Barometric Price Leadership

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Abstract

A dynamic Bertrand-duopoly model in which a firm leads price changes while its competitor always matches in equilibrium is developed. The firms produce a homogeneous product and are identical except for the information they possess. The market price follows a Markov process. One firm always knows the demand while the other only knows its distribution. Under some conditions, leadership allows firms to increase joint profits. A new feature is that sequential pricing is not needed for a firm to behave as a leader in equilibrium.

1 Introduction

Price leadership has kept the attention of both economists and regulators for many years because of its prevalence in oligopolistic behavior. When describing the nature of price leadership, Stigler (1947) and Markham (1951) classified cases into **dominant firm price leadership** and **barometric price leadership**. According to them, dominant firm price leadership occurs when the largest firm in the industry leads price changes as a consequence of the industry structure. In contrast, barometric price leadership occurs only because of information asymmetries. As Stigler states, the barometric price leader *"commands adherence of rivals to his price only because, and to the extent that, his price reflects market conditions with tolerable promptness"*.[1] Our goal is to develop a new model of barometric price leadership.

For clarity, we define a dominant firm price leadership model as one in which a price leader emerges as a result of differences in firms' characteristics that are unrelated to information, whereas we define a barometric price leadership model as one in which the firms differ only in information.[2]

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[1] Stigler (1947), p. 446.

[2] For example, dominant firm price leadership models rely on the existence of a competitive fringe of firms (Ono, 1982, D'Aspremont et al., 1983), different capacity constraints (Deneckere and Kovenock, 1992) or technological differences (Yano and Komatsubara, 2006, 2012).

Previous models have identified information asymmetries as a cause for a leader to arise. Cooper (1997) works with a static price-setting duopoly with differentiated goods. At the beginning the firms are uncertain about the market size but can purchase all the relevant information about demand at a positive cost without the competitor observing. Then, there is a finite number of price-posting periods before competition. Once a firm posts its price it cannot be changed, and delaying the posting costs a firm an infinitesimal amount of sales. In equilibrium, only one firm purchases the information and posts a price immediately, while the other firm waits to set its price after the other posts.

Repeated games that capture the essence of barometric price leadership were developed with a common feature - that at each stage game firms would be allowed to set prices sequentially before competition occurs (Rotemberg and Saloner, 1990, Mouraviev and Rey, 2011). One of the implications of this setting is that if we were to observe the sequence of prices and sales, prices would be the same at each period. Also, the fact that both firms set prices sequentially before competition can be interpreted in at least two ways that are problematic with antitrust authorities. First, we can think it as firms communicating their price intentions before revealing them to consumers, which is a practice that is illegal under the Sherman Act. Second, we can understand it as firms making price announcements that are not effective immediately. But the fact that some oligopolies, like vitamins (Marshall et al., 2008), have relied on not-immediately-effective price announcements to achieve supracompetitive profits may lead to suspicions of collusion by both authorities and consumers. Nevertheless, sequential pricing has been seen as necessary for price leadership to be an equilibrium. For example, Mouraviev and Rey (2011) state that *"most of the literature on collusion assumes that firms set their prices or quantities simultaneously, thus excluding the possible emergence of a leader"*.[3]

Our purpose is to develop a model of barometric price leadership in which firms set prices simultaneously and use leadership as a way to increase joint profits. Our model will introduce information asymmetries into a two firm and two state version of Kandori (1991). Overt communication is not allowed. At each period, firms compete in a homogeneous product Bertrand market and are identical except for the fact that one knows the market size while the other does not. In Rotemberg and Saloner (1990) and Mouraviev and Rey (2011), the market size is *i.i.d* across periods, whereas in our model the market size follows a Markov process; therefore the price that the informed firm sets today may provide information about the market size tomorrow. Then, we define price leadership as a sequence of prices in which the uninformed firm always matches the informed firm's previous price, while the informed firm sets different prices for different states. We show that there are prices that support price leadership with stage Nash reversal as an equilibrium for any parameter specification as firms become patient. Moreover, the price leadership with monopolistic prices can be supported as a PBE if the value of the information is large enough.

[3] Mouraviev and Rey (2011), p. 705.

2 Model

Consider a market with two firms who costlessly produce a homogeneous output. At the beginning of every period of the infinitely repeated game, a state s is drawn from the set $\{s_l, s_h\}$ with $0 < s_l < s_h$. The state is persistent in the sense that it follows a Markov process with transition matrix

$$\begin{bmatrix} p_{ll} & p_{lh} \\ p_{hl} & p_{hh} \end{bmatrix} = \begin{bmatrix} 1-\epsilon & \epsilon \\ \epsilon & 1-\epsilon \end{bmatrix}$$

for some $\epsilon < \frac{1}{2}$. After the draw, at each period t, Firm 1 observes the realization of the state s^t while Firm 2 only knows its distribution. Then, firms simultaneously set prices p_1^t and p_2^t from the support $[0, \overline{p}]$ where $\overline{p} > s_h$. The quantity demanded is given by

$$\max\{s^t - \min\{p_1^t, p_2^t\}, 0\}.$$

If the two firms set the same price, the quantity demanded is evenly split. If the firms set different prices, the firm with the lowest price gets the whole demand. After competition, each firm observe both prices and only its own quantity. Note that while the informed firm possess private information, the uninformed firm does not. Therefore, for a set of prices $(p_I, p_U) \in [0, \overline{p}]^2$ given a state $s \in \{s_l, s_h\}$ firm i's stage game profits, π_i, are

$$u_i(p_i, p_j; s) = \begin{cases} \pi(p_i; s) & \text{if } p_i < p_j, j \neq i; \\ \frac{\pi(p_i;s)}{2} & \text{if } p_i = p_j, j \neq i; \\ 0 & \text{otherwise.} \end{cases}$$

where $\pi(p; s) = \max\{p(s-p), 0\}$.

Finally, the common discount factor is given by $\delta \in (0,1)$. Firm i's payoff from a sequence of prices $\{(p_I^t, p_U^t)\}_{t=0,1,\ldots}$ and a sequence of states $\{s^t\}_{t=0,1,\ldots}$ is

$$(1-\delta)\sum_{t=0}^{\infty} \delta^t u_i(p_I^t, p_U^t; s^t).$$

Note that irrespective of the realization of s, the only stage game Nash equilibrium prices are given by $p_1 = p_2 = 0$ and the unique stage Nash equilibrium payoffs are therefore zero.

Then, given a sequence of prices $\{(p_I^t, p_U^t)\}_{t=0,1,\ldots}$ and a sequence of

The set of period t public histories, $\mathcal{H}^t$, is given by the sequences of both prices and the uninformed firm's quantity. The uninformed firm has no private information. The informed firm period t private history, $\mathcal{H}^t$, contains on top of the public history the sequence of its own quantity and state realizations up to period t and the state realization at period t. That is,

$$\mathcal{H}^t = \left[[0, \overline{p}]^2 \times [0, s_h]\right]^t$$

and

$$\mathcal{H}_I^t = \left[[0, \overline{p}]^2 \times [0, s_h]^2 \times \{s_l, s_h\}\right]^{t+1}.$$

The set of all possible public histories is given by

$$\mathcal{H} = \bigcup_{t=0}^{\infty} \mathcal{H}^t$$

and the set of all possible informed firm private histories is given by

$$\mathcal{H}_I = \bigcup_{t=0}^{\infty} \mathcal{H}_I^t.$$

Then, a pure strategy for the uninformed firm is a mapping

$$P_U : \mathcal{H} \to [0, \overline{p}].$$

A pure strategy for the informed firm is given by

$$P_I : \mathcal{H}_I \to [0, \overline{p}].$$

Also, the myopic monopoly price is given by $p_l = \frac{s_l}{2}$ when the state is sl and $p_h = \frac{s_h}{2}$ when the state is s_h. The myopic monopoly profits are given by $\pi_h = \frac{s_h^2}{4}$ when the state is s_h and $\pi_l = \frac{s_l^2}{4}$ when the state is s_l.

2.1 Price Leadership

We can observe that strategies can be extremely complicated objects. Now we will introduce a simple strategy profile of pricing rules that will be denoted as price leadership, or (PL) now on.

Definition 1 (Price Leadership)**.** At any period $t \geq 1$,

I. the informed firm sets a price p_I^t equal to p_l if the state is s_l and equal to p_h if the state is p_h with $0 \geq p_l < p_h$;

U. the uninformed firm always sets a price p_U^t that matches the informed firm's previous price, that is, $p_U^t = p_I^{t-1}$.

If a firm detects a deviation from the previous rule in the past, then sets a price equal to 0 forever.

2.2 Payoffs from following Price Leadership

Lets proceed to analyze the payoff that each of the firms obtain from following price leadership given that the other firm also follows. Define V_s^U for $s \in \{l, h\}$ as the expected discounted payoff that the uninformed firm gets from following price leadership and given that the informed firm set a price p_s in the previous period and is also following price leadership. Then,

- if the informed firm set a price p_l in the previous period and is following price leadership, then it must be the case that the market size was s_l in the previous period. If the uninformed firm follows price leadership it will set a price p_l and with probability $(1-\epsilon)$ the state will be s_l again which will lead to share monopoly profits today and get V_l^U tomorrow; but with probability ϵ the state today is s_h but then the uninformed will get the whole market today, obtaining $\pi(p_l; s_h)$ plus V_h^U tomorrow. That is,

$$V_l^U = (1-\delta)\left[(1-\epsilon)\frac{\pi(p_l; s_l)}{2} + \epsilon\pi(p_l; s_h)\right] + \delta\left[(1-\epsilon)V_l^U + \epsilon V_h^U\right]$$

- the informed firm set a price p_h in the previous period, the expected discounted payoff for the uninformed firm is given by

$$V_h^U = (1-\delta)\left[(1-\epsilon)\frac{\pi(p_h; s_h)}{2}\right] + \delta\left[\epsilon V_l^U + (1-\epsilon)V_h^U\right].$$

The informed firm possess more information than the uninformed at the time the prices are set, so we will defined the informed firm's intertemporal utility accordingly. Let $V_{ss'}^I$ with $s, s' \in \{l, h\}$ be the informed firm's expected discounted payoff starting today provided that today's state is s' and yesterday's was s. Then,

- when the market size went from low to low, the informed firm knows that the uninformed is going to set p_l as its price since both are following (PL) and the state yesterday was s_l. Therefore, since the state today is also s_l the informed firm is supposed to also set a price equal to p_l and get a payoff equal to V_{ll}^I with certainty today. With probability $(1-\epsilon)$ the state tomorrow will be s_l so the informed firm will get a discounted payoff equal to V_{ll}^I and with probability ϵ the state changes and the discounted payoff is equal to V_{lh}^I. That is,

$$V_{ll}^I = (1-\delta)\frac{\pi(p_l; s_l)}{2} + \delta\left[(1-\epsilon)V_{ll}^I + \epsilon V_{lh}^I\right]$$

- when the demand went from low to high, the expected discounted payoff for the informed firm is

$$V_{lh}^I = \delta\left[\epsilon V_{hl}^I + (1-\epsilon)V_{hh}^I\right]$$

- when the demand went from high to low, the expected discounted payoff for the informed firm is

$$V_{hl}^I = (1-\delta)\pi(p_l; s_l) + \delta\left[(1-\epsilon)V_{ll}^I + \epsilon V_{lh}^I\right]$$

- when the demand stayed high, the expected discounted payoff for the informed firm is

$$V_{hh}^I = (1-\delta)\frac{\pi(p_h; s_h)}{2} + \delta\left[\epsilon V_{hl}^I + (1-\epsilon)\epsilon V_{hh}^I\right].$$

3 Results

Throughout this section, a result that will be helpful is the first deviation principle.

Proposition 1. *In our setting the* ***first deviation principle*** *holds.*

The previous proposition is not that surprising since every deviation from the uninformed firm is detected and from the perspective of the uninformed this is a dynamic Markov game with imperfect monitoring. So using it, we will start by stating all the potentially profitable deviations from (PL) and deriving the incentive constraints that would prevent them in the next subsection.

3.1 Deviations from Price Leadership

- Lets start by analyzing the potential deviations by the uninformed firm.
  - I the informed firm previous price was p_l, the uninformed firm can deviate to
    - charge a slightly lower price than p_l and get an expected payoff arbitrarily close to $(1-\delta)[(1-\epsilon)\pi(p_l;s_l)+\epsilon\pi(p_l;s_h)]$. Then, for that deviation not to be profitable we would require that

$$V_l^U \geq (1-\delta)[(1-\epsilon)\pi(p_l;s_l)+\epsilon\pi(p_l;s_h)]. \tag{IC1}$$

    - charge a slightly lower price than p_h and get an expected payoff arbitrarily close to $(1-\delta)\epsilon\pi(p_h;s_h)$. That deviation is not profitable as long as,

$$V_l^U \geq (1-\delta)\epsilon\pi(p_h;s_h). \tag{IC2}$$

  - If the informed firm previous price was p_h, the uninformed firm can deviate to
    - charge a slightly lower price than p_l and get an expected payoff arbitrarily close to $(1-\delta)[(1-\epsilon)\pi(p_l;s_l)+\epsilon\pi(p_l;s_h)]$. That deviation is not profitable as long as,

$$V_h^U \geq (1-\delta)[(1-\epsilon)\pi(p_l;s_l)+\epsilon\pi(p_l;s_h)]. \tag{IC3}$$

    - charge a slightly lower price than p_h and get an expected payoff arbitrarily close to $(1-\delta)(1-\epsilon)\pi(p_h;s_h)$. That deviation is not profitable if

$$V_h^U \geq (1-\delta)(1-\epsilon)\pi(p_h;s_h). \tag{IC4}$$

- Informed firm.
  - The demand goes from s_l to s_l. In this case, the only potentially profitable deviation is for the informed firm is to charge a price slightly below p_l and get an expeceted payoff very close to $(1-\delta)\pi(p_l;s_l)$.

$$V_{ll}^I \geq (1-\delta)\pi(p_l;s_l) \tag{IC5}$$

- The demand goes from s_l to s_h. In this case, the only potentially profitable deviation is for the informed firm is to charge a price slightly below p_l and get an expeceted payoff very close to $(1-\delta)\pi(p_l; s_h)$.

$$V_{lh}^I \geq (1-\delta)\pi(p_l; s_h) \tag{IC6}$$

- The demand goes from s_h to s_l. In this case, the informed firm is obtaining the whole informed monopoly profits so there is no potential profitable deviation.
- The demand goes from s_h to s_h. In this case, there are two potential profitable deviations for the informed firm,
  * it can charge a price slightly below p_h and get an expeceted payoff very close to $(1-\delta)\pi(p_h; s_h)$.
  $$V_{hh}^I \geq (1-\delta)\pi(p_h; s_h) \tag{IC7}$$
  * it can charge a price p_l without being detected today. The informed firm will never be detected if in the next period the demand state is s_l otherwise he will end up being detected and therefore punished. Then, tomorrow with probability ϵ, the demand will be s_l and assuming it continues playing on equilirbium then the informed firm obtains a discounted payoff of V_{ll}^I. Otherwise, with probability $(1-\epsilon)$, the state tomorrow will be s_h and the firm will not be detected only if it sets a price p_h and therefore obtaining a expected discounted one period payoff of V_{lh}^I. Then, the expected payoff of the previous deviation is
  $$(1-\delta)\pi(p_l; s_h) + \delta[\epsilon V_{ll}^I + (1-\epsilon)V_{lh}^I]$$
  Then, the informed firm is better by following PL than deviating in the previous way if
  $$V_{hh}^I \geq (1-\delta)\pi(p_l; s_h) + \delta[\epsilon V_{ll}^I + (1-\epsilon)V_{lh}^I] \tag{IC8}$$

Lemma 1. (IC8) *holds if and only if*

$$[1+\delta(1-\epsilon)]\pi(p_h; s_h) - 2\pi(p_l; s_h) + \delta\epsilon\pi(p_l; s_l) \geq 0. \tag{IC8'}$$

3.2 Price Leadership as a PBE

Now, we will try to sustain price leadership as a PBE for patient firms. First note that, as long as in each state prices are positive, the incentive constraints (IC1)-(IC7) hold if the firms are patient enough since the respective deviations are always detected and therefore lead to a Nash reversal. As a consequence, when considering arbitrarily patient firms we need only to worry about (IC8').

So is natural to start by asking under which conditions can price leadership with the monopolistic prices can be sustained as a PBE. The following inequality will guarantee that there exists a δ such that this is the case.

$$\frac{s_l}{s_h} < \frac{2-\epsilon}{2+\epsilon} \tag{$\star$}$$

The previous inequality can be understood as the value of the information that the informed firm posses being high. For example, as the shock is less persistent (ϵ goes to $1/2$) the inequality becomes more binding. Similarly, if the gain from adjusting the price is very low (s_l/s_h close to 1) the inequality becomes more binding.

Proposition 2. *If condition ($\star$) holds, there exists $\underline{\delta}$ such that for $\delta > \underline{\delta}$, price leadership with monopolistic prices is a PBE.*

In the next proposition, we argue that even when monopolistic prices are not sustainable in a price leadership equilibrium there are prices that can be sustained as the firms become more patient. Looking back at (IC8'), we can see that the price that the firms set in the low market size state must be lowered to reduce the incentives of the informed firm to lie about the state when going from high to high market size.

Proposition 3. *If condition ($\star$) does not hold, there exists a price p_l with $0 < p_l < \frac{s_l}{2}$, such that price leadership with prices p_l and $p_h = \frac{s_h}{2}$ is a PBE if firms are patient enough.*

Then, price leadership can be sustained as an equilibrium for any specification of the parameters as long as firms are patient. Also note that the last result does not necessarily depend on firms being patient because for any $\delta > 0$ prices can be chosen in a way that all the incentive constraints hold.

3.3 Efficiency

It is fair to question how well does price leadership does in terms of joint profits. We know that the most efficient outcome is achieved by an informed monopolist. That firm will always charge the price $p_l = s_l/2$ whenever the state is s_l and a price $p_h = s_h/2$ whenever the state is s_h. So let V_l^M and V_h^M denote the expected payoff of a monopolist given that it does not know the realization of today's state given that the previous state was s_l and s_h respectively. Then,

$$V_l^M = \frac{(1-\epsilon)s_l^2 + \epsilon s_h^2}{4} + \delta[(1-\epsilon)V_l^M + \epsilon V_h^M]$$

and

$$V_h^M = \frac{\epsilon s_l^2 + (1-\epsilon)s_h^2}{4} + \delta[\epsilon V_l^M + (1-\epsilon)V_h^M].$$

Now we will consider if in our setting the firms joint profits can be equal to informed monopolist profits in equilibrium. Note that if there is such an equilibrium the informed always sets the informed monopolists price and the uninformed firm cannot set a price smaller than p_h^M at any period since s_l occurs with positive probability. Also, any equilibrium in which the uninformed always charges a price above p_h^M is joint-profit-equivalent to and more difficult to sustain than to one in which it always charges p_h^M. Then, we obtain the next result.

Proposition 4. *If $\frac{s_l}{s_h} <\leq \frac{4-\sqrt{8}}{4} \approx 0.2929$, firms can achieved joint profits equal to the informed monopolist profits in equilibrium.*

The condition in the previous proposition also guarantees that price leadership with monopolistic price is an equilibrium. In contrast, price leadership with monopolistic prices is an equilibrium in situations in which joint profit efficiency cannot be achieved.

A Proofs

Lemma 1.

$$\begin{aligned} V^I_{hh} \geq& (1-\delta)\pi(p_l;s_h) + \delta[\epsilon V^I_{ll} + (1-\epsilon)V^I_{lh}] \\ (1-\delta)\frac{\pi(p_h;s_h)}{2} + \delta[\epsilon V^I_{hl} + (1-\epsilon)V^I_{hh}] \geq& (1-\delta)\pi(p_l;s_h) + \delta[\epsilon V^I_{ll} + (1-\epsilon)V^I_{lh}] \\ (1-\delta)\left[\frac{\pi(p_h;s_h)}{2} - \pi(p_l;s_h)\right] \geq& \delta\left\{\epsilon\left[V^I_{ll} - V^I_{hl}\right] + (1-\epsilon)\left[V^I_{lh} - V^I_{hh}\right]\right\} \\ (1-\delta)\left[\frac{\pi(p_h;s_h)}{2} - \pi(p_l;s_h)\right] \geq& -\delta(1-\delta)\left[\frac{\epsilon\pi(p_l;s_l) + (1-\epsilon)\pi(p_h;s_h)}{2}\right] \end{aligned}$$

Then, this is equivalent to requiring that,

$$[1+\delta(1-\epsilon)]\pi(p_h;s_h) - 2\pi(p_l;s_h) + \delta\epsilon\pi(p_l;s_l) \geq 0.$$

□

Proposition 2. First note that the incentive constraints (IC1)-(IC8) reflect situations in which following PL payoff dominates detectable deviations. Since these deviations are always detected - and lead to a stage Nash repetition instead of a stream of positive payoff at each period - there exists a $\overline{\delta}_1 < 1$ such that for any $\delta \in (\overline{\delta}_1, 1)$, the incentive constraints (IC1)-(IC8) hold.

Now, it remains to show that the incentive constraint (IC9) holds if firms are patient enough. The proof will consist of the following steps,

1. When $p_l = s_l/2$ and $p_h = s_h/2$, (IC8') is equivalent to

$$[1+\delta(1-\epsilon)] - 4k + (2+\delta\epsilon)k^2 \geq 0$$

where $k = \frac{s_l}{s_h}$.

2. The LHS is strictly positive for $\delta = 1$ whenever ($\star$) holds.

3. Since LHS is continuous and increasing on δ there exists a $\overline{\delta}_9 < 1$ such that for any $\delta \in (\overline{\delta}_9, 1)$, the LHS is positive.

1. Take (IC8') with the monopolist prices,

$$[1+\delta(1-\epsilon)]\pi(s_h/2;s_h) - 2\pi(s_l/2;s_h) + \delta\epsilon\pi(s_l/2;s_l) \geq 0.$$

   Just plugging the profits,

$$[1+\delta(1-\epsilon)]\frac{s_h^2}{4} - s_l\left(s_h - \frac{s_l}{2}\right) + \delta\epsilon\frac{s_l^2}{4} \geq 0$$

   or

$$[1+\delta(1-\epsilon)]\frac{s_h^2}{4} - s_l s_h + (2+\delta\epsilon)\frac{s_l^2}{4} \geq 0.$$

   Then, multiplying by $\frac{4}{s_h^2}$,

$$[1+\delta(1-\epsilon)] - 4k + (2+\delta\epsilon)k^2 \geq 0$$

   where $k = \frac{s_l}{s_h}$.

2. Denote the left-hand-side as the function $f(\delta;k,\epsilon)$. This function is increasing on δ since

$$\frac{\partial f}{\partial \delta}(\delta;k,\epsilon) = (1-\epsilon) + \epsilon k^2 > 0$$

   and decreasing on k whenever ($\star$) holds since

$$\frac{\partial f}{\partial k}(\delta;k,\epsilon) = 2(2+\epsilon)k - 4 < 2(2+\epsilon)\left(\frac{2-\epsilon}{2+\epsilon}\right) - 4 = -2\epsilon < 0.$$

   Then, for any k and ϵ satisfying ($\star$),

$$\begin{aligned} f(1;k,\epsilon) &= (2-\epsilon) - 4k + (2+\epsilon)k^2 \\ &> (2-\epsilon) - 4\left(\frac{2-\epsilon}{2+\epsilon}\right) + (2+\epsilon)\left(\frac{2-\epsilon}{2+\epsilon}\right)^2 \\ &= \left(\frac{2-\epsilon}{2+\epsilon}\right)[(2+\epsilon) - 4 + (2-\epsilon)] \\ &> 0 \end{aligned}$$

   where the first inequality comes form ($\star$) and the fact that f is decreasing on k.

3. Since f is continuous - and increasing - on δ, there exists a $\overline{\delta}_2$ such that for any $\delta \in (\overline{\delta}_2, 1)$, $f(\delta;k,\epsilon) > 0$.

   Therefore, for any $\delta \in (\overline{\delta}_2, 1)$, (IC8) holds.

Letting $\overline{\delta} = \max\{\overline{\delta}_1, \overline{\delta}_2\}$, we have that for any for any $\delta \in (\overline{\delta}, 1)$, the incentive constraints (IC1)-(IC8) hold and therefore PL is a PBE. $\square$

Proposition 2. First, lets argue that given $p_h = p_h^M$ there exists $\underline{p}_l > 0$ such that for any $p_l' \in (0, \underline{p}_l)$ the incentive contraint holds for any $\delta \in (0,1)$. To show it, we will follow the next steps,

1. Given $p_h = \frac{s_h}{2}$, (IC8') becomes

$$[1 + \delta(1-\epsilon)]\frac{s_h^2}{2} - (2s_h - \delta\epsilon s_l)p_l + (2-\delta\epsilon)p_l^2 \geq 0$$

2. the left-hand-side of the previous inequality is decreasing on p_l,

1. Lets start with (IC8'),

$$[1 + \delta(1-\epsilon)]\pi(p_h; s_h) - 2\pi(p_l; s_l) + \delta\epsilon\pi(p_l; s_l) \geq 0$$

   and plug $p_h = s_h/2$,

$$[1 + \delta(1-\epsilon)]\frac{s_h^2}{4} - 2p_l(s_h - p_l) + \delta\epsilon p_l(s_l - p_l) \geq 0$$

   which is equivalent to

$$[1 + \delta(1-\epsilon)]\frac{s_h^2}{4} - [2s_h - \delta\epsilon s_l]p_l + (2-\delta\epsilon)p_l^2 \geq 0.$$

2. Denote the left-hand-side of the previous inequality as LHS and note that

$$\frac{\partial LHS}{\partial p_l} = 2(2-\delta\epsilon)p_l - (2s_h - \delta\epsilon s_l)$$

   but for $p_l' < \frac{s_l}{2}$

$$\frac{\partial LHS}{\partial p_l} < (2-\delta\epsilon)s_l - (2s_h - \delta\epsilon s_l) = 2(s_l - s_h) < 0.$$

   Also,

$$LHS(0) = [1 + \delta(1-\epsilon)]\frac{s_h}{2} > 0.$$

   Therefore, there must exist $\underline{p}_l > 0$ such that for any $p_l' \in (0, \underline{p}_l)$, $LHS(p_l') > 0$.

Now, fixing $p_l \in (0, \underline{p_l})$ and $p_h = p_h^M$, lets argue that if firms are patient enough the incentive constraints (IC1)-(IC8) hold. Note that deviations of the form described in incentives contraints (IC1)-(IC8) are always detected, therefore leading to a static Nash reversal. So, if firms are patient enough this deviations are not profitable. Then, it must exist a $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}, 1)$, (IC1)-(IC8) hold. And for any such a discount factor and prices, price leadership is a PBE. □

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