A Theory of Bargaining Deadlock*

Ilwoo Hwang†

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Abstract

I study a dynamic one-sided-offer bargaining model between a seller and a buyer under incomplete information. The seller knows the quality of his product while the buyer does not. During bargaining, the seller may receive an outside option, the value of which depends on the quality of the product. If the outside option is sufficiently important, there is an equilibrium in which the uninformed buyer fails to learn the product's quality and continues to make the same randomized offer throughout the bargaining process. As a result, the equilibrium behavior produces an outcome path that resembles the outcome of a bargaining deadlock and its resolution. The equilibrium with deadlock has inefficient outcomes, such as a delay in or breakdown of the negotiation. Bargaining delays do not vanish even with frequent offers, and they may exist when there is no static adverse selection problem. The mechanism behind the limiting delay is novel in existing bargaining literature. Under stronger parametric assumptions, the equilibrium with deadlock is the only one in which behavior is monotonic in the buyer's belief. Further, under these restrictions, all equilibria exhibit inefficient outcomes.

Keywords: bargaining game, asymmetric information, bargaining deadlock, delay, Coase conjecture.

JEL Classification number: C78, D82, D83.

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†Department of Economics, University of Miami. e-mail:ihwang@bus.miami.edu

1 Introduction

Bilateral bargaining is rarely exclusive. It is not uncommon for bargainers to receive outside offers during the negotiation process. For example, consider an entrepreneur who negotiates the sale of his company to an equity fund. The entrepreneur knows the company's fundamentals, but is unable to verify them for the equity fund. During the bargaining process, a competitor might arrive and make an offer to buy the entrepreneur's firm. Suppose that the competitor is better informed than the equity fund, so that his offer is correlated with the state of the fundamentals of the company.[1] In this example, when a bargainer is deciding whether or not to take the outside option, he must take into account the fact that choosing not to opt out may signal his private information. This paper analyzes the interplay of outside options and incomplete information in bargaining. Specifically, I analyze the equilibrium effects of additional information provided by how bargainers respond to an outside option.

I study a model of an infinite-horizon bargaining game between a seller (he) and a buyer (she). The seller privately knows his type, i.e., the quality of his product, which is labeled as either high or low. In each period, the buyer offers a price and the seller decides whether or not to accept the offer. After rejection, the seller's outside option privately arrives with positive probability. Then the seller who receives an outside option decides whether or not to opt out by taking that option. The value of the outside option is correlated with the seller's type. If the seller does not receive an outside option or he chooses not to opt out of the game by accepting the outside offer, bargaining continues into the next period.

There are two sources of information which the buyer uses to update her belief about the seller's type: the seller's decision to accept/reject the buyer's offer (acceptance behavior) and his decision about whether to take the outside option (opting-out behavior). If the buyer's offer is rejected, then she believes that the seller is more likely to be a high type since a high-type seller places a higher reservation value on his product. This informational effect of acceptance behavior is commonly incorporated into the standard models of incomplete-information bargaining (Ausubel and Deneckere, 1989; Deneckere and Liang, 2006). Such models consider only the seller's acceptance behavior, as there is no outside option built into them. As a result, the buyer's equilibrium belief about the seller's type increases over time, as does the equilibrium price. This is the well-known *skimming property* (Fudenberg, Levine and Tirole, 1985).

[1] In corporate finance, buyers of businesses are generally classified into two different categories: financial buyers and strategic buyers. Financial buyers are mostly equity funds interested in the return they can achieve by buying a business. Strategic buyers are typically a competitor or a company in the same industry, and they look for companies that will create a synergy with their existing businesses. The buyer type often affects an offer to a seller because strategic offers are often more financially lucrative.

However, in the model studied in this paper, additional information is provided by the seller's opting-out behavior—and it has an opposite effect on belief updating. The seller's continued presence in the negotiation might indicate that the seller has yet to receive an outside option or that he has received an outside option that he did not take. However, the buyer expects that a high-type seller will receive more lucrative outside offers, which makes him more likely to leave. Therefore, the buyer's belief of the seller's type may decrease if the seller stays in the negotiation.

I show that when the outside option arrives frequently, there is an equilibrium in which the two countervailing forces in belief updating exactly offset one another. As a result, the buyer's belief does not change over time and she continues to make the same randomized offer throughout the bargaining process. Since the buyer does not make more generous offers in response to continued rejections, and the seller's behavior does not change, the equilibrium behavior produces an outcome path that resembles the outcome of a *bargaining deadlock*.[2] For simplicity, I refer to such an equilibrium as a *deadlock equilibrium*.

In the deadlock equilibrium, there is a threshold belief point (called *deadlock belief*) at which the players' behavior does not change over time. If the buyer's prior belief is lower than the deadlock belief, she starts by offering a low price and makes an agreement only with a low-type seller. In this phase the buyer increases her belief in each period. Once the buyer's posterior reaches the deadlock belief, she uses a mixed strategy between offering the bargaining-ending high price (with a small probability) and a low price. In response to the buyer's low price offer, only the low-type seller accepts, with a probability equal to the arrival probability of the outside option. Then only the high-type seller exercises the outside option if it arrives. Since both types of sellers exit the game with the same probability, the posterior belief of the buyer remains the same in the next period, and the players continue to play in the same way.

Next, I analyze the limiting outcome of the deadlock equilibrium when the length of the time between the successive offers vanishes. I show that the limiting outcome exhibits delay in real time: while the negotiation reaches the deadlock phase almost immediately, the deadlock phase lasts for a nontrivial amount of time. Furthermore, the real-time delay may exist even when the static incentive constraints permit first-best efficiency, or equivalently, when there is no static adverse selection problem.[3] This contrasts with the result in existing literature on bargaining with interdependent values (Deneckere and Liang, 2006).

[2] I use the notion of "bargaining deadlock" instead of "bargaining delay" to describe a situation in which the agreement is not only delayed, but in which there also seems to be no progress in the negotiation because the bargainers' offer-and-response pattern does not change over time.

[3] A static adverse selection problem arises when the average value of the product is lower than the highest possible reservation value of the seller (Akerlof, 1970).

The mechanism behind the limiting delay in the deadlock equilibrium is novel in existing bargaining literature. In order to have a real-time delay, the buyer must sustain a low-price offer. Typically such strategy is not credible when the buyer can make successive offers frequently, because once the buyer's low-price offer is rejected, then her belief about the type of the seller increases, so that the buyer has an incentive to raise her price offer. In this paper, however, the buyer understands that the high-type seller is likelier to leave the market by taking an outside option. Therefore, the buyer's belief about the seller's type does not increase, which gives the buyer an incentive to sustain a low price. The above argument implies that the crucial mechanism for the limiting delay in this paper is the exit of the high-type seller during the bargaining process.[4]

In general, the model has multiple equilibria. There may exist an equilibrium where the informational effect of the acceptance behavior dominates that of the opting-out behavior, so that the equilibrium exhibits Coasian dynamics and thus is approximately efficient when offers are frequent. However, I show that under stronger parametric assumptions, the deadlock equilibrium is the only equilibrium that satisfies a natural monotonicity criterion that requires that the buyer's equilibrium offer be nondecreasing in the posterior belief of the seller's type. Moreover, I show that under the same condition, all equilibria exhibit similar characteristics, specifically the partial failure of learning and the inefficiency in the bargaining outcome, so that neither source of information dominates one another.

1.1 Literature Review

This paper contributes to a rich literature on dynamic bargaining with incomplete information. Standard models of incomplete-information bargaining either do not model outside options[5] or model them as an exogenous breakdown.[6] Since the players in these standard models do not have an opting-out decision, information is revealed only through the acceptance behavior. On the other hand, the arrival of an outside option in the present paper provides a second source of information from the opting-out behaviors, which in turn leads to the bargaining deadlock.

As a seminal contribution to the understanding of bargaining delay, Deneckere and Liang (2006) analyze a dynamic bargaining game in which a seller and a buyer have interdependent values. They show that the unique equilibrium exhibits a limiting delay if and only if the static

[4]See Section 5 for the other possible models which could endogenize the exit of the high-type seller.

[5]See Gul, Sonnenschein and Wilson (1986) and Ausubel and Deneckere (1989) for a durable goods monopoly; Deneckere and Liang (2006) for bargaining with interdependent values; Cho (1990) for two-sided private information; and Abreu and Gul (2000) for reputational bargaining.

[6]See Sobel and Takahashi (1983); Spier (1992) and Fuchs and Skrzypacz (2013).

adverse selection problem arises. In their model, the binding individual rationality constraint of either of the bargainers provides the buyer with the commitment device to generate the limiting delay. In contrast, the limiting delay in my paper can happen when there is no binding constraint. Further, the buyer's incentive to sustain a low price offer comes from the seller's opting-out behavior, which is a distinct characteristic of this paper.

Fuchs and Skrzypacz (2010) study a bargaining game with asymmetric information and a random arrival of events which exogenously ends the bargaining. In contrast to their model, I assume that the buyer has an acceptance decision if an outside option arrives, and that the game continues in the case of rejection. This characteristic is crucial in generating the information from the opting-out behavior and resulting bargaining deadlock. Moreover, the forces driving the limiting delay is distinct from theirs.

A few papers have equilibrium dynamics similar to the bargaining deadlock studied here, although the mechanisms underlying them are different. Evans (1989) and Hörner and Vieille (2009) consider bargaining with interdependent values and show that bargaining may result in an impasse when the buyer is too impatient (or short-lived) relative to the seller. In contrast, the present paper assumes a common discount factor, and a bargaining deadlock may exist even in the private value case. Abreu and Gul (2000) study a reputational bargaining game where each agent may be a behavioral type which demands a certain share of the pie, and show that the equilibrium has a "war of attrition" structure which exhibits a deadlock. Compared to that of Abreu and Gul (2000), the present model does not assume behavioral types, and a bargaining deadlock is associated with the uninformed buyer's failure of learning. Also, it is known that introducing an outside option into Abreu and Gul's model may eliminate the deadlock and delay (Board and Pycia, 2014), while deadlock in the present paper is a result of an interplay between the outside options and incomplete information.[7]

Board and Pycia (2014) analyze an incomplete-information bargaining model with a non-stochastic outside option for the buyer.[8] In their equilibrium, the bargainers immediately agree or the buyer immediately opts out, so the equilibrium does not exhibit a real-time delay and the Coase conjecture does not hold. In this paper, I show that the stochastic arrival of outside options leads to nontrivial and inefficient equilibrium dynamics. Furthermore, I show that the outcome of the deadlock equilibrium converges to that of Board and Pycia's equilibrium when

[7]Other models that explain bargaining delay include Merlo and Wilson (1995), in which the authors consider a complete information bargaining game where the bargaining surplus stochastically changes over time; they derive an equilibrium delay. Yildiz (2004) considers a sequential bargaining model in which players are optimistic about their bargaining power and shows that there exists a uniquely predetermined settlement date as players learn over time.

[8]In their model, the buyer has one-sided private information.

the arrival rate of outside options becomes arbitrarily high.

Lee and Liu (2013) study a repeated bargaining game between a long-run player and a sequence of short-run players, where a stochastic disagreement outcome in each bargaining period partially reveals the private information of the long-run player. The authors focus on the incentive of the long-run player to build a reputation by choosing to gamble with the outside option, while this paper analyzes the bargaining inefficiency caused by the informational effect of the outside options.[9] Finally, in a complementary paper, Hwang and Li (2014) study the effect of the transparency of outside options in bargaining and compare the deadlock equilibrium seen in this paper to the equilibrium when the arrival of outside options is public.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 constructs the deadlock equilibrium. Subsection 3.1 analyzes the equilibrium behavior under the limit case of frequent offers and shows the existence of real-time delay. Section 4 provides sufficient conditions under which the deadlock equilibrium is the only equilibrium that satisfies a natural monotonicity criterion, and under which all equilibria have similar characteristics. Section 5 discusses the role of assumptions and the robustness of the result under several extensions. Section 6 concludes. Some of the proofs are relegated to the Appendix.

2 Model

A seller (he) and a buyer (she) play an infinite-horizon, discrete-time bargaining game over the seller's product. Periods are indexed by $n = 0, 1, 2, \ldots$. Let Δ be the length of the time interval between two successive periods, so that period k occurs at time $k\Delta$.[10] Let $\delta = e^{-r\Delta}$ be a common discount factor, where $r > 0$ is a discount rate. Note that the discount factor becomes arbitrarily close to one as Δ converges to zero.

The seller's product is indivisible and can be either high type (H) or low type (L). The type of the product is the seller's private information, and the buyer forms a prior belief $\pi_0 \in (0,1)$ that $\theta = H$. The buyer's value of the type-θ product is $u_\theta > 0 (u_H \geq u_L)$. For simplicity, assume that the seller has a production cost of zero.[11]

Each period consists of an offer stage and an outside option stage. In the offer stage, the buyer offers a price p to the seller. Then the seller decides either to accept or reject the offer. If he accepts the offer, the game ends and the seller and the buyer obtain payoffs of p and $u_\theta - p$,

[9]Compte and Jehiel (2004) raise the opposite question about bargaining dynamics and identify a source of gradualism in bargaining and contribution games.

[10]This is a common modeling scheme in the literature on bargaining theory. The literature mainly considers the case where Δ is arbitrarily small, so that the commitment power of the uninformed player disappears.

[11]The robustness of the result to the case of a positive production cost is discussed in Section 5.

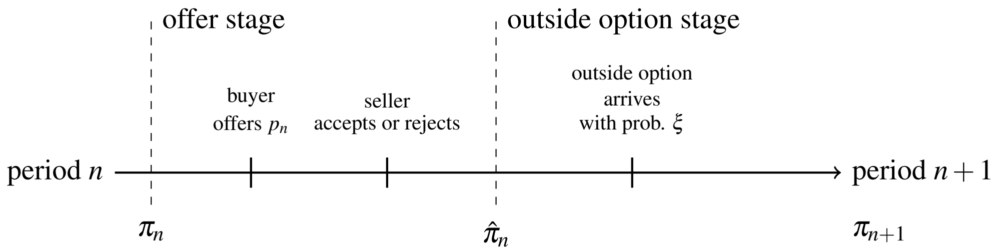


Figure 2.1: Timeline

respectively. If he rejects the offer, the game continues to the outside option stage, during which the seller receives an outside option with probability $\xi = 1 - e^{-\lambda\Delta}$.[12] The arrival of the outside option is information private to the seller. If the seller opts out by accepting the outside option, the game ends and the seller and the buyer obtain payoffs of v_θ and zero, respectively. Assume that $v_H > v_L > 0$, and that the buyer's value of the product is no less than the seller's value from the outside option $(u_\theta \geq v_\theta)$. If either no outside option arrives or the option is rejected by the seller, the game continues into the next period. Figure 2.1 describes the timeline of the game.

A public history $h^n \in \mathscr{H}$ is a sequence of rejected offers $\{p_k\}_{k=0}^n$. The seller's private history $h_S^n \in \mathscr{H}_S$ consists of $\{p_k\}_{k=0}^n$ and the availability of outside options in the past. Define $o_k = 1\{\text{an outside option is available in period } k\}$, then $h_S^n = \{p_k, o_k\}_{k=0}^{n-1}$.

The buyer's strategy is her offer $P : \mathscr{H} \to \Delta(\mathbb{R}_+)$ where $p_n = P(h^n)$. The seller's strategy is a function $\sigma : \mathscr{H}_S \times \{L, H\} \times \mathbb{R}_+ \to [0,1]^2$, which specifies the probability of accepting the current offer, $\sigma_1(h_S^n, \theta, p_n)$, and the probability of opting out, $\sigma_2(h_S^n, \theta)$, given that an outside option is available. Finally, define $\pi_n = \Pr(\theta = H | h^n)$ as a posterior belief of the buyer in period n.

The equilibrium concept is a perfect Bayesian equilibrium (PBE) as defined in Fudenberg and Tirole (1991, Definition 8.2).[13] PBE implies that upon receiving an out-of-equilibrium offer, the continuation strategy of the seller is optimal.

Consider a seller's strategy in which he rejects any offer from the buyer and opts out whenever the outside option arrives. Then the type-θ seller's expected payoff is

$$v_\theta^* \equiv \xi v_\theta + \delta(1-\xi)\xi v_\theta + \cdots = \frac{\xi}{1-\delta(1-\xi)} v_\theta.$$

Note that $v_\theta^* < v_\theta$ since delay is costly. As we define $\eta = \frac{\xi}{1-\delta(1-\xi)} \in (0,1)$ to be the *effective*

[12] Note that $\lambda > 0$ represents a Poisson arrival rate of the outside options.

[13] Formally speaking, Fudenberg and Tirole (1991) defined perfect Bayesian equilibria for finite games of incomplete information. The suitable generalization of their definition to infinite games is straightforward and is omitted.

arrival rate of outside options, then $v^*_\theta = \eta v_\theta$. In any equilibrium, the payoff of the type-θ seller is bounded below by v^*_θ, so the seller always rejects any offer less than v^*_θ. Hereafter I call v^*_θ the *reservation price* of the type-θ seller.

The paper focuses on the environment which satisfies the following assumption:

Assumption. (A1)

$$\delta\eta^2 v_H > \eta v_L + (1-\eta)u_L.$$

Assumption 1 requires that the effective arrival rate of outside options (η) is sufficiently high. In this model, the seller's reservation price comes from the value of the future outside options. Thus, frequently arriving outside options generate a sufficiently heterogeneous bargaining position of the seller according to his type. Note that (A1) does not require interdependency of the good's value for the buyer (u_θ) and its value for the seller (v_θ). Specifically, (A1) encompasses a case in which the buyer has a private value $(u_H = u_L)$.

2.1 Preliminary Observations

In the case of complete information, the bargaining ends immediately at a price v^*_θ. The main intuition behind the following proposition is similar to Diamond's paradox.

Proposition 1. *(Complete information) Suppose that the seller is type θ with probability one. Then there exists a unique subgame perfect equilibrium in which the buyer always offers v^*_θ, and the seller accepts any offer greater than or equal to v^*_θ.*

The following lemma shows that in any (perfect Bayesian) equilibrium, the buyer's equilibrium offer is bounded above by the high type's reservation value.

Lemma 1. *In any equilibrium, after any history h^n, the buyer never offers $p_n > v^*_H$.*

Corollary 1. *In any equilibrium:*

- *The high-type seller accepts any $p \geq v^*_H$, rejects any $p < v^*_H$, and takes the outside option whenever the option arrives.*
- *The low-type seller accepts any $p \geq \delta v^*_H$.*

Note that Corollary 1 completely describes the high-type seller's equilibrium behavior after any history. Therefore, it only needs to specify the behaviors of the low-type seller and the buyer to describe the equilibrium profile.

Lemma 1 and Corollary 1 describe how the bargaining ends in any equilibrium. After any history, the buyer offers either $p_n = v_H^*$ or $p_n < v_H^*$. If she offers v_H^*, then both types of sellers accept it for sure, and the bargaining ends in period n with probability one. On the other hand, if $p_n < v_H^*$ then the high type rejects it for sure and takes the outside option if the option arrives. Therefore, the bargaining continues into the next period with a positive probability, as the outside option does not arrive with probability one.

3 Deadlock Equilibrium

In this section I construct an equilibrium of interest. A heuristic argument for the equilibrium construction is provided here, while the complete equilibrium construction is provided in the Appendix. The following definition describes several important behaviors of the equilibrium of interest.

Definition. A perfect Bayesian equilibrium is called a *deadlock equilibrium* if the agents' behavior on the equilibrium path satisfies the following properties:[14] there exist $\pi^* \in (0,1)$, $\hat{p} < v_H^*$, and $q \in (0,1)$ such that

1. If $\pi_n > \pi^*$, the buyer offers v_H^* for sure; bargaining ends immediately.
2. If $\pi_n = \pi^*$,
   - the buyer offers either v_H^*, or $\hat{p}$, or randomizes between the two;
   - if $p_{n-1} = \hat{p}$, he offers v_H^* or $\hat{p}$ with probability q and $1-q$, respectively;
   - the high type rejects $\hat{p}$ and the low type accepts $\hat{p}$ with probability ξ;
   - the high type opts out for sure and the low type does not opt out;
   - $\pi_{n+1} = \pi^*$.
3. If $\pi_n < \pi^*$,
   - the buyer offers some $p \leq \hat{p}$;
   - the low type accepts p with positive probability; the high type rejects p;
   - $\pi_{n+1} \in (\pi_n, \pi^*]$.

[14] Behavior on the equilibrium path, as well as the behavior off the equilibrium path, is left unspecified since it is not relevant for the later analysis. See the Appendix for the complete description.

In the deadlock equilibrium, there exists a cutoff belief π^* where the posterior belief, given that the bargaining continues, does not change once it reaches π^*. I call π^* a *deadlock belief* since the players' behaviors do not change once the posterior reaches π^*.

How are the values of π^*, $\hat{p}$, and q determined? First, I use the following arguments to claim that a profile is a deadlock equilibrium only if $\hat{p} = v_L$.

- $\hat{p} \geq v_L$: Otherwise, the low-type seller will opt out since the outside option's value is greater than the buyer's offer.

- $\hat{p} \leq v_L$: Since the low-type seller accepts any offer greater than δv_H^* with probability one, $\hat{p} \leq \delta v_H^*$. Now suppose that $\hat{p} \in (v_L, \delta v_H^*)$. Then the buyer has an incentive to decrease her offer slightly as the seller accepts the lower offer with same probability.[15]

Next, the deadlock belief π^* is uniquely determined by the the buyer's indifference condition: At $\pi_n = \pi^*$, the buyer must be indifferent between offering v_H^* and $\hat{p} = v_L$. The buyer's expected payoff from offering the bargaining-ending price v_H^* is

$$U^* \equiv (1-\pi^*)(u_L - v_H^*) + \pi^*(u_H - v_H^*). \tag{1}$$

On the other hand, if the buyer offers v_L, the buyer obtains

$$(1-\pi^*)\xi(u_L - v_L) + \delta(1-\xi)U^*. \tag{2}$$

Combining the above two formulas gives

$$\pi^* = \frac{(v_H^* - u_L) + \eta(u_L - v_L)}{(u_H - u_L) + \eta(u_L - v_L)}. \tag{3}$$

Last, the value of q is uniquely determined from the low-type seller's indifference condition. Since the low type is indifferent between acceptance and rejection when the buyer offers v_L at $\pi_n = \pi^*$,

$$v_L = \delta(qv_H^* + (1-q)v_L), \tag{4}$$

[15] To see this in action, fix a history h^n with $\pi_n = \pi^*$. Let $\varepsilon > 0$ be small enough that $\hat{p} - \varepsilon > \max\{v_L, \delta\hat{p}\}$. Consider the buyer's deviation at h^n to offer $\hat{p} - \varepsilon$. Then by Corollary 1, the high-type seller exits the game with a probability ξ. I claim that in response to $\hat{p} - \varepsilon$, the low-type seller also exits the game with probability ξ. If he exits with probability greater than ξ, the buyer's posterior becomes $\pi_{n+1} > \pi^*$. Hence the buyer offers v_H^* in period $n+1$. If that is the case, then it is strictly optimal for the low type *not* to exit in period n, so his behavior is inconsistent with the belief. If he exits with probability less than ξ, then $\pi_{n+1} < \pi^*$, so the buyer offers $p_{n+1} \leq \hat{p}$ in period $n+1$. However, then it is strictly optimal for the low type to accept p_n at period n. Therefore, the low type must accept $\hat{p} - \varepsilon$ with probability ξ and not take the outside option, because $\hat{p} - \varepsilon > v_L$. Hence offering $\hat{p} - \varepsilon$ is a profitable deviation for the buyer.

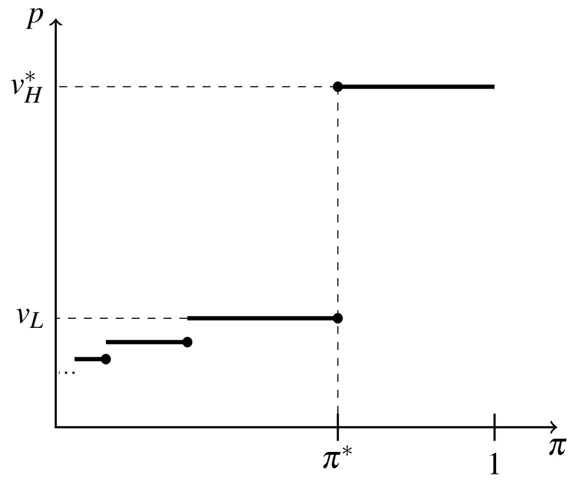


Figure 3.1: Buyer's equilibrium offer

which uniquely determines q.

I show in the Appendix that if the environment satisfies (A1), the above profile is an equilibrium. (A1) guarantees a high enough arrival rate of the outside option that the buyer does not have an incentive to "break the deadlock" by offering a price higher than x. Given the profile, the buyer needs to raise the price to at least δv_H^* in order to increase the seller's acceptance probability. This is because the low-type seller has no incentive to accept some $p < \delta v_H^*$ with probability greater than ξ, because then the belief becomes greater than π^* and the buyer offers v_H^* in the next period. If the buyer offers δv_H^*, by Corollary 1 the low-type seller accepts the offer with probability one. If the seller is the high type, however, he rejects the offer and opts out if the option is available, and the buyer therefore receives zero payoff. So if the outside option arrives with a high probability, the buyer's cost of losing the high-type seller is greater than the benefit from breaking the deadlock and expediting trading with the low type.

The following proposition summarizes the argument:

Proposition 2. *Suppose* (A1) *holds. Then the model generically has a unique deadlock equilibrium.*

Figure 3.1 describes the buyer's equilibrium offer as a function of the posterior belief. If the buyer's belief is greater than the deadlock belief π^*, she offers v_H^* and the bargaining ends immediately. If the buyer's belief is less than π^*, she targets only the low-type seller and offers a price that is no more than v_L. Her offer is nondecreasing in the belief.

Equilibrium behavior at the deadlock belief $\pi_n = \pi^*$ is depicted in Figure 3.2. At the offer stage (left panel), the buyer offers either v_H^* or v_L. Had the buyer offered v_L in the previous period, she would then randomize between offering v_H^* and v_L in which the randomization

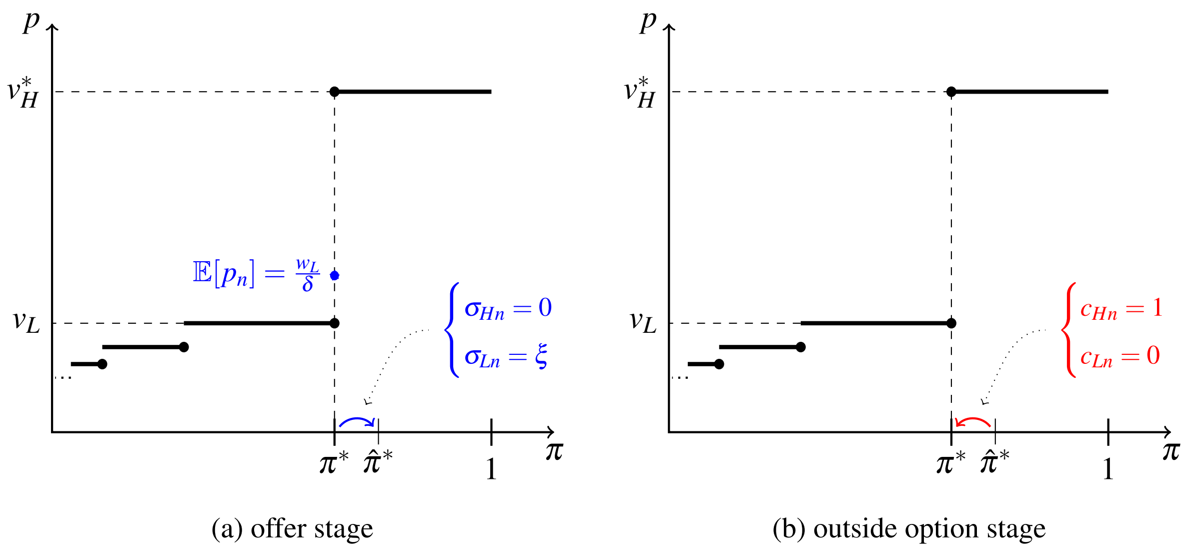


Figure 3.2: Equilibrium behavior at the deadlock belief $\pi = \pi^*$

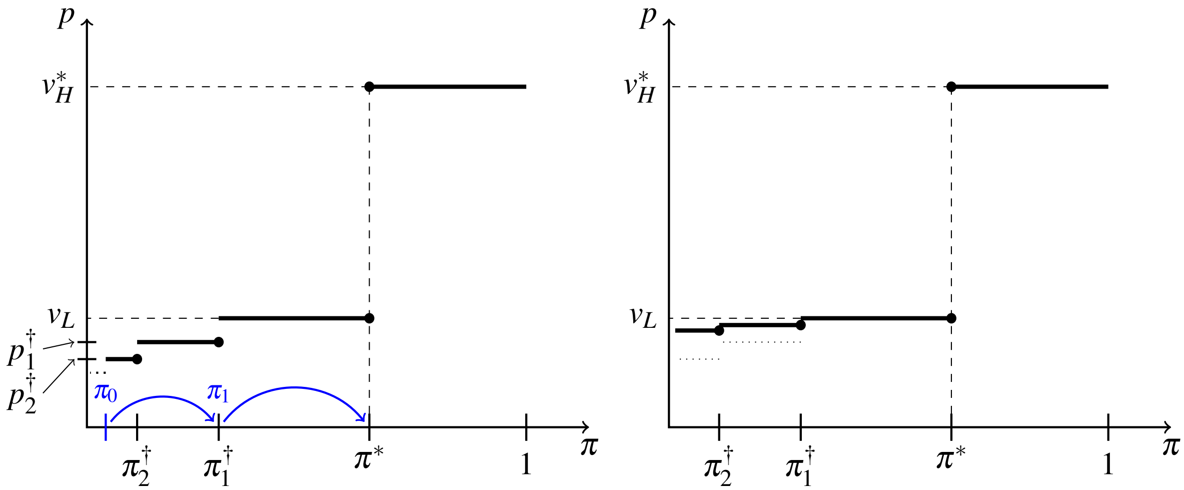
probability is determined by (4). If the buyer offers v_H^*, then both types of sellers accept her offer and the bargaining ends. If the buyer offers v_L, then the high type rejects it and the low type accepts the offer with probability ξ. So after the seller rejects the offer, the buyer's belief increases to $\hat{\pi}^* \equiv \frac{\pi_n}{\pi_n+(1-\pi_n)(1-\xi)} > \pi^*$. At the outside option stage (right panel), an outside option arrives with probability ξ, which only the high type exercises Therefore, the posterior belief π_{n+1} falls back to π^*. From then on, the bargaining parties repeat the same behavior in each period: The buyer randomizes between offering v_H^* and v_L; the low type accepts v_L with probability ξ while the high type rejects it; and only the high type opts out. Note that the buyer's belief does not change unless bargaining ends, since the information from the seller's acceptance behavior and his opting-out behavior exactly offset one another.

If $\pi_n < \pi^*$, then the equilibrium behavior exhibits "Coasian dynamics" (Fudenberg and Tirole, 1991). The buyer gradually increases the offer price over time, and her belief increases over time until it reaches the deadlock belief.[16] In the Appendix, I construct decreasing sequences of prices $\{p_k^\dagger\}(p_0^\dagger = v_L)$ and cutoff beliefs $\{\pi_k^\dagger\}(\pi_0^\dagger = \pi^*)$ such that the buyer's equilibrium price offer when $\pi < \pi^*$ is

$$P(h^n) = p_{k-1}^\dagger \qquad \text{if } \pi_n \in [\pi_k^\dagger, \pi_{k-1}^\dagger).$$

For any prior $\pi_0 < \pi^*$, there exists $N \in \mathbb{N} \cup \{0\}$ such that $\pi_{N+1}^\dagger \leq \pi_0 < \pi_N^\dagger$. Take, for example, the generic case that $\pi_{N+1}^\dagger < \pi_0$ (the left panel of Figure 3.3 describes the belief dynamics when $N = 2$). In the equilibrium, the buyer offers $p_N^\dagger$ in the first period. Then the low type uses a

[16] Fudenberg and Tirole (1991) define Coasian dynamics in sequential bargaining where the seller offers a price and a buyer has a private information about her valuation.



(a) when $\pi_0 < \pi^*$

(b) offer function in the limit case

Figure 3.3: Equilibrium behavior

mixed acceptance strategy so that the buyer's belief after the rejection becomes $\pi_{N-1}^\dagger$. In the outside option stage, both types of sellers opt out if the outside option arrives, so the belief does not change at $\pi_{N-1}^\dagger$. In the second period, the buyer increases his offer to $p_{N-1}^\dagger$; the low type randomizes and both types opt out; and the posterior becomes $\pi_{N-2}^\dagger$. This behavior continues until the posterior reaches $\pi_0^\dagger = \pi^*$. Therefore, it takes $\max\{N, 1\}$ periods for the posterior to reach the deadlock belief. Note that the buyer receives new information only from seller's acceptance behavior.

So if $\pi_0 < \pi^*$, the equilibrium behavior produces an outcome path with the following characteristics:

- Bargaining starts with a *pre-deadlock phase*. The buyer offers a price less than v_L, and her offer increases over time. Bargaining can end in the pre-deadlock phase by the low-type seller accepting the buyer's offer, or both types of sellers exercising the outside option.

- Once the buyer offers v_L, a *deadlock phase* begins. In this phase the outcome path features bargaining deadlock: The buyer offers the same offer of v_L in every period and is repeatedly rejected by the seller. Bargaining ends after a finite number of periods, by either 1) the buyer offering the bargaining-ending price v_H^*, 2) the low-type seller accepting v_L, or 3) the high-type seller exercising the outside option.

3.1 Frequent Offers

Consider the case in which the time between periods Δ is arbitrarily small, hence the buyer's commitment power vanishes. Note that as Δ goes to zero, the effective arrival rate of outside options converges to

$$\eta = \frac{\xi}{1-\delta(1-\xi)} = \frac{1-e^{-\lambda\Delta}}{1-e^{-(r+\lambda)\Delta}} \to \eta^* \equiv \frac{\lambda}{r+\lambda}.$$

The next proposition summarizes the behavior of the deadlock equilibrium under frequent offers.

Proposition 3. *Suppose $(\eta^*)^2 v_H > \eta^* v_L + (1-\eta^*)u_L$. Then,*

1. *The deadlock equilibrium exists for sufficiently small Δ.*

2. *In the deadlock equilibrium, as Δ converges to zero,*

   - *the buyer's offer for any $\pi < \pi^*$ converges to v_L;*
   - *for any prior $\pi_0 < \pi^*$, the length of the pre-deadlock phase (measured in real time) shrinks to zero;*
   - *the expected delay of the deadlock phase does not converge to zero.*

Proof.

1. Since $\delta \to 1$ and $\eta \to \eta^*$ as Δ goes to zero, (A1) is satisfied under sufficiently small Δ.

2. See the Appendix.

□

In the pre-deadlock phase, the equilibrium exhibits the Coasian dynamics at a price v_L. Since the discount factor goes to one as Δ converges to zero, the difference between the buyer's successive offers vanishes as the buyer makes the low-type seller indifferent between acceptance and rejection. Moreover, the same force behind the Coase conjecture results in the pre-deadlock phase shrinking to zero. The right panel of Figure 3.3 describes the limit equilibrium offer by the buyer when Δ converges to zero.

However, the deadlock phase does not shrink as $\Delta \to 0$ and the expected length of delay remains positive. Specifically, the resolution time of the deadlock phase converges to a Poisson arrival process with a finite arrival rate. The indifference condition (4) implies that as $\Delta \to 0$,

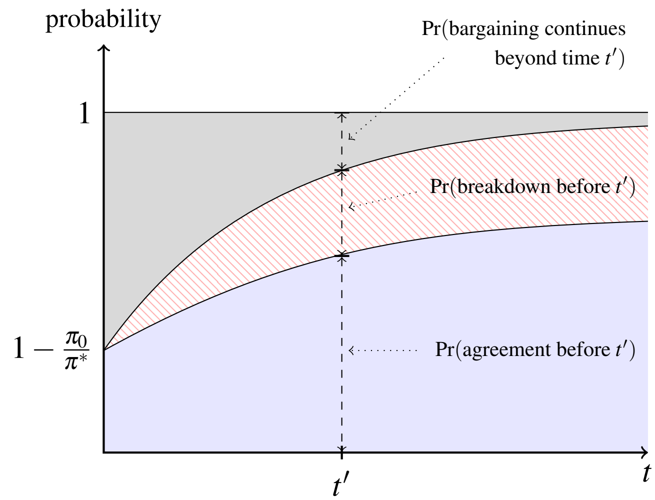


Figure 3.4: Limit distribution of the equilibrium outcome

the buyer's equilibrium offer path (in real time) converges to the base offer of v_L with the endogenous Poisson arrival of v_H^*.[17] Moreover, the low type's acceptance of offer v_L and the high type's opt-out occurs with probability $\xi = 1 - e^{-\lambda\Delta}$; thus they converge to Poisson processes with parameter λ. Note that the Poisson arrivals of the resolution behaviors are independent of each other.

The intuition for the real-time delay is as follows. The seller's equilibrium behavior makes the buyer's discount factor type-dependent: If the seller is high-type, the buyer is more impatient. This works as a commitment device for the buyer such that sustaining a low price is a credible strategy. Note that this mechanism is different from one in the bargaining with interdependent values, where adverse selection prevents the buyer from offering a bargaining-ending high price. Indeed, the real-time delay holds for the case where the static adverse selection condition does not hold ($u_L > v_H$) and even in the private value case ($u_L = u_H$), as long as (A1) is satisfied.

Figure 3.4 shows the limit distribution of the equilibrium outcome when $\Delta \to 0$ as a function of (real) time. At any time t', the height of the bottom (middle) area of the figure indicates the probability that the agreement (breakdown) happens at any time before t'. The height of the top

[17]To see this, note that

$$
\begin{aligned}
q &= \frac{v_L/\delta - v_L}{v_H^* - v_L} \\
&= \frac{v_L}{v_H^* - v_L}(e^{r\Delta} - 1) = \frac{v_L r}{v_H^* - v_L}\Delta + o(\Delta),
\end{aligned}
$$

so as $\Delta \to 0$, the arrival of the buyer's offer $p = v_H^*$ converges to a Poisson process of rate $\frac{v_L r}{v_H^* - v_L}$.

area is the probability that the bargaining continues beyond time t'. Note that for any finite t, bargaining will continue beyond time t with positive probability.

Expected Length of Delay and the Probability of Breakdown Since the limit distribution of the resolution behavior is a Poisson distribution, several key values of the equilibrium can be derived in a closed-form. The following proposition states the limit of the expected length of delay in real time.

Proposition 4. *Define T_d to be the (unconditional) expected length of delay in the deadlock equilibrium, and let $\hat{T}_d$ be the expected length of delay conditional on reaching the deadlock stage. Suppose that prior belief π_0 is less than the deadlock belief π^*. Then as Δ converges to zero,*

$$\hat{T}_d \to \frac{Z}{Z+\mu} \cdot \frac{1}{\lambda}, \text{ and } \qquad T_d \to \frac{\pi_0}{\pi^*}\hat{T}_d,$$

where $Z = \frac{v_H^* - v_L}{v_L}$ and $\mu = \frac{r}{\lambda}$.

Another source of inefficiency in the deadlock equilibrium is the possibility of a breakdown resulting from the high type's opt-out.

Proposition 5. *Let P_b be the ex ante probability of a breakdown in the deadlock equilibrium, and $\hat{P}_b$ be the breakdown probability conditional on reaching the deadlock stage. Then as Δ converges to zero,*

$$\hat{P}_b \to \pi^* \frac{Z}{Z+\mu}, \text{ and } \qquad P_b \to \pi_0 \frac{Z}{Z+\mu},$$

where $Z = \frac{v_H^* - v_L}{v_L}$ and $\mu = \frac{r}{\lambda}$.

3.2 Equilibrium Behavior when $\xi \to 1$

When the arrival rate of outside options becomes arbitrarily high, the players' behavior in the deadlock equilibrium converges to that of a static bargaining game.

Proposition 6. *Fix δ such that $\delta v_H > v_L$, then the deadlock equilibrium exists for sufficiently high ξ. As $\xi \to 1$, the limit of the outcome distribution of the deadlock equilibrium is the following:*

- *when $\pi_0 > \frac{v_H - v_L}{u_H - v_L}$, the bargaining ends immediately by the agreement at price v_H.*
- *when $\pi_0 < \frac{v_H - v_L}{u_H - v_L}$, if the seller is low-type, the bargaining ends immediately by an agreement at price v_L. If the seller is high type, the bargaining ends immediately with the seller exercising the outside option.*

When ξ is close to one, the reservation value of a type-θ seller (v_θ^*) converges to the value of his outside option (v_θ). Knowing that there is no second chance, the buyer maximizes her static payoff. This result also supports the intuition behind the "monopoly pricing equilibrium" of Board and Pycia (2014).

4 Uniqueness

In general, there are multiple equilibria for this model. In particular, there may exist an equilibrium where the buyer uses an offer strategy similar to the "Coasian" pricing (Fudenberg, Levine, and Tirole, 1985).[18] In this equilibrium, as the time between the periods becomes vanishingly small, the buyer's offer converges to v_H^* and the expected delay converges to zero, so the equilibrium outcome is approximately efficient. In the equilibrium with Coasian dynamics, although there are two sources of information, the information revealed by the seller's acceptance behavior dominates the information revealed by his opting-out behavior.

Then the question becomes whether the deadlock equilibrium is one equilibrium of the model where two sources of information happen to offset one another. In this section, I show that under a stronger parametric assumption, the offsetting effect can be found in all PBE of the model. First, I present the parametric assumption stronger than (A1).

Assumption. (A2)

$$\delta\eta^2 v_H > u_L.$$

A necessary condition for (A2) is $v_H^* > u_L$. Since $u_H \geq v_H$, the private value case $(u_H = u_L)$ does not satisfy (A2). More importantly, $v_H^* > u_L$ is a necessary condition for the existence of the static adverse selection problem. Suppose there is a static market where the buyer's value is u_θ and the seller's reservation value is v_θ^*. Then adverse selection in the trade exists if and only if $\mathrm{E}[u_\theta] < v_\theta^*$. Therefore, if $v_H^* > u_L$, the adverse selection problem arises for sufficiently low π_0.

Since (A2) implies (A1), (A2) guarantees the existence of the deadlock equilibrium. The following proposition states that under (A2), in every equilibrium neither source of information

[18] In the equilibrium with Coasian pricing, the buyer plays a pure pricing strategy at any history on the equilibrium path. The buyer gradually increases his offer over time. On the equilibrium path, the high type rejects the buyer's offer in all but the final period, and the low type uses a mixed acceptance strategy. If the initial offer is high enough, only the high type opts out in every period before the game ends. If the initial offer is low, then both types take the outside option until the offer exceeds some cutoff where it becomes suboptimal for the low type to opt out.

dominates the other, so the equilibrium has characteristics similar to those of the deadlock equilibrium.

Proposition 7. *Suppose* (A2) *holds. Then in every perfect Bayesian equilibrium of the game, if the prior belief is low enough,*

(1) the posterior belief π_n never exceeds the deadlock belief π^* (defined in Section 3) conditional on the bargaining continues, and

(2) for any finite n, bargaining continues beyond period n with positive probability.

Proof. See the Appendix. □

The next proposition shows that under (A2), the deadlock equilibrium is the only PBE satisfying a monotonicity property. The property, called *nondecreasing offers*, requires that when the buyer's expected quality is higher, she tends to offer a higher price to the seller.

Definition. Let $P(h^n)$ be a set of offer prices assigned positive probability at history h^n. A strategy profile satisfies *nondecreasing offers* if for any history $h^n, h'^{n'}$ with $\pi_n < \pi_{n'}$, if $p \in P(h^n)$ and $p' \in P(h'^{n'})$, then $p \leq p'$.

Proposition 8. *Suppose* (A2) *holds. Then the deadlock equilibrium is the unique perfect Bayesian equilibrium that satisfies nondecreasing offers.*

Proof. See the Appendix. □

5 Discussions

Arriving Outside Options One of the important assumptions of this paper is the random arrival of outside options. It describes a negotiation with stochastic payoffs from opting out: In many environments, the value of the best available outside option tends to stochastically change over time. Theoretically, the random arrival of outside options generates new bargaining dynamics, as the seller's behavior in deciding not to exercise the outside option may provide additional information for the buyer. This is in contrast to the literature, which typically assumes that the outside option is available in every period, and thus that bargaining ends immediately since the seller accepts the buyer's offer or exercises outside option in the first period.

Type-dependent Production Cost and Arrival Rate of Outside Options The results of the paper are robust to the case where the seller has a type-dependent production cost. Suppose the type-θ seller has a production cost of $c_\theta > 0$. Recall that the seller's payoff is v_θ when he takes an outside option. Then the type-θ seller never accepts an offer if

$$p - c_\theta < \frac{\xi}{1-\delta(1-\xi)} v_\theta = v_\theta^*,$$

or $p < c_\theta + v_\theta^*$.

As long as the high-type seller has a stronger incentive to opt out than the low type, the deadlock equilibrium exists. For this, I assume a modified version of (A1):

Assumption. (A1a) $\delta(c_H + \eta v_H) > \delta c_L + v_L + \frac{1-\eta}{\eta}(u_L - c_L)$.

Note that if $c_H = c_L = 0$, (A1) and (A1a) are equivalent.

Proposition 9. *Suppose* (A1a) *holds. Then there exists a deadlock equilibrium of the model with a positive production cost.*

Similarly, the deadlock equilibrium is robust under the type-dependent arrival rate of the outside option. For example, the deadlock equilibrium may still exist in the model in which only the high-type seller receives outside options.

Permanent Outside Option The results of the paper are robust to the case in which the seller can exercise his outside option at any time after it arrives. The key intuition is that both types of sellers have no incentive to keep the outside option and exercise it at later time. The high-type seller always exercises the outside option immediately, since the buyer's maximum offer is strictly smaller than the value of the outside option $(v_H^* < v_H)$. On the other hand, the low-type seller has no incentive to signal that he has an outside option, because doing so would disclose his type and lose his information rent. Therefore, there is no reason to keep the outside option as the buyer's offer is increasing over time.

Alternative Timelines One can consider two different timelines of the game. In both timelines, the seller receives an outside option *before* the buyer offers a price. First, if the seller decides whether to opt out before the buyer makes an offer, then it is easy to show that the equilibrium structure is unchanged. This is not surprising since the effect of switching the offer stage and the outside option stage only accounts for the discount factor. Second, if the seller's opting-out decision comes after the buyer makes an offer, there may exist multiple equilibria even if there is complete information about the seller's type. Under some parameter ranges, the buyer's offer and the seller's opting-out decision have a self-fulfilling effect on each other. If the arrival rate of the outside option is low, there exists an equilibrium where the buyer makes an offer that is accepted only by the seller without the outside option, and the seller with the outside option rejects the offer and opts out. If the arrival rate of the outside option is high, there exists another type of equilibrium where the buyer makes an offer high enough so that the seller accepts it. And for the intermediate arrival rate, both equilibria may exist.

Continuum of Types If the model assumes a continuum of types of sellers, the main difficulty in the analysis is tracking the belief. Since the outside option does not arrive with probability one, the belief after an outside option stage has the same support as the one before the stage; however, the belief about the high type would decrease. Therefore, the posterior belief is not a truncation of the prior and therefore cannot be simplified to a single state variable. So the equilibrium profile must describe the bargainer's behavior for any possible posterior belief.[19] I hypothesize that as in the two-types case, there are two countervailing forces in belief updating: The lower types tend to accept the buyer's offer and the higher types tend to opt out. However, it is unclear whether these countervailing forces would lead to a bargaining deadlock or to another equilibrium dynamic.

6 Concluding Remarks

One interesting extension of the work presented in this paper is to assume a random value of the outside options instead of random availability. Consider a model where the type-θ seller receives an outside option in each period, and the value of the outside option is randomly drawn from distributions F_θ. Assume that F_H is first-order stochastic dominant over F_L. I hypothesize that under some conditions on the distributions, there exists a deadlock equilibrium. In this case, neither type of seller plays a mixed strategy with regard to the outside option. Moreover, the low-type seller would also opt out if he received a good enough outside option. Similarly to the benchmark model, however, not taking the outside option conveys a bad signal about the seller's type. It would be interesting to investigate whether and how a bargaining deadlock occurs in this extension.

[19] Fuchs, Öry, and Skrzypacz (2012) analyzed an equilibrium where the posterior belief is an addition of multiple truncated beliefs. In their paper, such beliefs are formed when the future buyer cannot observe past offers, so the price history does not affect future buyers' beliefs and hence it does not affect their strategies. In this paper, there is a single buyer and he observes history of past offers. So an out-of-equilibrium offer affects the future belief of the buyer, which makes the analysis difficult.

Appendix

Proof of Proposition 1

It is sufficient to show that in any equilibrium, after any history the buyer never offers a price p greater than v_θ^*. First, observe that the buyer never makes an offer above u_θ, since his equilibrium payoff must be nonnegative. Given that, the seller's expected payoff after the rejection is no more than

$$z_1 \equiv \max\{\delta u_\theta, \xi v_\theta + \delta(1-\xi)u_\theta\},$$

where the first (second) term in the bracket denotes the seller's maximum expected payoff when it is optimal for him to reject (accept) an outside option. Note that $z_1 < u_\theta$. So the seller accepts any offer $p > z_1$ after any history; thus such offer is suboptimal for the buyer, since she can always make a lower offer $(p - \varepsilon > z_1)$ and buy the product.

Proceeding with the same argument, given that the buyer's offer is bounded by z_m, the seller always accepts any offer above

$$z_{m+1} \equiv \max\{\delta z_m, \xi v_\theta + \delta(1-\xi)z_m\} < z_m.$$

Thus, any offer greater than z_{m+1} is suboptimal for the buyer. Since the sequence $\{z_m\}$ is decreasing and converges to v_θ^*, for any $\varepsilon > 0$ the buyer's offer $v_\theta^* + \varepsilon$ is accepted by the seller, and therefore is suboptimal.

Proof of Lemma 1

Let $z_0 = u_H$. It is clear that the buyer never offers a price p greater than z_0. Now suppose that the buyer never offers more than $z_m > v_H^*$ in the equilibrium. Then for the type-θ buyer, her maximum payoff after the rejection is no more than $\max\{\delta z_m, \xi v_\theta + \delta(1-\xi)z_m\}$. Therefore, both types of sellers accept the buyer's offer higher than

$$z_{m+1} \equiv \max\{\delta z_m, \xi v_H + \delta(1-\xi)z_m\}.$$

Since the sequence $\{z_m\}$ is decreasing and converges to v_H^*, making an offer $v_H^* + \varepsilon$ for any $\varepsilon > 0$ is suboptimal for the buyer.

Proof of Proposition 2: Construction of the Deadlock Equilibrium

Sequences of the Cutoff Prices and Beliefs

I construct sequences of prices $\{p_k^\dagger\}_{k=-1}^\infty$ and cutoff beliefs $\{\pi_k^\dagger\}_{k=-1}^\infty$ that describe behaviors of the deadlock equilibrium. I use the following necessary conditions to construct the sequences:

- $p_{-1}^\dagger = v_H^*, p_0^\dagger = v_L; \pi_{-1}^\dagger = 1, \pi_0^\dagger = \pi^*$;
- When the buyer's belief is $\pi_k^\dagger (k \geq 0)$, she is indifferent between offering $p_k^\dagger$ and $p_{k-1}^\dagger$;
- If the buyer offers $p_k^\dagger (k \geq 0)$, then the seller uses a (possibly mixed) strategy that increases the buyer's belief to $\pi_{\max\{k-1,0\}}^\dagger$; and
- The low-type seller exercises her outside option if and only if $p < p_0^\dagger = v_L$.

Recall that the deadlock phase begins when the buyer offers v_L, thus $p_k^\dagger$ is the equilibrium price when there are k periods until the bargaining reaches the deadlock phase. The low-type seller is indifferent between accepting $p_k^\dagger$ and rejecting it. In the case of rejection, he opts out immediately if he receives an outside option since $p_k^\dagger < p_0^\dagger$ for all $k \geq 1$. Then the low type's indifference condition is given by

$$p_k^\dagger = \xi v_L + \delta(1-\xi) p_{k-1}^\dagger, \tag{5}$$

which gives a recursive equation for $\{p_k^\dagger\}_{k=0}^\infty$.

The construction of $\pi_0^\dagger = \pi^*$ is discussed in Section 3. For the construction of $\pi_k^\dagger (k \geq 1)$, I define notations that make the analysis simpler. Let $\beta(\pi, \pi')$ be the low-type seller's acceptance probability that changes the posterior belief from π to π', given that both types of sellers opt out. That is,

$$\frac{\pi'}{1-\pi'} = \frac{\pi}{1-\pi} \cdot \frac{1}{1-\beta(\pi,\pi')} \Leftrightarrow \beta(\pi,\pi') = 1 - \frac{\pi}{1-\pi} \cdot \frac{1-\pi'}{\pi'}.$$

On the other hand, let $\tilde{\beta}(\pi, \pi')$ be the low type's acceptance probability that changes the posterior from π to π', given that only the high type opts out. Then $\tilde{\beta}(\pi, \pi')$ satisfies

$$\frac{\pi'}{1-\pi'} = \frac{\pi}{1-\pi} \cdot \frac{1-\xi}{1-\tilde{\beta}(\pi,\pi')} \Leftrightarrow \tilde{\beta}(\pi,\pi') = 1 - \frac{\pi}{1-\pi} \cdot \frac{1-\pi'}{\pi'}(1-\xi).$$

When the buyer's belief is $\pi_1^\dagger$, she is indifferent between offering $p_0^\dagger = v_L$ and $p_1^\dagger$. By Lemma 1 the high-type seller plays $(\sigma_1, \sigma_2) = (0, 1)$. The low-type uses a randomized strategy that increases the belief to $\pi_0^\dagger = \pi^*$: If the buyer offers $p_1^\dagger$, the low type seller's response is

$(\sigma_1, \sigma_2) = (\beta(\pi_1^\dagger, \pi_0^\dagger), 1)$; if the buyer offers $p_0^\dagger = v_L$, only the high type opts out, and the low type accepts with probability $(\tilde{\beta}(\pi_1^\dagger, \pi_0^\dagger), 0)$.

Therefore, the payoff to the buyer when she offers $p_1^\dagger$ is

$$\left(1 - \frac{\pi_1^\dagger}{\pi^*}\right)(u_L - p_1^\dagger) + \frac{\pi_1^\dagger}{\pi^*}\delta(1-\xi)U^*,$$

where $U^* = (1-\pi^*)(u_L - v_H^*) + \pi^*(u_H - v_H^*)$. The payoff when she offers $p_0^\dagger = v_L$ is

$$\left(1 - \frac{\pi_1^\dagger}{\pi^*}\right)(u_L - p_0^\dagger) + \frac{\pi_1^\dagger}{\pi^*}\xi(1-\pi^*)(u_L - p_0^\dagger) + \frac{\pi_1^\dagger}{\pi^*}\delta(1-\xi)U^*.$$

The indifference condition gives

$$(1 - \frac{\pi_1^\dagger}{\pi^*})(p_0^\dagger - p_1^\dagger) = \frac{\pi_1^\dagger}{\pi^*}\xi(1-\pi^*)(u_L - p_0^\dagger).$$

Note that the left-hand side of the equation above is the benefit of screening the low type at a lower price, while the right-hand side is the cost of the low type's opting-out. Hence $\pi_1^\dagger$ is given by

$$\frac{\pi_1^\dagger}{\pi^*} = \frac{(1-\delta)(1-\xi)v_L}{(1-\delta)(1-\xi)v_L + \xi(1-\pi^*)(u_L - v_L)}. \tag{6}$$

For $k > 1$, when the posterior is $\pi_k^\dagger$ the buyer is indifferent between offering $p_k^\dagger$, which the low type accepts with probability $\beta(\pi_k^\dagger, \pi_{k-1}^\dagger)$, and offering $p_{k-1}^\dagger$, which the low type accepts with probability $\beta(\pi_k^\dagger, \pi_{k-2}^\dagger)$. Let $W(\pi)$ be the buyer's expected payoff in the equilibrium when the posterior is π. Then

$$\begin{aligned} W(\pi_k^\dagger) &= (1-\gamma_k)(u_L - p_k^\dagger) + \delta\gamma_k(1-\xi)W(\pi_{k-1}^\dagger) && (7) \\ &= (1-\gamma_k)(u_L - p_{k-1}^\dagger) + \gamma_k W(\pi_{k-1}^\dagger), && (8) \end{aligned}$$

where $\gamma_k = \frac{\pi_k^\dagger}{\pi_{k-1}^\dagger}$. Note that (7) and (8) are the buyer's expected payoff when she offers $p_k^\dagger$ and $p_{k-1}^\dagger$, respectively. The indifference condition gives

$$\gamma_k W(\pi_{k-1}^\dagger) = (1-\gamma_k)(p_{k-1}^\dagger - v_L^*). \tag{9}$$

Then plugging (9) into (8) gives

$$W(\pi_k^\dagger) = (1-\gamma_k)(u_L - v_L^*).$$

Finally, plugging the equation above into (9) leads to

$$\frac{1}{\gamma_k} = 1 + (1-\gamma_{k-1})\frac{u_L - v_L^*}{p_{k-1}^\dagger - v_L^*}. \tag{10}$$

Note that since $\lim_{k\to\infty} p_k^\dagger = v_L$, γ_k converges to zero as k goes to infinity. Therefore, for any $\pi_0 \in (0,\pi^*)$, there exists $K \in \mathbb{N}$ such that $\pi_K^\dagger \le \pi_0 < \pi_{K-1}^\dagger$. Here I consider the generic case that $\pi_{K+1}^\dagger < \pi_0$.

Strategy Profile

Using the sequences $\{p_k^\dagger\}_{k=-1}^\infty$ and $\{\pi_k^\dagger\}_{k=-1}^\infty$ I present the strategy profile of the deadlock equilibrium both on and off the equilibrium path. The seller's strategy is Markovian in the sense that it only depends on the current belief π_n and the current offer p_n. The buyer's strategy on the equilibrium play depends only on π_n, and off the equilibrium path it depends on π_n and the previous price p_{n-1}.

The strategy profile is similar to the stationary equilibrium in the literature on bargaining with incomplete information (Ausubel and Deneckere, 1989; Deneckere and Liang, 2006; Fuchs and Skrzypacz, 2010) with the following differences: The buyer's belief does not change once it reaches $\pi_0^\dagger = \pi^*$, and the buyer may use a randomized strategy both on and off the equilibrium path.

- Buyer:

$$P(h^n) = \begin{cases} p_{-1}^\dagger & \text{if } \pi_n > \pi_0^\dagger, \\ q_0(p_{n-1},\pi_{n-1}) \circ p_{-1}^\dagger + (1-q_0(p_{n-1},\pi_{n-1})) \circ p_0^\dagger & \text{if } \pi_n = \pi_0^\dagger, \\ p_{k-1}^\dagger & \text{if } \pi_n \in (\pi_k^\dagger, \pi_{k-1}^\dagger) \text{ for } k \ge 1, \\ q_k(p_{n-1}) \circ p_{k-1}^\dagger + (1-q_k(p_{n-1})) \circ p_k^\dagger & \text{if } \pi_n = \pi_k^\dagger \text{ for } k \ge 1, \end{cases}$$

where

$$q_0(p_{n-1},\pi_{n-1}) = \begin{cases} \frac{p_{n-1}/\delta - p_0^\dagger}{p_{-1}^\dagger - p_0^\dagger} & \text{if } \pi_{n-1} \le \pi_0^\dagger, \\ \frac{p_0^\dagger/\delta - p_0^\dagger}{p_{-1}^\dagger - p_0^\dagger} & \text{if } \pi_{n-1} > \pi_0^\dagger, \end{cases} \qquad q_k(p_{n-1}) = \frac{\frac{p_{n-1}-\xi v_L}{\delta(1-\xi)} - p_k^\dagger}{p_{k-1}^\dagger - p_k^\dagger}.$$

- Low-type seller:

$$(\sigma_1,\sigma_2)(h_S^n, L, p_n) = \begin{cases} (1,0) & \text{if } p_n \ge \delta v_H^*, \\ (\rho_0(\pi_n), 0) & \text{if } p_n \in [p_0^\dagger, \delta v_H^*), \\ (\rho_k(\pi_n), \kappa(\pi_n)) & \text{if } p_n \in [p_k^\dagger, p_{k-1}^\dagger) \text{ for } k \ge 1, \\ (0, \kappa(\pi_n)) & \text{if } p_n < v_L^*, \end{cases}$$

where

$$\rho_0(\pi_n) = \begin{cases} \tilde{\beta}(\pi_n, \pi_0^\dagger) & \text{if } \pi_n \le \hat{\pi}^*, \\ 0 & \text{if } \pi_n > \hat{\pi}^*, \end{cases} \qquad \rho_k(\pi_n) = \begin{cases} \beta(\pi_n, \pi_{k-1}^\dagger) & \text{if } \pi_n \le \pi_{k-1}^\dagger, \\ 0 & \text{if } \pi_n > \pi_{k-1}^\dagger, \end{cases}$$

where $\hat{\pi}^* = \frac{\pi^*}{\pi^* + (1-\pi^*)(1-\xi)}$, and

$$\kappa(\pi_n) = \begin{cases} 1 & \text{if } \pi_n \le \pi_0^\dagger, \\ \max\left[0, \frac{\tilde{\beta}(\pi_n, \pi_0^\dagger)}{\xi}\right] & \text{if } \pi_n > \pi_0^\dagger. \end{cases}$$

- High-type seller: by Lemma 1, $(\sigma_1, \sigma_2)(h_S^n, H, p_n) = (1, 0)$ if $p_n \ge v_H^*$ and $(0, 1)$ if $p_n < v_H^*$.

Optimality of the Profile

First look at the buyer's optimality. The construction of $\{\pi_k^\dagger\}_{k=-1}^\infty$ in (3), (6), and (10) requires that if $\pi = \pi_k^\dagger (k \ge 0)$ then the buyer is indifferent between offering $p_{k-1}^\dagger$ and $p_k^\dagger$. Then the single-crossing condition implies that offering $p_{k-1}^\dagger$ is better than $p_k^\dagger$ if and only if $\pi > \pi_k^\dagger$, which completes the buyer's optimality for any $\pi < \pi_0^\dagger = \pi^*$. For $\pi \ge \pi^*$, it suffices to show that offering δv_H^* is suboptimal at $\pi = \pi^*$. In this case, the low type accepts the offer for sure and the high type opts out when the option is available, then the buyer offers v_H^* in the next period and the bargaining ends. Hence the buyer's expected payoff is

$$(1-\pi)(u_L - \delta v_H^*) + \pi\delta(1-\xi)(u_H - v_H^*).$$

If the buyer instead offers v_H^* and ends the bargaining immediately, she obtains $U(\pi) = (1-\pi)(u_L - v_H^*) + \pi(u_H - v_H^*)$. Then offering v_H^* yields greater payoff if and only if

$$\pi > \tilde{\pi} \equiv \frac{(1-\delta)v_H^*}{(1-\delta(1-\xi))(u_H - v_H^*) + (1-\delta)v_H^*}.$$

A simple calculation shows that (A1) holds if and only if $\pi^* > \tilde{\pi}$.

To check the optimality of the low-type seller, we consider the following three cases:

- If $p_n \ge \delta v_H^*$: By Corollary 1, the low type accepts any $p_n \ge \delta v_H^*$ for sure.

- If $p_n \in [v_L, \delta v_H^*)$: Not taking the outside option ($\sigma_2 = 0$) is optimal since $p_n \ge v_L$. If $\pi_n \le \hat{\pi}^*$, accepting the offer with probability $\sigma_1 = \tilde{\beta}(\pi_n, \pi^*)$ induces $\pi_{n+1} = \pi^*$. If $\pi > \hat{\pi}^*$, the low type has a strict incentive to reject p_n and the outside option as $p_{n+1} = v_H^*$.

- If $p_n < v_L$: Taking the outside option ($\sigma_2 = 1$) is optimal since $p_{n+1} \le v_L$ according to the profile. The construction of the sequences $\{(\pi_k^\dagger, p_k^\dagger)\}_{k=0}^\infty$ implies that the low type is indifferent between acceptance and rejection by following the above strategy profile.

Proof of Proposition 3

Define sequences $\{\bar{p}_k\}$ and $\{\bar{\pi}_k\}$ such that $\bar{p}_k = \lim_{\Delta\to 0} p_k^\dagger$ and $\bar{\pi}_k = \lim_{\Delta\to 0} \pi_k^\dagger$ for all k. Then equation (5) implies that $\bar{p}_k = v_L$ for any k. Therefore, for sufficiently small Δ, the equilibrium offer in the pre-bargaining phase ($p_k^\dagger$ for some k) becomes arbitrarily close to v_L. From (10), the recursive equation for $\bar{\gamma}_k = \frac{\bar{\pi}_k}{\bar{\pi}_{k-1}}$ is given by

$$\bar{\gamma}_k = \frac{1}{1+(1-\bar{\gamma}_{k-1})\frac{u_L-\eta^* v_L}{v_L-\eta^* v_L}}.$$

Define $g(x) = \frac{1}{1+(1-x)\frac{u_L-\eta^* v_L}{v_L-\eta^* v_L}}$. Then $g(x)$ is convex on the interval $[0,1]$ and has fixed points of one and $\frac{v_L-\eta^* v_L}{u_L-\eta^* v_L} < 1$. Hence for any $\bar{\gamma}_1 \in (0,1)$, $\lim_{k\to\infty} \bar{\gamma}_k = \frac{v_L-\eta^* v_L}{u_L-\eta^* v_L}$. Therefore, for any belief $\pi \in (0,\pi^*)$ there exists a finite K such that $\bar{\pi}_K = \bar{\pi}_0 \cdot \prod_{m=1}^K \bar{\gamma}_m \leq \pi$. Therefore, as Δ goes to zero, the number of periods of the pre-deadlock phase is bounded above, hence the real-time length of the pre-deadlock phase shrinks to zero. In the deadlock phase, in each period the bargaining ends by 1) agreement at $p = v_H^*$ with probability q, 2) agreement at $p = v_L$ with probability $(1-q)(1-\pi^*)\xi$, or 3) opting-out with probability $(1-q)\pi^*\xi$. Therefore, the resolution period of the deadlock phase is a geometric distribution with parameter $q+(1-q)\xi$. Then the expected length of the deadlock phase (in real time) is $\frac{\Delta}{q+(1-q)\xi}$. Since $q = \frac{v_L r}{v_H^*-v_L}\Delta + o(\Delta)$ (see footnote 17), as Δ goes to zero, the expected length becomes

$$\frac{\Delta}{q+(1-q)\xi} \to \frac{1}{\frac{v_L r}{v_H^*-v_L}+\lambda} > 0,$$

and thus the expected length does not shrink to zero.

Proof of Propositions 4 and 5

The probability of agreement at $t = 0$ is $(1-\pi_0)\beta(\pi_0,\pi^*) = 1 - \frac{\pi_0}{\pi^*}$. The proof of Proposition 3 implies that $\hat{T}_d = \frac{\Delta}{q+(1-q)\xi}$, and letting $\Delta \to 0$ provides the desired result.

The probability of a breakdown conditional on the bargaining reaching the deadlock phase is

$$\frac{(1-q)\pi^*\xi}{q+(1-q)\xi},$$

so letting $\Delta \to 0$ provides the desired result.

Proof of Proposition 6

If $\delta v_H > v_L$, then (A1) is satisfied for sufficiently high ξ, so there exists a deadlock equilibrium. By (3), we have

$$\lim_{\xi \to 1} \pi^* = \lim_{\xi \to 1} \frac{(v_H^* - u_L) + \eta(u_L - v_L)}{(u_H - u_L) + \eta(u_L - v_L)} = \frac{v_H - v_L}{u_H - v_L},$$

and (5) and (6) implies that

$$p_1^\dagger \to v_L \text{ and } \pi_1^\dagger \to 0,$$

so we have proven the desired result.

Proof of Proposition 7

Suboptimality of "Two-Period Screening"

Lemma 2. *Suppose* (A2) *holds. Then in equilibrium,* $p_n \in [\delta v_H^*, v_H^*)$ *is never offered after any history.*

Proof. Recall from the proof of Proposition 2 that offering v_H^* yields a greater payoff than offering a sequence of prices $\delta v_H^*, v_H^*$ if and only if

$$\pi > \tilde{\pi} \equiv \frac{(1-\delta)v_H^*}{(1-\delta(1-\xi))(u_H - v_H^*) + (1-\delta)v_H^*}.$$

On the other hand, from the inequality

$$(1-\pi)(u_L - \delta v_H^*) + \pi\delta(1-\xi)(u_H - v_H^*) < 0,$$

the offer sequence $\delta v_H^*, v_H^*$ yields a negative payoff if and only if

$$\pi < \underline{\pi} \equiv \frac{\delta v_H^* - u_L}{\delta(1-\xi)(u_H - v_H^*) + (\delta v_H^* - u_L)}.$$

Suppose (A2) holds. Then a simple calculation shows that (A2) implies $\underline{\pi} > \tilde{\pi}$. Then for any $\pi \in [0,1]$, $p_n \in [\delta v_H^*, v_H^*)$ is not offered in equilibrium, since either $p = v_H^*$ or $p = 0$ is a profitable deviation. □

Upper Bound on the Equilibrium Posterior

Lemma 3. *In any equilibrium, there exists* $\bar{\pi} \in (0,1)$ *such that if* $\pi_n > \bar{\pi}$ *after any history, the buyer offers* $p_n = v_H^*$.

Proof. The maximum payoff of the buyer by screening the low type is

$$(1-\pi)\cdot(u_L - v_L^*) + \pi(1-\xi)\cdot\delta(u_H - v_H^*). \tag{11}$$

If instead the buyer offers v_H^*, then her payoff is $U(\pi)$. Therefore, if

$$\pi > \frac{v_H^* - v_L^*}{(1-\delta(1-\xi))(u_H - v_H^*) + v_H^* - v_L^*}$$

then the buyer strictly prefers to offer v_H^* regardless of the history. □

Lemma 4. *If $\pi_n \leq \bar{\pi}$ and $p_n < v_H^*$, $\pi_{n+1} \leq \bar{\pi}$.*

Proof. Suppose this is not the case; that is, there exists a history h^n where $\pi_n \leq \bar{\pi}$, $p_n < v_H^*$, and $\sigma_1(h_S^n, L, p_n) + (1-\sigma_1(h_S^n, L, p_n))\xi\sigma_2(h_S^n, L, p_n) > \tilde{\beta}(\pi_n, \bar{\pi})$. By Lemma 2, $p_n < \delta v_H^*$. Moreover, by Lemma 3, $p_{n+1} = v_H^*$. Then it is optimal for the low type to reject both p_n and an outside option and wait for the next period offer, which leads to a contradiction. □

Lemma 5. *(1) If $\pi_n \leq \bar{\pi}$ and $p_n \in (v_L, \delta v_H^*)$, then $\sigma_2(h_S^n, L, p_n) = 0$.*

(2) If $\pi_n = \bar{\pi}$ and $p_n < v_L$, then $\sigma_1(h_S^n, L, p_n) = 0$.

Proof. (1) Suppose this is not the case; that is, there exists a history h^n where $\pi_n \leq \bar{\pi}$, $p_n \in (v_L, \delta v_H^*)$, and $\sigma_2(h_S^n, L, p_n) > 0$. Then opting-out must be at least as good as waiting, so $u_L(h^{n+1}) \leq v_L/\delta$. Then it is strictly optimal to accept p_n, contradicting Lemma 4.

(2) Suppose that there exists a history h^n where $\pi_n = \bar{\pi}$ and $p_n < v_L$, and $\sigma_1(h_S^n, L, p_n) = 0. >$ 0. Then by Lemma 4 $\sigma_2(h_S^n, L, p_n) < 1$, which implies $u_L(h^{n+1}) \geq v_L/\delta$. But then accepting p_n is suboptimal, so this is a contradiction. □

Lemma 6. $\bar{\pi} \leq \pi^*$.

Proof. Suppose the opposite: That there exists an equilibrium with $\bar{\pi} > \pi^*$. Then it suffices to show that for all history with a belief smaller than but sufficiently close to $\bar{\pi}$, offering v_H^* is optimal for the buyer.

Define

$$\tilde{U}(\bar{\pi}) = (1-\bar{\pi})\xi(u_L - v_L) + \delta(1-\xi)U_F(\bar{\pi}),$$

and let $\hat{U}(h^n)$ be the supremum of the buyer's expected payoff at h^n, given that the buyer offers $p < v_H^*$ at h^n. I claim that for any history h^n with a belief $\bar{\pi}$, $\hat{U}(h^n) < \tilde{U}(\bar{\pi})$. Suppose the bargaining ends after k periods. Then by Lemma 4, the probability of an agreement between the low-type seller before bargaining ends is no more than $\frac{1-\xi^k}{1-\xi}$. Since the low type never accepts any offer less than v_L,[20] making an agreement at v_L with the least delay yields the highest possible payoff to the buyer. Therefore the buyer's payoff is bounded by

$$\tilde{U}_k(\bar{\pi}) = (1-\bar{\pi})\frac{\xi(1-\delta^k(1-\xi)^k)}{1-\delta(1-\xi)}(u_L - v_L) + \delta^k(1-\xi)^k U_F(\bar{\pi}).$$

Since $\bar{\pi} > \pi^*$, $(1-\delta(1-\xi))U_F(\bar{\pi}) > (1-\bar{\pi})\xi(u_L - v_L)$, so $k = 1$ is optimal.

Now consider histories with beliefs less than $\bar{\pi}$. Then the continuity of the previous argument implies that for any $\beta > 0$, there exists $\varepsilon > 0$ such that for any history h^n with a belief $\pi(h^n) \in (\bar{\pi} - \varepsilon, \bar{\pi})$, if the buyer offers $p < v_H^*$ with positive probability at h^n, then $U(h^n) < \tilde{U}(\bar{\pi}) + \beta$.

Equations (1) and (2) imply that $\bar{\pi} > \pi^*$ if and only if $\tilde{U}(\bar{\pi}) < U_F(\bar{\pi})$. Then since $U_F(\pi)$ are continuous, for sufficiently small $\beta > 0$, there exists $\varepsilon > 0$ such that for any history h^n with a belief $\pi(h^n) \in (\bar{\pi} - \varepsilon, \bar{\pi})$, if the buyer offers $p < v_H^*$ with positive probability at h^n, $U(h^n) < \tilde{U}(\bar{\pi}) + \beta < U_F(\bar{\pi} - \varepsilon)$. So the buyer's optimal offer is v_H^* for any history with $\pi \in (\bar{\pi} - \varepsilon, \bar{\pi})$, which contradicts the definition of $\bar{\pi}$. □

Proof of Proposition 8

Fix a perfect Bayesian equilibrium that satisfies nondecreasing offers.

Step 1 For any history h^n, $\pi_n \geq \pi_{n-1}$.

Proof. Suppose this is not the case; that is, there exists a history h^n such that $\pi_n < \pi_{n-1}$. Then in order to make the low type indifferent, the buyer's offer at h^n satisfies $\mathbb{E}[p_n] = p_{n-1}/\delta$. Therefore, the seller offers $p_n > p_{n-1}$ with positive probability, which violates the nondecreasing offers. □

In the proof of Proposition 7, I show that there exists $\bar{\pi} \leq \pi^*$ that bounds the posterior belief along the bargaining process. Then Step 1 implies that if $\pi_n = \bar{\pi}$ at some history h^n, then $\pi_{n+1} = \bar{\pi}$ after any $p_n < \delta v_H^*$, which implies that $\sigma_1(h_S^n, L, p_n) = \xi$ for any $p_n \in (v_L, \delta v_H^*)$.

Step 2 At $\pi = \bar{\pi}$, the buyer's equilibrium offer is either v_L or v_H^*.

[20]Suppose this is not the case; that is, there exists a history $\hat{h}^m$ where the buyer offers a price less than v_L and the low type accepts it with positive probability. Then, the low type must take the outside option for sure (if it is available) at every history between h^n and $\hat{h}^m$. Then at $\hat{h}^m$ the posterior is $\bar{\pi}$, which contradicts Lemma 5.

Proof. It is clear that offering $p < v_L$ is suboptimal for the buyer. Moreover, Lemma 2 says that any offer $p \in [\delta v_H^*, v_H^*)$ is suboptimal. It is then sufficient to show that if any $p \in (\tilde{p}, v_H^*)$ is not offered, the same goes for any $p \in (\max\{\delta\tilde{p}, v_L\}, \tilde{p})$. Suppose that at some history h^n, the buyer offers $p_n \in (\max\{\delta\tilde{p}, v_L\}, \tilde{p})$. Then in the next period, to make the low type indifferent, the buyer must use mixed offer between v_H^* and some (possibly multiple) $p \leq \tilde{p}$. Therefore the buyer's expected payoff at history h^n is

$$(1-\bar{\pi})\xi(1-p_n) + \delta(1-\xi)(1-v_H^*).$$

Now consider the deviation of the buyer to offer $p' = p_n - \varepsilon$, where ε is small enough such that $p' > \max\{\delta\tilde{p}, v_L\}$. Then the buyer can make an agreement with the low type at a lower offer with the same probability and still use a mixed offer in the next period. So offering p' is a profitable deviation, which proves that any $p \in (v_L, v_H^*)$ cannot be offered in equilibrium. □

Step 3 $\bar{\pi} = \pi^*$.

Proof. Suppose $\bar{\pi} < \pi^*$. First, I claim that if $\pi_n = \bar{\pi}$, the buyer's offer must be v_H^*. Suppose v_L is offered in some history h^n. Then in the next period the buyer must use a mixed strategy between v_H^* and some $p \leq v_L$. Therefore, the buyer's expected payoff at history h^n is

$$(1-\bar{\pi})\xi(1-v_L) + \delta(1-\xi)(1-v_H^*),$$

which is greater than $1 - v_H^*$ since $\bar{\pi} < \pi^*$. So offering v_L at h^{n+1} is strictly better than v_H^*, contradictory to the fact that the buyer uses a mixed strategy.

Since the equilibrium satisfies nondecreasing offers, it must be that for all history h^n with $\pi_n < \bar{\pi}$, the buyer never offers v_H^*. Let $\bar{p}$ be a supremum of the buyer's offer at history h^n with $\pi_n < \bar{\pi}$. Then by Lemma 2, $\bar{p} \leq \delta v_H^*$. Fix ε sufficiently small that $\bar{p} - \varepsilon > \delta\bar{p}$. Then there exists a history h^n with $\pi_n < \bar{\pi}$ where the buyer offers $p > \bar{p} - \varepsilon$ with positive probability. Suppose that $\pi_{n+1} \geq \bar{\pi}$; then $p_{n+1} = v_H^*$ and the low type is strictly better off by rejecting p_n. If $\pi_{n+1} < \bar{\pi}$, then accepting p_n is a strict best response of the low type, violating consistency. □

Step 4 Behavior at $\pi \leq \pi^*$ is determined uniquely.

Proof. By step 1, the equilibrium belief is nondecreasing. Therefore, the backward induction method in the proof of Proposition 2 yields unique equilibrium behavior. □

Proof of Proposition 9

The buyer's indifference condition at the deadlock belief π^* is given by

$$(1-\delta(1-\xi))\{(1-\pi^*)u_L+\pi^*u_H-(c_H+v_H^*)\}=(1-\pi^*)\xi(u_L-(c_L+v_L)),$$

so

$$\pi^*=\frac{(c_H+v_H^*-u_L)+\frac{\xi}{1-\delta(1-\xi)}(u_L-(c_L+v_L))}{(u_H-u_L)+\frac{\xi}{1-\delta(1-\xi)}(u_L-(c_L+v_L))}.$$

The buyer can conduct a two-period screening by offering $(1-\delta)c_L+\delta(c_H-v_H^*)$ to the low type then offering $c_H+v_H^*$ to the remaining high type. In this case, his payoff is

$$(1-\pi)[u_L-(1-\delta)c_L+\delta(c_H-v_H^*)]+\pi\delta(1-\xi)[u_H-(c_H+v_H^*)].$$

Hence the two-period screening yields a higher payoff than offering $c_H+v_H^*$ if

$$\pi<\tilde{\pi}\equiv\frac{(1-\delta)[(c_H+v_H^*)-c_L]}{(1-\delta)[(c_H+v_H^*)-c_L]+(1-\delta(1-\xi))[u_H-(c_H+v_H^*)]}.$$

It can be shown that (A1a) is satisfied if and only if $\pi^*>\tilde{\pi}$.

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