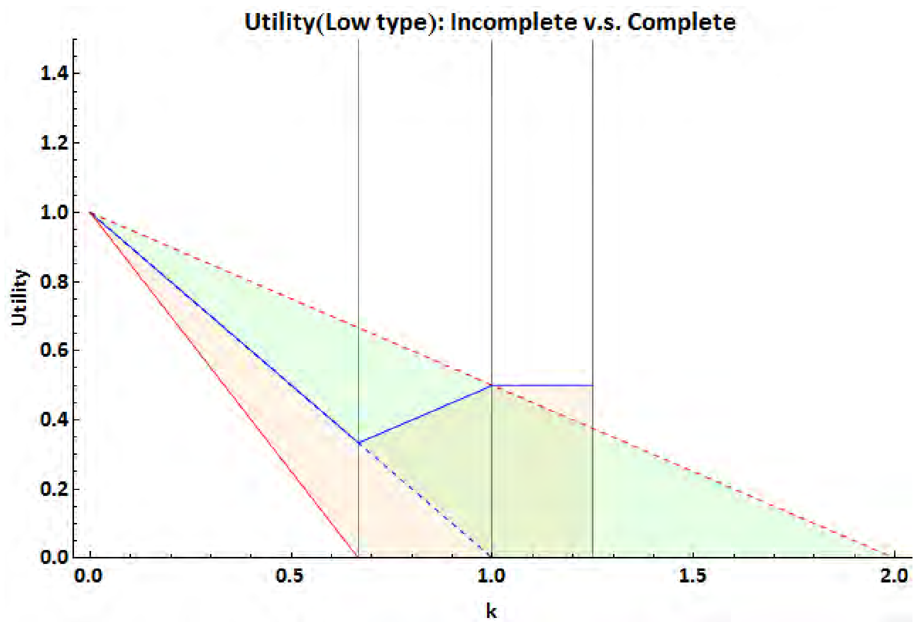

Figure 7: Utility of a low type worker.

Remark 4 (Workers' utility)**.**

- *If the market is sufficiently competitive,* $k < \frac{2}{3}$*, all workers are better off under the complete information because the publicly known location would just evoke the competition between firms for workers.*

- *If a worker is sufficiently desirable for both firms to hire, the worker is better off under the complete information. If not, the worker is better off under the incomplete information because*

    1. *the lessened competition lower the signaling cost under the incomplete information*
       or

    2. *the competition completely goes away for the workers near the ends under the complete information.*

We now conduct welfare analysis under both information structures.

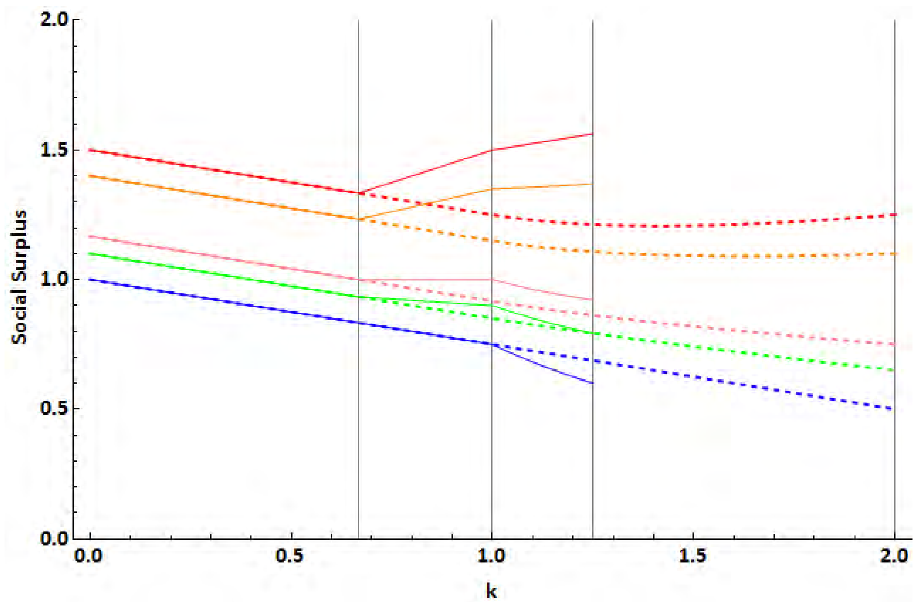

Figure 8: Total surplus by comparing with complete information.

Definition 6 (Workers' surplus: Producer surplus)**.**

$$WS(k) = \begin{cases} \frac{3}{2} - \frac{1}{2}q - \frac{5}{4}k, & k \leq \frac{2}{3} \\ 1 - q + \frac{3}{4}(q - \frac{2}{3})k, & \frac{2}{3} < k \leq 1 \\ \frac{5(1-q)}{4} - \frac{3(1-q)}{4}k + \frac{q}{4k}, & 1 < k \leq \frac{5}{4} \end{cases}$$

Definition 7 (Firms' surplus: Consumer surplus)**.**

$$FS(k) = \begin{cases} k, & k \leq \frac{2}{3} \\ q - \frac{3}{2}(q - \frac{2}{3})k, & \frac{2}{3} < k \leq 1 \\ \frac{q}{2k} + (1-q)k, & 1 < k \leq \frac{5}{4} \end{cases}$$

Definition 8 (Total surplus: Social welfare)**.**

$$SW(k) = \begin{cases} \frac{3}{2} - \frac{1}{2}q - \frac{1}{4}k, & k \leq \frac{2}{3} \\ 1 - \frac{3}{4}(q - \frac{2}{3})k, & \frac{2}{3} < k \leq 1 \\ \frac{5(1-q)}{4} + \frac{(1-q)}{4}k + \frac{3q}{4k}, & 1 < k \leq \frac{5}{4} \end{cases}$$

Remark 5 (Total surplus and the competitiveness)**.**

$$\frac{dSW(k)}{dk} > 0 \ \textit{if} \ (k, q) \in \left(\tfrac{2}{3}, 1\right] \times \left(0, \tfrac{2}{3}\right] \cup \left(1, \tfrac{5}{4}\right] \times \left(0, \tfrac{1}{4}\right]$$

If k is sufficiently high and q is sufficiently low, the total surplus increases in k. Moreover, if q is low enough, the total surplus is even higher with the highest possible $k = \frac{5}{4}$ than with $k = 0$. This implies that, in the Pareto optimal separating equilibrium, the society is better off under the least competitive market than under the perfectly competitive market.

When $k \geq \frac{2}{3}$, if k increases, then the competition rent for the high type workers decreases and the information rent for the low type workers increases. Hence, the incentive for the low type workers to mimic the high type workers decreases, and the high type workers take lower levels of costly education. As a result, the total surplus can even increase in k if there is enough high type workers.[10]

Remark 6 (Comparison of the welfare under two information structures)**.** *If k is sufficiently low or q is sufficiently large (the market is competitive enough or there are enough high type workers), then workers' surplus is higher under the complete information and firms' surplus is higher under the incomplete information.*

4 Extension to n firms

In this section we extend our analysis to any arbitrary finite number n of firms. In the horizontal dimension there are n firms. As depicted in Figure 9, the locations of firms evenly split the unit circle. Each firm's objective is to maximize the profit from hiring workers. Again we focus on symmetric equilibria in which each firm makes the same wage offers for each type of workers.

4.1 Sequentially Rational Wage Schedule

4.1.1 For $\frac{k}{n} \leq \frac{2}{3}\theta$

A worker (θ, x) is in between the firm i and j. For $\frac{k}{n} \leq \frac{2}{3}\theta$, we assume that, in a symmetric equilibrium, every worker in between the two firms is hired by one of those two firms. The worker would like to accept the offer of Firm i rather than that of Firm j, when the following condition holds.

$$w_\theta^i - kx \geq w_\theta^j - k(\frac{1}{n} - x)$$

[10]Of course, this results highly depends on the assumption of unproductive signaling. If we assume the education is costly but productive, then the total welfare could decrease in k even under relatively less competitive market because of the decreasing education level.

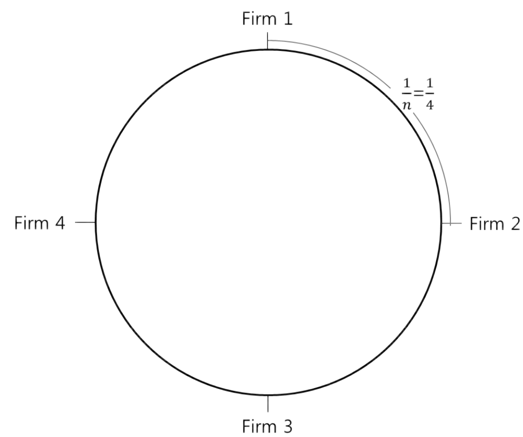

Figure 9: $n = 4$ Firms Case.

Then the market share of Firm i in between i and j, x_θ^{ij}, is,

$$x_\theta^{ij} = \left(\frac{w_\theta^i - w_\theta^j}{2k} + \frac{1}{2n} \right).$$

The profit maximization problem for Firm i which is located in between Firm j and m

$$\max_{w_\theta^i < \theta} \left(\frac{w_\theta^i - w_\theta^j}{2k} + \frac{1}{2n} \right) (\theta - w_\theta^i) + \left(\frac{w_\theta^i - w_\theta^m}{2k} + \frac{1}{2n} \right) (\theta - w_\theta^i)$$

From the first order condition,

$$\text{(F.O.C.)} \ \ \frac{1}{2k}(\theta - w_\theta^i) - (\frac{w_\theta^i - w_\theta^j}{k} + \frac{1}{n}) + \frac{1}{2k}(\theta - w_\theta^i) - (\frac{w_\theta^i - w_\theta^m}{k} + \frac{1}{n}) \leq 0$$

$$\text{with equality if } 0 < w_\theta^i < \theta.$$

we get the symmetric wage offer which is $w_\theta^i = w_\theta^j = w_\theta^m = w_\theta^n = \theta - \frac{k}{n}$. To be consistent with the full coverage assumptions, the wage schedule should satisfy the following constraint.

$$w_\theta^i - kx_\theta^{ij} = w_\theta^i - k \left(\frac{w_\theta^i - w_\theta^j}{2k} + \frac{1}{2n} \right) \geq 0$$

This constraint requires that the I.R. conditions for all workers are satisfied. With the symmetric wage offers, $w_\theta^i = w_\theta^j = w_\theta^n = \theta - \frac{k}{n}$, this condition holds for any $\frac{k}{n} \leq \frac{2}{3}\theta$.

$$w_\theta^1 - k\left(\frac{w_\theta^1 - w_\theta^2}{2k} + \frac{1}{2n}\right) = \theta - \frac{k}{n} - \frac{k}{2n} = \theta - \frac{3k}{2n} \geq 0$$

Therefore, the wage schedules when $\frac{k}{n} \leq \frac{2}{3}\theta$ are, for the low type and the high type respectively,

$$w_l^n = 1 - \frac{k}{n}, \quad \frac{k}{n} \leq \frac{2}{3} \tag{16}$$

$$w_h^n = 2 - \frac{k}{n}, \quad \frac{k}{n} \leq \frac{4}{3} \tag{17}$$

If we define $k' = \frac{k}{n}$ as the normalized degree of competition, then in terms of k' the wage schedules (16) and (17) are exactly the same as the symmetric wage schedules in the duopoly case where $k' = \frac{k}{2}$. This implies that the analysis of the n-firm case can be translated into the analysis of the duopoly case through normalizing k by n, and in terms of k' the solution to the n-firm model is the same as the solution to the duopoly model.

4.1.2 For $\frac{k}{n} > \theta$

As following the similar logic in the Duopoly analysis, we can find the symmetric wage schedules with n firms when $\frac{k}{n} > \theta$, which are

$$w_l^n = \frac{1}{2}, \quad 1 < \frac{k}{n}$$
$$w_h^n = 1, \quad 2 < \frac{k}{n}.$$

4.1.3 For $\frac{2}{3}\theta < \frac{k}{n} \leq \theta$

As following the similar logic in the Duopoly analysis, we conclude that, for $\frac{2}{3}\theta < k \leq \theta$, the sequentially rational wage schedule is $w_\theta^n = \frac{k}{2n}$.

4.2 Symmetric Equilibrium

As mentioned above, because the n-firm model can be translated into the analysis of the duopoly model through normalizing k by n, without checking deviation incentives and incentive compatibility conditions, we can simply identify symmetric equilibria in the n-firm case as follows.

Proposition 6 (Separating Equilibrium in the n-firm Model).
In the n-firm model with any value of $q \in (0,1)$, for every possible $\frac{k}{n} \leq \frac{5}{4}$, $k > 0$ and $n > 0$, and for any $e_h \in E(k,n)$, there is a separating equilibrium consisting of a strategy profile $\langle w(e;k,n), e(\theta,x;k,n)\rangle$ and supporting beliefs $\mu(\theta|e)$.
Specifically, for a given $\frac{k}{n} \leq 1$, a separating equilibrium is a triple, $\langle w(e;k,n), e(\theta,x;k,n), \mu(\theta|e)\rangle$, which satisfies

$$w_l(e=0;k,n) = \begin{cases} 1-\frac{k}{n}, & \frac{k}{n} \leq \frac{2}{3} \\ \frac{k}{2n}, & \frac{2}{3} < \frac{k}{n} \leq 1 \\ \frac{1}{2}, & 1 < \frac{k}{n} \leq \frac{5}{4} \end{cases}, \quad w_h(e=1;k,n) = 2-\frac{k}{n}, \quad \frac{k}{n} \leq \frac{5}{4}.$$

$$e(\theta,x;k,n) = \begin{cases} 0, & \theta = 1 \\ e_h, & \theta = 2 \end{cases}$$

$$\textit{where } e_h \in E(k,n) = \begin{cases} [1,2] & \frac{k}{n} \leq \frac{2}{3} \\ \left[2-\frac{3}{2}\frac{k}{n}, 4-3\frac{k}{n}\right], & \frac{2}{3} < \frac{k}{n} \leq 1 \\ \left[\frac{3}{2}-\frac{k}{n}, 4-3\frac{k}{n}\right], & 1 < \frac{k}{n} \leq \frac{5}{4} \end{cases},$$

$$\mu(\theta=1|e<e_h) = 1, \;\; \mu(\theta=1|e \geq e_h) = 0$$

Proposition 7 (Pareto Dominant Separating Equilibrium in n-firm model).
In duopoly case with any value of $q \in (0,1)$, for every $\frac{k}{n} \leq \frac{5}{4}$, there is a separating equilibrium consisting of a strategy profile $\langle w(e;k,n), e(\theta,x;k,n)\rangle$ and supporting beliefs $\mu(\theta|e)$. Specifically, for a given $\frac{k}{n} \leq \frac{5}{4}$, a separating equilibrium is a triple, $\langle w(e;k,n), e(\theta,x;k,n), \mu(\theta|e)\rangle$, which satisfies

$$w(e<e_h;k,n) = \begin{cases} 1-\frac{k}{n}, & \frac{k}{n} \leq \frac{2}{3} \\ \frac{k}{2n}, & \frac{2}{3} < \frac{k}{n} \leq 1 \\ \frac{1}{2}, & 1 < \frac{k}{n} \leq \frac{5}{4} \end{cases}, \; w(e \geq e_h;k,n) = 2-\frac{k}{n}, \;\; \frac{k}{n} \leq \frac{5}{4}$$

$$e(\theta,x;k,n) = \begin{cases} 0, & \theta = 1 \\ e_h(k,n), & \theta = 2 \end{cases}$$

$$\textit{where } e_h(k,n) = \begin{cases} 1, & \frac{k}{n} \leq \frac{2}{3} \\ 2-\frac{3}{2}\frac{k}{n}, & \frac{2}{3} < \frac{k}{n} \leq 1 \\ \frac{3}{2}-\frac{k}{n}, & 1 < \frac{k}{n} \leq \frac{5}{4} \end{cases},$$

$$\mu(\theta=1;e<e_h) = 1, \;\; \mu(\theta=1;e \geq e_h) = 0.$$

Proposition 8 (Pooling Equilibrium in the n-firm Model).
In duopoly case, for any $q \in (0,1)$, $k > 0$ and $n > 0$, and for any $e_h \in E(k,n)$, there exists a pooling equilibrium consisting of a strategy profile

$\langle w_p(e;k), e_p(\theta,x;k)\rangle$ and supporting beliefs $\mu_p(\theta|e)$.
Specifically, for given $q \in (0,1)$, $k > 0$ and $n > 0$, a pooling equilibrium is a triple, $\langle w_p(e;k), e_p(\theta,x;k), \mu_p(\theta|e)\rangle$, which satisfies

$$w_p(e=0;k,n) = \begin{cases} (2-q) - \frac{k}{n}, & \frac{k}{n} \leq \frac{2}{3}(2-q) \\ \frac{k}{2n}, & \frac{2}{3}(2-q) < \frac{k}{n} \leq (2-q) \\ \frac{(2-q)}{2}, & (2-q) < \frac{k}{n} \end{cases},$$

$$w_p(e=1;k,n) = \begin{cases} (2-r) - \frac{k}{n}, & \frac{k}{n} \leq \frac{2}{3}(2-r) \\ \frac{k}{2n}, & \frac{2}{3}(2-r) < \frac{k}{n} \leq (2-r) \\ \frac{(2-r)}{2}, & (2-r) < \frac{k}{n} \end{cases}.$$

$$e_p(\theta,x;k,n) = 0$$

$$\mu_p(\theta=1|e=0) = q, \;\; \mu_p(\theta=1|e>0) = r > q.$$

Therefore, we conclude that the duopoly model represents the more general n-firm case. Because there is an inverse relationship between k and n, by analyzing the effect of k, we can explain the effect of n in equilibrium.

5 Continuous Type and Education

In the basic duopoly model, we assume that there are only two types of workers; the high and the low type. In this section, we investigate the duopoly model with a continuous type space of workers, i.e., the type space $\Theta \equiv \left[\underline{\theta}, \overline{\theta}\right]$ is a subset of the set of positive real numbers, $\mathbb{R}_{++}$.

The existence of a separating equilibrium depends on the natures of the parameter space. We first show that, if the set of type is the entire set of positive real numbers, $\Theta = \mathbb{R}_{++}$, then there does not exist any separating equilibrium (Proposition 9 and Corollary 9.1). After that, we show that there exist separating equilibria for certain restricted type spaces (Proposition 10). In order to identify those sets and the corresponding equilibrium, we assume the followings.

Because we are mainly focusing on separating equilibrium, we assume a worker with a different value of θ chooses a different level of education $e(\theta)$. That is, in a separating equilibrium, $\theta \neq \theta'$ iff and only if $e(\theta) \neq e(\theta')$.

Firms' problems are exactly the same as that in the basic duopoly model. After a worker chooses $e(\theta')$, the firms know there is only one type, θ, in the information set, i.e., $\mu(\theta = \theta'|e = e(\theta')) = 1$. Hence, the sequentially rational wage schedule for a worker who takes the education level $e(\theta)$ turns out to be

$$w(e(\theta);k)=\begin{cases}\theta-k, & k\leq\frac{2}{3}\theta\\ \frac{k}{2}, & \frac{2}{3}\theta<k\leq\theta\\ \frac{\theta}{2}, & \theta<k\end{cases}.$$

Now, we check if there is a signaling strategy $e(\theta)$ which is incentive compatible and sequentially rational for every θ or not. To be sequentially rational, no workers should be excluded from the market for $k\leq\theta$, and every worker (θ,x) who has $x\in\left(\frac{\theta}{2k},1-\frac{\theta}{2k}\right)$ should be excluded from the market for $k>\theta$.

5.1 For $\theta\in\left(k,\frac{3}{2}k\right)$

First, for some k, let's assume there is a type-θ' which satisfies $\frac{2}{3}\theta'<k<\theta'$, and denote the neighborhood of the particular θ', $(\theta'-\epsilon,\theta'+\epsilon)\equiv\Theta_\epsilon$ with $\epsilon>0$. Then, for a small enough ϵ, the sequentially rational wage offer for $\theta\in\Theta_\epsilon$ is $\frac{k}{2}$.

To satisfy incentive compatibility, the following should be satisfied.

$$\forall\theta\in\Theta_\epsilon,\;\; U\left(\theta;\theta,k\right)=\max_{\hat{\theta}}U\left(\hat{\theta};\theta,k\right)=\frac{k}{2}-kx-\frac{e(\hat{\theta})}{\theta}$$

From the envelope theorem, $e(\theta)$ should satisfy the following condition for any $\theta\in\Theta_\epsilon$.

$$-\frac{e'(\theta)}{\theta}=0 \tag{18}$$

The only value that satisfy equation (18) is some constant $M\in\mathbb{R}$. However, $e(\theta)=M$ cannot be a separating equilibrium strategy because it does not depend on θ. In other words, if there is some $\theta\in\Theta$ which satisfies $k<\theta<\frac{3}{2}k$, there cannot be any separating equilibrium.

Then, it should be $\Theta\subseteq[0,k]$ or $\Theta\subseteq[\frac{3}{2}k,\infty)$ for a separating equilibrium to exist.

5.2 For $\Theta\subseteq[0,k]$

Let's now assume that $\Theta\subseteq[0,k]$. With the corresponding wage schedule, $w(e(\theta);k)=\frac{\theta}{2}$, incentive compatibility requires

$$\forall\theta\in\Theta\;\; U\left(\theta;\theta,k\right)=\max_{\hat{\theta}}U\left(\hat{\theta};\theta,k\right)=\frac{\hat{\theta}}{2}-kx-\frac{e(\hat{\theta})}{\theta}.$$

After applying the envelope theorem, we get the corresponding education level $e(\theta)$ for $\theta \in \Theta$,

$$e(\theta) = \frac{\theta^2}{4} + M \tag{19}$$

with some constant $M \in \mathbb{R}$.

However, this signaling strategy is not individually rational for a certain type of worker. As mentioned before, every worker (θ, x) who has $\theta < k$ and $x \in [0,1] \,/\, \left(\frac{\theta}{2k}, 1 - \frac{\theta}{2k}\right)$ should choose the education level $e(\theta)$. Let's consider a worker $(\theta, x) = (\theta, \frac{\theta}{2k})$ where $\theta > \underline{\theta}$, which is the marginal type worker who is not excluded from the market. The worker's utility from the market is

$$U(\theta; \theta, k) = \frac{\theta}{2} - kx - \frac{e(\theta)}{\theta} = \frac{\theta}{4} - kx - \frac{M}{\theta} = -\frac{\theta}{4} - \frac{M}{\theta}. \tag{20}$$

To be individually rational, (20) should be non-negative. It implies,

$$M \leq -\frac{\theta^2}{4}. \tag{21}$$

(21) should be satisfied for any $\theta \in \Theta$. Hence, M should be less than or equal to $-\frac{\overline{\theta}^2}{4}$.

With this condition, for the lowest type of worker $\underline{\theta}$

$$e(\underline{\theta}) = \frac{\underline{\theta}^2}{4} + M \leq \frac{\underline{\theta}^2}{4} - \frac{\overline{\theta}^2}{4} < 0, \tag{22}$$

by construction. However, the education level cannot be negative. Hence, the signaling strategy (19) cannot be valid. Therefore, we can conclude that, if $\Theta \subseteq [0, k]$, there cannot be any separating equilibrium.

We summarize the analyses of the above two subsections in following propositions.

Proposition 9 (Impossibility result)**.**
For some value of k, if $\exists \theta \in \left[\underline{\theta}, \overline{\theta}\right]$ such that $\theta < \frac{3}{2}k$, then there is no separating equilibrium.

This proposition says if there is some type of a worker which is in a specific interval, $\theta < \frac{3}{2}k$, then there is no separating equilibrium. This directly implies that there is no separating equilibrium if $\Theta = \mathbb{R}_{++}$.

Corollary 9.1 (Impossibility result with $\Theta = \mathbb{R}_{++}$)**.**
For any value of $k > 0$, if $\Theta = \mathbb{R}_{++}$, then there exists a $\theta \in \Theta$ which satisfies $\theta < \frac{3}{2}k$. Therefore, by Proposition 9, there is no separating equilibrium.

5.3 For $\Theta \subseteq [\frac{3}{2}k, \infty)$

Finally, let's assume $\Theta \subseteq [\frac{3}{2}k, \infty)$. Incentive compatibility requires

$$\forall \theta \in \Theta,\ U(\theta; \theta, k) = \max_{\hat{\theta}} U\left(\hat{\theta}; \theta, k\right) = \hat{\theta} - k - kx - \frac{e(\hat{\theta})}{\theta}. \tag{23}$$

By applying the envelope theorem, we get the incentive compatible education level $e(\theta)$ for $\theta \in \Theta$, which is as follows.

$$e(\theta) = \frac{1}{2}\theta^2 + M \tag{24}$$

Now, we check individual rationality of workers. The utility of a worker (θ, x) is

$$U(\theta; \theta, k) = \theta - k - kx - \frac{e(\theta)}{\theta} = \frac{\theta}{2} - k - kx - \frac{M}{\theta}. \tag{25}$$

For any worker $(\theta, x) \in \Theta \times \left[0, \frac{1}{2}\right]$, (25) should be nonnegative.

$$\frac{\theta}{2} - k - kx - \frac{M}{\theta} \geq \frac{\theta}{2} - \frac{3k}{2} - \frac{M}{\theta} \geq 0$$

Combining with nonnegative constraint for (24), we get the condition for the constant M.

$$\forall \theta,\ \tfrac{1}{2}\theta^2 - \tfrac{3k}{2}\theta \geq M \geq -\tfrac{1}{2}\theta^2$$

Because $\frac{1}{2}\theta^2 - \frac{3k}{2}$ is increasing in θ and $-\frac{1}{2}\theta^2$ is decreasing in θ, we get

$$\tfrac{1}{2}\underline{\theta}^2 - \tfrac{3k}{2}\underline{\theta} \geq M \geq -\tfrac{1}{2}\underline{\theta}^2. \tag{26}$$

Therefore, if $\Theta \subseteq [\frac{3}{2}k, \infty)$, we can identify a separating equilibrium with M which satisfies (26). Moreover, we can idenfy the Pareto dominant separating equilibrium with $M = -\frac{1}{2}\underline{\theta}^2$.

5.4 Symmetric Equilibrium

We summarize the above analysis in following propositions.

Proposition 10 (Highly competitive market)**.**
For some value of k, if all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ satisfy $\theta \geq \frac{3}{2}k$, or equivalently if $\Theta \subseteq [\frac{2}{3}k, \infty)$, then there is a separating equilibrium consisting of a strategy profile $\langle w(e;k), e(\theta, x; k)\rangle$ and supporting beliefs $\mu(\theta|e)$ with $M \in M(k) \equiv$

$\left[-\frac{1}{2}\underline{\theta}^2, \frac{1}{2}\underline{\theta}^2 - \frac{3k}{2}\underline{\theta}\right]$. *Specifically, for a given* k *and* $\Theta \subseteq [\frac{2}{3}k, \infty)$, *a separating equilibrium is a triple,* $\langle w(e;k), e(\theta, x; k), \mu(\theta|e)\rangle$,

$$w(e;k) = \sqrt{2\,(e-M)} - k, \quad e(\theta, x; k) = \frac{1}{2}\theta^2 + M,$$

$$\forall \tilde{\theta} \in \Theta, \ \ \mu(\theta = \tilde{\theta}; e = \frac{1}{2}\tilde{\theta}^2 + M) = 1.$$

Proposition 11 (Pareto Dominant Separating Equilibrium)**.**
For some value of k*, if all* $\theta \in [\underline{\theta}, \overline{\theta}]$ *satisfy* $\theta \geq \frac{3}{2}k$*, or equivalently if* $\Theta \subseteq [\frac{2}{3}k, \infty)$*, then there is a unique Pareto dominant separating equilibrium consisting of a strategy profile* $\langle w(e;k), e(\theta, x; k)\rangle$ *and supporting beliefs* $\mu(\theta|e)$*. Specifically, for a given* k *and* $\Theta \subseteq [\frac{2}{3}k, \infty)$*, a separating equilibrium is a triple,* $\langle w(e;k), e(\theta, x; k), \mu(\theta|e)\rangle$,

$$w(e;k) = \sqrt{2\left(e + \frac{1}{2}\underline{\theta}^2\right)} - k, \quad e(\theta, x; k) = \frac{1}{2}\theta^2 - \frac{1}{2}\underline{\theta}^2,$$

$$\forall \tilde{\theta} \in \Theta, \ \ \mu(\theta = \tilde{\theta}; e = \frac{1}{2}\tilde{\theta}^2 - \frac{1}{2}\underline{\theta}^2) = 1.$$

These propositions hold only when k is sufficiently small. It means that only when the market is highly competitive for firms, there could be a separating equilibrium.

This result is consistent with an example in Riley (1979), which is a generalized version of the basic model of Spence (1973). In the example, Riley (1979) investigates the same model as in ours only when the market is perfect competitive for the firms, which corresponds to our model with $k = 0$ and $\Theta = \mathbb{R}_+$. The equilibrium strategy is consistent with our model.

Corollary 11.1 (Consistency with the result of Riley (1979))**.**
When $k = 0$ *and* $\Theta = \mathbb{R}_+$*, the wage offer is* $w(e; k = 0) = \sqrt{2(e - M)}$ *and the signaling strategy is* $e(\theta, x; k) = \frac{1}{2}\theta^2$*. These are consistent with those in the example of Riley (1979).*

Our model can also explain the relationship between the degree of competition and the level of wage offer.

Corollary 11.2 (Degree of the Competition and level of the Wage Offer)**.**
When $\theta \geq \frac{3}{2}k$*, if* k *increases, then the level of wage offer decreases.*

$$\frac{dw(e;k)}{dk} = -1 < 0.$$

In words, when k is sufficiently small, workers can get higher wages in more competitive market.

6 Conclusion

We have explored imperfect competition among employers in a job market signaling model. We conclude that if competition among firms is severe enough, there exists a separating equilibrium. Moreover, we also show that there always exits a pooling equilibrium, but all pooling equilibria would fail the intuitive criterion if there exists a separating equilbrium.

These results are fairly intuitive. If the market converges to monopoly, so there is only one employer without any competition, then workers do not have any incentive to take costly education because they cannot extract any competition rent by doing that. Once a worker earns an education, the signaling cost will be sunk. So, a monopolist may offer a wage which only covers the transportation cost of the worker. Therefore, the level of the wage offer for the worker who took the costly education cannot be higher than the wage offer for someone who has not earned an education. However, if the market is competitive enough, to hire a certain type of worker, firms have to pay competitive wages which outweigh the signaling cost as well as the transportation cost in the sense of the competition rent. Therefore, there would be incentives for workers to take costly education in those competitive markets.

We have compared two information structures; the complete and incomplete information about the location of a worker x. When k is small enough, so the competition rent outweighs the information rent, the workers would prefer the complete information to the incomplete information. On the other hand, when k is too large, then competition rent goes away. Hence, the workers would prefer the incomplete information structure where the information rent exists.

In the welfare analysis, we have shown that, under some circumstances, when k increases, the social welfare increases. It is a bit striking because usually an increase in a cost has a bad influence on social welfare. However, in this case, the increase in k, which is the transportation cost, induces the decrease in the incentive for workers to take costly education. Therefore, it would decrease the total cost from signaling.

Of course the results rely on the assumption that education is unproductive. In future research, productive signaling can be considered. The procedure of the analysis would be similar to the unproductive signaling scenario, but the welfare analysis might change. When the signaling is productive, if the education level of a hired worker decreases, then the total surplus as well as the producer surplus would decrease. So, in a separating equilibrium, increasing k could decrease the total welfare by lessening the incentive to pursue higher level of education.

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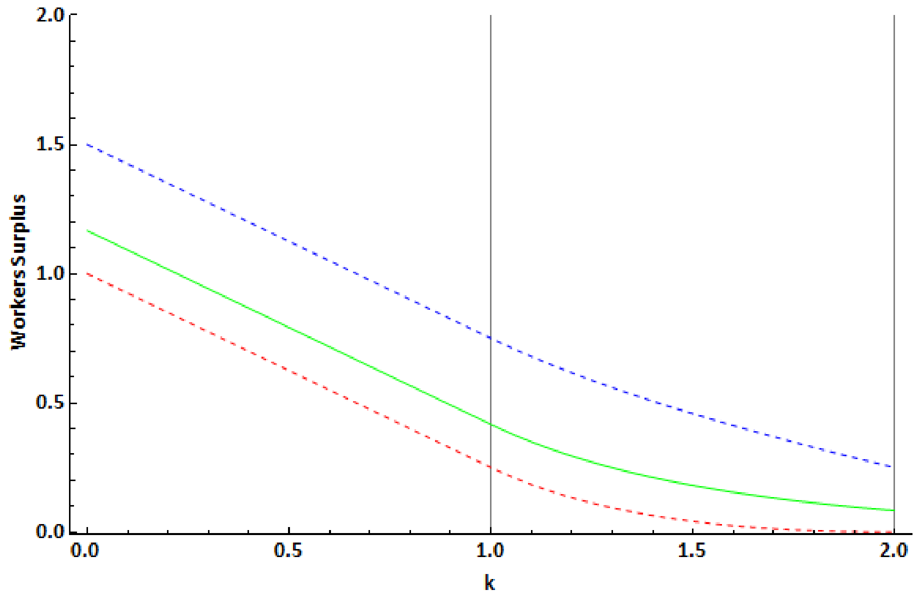

Figure 10: Workers' Surplus under complete information.

A Appendix

A.1 Complete Information

Definition 9 (Worker's payoff)**.**

$$u(e;\theta_h,x) = 2 - k(1-2x) - kx - \tfrac{e}{2} \tag{27}$$

$$= \begin{cases} 1-(1-x)\frac{k}{2}, & x \le 1-\frac{1}{k} \\ \frac{3}{2}-(1-x)k, & 1-\frac{1}{k} < x \le \frac{1}{2} \end{cases}$$

$$u(e;\theta_l,x) = \begin{cases} 0, & x \le 1-\frac{1}{k} \\ 1-(1-x)k, & 1-\frac{1}{k} < x \le \frac{1}{2} \end{cases} \tag{28}$$

Definition 10 (Workers' surplus: Producer surplus)**.**

$$WS(k) = q \times 2\int_0^{\frac{1}{2}} u(e;\theta_l,x,k)dx + (1-q)\times 2\int_0^{\frac{1}{2}} u(e;\theta_h,x,k)dx \tag{29}$$

$$= \begin{cases} q(1-\frac{3}{4}k) + (1-q)(\frac{3}{2}-\frac{3}{4}k) = \frac{3}{2}-\frac{1}{2}q-\frac{3}{4}k, & k \le 1 \\ q(\frac{1}{4}k - 1 + \frac{1}{k}) + (1-q)(\frac{1}{2}-\frac{1}{4}k+\frac{1}{2k}) & \\ \quad = \frac{1}{4}k - \frac{1}{2} - \frac{1}{2}q + \frac{1}{k}, & 1<k\le 2 \end{cases}$$

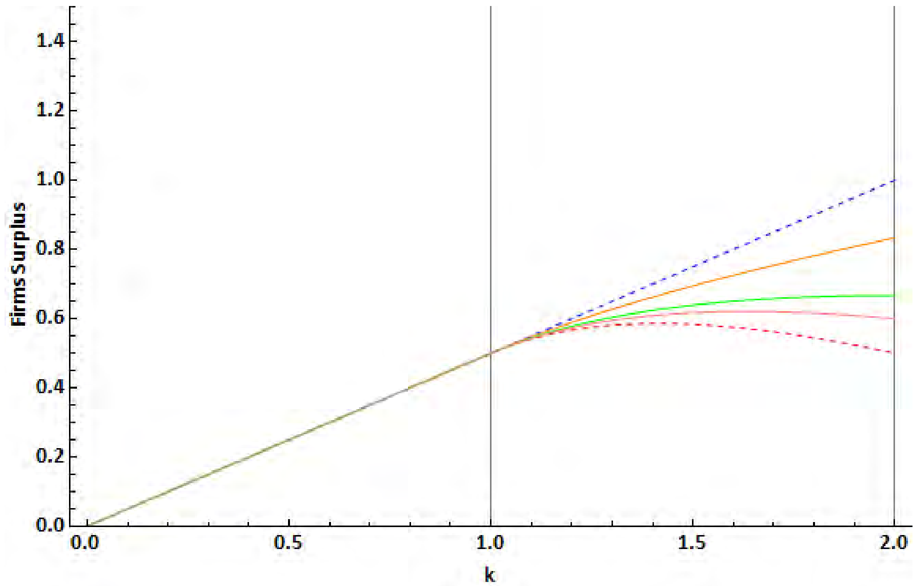

Figure 11: Firms' Surplus under complete information.

Definition 11 (Firms' surplus: Consumer surplus)**.**

$$FS(k) = q \times \Pi(w;\theta_l,k) + (1-q)\times\Pi(w;\theta_h,k) \tag{30}$$
$$= \begin{cases} q\frac{k}{2} + (1-q)\frac{k}{2} = \frac{k}{2}, & k \le 1 \\ q(2-\frac{k}{2}-\frac{1}{k}) + (1-q)\frac{k}{2} = 2q - \frac{q}{k} + \frac{k}{2} - kq, & 1 < k \le 2 \end{cases}$$

Definition 12 (Total surplus: Social welfare)**.**

$$SW(k) = WS(k) + FS(k) \tag{31}$$
$$= \begin{cases} \left(\frac{3}{2} - \frac{1}{2}q - \frac{3}{4}k\right) + \left(\frac{k}{2}\right) = \frac{3}{2} - \frac{1}{2}q - \frac{1}{4}k, & k \le 1 \\ \left(\frac{1}{4}k - \frac{1}{2} - \frac{1}{2}q + \frac{1}{k}\right) + \left(2q - \frac{q}{k} + \frac{k}{2} - kq\right) & \\ = \frac{3}{2}q - \frac{q}{k} - kq + \frac{3}{4}k + \frac{1}{k} - \frac{1}{2}, & 1 < k \le 2 \end{cases}$$

A.2 Separating equilibrium

A.2.1 Workers' signaling strategies

When $k \le \frac{2}{3}$, from the incentive compatibility conditions for each type of workers, we get the condition as follows.

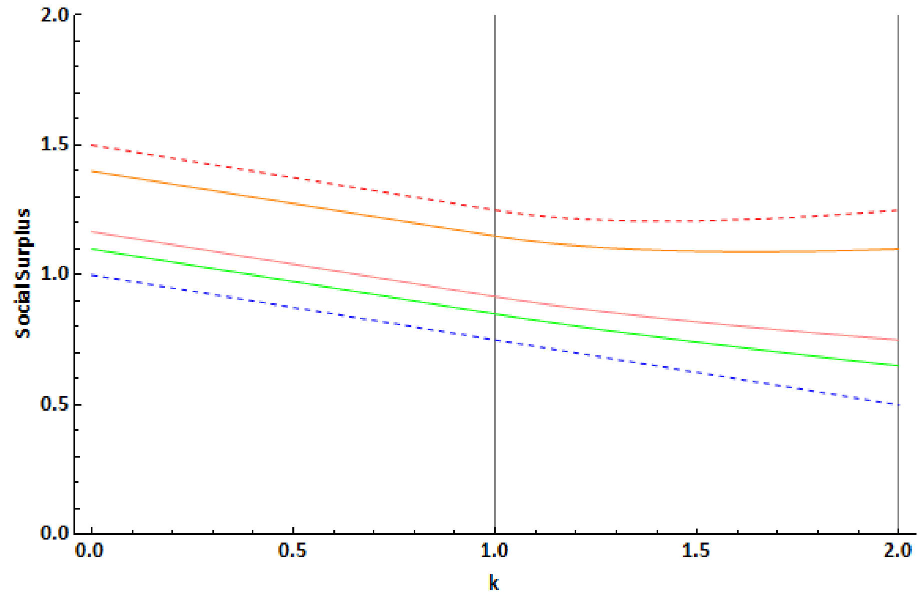

Figure 12: Total surplus under complete information.

1. Low type workers: $u(e_l;\theta_l,x) \geq u(e_h;\theta_l,x) \Leftrightarrow$
   $(1-k-kx) \geq (2-k-kx-e_h) \Leftrightarrow e_h \geq 1$

2. High type workers: $u(e_h;\theta_h,x) \geq u(e_l;\theta_h,x) \Leftrightarrow$
   $(2-k-kx) \geq (1-k-kx) \Leftrightarrow e_h \leq 2$

Hence, if $k \leq \frac{2}{3}$, the sequentially rational signaling strategy is $e_l = 0$ and $e_h \in [1,2]$.

When $\frac{2}{3} < k \leq 1$,

1. Low type workers: $u(e_l;\theta_l,x) \geq u(e_h;\theta_l,x) \Leftrightarrow$
   $\left(\frac{k}{2} - kx\right) \geq (2-k-kx-e_h) \Leftrightarrow e_h \geq 2 - \frac{3}{2}k$

2. High type workers: $u(e_h;\theta_h,x) \geq u(e_l;\theta_h,x) \Leftrightarrow$
   $\left(2-k-kx-\frac{e_h}{2}\right) \geq \left(\frac{k}{2} - kx\right) \Leftrightarrow e_h \leq 4-3k.$

Hence, if $\frac{2}{3} < k \leq 1$, the sequentially rational signaling strategy is $e_l = 0$ and $e_h \in \left[2-\frac{3}{2}k, 4-3k\right]$.

When $1 < k \leq \frac{5}{4}$,

1. Low type workers with $x \in \left[0, \frac{1}{2k}\right]$: $u(e_l;\theta_l,x) \geq u(e_h;\theta_l,x) \Leftrightarrow$
   $\left(\frac{1}{2} - kx\right) \geq (2-k-kx-e_h) \Leftrightarrow e_h \geq \frac{3}{2} - k$

2. Low type workers with $x \in \left(\frac{1}{2k}, \frac{1}{2}\right]$: $u(e_l;\theta_l,x) \geq u(e_h;\theta_l,x) \Leftrightarrow$ $0 \geq (2-k-kx-e_h) \Leftrightarrow e_h \geq 2-k-kx$

3. High type workers with $x \in \left[0, \frac{1}{2k}\right]$: $u(e_h;\theta_h,x) \geq u(e_l;\theta_h,x) \Leftrightarrow$ $\left(2-k-kx-\frac{e_h}{2}\right) \geq \left(\frac{1}{2}-kx\right) \Leftrightarrow e_h \leq 3-2k$

4. High type workers with $x \in \left(\frac{1}{2k}, \frac{1}{2}\right]$: $u(e_h;\theta_h,x) \geq u(e_l;\theta_h,x) \Leftrightarrow$ $\left(2-k-kx-\frac{e_h}{2}\right) \geq 0 \Leftrightarrow e_h \leq 4-2k-2kx$.

Hence, if $1 < k \leq \frac{5}{4}$, the sequentially rational signaling strategy is $e_l = 0$ and $e_h \in \left[\frac{3}{2}-k, 4-3k\right]$.

When $\frac{5}{4} < k \leq \frac{4}{3}$,

1. Low type workers with $x \in \left[0, \frac{1}{2k}\right]$: $u(e_l;\theta_l,x) \geq u(e_h;\theta_l,x) \Leftrightarrow$ $\left(\frac{1}{2}-kx\right) \geq (2-k-kx-e_h) \Leftrightarrow e_h \geq \frac{3}{2}-k$

2. Low type workers with $x \in \left(\frac{1}{2k}, \frac{1}{2}\right]$: $u(e_l;\theta_l,x) \geq u(e_h;\theta_l,x) \Leftrightarrow$ $0 \geq (2-k-kx-e_h) \Leftrightarrow e_h \geq 2-k-kx$

3. High type workers with $x \in \left[0, \frac{1}{2k}\right]$: $u(e_h;\theta_h,x) \geq u(e_l;\theta_h,x) \Leftrightarrow$ $\left(2-k-kx-\frac{e_h}{2}\right) \geq \left(\frac{1}{2}-kx\right) \Leftrightarrow e_h \leq 3-2k$

4. High type workers with $x \in \left(\frac{1}{2k}, \frac{1}{2}\right]$: $u(e_h;\theta_h,x) \geq u(e_l;\theta_h,x) \Leftrightarrow$ $\left(2-k-kx-\frac{e_h}{2}\right) \geq 0 \Leftrightarrow e_h \leq 4-2k-2kx$.

Hence, if $\frac{5}{4} < k \leq \frac{4}{3}$, e_h should be in $\left[\frac{3}{2}-k, 4-3k\right]$. However, $\frac{3}{2}-k > 4-3k$, so there is no symmetric separating equilibrium when $\frac{5}{4} < k \leq \frac{4}{3}$.

When $\frac{4}{3} < k \leq 2$, from the I.C. condition of a high type worker with $x \in \left(\frac{1}{2k}, \frac{1}{2}\right]$, we get,

$$u(e_h;\theta_h,x) \geq u(e_l;\theta_h,x) \Leftrightarrow \left(\frac{k}{2}-kx-\frac{e_h}{2}\right) \geq 0 \Leftrightarrow e_h \leq k-2kx \tag{32}$$

Hence e_h should be zero to satisfy this condition. Therefore, there is no symmetric separating equilibrium if $\frac{4}{3} < k \leq 2$.

When $2 < k$, from the I.C. condition of a high type worker with $x \in \left(\frac{1}{2k}, \frac{1}{2}\right]$, we get,

$$u(e_h;\theta_h,x) \geq u(e_l;\theta_h,x) \Leftrightarrow \left(1-kx-\frac{e_h}{2}\right) \geq 0 \Leftrightarrow e_h \leq 2-2kx \tag{33}$$

Hence, e_h should be negative to satisfy this condition. Therefore, there is no symmetric separating equilibrium if $2 < k$.

A.2.2 Deviation incentive in a separating equilibrium

Figure 3 illustrates the sequentially rational signaling for the duopoly model. The red line represents the sequentially rational signaling in the Pareto dominant equilibrium. In the Pareto dominant equilibria, I.C. conditions for the low-type workers bind for all the values of k. From the I.C. condition of the low-type worker,

$$w_l - kx - \frac{e_l}{\theta_l} \geq w_h - kx - \frac{e_h}{\theta_l}, \tag{34}$$

we define the deviation incentive of the low-type worker in the sense of the difference in utilities.

$$\Delta u_l = \left(w_h - kx - \frac{e_h}{\theta_l}\right) - \left(w_l - kx - \frac{e_l}{\theta_l}\right) = w_h - w_l - e_h \tag{35}$$

The next equation indicates the low type worker's deviation incentive in the Pareto dominant equilibrium.

$$\Delta u_l(e_h; k) = \begin{cases} 1 - e_h, & k \leq \frac{2}{3} \\ 2 - \frac{3}{2}k - e_h, & \frac{2}{3} < k \leq 1 \\ \frac{3}{2} - k - e_h, & 1 < k \leq \frac{5}{4} \end{cases} \tag{36}$$

For $k < \frac{2}{3}$, the incentive does not depend on k. For $\frac{2}{3} < k \leq 1$, high type workers get the competitive wage, $2 - k$, but low type do not. Hence, when k increases, the wage for low type workers increases, while the wage for high type workers decreases. Therefore, the deviation incentive of the low type workers decreases in k. In the last interval, $1 < k \leq \frac{5}{4}$, some low type workers would be excluded from the market. The corresponding wage level for the low type workers is constant over k, while the high type workers still get the competitive wage, $2 - k$. As a result, the deviation incentive of low type workers decreases in k as in the second interval.

A.2.3 Extra remark

Remark 7 (The maximum value of k, $\overline{k} = \frac{5}{4}$)**.**
If $k > \overline{k} = \frac{\theta_h}{2} + \frac{\theta_l}{4}$, there is no a separating equilibrium.

$$\frac{d\overline{k}}{d\theta_h} > 0, \ \ \frac{d\overline{k}}{d\theta_l} > 0.$$

If the high type workers are more productive, or the low type workers are less productive, then under the less competitive market there could be a separating equilibrium.

A.3 Pooling Equilibrium

Proof of Proposition 4.
It is obvious that if both types take the education $e_h > 0$, then the low type worker is worse off given firms' best response, which is exact same with the firms' wage schedules in equilibrium, than in pooling equilibrium.

Now, we identify the best response of the firm if the high type take the higher education, $e_h > 0$, which is as follows.

$$w^{BR}(e = e_h; k) = \begin{cases} 2-k, & k \leq \frac{4}{3} \\ \frac{k}{2}, & \frac{4}{3} < k \leq 2 \\ 1, & 2 < k \end{cases} , \tag{37}$$

Second, let us point out $q > 0 \Leftrightarrow \frac{2}{3}(2-q) < \frac{4}{3}$. Then, we now have two distinct intervals.

For $k \leq \frac{2}{3}(2-q)$, with any value of $e_h < 2q$,

$$u_h = (2-q) - k - kx < u_h(e = e_h) = 2 - k - kx - \tfrac{e_h}{2}.$$

For $\frac{2}{3}(2-q) < k < \frac{4}{3}$, with any value of $e_h < 4 - 3k$,

$$u_h = \frac{k}{2} - kx < u_h(e = e_h) = 2 - k - kx - \tfrac{e_h}{2}$$

Therefore, if $k < \frac{4}{3}$, all pooling equilibria fail the Intuitive Criterion. □

Proof of Proposition 5.
The best response of the firm if the high type take the higher education, $e_h > 0$, is same as (37).

For $\frac{4}{3} < k \leq (2-q)$ if $\frac{4}{3} < (2-q)$,

$$u_h = \frac{k}{2} - kx > u_h(e_h) = \frac{k}{2} - kx - \tfrac{e_h}{2}$$

For $\max\left\{\frac{4}{3}, (2-q)\right\} < k \leq 2$ and $x \in \left(\left[0, \frac{(2-q)}{2k}\right] \cup \left[1 - \frac{(2-q)}{2k}, 1\right]\right)$,

$$u_h = \frac{(2-q)}{2} - kx > u_h(e_h) = \frac{k}{2} - kx - \tfrac{e_h}{2}$$

if $e_h > k - (2-q)$.

For $\max\left\{\frac{4}{3}, (2-q)\right\} < k \leq 2$ and $x \in \left(\frac{(2-q)}{2k}, 1 - \frac{(2-q)}{2k}\right)$,

$$u_h = 0 > u_h(e_h) = \frac{k}{2} - kx - \tfrac{e_h}{2}$$

For $2 < k$ and $x \in \left(\left[0, \frac{(2-q)}{2k}\right] \cup \left[1 - \frac{(2-q)}{2k}, 1\right]\right)$,

$$u_h = \frac{(2-q)}{2} - kx > u_h(e_h) = 1 - kx - \tfrac{e_h}{2}$$

if $e_h > q$.

For $2 < k$ and $x \in \left(\frac{2-q}{2k}, \frac{1}{2k}\right] \cup \left[1 - \frac{1}{2k}, 1 - \frac{2-q}{2k}\right)$,

$$u_h = w_R = 0 > u_h(e_h) = 1 - kx - \tfrac{e_h}{2}$$

if $e_h > q$.

For $2 < k$ and $x \in \left(\frac{1}{2k}, 1 - \frac{1}{2k}\right)$,

$$u_h = w_R = 0 > u_h(e_h) = -\tfrac{e_h}{2}$$

Therefore, all pooling equilibria survive the intuitive criterion. □

A.4 Comparative statics

Definition 13 (Workers' surplus: Producer surplus)**.**

(38)

$$\begin{aligned} WS(k) =& q \times 2 \int_0^{\frac{1}{2}} u(e^D; \theta_l, x, k)dx + (1-q) \times 2 \int_0^{\frac{1}{2}} u(e^D; \theta_h, x, k)dx \\ =& \begin{cases} q(1 - \frac{5}{4}k) + (1-q)(\frac{3}{2} - \frac{5}{4}k) = \frac{3}{2} - \frac{1}{2}q - \frac{5}{4}k, & k \leq \frac{2}{3} \\ q\frac{k}{4} + (1-q)(1 - \frac{1}{2}k) = 1 - q + \frac{3}{4}(q - \frac{2}{3})k, & \frac{2}{3} < k \leq 1 \\ q\frac{1}{4k} + (1-q)(\frac{5}{4} - \frac{3}{4}k) = \frac{5(1-q)}{4} - \frac{3(1-q)}{4}k + \frac{q}{4k}, & 1 < k \leq \frac{5}{4} \end{cases} \end{aligned}$$

Definition 14 (Firms' surplus: Consumer surplus)**.**

(39)

$$\begin{aligned} FS(k) =& q \times \Pi(w^D; \theta_l, k) + (1-q) \times \Pi(w^D; \theta_h, k) \\ =& \begin{cases} q \times k + (1-q) \times k = k, & k \leq \frac{2}{3} \\ q \times (1 - \frac{k}{2}) + (1-q) \times k = q - \frac{3}{2}(q - \frac{2}{3})k, & \frac{2}{3} < k \leq 1 \\ q \times \frac{1}{2k} + (1-q) \times k = \frac{q}{2k} + (1-q)k, & 1 < k \leq \frac{5}{4} \end{cases} \end{aligned}$$

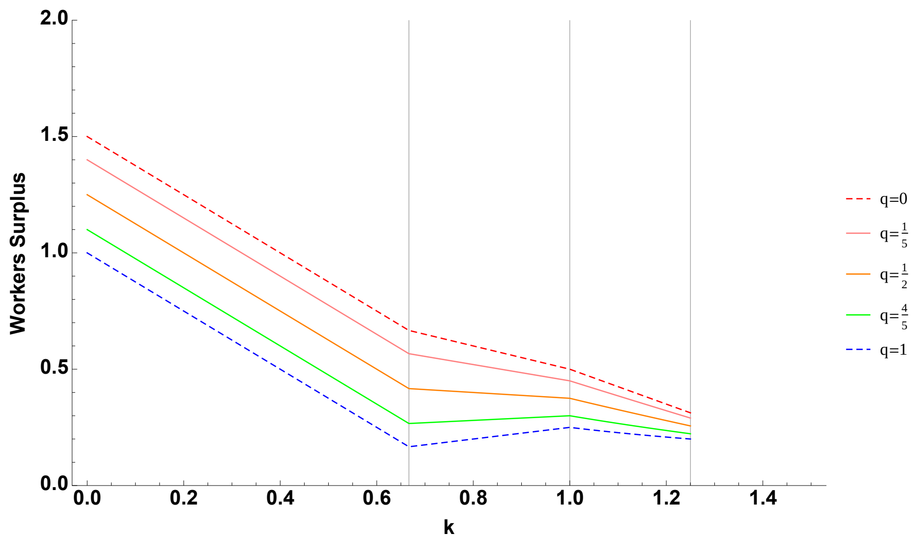

Figure 13: Workers' Surplus.

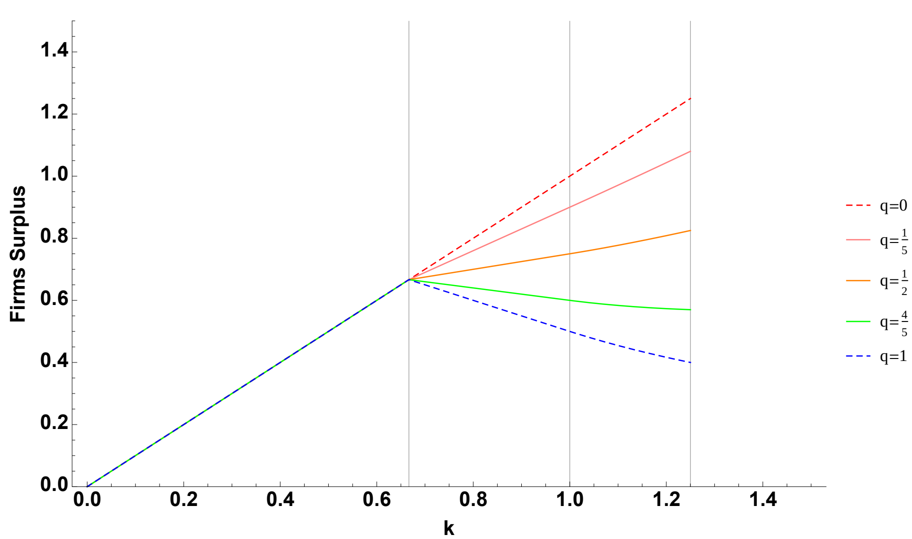

Figure 14: Firms' Surplus.

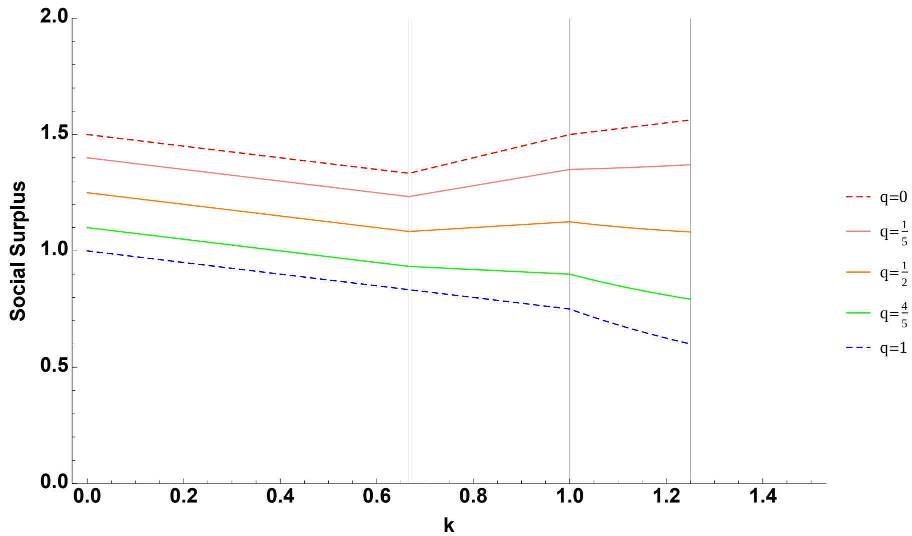

Figure 15: Total surplus.

Definition 15 (Total surplus: Social welfare)**.**

$$
\begin{aligned}
(40)\qquad SW(k) &= WS(k) + FS(k) \\
&= \begin{cases} \left[\frac{3}{2} - \frac{1}{2}q - \frac{5}{4}k\right] + k, & k \leq \frac{2}{3} \\ \left[1 - q + \frac{3}{4}(q - \frac{2}{3})k\right] + \left[q - \frac{3}{2}(q - \frac{2}{3})k\right], & \frac{2}{3} < k \leq 1 \\ \left[\frac{5(1-q)}{4} - \frac{3(1-q)}{4}k + \frac{q}{4k}\right] + \left[\frac{q}{2k} + (1-q)k\right], & 1 < k \leq \frac{5}{4} \end{cases} \\
&= \begin{cases} \frac{3}{2} - \frac{1}{2}q - \frac{1}{4}k, & k \leq \frac{2}{3} \\ 1 - \frac{3}{4}(q - \frac{2}{3})k, & \frac{2}{3} < k \leq 1 \\ \frac{5(1-q)}{4} + \frac{(1-q)}{4}k + \frac{3q}{4k}, & 1 < k \leq \frac{5}{4} \end{cases}
\end{aligned}
$$

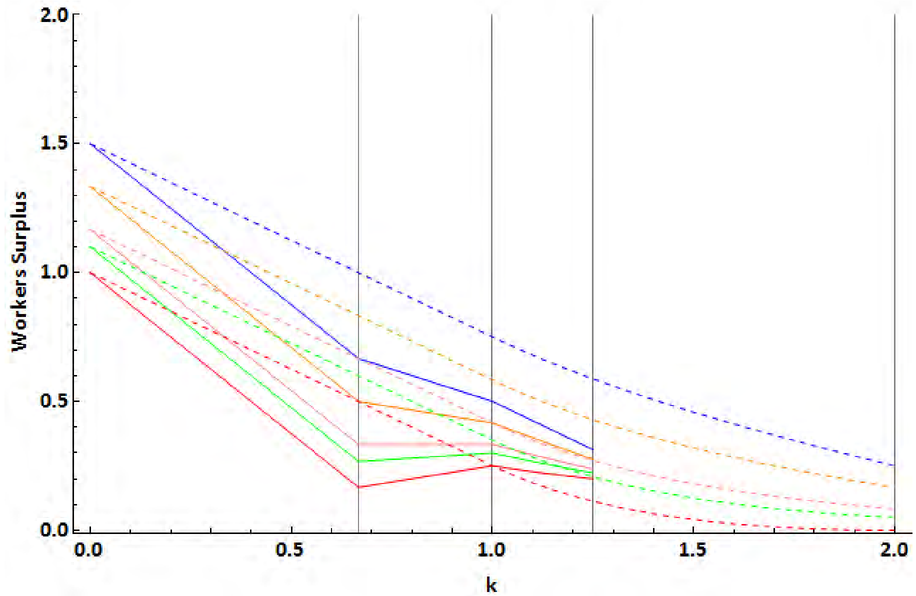

Figure 16: Workers' Surplus by comparing with complete information.

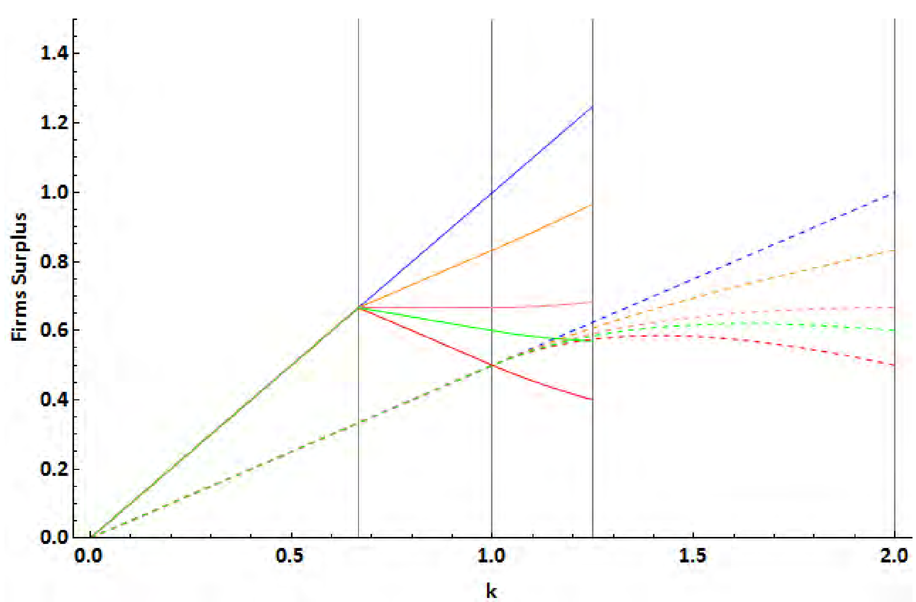

Figure 17: Firms' Surplus by comparing with complete information.