Procurement Auction with Corruption by Quality Manipulation

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Abstract

We studied an agency problem in procurement auction where the principal cares both price and quality. The project that can be delivered at a range of quality level chosen by the firms. The quality evaluation job is delegated to the auctioneer and a collusive auctioneer can manipulate firm’s quality score. The collusive firm thus has larger chance to win the contract and the principal’s cannot procure at best combination of price and quality. We construct a model to show the equilibrium and principal’s optimal procurement mechanism with presence of this corruption by quality manipulation. Depending on severeness of corruption, the principal pick a mechanism inducing efficient firm to win by giving out more rent, or allow the collusive firm wins but procure at lower quality. The model allows us to compare principal’s payoff and efficiency in two famous procurement auction format: scoring auction and minimum quality under corruption.

1 Introduction

In procurement of some differential product or project, the buyer cares about both price and a number of other quality factors. Therefore, most procurement auction are conducted as a scoring auction, where the multidimensional objectives of the principal are reflected in a scoring rule. By the law of tender in China, government and state-owned enterprises must conduct their procurement via open auction at city government’s public

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resource trading center. The law also requires the auction shall base on a scoring rule containing three factors:

\[
\text{score} = S(\text{economic factor, technical factor, business factor})
\]

The score function is usually based on linear weights and the firm with highest score wins the contract.

Another common way of procurement is called minimum quality. The principal announce a minimum level of quality standard (and other requirement, like experience of the company). For all those who meets the standard, their bids are evaluated purely on price factor.

Both procurement format involve evaluation of quality factors. Due to the complexity of the project, the principal usually cannot fully observe quality and has to rely on experts in the industry. Because of the complexity, delegation is necessary. Because of subjectivity of quality evaluation, a natural problem of corruption rises: the procurement agent and some firm may collude, and manipulate the quality evaluation score. The result is the contract can be granted to less efficient firm at a high price and low quality. That’s why we see so many “jerry-built” projects and low quality but expensive government procurement cases. In this paper, we study this particular kind of collusion. To distinct from bid rigging problem among bidders, we the problem corruption by quality manipulation between the auctioneer and firm(s).

A Motivating Story

The city government (principal, she) plans to build a bridge and cares about both the amount of spending and the quality of the bridge, for example, the number of lifespan, firmness, material, maximum weight, travel capacity, beauty etc.. The city government representative does not know the bridge building industry. In particular, she cannot evaluate quality of bridge according to the submitted construction plan in the procurement auction. Therefore, she delegates the procurement auction task (or only quality evaluation part) to an auctioneer (He) who knows the industry and can evaluate quality.

Suppose there are two firms \(i = 1, 2\) show up in the procurement auction. The auction rule ask them to submit a price quality combination as their bids. If the auctioneer is honest, he will simply report the quality evaluation to the principal and the problem becomes how to design the auction of two dimensional bid analyzed in [Che 1993].

If the auctioneer is corruptible, then he will exert its ability of quality evaluation in

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2Economic factor means price. In real practice, it is usually not just “the low the better”. Firm’s the economic factor score is related to the engineer estimate and other firm’s submitted prices. Technical factor is an evaluation on the project proposal or construction plan concerning its technology, quality standard, follow-up service etc.. Business factor is an evaluation of the firm itself, such as reputation, reliability, risk of bankruptcy.

3In reality, the procurement task is managed by a street level bureaucrat or representative within the principal’s enterprise. Then it is usually delegated to a procurement agency specialized in that industry. Then the auction is conducted in some government supervised trading center. The score evaluation process is done a committee of experts. So there is several layer of agency between the buyer and the seller. We abstract all of them as one auctioneer.
exchange for bribery. Suppose he has a collusion relation with firm 1, then he can give a high score on the quality firm 1’s construction plan and thus increase its chance to win. Moreover, firm 1 receives a higher compensation due to the reported high quality, while the actual quality delivered is lower. The rent is divided between the auctioneer and the collusive bidder. This hierarchy agency problem is illustrated in Table 1.

<table>
<thead>
<tr>
<th>Role</th>
<th>Main Action</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal (She)</td>
<td>buyer</td>
<td>set scoring rule</td>
</tr>
<tr>
<td>(government, corporation)</td>
<td>or minimum quality</td>
<td></td>
</tr>
<tr>
<td>Auctioneer (He)</td>
<td>procurement agency</td>
<td>evaluate score</td>
</tr>
<tr>
<td>(specialist, committee of experts)</td>
<td></td>
<td>F(θ), c(q, θ)</td>
</tr>
<tr>
<td>Firms (It)</td>
<td>sellers</td>
<td>submit bids</td>
</tr>
<tr>
<td>(producer, providers, contractor)</td>
<td>price quality combination</td>
<td></td>
</tr>
</tbody>
</table>

**Contributions**

In this paper, we construct a model to analyze this corruption by quality manipulation problem. This issue is much less studied comparing to bidding ring and bid revision corruption. But we think it happens much more frequently in reality, especially in developing countries. “Jerry-built” projects, reports of low quality but expensive government procurement, court cases of bribery in procurement are prominent phenomena everywhere. Relevant economic theoretical model, empirical analysis and corruption detection technique needs to be developed to understand and restrict this corruption problem.

As the first step theoretical model, we make the following contribution and findings:

1. In literature, when collusion relation is endogenous, the model is mostly focusing on bribery competition and how it is different than market competition. By allowing collusion relation to be exogenous, we focus on the issue of corruption. The model clearly define and separate technological advantage and corruption advantage in the competition of procurement auction. We show that the equilibrium bids and outcome critically depends on the relative magnitude of these two advantages, both in complete information and incomplete information case.

2. In literature of favoritism, usually the principal is passive and the auctioneer/agent is the mechanism designer. Hence the principal just passively suffers a loss when corruption is present. By allowing exogeneity of collusion relation, we take the principal as mechanism designer and treat auctioner as passive. We show that how the principal designs a optimal mechanism to fight against corruption, in particular, making two advantages offset each other. We show that her payoff could be higher with corruption than without in some cases.

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4It may come from long run relation, favoritism or other reason outside of the model.
3. With presence of corruption, the optimal mechanism in both scoring rule auction and minimum quality have the following feature. When the size of quality manipulation is relative small, the principal may still design the mechanism so the efficient firm wins the contract. But when the size of quality manipulation is large, a mechanism guarantee the efficient firm’s winning will give out too much rent. So instead, the principal will choose to procure at lower quality level and allow which ever firm is collusive to win.

4. Concerning the optimal mechanism, literature consistently predict that the principal shall use a mechanism that under-report her preference on quality. So the efficient firm is “Handicapped” and thus its rent is suppressed. In our model, we show that, to fight against corruption, the principal may over-report her preference on quality.

5. In comparison of scoring auction and minimum quality mechanism, models in literature predict that scoring auction always dominates. We study this issue with presence of corruption and show that in some situation, minimum quality can be better than weighted scoring rule. Che (1993) and Asker and Cantillon (2008) show that scoring auction strictly dominate price-only auction with minimum standards. However, the empirical work by Tran (2009) show the opposite. They use a bribing company’s internal data and find that switching from scoring auction to minimum quality reduce amount of bribery. In our model, we provide a little theoretical foundation that why in some cases, minimum quality can dominate scoring auction and reduce room of corruption.

6. We also construct an explicit model of incomplete information of collusion relation. We show that how mixed strategy equilibrium rises and the outcome become unsure.

1.1 Literature Review

Bidding ring/bid rigging - collusion among bidders

This is the most well-studied form of collusion in auction. It is a collusion among a group of bidders. They form a cartel, reduce competition and try to win the contract at higher price (lower price if they are buyers). The theoretical works is covered in the textbook by Krishna (2009). Empirical works and collusion detection are studied by Porter and Zona (1993), Bajari and Ye (2003), Marmer (2014), Aryal and Gabrielli (2013).

Auctioneer-bidder corruption by bid revision

To distinct from collusion among bidders like bidding ring, we call the collusion between auctioneer and bidder corruption. Another collusion problem in procurement is the principal-agent problem when the principal delegate the auction to an auctioneer. In literature, starting by Laffont and Tirole (1991) and Laffont (1993), they call it favoritism, where they take the auctioneer as mechanism designer. The auctioneer is in favor of a firm and design a unfair procurement rule, thus the principal’s interest is jeopardized.
In auction setting, auctioneer-bidder collusion is mostly modeled by bid/price revision. This form of collusion is sometimes called “magic-number”, because the auctioneer will change the collusive firm’s bid in a way that it barely wins the contract. There are several paper explore this issue, like Lengwiler and Wolfsatter (2006), Burguet and Perry (2009) and Burguet and Perry (2014). There are empirical works like Büchner et al. (2008), Cai et al. (2013) and collusion detection methods by Ingraham (2005). Tian and Liu (2008) propose another way to model bid revision: they assume the collusive player can submit two bids.

Auctioneer-bidder corruption by quality manipulation

This format of corruption is done by the auctioneer changing the collusive bidder’s quality evaluation report. It fits better into real procurement auction where quality is a major concern and evaluating quality is both complex and subjective. It also explain phenomenon like jerry-built project, bribery in “fair” auction, low quality but expensive procurement etc..

To our knowledge, Burguet and Che (2004) and Celentani and Gauza (2002) are the only existing economic papers on this issue. Burguet and Che (2004) treat the collusion relationship as a endogenous result of bribery competition. Instead of only bidding for contract, bidders also bid for collusive relationship with the auctioneer. Essentially, it adds another round of competition. At first glance, because the most efficient firm typically has the largest advantage of acquiring collusion relationship from bribery. But they show that because the inefficient firm can use a mixed strategy on bribery competition and price competition, so the efficient firm cannot guarantee its winning in every possibility. So inefficient allocation arises.

Celentani and Gauza (2002) treats the collusion relationship formation as a random matching process. The procurement agency randomly select a firm to offer side contract, without knowledge of its type. Then dishonest firm and other honest firms compete together in a procurement auction. Once a firm become collusive, it surely wins the contract. Celentani and Gauza (2002) then analyze how the principal designs an optimal mechanism to minimize her lost.

Collusion relation, exogenous or endogenous?

Burguet and Perry (2007) categorized the formation of collusion relation in two types. Type I corruption: collusion relation is endogenous. So any bidder can choose to bribe, the auction turns to a bribery auction/competition. Usually, the presence of corrupted auctioneer has no effect on allocation of good because the bidder with highest value also have strongest incentive to bribe. Bribery simply results in a transfer of rents from the buyer to the auctioneer. Most corruption literature take this setting, for example Burguet and Che (2004), Lengwiler and Wolfsatter (2006), Burguet and Perry (2009) and Burguet and Perry (2014).

Type I corruption: collusion relation is exogenous. The model specifies only one supplier can bribe the auctioneer, but others cannot. In the actual analysis of auction, collusion relation is already fixed by side-contract. Most bidding ring literature take this setting, for example Porter and Zona (1993), Bajari and Ye (2003), Marmer (2014), Aryal.
and Gabrielli (2013). In Krishna (2009) textbook, he discusses how bidding ring (side contract) can be supported by incentive within the ring.

In this paper, we take the second framework and assume there is one bidder has an exclusive collusive relationship with the auctioneer. They are the only pair among the agents that are able to collude. The model can be easily generalized to there is a group of bidders have collusive relationship with the auctioneer, and the result won’t be significantly different. This collusion relation may come from some long run relationship, auctioneer’s favoritism (like prefer a domestic firm) and a before-auction exclusive side-contract.

**Multidimensional / scoring auction**

To analyze quality manipulation in procurement auction, we must be studied a multidimensional auction. The literature is well developed by theorist, like Rogerson [1990], Che [1993], Branco [1997], Asker and Cantillon [2008] and Asker and Cantillon [2010]. We take the framework developed in Che [1993]. Empirical works on multidimensional is still rare. Lewis and Bajari [2011] explore a highway contract procurement data set of “A+B auction”, where bids are evaluated both by price and time of delivery.

## 2 Model

For the main model, to focus on the effect of corruption, we set up the model with two firms under complete information on cost structure and collusion relation. The agency structure is described in Table 1.

### 2.1 Procurement by Scoring Rule

#### 2.1.1 Scoring rule auction without corruption

**Model setup**

We start the model by a benchmark case of no corruption\(^5\). The principal seeks procurement of a project with different level of quality \(q \in [0, \infty)\). With quality \(q\) and price \(p\), her payoff is linear

\[
V(q, p) = q - p
\]

The linearity of payoff function does not lose any generality (as long as we allow cost function to be nonlinear) because quality is a subjective measure, so can always rescale \(q\) to make it enter the payoff function linearly. The principal’s tool to screen the firms is by designing the scoring rule of the procurement auction. It is a function mapping quality and price as a score \(S(q, p) : \mathbb{R}_+^2 \rightarrow \mathbb{R}\). We focus on linear scoring rule

\[
S(q, p) = \alpha q - p, \ \alpha \geq 0.
\]

If \(\alpha = 0\), the multi-dimensional auction reduced to a normal procurement auction with cost as each firm’s type. If \(\alpha = 1\), we say the scoring rule is truthful. In general, we expect \(\alpha \in (0, 1)\). This is the prediction from Che [1993], Asker and Cantillon [2008] and Asker and Cantillon [2010], where they show that it is optimal for the principal to “handicap”

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\(^5\) Either the principal can observe quality, so there is no delegation, or the auctioneer is honest, so he always report the true observed quality.
the efficient firm to achieve higher expected payoff. We also implicitly assume the principal has the ability to commit to select the winning firm by this scoring rule and use the report of the auctioneer to evaluate each firm’s score.

Linearity argument: in reality... \( S(q, p) = w_1 q - w_2 p \)

There are two firms characterized by their type as efficiency parameters \( \theta_1 \) and \( \theta_2 \), drawn from distribution \( F_\theta \). Without loss of generality, take \( \theta_1 < \theta_2 \) and refer firm 1 as efficient and firm 2 as inefficient. The cost function \( c(q, \theta) \) satisfies the following following the assumptions:

A1 - Standard properties: \( c(0, \theta) = 0 \), \( c_q > 0 \), \( c_{qq} > 0 \), \( c_{q\theta} > 0 \), \( c_{qq\theta} > 0 \).

A2 - Cost function is steep enough: \( c_{qqq} > -c_{qq}^2 \).

A3 - The “inverse” of cost function also satisfied Spence-Mirrlees single crossing property: define function \( q = f(C, \theta) \) where \( C = c(q, \theta) \), \( f \) satisfies \( f_{C\theta} > 0 \).

Under complete information of cost structure, both firms observes \( \theta_1, \theta_2 \). Facing the scoring rule and given its type, each firm chooses \( q \) and \( p \) simultaneously as its bid. Each firm has payoff as

\[
\pi(q, p) = \begin{cases} p - c(q, \theta), & \text{if wins the contract} \\ 0, & \text{otherwise} \end{cases}
\]

The auctioneer’s role is passive. He reports submitted quality honestly to the principal. The following timeline describes the game of procurement by scoring rule:

\[ t = 1 \], each firm \( i \) has its type realized as \( \theta_i \) and observe other firm’s type.
\[ t = 2 \], the principal choose a scoring rule by specifying a \( \alpha \) and announce it.
\[ t = 3 \], each firm submit a sealed bid as a price-quality combination \( (p, q) \).
\[ t = 4 \], the auctioneer evaluates each firm’s bid according to the scoring rule \( S(q, p) = \alpha q - p \). The firm with highest score wins the contract and it deliver the project at quality \( q \) and will be compensated by price \( p \) according to its winning bid.

Equilibrium

Given the result in analysis of multidimensional auction, when scoring rule is additively separate in quality and price, then the decision on quality and price can be separated. The following Lemma is a special case of Che (1993):

**Lemma 1**: Given \( S(q, p) = \alpha q - p \), it is a weakly dominant strategy for firm with type \( \theta \) to choose quality as

\[
q^* = \arg \max_q \alpha q - c(q, \theta).
\]

\( q^* \) satisfies first-order condition \( \alpha = c_q(q^*, \theta) \) and \( q^* \) is unique because the second-order condition \( -c_{qq} < 0 \) always hold under convex cost function. Treating \( \alpha \) as parameter, we denote the equilibrium quality as \( q_1(\alpha) = \arg \max_q \alpha q - c(q, \theta_1) \) and \( q_2(\alpha) = \arg \max_q \alpha q - c(q, \theta_2) \). They satisfy the following property:

**Lemma 2**: Equilibrium quality as a function of \( \alpha \) has the following properties:

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The assumptions hold for this parametric cost function: \( c_1 = \theta_1 q^\gamma \), \( c_2 = \theta_2 q^\gamma \), \( \gamma > 2 \), \( \theta_2 \geq \theta_1 \).

For uniqueness of solution in theorem 2.
(i) If the principal announces a higher quality weight $\alpha$, then the optimal quality increases, i.e. $q'_i(\alpha) > 0$.

(ii) The efficient firm picks higher quality under the same $\alpha$ and is more responsive to change in $\alpha$, i.e. $q_1(\alpha) > q_2(\alpha)$ and $q'_1(\alpha) - q'_2(\alpha) > 0$.

(iii) $q_i(0) = 0$.

(iv) Define each firm’s cost at producing equilibrium quality as $c_1(\alpha) \equiv c(q_1(\alpha), \theta_1)$, $c_2(\alpha) \equiv c(q_2(\alpha), \theta_2)$, $c_1(\alpha) > c_2(\alpha)$.

**Proof of Lemma 1:**
Suppose firm $i$ submit $(q, p)$ such that $q \neq q^*$, due to the monotonicity of scoring rule in both quality and price, there always exist another bid $(q^*, p')$ such that $\alpha q^* - p' = \alpha q - p$. Because $S(q^*, p') = S(q, p)$, so both $(q, p)$ and $(q^*, p')$ wins the contract with same probability. Note $p' = \alpha q^* - \alpha q + p$, the payoff if the firm wins the contract has the following relation

$$\pi(q^*, p') - \pi(q, p) = p' - c(q^*, \theta) - p + c(q, \theta)$$

$$= \alpha q^* - \alpha q + p - c(q^*, \theta) - p + c(q, \theta)$$

$$= \alpha q^* - c(q^*, \theta) - [\alpha q - c(q, \theta)] > 0,$$

by definition and uniqueness of $q^*$. So deviating from $q^*$ does not change winning probability but reduce firm’s payoff.

*Q.E.D.*

**Proof of Lemma 2:**

(i) $q_i(\alpha)$ satisfies equation $f(\alpha, q) = \alpha - c_q(q, \theta_i) = 0$. By implicit function theorem and $c_{qq} > 0$, $q'_i(\alpha) = -\frac{\partial f}{\partial q} = \frac{1}{c_{qq}(q, \theta_i)} > 0$.

(ii) Because $c_{q\theta} > 0$, $\frac{\partial^2 q^*}{\partial \theta} = -\frac{\partial f}{\partial \theta} = -\frac{c_{q\theta}}{c_{qq}} < 0$. By assumption $\theta_1 < \theta_2$, so $q_1(\alpha) > q_2(\alpha)$. Because $c_{qq\theta} > 0$, $c_{qq}(q, \theta_1) < c_{qq}(q, \theta_2)$ and

$$q'_1(\alpha) - q'_2(\alpha) = \frac{1}{c_{qq}(q, \theta_1)} - \frac{1}{c_{qq}(q, \theta_2)} > 0.$$

(iii) Because when $\alpha = 0$, $q^* = \arg\max_q 0 \times q - c(q, \theta) = \arg\max_q \{-c(q, \theta)\}$. The monotonic increasing function $c(q, \theta)$ take minimum at $q = 0$, so $q_i(0) = 0$.

(iv) Recall assumption A1 and A3, $c_{q\theta} > 0$ and $f_{C\theta} > 0$ are the Spence-Mirrlees single crossing property. Let $C = c_1(\alpha)$, $q_2(C) = f(C, \theta_2)$. $f_{C\theta} > 0 \Rightarrow c_2(\alpha) < c(q_2(C), \theta_2) = C = c_1(\alpha)$. Hence, $c_1(\alpha) > c_2(\alpha)$.

*Q.E.D.*

After pinning down quality choice, firms choose their prices in a way of Bertrand competition and the firm 1 can always wins by slightly out-bidding firm 2 in its score.
**Theorem 1**: The equilibrium prices are

\[
\begin{align*}
    p_1(\alpha) &= \alpha q_1(\alpha) - \alpha q_2(\alpha) + c_2(\alpha) = c_1(\alpha) + \Delta_S(\alpha), \\
    p_2(\alpha) &= c_2(\alpha)
\end{align*}
\]

where \(\Delta_S(\alpha) \equiv \alpha q_1(\alpha) - c_1(\alpha) - \alpha q_2(\alpha) + c_2(\alpha)\) represents firm 1’s rent and we call it *technical advantage* (under scoring rule auction). In the equilibrium, \(\Delta_S(\alpha) \geq 0, \Delta_S'(\alpha) > 0\) and \(\Delta_S''(\alpha) > 0\).

**Proof of Theorem 1:**

Let \(\epsilon\) be some small positive number. Under Bertrand competition, firm 2 chooses price as its cost \(p_2 = c_2(\alpha)\) in equilibrium. Firm 1 chooses \(p_1(\alpha) = \alpha q_1(\alpha) - \alpha q_2(\alpha) + c_2(\alpha) - \epsilon\) to match firm 2’s score and slightly over-bids it.

\[
S_1 = \alpha q_1(\alpha) - p_1(\alpha)
\]

\[
= \alpha q_1(\alpha) - [\alpha q_1(\alpha) - \alpha q_2(\alpha) + c_2(\alpha) - \epsilon]
\]

\[
= \alpha q_2(\alpha) + c_2(\alpha) + \epsilon
\]

\[
= \alpha q_2(\alpha) + p_2(\alpha) + \epsilon > S_2
\]

Given \(p_1(\alpha) = \alpha q_1(\alpha) - \alpha q_2(\alpha) + c_2(\alpha) - \epsilon\), firm 2 cannot decrease its price below its marginal cost and don’t have incentive to increase its price. Ignoring \(\epsilon\) for conciseness, \(p_2(\alpha) = c_2(\alpha)\) and \(p_1(\alpha) = \alpha q_1(\alpha) - \alpha q_2(\alpha) + c_2(\alpha)\) are equilibrium prices.

Because in the equilibrium, firm 1 earns a positive rent, \(\Delta_S(\alpha) = p_1(\alpha) - c_1(\alpha) > 0\). By first-order condition in Lemma 1,

\[
\Delta_S'(\alpha) = q_1(\alpha) + \alpha q_1'(\alpha) - c_q(q, \theta_1)q_1'(\alpha) - q_2(\alpha) - \alpha q_2'(\alpha) + c_q(q, \theta_2)q_2'(\alpha)
\]

\[
= q_1(\alpha) - q_2(\alpha) + q_1'(\alpha) [\alpha - c_q(q, \theta_1)] - q_2'(\alpha) [\alpha - c_q(q, \theta_2)]
\]

\[
= q_1(\alpha) - q_2(\alpha) > 0
\]

\[
\Delta_S''(\alpha) = q_1'(\alpha) - q_2'(\alpha) > 0
\]

Q.E.D.

We can use the following figure to illustrate this equilibrium. In quality-price space, \(q\) and \(\alpha q\) represents principal’s benefit and first term of the score of quality \(q\) respectively. Firm’s quality choice depends on slope of scoring rule and its cost function. the maximum score firm 2 can get is \(S_2 = \alpha q_2(\alpha) - c_2(\alpha)\), while firm 1 matches firm 2’s score to beat firm 2 by score \(S_1\). They are both illustrated by the red part. Hence, in the equilibrium, firm 1’s rent being its technological advantage is represented by \(\Delta_S(\alpha)\) and principal’s pay-off \(V_H(\alpha)\). When principal is the mechanism designer, he use \(\alpha\) to make trade-off between reaching a desired quality level and firm 1’s rent.
Principal’s payoff and optimal scoring rule

With honest auctioneer and quality weight $\alpha$, at the equilibrium $\{q_1(\alpha), p_1(\alpha), q_2(\alpha), p_2(\alpha)\}$, firm 1 always wins and the principal’s payoff is

$$V_H(\alpha) = q_1(\alpha) - p_1(\alpha) = q_1(\alpha) - c_1(\alpha) - \Delta_S(\alpha) = (1 - \alpha)q_1(\alpha) + \alpha q_2(\alpha) - c_2(\alpha)$$  \hspace{2cm} (2)

**Theorem 2:** There exist a unique quality weight $\alpha_H$ characterizes the optimal (linear) scoring rule which maximizes $V_H(\alpha)$ and $0 < \alpha_H < 1$, i.e. the efficient firm is “handicapped”.

\[\text{A similar result is shown in } \text{Burguet and Che (2004) section 5.}\]
Proof of Theorem 2:

\( \alpha_H \) satisfies the following first-order condition:

\[
V'_H(\alpha) = (1 - \alpha)q'_1(\alpha) - q_1(\alpha) + \alpha q'_2(\alpha) + q_2(\alpha) - c_q(q, \theta_2)q'_2(\alpha)
\]

\[
= (1 - \alpha)q'_1(\alpha) - q_1(\alpha) + q_2(\alpha) + q'_2(\alpha) \left[ \alpha - c_q(q, \theta_2) \right]
\]

\[
= (1 - \alpha)q'_1(\alpha) - q_1(\alpha) + q_2(\alpha) = 0,
\]

By Lemma 2 and \( c_q > 0 \),

\[
V'_H(0) = \frac{1}{c_{qq}(q, \theta_1)} - q_1(0) + q_2(0) > 0,
\]

\[
V'_H(1) = 0 - q_1(1) + q_2(1) < 0.
\]

\( V_H \) is continuous in \( \alpha \), therefore, there exists at least one \( \alpha_H \in (0, 1) \) satisfies \( V'_H(\alpha) = 0 \). So the solution to \( V'_H(\alpha) = 0 \) lies within \( (0, 1) \). By Lemma 2 (ii), \( q'_1(\alpha) - q'_2(\alpha) > 0 \),

\[
q''_1(\alpha) = \frac{d}{d\alpha} \left( \frac{1}{c_{qq}(\alpha, \theta_1)} \right) = -\frac{c_{qq}}{c_{qq}^2} \times q'_1(\alpha) = -\frac{c_{qq}}{c_{qq}^2} \times \frac{1}{c_{qq}} = -\frac{c_{qq}}{c_{qq}^2}.
\]

The second-order condition of concavity is

\[
V''_H(\alpha) = (1 - \alpha)q''_1(\alpha) - q'_1(\alpha) - \left[ q'_1(\alpha) - q'_2(\alpha) \right].
\]

\[
< (1 - \alpha)q''_1(\alpha) - q'_1(\alpha)
\]

\[
= - (1 - \alpha) \frac{c_{qq}}{c_{qq}^3} - \frac{1}{c_{qq}} < 0
\]

\[
\Leftrightarrow c_{qq} > -\frac{c_{qq}}{1 - \alpha}.
\]

Therefore, a sufficient condition for the second-order condition to hold on \( \alpha \in [0, 1] \) is

\[ c_{qq} > -\frac{c_{qq}^2}{1 - \alpha}. \]

Q.E.D.

2.1.2 Scoring rule auction with corruption

Model setup

As we describe in the motivating story, because of the principal lacks the ability to evaluate quality at the stage of determining the procurement winner, so she has to delegate the job of score evaluation to the auctioneer. The auctioneer is given some discretion/manipulation power on quality evaluation. Following Burguet and Che (2004), we
assume that he can raise the collusive firm’s quality score by $m$. Burguet and Che (2004) show that, if there is no monitoring or the monitoring is not related to $m$, then the auctioneer will always exert full manipulation power. So the result is the collusive firm’s reported $q$ on the bid will be changed to $q + m$.

This form of quality manipulation can reflect a wide class of corruption:

(i) Direct quality manipulation by the auctioneer (Burguet and Che (2004)).

(ii) The quality evaluation process is fair, but the quality requirement design by the auctioneer is biased, so that it is in favor of the collusive firm’s technological factor (Lafont and Tirole (1991)).

(iii) Both the quality evaluation process and quality requirement are fair. But as the collusive firm wins a fair auction, afterward, the auctioneer allows the firm to deliver a quality lower than the one written on the bid (Celentani and Ganuza (2002)).

We restrict quality manipulation by a fixed number for two reasons. First, the principal receives the project will learn the quality ex post. Due to the complexity of the project, some discrepancy between reality and the evaluation report in procurement is reasonable and can be caused by unintended mistake of the auctioneer. This allowance is of course limited and the auctioneer don’t want to trigger investigation by manipulating quality too much. Second, the auctioneer is typically liable if the project went wrong, for example, the bridge collapses. So he does not want to announce the quality too high and the excessive use of low quality project reveals problem in the future.

Concerning collusion relation, we assume the principal knows that the auctioneer is matched with firm 1 (the efficient firm) with probability $p$ and firm 2 with probability $1 - p$, but not exactly who he matched. We also assume complete information of collusion relation among the auctioneer and both firms: if a firm knows it has the collusion relation, it knows the other firm don’t, vice versa. This assumption is relaxed in extension section on incomplete information and many firms.

The following timeline describes the game of procurement by scoring rule $(S)$:

$t = 1$, firm types and become common knowledge among firms. Collusion relation realized and become common knowledge among firms and the auctioneer.

$t = 2$, the principal choose a scoring rule by specifying a $\alpha$ and announce it.

$t = 3$, each firm submit a sealed bid as a price-quality combination $(p, q)$.

$t = 4$, the auctioneer raises the collusive firm’s quality by $m$, then evaluate each bids by scoring rule $S(q, p) = \alpha q - p$. So the collusive firm submitting $(q, p)$ bid receives a score $\alpha(q + m) - p$.

**Equilibrium**

We assume the manipulation power $m$ satisfies $m \leq q_2(\alpha) - c_2(\alpha)$, which means that the sufficient condition that the principal want the project despite corruption.

**Theorem 3**: In a scoring rule auction with corruption, firms still choose equilibrium following $[1]$. The equilibrium prices, outcome and principal’s payoffs

---

9. We assume the auctioneer does not use manipulation power to extort the other firm, “if you don’t pay me a bribe, I will reduce your quality score”. When both bribery and extortion exists, the problem become complicated, see Fahad Jacques paper.
<table>
<thead>
<tr>
<th></th>
<th>outcome</th>
<th>price</th>
<th>principal's payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1 collateral [p]</td>
<td>firm 1 wins</td>
<td>( p_1(\alpha) = c_1(\alpha) + \Delta_S(\alpha) + \alpha m ) ( q_1(\alpha) - p_1(\alpha) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( p_2(\alpha) = c_2(\alpha) )</td>
<td></td>
</tr>
<tr>
<td>Firm 2 collateral [1 - p]</td>
<td>firm 1 wins</td>
<td>( p_1(\alpha) = c_1(\alpha) + \Delta_S(\alpha) - \alpha m ) ( q_1(\alpha) - p_1(\alpha) )</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>( p_2(\alpha) = c_2(\alpha) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>firm 2 wins</td>
<td>( p_1(\alpha) = c_1(\alpha) ) ( q_2(\alpha) - p_2(\alpha) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( p_2(\alpha) = c_2(\alpha) - \Delta_S(\alpha) = \alpha m )</td>
<td></td>
</tr>
</tbody>
</table>

Note that \( \alpha m \) represents the extra score gain by collusive firm and we call it *corruption advantage* (under scoring rule auction). When the efficient firm is collusive, it wins the contract for sure. When the inefficient firm is collusive, the outcome depends on the relative magnitude of technological advantage \( \Delta_S \) and \( \alpha m \).
Proof of Theorem 3:

(1) Equilibrium quality
Suppose firm $i$ is collusive and $(q,p)$ such that $q \neq q^*$ defined by (Ⅰ), there always exist another bid $(q^*,p')$ such that $\alpha q^* - p' + \alpha m = \alpha q - p + \alpha m$. Because $S(q^*,p') = S(q,p)$, so both $(q,p)$ and $(q^*,p')$ wins the contract with same probability. Following the same method in the proof of Lemma 1, can show that $\pi(q^*,p') - \pi(q,p) > 0$, so $q^*$ is optimal for firm $i$. For the non-collusive firm, the proof is similar.

(2) If firm 1 is corrupted
Let $\epsilon$ be some small positive number. Under Bertrand competition, firm 2 chooses price as its cost $p_2 = c_2(\alpha)$ in equilibrium. Firm 1 chooses $p_1(\alpha) = c_1(\alpha) + \Delta_S(\alpha) + \alpha m - \epsilon$ to match firm 2’s score and slightly over-bids it

$$S_1 = \alpha q_1(\alpha) - p_1(\alpha) + \alpha m$$
$$= \alpha q_1(\alpha) + \alpha m - [c_1(\alpha) + \Delta_S(\alpha) + \alpha m]$$
$$= \alpha q_1(\alpha) + \alpha m - [\alpha q_1(\alpha) - \alpha q_2(\alpha) + c_2(\alpha) + \alpha m - \epsilon]$$
$$= \alpha q_2(\alpha) + c_2(\alpha) + \epsilon$$
$$= \alpha q_2(\alpha) + p_2(\alpha) + \epsilon > S_2.$$

Principal’s payoff in this case is $q_1(\alpha) - p_1(\alpha) = q_1(\alpha) - c_1(\alpha) - \Delta(\alpha) - \alpha m$.

(3) If firm 2 is corrupted
When $\alpha m \leq \Delta_S(\alpha)$, technological advantage dominates corruption advantage. Firm 2 still has no chance to win. It chooses $p_2 = c_2(\alpha)$ and received raised score $S_2 = \alpha q_2(\alpha) - p_2(\alpha) + \alpha m$. Firm 1 chooses $p_1(\alpha) = c_1(\alpha) + \Delta_S(\alpha) - \alpha m - \epsilon$ to match firm 2’s score and slightly over-bids it

$$S_1 - S_2 = \alpha q_1(\alpha) - p_1(\alpha) + \epsilon - [\alpha q_2(\alpha) - p_2(\alpha) + \alpha m]$$
$$= \alpha q_1(\alpha) - c_1(\alpha) - \Delta_S(\alpha) + \alpha m + \epsilon - \alpha q_2(\alpha) + c_2(\alpha) - \alpha m$$
$$= \alpha q_1(\alpha) - c_1(\alpha) - \alpha q_2(\alpha) + c_2(\alpha) - \Delta_S(\alpha) + \epsilon$$
$$= \epsilon > 0.$$

Principal’s payoff is $q_1(\alpha) - p_1(\alpha) = q_1(\alpha) - c_1(\alpha) - \Delta_S(\alpha) + \alpha m$. When $\alpha m > \Delta_S(\alpha)$, corruption advantage dominates technological advantage. Firm 1 now has no chance to win. It chooses $p_1 = c_1(\alpha)$ and received score $S_1 = \alpha q_1(\alpha) - p_1(\alpha)$. Firm 2 chooses $p_2(\alpha) = c_2(\alpha) - \Delta_S(\alpha) + \alpha m - \epsilon$ to match firm 1’s score and slightly over-bids it

$$S_2 - S_1 = \alpha q_2(\alpha) - p_2(\alpha) + \alpha m - [\alpha q_1(\alpha) - p_1(\alpha)]$$
$$= \alpha q_2(\alpha) - c_2(\alpha) + \Delta_S(\alpha) - \alpha m + \epsilon + \alpha m - \alpha q_1(\alpha) + c_1(\alpha)$$
$$= \alpha q_2(\alpha) - \alpha q_1(\alpha) + c_1(\alpha) - c_2(\alpha) + \Delta_S(\alpha) + \epsilon$$
$$= \epsilon > 0$$

Principal’s payoff is $q_2(\alpha) - p_2(\alpha) = q_1(\alpha) - c_2(\alpha) + \Delta_S(\alpha) - \alpha m$.

$Q.E.D.$
The principal's payoff at the equilibrium is computed by considering two cases, referred as outcome A and B hereafter. Outcome A: When technological advantage dominates corruption advantage, i.e. \( \alpha m \leq \Delta_S(\alpha) \). Firm 1 wins for sure regardless collusion relation. By Theorem 3 and \([2]\), we can write

\[
V_A(\alpha) = p[q_1(\alpha) - c_1(\alpha) - \Delta_S(\alpha) - \alpha m] + (1 - p)[q_1(\alpha) - c_1(\alpha) - \Delta_S(\alpha) + \alpha m]
\]

\[
= q_1(\alpha) - c_1(\alpha) - \Delta_S(\alpha) - (2p - 1)\alpha m
\]

\[
= V_H(\alpha) - (2p - 1)\alpha m
\]  \((3)\)

Outcome B: When corruption advantage dominates technology advantage, i.e. \( \alpha m > \Delta_S(\alpha) \). The collusion relation determines which firm wins the contract,

\[
V_B(\alpha) = p[q_1(\alpha) - c_1(\alpha) - \Delta_S(\alpha) - \alpha m] + (1 - p)[q_2(\alpha) - c_2(\alpha) + \Delta_S(\alpha) - \alpha m]
\]

\[
= p[q_1(\alpha) - c_1(\alpha) - \Delta_S(\alpha)] + (1 - p)[q_2(\alpha) - c_2(\alpha) + \Delta_S(\alpha)] - \alpha m
\]

\[
= pV_H(\alpha) + (1 - p)[q_2(\alpha) - c_2(\alpha) + \Delta_S(\alpha)] - \alpha m.\]  \((4)\)

Given a fixed manipulation power \( m \), the occurrence of case A or B is determined by the relative magnitude of \( \alpha m \) and \( \Delta_S(\alpha) \), which in turn depends on the quality weight choice of the principal. There exists a threshold quality weight \( \tilde{\alpha} \) satisfies the following condition:

**Lemma 3:** Define \( \tilde{\alpha} \) as the positive solution of equation \( \alpha m = \Delta_S(\alpha) \).

(i) Define \( k(\alpha) \equiv \frac{\Delta_S(\alpha)}{\alpha} \). If \( m < \sup_{\alpha \in [0, \infty)} k(\alpha) \), then there exists a unique \( \tilde{\alpha} > 0 \) such that \( \tilde{\alpha} m = \Delta_S(\tilde{\alpha}) \).

(ii) For \( \alpha < \tilde{\alpha} \), \( \alpha m > \Delta_S(\alpha) \); for \( \alpha \geq \tilde{\alpha} \), \( \alpha m \leq \Delta_S(\alpha) \).

(iii) \( \tilde{\alpha} \) is monotonically increasing in \( m \).

Note that when \( m > \sup_{\alpha \in [0, \infty)} k(\alpha) \), the manipulation power is so large that the corruption advantage always dominates. In this case, we can treat \( \tilde{\alpha} = \infty \).

**Proof of Lemma 3:**

(i) By Lemma 2 (iv), \( k'(\alpha) = \frac{1}{\alpha^2} [\Delta'_S(\alpha) - \Delta_S(\alpha)] = \frac{1}{\alpha^2} [c_1(\alpha) - c_2(\alpha)] > 0 \), \( k(\alpha) \) is increasing. If \( m < \sup_{\alpha \in [0, \infty)} k(\alpha) \), by continuity of \( k(\alpha) \), there exists a unique \( \tilde{\alpha} > 0 \) such that \( m = k(\tilde{\alpha}) \iff \tilde{\alpha} m = \Delta_S(\tilde{\alpha}) \).

(ii) Following monotonicity of \( k(\alpha) \), obviously, for \( \alpha < \tilde{\alpha} \), \( k(\alpha) < m \iff \alpha m > \Delta_S(\alpha) \); for \( \alpha \geq \tilde{\alpha} \), \( k(\alpha) \geq m \iff \alpha m \leq \Delta_S(\alpha) \).

(iii) By identity \( m - k(\tilde{\alpha}) = 0 \)

\[
\tilde{\alpha}'(m) = -\frac{1}{k'(\alpha)} > 0.
\]

Q.E.D.

\(^{10}\alpha = 0 \) always makes \( \alpha m = \Delta_S(\alpha) \), but the principal does not want to induce zero quality.
With Lemma 3, principal’s payoff function $V(\alpha)$ depends on parameter $m$ and her choice of $\alpha$,

$$V(\alpha) = \begin{cases} V_A(\alpha) = V_H(\alpha) - (2p - 1)am & \text{if } \alpha \geq \bar{\alpha} \Leftrightarrow \alpha m \leq \Delta_S(\alpha) \\ V_B(\alpha) = pV_H(\alpha) + (1 - p) [q_2(\alpha) - c_2(\alpha) + \Delta_S(\alpha)] - \alpha m & \text{if } \alpha < \bar{\alpha} \Leftrightarrow \alpha m > \Delta_S(\alpha) \end{cases}$$

(5)

Given a $m$, the principal can induce the occurrence of outcome A or B. Because technological advantage is a convex function of $\alpha$ and corruption advantage is linear of $\alpha$, if the principal chooses a large quality weight, firm 1’s technological advantage dominates firm 2’s corruption advantage, thus firm 1 is guaranteed as winner. If the principal chooses a small quality weight, firm 2 dominates firm 1 when it is corrupted.

**Optimal scoring rule**

**Lemma 4:** Recall (3) and (8), $V_A$ and $V_B$ have the following properties

(i) $V_A(\cdot)$ has a unique maximum $\alpha_A$.

(ii) $V_B(\cdot)$ is not generally concave. Because $V_B$ is continuous, we can still define its maximum on the compact interval $\alpha_B \in \operatorname{arg\,max}_{\alpha \in [0, \bar{\alpha}]} V_B(\alpha)$.

(iii) For $p > \frac{1}{2}$, $\alpha_A < \alpha_H$ and $\alpha_A$ decreases in $m$; for $p < \frac{1}{2}$, $\alpha_A > \alpha_H$ and increases in $m$.

(iv) $\forall \alpha \in [0, \bar{\alpha}), V_A(\alpha) > V_B(\alpha)$. 

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Proof of Lemma 4:

(i) The second-order condition for maximization is satisfied because

\[ V'_A(\alpha) = V'_H(\alpha) - (2p - 1)m = 0 \]
\[ V''_A(\alpha) = V''_H(\alpha) < 0. \]

So there exists a unique maximum \( \alpha_A \).

(ii)

\[
\begin{align*}
V'_B(\alpha) &= pV'_H(\alpha) - (1 - p) \left[ q'_2(\alpha) - c'_2(\alpha) + \Delta'_s(\alpha) \right] - m \\
V''_B(\alpha) &= p \left[ (1 - c_2(q, \theta)) q'_2(\alpha) - c_{qq}(q, \theta_1)(q'_1(\alpha))^2 \right] \\
&\quad + (1 - p) \left[ (1 - c_2(q, \theta_2)) q'_2(\alpha) - c_{qq}(q, \theta_2)(q'_2(\alpha))^2 \right] \\
&\quad - (1 - 2p) \left[ q'_1(\alpha) - q'_2(\alpha) \right].
\end{align*}
\]

The sign of \( V''_B(\alpha) \) is not deterministic. A numerical example is shown in Appendix B.

(iii) For \( p > \frac{1}{2} \), \( (2p - 1)m > 0 \), \( V'_A(\alpha) < V'_H(\alpha) \). Because \( V'_A(\alpha_A) = 0 \), \( V'_H(\alpha_H) = 0 \) and both \( V'_A(\alpha) \) and \( V'_H(\alpha) \) is are decreasing function, hence \( \alpha_A < \alpha_H \). For \( p < \frac{1}{2} \), similarly, we can show that \( \alpha_A > \alpha_H \).

By implicit function theorem and identity \( V'_A(\alpha_A) = V'_H(\alpha_A) - (2p - 1)m = 0 \)

\[
\frac{d\alpha_A}{dm} = \frac{2p - 1}{V''_H(\alpha)} \begin{cases} < 0, & \text{if } p > \frac{1}{2} \\ > 0, & \text{if } p < \frac{1}{2} \end{cases}
\]

(iv)

\[
\begin{align*}
V_A(\alpha) - V_B(\alpha) &= V_H(\alpha) - (2p - 1)m - pV_H(\alpha) - (1 - p) \left[ q_2(\alpha) - c_2(\alpha) + \Delta_s(\alpha) \right] + \alpha m \\
&= (1 - p) \left[ V_H(\alpha) - q_2(\alpha) + c_2(\alpha) - \Delta_s(\alpha) + 2\alpha m \right] \\
&= (1 - p) \left\{ q_1(\alpha) - c_1(\alpha) - \left[ q_2(\alpha) - c_2(\alpha) \right] + 2 \left[ \alpha m - \Delta_s(\alpha) \right] \right\} > 0.
\end{align*}
\]

Q.E.D.

Intuitively, when efficient firm is more likely to be corrupted, choose a low \( \alpha \) to reduce both technological advantage and corruption advantage. When efficient firm is less likely to be corrupted, choose a high \( \alpha \) to possibly use firm 2’s corruption advantage offsetting firm 1’s technological advantage.

Theorem 4: Define \( \tilde{m}_S \) as the solution\(^{11} \) of \( \alpha(\tilde{m}) = \alpha_A(\tilde{m}) \). The optimal choice of quality weight \( \alpha^* \) in scoring rule auction is described below:

\(^{11}\)\( \tilde{\alpha} \) and \( \alpha_A \) are defined in Lemma 3 and Lemma 5. More rigorously, define \( \tilde{m}_S \) as \( \tilde{m}_S = \begin{cases} m_0 \text{ such that } \alpha(m_0) = \alpha_A(m_0) \\ \infty \text{ if } \alpha_A(m) > \tilde{\alpha}(m), \forall m \end{cases} \).
(i) \( m \leq \bar{m}_S \), \( \alpha^* = \max\{\tilde{\alpha}, \alpha_A\} \) [diagram (a)].

(ii) If \( m > \bar{m}_S \), there exists a unique \( \hat{\alpha} > \alpha_A \) such that \( V_A(\hat{\alpha}) = V_B(\alpha_B) \).

If \( \tilde{\alpha} \leq \hat{\alpha} \), \( \alpha^* = \tilde{\alpha} \) [diagram (b)].

If \( \tilde{\alpha} > \hat{\alpha} \), \( \alpha^* = \alpha_B \) [diagram (c)].

(iii) The relative magnitude of \( \tilde{\alpha} \) and \( \hat{\alpha} \) depends on parameter \( m \) and \( p \).

\[ \begin{align*}
\text{Figure 2: Principal’s Payoffs under Scoring Rule} \\
\end{align*} \]

Proof of Theorem 4:

(1) If \( m \leq \bar{m}_S \), \( \tilde{\alpha} \leq \alpha_A \), the maximum of \( V_A \) can be achieved. In Lemma 4, \( \forall \alpha \leq \tilde{\alpha}, V_B(\alpha) \leq V_A(\alpha) < V_A(\alpha_A) \). So \( \alpha^* = \alpha_A \).

If \( m > \bar{m}_S \), \( \tilde{\alpha} > \alpha_A \), the maximum of \( V_A \) can be achieved. Inducing outcome B yields the principal at most \( V_B(\alpha_B) \). \( V_B(\alpha_B) < V_A(\alpha_B) \leq V_A(\alpha_A) \). By concavity of \( V_A \), \( V_A \) is decreasing on \( [\alpha_A, \infty) \), so there exists a unique \( \hat{\alpha} > \alpha_A \) such that \( V_A(\hat{\alpha}) = V_B(\alpha_B) \). The optimal \( \alpha^* \) now depends on the relative magnitude of \( \tilde{\alpha} \) and \( \hat{\alpha} \).

If \( \tilde{\alpha} \leq \hat{\alpha} \), \( V_A(\tilde{\alpha}) \geq V_A(\hat{\alpha}) = V_B(\alpha_B) \), then \( \alpha^* = \tilde{\alpha} \).

If \( \tilde{\alpha} > \hat{\alpha} \), \( \forall \alpha \in [\tilde{\alpha}, \infty) \), \( V_A(\alpha) < V_A(\tilde{\alpha}) < V_A(\hat{\alpha}) = V_B(\alpha_B) \), the optimal solution is \( \alpha^* = \alpha_B \).

(2) Discussion on \( \tilde{\alpha} \) and \( \hat{\alpha} \)

Define \( \hat{m}_S \) as \( \hat{\alpha}(m) = \hat{\alpha}(m) \).

Q.E.D.

\[ \begin{align*}
\text{Figure 2: Principal’s Payoffs under Scoring Rule} \\
\end{align*} \]

We haven’t been able to characterize the set of parameter that makes \( \tilde{\alpha} > \hat{\alpha} \), so the principal induces outcome B.

Theorem 4 tell us the following intuitive results. When corruption is present, there are two effects: when firm 1 is corrupted, the principal suffers a loss; but when firm 2 is corrupted, firm 1’s rent is suppressed and the principal benefits from it.

If manipulation power is relative small, the second effect dominates, the principal chooses a high \( \alpha \) to induce outcome A, where firm 1 always win. In particular, when \( p < \frac{1}{2} \) and \( m \) is small, firm 1 wins the contract with less rent and the principal’s payoff is higher than no corruption case. In the case that \( p < \frac{1}{2} \) and \( \alpha_A > \alpha_H \), the optimal quality weight can exceed 1. So the principal over-report his quality preference.

\[ \begin{align*}
\text{Figure 2: Principal’s Payoffs under Scoring Rule} \\
\end{align*} \]

12In Appendix B, we show that there is cases when \( \tilde{\alpha} > \hat{\alpha} \) and thus the principal induces outcome B.
However, when manipulation power is large, the first effect dominates and allowing a noncollusive firm 1 win will require a very high $\alpha$. Doing so, the rent given up is larger than the loss from allowing corrupted firm 2 win. The principal will rather set a low $\alpha$ and induce outcome B, where the collusive firm will win.

2.2 Procurement Auction by Minimum Quality

Besides scoring rule, the other focus format of procurement auction is imposing a minimum quality requirement. Each participating firms needs to meet a fixed quality level to be a valid bidder in the auction. Once a firm passes the level, then the competition is fully depend on price. Similar quality manipulation problem may arise because the collusive firm can enter by a lower quality than other firms.

In the literature, scoring auction and minimum quality are compared without corruption and the prediction is that scoring auction always dominates minimum quality. We show that with quality manipulation corruption, this result still holds in outcome A, but may not hold if the principal induces outcome B.

2.2.1 Minimum quality auction without corruption

Model setup

The principal sets up a minimum quality requirement $q$. Firms need to meet this quality $q \geq q$. The following timeline describes the game of procurement by minimum quality:

$t = 1$, firm types realized and become common knowledge among firms.
$t = 2$, the principal choose a minimum quality $q$ and announce it.
$t = 3$, both firms submit their bid $(q, p)$ and the quality is checked by the auctioneer
$t = 4$, among firms that meet minimum quality $q$, the firm with lowest price wins the contract.

If the principal commits on only considering price besides minimum quality requirement, there no bonus to for firms to make a higher quality than $q$. So if a firm decides to bid, it will pick quality just meet the requirement, i.e. $q = q$. Therefore, the principal is just picking procurement quality $q$ as her strategy space. Assume that $q$ is not too large, so the inefficient firm 2 is not “shut down”: it makes profit given upon winning. The cost structure is assume to be the same as previous models.

Equilibrium and optimal minimum quality

When there is an honest auctioneer, provided the principal sets $q$, assume both firms choose to enter the auction. Denote their costs as $c_1(q) \equiv c(q, \theta_1)$ and $c_2(q) \equiv c(q, \theta_2)$. Because $c_1(q) < c_2(q)$, firm 1 wins for sure and equilibrium prices are

$$p_1(q) = c_2(q), \quad p_2(q) = c_2(q).$$

The proof is similar to Bertrand competition model and is trivial. Firm 1 earns a rent $\Delta_M(q) \equiv c_1(q) - c_2(q)$: the technological advantage under minimum quality auction. Prin-
Firm 1 collusive \[ p \]

<table>
<thead>
<tr>
<th>Firm 1 collusive [ p ]</th>
<th>outcome</th>
<th>price</th>
<th>principal’s payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>firm 1 wins</td>
<td>( p_1(q) = c_2(q) )</td>
<td>( q - m - c_2(q) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( p_2(q) = c_2(q) )</td>
<td></td>
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</tbody>
</table>

Firm 2 collusive \[ 1 - p \]

<table>
<thead>
<tr>
<th>Firm 2 collusive [ 1 - p ]</th>
<th>outcome</th>
<th>price</th>
<th>principal’s payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>firm 1 wins ( c_1(q) \leq c_2(q - m) )</td>
<td>( p_1(q) = c_2(q - m) )</td>
<td>( q - c_2(q - m) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( p_2(q) = c_2(q - m) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>firm 2 wins ( c_1(q) &gt; c_2(q - m) )</td>
<td>( p_1(q) = c_1(q) )</td>
<td>( q - m - c_1(q) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( p_2(q) = c_1(q) )</td>
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Principal’s payoff is then

\[
U_H(q) = q - c_2(q) \tag{6}
\]

The optimal minimum quality \( q_H \) satisfies first-order condition \( U'_H(q) = 1 - c'_2(q) = 0 \). It is unique maximum because \( U''_H(q) = -c''_2(q) = -c_{qq}(q, \theta_2) < 0 \).

2.2.2 Minimum quality auction with corruption

Model setup

Keep the same assumption on cost structure, collusion relation and information on them.

The game of procurement by minimum quality under corruption is described as:

\( t = 1 \), firm types and become common knowledge among firms. Collusion relation realized and become common knowledge among firms and the auctioneer.

\( t = 2 \), the principal choose a minimum quality \( q \) and announce it.

\( t = 3 \), both firms submit their bid \((q, p)\) and the quality is checked by the auctioneer. The auctioneer raises the quality evaluation of the collusive firm so it can pass the minimum quality by only producing at \( q - m \).

\( t = 4 \), among firms that meet minimum quality \( q \), the firm with lowest price wins the contract.

Similar to the case without corruption, the principal is essentially choosing its procurement quality \( q \) and she knows if the collusive firm wins, she will only receive \( q - m \).

Equilibrium

**Theorem 5**: Given \( q \), with corruption, the non-collusive firm chooses quality \( q \) and collusive firm chooses quality \( q - m \). The equilibrium prices, outcome and principal’s payoffs are

The result is similar to scoring rule auction. When the efficient firm 1 is collusive, it wins the contract for sure. When the inefficient firm 2 is collusive, the outcome depends on the relative magnitude of firm 1’s technological advantage and firm 2’s corruption advantage. The corruption advantage is cost saved by the collusive firm 2: \( c_2(q) - c_2(q - m) \). Hence, the relative magnitude of them depends on \( c_1(q) \) and \( c_2(q - m) \).
**Proof of Theorem 5:**

The proof is straightforward. When firm 1 is corrupted, firm 2 bid at its cost $c_2(q)$, firm 1 can match and slightly over-bid it by $p_1 = c_2(q) - \epsilon$. The principal thus receives a distorted quality compared to no collusion case: $q - m - p_2(q) = q - m - c_2(q)$.

When firm 2 is corrupted, the outcome is determined by the relative magnitude of $c_1(q)$ and $c_2(q - m)$. When $c_1(q) \leq c_2(q - m)$, technological advantage still dominates, firm 1 wins by slightly over-bidding firm 2 at its cost $c_2(q - m) - \epsilon$. The principal receives the non-distorted quality $q$ produced by firm 1 at a lower cost, $q - c_2(q - m)$. On the other hand, when $c_1(q) > c_2(q - m)$, corruption advantage now dominates technological advantage, firm 2 wins for sure by slightly over-bidding firm 2 at its cost $c_1(q) - \epsilon$. The principal receives the project from firm 2 with distorted quality. The payoff is then $q - m - c_1(q)$.

*Q.E.D.*

The principal’s payoff at the equilibrium, similar to scoring rule, can be divided into two outcomes:

(A) When technological advantage dominates corruption advantage, i.e. $c_1(q) \leq c_2(q - m)$. Firm 1 wins for sure regardless collusion relation. By Theorem 5 and (6)

$$U_A(q) = p[q - m - c_2(q)] + (1 - p)[q - c_2(q - m)]$$

$$= q - pc_2(q) + (1 - p)c_2(q - m) - pm$$

$$= q - c_2(q) + (1 - p)c_2(q) - (1 - p)c_2(q - m) - pm$$

$$= U_H(q) + (1 - p)[c_2(q) - c_2(q - m)] - pm$$

(7)

(B) When corruption advantage dominates technology advantage, i.e. $c_1(q) > c_2(q - m)$. The collusion relation determines which firm wins the contract,

$$U_B(q) = p[q - m - c_2(q)] + (1 - p)[q - m - c_1(q)]$$

$$= q - pc_2(q) - (1 - p)c_1(q) - m$$

$$= q - c_2(q) + (1 - p)c_2(q) - (1 - p)c_1(q) - m$$

$$= U_H(q) + (1 - p)[c_2(q) - c_1(q)] - m$$

(8)

Given a fixed manipulation power $m$, the occurrence of outcome A or B can be determined by the relative magnitude of $c_1(q)$ and $c_2(q - m)$, which in turn depends on the minimum quality choice of the principal. There exists a threshold minimum quality $\tilde{q}$ satisfies the following condition:

**Lemma 5:** For any $q > 0$, because $c_2(q) > c_1(q) > 0$ and $c'_2(q) > c'_1(q) > 0$, there exists a unique $t > 0$, such that $c_2(q - t) = c_1(q)$. We define this $t$ as a function of $q$.

(i) If $m < \sup_{q \in [0, \infty)} t(q)$, there exists a unique $\tilde{q} > 0$ such that $c_1(\tilde{q}) = c_2(\tilde{q} - m)$.

(ii) For $q \geq \tilde{q}$, $c_1(q) \leq c_2(q - m)$; for $q < \tilde{q}$, $c_1(q) > c_2(q - m)$.
(iii) $\tilde{q}$ increases in $m$. If $m = 0$, $\tilde{q} = 0$.

(iv) For $q \geq \tilde{q}$, $c'_2(q - m) > c'_1(q)$.

Note that, similar to scoring rule auction, when $m > \sup_{q \in [0, \infty)} t(q)$, the manipulation power is so large that the corruption advantage always dominates. In this case, we can treat $\tilde{q} = \infty$. 

Figure 3: Cost and Threshold Qualities
Proof of Lemma 5:

(i) By assumption A3, given that \( c_2(q-t) = c_1(q) = C \), \( c'_2(q-t) - c'_1(q) > 0 \). So \( t(q) \) is monotonically increasing because

\[
t'(q) = \frac{c'_2(q-t) - c'_1(q)}{c'_2(q-t)} > 0.
\]

By the continuity of \( t(q) \), if \( m < \sup_{q \in [0, \infty)} t(q) \), then there exists a unique \( \bar{q} > 0 \) such that \( t(\bar{q}) = m \), and thus \( c_1(\bar{q}) = c_2(\bar{q} - m) \).

(ii) Because \( t(q) \) is increasing, for \( q \geq \bar{q} \), \( t(q) \geq t(\bar{q}) = m \), \( c_2(q-m) = c_2(q-t(\bar{q})) \geq c_2(q-t(q)) = c_1(q) \); for \( q < \bar{q} \), \( t(q) < t(\bar{q}) \), \( c_2(q-m) = c_2(q-t(\bar{q})) < c_2(q-t(q)) = c_1(q) \).

(iii) By assumption on cost function, at a fixed cost level \( C = c_2(q-m) = c_1(q) \) and \( c'_2(q) > c'_1(q) \). So \( c'_2(q-m) - c'_1(q) = c'_2(q-t(q)) - c'_1(q) > c'_2(q) - c'_1(q) > 0 \). Because \( \bar{q} \) satisfies identity \( c_1(q) - c_2(q-m) = 0 \),

\[
\frac{d\bar{q}}{dm} = \frac{c'_2(q-m)}{c'_2(q-m) - c'_1(q)} > 0.
\]

When \( m = 0 \), \( c_1(\bar{q}) = c_2(\bar{q}) \) has a unique solution at \( \bar{q} = 0 \).

(iv) \( Q.E.D. \)

A fuzzy proof:

Define \( \psi(q) = c_2(q-m) - c_1(q) \)

When \( q = m \), \( \lim_{q \to m} \psi(q) = c_2(0) - c_1(m) < 0 \); when \( q \to \infty \), \( \lim_{q \to \infty} \psi(q) = c_2(\infty) - c_1(\infty) > 0 \).

\[
\psi'(q) = c'_2(q-m) - c'_1(q)
\]

When \( q = m \), \( \lim_{q \to m} \psi'(q) = c'_2(0) - c'_1(m) < 0 \); when \( q \to \infty \), \( \lim_{q \to \infty} \psi'(q) = c'_2(\infty) - c'_1(\infty) > 0 \).

With Lemma 5, principal’s payoff function \( U(q) \) depends on value of parameter \( m \) and her choice of \( q \)

\[
U(q) = \begin{cases} 
U_A(q) = U_H(q) + (1-p) [c_2(q) - c_2(q-m)] - pm & \text{if } q \geq \bar{q} \iff c_1(q) \leq c_2(q-m) \\
U_B(q) = U_H(q) + (1-p) [c_2(q) - c_1(q)] - m & \text{if } q < \bar{q} \iff c_1(q) > c_2(q-m)
\end{cases}
\]

(9)

Intuitively, due to the convexity of cost function, the gap between \( c_1(q) \) and \( c_2(q) \) increases as the minimum quality rises. So when \( q \) is large, firm 1’s technological advantage dominates firm 2’s corruption advantage, thus guarantee its as winning firm. When \( q \) is small, firm 2 dominates firm 1 when it is corrupted. So given\(^{[3]} \) a \( m > 0 \), the principal can induce the occurrence of outcome A or B by choosing \( q \) above or below \( \bar{q} \). Given a higher manip-

\(^{[3]} \) When there is no corruption \( (m = 0) \), \( U(q) \) take \( U_A(q) \) on its whole support and the principal cannot (and won’t) induce outcome B.
ulation power, \( \tilde{q} \) is higher, which means the principal needs to set up a higher \( q \) to make sure firm 1 wins. The optimal choice of \( q \) depends on parameter \( m \) and \( p \).

**Optimal minimum quality**

By adjusting \( q \), the principal controls the outcome leading to \( U_A \) or \( U_B \). We first discuss some basic property of \( U_A \) and \( U_B \).

**Lemma 6** : These two payoff function have the following property:

(i) \( U_A(q) \) has a unique maximum \( q_A \). \( U_B(q) \) has a unique maximum \( q_B \) and \( q_B \) does not depend on \( m \).

(ii) \( q_A > q_H \) and \( q_B > q_H \).

(iii) \( \forall q \in [0, \tilde{q}], U_A(q) > U_B(q) \).
Proof of Lemma 6:

(i) By Lemma 6 (ii), for \(q < \tilde{q}\), \(c_2(q - m) - c_1(q) < 0\)

\[
U_B(q) - U_A(q) = (1 - p) \left[ c_2(q - m) - c_1(q) - m \right] < 0.
\]

(ii) For \(U_A\), the second order condition satisfies because

\[
\begin{align*}
U'_A(q) &= 1 - pc'_2(q) - (1 - p)c'_1(q) \\
U''_A(q) &= -p c''_2(q) - (1 - p) c''_1(q) < 0.
\end{align*}
\]

So there exists a unique maximum \(q_A\).

For \(U_B\), the second order condition satisfies because

\[
\begin{align*}
U'_B(q) &= 1 - pc'_2(q) - (1 - p)c'_1(q) \\
U''_B(q) &= -p c''_2(q) - (1 - p) c''_1(q) < 0.
\end{align*}
\]

So there exists a unique maximum \(q_B\). Because the first-order condition \(U'_B(q) = 0\) does not involve \(m\), so \(q_B\) does not depends on \(m\).

(iii)

\[
\begin{align*}
U'_A(q) &= U'_H(q) + (1 - p) \left[ c'_2(q) - c'_2(q - m) \right] = 0, \\
U'_B(q) &= U'_H(q) + (1 - p) \left[ c'_2(q) - c'_1(q) \right] = 0.
\end{align*}
\]

Because \(q_H\) satisfies \(U'_H(q) = 0\), hence \(q_A > q_H\) and \(q_B > q_H\).

Q.E.D.

(iv) \(\forall q \geq \tilde{q}\), there exists a unique \(\hat{q}\) such that for \(q \leq \hat{q}\), \(U_A(q) \geq U_B(q)\); \(q > \hat{q}\) for \(U_A(q) < U_B(q)\).

(v) \(\hat{q}\) increases in \(m\). For any \(m\), \(\hat{q} > \tilde{q}\).

(iv) For \(q \geq \tilde{q}\), \(U_B(q) - U_A(q) = (1 - p) \left[ c_2(q - m) - c_1(q) - m \right]\), cannot determine the sign without discussing \(m\). By Lemma 6 (ii) and (iv), for \(q \geq \tilde{q}\), \(c_2(q - m) - c_1(q) \leq 0\), \(c'_2(q - m) - c'_1(q) > 0\). So there exists a unique \(\hat{q}\) such that \(c_2(q - m) - c_1(q) \leq m\) for \(q \leq \hat{q}\); \(c_2(q - m) - c_1(q) > 0\), for \(q > \hat{q}\), which in turn gives the result.

(v) By identity \(c_2(q - m) - c_1(q) - m = 0\)

\[
\frac{d\hat{q}}{dm} = \frac{c'_2(q - m) + 1}{c'_2(q - m) - c'_1(q)} > 0
\]

It is easy to see that when \(m = 0\), \(\hat{q} > 0\), \(\hat{q} \geq 0\). For any \(m\), \(\frac{d\hat{q}}{dm} > \frac{d\tilde{q}}{dm} > 0\), so \(\hat{q} > \tilde{q}\).
The last property implies that the optimal quality choice rises when corruption arises. While in Lemma 5, the optimal quality weight could rise or fall depending on \( p \), the principal’s belief of the probability that the efficient firm is collusive. With the result in Lemma 5 and 6, we reach the following theorem on optimal minimum quality choice.

**Theorem 6**: Define \( \tilde{m}_M \) as the solution\(^{14}\) of \( \tilde{q}(m) = q_B \). The optimal choice of minimum quality \( q^* \) in minimum quality auction is describe below:

(i) If \( m \leq \tilde{m}_M \), \( q^* = \max\{\tilde{q}, q_A\} \) [diagram (a)].

(ii) If \( m > \tilde{m}_M \), there exists a unique \( \hat{q} > q_A \) such that \( U_A(\hat{q}) = U_B(q_B) \).

If \( \tilde{q} \leq \hat{q} \), \( q^* = \max\{\tilde{q}, q_A\} \) [diagram (b)].

If \( \tilde{q} > \hat{q} \), \( q^* = q_B \) [diagram (c)].

(iii) The relative magnitude of \( \tilde{q} \) and \( \hat{q} \) depends on parameter \( m \) and \( p \).\(^{15}\)

---

\(^{14}\)\( \tilde{q} \) is defined in Lemma 6.

\(^{15}\)In Appendix B, we show that there is cases when \( \tilde{q} > \hat{q} \) and thus the principal induces outcome B.
Proof of Theorem 6

(1) $q_B$ is fixed and does not depend on $m$. $\tilde{q}$ rises in $m$.
If $m \leq \bar{m}_M$, $\tilde{q} \leq q_B$, by Lemma 5 $U_B(\tilde{q}) < U_A(\tilde{q}) \leq U_A(q_A)$. The principal will choose $q$ such that firm 1 always win. If $\tilde{q} \leq q_A$, then can reach the maximum of $U_A$ by picking $q_A$. If $\tilde{q} > q_A$, $U_A$ is decreasing on $[\tilde{q}, \infty)$, hence pick $\tilde{q}$. In summary, $q^\ast = \max\{\tilde{q}, q_A\}$. (Figure )

If $m > \bar{m}_M$, $\tilde{q} > q_B$, inducing outcome B can at most get the principal $U_B(q_B) < U_A(q_B) \leq U_A(q_A)$. By concavity of $U_A$, there exists a unique $\hat{q} > q_A$ such that $U_A(\hat{q}) = U_B(q_B)$. $q^\ast$ depends on the relative magnitude of $\tilde{q}$ and $\hat{q}$.
If $\tilde{q} < \hat{q}$, then inducing outcome A yields higher payoff. For the similar argument above, $q^\ast = \max\{\tilde{q}, q_A\}$. (Figure )
If $\tilde{q} > \hat{q}$, because $U_A$ is decreasing, $\forall q > \hat{q}$, $U_A$ is decreasing. So inducing outcome B yields higher payoff, $q^\ast = q_B$. (Figure )

(2) Discussion on relative magnitude of $\tilde{q}$, $\hat{q}$
By identity $U_A(\hat{q}(m)) - U_B(q_B) = 0$, $\frac{d\hat{q}}{dm} = \frac{c'_2(q-m)}{c'_2(q-m)-c'_1(q)} > 0$. Recall from the proof of Lemma 6, $\frac{\tilde{q}}{dm} = \frac{1}{(1-p)c'_2(q-m)+p} > 0$. Both $\tilde{q}$ and $\hat{q}$ are monotonically increasing.
Q.E.D.

Theorem 6 has the same structure and implication as theorem 4. If manipulation power is relative small, the second effect dominates, the principal chooses a high $q$ to induce outcome A, where firm 1 always win. However, when manipulation power is large, allowing a noncollusive firm 1 win will require a very high $q$. Doing so, the rent given up is too large that the principal would rather pick a low $q$ to induce outcome B.

2.3 Scoring rule or minimum quality?

2.3.1 Comparison of principal’s payoff

The comparison is based on the expected payoff received by the principal under optimal mechanism in scoring rule and minimum quality. When the auctioneer is honest and there is no corruption, we can show that the principal receive higher expected payoff by using scoring rule auction than using minimum quality. The basic intuition is, under scoring rule, the principal can better suppress the technology advantage of the efficient firm.

**Theorem** [16] Given the same manipulation power $m$, cost structure and collusion relation structure, the principal’s payoff from the procurement auction is higher when she uses optimal scoring rule than optimal minimum quality.

---

[16] There is a short proof. When $\alpha = 1$, then $q_2(\alpha = 1) = \arg\max_q q - c_2(q) = q_H$, then $V_H(\alpha = 1) = q_2(\alpha = 1) - c_2(q_2(\alpha = 1)) = U_H(q_H)$. Because $\alpha_H < 1$, $V_H(\alpha_H) > V_H(\alpha = 1) = U_H(q_H)$, scoring rule dominates.
Proof of Theorem 7:

Recall (2) and (6), with honest auctioneer, under scoring rule

\[ V_H(\alpha_H) = (1 - \alpha_H)q_1(\alpha_H) + \alpha q_2(\alpha_H) - c_2(\alpha_H). \]

Under minimum quality

\[ U_H(q_H) = q_H - c_2(q_H). \]

To compare these two mechanism, we transfer scoring auction payoffs \( V \) from function of \( \alpha \) to a function of \( q \), the actual procurement quality. If firm 1 wins, his quality is optimal under weight \( \alpha \) is \( q = q_1(\alpha) \). Because \( q_1(\alpha) \) in monotonically increasing, can define the inverse function as \( \alpha(q) = q_1^{-1}(q) \). By Lemma 2 Note \( \frac{d\alpha(q)}{dq} > 0 \) and \( q_2(\alpha) < q_1(\alpha) \), Define \( q_2(\alpha(q)) \equiv q_2(q) \) and we have \( q_2(q) < q \).

By definition of \( q_2(\alpha) \), it satisfies 2's FOC

\[ \alpha(q) = c'_2(q_2(q)) \]

Then \( \frac{d\alpha(q)}{dq} = c''_2(q_2(q)) \frac{dq_2(q)}{dq} \), \( \frac{dq_2(q)}{dq} > 0 \).

Principal’s payoff

\[ V_H(\alpha(q)) \overset{d}{=} V_H(q) = q - c_1(q) - \Delta(\alpha(q)) \]

\[ U_H(q) = q - c_2(q) = q - c_1(q) - [c_2(q) - c_1(q)] \]

2nd order Taylor expansion of \( c_2(q) \) at \( q_2(q) \)

\[ c_2(q) = c_2(q_2(q)) + c'_2(q_2(q)) [q - q_2(q)] + \frac{1}{2} c''_2(q_2(q)) [q - q_2(q)]^2 \]

where \( \tilde{q} \) is between \( q \) and \( q_2(q) \).

\[ \Rightarrow c_2(q) - c_2(q_2(q)) = c'_2(q_2(q)) [q - q_2(q)] + \frac{1}{2} c''_2(q_2(q)) [q - q_2(q)]^2 \]

Technology advantage of firm 1 under scoring rule and minimum quality

\[ \Delta^S(\alpha(q)) \overset{d}{=} \Delta^S(q) = \alpha(q)[q - q_2(q)] + c_2(q_2(q)) - c_1(q) \]

\[ \Delta^M(q) = c_2(q) - c_1(q) \]

\[ \Delta^S(q) - \Delta^M(q) = \alpha(q)[q - q_2(q)] + c_2(q_2(q)) - c_2(q) \]

\[ = -\frac{1}{2} c''_2(q_2(q)) [q - q_2(q)]^2 < 0 \]

So, for any induced quality \( q > 0 \), we have the relation of technology advantage as \( \Delta^S(q) < \Delta^M(q) \), so \( V_H(q) > U_H(q) \).

Q.E.D.
**Theorem 8**: Principal’s payoff is higher when she uses optimal scoring rule than optimal minimum quality when outcome A (firm 1 wins) occurs.

**(Theorem 8 is not finished because the occurrence of outcome B haven’t been characterized by parameter value)**

Case 1, firm 1 is corrupted
Following the notation above (**)

\[
V(q) = q - \alpha(q)(q - q_2(q)) - c_2(q_2(q)) - \alpha m
\]

\[
U(q) = q - c_2(q) - m
\]

\[
V(q) - U(q) = -\alpha(q)(q - q_2(q)) - c_2(q_2(q)) - \alpha m + c_2(q) + m
\]

Hence \(V(q) > U(q)\). So when firm 1 is corrupted, scoring rule always dominates minimum quality.

Case 2, firm 2 is corrupted, but \(m\) is small, firm 1 wins in equilibrium.

\[
m \leq \min\{\hat{m}_S, \hat{m}_M\}
\]

\[
V(q) = q - \alpha(q)(q - q_2(q)) - c_2(q_2(q)) + \alpha m
\]

\[
U_H(q) = q - c_2(q - m)
\]

\[
V(q) - U(q) = -\alpha(q)(q - q_2(q)) + c_2(q - m) - c_2(q_2(q)) + \alpha m
\]

\[
\frac{\partial g(q,m)}{\partial q} = -\alpha'(q)[q - q_2(q)] - \alpha(q) + \alpha(q)q'_2(q) + c'_2(q - m) - c'_2(q_2(q))q'_2(q) + m\alpha'(q)
\]

\[
= -\alpha'(q)[q - q_2(q) - m] - \alpha(q) + c'_2(q - m) + \left[\alpha(q) - c'_2(q_2(q))\right]q'_2(q)
\]

\[
= -\alpha'(q)[q - q_2(q) - m] - \alpha(q) + c'_2(q - m)
\]

Define \(q_0\) as when \(q_0 - m = q_2(q_0)\).
When \(q = q_0\), \(c'_2(q - m) = c'_2(q_2(q)) = \alpha(q)\), then both \(g(q,m) = 0\) and \(\frac{\partial g(q,m)}{\partial q} = 0\).

\[
\frac{\partial^2 g(q,m)}{\partial q^2} = -\alpha''(q)[q - q_2(q) - m] + \alpha'(q)q'_2(q) + c''_2(q - m)
\]
Case 3, firm 2 is corrupted, \( m \) is large, firm 2 win in equilibrium.

\[
m \geq \max \{ \hat{m}_S, \hat{m}_M \}
\]

Induce a \( q \) to let firm 2 wins.

\[
q = q_2(\alpha) \\
\alpha(q) = q_2^{-1}(q) \\
q_1(q) > q
\]

\[
V(q) \overset{d}{=} V(\alpha(q)) = q - c_2(q) + \Delta(q) - \alpha m \\
= q - c_2(q) + \alpha q_1(q) - \alpha q - c_1(q_1(q)) + c_2(q) - \alpha m \\
= q - \alpha q + \alpha q_1(q) - c_1(q_1(q)) - \alpha m \\
U(q) = q - m - c_1(q)
\]

\[
V(q) - U(q) = \alpha q_1(q) - \alpha q - c_1(q_1(q)) + c_1(q) - \alpha m + m \\
= \alpha[q_1(q) - q] + [c_1(q) - c_1(q_1(q))] + (1 - \alpha)m
\]

Also compare efficiency and probability of corruption happening?

Compare the possibility that corruption advantage dominates technology advantage \( \hat{m}_S \) and \( \hat{m}_M \)

\[
\alpha m = \alpha(q_1 - q_2) + (c_1(q_1) - c_2(q_2)) \\
c_1(q) = c_2(q - m)
\]

### 2.3.2 “Optimal” corruption level

When \( p < \frac{1}{2} \), that is the efficient firm has lower probability to win, the principal’s payoff is maximized at some \( m > 0 \). In both scoring rule and minimum quality, we can show that

\[
\left. \frac{\partial (V(\alpha_A(m)))}{\partial m} \right|_{m=0} = \cdots > 0.
\]

\[
\left. \frac{\partial (U(q_A(m)))}{\partial m} \right|_{m=0} = \cdots > 0.
\]

Intuitively, without corruption, firm 1 can reap all its technological advantage as rent. But with a positive amount of corruption, the principal will design the mechanism so that because corruption of the other firm has To the principal, firm 2 serve as the role of phony bidder.

\[ \arg \max_{m} V(m), \ m^* > 0. \]

A positive amount of bribery can increase principal’s payoffs.

When \( m \) is too large, \( m \geq \hat{m}_S \), then there is efficiency loss. Firm 2 wins in outcome B.
2.3.3 Other comparison

**Not finished

Comparison of efficiency

Compare the probability of the efficient firm wins the contract.

Comparison of frequency of corruption

Compare the probability that the collusive firm wins the contract (Outcome B happens)

Besides principal’s payoff, the efficiency and probability of corruption happening is also of interest.

\[(1 - p)(1 - F(m))\]

Compare the possibility of having firm 2 wins.

That is compare \(\hat{m}_S\) and \(\hat{m}_M\), when \(m\) is above these threshold, \(V_B(\alpha_B)\) and \(U_B(q_B)\) is induced and firm 2 has positive probability to win the contract.

If \(\hat{m}_S > \hat{m}_M\), it means minimum quality, more likely firm 2 can win.

Very hard to compare

3 Extensions

**Not finished

3.1 Incomplete information on Collusion Relation

A desirable model of collusion shall consider the case that both the cost structure and collusion relation are private information. However, setting up an economic model with both cost and collusion relation as private information is very difficult. In the literature, the explicit assumption on incomplete information of collusion relation is usually avoid. Bajari and Ye (2003) assume collusion relation is common knowledge (the identity of firms involve in the bidding ring is known to all bidders), Aryal and Gabrielli (2013) and Tian et al. (2008) assume bidders outside the bidding ring is completely unaware of the ring, so they follow their equilibrium strategy without collusion. Other literature (?) assume bidders outside the ring believe there is collusion with a fixed probability.

In one of the extension, we analyze two cases under incomplete information. In the first case, we give an explicit structure of incomplete information on collusion relation and keep cost as common knowledge. In the second case, we allow incomplete information on cost while keeping non-collusive bidders follow their equilibrium strategy without collusion.

In the main model, we assume complete information on both collusion relation and cost structure. Modeling will be very difficult if we allow two of them become incomplete information. So as two extension, we allow collusion relation to be incomplete information first in this section. Then in next section, we study the problem of incomplete information on cost.

It is not trivial to spell out the information structure of each agent’s knowledge on the collusion relation. In modeling bidding ring, to our knowledge, there is no paper explic-
itly models the incomplete information structure of collusion relation. Literature take two paths circumvent the problem. The first one is assuming common knowledge on collusion relation, like [Krishna (2009) and Bajari and Ye (2003)]. The other is assuming that collusion is completely unaware to non-collusive bidders, like [Aryal and Gabrielli (2013) and Tian et al. (2008)]. Taking a step forward, we explicitly model the information on collusion relation.

Model setup

The procurement auction game is the same as the counterpart in section 2. There are two firms. Firm 1 is efficient and firm 2 is inefficient. Cost structure are common knowledge among firms. Assume that there is at most one collusive firm. The auctioneer and the collusive firm of course knows their relation, but the principal and non-collusive firms do not know it for sure. Let $(1, 0), (0, 1)$ and $(0, 0)$ denote three state in state space of collusion relation, the information structure is described in the following table

<table>
<thead>
<tr>
<th>state indicator $(t_1, t_2)$</th>
<th>probability of being collusive</th>
<th>firm 1’s belief $p_1$</th>
<th>firm 2’s belief $p_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1 is collusive $(1, 0)$</td>
<td>$p_1$</td>
<td>$1$</td>
<td>$\frac{p_1}{1-p_2}$</td>
</tr>
<tr>
<td>Firm 2 is collusive $(0, 1)$</td>
<td>$p_2$</td>
<td>$\frac{p_2}{1-p_1}$</td>
<td>$1$</td>
</tr>
<tr>
<td>No collusion $(0, 0)$</td>
<td>$1-p_1 - p_2$</td>
<td>$h \equiv \frac{1-p_1-p_2}{1-p_1}$</td>
<td>$\frac{1-p_1-p_2}{1-p_2}$</td>
</tr>
</tbody>
</table>

We restrict $p_1 + p_2 \in (0, 1)$. When $p_1 + p_2 = 0$ and $p_1 + p_2 = 1$, the model reduced to the without corruption and with corruption counterparts in section 2.

Equilibrium

If firm 1 is non-collusive, its belief on the probability of firm 2 also non-collusive is denoted as $h \equiv \frac{1-p_1-p_2}{1-p_1}$. Given $m$, we have the following theorems of the equilibrium of procurement auction:

**Theorem 9:** Under scoring rule $S(q, p) = \alpha q - p$, equilibrium quality follows $q_i = \arg \max_q \alpha q - c(q, \theta_i), i = 1, 2$. The equilibrium outcome and prices are where $f_S(p_1)$ is some density function with support $[c_1(\alpha) + h\Delta_S(\alpha), c_1(\alpha) + \Delta_S(\alpha)]$ and $g_S(p_2)$ is some density function with support $[c_2(\alpha) - (1-h)\Delta_S(\alpha) + \alpha m, c_2(\alpha) + \alpha m]$.  

<table>
<thead>
<tr>
<th>outcome</th>
<th>price</th>
<th>state</th>
</tr>
</thead>
<tbody>
<tr>
<td>firm 1 wins</td>
<td>$p_1(\alpha) = c_1(\alpha) + \Delta_S - \alpha m$</td>
<td>firm 1 collusive</td>
</tr>
<tr>
<td></td>
<td>$p_1(\alpha) = c_1(\alpha) + \Delta_S(\alpha) + \alpha m$</td>
<td>firm 1 not collusive</td>
</tr>
<tr>
<td></td>
<td>$p_2(\alpha) = c_2(\alpha)$</td>
<td>firm 2 collusive</td>
</tr>
<tr>
<td></td>
<td>$p_2(\alpha) = c_2(\alpha)$</td>
<td>firm 2 not collusive</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>outcome</th>
<th>price</th>
<th>state</th>
</tr>
</thead>
<tbody>
<tr>
<td>firm 1 wins</td>
<td>$p_1(\alpha) = c_1(\alpha) + \Delta_S(\alpha) + \alpha m$</td>
<td>firm 1 collusive</td>
</tr>
<tr>
<td></td>
<td>$p_1(\alpha) \sim f_S(p_1)$</td>
<td>firm 1 not collusive</td>
</tr>
<tr>
<td></td>
<td>$p_2(\alpha) \sim g_S(p_2)$</td>
<td>firm 2 collusive</td>
</tr>
<tr>
<td>firm 1 wins</td>
<td>$p_2(\alpha) = c_2(\alpha)$</td>
<td>firm 2 not collusive</td>
</tr>
<tr>
<td>( c_2(q - m) \geq c_1(q) + h\Delta_M(q) )</td>
<td>outcome</td>
<td>price</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>firm 1 wins</td>
<td>( p_1(q) = c_2(q) )</td>
<td>firm 1 collusive</td>
</tr>
<tr>
<td></td>
<td>( p_1(q) = c_2(q - m) )</td>
<td>firm 1 not collusive</td>
</tr>
<tr>
<td></td>
<td>( p_2(q) = c_2(q - m) )</td>
<td>firm 2 collusive</td>
</tr>
<tr>
<td></td>
<td>( p_2(q) = c_2(q) )</td>
<td>firm 2 not collusive</td>
</tr>
<tr>
<td>( c_2(q - m) &lt; c_1(q) + h\Delta_M(q) )</td>
<td>firm 1 wins</td>
<td>( p_1(q) = c_2(q) )</td>
</tr>
<tr>
<td></td>
<td>?</td>
<td>( p_1(q) \sim f_M(p_1) )</td>
</tr>
<tr>
<td></td>
<td>?</td>
<td>( p_2(q) \sim g_M(p_2) )</td>
</tr>
<tr>
<td></td>
<td>firm 1 wins</td>
<td>( p_2(q) = c_2(q) )</td>
</tr>
</tbody>
</table>

**Proof of Theorem 9:**

*Q.E.D.*

**Comments**

**Theorem 10:** Under minimum quality \( q \), equilibrium quality follows \( q_i = \text{arg max}_q \alpha q - c(q, \theta_i), \ i = 1, 2 \). The equilibrium outcome and prices are where \( f_M \) and \( g_M \) are some density functions with property that

\[
f_M(p_1) = \begin{cases} 
1 - \frac{1}{p_1 - c_2(q-m)} & \text{if } (1-h)c_1(q) + hc_2(q) \leq p_1 < c_2(q) \\
1 & \text{if } p_1 = c_2(q)
\end{cases},
\]

\[
g_M(p_2) = \begin{cases} 
1 + h - \frac{1}{p_2 - c_1(q)} & \text{if } (1-h)c_1(q) + hc_2(q) \leq p_1 < c_2(q) \\
1 & \text{if } p_1 = c_2(q)
\end{cases}.
\]

**Proof of Theorem 10:**

*Q.E.D.*

**Comments**

Mixed strategy

Compare to complete information

Comparing scoring rule and minimum quality

3.2 Incomplete information on cost structure

3.3 Many Firms

Go back to the assumption with complete information on both cost and corruption relation

2 firms case to \( n \) firms case

General prediction: attracting more non-collusive firm is beneficial to the principal
3.4 Discussion on auctioneer and collusion

3.4.1 Discussion on the auctioneer

Auctioneer’s incentive to introduce a “not going to win” collusive firm 2. What if auctioneer is making scoring rule.

3.4.2 Endogenous manipulation power and bribery

Bribery payment \( y \leq \pi_C - \pi_H \cdot y(m) \)

3.5 Empirical work

4 Conclusion

Empirical work following

Explain exclusion phenomenon. The entry threshold of large procurement auction is usually set high. Nearly all procurement auction require bidders have certain certificates or reaching certain standards. Typically, this kind of document is long and complex. As a result, there won’t be many bidders in a procurement auction. It seems the auctioneer don’t want to maximize entry and competition - as auction theory suggested. The procurement law usually requires a minimum number of bidders. (In China, this minimum number is 3 for public sector projects)

Appendix

Appendix A - Proofs

Appendix B - Numerical Example

Setup

Cost function \( c(q, \theta) = \theta q^\gamma \). Take efficiency parameter \( \theta_2 = \theta > \theta_1 = 1 \).

\( c_1 = q^\gamma, \quad c_2 = \theta q^\gamma, \quad \gamma > 2 \)

Quality choice under scoring rule auction

References


