Economies of scale and the development of market structure

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Abstract

Oligopolistic industries, in which firms have to cumulatively build up lumpy capacity, lead to preemption games: firms engaging in cutthroat competition for who gets to make the next profitable investment. Such Markov-perfect preemption has been believed to drive profits for each plant and the entire industry to zero, and to render market structure irrelevant. This paper shows that, in the canonical framework, these results depend on firms coordinating on unreasonably aggressive equilibrium strategies. With limited entry, more reasonable equilibria involve tacit collusion without threat strategies: dominant firms let entrants in to make them less hungry. Clustering of investments may occur despite complete information and the absence of uncertainty. When economies of scale with respect to plant size are considered, new types of equilibrium investment events arise. Some of these involve second-mover advantage, with rent equalisation not necessarily holding.

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1 Introduction

Many industries are characterised by three features. Firstly, demand for the product may be growing over time. This might result from economic growth driving aggregate consumption up. Alternatively, new industries typically experience fast growth followed by maturation. Secondly, production may involve large (up-front) capital costs but small variable costs. This is typical in many manufacturing sectors or in electricity generation.[1] Thirdly, there may exist economies of scale with respect to the plant size. For example, coal-fired and nuclear power stations typically display substantial economies of scale.[2] Similarly, different types of power generation plants may vary significantly in their capital costs per unit of capacity: coal-fired generation capacity is often said to involve much higher capital costs per unit than gas turbines.

How does the market structure of a growing industry develop? Initially, a budding industry may support very few plants, and thus also very few firms. With time, new entrants are attracted to the sector by the expanding opportunities to make profits. However, it has been suggested that cutthroat competition for such opportunities may yield very stark results. Preemptive investment—so called as firms always seek to preempt a profitable investment planned by a competitor—can eat away industry profits. Firms always seek to cut their competitors out of the market by investing earlier than them, much as firms in price competition seek to undercut their competitors on price. Gilbert and Harris (1984) (hereafter GH84) show that, as long as there are at least two firms in a market, preemptive competition fully dissipates rents, for the industry and for each individual plant. Further, the results imply that market structure is irrelevant: any oligopoly always yields the same outcome, irrespective of how many firms are active, or how existing capacity is distributed across active firms.

Preemption concerns can thus affect how actors in growing sectors expand. For example, in the energy sector, a glaring example of preemptive behaviour is the manoeuvering of Russia in the European gas supply sector. Plans to diversify gas supply in southeastern Europe, by the construction of the Nabucco pipeline from Turkey to Austria, have been countered with threats by Russia to expand its supply capacity by the construction of a competing South Stream

[1] This is also the case in the pharmaceutical industry: however, a large fraction of the cost of producing new medicines is related to the R&D work. The focus of this paper is on expanding production capacity.

[2] The capital cost per kilowatt of coal-fired generation capacity falls by some 30% as capacity increases from 400 MW to 900 MW (Sargent & Lundy LLC, 2008).

pipeline. Such preemption may be intended to retain a monopolistic position in the market. On the other hand, the GH84 model suggests that the mere potential of a competing investment project will pressurise the monopolist to expand more rapidly than they would prefer to, forced to relinquish some of their monopoly rents.

The present paper first demonstrates that the stark results of Gilbert & Harris follow from the assumption that firms coordinate on very aggressive strategies. Such equilibria may be considered unreasonable as they are Pareto-dominated by alternative, tacitly collusive Markov-perfect equilibria. Such tacit collusion involves no threat strategies, only the recognition of the competitor's response to increased capacities.[3] In fact, with slightly modified assumptions regarding the timing of the game and the very long-run evolution of the market, the equilibria proposed by GH84 do not arise at all. The tacitly collusive equilibria I consider feature equal, positive rents. Market structure is again made relevant, with the ultimate industry structure featuring concentration or fragmentation depending on the scenario.

I then incorporate economies of scale with respect to plant size into the model. This introduces new types of equilibrium investment episodes: in particular, some investment episodes feature second-mover advantage, with one or both firms wanting the other to make the first investment. Such episodes may imply that rent equalisation no longer holds, as in the existing literature. Furthermore, even with a very large pool of potential entrants, which leads to rent dissipation in the manner of GH84, not all individual plants necessarily make zero profits; instead, some may make profits, and others offsetting losses.

This paper is structured as follows. Section 2 will present the general model and discusses the equilibrium in the absence of economies of scale, with particular reference to GH84. Section 3 introduces scale economies and develops the equilibrium. Section 4 presents numerical examples. Section 5 concludes. I will begin by describing the existing literature.

1.1 Literature review

Preemptive capacity investment games were introduced by GH84. However, as I demonstrate below, they focus on an equilibrium which involves extremely aggressive strategies. In particular, the equilibrium features 'purely self-defensive'

[3]Gilbert and Harris (1984) do recognise that equilibria other than the one they consider may well exist, possibly involving threat strategies, but do not discuss these further.

investment (in the sense of Fudenberg and Tirole, 1985): situations in which both players are happy to invest only because the other one also intends to invest, even though both players would prefer to have no further investment made by either firm. Such equilibria may not seem very reasonable, and the stark results are ultimately not very surprising.[4] The key issue is the willingness of the smaller firm to invest: an entrant with no capacity is very hungry and wants to enter the market, while a small firm with some capacity is less keen to expand as it has inframarginal revenues to protect. In the present paper, I focus on equilibria which are not 'self-defensive'. I also explore the effect of alternative assumptions regarding the timing of the game, as well as the long-run evolution of the market. In particular, I show that with non-simultaneous timing the self-defensive equilibrium is not robust to an eventual market contraction.

Boyer et al. (2012) develop a model of duopolistic capacity investment with stochastically growing demand, with quantity competition in the product market. Some of their results are fairly similar to the ones I present: both firms may enter the market, making positive profits. Their results follow from Cournot competition with a demand specification which departs from the GH84 canonical model. More specifically, they assume that consumers' willingness-to-pay grows indefinitely, but that the number of consumers and the Cournot quantity are constant, implying that no firm will ever supply arbitrarily large quantities, and that an entrant can eventually force the incumbent to accommodate entry. As the model structures differ in several dimensions, it is not immediately clear what drives the difference between their results and those of GH84. I show that, in the canonical model, many of the results of Boyer et al. (2012) can be reproduced by the assumption that demand growth eventually peters out, and by focusing on non-self-defensive equilibria. Thus the rationale for the results I obtain is subtly different. In the absence of scale economies, the present paper yields ambiguous results with respect to the degree of ultimate industry concentration; in contrast to Boyer *et al.*, who predict that smaller entrants will always tend to catch up with the incumbent, so that industries ultimately feature a group of roughly similar-sized firms. The introduction of scale economies in the present paper also goes beyond the analysis of Boyer et al. (2012).

[4] GH84 also impose a small, exogenous timing advantage on one of the firms, in order to break ties in cases in which both firms want to invest at a given time, conditional on the other not doing so. The firm with the advantage ends up making all investments. However, this is not crucial for their key results. Changing the tie-breaking rule to e.g. a coin flip, while retaining the same strategies, would yield essentially the same equilibrium.

Katz and Shapiro (1987) extend the methods of Fudenberg and Tirole (1985) to consider asymmetric situations of preemptive competition. In the present paper, such asymmetry rises endogenously as firms expand their capacities: I use very similar methods to describe the equilibrium investment events in any stage of the preemption game. However, with scale economies, I get many more types of investment events than the ones considered by Katz and Shapiro (1987).

Mills (1990) extends GH84 by incorporating economies of scale. However, he assumes the firms do not consider the effects of capacity expansion on their inframarginal revenues. The results thus omit some of the most interesting strategic interactions in the game.[5] I consider the equilibrium in which the firms take into account all strategic interactions.

Several authors have recently studied preemption games. Hoppe and Lehmann-Grube (2005) use the Simon and Stinchcombe (1989) continuous-time framework to develop an algorithm to solve a larger class of preemption games than the previous literature has focused on. The model in the present paper does not fall within their assumptions, however; furthermore, the game I study of course involves a sequence of strategic investment decisions, instead of just one. Argenziano and Schmidt-Dengler (forthcoming) use the same framework to show that, with more than two players, equilibrium investments may be clustered (i.e. occur in waves with more than one player investing simultaneously) even in a game with full information.[6,7]

Besanko and Doraszelski (2004) use the Ericson and Pakes (1995) numerical framework to consider capacity investment dynamics with price or quantity competition, capital depreciation and firm-specific shocks. Their prediction, under quantity competition, is that firm sizes tend to even out, much as in Boyer et al. (2012). Under price competition, they show that firms tend to engage in tough preemptive competition to obtain a leading position; once one firm manages to obtain a dominant position, firm sizes start diverging. The

[5] Were the firms to consider these interactions, the equilibrium in Mills (1990) would no longer be an equilibrium. To see this, note that there is a profitable deviation to the equilibrium given for Example 7 in that paper: namely, following the first investment in a small plant, for the now-incumbent to build another small plant at time $t = 4.07 - \epsilon$, for some small ϵ.

[6] See Murto and Välimäki (2013) for an example of a model in which clustering occurs with private information; in their model, observed investment decisions lead players to update their beliefs on an underlying state variable, resulting in investment waves.

[7] I do not resort to the Simon and Stinchcombe (1989) framework, instead reporting the equilibrium of a sequential-move game with decision intervals becoming arbitrarily short. Ongoing work will describe the equilibrium of an asynchronous-move game, in which players move at decision opportunities arriving as independent Poisson processes (for asynchronous-move games, see Calcagno et al., 2013).

present paper demonstrates a different rationale for asymmetric firm sizes with *ex ante* identical firms.

2 The model

2.1 Technology and demand

Let there be two firms, indexed by $k \in \{1, 2\}$, with supply at time t by firm k given by $q^k(t) \leq Q^k(t)$, $Q^k(t)$ denoting capacity of firm k at time t. Aggregate supply is denoted by $q(t) = \sum_k q^k(t)$. I will from now on refrain from explicitly denoting the dependence of all variables on time. Let the marginal cost of producing output be zero. It will become clear below that, given my assumptions on demand, both firms will always supply up to capacity: I will hence dispose of the variable Q^k and instead denote the vector of output and capacities both by $\boldsymbol{q} \equiv (q^1, q^2)$. Investment quantities (increments to capacity) will be denoted similarly $\overline{\boldsymbol{q}} \equiv (\overline{q}^1, \overline{q}^2)$. Assume building a single unit of capacity involves a cost of I, so that the investment cost, incurred at the time of investment, is $c(\overline{q}) = \overline{q}I$. Firms will be able to build multiple units at the same point in time, so that $\overline{q}^k \in \mathbb{Z}^+ \equiv \{0, 1, \dots\}$.

Let demand be given by

$$p(q,t) = \begin{cases} p_0(q)f(t) & \text{if } t \leq T \\ 0 & \text{otherwise.} \end{cases}$$

satisfying $f'(t) > 0$, $\frac{\partial q p_0(q)}{\partial q} > 0$. The time horizon T is interpreted as a (deterministic) date at which the market disappears as the good in question becomes obsolete.[8] I also allow $T = \infty$. As marginal revenue is assumed strictly positive, it is apparent that, as claimed above, firms will always produce up to their capacity.

I also assume that in the infinite-horizon case demand eventually plateaus: $\lim_{t\to\infty} f'(t) = 0$. This ensures that the ultimate number of investments is finite, irrespective of whether T is finite or infinite. In particular, an upper

[8]More realistically, the market could disappear gradually. The main purpose of a finite horizon is to allow the possibility of backward induction. For this purpose, the precise manner of the market disappearing does not matter and a simple cut-off date suffices.

bound to aggregate capacity is given by

$$n \equiv \min \left\{ q : \int_{t'}^{T} e^{-\rho(\tau - t')} p(q+1,t)\,\mathrm{d}t - I < 0, \forall t' \in [0,T] \right\}$$

The bound on capacity is thus derived as the maximal capacity for which the increment of one further unit of capacity, with no further investment, will yield a negative profit for an entrant with no existing capacity, irrespective of the investment date. No firm could ever make a profit making such an investment.[9]

Given these assumptions, with $T = \infty$ the model conforms closely to that of GH84. The only departure from their model is the separability of the demand function; the present model in a special case of GH84. This assumption is made here for analytical convenience.

2.2 Timing assumptions

I will assume the timing of the game is as follows: time flows continuously, but the firms get to make choices at discrete moments, separated by intervals of length κ. At each moment of time, firms choose their actions sequentially. This could reflect a vanishing observational advantage as in GH84. To allow better comparisons with their model, in this section, in which capacity investment does not involve economies of scale, the identity of the firm moving first each period is exogenously fixed.[10] I allow the length of the decision interval become arbitrarily small ($\kappa \to 0$), to capture the fact that investment dates are in reality chosen from a continuous set.[11,12]

Note that the GH84 model can be replicated here by imposing simultaneous

[9] The present paper's commentary on GH84 holds only for the case in which capacity expansion eventually stops. This is arguably a more realistic and relevant case than the one in which capacities ultimately tend to infinity.

[10] The timing assumptions are close to those made by Gerlagh and Liski (2011); Katz and Shapiro (1987) use similar assumptions but with simultaneous moves.

[11] I do not work with a purely continuous-time model as defined by Simon and Stinchcombe (1989); see Hoppe and Lehmann-Grube (2005) and Argenziano and Schmidt-Dengler (forthcoming) for applications of this framework to preemption games. Instead, I show that the continuous-time formulation is an arbitrarily precise approximation to the discrete-decision-interval game as the period length vanishes. Implementation of the model in the asynchronous-move framework of Calcagno et al. (2013) is work in progress.

[12] The game could alternatively be set up in continuous time, as a sequence of stopping games (as in Murto and Välimäki, 2013). Due to numerous open-set issues, particular restrictions on strategies and a carefully selected tie-breaking rule (for situations in which both players try to invest at the same time) would be required to ensure the existence of equilibrium. Furthermore, a multiplicity of equilibria would still exist, although the equilibria would be very closely related to the ones presented here. I have chosen the discrete decision interval framework as the assumptions required are more transparent.

moves at any decision moment, as in Katz and Shapiro (1987), together with a tie-breaking rule which exogenously favours one of the firms; i.e. if both firms attempt to invest at the same moment, then the favoured firm gets to go ahead while the other firm can (but does not have to) cancel its investment plans, with an abortive investment attempt carrying no costs.

2.3 Strategies

I assume the players condition their actions on the history of the game. However, generically, this is equivalent to the players conditioning their actions only on the current state $\boldsymbol{q}$, the calendar date t and the identity of the player moving first in period in question; in other words, the players play Markov-perfect strategies (in the sense of Maskin and Tirole, 2001).[13] I thus focus on the case in which strategies are a function ϕ only of the calendar date, the existing distribution of capacity and the binary variable indicating move order: $q^{k,*} = \phi^k(\boldsymbol{q}, t, \mathbf{1}(k \text{ moves first}))$.

Given any initial state $(\boldsymbol{q}_0, t_0)$, I will denote the equilibrium capacity, investment quantity and investment date sequences (for $j \geq 1$) by

$$\begin{aligned}\boldsymbol{q}^* &\equiv \{\boldsymbol{q}_j^*\} \equiv \{(q_j^{1,*}, q_j^{2,*})\} \\ \overline{\boldsymbol{q}}^* &\equiv \{\overline{\boldsymbol{q}}_j^*\} \equiv \{(\overline{q}_j^{1,*}, \overline{q}_j^{2,*})\} \\ t^* &\equiv \{t_j^*\}\end{aligned}$$

respectively, so that

$$q_{j+1}^{k,*} = q_j^{k,*} + \overline{q}_{j+1}^{k,*}.$$

If player i makes the jth investment, then $\overline{q}_j^{-i,*} \equiv 0$ (with the superscript $-i$ denoting 'the player other than i' as is conventional). As before, aggregate equilibrium capacity is given by $q_j^* \equiv \sum_k q_j^{k,*}$.

For convenience of notation, also define

$$\begin{aligned}t_0^* &\equiv t_0 \\ t_{|\overline{\boldsymbol{q}}^*|+1}^* &\equiv T \\ \overline{q}_{|\overline{\boldsymbol{q}}^*|+1}^{k,*} &\equiv 0\end{aligned}$$

with $|\overline{\boldsymbol{q}}^*|$ denoting the number of investments in equilibrium.

[13] I discuss this in footnote 25, after first setting up the structure of the game.

2.4 Aggressive preemption

Without developing the equilibrium fully, I now want to discuss the GH84 results in the above framework. This section is intended purely to illustrate how key results in that paper follow from the very aggressive strategies which the firms are assumed to employ. In the next section I develop the full equilibrium with scale economies in plant construction. I index, in this section, by a the firm with the exogenous advantage, and by $-a$ the other firm.

Assumption 1. *Suppose the initial level of demand is arbitrarily low. Further, suppose that for all states satisfying $\boldsymbol{q}(0) = (q^a, 0)$, either the equilibrium investment quantity for $t = 0$ is 1, or there is no investment. Finally, let $T = \infty$.*

Assumption 1 guarantees the model conforms to GH84. The first part is required to ensure there is no immediate investment; the second part ensures all investment events are separated by a strictly positive interval of time. In other words, $t^*_{j+1} > t^*_j$ for all $j \in \{0, \ldots, n\}$, with n the number of investments on the GH84 equilibrium path. To be satisfied, it requires that investment units are sufficiently large so that the unit price falls substantially after any capacity increment.

GH84 provide an equilibrium path of investment dates $\{t^*_j\}$ characterised by the following conditions:

$$\sum_{z=j}^{n} \int_{t^*_z}^{t^*_{z+1}} e^{-\rho\tau} p(q(\tau), \tau)\, \mathrm{d}t - I = 0, \qquad j \in \{1, \ldots, n\}$$

with $t^*_{n+1} \equiv \infty$. These conditions just state that each plant built makes exactly zero profits during its lifetime. In equilibrium, all plants are built by the firm with the advantage, with the second firm ready to make an investment as soon as an opportunity turns up (although, in equilibrium, one never does).

To see the logic behind the equilibrium, suppose aggregate capacity is $n-1$, so that at most one more investment will be made. For any capacity vector $\boldsymbol{q}$ such that aggregate capacity is $q = n - 1$, and initial date t (for example, $t = t^*_{n-1}$) consider the profits supposing firm k invests at time t':

$$\pi^{k,L}_{\boldsymbol{q},t}(t') = \int_t^{t'} e^{-\rho(\tau-t)} p(n-1, \tau) q^k \,\mathrm{d}\tau + \int_{t'}^{\infty} e^{-\rho(\tau-t)} p(n, \tau)(q^k+1)\,\mathrm{d}\tau - e^{-\rho(t'-t)} I$$

Similarly, the profits for following (the other player investing at t') are

$$\pi^{k,F}_{\boldsymbol{q},t}(t') = \int_t^{t'} e^{-\rho(\tau-t)} p(n-1,\tau) q^k \,\mathrm{d}\tau + \int_{t'}^{\infty} e^{-\rho(\tau-t)} p(n,\tau) q^k \,\mathrm{d}\tau$$

Finally, the profits for abstention by both player from further investment are given by

$$\pi^{k,A}_{(q^k,n-1-q^k),t}(t') = \int_t^{T} e^{-\rho(\tau-t)} p(n-1,\tau) q^k \,\mathrm{d}\tau$$

which is of course constant with respect to t'.

Some remarks can be made regarding these functions. Profit from leading is quasiconcave in t', seen easily by differentiating the function twice, then setting the first derivative equal to zero and observing that, due to the separability of $p(q,t)$, this means the second derivative has to be negative.[14] Profits from following are strictly increasing. Both approach profits from abstaining as t' becomes very large: $\lim_{t'\to\infty} \pi^{L,k} = \lim_{t'\to\infty} \pi^{F,k} = \pi^{A,k}$. If $q^k = 0$, then $\pi^{F,k} = \pi^{A,k} = 0$. If $q^k > 0$, then $\pi^{A,k} > 0$; whether $\pi^{L,k}$ approaches it from above or below depends on the exact specification of the model.

These curves are illustrated in Figure 1. Which type of π^L-curve applies depends on the parameters of the situation; the key distinction regarding the current discussion is whether either firm would prefer abstention to leading at any t'. For zero capacity, π^A coincides with π^F and profits from leading must be positive for some t', by the definition of n, so that $\pi^{L''}$ applies.

Note also that

$$\begin{aligned}
\pi^{i,L} - \pi^{i,F} &= \int_{t'}^{T} e^{-\rho(\tau-t)} p(n,\tau) \,\mathrm{d}\tau - e^{-\rho(t'-t)} I \\
&= e^{-\rho(t'-t)} \left(\int_{t'}^{T} e^{-\rho(\tau-t')} p(n,\tau) \,\mathrm{d}\tau - I \right) \\
\pi^{i,L} - \pi^{i,A} &= \pi^{i,L} - \pi^{i,F} + \int_{t'}^{T} e^{-\rho(t'-t)} \left(p(n,\tau) - p(n-1,\tau) \right) q^k \,\mathrm{d}\tau \\
&\leq \pi^{i,L} - \pi^{i,F}
\end{aligned}$$

The first equation gives the difference between profits from leading and following. Note that this is independent of the existing capacity a firm has: if the price will fall (due to either k or $-k$ investing) at time t', the impact on inframarginal revenues is independent of who invests. The second equation yields the differ-

[14] See Section 3.3.1.

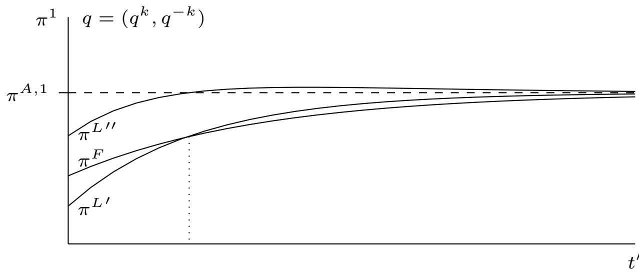

Figure 1: Profits for leading, following and abstaining for the last investment, as a function of the investment date. I illustrate, for $q^k > 0$, profits for following (π^F), two alternative possible profits for leading (π^L), and profits for abstaining (π^A). Note that firm k would prefer abstention to following, i.e. firm $-k$ investing. Whether firm would prefer abstention to leading depends on which π^L-curve applies. If $q^k = 0$, π^F is actually given by $\pi^A = 0$, and $\pi^{L''}$ applies.

ence between leading and abstention by both firms. This can be conveniently expressed as the difference between leading and following, plus a term which does depend (negatively) on the existing capacity of firm k. The inequality holds weakly if $q^k = 0$, and is otherwise strict.

I now show the sense in which the GH84 equilibrium may be considered overly aggressive. Note that if capacities are $(n-1, 0)$, then player $-a$ faces a scenario illustrated in Figure 1 with $\pi^{L''}$: it will strictly prefer investment at some dates to abstention.

Suppose now $q^k = n-2$, $q^{-k} = 1$. Then I can find an instance of the model in which the maximum possible difference between leading and following, or $\max_{t'} \pi^{L,-k} - \pi^{F,-k}$, is very small. In particular, there is an instance of the model in which $\pi^{L,-k} - \pi^{A,-k} < 0$; that is, the small firm (with only one unit of capacity) would prefer for neither firm to ever invest again. It is clear that the difference $\pi^{L,k} - \pi^{A,k}$ is increasing in q^k, so that the larger firm would also have a strict preference for abstention. To belabour the point, if either firm believed the other will never invest, it would not want to invest either. This corresponds to the scenario with $\pi^{L'}$ in Figure 1. However, if both firms believe the other firm is going to invest at some point for which $\pi^L > \pi^F$, each would want to preempt its competitor. In the preemptive equilibrium, both firms believe the other firm will invest at t_n^*, defined by $\pi^L(t^*) = \pi^F(t^*)$; and both will be happy to invest at this point, given their beliefs.

This seems like very aggressive behaviour. Each firm is (weakly) willing to invest only because it believes the other is going to invest; and both firms would be strictly better off by coordinating on the equilibrium in which both abstain from investment, with ultimate capacity remaining at $n-1$ forever. Fudenberg and Tirole (1985) call this type of an equilibrium a 'self-defensive' equilibrium and consider it to be somewhat unreasonable, as it is Pareto-dominated by the abstention equilibrium. Note that both firms make strictly positive profits if they are able to coordinate on the abstention equilibrium.

It can be shown that, from the state $(n-2,0)$, the incumbent with the exogenous advantage has an incentive to allow the entrant to enter: furthermore, if the entrant knows that from $(n-2,1)$ the firms are able to coordinate on abstaining, there is then no equilibrium in which the incumbent builds a unit of capacity, taking the equilibrium to $(n-1,0)$ and then to $(n,0)$. This can be considered tacit collusion, but note that it does not depend on threat strategies. The incumbent simply realises that allowing the entrant into the sector makes the entrant less hungry. A small firm with inframarginal revenues to protect no longer wants to expand as aggressively as an entrant.

The GH84 result holds with an infinite horizon or with a simultaneous move order in each period, but remains 'self-defensive' in either case. However, if the market is expected to decline after some point, so that there is a guaranteed date following which neither firm want to invest any more, then a sequential move order will allow the firms to coordinate on abstention, by backward induction, eliminating the GH84 equilibrium. In this sense the GH84 is sensitive to the time horizon of the game (an effectively infinite horizon implies that a firm can never rule out that the other might invest at some distant future date); and to the timing assumptions (fully simultaneous moves allow self-defensive equilibria even with a finite time horizon).[15]

Note that these results do not depend on the exogenous advantage GH84 assign to one of the firms. Suppose ties were broken by the flip of a coin, another common device is these types of models. In that case the self-defensive equilibrium would still yield an ultimate capacity of n, whereas the possibility of coordination on abstention would allow the firms to tacitly collude.

This type of tacit collusion is possible as long as some distribution of $n-1$ units of capacity implies both firms prefer abstention to investment. Hence it

[15] The next iteration of this paper considers the game with asynchronous moves, i.e. with decision moments for the players arriving as two independent Poisson processes, at an arbitrarily high rate. Preliminary results indicate this formulation also eliminates the GH84 equilibrium, provided the market eventually contracts; that is, the equilibrium seems likely to be identical to the one presented here. However, the results of the next section may be slightly modified.

is easy to check whether a coordinated-abstention equilibrium exists: if a firm prefers abstention at aggregate capacity $\frac{n-1}{2}$ (rounded down), then there exists a coordinated-abstention equilibrium which Pareto-dominates the self-defensive equilibrium. Moreover, even lower levels of ultimate capacity can be supported in equilibrium. The firm with the advantage chooses any supportable level of ultimate capacity it prefers, taking into account that it has to yield some minimal number of capacity units to the entrant for any given level of ultimate capacity.

3 Equilibrium with economies of scale

As outlined in the introduction, in many industries scale economies reach beyond the lumpiness of investment units. Take, for example, the power generation industry. It is of course not economical to build an infinitesimal coal-fired power station.[16] However, even at empirically relevant plant sizes, doubling plant size (for example from 500 MW to 1 GW) may have a substantial impact on costs per unit of capacity.[17] Preemptive competition could be seen as implying firms might tend to build inefficiently small plants in an effort to keep entrants at bay, leading to wasteful investment in fixed costs.

I thus now turn to developing the full model, taking into account that, in addition to indivisibilities, there could be economies of scale in capacity investment. I will only state the differences below, with the specification of the previous section holding otherwise.

3.1 Demand and capacity technology

I will here, for simplicity, make the additional assumption that demand does not grow very rapidly: $f'(t) \leq \rho f(t)$. Further, I assume in this section that T is finite.

Let the cost structure be given by a linear specification, with zero cost for no investment explicitly included to ease notation later:

$$c(\overline{q}) = \begin{cases} \alpha_1 + \alpha_2 \overline{q} & \text{if } \overline{q} > 0; \\ 0 & \text{if } \overline{q} = 0. \end{cases}$$

[16] For other technologies, such as wind and solar power, capacity may be more fine-grained. Considering rooftop solar panels might make capacity a nearly continuous variable.

[17] Sargent & Lundy LLC (2008) report a 30% decrease in unit capacity cost as the size of a coal-fired plant is increased from 400 MW to 900 MW.

Let the investment quantity be chosen from a discrete set: $\overline{q} \in \{0, \delta, 2\delta, \ldots, n\delta\}$.[18] I assume that $n\delta$ is sufficiently large to cover all investment quantities the players could desire in equilibrium, so that a player never wants to make two consecutive investments at the same moment, but would rather prefer one larger investment to save on the fixed costs.

3.2 Timing and strategies

The timing assumptions are as in the previous section; however, at each decision moment, the identity of the player moving first is randomised, with each player having an equal probability of being the first to move.[19] When either player invests, time is immediately incremented by the decision interval, the state changes, and the game continues with the randomisation of the player with the advantage.

From the structure of the game it is apparent that the equilibrium will feature symmetric strategies.

3.3 Value functions

Recalling notation from Section 2.3, I can now state the value[20] of the equilibrium to player k:

$$V^k(\boldsymbol{q}_0, t_0) = \mathbb{E}\left\{\sum_{j=0}^{|\overline{\boldsymbol{q}}^*|}\int_{t_j^*}^{t_{j+1}^*} e^{-\rho(t-t_0)} p\left(q_j^*, t\right) q_j^{k,*} \,\mathrm{d}t - e^{-\rho(t_{j+1}^*-t_0)} c\left(\overline{q}_{j+1}^{k,*}\right)\right\}$$

This is just the discounted stream of revenues from selling at full capacity, less any investment costs. The expectation is taken with respect to the randomisa-

[18] I could easily use some other discrete set, with arbitrary capacity increments. The key assumption is one of a discrete, rather than continuous, capacity choice set: the latter presents substantial difficulties in terms of obtaining the appropriate first- and second-order conditions, as the equilibrium often involves corner solutions. Furthermore, the full model has to be solved numerically and a discrete choice set makes this much easier.

[19] Alternative assumptions include having a fixed move order; or randomising e.g. after each investment. These choices would in most cases not make a difference, but would change the equilibrium with some parameter combinations. A move order fixed from period to period implies that firms know with certainty which firm has a very marginal advantage in the future and can plan accordingly, and/or that there might be complex correlations in terms of who has the advantage across time. I elaborate on this below. While fixing the move order would be a more standard approach, I find it less plausible. For this reason, and with the added benefit of a simplified computational algorithm, I choose per-period randomisation instead.

[20] Recall that this is an approximation of the value function as the time interval between decisions κ goes to zero. As time is defined to run continuously, the integrals are exact; approximation errors arise only from the investment dates being forced to lie on the grid of decision points. As $\kappa \to 0$, these approximation errors go linearly to zero (see Appendix A).

tion of the first mover at the beginning of each period, which is the only source of uncertainty. Observe that by the definition of the cost function and the investment vectors, if equilibrium investment j is made by player i, the cost for player $-i$ is (of course) zero.

Slightly abusing notation, I will also denote the next equilibrium investment, given any initial state $(\boldsymbol{q}_0, t_0)$ by $\overline{\boldsymbol{q}}^*_{\boldsymbol{q}_0,t_0}$, $t^*_{\boldsymbol{q}_0,t_0}$. I can then express the value function recursively:

$$V^k(\boldsymbol{q}_0, t_0) = \mathbb{E}\left\{ \int_{t^0}^{t^*_{\boldsymbol{q}_0,t_0}} e^{-\rho(t-t_0)} p\left(q_0^*, t\right) q_0^k \,\mathrm{d}t - e^{-\rho(t^*_{\boldsymbol{q}_0,t_0}-t_0)} c\left(\overline{q}^{k,*}_{\boldsymbol{q}_0,t_0}\right) \right.$$
$$\left. + e^{-\rho(t^*_{\boldsymbol{q}_0,t_0}-t_0)} V^k(\boldsymbol{q}_0 + \overline{\boldsymbol{q}}^*_{\boldsymbol{q}_0,t_0}, t^*_{\boldsymbol{q}_0,t_0}) \right\}$$

This recursive formulation allows me to use the framework developed by Fudenberg and Tirole (1985) and Katz and Shapiro (1987), and thus yields clearer insight to the equilibrium than a simple brute-force numerical approach.

3.3.1 Recursive equilibrium investment

I will now construct the equilibrium outcome and the value functions using the recursive structure above. Given initial state $(\boldsymbol{q}_0, t_0)$, the profit function for player k, as a function of the next investment date t' and investment quantity vector $\overline{\boldsymbol{q}}'$, is

$$\begin{aligned}
&\pi^k_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t') \\
&\quad = \int_{t_0}^{t'} e^{-\rho(t-t_0)} p(q_0, t) q_0^k \,\mathrm{d}t + \int_{t'}^{t^*_{\boldsymbol{q}',t'}} e^{-\rho(t-t_0)} p(q', t) q'^k \,\mathrm{d}t \\
&\qquad - e^{-\rho(t'-t_0)} c(\overline{q}'^k) \\
&\qquad + e^{-\rho(t^*_{\boldsymbol{q}',t'}-t_0)} \mathbb{E}\left(V^k(\boldsymbol{q}' + \overline{\boldsymbol{q}}^*_{\boldsymbol{q}',t'}, t^*_{\boldsymbol{q}',t'}) - c(\overline{q}^{k,*}_{\boldsymbol{q}',t'}) \right)
\end{aligned} \tag{1}$$

with $\boldsymbol{q}' \equiv \boldsymbol{q}_0 + \overline{\boldsymbol{q}}'$. I will denote the profit function if player k leads (i.e. with $\overline{q}'^k \neq 0$, $\overline{q}'^{-k} = 0$) by $\pi^{k,L}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t')$. Similarly, if player k follows (with $\overline{q}'^k = 0$, $\overline{q}'^{-k} \neq 0$), I will denote the resulting profit function by $\pi^{k,F}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t')$. I will from now on suppress the subscripts indexing the values to a given state.

Assume, in what follows, that the equilibrium value has been determined for all $\tilde{\boldsymbol{q}} \geq \boldsymbol{q}_0$, i.e. all states with weakly higher capacity (strictly for at least one firm) and for all $\tilde{t} \in [t_0, T]$. To determine the equilibrium for state $(\boldsymbol{q}_0, t_0)$, I will need to construct the profits for either player following and leading at any t'. I can then construct the equilibrium outcome for this particular state by backward induction.

I will first show how to construct the profits conditional on firm i leading on the next investment.[21] Fix a scalar $\overline{q}'$ and let this be the ith component of $\overline{\boldsymbol{q}}'$ (with the other component zero). Then, for any t', the subsequent investment date $t^*_{\boldsymbol{q}',t'}$ and the corresponding value are known by assumption. For now, consider only cases such that $t^*_{\boldsymbol{q}',t'} > t'$, i.e. there is an interval of strictly positive length separating the investment under consideration from the subsequent one. Then, by subgame perfection, $\frac{\mathrm{d}t^*_{\boldsymbol{q}',t'}}{\mathrm{d}t'} = 0$, i.e. a small delay in investment will not affect the subsequent investment date.[22]

It is now straightforward to show that the profit from leading is quasiconcave in t'. To see this, note that

$$\frac{\partial \pi^{L,i}}{\partial t'} = e^{-\rho(t'-t_0)} \left(p(q_0, t')q_0 \frac{q_0^i}{q_0} - p(q', t')q' \frac{q'^i}{q'} + \rho c \left(\overline{q}'^i\right) \right) \tag{2}$$

$$\frac{\partial^2 \pi^{L,i}}{\partial t'^2} = -\rho \frac{\partial \pi^{L,i}}{\partial t'} + e^{-\rho(t'-t)} \left(p_t(q_0, t')q_0^i - p_t(q', t')q'^i \right) \tag{3}$$

Suppose $\frac{\partial \pi^{L,i}}{\partial t'} = 0$, $\frac{\partial^2 \pi^{L,i}}{\partial t'^2} \geq 0$; then clearly $p_t(q_0, t')q_0^i - p_t(q', t')q'^i \geq 0$. But, by the assumed separability of $p(q,t)$, this implies $p_0(q_0)q_0^i - p_0(q')q'^i \geq 0$, so the bracketed term in (2) has to also be positive, and thus $\frac{\partial \pi^{L,i}}{\partial t'} > 0$, contrary to the initial assumption. This yields quasiconcavity. To repeat, the π^L-curves given here are defined only for t' satisfying $t^*_{x',t'} > t'$.

The choice of investment quantity $\overline{q}'$ is simple to determine. Given that player i gets to lead and given t', she will choose $\overline{q}'$ which yields the highest possible profit. In other words, from the perspective of moment t_0, as long as player i leads on the next investment, the optimal value is given by the upper envelope, with respect to $\overline{q}'$, of the curves $\pi^{L,i}(t', \overline{\boldsymbol{q}}')$ (Figure 2). Three features are worth observing. Firstly, as the profit curves are differentiable, any local maximum of the upper envelope will also be a critical point, i.e. there will exist no 'kinked' maxima. Secondly, the curves' concavity is increasing in $\overline{q}'$, in the

[21] I refer by the index i to the investing firm, by $-i$ to the non-investing firm, and by k to any firm.

[22] The time derivatives of various quantities have to be defined so that $\mathrm{d}t$ and the grid size κ approach zero in lockstep. Note that I use the derivatives to characterise the equilibrium, but the agents in the model do not need to calculate them, so approximation errors in the derivatives do not affect the equilibrium outcome.

sense that for $\overline{q}' > \overline{q}''$,

$$\left.\frac{\partial \pi^{L,i}}{\partial t'}\right|_{\overline{q}'} = \left.\frac{\partial \pi^{L,i}}{\partial t'}\right|_{\overline{q}''} \Rightarrow \left.\frac{\partial^2 \pi^{L,i}}{\partial t'^2}\right|_{\overline{q}'} < \left.\frac{\partial^2 \pi^{L,i}}{\partial t'^2}\right|_{\overline{q}''}$$

implying that any two profit curves cross at most twice. Finally, note also that, as $\overline{q}'$ increases, for any t', the first integral term in (1) is unchanged; the second integral term increases; the investment cost becomes larger; and the continuation value may either increase or decrease. Hence, in general, it is not possible to order π_L with respect to $\overline{q}'$. In particular, the continuation values may change non-monotonically with respect to $\overline{q}'$, as the continuation equilibrium path may vary non-trivially with the continuation state.

On the other hand, the profits player $-i$ gets for following when player i invests $\overline{q}'$ at time t' are strictly increasing:

$$\frac{\partial \pi^{F,-i}}{\partial t'} = e^{-\rho(t'-t_0)} \left(p(q_0, t') - p(q', t')\right) q_0^{-i} > 0 \tag{4}$$

$$\frac{\partial^2 \pi^{F,-i}}{\partial t'^2} = -\rho \frac{\partial \pi^{F,-i}}{\partial t'} + e^{-\rho(t'-t_0)} \left(p_t(q_0, t') - p_t(q', t')\right) q_0^{-i} \tag{5}$$

where $q'^{-i} = q_0^{-i}$ as player $-i$ does not invest; $\frac{\partial^2 \pi^{F,i}}{\partial t'^2} = \frac{\partial \pi^{F,i}}{\partial t'} \left(-\rho f(t) + f'(t)\right) \leq 0$ if $\frac{p_t}{p} = \frac{f'(t)}{f(t)} \leq r$, which holds by assumption. In words, given that the other player is next to invest, the later this occurs, the better: the non-investing player prefers the price drop associated with new capacity to be delayed.[23] I also illustrate some π^F-curves in Figure 2. Note that $\lim_{t' \to T} \pi^F$ is independent of $\overline{q}'$ and equal to the profits obtained if no further investment is undertaken.

The solution is similarly straightforward in the case of simultaneous investment, i.e. if the investment subsequent to the next is immediate, or $t^*_{\boldsymbol{q}',t'} = t' + \kappa$ (recall that after any investment, the time period advances by one). As $\kappa \to 0$, then, $t^*_{\boldsymbol{q}',t'} \to t'$; I will henceforth only consider this limit. The sequence of investments at a given moment results in state $(\boldsymbol{q}'', t')$. Thereafter the following investment occurs a strictly positive interval of time later: $t^*_{\boldsymbol{q}'',t'} > t'$.

For simultaneous investments, the profit curve π^S is given by (1), with $\boldsymbol{q}'$ replaced by $\boldsymbol{q}''$, and the investment costs for both players given by the sum of the costs of their respective investments in the sequence.[24] Quasiconcavity

[23] The model thus departs from the framework of Hoppe and Lehmann-Grube (2005), who only consider cases such that profits for following decrease over time.

[24] One might think that simultaneous investments always feature exactly one investment by each player, as this would minimise total costs given the total capacity increment by each player. Due to the sequential timing assumptions, this is difficult to demonstrate analytically.

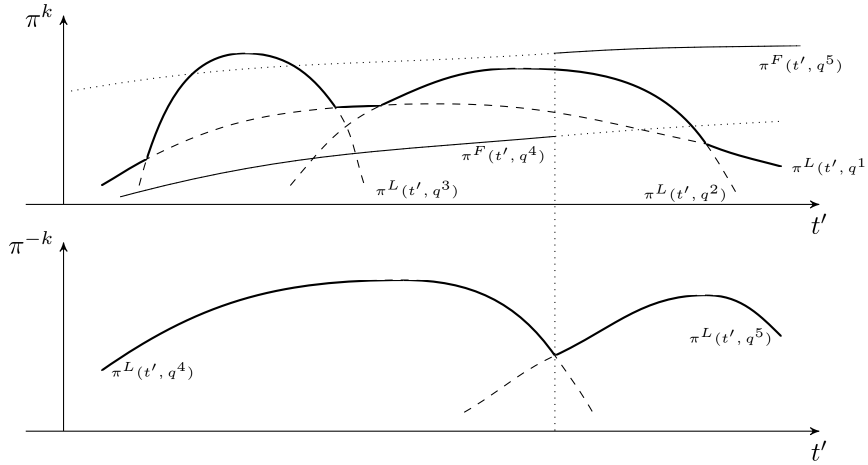

Figure 2: The profits for leading, as function of time, are given by the upper envelope (*thick line*) of the π^L-curves (*dashed lines*) for different investment quantities. For the top player, curves displaying profits for different lead quantities chosen by the other player (*dotted lines*) are also shown. The actual π^F-curve (*solid line*) may be discontinuous at points at which the other player's lead quantity changes.

holds by the same arguments used previously. Finally, fix any $\overline{\boldsymbol{q}}'$, and denote by $\tilde{t}$ the point such that, in a neighbourhood of $\tilde{t}$, $t^*_{\boldsymbol{q}',t} > t$ for $t < \tilde{t}$, but $t^*_{\boldsymbol{q}',t} = t$ for $t > \tilde{t}$. That is, $\tilde{t}$ is a point at which, given an investment quantity, the outcome switches from the subsequent investment being delayed (unilateral investment) to immediate subsequent investment (simultaneous investment). Then

$$\lim_{t\uparrow\tilde{t}} \pi^{L,i}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t) = \lim_{t\downarrow\tilde{t}} \pi^{S,i}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t)$$
$$\lim_{t\uparrow\tilde{t}} \frac{\partial \pi^{L,i}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t)}{\partial t'} < \lim_{t\downarrow\tilde{t}} \frac{\partial \pi^{S,i}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t)}{\partial t'}.$$

In words, at the moment at which investment becomes simultaneous, the π^L- and π^S-curves join up but there is an upward kink.

However, numerical experiments have not revealed outcomes in which multiple investments are made by one player at a given moment in time.

3.4 Subgame-perfect equilibrium

I will now describe the subgame-perfect equilibrium to the game. Existence of such an equilibrium holds trivially, by Zermelo's Theorem (Fudenberg and Tirole, 1991). The equilibrium also seems to be generically unique.[25]

Note that the timing structure I utilise here, together with the finite horizon, rules out self-defensive equilibria, unlike various continuous-time or simultaneous-move formulations (Gilbert and Harris, 1984; Fudenberg and Tirole, 1985; Katz and Shapiro, 1987). The timing assumptions in the present paper—in particular, the sequential move order—ensure that Pareto-dominated equilibria to any subgame are never played (in the limit as $\kappa \to 0$).[26]

The construction of the subgame perfect equilibrium is not difficult, if a little tedious. Equilibrium investments can be characterised and interpreted in a relatively tidy fashion. I will here only enumerate and intuitively describe the various types of equilibrium investment; the formal characterisation is relegated to Appendix B.

3.4.1 Unilateral investment

Given the state $(\boldsymbol{q}_0, t_0)$, consider unilateral equilibrium investment, that is, investment such that the subsequent investment takes place only after a strictly positive interval of time: $t_2^* > t_1^*$. Any such outcome can be classified as belonging to one of seven different types. These types are illustrated in Figure 3, in terms of the two players' respective profit curves $\pi^{k,L}$ and $\pi^{k,F}$. These points are equilibrium *candidates* only; the actual equilibrium outcome is determined from the exact sequence of such points by backward induction (see Section 4 for

[25]At each decision node, a player will have exactly two choices: investing at the optimal quantity, or letting the game continue, with the continuation payoff obtained by backward induction. The optimal quantity is uniquely determined. Firm i investing on any given date t' will choose to invest at a quantity which maximises $\pi^{L,i}(\overline{\boldsymbol{q}}', t')$, i.e. its profits for leading. Two or more values of $\overline{q}'^i$ yield the same profits, by definition, at a crossing of the respective $\pi^{L,i}$-curves. As decision moments are discrete, the date t' coincides with such an intersection only by chance. Such a coincidence would not be robust to a small perturbation in any key parameter, e.g. the period length.

This heuristic argument is based on the failure to find any *a priori* mechanisms systematically causing such equalities to hold, or any numerical examples in which they do hold. Should the argument fail, uniqueness could be ensured by adding a very small stochastic perturbation to e.g. the cost functions. The magnitude of such a perturbation would have to fall sufficiently rapidly with the period length to ensure the perturbation would never outweigh any approximation errors, so that the continuous-time approximation would still hold.

[26]Of course, in a simultaneous-move game, coordination on a self-defensive Pareto-dominated equilibrium in a particular subgame might be used as a threat strategy; I do not imply that the equilibrium picked out by my assumptions might be Pareto-dominant for the full game.

the algorithm to determine the equilibrium investment outcome).

The first three cases consider equilibrium investments in situations in which both players' optimal actions are continuous, that is, neither player's optimal lead quantity is about to change. The next three cases describe equilibrium investment driven by a change in this optimal lead quantity. The last case completes the list. I will refer to the two players as 'top' and 'bottom', with reference to Figure 3.

(i) Preemption. (Figure 3, top left.) This is most basic case: bottom (weakly) prefers to lead, while top is indifferent between following and leading. Note that any potential equilibrium in which investment were to occur a short time later would unravel by the desire of both players to preempt the other. If top strictly prefers to lead, she will get to invest. If both players are indifferent between following and leading, the player who moves first invests at the first moment following the crossing of the two curves.

(ii) Unilateral investment without preemption. (Figure 3, top right.) In this case top prefers leading to following, while bottom prefers following to leading. Further, top's π_L-curve has a local maximum, that is, top can choose an interior optimum, with constraints imposed by preemption not binding.

(iii) Forced investment. (Figure 3, middle left.) Bottom prefers following to leading or continuation, and wants to force top to invest; top would prefer to continue, but would rather lead than follow. In other words, bottom faces a second-mover advantage, and top a first-mover advantage; but top would prefer for the game to continue.[27] Bottom can force top to invest at the point at which bottom's profits from leading are just about to fall below the value she obtains from continuing the game. At this point, bottom's profits from leading have to be decreasing; otherwise bottom could leave the threat until a while later and get even higher profits for following. Investment is determined by the fact that, a moment later, the threat to force investment would no longer be credible.

(iv) Symmetric forced investment. (Figure 3, middle right.) This case occurs, generically, only when the players both have equal capacity, at

[27] Note that when discussing first- and second-mover advantage here, I refer to whether the players would like to 'invest first', (i.e. lead), or to 'invest second' (follow). In particular, these terms do not refer to the move order in any particular period.

a moment at which the optimal lead quantity changes.[28] Both players would prefer the other player to lead with the quantity optimal running up to the investment point. Following this point, both prefer leading to following, and preemption forces immediate investment. In other words, both players face a situation with second-mover advantage, about to expire. Thus, the player who moves last just before the crossing is forced to invest; the player who moves first has the first option to decline to invest. Note that the π^L-curves might also be decreasing.

(v) Preemption with discontinuity. (Figure 3, bottom left.) This case is the standard preemption case, except that bottom's π^F-curve crosses her π^L-curve discontinuously, due to top's optimal investment quantity changing. The threat of impending preemption means top invests at the last possible moment before the curves cross.

(vi) Forced investment with discontinuity. (Figure 3, bottom right.) This case is like the forced investment case; top strictly prefers following to leading, and leading to continuing. However, in the previous case, investment is forced by the expiration of the threat's credibility. Here, the threat itself expires; a moment later, bottom would rather follow than lead, and can so no longer be threatened into investing.

(vii) Immediate investment. Given the date t_0, i.e. the start of the subgame, it is always possible that the optimal investment occurs immediately.

I will make a few comments about the potential equilibrium investment dates. Types (i) and (ii) were investigated by Katz and Shapiro (1987). For type (iii), their assumptions on timing (simultaneous moves) and tie-breaking (a coin flip) led to non-existence of equilibrium; in the present paper, with sequential moves, the equilibrium exists. The discontinuous cases have not, to my knowledge, been previously considered in the literature.

Case (iv) may seem odd: the firms are symmetric, but both would rather follow than lead. In this, the equilibrium resembles a war of attrition (Hendricks et al., 1988). Numerical examples demonstrate that such cases are not impossible. In such a case, both firms want someone to invest, as continuation would eventually (or immediately) lead to higher ultimate capacity. However, both would still strictly prefer the other firm to be the first investor, knowing they

[28] With unequal capacities, the case in which both players' optimal actions change at the same moment is not robust to a small perturbation of the model parameters.

will be allowed into the market later in the subsequent equilibrium, possibly with higher capacity and saving the opportunity cost of investment funds for the time being.

Per-period randomisation of the move order is important here. With a fixed move order, the firm moving first would have a first-mover advantage in any symmetric pre-emptive investment outcomes, but its opponent would hold a second-mover advantage in a symmetric forced investment case. Such persistence and asymmetry of the infinitesimal advantage would alter the continuation payoffs, and thus lead to a very different equilibrium. I feel it is unrealistic to assume one of the firms is able to consistently hold on to a very small (dis-)advantage.

3.4.2 Simultaneous investment

In the case of simultaneous investment, the profits from leading and following can be calculated as outlined above. The equilibrium types are thus as above, except that the profit curves for the other player leading first (equivalent to 'following') may now also decrease. One particular special case deserves highlighting.

Suppose there is some time t' at which the players optimally make one investment each simultaneously. Denote the optimal investment quantities by players 1 and 2, respectively, by $\overline{q}'$ and $\overline{q}''$, the corresponding unilateral investment vectors by $\overline{\boldsymbol{q}}' \equiv (\overline{q}', 0)$ and $\overline{\boldsymbol{q}}'' \equiv (0, \overline{q}'')$, and the corresponding next states by $\boldsymbol{q}' \equiv \boldsymbol{q} + \overline{\boldsymbol{q}}'$ and $\boldsymbol{q}'' \equiv \boldsymbol{q} + \overline{\boldsymbol{q}}''$. Suppose now that the optimal responses to these investments are $\overline{\boldsymbol{q}}^*_{\boldsymbol{q}',t'} = \overline{\boldsymbol{q}}''$ and $\overline{\boldsymbol{q}}^*_{\boldsymbol{q}'',t'} = \overline{\boldsymbol{q}}'$, respectively. Then, irrespective of who invests first, the continuation state is going to be $\boldsymbol{q} + \overline{\boldsymbol{q}}' + \overline{\boldsymbol{q}}''$, and both players get the same profits irrespective of whether they follow or lead.

Over an interval on which this holds, both players' profits from simultaneous investment, leading, coincide with her profits from simultaneous investment, following. On such an interval, equilibrium investment takes place at the earliest moment on which the slope of either player's profit curve is (weakly) negative.

In other words, it may be that under simultaneous investment, the players both delay investment, as in the joint adoption outcome described by Fudenberg and Tirole (1985). Note that the presence of such an outcome—both players tacitly delaying and then investing at the same time—depends crucially on the incentives for any active players to let its competitor enter or expand capacity. Without such incentives, such a tacitly collusive outcome would not exist; either player would have an incentive to preempt, by building $\overline{q}' + \overline{q}''$ units just before

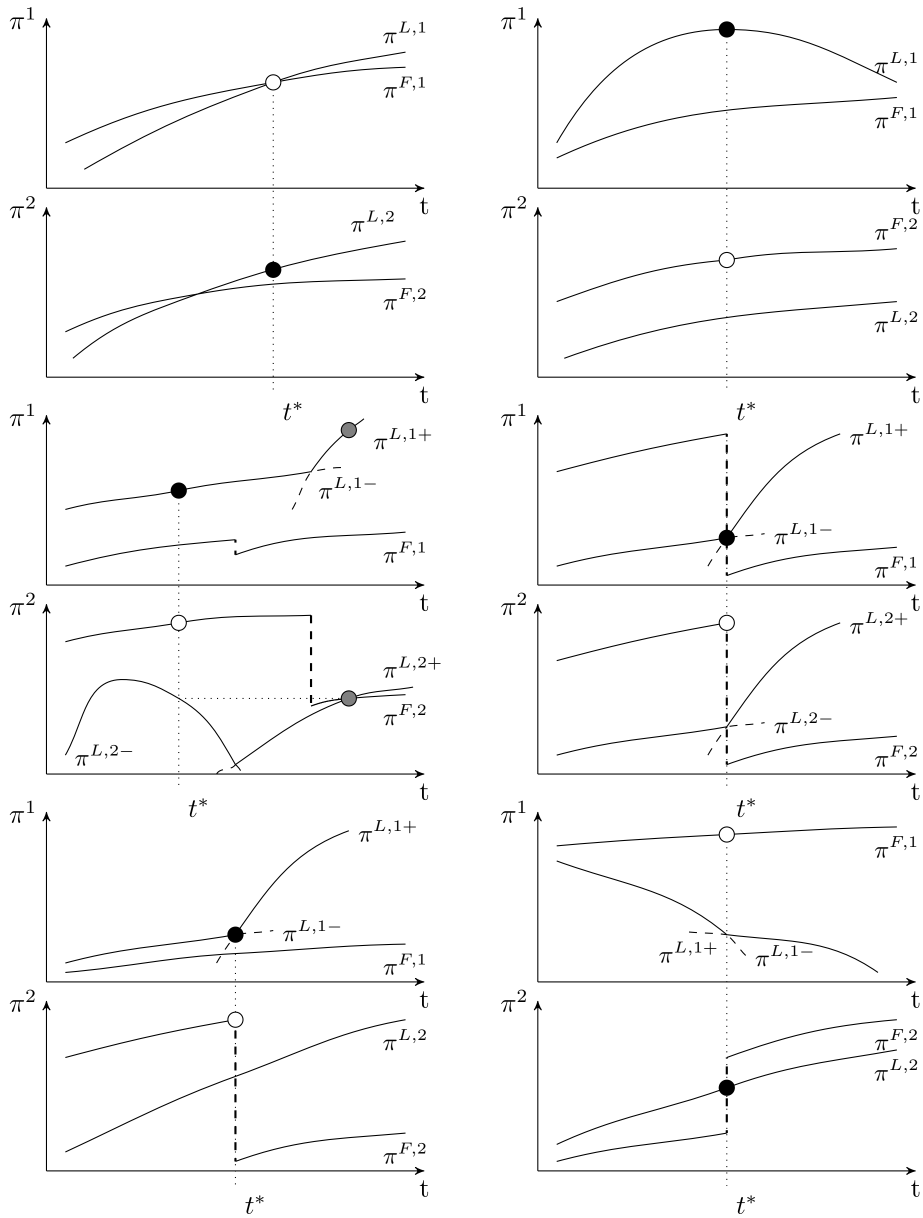

Figure 3: Equilibrium candidates. Black (white/grey) dot indicates equilibrium profit for leading (following/continuation). (*top left*) Preemptive investment. (*top right*) Investment without preemption. (*center left*) Forced investment due to a credible threat. (*center right*) Symmetric forced investment. (*bottom left*) Preemption with an asymmetric discontinuity. (*bottom right*) Forced investment with an asymmetric discontinuity.

the equilibrium investment date.[29]

4 Numerical examples

In this section, I present a simple numerical algorithm which is used to solve the model, and some computed examples.

4.1 The computational algorithm

The model can be solved using a straightforward computational algorithm. Take any $\boldsymbol{q}$ such that the equilibrium is known for all $\tilde{\boldsymbol{q}}$ with higher aggregate capacity (i.e. $\tilde{q} > q$). I partition the timeline $[0, T]$ into a collection $\mathcal{T}$ of disjoint intervals, with the elements separated:

- at points at which optimal lead quantities change;
- at critical points of the π^L-curves; and
- at all the points at which the continuation outcome changes (either changing from delayed investment to immediate subsequent investment, or, with immediate subsequent investment, changing from one quantity to another).

The backward induction algorithm will further keep partitioning the time intervals as the continuation value changes; this is explained below. The functional assumptions made allow the number of crossings of any two π^k-curves, or of π^k and a constant, to be determined analytically, with some crossing points solved in closed-form and others numerically.[30]

In this way, I obtain a sequence of disjoint time intervals, ordered in time. Each of these elements of $\mathcal{T}$ can be classified in terms of: a) the ordering of $\pi^{k,L}$, $\pi^{k,F}$ and the continuation value $V^{k,C}$; and b) the slope of $\pi^{k,L}$. The optimal

[29] Fudenberg and Tirole (1985), by assumption, restrict cumulative investment to one unit each for both players as they consider technology adoption rather than capacity build-up. This is why their model can have simultaneous, or clustered, investments. Argenziano and Schmidt-Dengler (forthcoming) show that clustering can occur for an alternative mechanism for three or more players. Mills (1990) also obtains clustering for some cases, but only because the maximum investment size in his model is capped at 2. The mechanism presented here, also demonstrated by Boyer et al. (2012) in a subtly different context, is different to any of these.

[30] As decisions are taken at discrete intervals, it is immaterial to which element the cutting point is assigned.

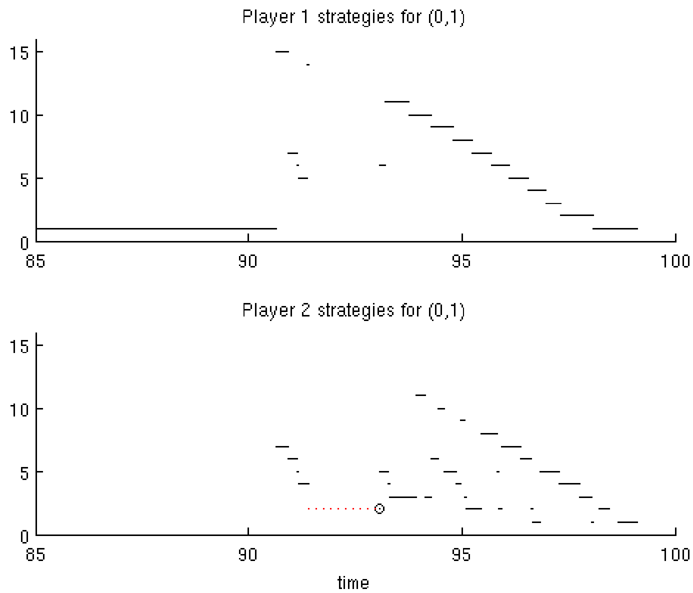

Figure 4: Equilibrium strategies for state $(0, 1)$ for a particular instance of the model ($T = 100$, $n = 25$). The strategies are shown only for $t \in [85, 100]$. The red dotted line indicates a period of waiting until the end, at which point player 2 invests with $\bar{q}^2 = 2$. Clearly strategies are difficult to characterise intuitively.

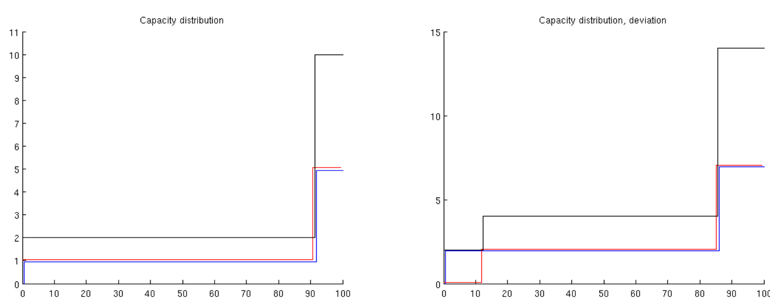

Figure 5: (*left*) Equilibrium capacity paths for an instance of the model ($T = 100$, $n = 25$); red and blue indicate capacities of individual firms, with black indicating aggregate capacity. In this case, firms immediately invest in one unit each, then wait until $t = 91$ before investing in four additional units each. Tacit collusion ensures aggregate capacity remains well below n. (*right*) A deviation by one firm to build two units a time $t = 0$ leads the competitor to catch up with also two units at $t = 12$. A further five units each are built at $t = 86$. The attempt to capture the market backfires as both firms tend to overinvest later.

strategies and equilibrium outcomes on all of these intervals are straightforward to classify (see Appendix B).

Close to the end of the game, investment is no longer profitable, so the default candidate equilibrium from which to start iterating is $t^C = T$, $\overline{\boldsymbol{q}}^C = (0,0)$, with the continuation values given by the abstention profits: $V^{k,C} = \pi^{k,A}$. Take the last element of $\mathcal{T}$, and denote by $\underline{t}$ the starting point of this element.

Then:

1. Based on the ordering of π_L^k, π_F^k and $V^{k,C}$, and the slope $\frac{\partial \pi^{k,L}}{\partial t'}$, determine the equilibrium strategies for the players in this interval. If the equilibrium outcome is continuation, go to step 4.

2. Update the candidate investment time t^C and values $V^{k,C}$.

3. For both players, project $V^{k,C}$ backwards from t^C to determine the point at which it crosses the $\pi^{k,L}$-curve, or $\max\{t \in [0,\underline{t}] : \forall \tau \in [t,\underline{t}), \pi^{k,L} \leq V^{k,C}\}$. Partition the corresponding element at this point.

4. If $\underline{t} > 0$, move to the next (earlier) interval element, updating $\underline{t}$. Otherwise stop.

The outer nest of the algorithm will simply backward induct with respect to aggregate capacity. I have shown above that there exists a cap n to aggregate capacity. I can then solve for the equilibrium for all $\boldsymbol{q} \times [0,T]$ such that $q = n-1$, and so work backward all the way to $q = 0$.

4.2 Results

I specify the demand function to take the isoelastic form, growing exponentially: $f(t) = e^{\gamma t}$, $p_0(q) \equiv Aq^{-\frac{1}{\sigma}}$, with $\sigma > 1$, $\gamma < \rho$. The scaling parameter A is, in principle, redundant and could be eliminated by a convenient choice of units. Demand is thus isoelastic, with elasticity greater than unity, with the level of demand growing at rate γ until T, after which the market disappears.

For arbitrary specifications of the model, the equilibria can become quite complex. All of the different equilibrium investment types characterised above can be observed. Typical equilibrium outcomes involve tacit collusion, so that the first firm to enter will later allow the competitor into the market. Clustering of investments—both firms investing at the same moment—is common. In many cases, the equilibrium outcome is perfectly symmetric so that both firms make equal investments at the same moment.

I illustrate the potential complexity in Figure 4. In general, for aggregate quantities close to n (equal to 25 in this example) the equilibrium strategies are relatively simple and intuitive. However, the complexity is compounded as the algorithm proceeds backwards through the state tree, as points of discontinuity in the strategies start to add up. It becomes difficult to draw general conclusions from the results; I illustrate equilibrium strategies for one particular vector of capacities. Figure 5 illustrates the equilibrium path for this instance of the model, and one possible deviation to show how tacit collusion pays off. Note that along the equilibrium path ultimate capacities remain at around 40% of n. The attempt by one player to capture a larger share of the market is simply followed by aggressive catching up by the hungry entrant; later, both firms invest in even more capacity as they care less for the resulting price decreases. This deviation leads to ultimate capacity reaching almost 60% of n.

4.3 Preemptive CO_2 pipeline investment

Finally, I present a little illustrative experiment related to duopolistic supply of carbon storage capacity. This experiment considers CO_2 storage under the North Sea and the required expansion of pipeline capacity to transport the pollutant (Jaakkola, 2013). Storage capacity is the product being sold here, with demand for storage capacity arising from the presence of a climate policy which prices carbon. The firms here represent Norway and Scotland, the two parties with very large undersea storage potential in northwestern Europe.

I offer two illustrative numerical experiments, under somewhat *ad hoc* assumptions regarding CO_2 storage demand. The parameters are as follows. Resource demand is now parameterised with $\sigma = 1.3$ (1.5 in the alternative experiment), $A = 15$ (50 similarly), $\gamma = .29$, $T = 80$. The cost function is given by parameters $\alpha_1 = 1.066$, $\alpha_2 = .038$, with capacity measured in millions of tonnes of CO_2 ($MtCO_2$) transported per annum. The discount rate is $\rho = .03$. I offer extended justification for these parameters in Jaakkola (2013), except for the demand specification.[31]

The equilibrium paths of the experiments are shown in Figure 6. I use pipeline increments of 20 $MtCO_2$ per annum. In the top case, elasticity of demand is low ($\sigma = 1.3$). The equilibrium features preemption, with both firms waiting until $t = 4.5$, at which point both want to invest into a 100 $MtCO_2$ pipeline. Only one of these investments takes place; no further investments are

[31] A calibration for the CO_2 storage demand curve is work-in-progress.

made, and both firms make zero profits. The socially optimal outcome is to immediately build an 80 $MtCO_2$ pipeline, followed by a further 180 $MtCO_2$ of capacity at time $t = 38$. A monopolist would build a very small, 40 $MtCO_2$ pipeline immediately and refrain from further investment. Thus, in this case, preemption holds; all profits are zero, and only one firm is active in the market. Nevertheless, total capacity is held back compared to the efficient case as the duopolists do not consider consumers' surplus in their decisions.

With more elastic demand ($\sigma = 1.5$), and also a higher implicit carbon price, so that a social planner's cumulative investment would reach 580 $MtCO_2$, the equilibrium outcome features tacit collusion with a second-mover advantage. That is, both firms want a 100 $MtCO_2$ pipeline to be built at time $t = 11$, but moreover both prefer this pipeline to be built by the other firm. The firm which follows on the first investment then enters by building 140 $MtCO_2$ of capacity at time $t = 34$. The firm forced to make the first investment is the one who moves second at the last instant before the investment date. Immediately following this date, it would be optimal to lead by building a much larger capacity pipeline, with no further investments in equilibrium. Neither firm wants to take their chances under this preemptive outcome as they worry they will be the one left outside the market. Both firms make positive profits; in particular, the firms' roles in the market are reversed along the equilibrium path, with the initial incumbent ending up smaller than its competitor, and also making lower profits. Thus, rent equalisation does not hold. Following the first investment, it will not make sense for the incumbent to preempt the second entrant as a very large pipeline is required to keep the other firm permanently out of the market, with ultimate capacities rising to $q(T) = 360$, instead of $q(T) = 220$ as in equilibrium.

5 Conclusions

Many industrial sectors involve capital-intensive production technologies. Substantial economies of scale imply capital investments in production plants tend to be lumpy. In such a context, oligopolistic competition may lead to firms seeking to preempt each other whenever profitable investment opportunities arise. Previous literature has emphasised the possibility that such competition may involve very aggressive strategies: if firms always expect their competitors to invest aggressively, the best response is to respond in kind. Such tough compe-

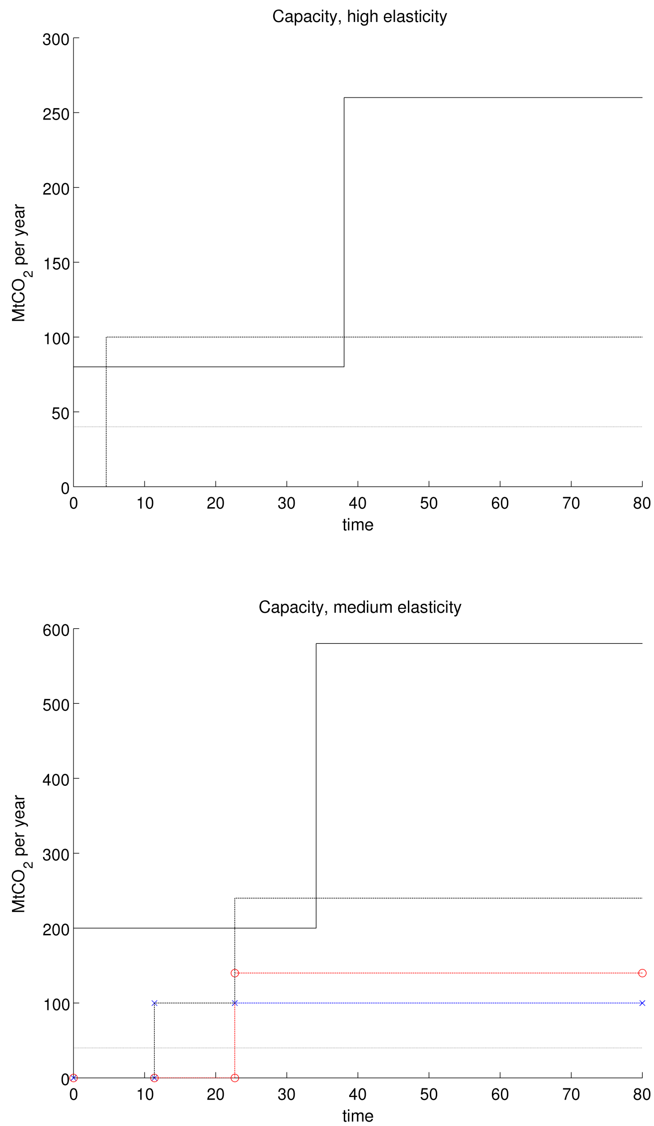

Figure 6: Comparison of the preemptive investment schedule (*dashed*: *blue with crosses, red with circles*: individual firms, *black*: aggregare), with the socially optimal (*solid*) and monopolistic (*dotted*) outcomes. The two graphs differ with respect to demand elasticity and the implicit carbon price: the bottom case involves more elastic demand, but also a higher carbon price, in the sense of leading to higher cumulative investment. As the top example features preemption under duopoly, only aggregate capacity is shown.

tition may end up wasting a large fraction of the available rents.

In the present paper, I have shown that such races need not lead to excessively competitive outcomes. In fact, if firms recognise that their opponents are not likely to behave in such an overly aggressive manner, they can themselves also relax a little. Without employing threat strategies, the firms are able to coordinate on a tacitly collusive outcome which ensures ultimate capacity is held back. This ultimately relies on a recognition that, in this setting, capacities are strategic complements: building less capacity tempers competing firms' desire to build capacity themselves. Other authors (in particular, Boyer et al., 2012) have previously demonstrated similar results. As their model introduces multiple modifications of the models used in previous literature, it has not been clear what has driven these results. I have shown that similar results obtain in the canonical framework of Gilbert and Harris (1984); all that is required is a focus on an alternative equilibrium.

I have then extended the model to account for economies of scale in the amount of capacity added. This makes the size of plants central: investment can be triggered by the intention of the other firm to start planning larger or smaller plants. In particular, unlike in previous literature, this implies that investment races may sometimes not be about who gets to invest first, but rather who is forced to invest first. Such a possibility, observed in my numerical results, may mean that rent equalisation—a ubiquitous feature in previous models of preemptive competition—may not hold. The present paper has also shown that the ultimate distribution of capacity may be dependent on the model particulars, in some cases tending towards domination by a large incumbent, in others towards more equally divided capacities.

This paper has demonstrated the existence of new types of equilibria in capacity expansion games. However, at present there is little to say regarding the economic significance of such equilibria. Ongoing work, both analytical and numerical, will seek to provide insight into the key determinants of these equilibria; to evaluate the welfare effects of tacit collusion, and any inefficiencies related to suboptimal plant sizes (as well as investment timings) when scale economies are present; and to identify clear testable predictions from the model and compare these with empirical observations.

Appendices

A Continuous-time approximation

In the main text, I claim that the discrete-time equilibrium can, as the time period becomes very small, be characterised by the continuous-time description of the various equilibrium points. More precisely, I claim that the equilibrium investment times and quantities are given, in the limit, by the crossing points of the various continuous-time profit curves.

Lemma 1. *In the approximation of the discrete-time framework by the continuous-time formulae, the approximation errors in* $\pi^k_{\boldsymbol{q_0},t_0}(\overline{\boldsymbol{q}}', t')$ *are linear in the period length* κ *(as the period length becomes sufficiently small).*

Proof. Take the continuous-time approximation of the π^k-curves as the benchmark; for any κ, the values at the decision points do not lie exactly on this curve. I want to establish a bound on the difference ϵ^π between the values in the discrete-time formulation and the continuous-time formulation, and show that this value goes to zero as $\kappa \to 0$.

Take any initial state $(\boldsymbol{q}_0, t_0)$. Assume that, as we change κ, the sequence of states in the rest of the game does not change. I will later show how to ensure this is the case.

As time has been defined to run continuously, any integrals between two dates $\tilde{t}_1$ and $\tilde{t}_2$ hold exactly. Thus, there are two sources of approximation error in (1). The first is the continuation value term $e^{-\rho(t^*_{\boldsymbol{q}',t'}-t_0)}V^k(\boldsymbol{q}', t^*_{\boldsymbol{q}',t'})$ (here described before the subsequent investment), in which the value of the subsequent state is approximate, and the subsequent investment date $t^*_{\boldsymbol{q}',t'}$ is not constrained to lie on the grid of decision points (as it is in the true, discrete-time model). Call the total error ϵ^{V+}. The second source is the error in the second revenue term; the upper limit of integration is given by $t^*_{\boldsymbol{q}',t'}$, which carries an error of magnitude ϵ^{t+}. Call the second error ϵ^R. In reality, the two terms offset each other, but I conservatively add them up when obtaining error bounds. Both error terms are a function of κ, but, given κ, constant with respect to t'. Thus, this error implies that, for any κ,

$$\pi^k_{\mathrm{TRUE}}(\overline{\boldsymbol{q}}', t') = \pi^k(\overline{\boldsymbol{q}}', t') + \epsilon^{V+}(\kappa) + \epsilon^R(\kappa)$$

i.e. the true profits are given by a sequence of points which has the shape of the continuous-time approximation, but has been shifted up or down. As I have argued in the text that the continuous-time curves have at most two crossings, so the shifted curves also cross at most twice—and thus the true values also.

These vertical shifts induce an error on the crossing point. Furthermore, there is an additional source of error: the true discrete-time formulation requires the equilibrium to lie on the grid of decision points. In fact, the equilibrium point itself may not be in the node immediately next to the crossing, but in the next one, that is, a distance of up to 2κ from the true crossing point. Hence, the approximation error in the investment timing ϵ^t, for small κ, is bounded by

$$|\epsilon^t| < 2\kappa + \frac{\left|\epsilon^{V+}(\overline{\boldsymbol{q}}^1;\kappa)\right| + \left|\epsilon^{V+}(\overline{\boldsymbol{q}}^2;\kappa)\right|}{\left|\frac{\mathrm{d}\pi^k(\overline{\boldsymbol{q}}^1)}{\mathrm{d}t'}\right| + \left|\frac{\mathrm{d}\pi^k(\overline{\boldsymbol{q}}^2)}{\mathrm{d}t'}\right|}$$

in which the continuation error terms refer to the two curves whose crossing we are considering (these could be from following, leading or simultaneous investment). The denominator contains the slopes of the curves at the crossing; with small enough κ, the curves are close to linear and the above establishes an upper bound on the error term.[32]

Note that if the terms ϵ^{V+} are linear in κ, so is the bound on the timing error ϵ^t. Suppose that this is the case, and also that the approximation error ϵ^{t+} is similarly linear. With the initial state still $(\boldsymbol{q}_0, t_0)$, denote by $\bar{t}_j^* \equiv \max(t_j^*, t_{j,\mathrm{TRUE}}^*)$ the latter of the continuous-time crossing date and the true investment date for the jth equilibrium investment, and, correspondingly, by $\underline{t}_j^*$ the earlier. Then the error in the optimal value for a state, ϵ^V, is also bounded and this bound is linear in κ:

$$\begin{aligned}
\left|\pi^k - \pi^k_{\mathrm{TRUE}}\right| \leq{}& \left|\int_{\underline{t}_1^*}^{\bar{t}_1^*} e^{-\rho(t-t_0)}\left(p(q,t)q^k - p(q',t)q'^k\right)\,\mathrm{d}t\right| \\
&+ \left|\int_{\underline{t}_2^*}^{\bar{t}_1^*} e^{-\rho(t-t_0)}\left(p(q',t)q'^k\right)\,\mathrm{d}t\right| \\
&+ \left(e^{-\rho\left(\underline{t}_1^*-t_0\right)} - e^{-\rho\left(\bar{t}_1^*-t_0\right)}\right)c(\overline{\boldsymbol{q}}'^k) + \epsilon^{V+} \\
\leq{}& e^{-\rho\left(\underline{t}_1^*-t_0\right)}\left|p(q,\bar{t}_1^*)q^k - p(q',\bar{t}_1^*)q'^k - c(q_1^*)\right|\epsilon^t \\
&+ \epsilon^{V+} + e^{-\rho\left(\underline{t}_2^*-t_0\right)}p(q',\bar{t}_2^*)q'^k\epsilon^{t+}
\end{aligned}$$

The final step is to observe that, for any state and investment quantity such

[32] The case in which both slopes were zero would not be a crossing of interest, as the π_L-envelope would not change at this point.

that the following state yields no further investment in equilibrium, the errors in the continuation value, continuation timing and current timing are $|\epsilon^{V+}| = 0$, $|\epsilon^{t+}| = 0$, $|\epsilon^{t}| < 2\kappa$. By induction on the state space, all approximation errors are then linear in κ.

The sequence of states following any given $(\boldsymbol{q}_0, t_0)$ given by the continuous-time approximation coincides with the discrete-time approximation, provided the period length is small enough so that no potential equilibrium investment point is 'jumped over' and that the approximation errors are made sufficiently small. It is straightforward to establish this argument formally by induction. □

Based on the above Lemma, it is apparent that the continuous-time equilibrium gives an arbitrarily good approximation to the discrete-time equilibrium as the period length becomes infinitesimal. The characterisation of the equilibrium below makes it clear that equilibrium points are determined by intersections of the various π^k-curves, and the equilibrium strategies hinge on various inequalities between π^L, π^F and the continuation value V^C. These quantities will be correctly ordered provided that κ is small enough. The only potential problem in the limit would be if the continuous-time approximations of two of these quantities would be exactly equal: then approximation errors could result in the discrete-time equilibrium not converging as $\kappa \to 0$. However, such a case would be a knife-edge case, not robust to a small perturbation of e.g. the cost parameters.

B Characterisation of candidate equilibrium investments

Observe first that any point t' at which the π^L-curves are continuous for both players can only be an equilibrium if $\pi^{k,F} > \pi^{k,L}$ for at least one k; otherwise preemption unravels the equilibrium (unless $t' = t_0$, the starting moment of the game under consideration). Investment will occur only for cases (i)-(iii); if the conditions described do not hold, then the investing player always has an incentive to either bring investment forward or to delay it.

The only other moments at which equilibrium investment can take place involve discontinuities in one player's optimal lead quantity. I do not consider cases in which both players have discontinuous lead quantities at the same moment as these are not robust to a perturbation in model parameters, except in the case in which the players both have equal capacity.

To characterise potential equilibrium investments at a point of discontinuity, I classify possible equilibrium candidates by: the continuation value $V^{k,C}$, relative to $\pi^{k,L}$, for both k; the slopes of the $\pi^{k,L}$-curves for both players; the ordering of $\pi^{-i,L}$, $\lim_{t\uparrow t'} \pi^{k,F}$ and $\lim_{t\downarrow t'} \pi^{k,F}$ for the noninvesting player $-i$. I go through these candidates one by one to rule out all scenarios which cannot be an equilibrium. Many of these are easy to rule out. For example, no equilibrium investment can (obviously) take place at which both players get a higher payoff by continuation to the next candidate. Similarly, no investment can take place where the identity of the next investor is known with certainty, and that player's π^L-curve is decreasing.

The remaining candidates have to be worked through one by one. As an example, cases (v) and (vi) are illustrated in Figure 7. The lines are given by the continuous-time approximations of the profit curves, which are very close to the true values. I show a few decision moments around the continuous-time 'equilibrium point'. Each period, the black dot gives the value at the beginning of the period if top moves first; the circle gives the value if bottom moves first. These are easy to confirm by constructing the decision tree in both cases. The diamond illustrates the expected continuation value in the previous period, which is just the mean between the two outcomes. It is straightforward to work through these examples to confirm that the discrete-time equilibrium investment takes place at the earliest depicted timestep (in the upper case, top invests just before the crossing; in the lower case, bottom does so but at the second step before the crossing).

This process results in a set of conditions for equilibrium candidates (the candidates have been more intuitively described in the main text). Take some candidate moment for equilibrium investment $\tilde{t}^*$. Denote the limiting optimal lead quantities in the neighbourhood of $\tilde{t}^*$ by $\overline{q}'^{k-} \equiv \lim_{t\uparrow\tilde{t}^*} \arg\max_{\overline{q}'^k;\overline{q}'^{-k}=0} \pi^{k,L}(t,\overline{\boldsymbol{q}}')$, $\overline{q}'^{k+} \equiv \lim_{t\downarrow\tilde{t}^*} \arg\max_{\overline{q}'^k;\overline{q}'^{-k}=0} \pi^{k,L}(t,\overline{\boldsymbol{q}}')$. Denoting the investing player with i, and evaluating all profit functions at the optimal lead quantities, equilibrium candidates then satisfy one of the following conditions:

(i) $\tilde{t}^* > t_0$, $\overline{q}'^{k-} = \overline{q}'^{k+}$, $\forall k \in \{1,2\}$: a) $\frac{\partial \pi^{i,L}}{\partial t'} \geq 0$, b) $\pi^{-i,L} - \pi^{-i,F} = 0$, and c) $\frac{\partial \pi^{-i,L}}{\partial t'} \geq 0$;

(ii) $\tilde{t}^* > t_0$, $\overline{q}'^{k-} = \overline{q}'^{k+}$, $\forall k \in \{1,2\}$: a) $\frac{\partial \pi^{i,L}}{\partial t'} = 0$, b) $\pi^{-i,L} - \pi^{-i,F} \leq 0$, and c) $\frac{\partial \pi^{-i,L}}{\partial t'} \geq 0$ if (i-b) holds with equality;

(iii) $\tilde{t}^* > t_0$, $\overline{q}'^{k-} = \overline{q}'^{k+}$, $\forall k \in \{1,2\}$: a)[33] $\pi^{i,L} - \pi^{i,F} > 0$ and $\frac{\partial \pi^{i,L}}{\partial t'} \geq 0$, b) $\frac{\partial \pi^{-i,L}}{\partial t'} \leq 0$ and $\pi^{-i,L} - \pi^{-i,F} < 0$, and c) $\pi^{-i,L} = \lim_{\tau \downarrow \tilde{t}^*} V^{-i,C}(x, \tau)$;

(iv) $\tilde{t}^* > t$, $q_0^k = q_0^{-k}$, $\overline{q}'^* = \overline{q}'^{k-} \neq \overline{q}'^{k+}$, for $i, k \in \{1,2\}$: a) $\lim_{t \uparrow \tilde{t}^*} \pi^{k,L} - \pi^{k,F} < 0$ and $\lim_{t \downarrow \tilde{t}^*} \pi^{k,L} - \pi^{k,F} > 0$, and b) $\lim_{t \downarrow \tilde{t}^*} V^{k,C}(\boldsymbol{q}_0, t) < \pi^{k,L}$;

(v) $\tilde{t}^* > t$, $\overline{q}'^* = \overline{q}'^{k-} \neq \overline{q}'^{k+}$: a) $\lim_{t \uparrow \tilde{t}^*} \frac{\partial \pi^{i,L}}{\partial t'} \geq 0$, b) $\pi^{i,L} - \pi^{i,F} > 0$, $\lim_{t \uparrow \tilde{t}^*} \pi^{-i,L} - \pi^{-i,F} < 0$ and $\lim_{t \downarrow \tilde{t}^*_{x,t}} \pi^{-i,L} - \pi^{-i,F} > 0$, and c) $\lim_{t \downarrow \tilde{t}^*} V^{k,C}(\boldsymbol{q}_0, t) < \pi^{k,L}$ for $k \in \{1,2\}$;

(vi) $\tilde{t}^* > t$, $\overline{q}'^* = \overline{q}'^{k-} \neq \overline{q}'^{k+}$: a) $\lim_{t \uparrow \tilde{t}^*} \frac{\partial \pi^{i,L}}{\partial t'} \geq 0$ and $\lim_{t \uparrow \tilde{t}^*} \frac{\partial \pi^{-i,L}}{\partial t'} \leq 0$, b) $\lim_{t \uparrow \tilde{t}^*} \pi^{i,L} - \pi^{i,F} > 0$, $\lim_{t \downarrow \tilde{t}^*} \pi^{i,L} - \pi^{i,F} < 0$, and $\pi^{-i,L} - \pi^{-i,F} < 0$; c) $\lim_{t \downarrow \tilde{t}^*} V^{i,C}(\boldsymbol{q}_0, t) > \pi^{i,L}$ and $\lim_{t \downarrow \tilde{t}^*} V^{-i,C}(\boldsymbol{q}_0, t) < \pi^{-i,L}$; or

(vii) $\tilde{t}^* = t_0$.

Note that these conditions simply summarise the formal conditions which are discussed more intuitively in the main text.

B.1 Numerical equilibrium types

The numerical algorithm considers intervals $[\underline{t}, \overline{t}]$ along which, for each player: a) the ordering of $\pi^{k,L}$, $\pi^{k,F}$ and $V^{k,C}$ are constant, where the continuation value refers to continuation beyond the interval in question; b) the slope $\frac{\mathrm{d}\pi^{k,L}}{\mathrm{d}t'}$ does not change sign; c) the optimal lead quantity $\arg\max_{\overline{q}'} \pi^{k,L}(\overline{\boldsymbol{q}}', t')$ is constant; d) the equilibrium strategies for the continuation state are constant.

Note that if $V^{k,C} > \pi^{k,L}$ for both k, then the equilibrium outcome for each player is to continue. Thus, at least one player must have the opposite hold for investment to take place. I characterise the equilibrium outcomes in Table 1 below.

References

Argenziano, R., Schmidt-Dengler, P., forthcoming. Clustering in N-player preemption games. Journal of the European Economic Association.

Besanko, D., Doraszelski, U., 2004. Capacity dynamics and endogenous asymmetries in firm size. RAND Journal of Economics, 23–49.

[33] I ignore, as non-generic, the case $\pi^{i,L} = \pi^{i,F}$.

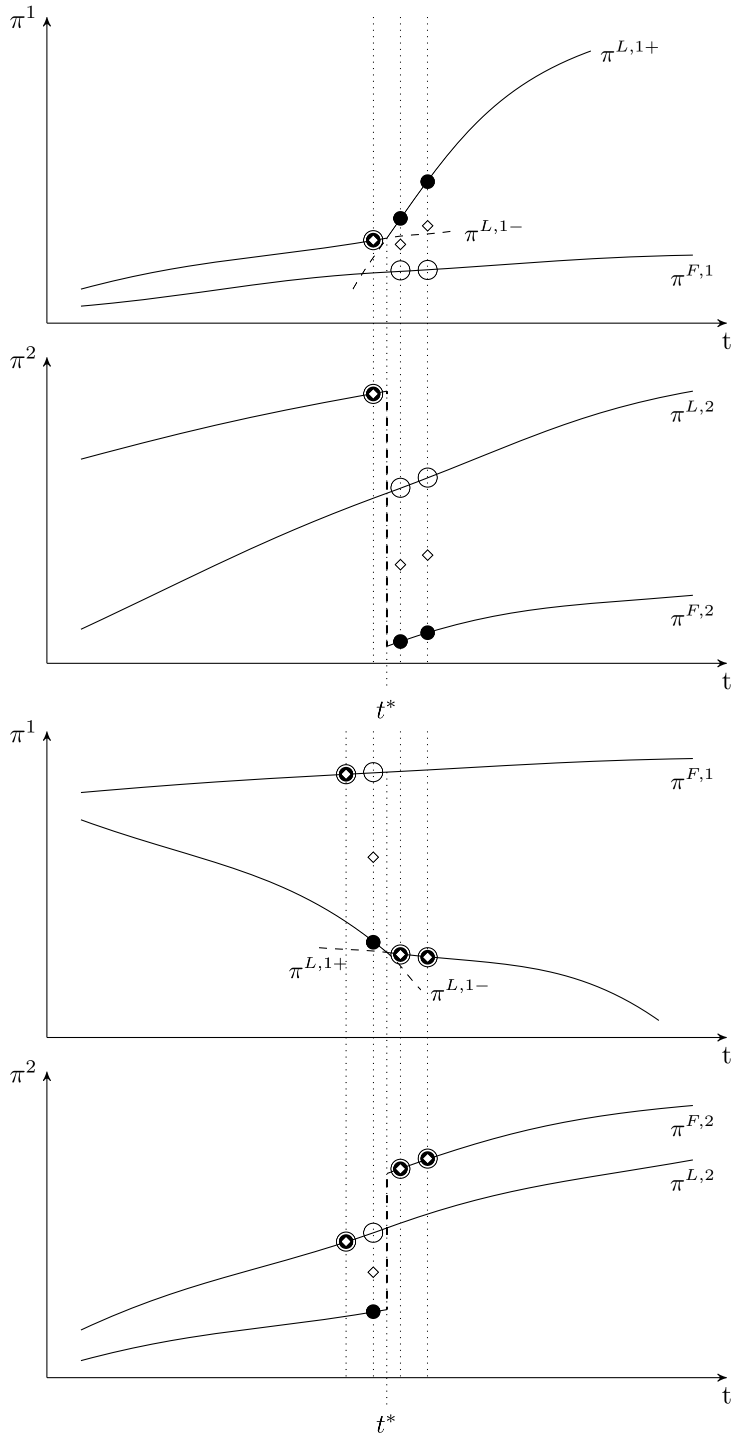

Figure 7: Example of the determination of the discrete-time equilibrium. The black dots (white circles) denote the outcome for both players when top (bottom) leads in a given period. The diamonds denote the expected continuation payoffs in the previous period.

	$\pi^L > V^C > \pi^F$	$\pi^L > \pi^F > V^C$	$\pi^F > \pi^L > V^C$
$\pi^L > V^C > \pi^F$ $\pi^L > \pi^F > V^C$ $V^C > \pi^L > \pi^F$	*Row* / *Col* (equal prob), $\underline{t}$.		*Col*, t_{MAX}.
$V^C > \pi^F > \pi^L$ $\pi^F > V^C > \pi^L$	*Row*, t_{MAX}.		*Col*, t_{MAX}.
$\pi^F > \pi^L > V^C$			*Row*/*Col* (equal prob), $\bar{t}$

Table 1: Equilibrium investor identity and investment date, given the rankings of lead profits π^L, follower profits π^F and continuation value V^C for both players. If both players have $V^C > \pi^L$ then no investment takes place, so these cases are omitted. t_{MAX} indicates the moment at which the investing player's π^L is maximised along the interval; as intervals are also broken with respect to critical points of π^L, this will be either the beginning or the end of the interval.

Boyer, M., Lasserre, P., Moreaux, M., 2012. A dynamic duopoly investment game without commitment under uncertain market expansion. International Journal of Industrial Organization.

Calcagno, R., Kamada, Y., Lovo, S., Sugaya, T., 2013. Asynchronicity and coordination in common and opposing interest games. Theoretical Economics.

Ericson, R., Pakes, A., 1995. Markov-perfect industry dynamics: A framework for empirical work. The Review of Economic Studies 62 (1), 53–82.

Fudenberg, D., Tirole, J., 1985. Preemption and rent equalization in the adoption of new technology. The Review of Economic Studies 52 (3), 383–401.

Fudenberg, D., Tirole, J., 1991. Game theory. MIT Press.

Gerlagh, R., Liski, M., 2011. Strategic resource dependence. Journal of Economic Theory 146, 699–727.

Gilbert, R. J., Harris, R. G., 1984. Competition with lumpy investment. RAND Journal of Economics 15 (2), 197–212.

Hendricks, K., Weiss, A., Wilson, C., 1988. The war of attrition in continuous time with complete information. International Economic Review 29 (4), 663–680.

Hoppe, H. C., Lehmann-Grube, U., 2005. Innovation timing games: a general framework with applications. Journal of Economic Theory 121 (1), 30–50.

Jaakkola, N., 2013. Monopolistic sequestration of European carbon emissions, unpublished chapter of doctoral thesis, University of Oxford.

Katz, M. L., Shapiro, C., 1987. R & D rivalry with licensing or imitation. The American Economic Review, 402–420.

Maskin, E., Tirole, J., 2001. Markov perfect equilibrium: I. Observable actions. Journal of Economic Theory 100 (2), 191–219.

Mills, D. E., 1990. Capacity expansion and the size of plants. RAND Journal of Economics, 555–566.

Murto, P., Välimäki, J., 2013. Delay and information aggregation in stopping games with private information. Journal of Economic Theory 148, 2404–2435.

Sargent & Lundy LLC, 2008. New coal-fired power plant: Performance and cost estimates. Report downloaded from at http://www.epa.gov/airmarkets/resource/docs/CoalPerform.pdf.

Simon, L. K., Stinchcombe, M. B., 1989. Extensive-form games in continuous time: Pure strategies. Econometrica 5, 1171–1214.