Fight or Surrender: Experimental Analysis of Last Stand Behavior

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Abstract

In a dynamic contest where it is costly to compete, a player on a losing trajectory must decide whether to surrender or to keep fighting in the face of bleak odds. We experimentally examine the prediction of last stand behavior in a multi-battle contest with a winning prize and losing penalty, as well as the contrasting prediction of surrendering in the corresponding contest with no penalty. As predicted, we find that players nearing defeat compete more fiercely when they face a large penalty, but that they taper their effort when losing is costless. This behavior impacts winning margins: neck-and-neck victories are more common when the penalty is relatively large, while landslide victories occur more frequently when the penalty is small. We find that the winner of the initial battle will typically also win the overall contest. Subjects with previous experience in related experiments tend to mirror theoretical predictions more closely than those without.

Keywords: Dynamic Contest, Multi-Battle Contest, Experiment, Last Stand, Winning Margin JEL: C73, C92, D44, D72, D74

1. Introduction

The notion of a last stand has become iconically linked with the tale of General George A. Custer—a celebrated Civil War hero who, in 1876, in the Territory of Montana, fought to the death alongside his men as they were heavily outnumbered by Northern Cheyenne and Lakota warriors. As in the case of General Custer, last stand behavior is fundamentally characterized as a strong defensive push in the face of bleak odds. Making such a push, however, is necessarily costly, which raises a question about the optimality of a last stand. When would a player on a losing trajectory be willing to incur the cost of a last stand rather than simply accepting defeat? Put differently, when is it better to fight or surrender?

Theoretically, Gelder (2013) provides sufficient conditions under which last stand behavior is optimal. If there is a cost to losing (beyond merely forgoing the winning prize) and if players have a time preference for when they win or lose, then making a last stand becomes a best response for a player on a losing trajectory. The optimality of last stand behavior contrasts with earlier work on dynamic contests where players slacken their effort or even give up entirely if they fall behind. An early example is Fudenberg et al. (1983) who examine preemption in patent races by firms who have a marginal lead over their competitors. A more recent example is Konrad and Kovenock (2009), who, like Gelder (2013), examine a contest where players compete in several successive battles until one of the players achieves a critical number of victories. Konrad and Kovenock find that a player who is behind will completely give up—essentially surrendering—unless there is a separate intermediate prize for winning each of the individual battles. This paper tests the theoretical predictions of Gelder (2013) and Konrad and Kovenock (2009). We find evidence to support the basic predictions of each of these models.

Experimentally, this paper examines competitive behavior in a best-of-seven tournament, a special case of the contest structure in Gelder (2013) and Konrad and Kovenock (2009). This type of a contest is known more generally as a two-player race and stems back to the patent race model of Harris and Vickers (1987). Although previous experimental work has addressed best-of-three tournaments (see Mago et al. 2013; Mago and Sheremeta 2012; Irfanoglu et al. 2010; and Sheremeta 2010), the full dynamics of last stand behavior can only be realized with a larger number of battles. The best-of-seven tournament provides a structure that is large enough to capture the desired dynamics, but small enough to keep the experiment simple. It is also a natural choice as it is used in many championship settings, such as the World Series.

The last stand environment of Gelder (2013) differs from Konrad and Kovenock (2009) through the introduction of a losing penalty and discounting. Since the behavioral predictions of Konrad and Kovenock are innocuous to the introduction of discounting, we can compare the two models solely on the basis of the losing penalty. We examine three prize-penalty combinations: one in which there is no penalty, one with a prize and penalty of equal magnitude, and a third in which the penalty is dramatically larger than the prize. For each of these three cases, the net difference between the prize and penalty is identical; so in a one-shot contest, equilibrium behavior would be the same in each case. The diverging predictions of surrendering versus making a last stand only come into play in a dynamic setting.

A consequence of last stand behavior is that the overall winning margin can be predicted based on the magnitude of the losing penalty in comparison to the winning prize. Neck-and-neck outcomes are more likely to be observed when the losing penalty is relatively large, while landslide victories are more probable when the winning prize is dominant. We find evidence to support this hypothesis.

We model each battle of the best-of-seven tournament as an all-pay auction—the highest bidder wins, but all players incur the cost of their own bid (see Hillman and Riley 1989; and Baye et al. 1996). Although the winning prize and losing penalty are fixed, allowing players to expend or conserve resources through the size of their bids makes the tournament a non-constant-sum game. Given the strategic complexity of this environment, we wanted to see if prior experience in other contest related experiments affected performance. To do this we placed restrictions on the subject pool from which participants were drawn. Some sessions were conducted solely with subjects who had previously participated in another contest related experiment; other sessions were entirely comprised of subjects who had no such experience. A third set of sessions did not have any such constraints and so allowed for a mix of subjects tended to behave more like the theoretical predictions.

Our paper fits within a small but emerging literature on dynamic contest experiments, as well as within a broader literature on contests and tournaments (see Dechenaux et al. 2012 for an extensive survey on experiments involving contests). In terms of "best-of" experiments, we bridge the gap between the work on bestof-three tournaments mentioned previously and the best-of-19 tournament in Zizzo (2002), which was explicitly patterned after Harris and Vickers (1987). Our paper is also closely related to the game of siege experiment by Deck and Sheremeta (2012). In their experiment, players are positioned asymmetrically so that one player (the defender) needs to win two successive battles to be victorious, while the attacker only needs to win one (this is the dynamic counterpart of a weakest link contest). This asymmetric starting point can be reached as an intermediate stage in a bestof-three and a best-of-seven tournament.

We begin by giving a brief description of the theoretical framework (Section 2) and then describe how we set up the experiment (Section 3). Section 4 presents our results, and Section 5 concludes.

2. Theory and Hypotheses

The winner of a best-of-seven tournament is the first player to win four battles. To keep track of each player's progress, we can model the state space as a pair (i, j) where i is the number of battles that Player A still needs to win and j is the

number of battles that Player B still needs to win.¹ Hence, the tournament begins at state (4,4) and proceeds until it reaches (0, j) for (i, 0) for $i, j \in \{1, 2, 3, 4\}$. This is depicted in Figure 1. Once a player has won four battles, he receives a prize $Z \ge 0$ and his opponent incurs a penalty $L \le 0$. Each battle consists of players competing in an all-pay auction with the winner of the auction advancing one state closer to victory.² The unique equilibrium of the two-player all-pay auction is in mixed strategies with players randomizing their bids between 0 and the smaller of the two players' valuation of the prize (Baye et al. 1996). While both players randomize over this interval, the player with the lower valuation will bid 0 with positive probability. That is, if ζ_H and ζ_L are the high and low valuations of the prize ($\zeta_H \ge \zeta_L > 0$), then the equilibrium bidding distributions are as follows:

$$F_{H}(h) = \begin{cases} h/\zeta_{L} & \text{if } h \in [0, \zeta_{L}] \\ 1 & \text{if } h > \zeta_{L} \end{cases}$$

$$G_{L}(\ell) = \begin{cases} (\zeta_{H} - \zeta_{L} + \ell)/\zeta_{H} & \text{if } \ell \in [0, \zeta_{L}] \\ 1 & \text{if } \ell > \zeta_{L} \end{cases}$$
(1)

Given these distributions, then the expected payoffs are $u_H = \zeta_H - \zeta_L$ and $u_L = 0$; the winning probabilities are $p_H = 1 - \frac{\zeta_L}{2\zeta_H}$ and $p_L = \frac{\zeta_L}{2\zeta_H}$; and the expected bids are $\mathbb{E}[e_H] = \frac{\zeta_L}{2}$ and $\mathbb{E}[e_L] = \frac{\zeta_L^2}{2\zeta_H}$.

The bulk of the analysis in Gelder (2013) and in Konrad and Kovenock (2009) is in extending the one-shot all-pay auction to a dynamic structure where an actual prize is awarded only after a player has achieved a critical number of wins. Hence, it becomes necessary to identify the prize valuations at each interior state (i, j)where i, j > 0. These prize valuations are implicitly defined based on the marginal benefit of winning at (i, j) and being one state closer to overall victory versus losing and being one state closer to defeat. When losing is costless—as in Konrad and Kovenock—a player who is behind has a prize valuation of zero, so there is no incentive to compete.³ In Gelder's framework on the other hand, when there is a cost to

¹Tracking the absolute number of wins for each player requires a two dimensional state space an alternative model, known as the tug-of-war, tracks the relative number of wins with a unidimensional state space. Within the tug-of-war setting, Konrad and Kovenock (2005) predict that laggards surrender when there is no losing penalty, while Agastya and McAfee (2006) find that last stand behavior is possible when there is a penalty.

²Although an arbitrary tie-breaking rule typically suffices, the equilibrium in Konrad and Kovenock (2009) requires that ties be awarded to the player who is ahead in the tournament. This assumption allows the frontrunner to coast to victory with a bid of zero when the laggard surrenders. Since this is a rather technical requirement, we use a fifty-fifty tie breaking rule in the experiment.

³Since the player who is behind receives zero from continuing to lose, and since the expected

		Βv	vins			
0	(4, 0)	(3,0)	(2, 0)	(1, 0)		
1	(4, 1)	(3, 1)	(2, 1)	(1, 1)	(0, 1)	
2	(4, 2)	(3, 2)	(2, 2)	(1, 2)	(0, 2)	А
3	(4, 3)	(3,3)	(2, 3)	(1, 3)	(0,3)	wins
4	(4, 4)	(3, 4)	(2, 4)	(1, 4)	(0, 4)	01
	4	3	2	1	0	

Figure 1: Best-of-seven tournament

losing and when players would prefer to win early and lose late, the prize valuations are always strictly positive so that players actively compete at every interior state.⁴ The magnitudes of the prize valuations do, however, vary from state to state and across players. Gelder finds that there is a collection of states where the player who is behind in the tournament actually has the higher prize valuation and therefore tends to compete more aggressively. This heightened degree of competition from the underdog is what Gelder terms the last stand.

In terms of incentives, the last stand represents the position in the tournament where the underdog's incentive to avoid losing is stronger than the frontrunner's incentive to win. A player who must avoid losing today, or else incur a sufficiently large penalty, has a stronger motive to compete than the opposing player who may secure the victory tomorrow if not today. The precise collection of states where a last stand occurs depends on the ratio of the winning prize to the losing penalty, as well as the discount factor. The larger the penalty, the closer to the end of the tournament the last stand occurs. The likelihood of the underdog catching up after an unsuccessful last stand is minimal at best. In addition to the last stand, Gelder also finds that the frontrunner will defend his overall lead in the tournament if it is threatened. The "defense of the lead" occurs when the frontrunner only has a

payoff from winning a single state is also zero, then the prize valuation is zero as well. Konrad and Kovenock also examine the case where there is an intermediate prize for winning each battle. In that setting, the prize valuation for a player who is behind is solely based on the intermediate prize.

⁴An example of when these assumptions may be satisfied is the US presidential primaries. Candidates would typically prefer to secure their party's nomination early in the election cycle to have more time to prepare for the general election. On the losing side, the potential loss of political capital is likely higher for candidates who unmistakably lose at an early stage and are not able to demonstrate their viability for future campaigns.

one-state lead in the tournament, and it entails a much higher expenditure from the frontrunner than from the underdog in expectation. Thus the last stand acts as a defensive push, while the defense of the lead acts as an offensive one.

Based on the theoretical predictions, there are five main hypotheses that we will examine in this experiment:

Hypothesis 1. Players on a losing trajectory will make a last stand if the penalty for losing is large relative to the winning prize.

Hypothesis 2. If there is no losing penalty, then a player who is behind will surrender (or cease to compete).

Hypothesis 3. Players with a one battle lead in the tournament will compete more aggressively than their opponent in order to maintain their lead.

Hypothesis 4. The expected winning margin is increasing in the size of the winning prize relative to the losing penalty.

Hypothesis 5. The winner of the initial battle will win the tournament the majority of the time.

3. Methodology

We conducted 18 experimental sessions, each composed of 12 subjects. These sessions were conducted at the Economic Science Institute, Chapman University, in computer labs where the computers were separated by partitions for privacy. The experiment began with subjects reading the instructions on their computer (a copy of the instructions is provided in the appendix). After reading the instructions, subjects were given a short quiz comprised of three possible scenarios for how a best-of-seven tournament could unfold. Subjects were then asked to compute the payoff for each scenario. The purpose of this short quiz was to ensure that subjects had a basic level of comprehension about the structure of the game. The quiz was immediately followed by a short risk preference lottery à la Holt and Laury (2002). During the main portion of the experiment, subjects participated in 20 best-of-seven tournaments. Subjects were randomly and blindly paired and re-paired for each of these tournaments via the computer network. At the conclusion of the experiment, subjects completed a demographics survey and were paid in cash based on their performance in two randomly selected tournaments.

Each battle of a best-of-seven tournament was treated as an all-pay auction: subjects placed bids simultaneously and the highest bidder won (ties were broken randomly).

In the all-pay fashion, the sum of a player's bids throughout a tournament was deducted from his or her payoff for that tournament. Additionally, the winner of the tournament received a prize and the loser incurred a penalty. Since time preferences for winning or losing in Gelder (2013) were implemented through a discount factor, and since discounting is difficult to replicate in a short laboratory experiment, we followed a common practice from macroeconomic experiments by implementing discounting via a continuation probability (see, for instance, Duffy 2008, and Noussair and Matheny 2000). Until a player had succeeded in winning four battles, there was a 90% probability that the tournament would actually continue from one battle to the next. If a tournament ended prematurely, neither player would receive a prize or a penalty, but players still had to pay their bids. Our justification for this approach is that a continuation probability is equivalent to discounting in terms of the expected payoffs.⁵

Experimental sessions varied along two treatment variables: (1) payoffs in the bestof-seven tournament and (2) subjects' prior exposure to contest related experiments. We conducted three separate payoff scenarios: the first with a substantial losing penalty and meager winning prize (Win 15 Lose 285), the second with an equal prize and penalty (Win 150 Lose 150), and the third with a sizable prize but no penalty (Win 300 Lose 0). Prizes, penalties, as well as all bids, were denominated in an experimental currency called rupees, where 50 rupees = \$1 US dollar. In order to make the stakes comparable across treatments, we fixed the difference between the positive prize and the negative penalty at 300 rupees. The two treatments with non-zero penalties coincide with the Gelder (2013) model, while the treatment with no losing penalty fits the Konrad and Kovenock (2009) model.

For the second treatment variable, we varied the subject pool from which participants were recruited according to whether subjects had previously participated in a separate contest related experiment. The Economic Science Institute at Chapman University conducts a large volume of economic experiments. Since records are kept of the experiments that subjects participate in, we were able to place restrictions

⁵The random ending rule may also be thought of as the potential that some exogenous factor suddenly disrupts the conflict (such as the cavalry coming to save the day). An alternative method for implementing discounting is to adjust the size of the prize and the penalty according to the winning margin. Since the winning margin is based on the number of rounds that players compete, this is a present value interpretation of discounting. A benefit of using the random ending rule in an experimental setting is that the order of magnitude of expenditures early in the tournament remains comparable to that of the prize and penalty at later stages of the tournament. Noussair and Matheny (2000) compared both the random ending rule and the present value interpretation of discounting in an experiment involving a single agent dynamic optimization problem.

Sessions	Subject Pool	Prize	Penalty	Bid Observations
3	Experienced	15	285	2786
3	Experienced	150	150	2536
3	Mixed	15	285	2616
3	Mixed	150	150	2692
3	Mixed	300	0	2456
1	Inexperienced	15	285	920
1	Inexperienced	150	150	874
1	Inexperienced	300	0	854

 Table 1: Sessions by Treatment

on the subject pool based on whether subjects had participated in an experiment that had involved contests or contest theory. In some of our sessions, we restricted the subject pool to individuals who had previous experience in a contest related experiment. Other sessions were specifically limited to those without experience. We also ran control sessions in which we placed neither of these restrictions on the subject pool, allowing for a mix of participants with and without prior experience. We will refer to these different treatments as experienced, inexperienced, and mixed. A summary of the experimental sessions by treatment is shown in Table 1.

During a best-of-seven tournament, subjects could see both their own and their opponent's previous bids. They also could see how many rounds they had won or lost, as well as the sum of their bids up to that point in the tournament. An example of the bidding screen is shown in Figure 2. The bidding screen would also alert subjects when a tournament had finished, either by a player winning four rounds or by the computer ending the tournament early. After displaying the final outcome and payoffs for the tournament, subjects would then be randomly re-matched to begin a new tournament.

In order to cover bids and potential losses, each subject received an initial endowment of rupees at the start of the experiment. At the end of the experiment, two of the best-of-seven tournaments were randomly selected for payment. Subjects were then paid, with winning prizes in the two tournaments being added to their endowment, but bids and losing penalties being subtracted from it. Since losing penalties varied across treatments, and since bids and losing penalties were both deducted from the same account, we wanted to make the treatments comparable in terms of the bidding budget. We accomplished this by varying the initial endowment across treatments so that it was composed of an effective bidding budget (700 rupees) plus



Figure 2: Bidding screen during a best-of-seven tournament

the size of the losing penalty. Thus, for penalties of 285, 150, and 0, the endowment was 985, 850, and 700. In paying for two tournaments, we decided to set the bidding budget at more than twice the prize-penalty spread of 300 rupees. We did not want subjects to be budget constrained—especially in tournaments that continued onto the seventh round. In most cases, the bidding budget was more than sufficient. For each round of a best-of-seven tournament, we allowed players to bid between 0 and 300 inclusive (with up to one decimal place).

4. Results

4.1. Summary Statistics

Before analyzing the main hypotheses, we will briefly highlight the major summary statistics. The fundamental level of observation for each treatment is a player's bid at a particular state (i, j) within tournament t. As a whole, the data form a panel with 20 tournament observations per subject and up to seven bid observations per tournament. Variance in the bidding observations is considerably higher during the initial tournaments of the experiment since subjects are learning the structure of the game. The analysis in this paper omits all observations from the first three (of 20) tournaments.⁶ Here we will summarize observations, winning probabilities, and

⁶Omitted bid observations are excluded from the counts in Table 1. We also omitted two additional tournaments from one particular subject who clearly did not know what was going

q	0	59	55	53	4	40	65	47	48	;	26						
nce	1	104	109	99) 8	30	96	85	76	;	52						
rieı	2	165	136	11	6		167	118	98	;							
¢ре	3	282	192				271	148									
Ē	4	608		-			612										
								-									
	0	62	53	41		36	78	57	3	4	41		82	44	40)	21
ed	1	101	94	82	2 7	72	123	94	8	1	82		96	64	65	5	42
Ліх	2	160	120	10	6		187	115	10	0		_	157	119	10	8	
4	3	275	162				278	142			-		267	158			
	4	612		_			612						612				
								-									
ed	0	19	19	15	15		16	22	14	1	2		19	22	11	11	
enc	1	27	29	30	30		27	32	27	2	24		22	28	23	22	
eri	2	43	54	52		_	49	47	44				48	50	40		
dxe	3	91	86				89	60		-			90	66			
Ine	4	204					204						204				
		4	3	2	1		4	3	2		1		4	3	2	1	

Table 2: Bidding Observations by State (i, j) and by Treatment

Win 150 Lose 150

Win 300 Lose 0

average bids by state and treatment.

Win 15 Lose 285

Table 2 shows the number of observations in each state for each treatment. Due to the symmetry of the tournament—whenever one player is ahead, the other player is behind by the same margin—observations are only shown for states where a player is behind or the tournament is tied (i.e. states (i, j) such that $i \ge j$). The random ending rule causes the total number of observations to decrease by roughly 10% after each of the first four battles. In successive states, the number of observations continues to decrease through the random ending rule, but also decreases through players winning or losing tournaments.

on (we omitted their opponent's bids as well). This subject was contacted midway through the experiment and was asked if they needed any clarification. We kept all subsequent observations from this subject.

V	1	56.4	52.4	5.2	50		26.3	26.3	2.6	50	0	0	0	50
eor	2	52.9	5.6	50			27.8	2.8	50		0	0	50	
$\mathbf{Th}_{\mathbf{t}}$	3	5.6	50				2.9	50			0	50		
2	4	50					50				50			
								-						
ced	1	43.3	49.5	46.5	50		32.3	44.7	36.8	50				
iene	2	32.1	44.9	50		_	34.1	46.6	50					
)erj	3	37.2	50				32.1	50		_				
Ext	4	50					50							
—														
_	1	38.6	44.7	50.0	50		36.6	39.4	58.0	50	14.6	31.2	38.5	50
xec	2	32.5	46.7	50			29.4	47.8	50		29.9	49.6	50	
Mi	3	33.8	50				27.7	50		_	33.3	50		-
	4	50					50				50		-	
7								-				-		
Cec	1	29.6	34.5	50.0	50		40.7	31.2	48.1	50	13.6	21.4	52.2	50
ien	2	34.9	55.6	50		_	40.8	48.9	50		43.8	42.0	50	
per	3	50.5	50				39.3	50		_	40.0	50		-
лех	4	50		-			50		-		50		-	
Ir		4	3	2	1		4	3	2	1	4	3	2	1

Table 3: Winning Percentages in State (i, j): Theoretical and Observed

Win 15 Lose 285

Win 150 Lose 150

Win 300 Lose 0

Theoretical and observed probabilities of winning a battle at each state are shown in Table 3. Symmetry allows us to again focus on the states where a player is behind or the tournament is tied. The major patterns of competition can be seen by examining the theoretic winning probabilities. For instance, the defense of the lead is reflected by the remote winning probabilities at states (4,3), (3,2), and (2,1)in the Win 15 Lose 285 and the Win 150 Lose 150 treatments. The last stand is evidenced by the underdog having the higher winning probability at states (4,2), (4,1), and (3,1) of the Win 15 Lose 285 treatment. Although not as strong, the underdog is still expected to win roughly a quarter of the time at these three states

in the Win 150 Lose 150 treatment.⁷ Finally, the tendency to surrender is depicted in the Win 300 Lose 0 treatment by the zero probability of winning a battle when a player is behind in the tournament.

Although the observed probabilities from the laboratory fail to capture the defense of the lead, the basic contrast between making a last stand and surrendering can be seen. For instance, in the Win 300 Lose 0 treatment, the winning probability falls below 15% at state (4, 1) for both the Mixed and Inexperienced treatments—a considerable drop from winning probabilities at (4, 3) and (4, 2) of 30% to 44%. On the other hand, there is a jump in the winning probability between states (4, 2) and (4, 1) in the two Mixed treatments with a losing penalty, as well as in the Experienced Win 15 Lose 285 treatment, with differences ranging from 6 to 11 percentage points.

While the winning probabilities provide information about the ratio of bids at (i, j)relative to (j, i), it is also informative to have a measure of the absolute magnitude of the bids at the different states. The expected bids, as predicted by theory, as well as the average bids in the experiment are given in Table 4, by state and by treatment. There are a few patterns in the bidding magnitudes that merit some attention. For instance, when the tournament is tied, theory predicts that bids will far and away exceed those at any other state—even when a losing penalty is present. That is not the case in the experimental data. Average bids when the tournament is tied are never as large as predicted. Likewise, bids by a player who is trailing in the tournament by a single state do not plunge precipitously as theory suggests. What does occur in every experimental treatment is that bids progressively increase as both players win battles (that is, as the tournament proceeds to the Top-Right). As if there were a minimization function, with the number of battles each player has won as inputs, bids tend to plateau until the function reaches a higher value. There are some minor fluctuations (such as the drop at (4, 1) in the Win 300 Lose 0 treatment with the mixed subject pool), but it generally appears that a winning player will not update her bid until she loses, and a losing player will not update his bid until he wins. At least that is the story in the aggregate. The tendency to make a last stand or to surrender can be more credibly assessed using fixed effects models which control for the idiosyncratic characteristics of the individual tournaments.

⁷For the Win 150 Lose 150 treatment, a best-of-seven tournament is not large enough to include the states where the player who is behind wins battles with more than one-half probability.

v	1	13.0	13.6	1.5	150		3.9	3.9	0	.4	150		0	0	0	150
OL	2	11.0	1.4	122.2	14.3		3.8	0.4	12	8.3	7.5		0	0	135	0
Γh€	$\mathcal{3}$	1.1	99.0	12.2	12.9		0.4	109.4	1 6	.8	7.5	1	0	121.	5 0	0
	4	80.2	9.9	10.4	11.3		92.9	6.1	6	.8	7.5	1	109.4	0	0	0
						_						_				
ced	1	15.1	36.4	67.5	93.4		7.6	21.8	31.7	48	3.7					
iene	2	11.4	30.2	45.3	72.1		7.6	20.3	25.1	33	3.6					
ieri	3	11.2	25.5	34.2	37.4		7.9	18.0	21.3	26	5.3					
Ext	4	11.7	16.8	16.0	16.9		9.8	12.4	11.6	13	3.7					
—								·								
_	1	13.2	28.8	47.1	70.1		6.8	22.0	47.1	L 7	72.7		5.7	22.6	43.9	74.8
xec	2	7.8	21.9	36.0	47.4		6.9	24.4	34.8	3 4	12.3		11.9	32.0	41.3	53.5
Mi	$\mathcal{3}$	8.7	19.0	23.0	28.9		8.3	22.0	23.'	7 2	26.7		13.0	26.0	35.4	37.3
	4	9.4	12.7	13.1	15.1		12.8	20.2	18.9	9 2	21.5		15.0	21.9	23.9	25.5
_																
cec	1	20.1	26.0	66.2	114.3		26.0	41.2	56.'	7 7	73.0		11.2	25.5	62.0	66.0
ien	2	21.6	42.6	46.4	60.8		27.9	43.5	53.3	3 5	58.2		17.0	29.7	50.1	62.2
per	3	17.7	29.7	39.9	55.7]	24.5	40.0	47.0	3 5	55.5		13.4	27.8	36.5	50.8
iex	4	19.9	21.1	26.4	37.1]	22.9	32.8	37.9	9 4	12.4		9.6	17.1	20.7	29.8
Ir		4	3	2	1	-	4	3	2		1		4	3	2	1

Table 4: Expected Theoretical Bids and Average Observed Bids

Win 150 Lose 150

Win 300 Lose 0

4.2. Fight or Surrender

Win 15 Lose 285

In analyzing bidding behavior, it is pertinent to identify whether the frontrunner or the underdog is bidding more aggressively at each stage of the tournament—or even if there is a difference. If the underdog is engaging in last stand behavior, then we would expect his bids to increase relative to his opponent's as he nears an overall loss. Therefore, we want to compare the underdog's bid at state (i, j) with the frontrunner's bid at the symmetric state (j, i). To do this, we use a fixed effects regression model with cluster robust standard errors where the dependent variable is a player's bid. For the independent variables, we consider two models: one that only accounts for the different states, and another that also factors in the previous state of the tournament. To account for the different states within the tournament, we include a set of dichotomous variables, equal to one if a bid is made from that particular state and zero otherwise. Since we are interested in comparing bidding behavior at state (i, j)with state (j, i), we take advantage of the fact that when a categorical variable with N distinct values is represented by a set of N - 1 dichotomous variables, the coefficients of the N - 1 dichotomous variables can be interpreted directly in reference to the omitted N^{th} value. Thus, we are interested in the coefficient for state (i, j)in a regression where (j, i) is the omitted state.⁸

In factoring in the previous state, we are interested in a similar comparison. The current and previous state can be represented by a triple (i, j, h) where h denotes whether Player A arrived at (i, j) by winning or losing the previous round. Equilibrium bidding strategies are invariant to the history of the tournament. However, we are interested in seeing whether there are any clear effects (psychological or otherwise) from having won the previous round. Thus we want to compare the coefficient for (i, j, won) when (j, i, lost) is the omitted state with the coefficient for (i, j, won) is omitted.

Let $\mathbf{s}(\mathbf{j}, \mathbf{i})$ be the vector of dichotomous state variables which omits state (j, i). Similarly, let $\mathbf{q}(\mathbf{j}, \mathbf{i}, \mathbf{h})$ be the vector representing current and previous states, but omitting (j, i, h).⁹ With this notation, we use the following two fixed effects models to predict player k's bid at time t within the experiment:¹⁰

Model 1. $\widehat{bid}_{k,t} = \beta_0 + \mathbf{s}(\mathbf{j}, \mathbf{i})'_{k,t} \boldsymbol{\beta}_s + f_k + \varepsilon_{k,t}$ Model 2. $\widehat{bid}_{k,t} = \beta_0 + \mathbf{q}(\mathbf{j}, \mathbf{i}, \mathbf{h})'_{k,t} \boldsymbol{\beta}_q + f_k + \varepsilon_{k,t}$

Table 5 shows the coefficients from Model 1 for bids made at (i, j) relative to (j, i) for i > j. Corresponding results for Model 2 are given in Tables 6 and 7—the former comparing (i, j, lost) to (j, i, won), while the latter compares (i, j, won) with (j, i, lost). Some states cannot be reached by winning, so Table 7 does not include the states where Player A has yet to win a round (i = 4). Yet, unlike Tables 5 and 6, it does include the states where the players are tied (i = j).

⁸Alternatively, for two states that are not omitted, we could obtain the relative difference by subtracting one of the coefficients from the other, taking care to compute the appropriate standard error for the difference.

⁹State (4,4) is not included because it has no previous state. Also, for $j \in \{1, 2, 3\}$, Player A can only reach state (4, j) by losing, while (j, 4) can only be reached by winning.

¹⁰There are two time components: the tournament number and the bids within each tournament. Since the number of bids per tournament may vary between one and seven, we interpolate the timing of each bid to be at one-seventh intervals between tournaments.

Under Model 1, Table 5 shows that for the treatments with a losing penalty, it is common for players to bid more aggressively when they are behind. This is particularly true in the experienced and mixed sessions. Moreover, in the Win 15 Lose 285 treatment, subjects tend to bid increasingly more aggressively as they fall farther and farther behind in the tournament. In the experienced group, for instance, players tend to bid 1.89 more rupees at (4, 3) than at (3, 4); this difference then increases until by (4, 1), players are submitting bids that average 9.30 rupees higher than at (1, 4). The mixed group for the Win 15 Lose 285 treatment has a similar increase. Bids at (4, 3) are 5.02 more than at (3, 4)—a difference that then rises to 13.42 between (4, 1) and (1, 4). In terms of magnitude, the differences of 9.30 and 13.42 between states (4, 1) and (1, 4) are especially sizable when considering that the average bids at these states and for these treatments range from 13.2 to 16.9 (see Table 4). We view this as evidence in support of our last stand hypothesis.

There are also a couple of treatments where a last stand happens at (2, 1). A last stand at (2, 1) is interesting as it occurs at a point where players have a credible chance of winning the entire tournament; last stands at (4, 1) and (3, 1), on the other hand, are more likely motivated by the probability that the tournament will end prematurely. In the Win 15 Lose 285 treatment of the inexperienced group, the bidding behavior flips from bidding 20 rupees less at (3, 1) than at (1, 3) to bidding 14 rupees more at (2, 1) than at (1, 2). Although less dramatic, in the Win 150 Lose 150 treatment of the mixed group, there is no significant difference between bids at (4, 1) and (1, 4). However, by (2, 1), players are bidding significantly more than at (1, 2) by 7.70 rupees.

Last stand behavior continues to persist under Model 2, but with a few interesting changes. The most prominent pattern is that players who are behind in the tournament compete with renewed vigor when they have won the previous round. For instance, in the Win 15 Lose 285 treatment of the mixed group, if a player arrives at (3, 1) by losing at (3, 2), he will bid an average of 5.88 rupees more than at (1, 3)after winning at (2, 3). However, this difference increases to 23.46 rupees when the underdog reaches (3, 1) by winning at (4, 1). The bidding difference in Model 1 sits between at 10.30 rupees. In most cases, the coefficients in Table 7 are larger than the corresponding coefficients in Tables 5 and 6 for states (3, 2), (3, 1) and (2, 1). The Table 7 coefficients are also more likely to be significant. Again, using the Win 15 Lose 285 treatment of the mixed group as an example, the difference between bids at (2, 1) and (1, 2) is not significant in Model 1. Neither is the difference significant when a player arrives at (2, 1) by losing at (2, 2). However, a player reaching (2, 1) by winning at (3, 1) will bid significantly more than at (1, 2) after losing at (1, 3) by an average of 8.70 rupees.

		Win 1	5 Lose 2	285		Win 1	50 Lose	150	Win 300 Lose 0
		4	3	2		4	3	2	
		9.30	8.69	-1.19		8.93	7.24	2.31	
F	1	(1.99)	(4.16)	(4.08)		(2.53)	(3.61)	(2.49)	
cec		> 0***	> 0**	、 <i>、</i> /		> 0***	> 0**		
ien		5.66	1.69]		10.06	5.72		
erj	2	(1.41)	(1.84)			(1.57)	(1.70)		
lxp		> 0***				$> 0^{***}$	> 0***		
Γ.		1.89]			6.90			
	3	(1.04)				(1.75)			
		$> 0^{**}$				$> 0^{***}$			
		<u> </u>	1				1		
		13.42	10.30	4.08		1.17	2.66	7.70	-3.41 -8.33 -8.78
	1	(3.09)	(3.54)	(3.47)		(4.29)	(1.39)	(3.03)	(3.31) (3.37) (4.21)
		$> 0^{***}$	$> 0^{***}$				$> 0^{**}$	$> 0^{***}$	$ < 0^{***} < 0^{**}$
ed		8.62	4.73			-0.16	0.01		2.09 0.92
Iix	2	(1.50)	(1.30)			(2.75)	(1.43)		(2.19) (3.24)
4		$> 0^{***}$	$> 0^{***}$						
		5.02				-2.38			2.29
	3	(0.85)				(2.03)			(1.38)
		$> 0^{***}$							> 0**
			1		_		1		
		-5.18	-20.23	14.26		4.17	-3.20	1.44	-10.55 -20.78 -1.34
	1	(9.39)	(12.23)	(7.34)		(6.62)	(5.35)	(6.64)	(3.09) (4.33) (6.99)
ed			< 0**	> 0**					< 0*** < 0***
inc		9.46	9.33			8.71	3.23		2.25 -4.82
rie	2	(3.02)	(3.40)			(4.09)	(4.57)		(2.93) (2.17)
tpe		$> 0^{***}$	$> 0^{***}$			$> 0^{**}$			< 0**
vev	-	6.94				8.19			
Π	3	(2.61)				(2.57)			(1.39)
		$> 0^{***}$				$> 0^{***}$]		
		4	3	2		4	3	2	4 3 2

Table 5: Bidding Behavior at State (i,j) Compared to $(j,i) {\rm :}$ Model 1 (baseline)

Significance levels: * 10%, ** 5%, *** 1%.



Table 6: Losing to get to (i, j) vs. Winning to get to (j, i)

Significance levels: * 10%, ** 5%, *** 1%.

Contrasting the last stand behavior that frequently accompanies large losing penalties, there is a marked tendency in the Win 300 Lose 0 treatment for players to bid less aggressively when they are nearing defeat. Beginning in Table 5 with Model 1, players in the mixed experience group at (3, 1) and (2, 1) submit bids that are significantly lower than those at (1, 3) and (1, 2) by roughly 8 to 9 rupees. The bid reduction can be more than 20 rupees in the inexperienced session. In all of these cases, there is a clear pattern of relative retreat. This pattern helps to substantiate our hypothesis of a player surrendering when losing is costless.

Results for the Win 300 Lose 0 treatment are similar with Model 2, but with the exception that winning the previous round again leads to more aggressive bidding.



Table 7: Winning to get to (i, j) vs. Losing to get to (j, i)

Significance levels: * 10%, ** 5%, *** 1%.

In the mixed group, for example, the coefficients in Table 6 for when a player arrives at state (i, j) by losing closely mirror the Model 1 results—both in terms of magnitude and significance. Table 7, however, reveals that winning the previous round leads to substantial changes in the bidding coefficients. Players no longer bid significantly less at (3, 1) and (2, 1) than at (1, 3) and (1, 2), and players actually bid significantly more at (3, 2) than at (2, 3).

A final aspect of Table 7 that is worth mentioning is that it allows for bidding behavior to be analyzed at states (1,1), (2,2), and (3,3) based on whether a player brought the tournament to a tied position by winning or losing. As with all of the comparisons involving Model 2, whether a player won or lost the previous round should not alter the bidding behavior from a theoretical standpoint. After all, equilibrium bidding distributions are Markov perfect in the sense that they are based on the present state of the tournament, not on how players arrived at that state. Here, however, there are many instances where winning or losing the previous round does indeed matter. In the Win 15 Lose 285 treatment of the experienced group, players that lost at (4, 4) but then won at (4, 3) bid more aggressively at (3, 3) than had they won at (4, 4) but lost at (3, 4). There is an element of the defense of the lead at (2, 2) in the mixed Win 150 Lose 150 treatment. Players that won at (3, 2) to tie the tournament are likely to be outbid by their counterparts who lost their lead by losing at (2, 3). A similar defense of the lead occurs at (1, 1) in the Win 300 Lose 0 treatment of the mixed group.

4.3. Winning Margins and Initial Leads

Our final two hypotheses address the implications of making a last stand verses surrendering—specifically in terms of the size of the winning margin, and also in terms of the importance of winning the initial battle of the tournament. Last stands are more pronounced as the relative size of the losing penalty increases, and as a result the winning margin in the tournament should decrease. The opposite should occur when players are prone to surrendering. Table 8 reports the distribution of winning margins for each treatment.¹¹ The most pronounced changes in these distributions occur at the endpoints where the winning margin is either one or four battles. For instance, in the experienced group, the percent of tournaments with a winning margin of two or three is hardly affected by moving between the Win 15 Lose 285 and the Win 150 Lose 150 treatments. However, there is a six percentage point increase in tournaments that end by a landslide and a corresponding six percentage point decrease in neck-and-neck tournaments as the relative size of the losing penalty decreases. This is in line with our prediction. The result is similar with the mixed experience group. Landslide victories increase from 28.1% in the Win 15 Lose 285 treatment to 32.6% in the Win 150 Lose 150 treatment before finally reaching 39.2% in the Win 300 Lose 0 treatment. Neck-and-neck victories likewise decrease from 22.4% to 13.8% between the Win 15 Lose 285 and the Win 300 Lose 0 treatments. There is a curious drop to 17.5% at a winning margin of 2 in the Win 150 Lose 150 (Mixed) treatment that then rebounds to 23.5% for the final winning margin of 1. The last stand at (2, 1), which was previously noted for this treatment, accounts for this dip. Patterns for the inexperienced group are less distinct (although there is a 5.1 percentage point decline in neck-and-neck outcomes

¹¹The counts underlying the percentages in Table 8 have been adjusted to account for attrition with the random ending rule (e.g. a winning margin of 4 is equal to $1/\delta$ winning margins of 3).

		W	Vinning	g Marg	in
		4	3	2	1
Experienced	L285	24.5	25.4	27.2	22.8
	L150	30.6	24.6	27.9	16.8
Mixed	L285	28.1	26.7	22.9	22.4
	L150	32.6	26.4	17.5	23.5
	LO	39.2	23.4	23.6	13.8
Inexperienced	L285	24.0	26.7	23.4	26.0
	L150	21.6	32.9	23.3	22.2
	L0	26.3	33.9	18.8	20.9

Table 8: Winning Margins by Treatment (in %)

from the Win 15 Lose 285 treatment to the Win 300 Lose 0 treatment).

The initial battle is often viewed as pivotal in deciding the ultimate outcome of a dynamic contest.¹² It is strongly advantageous in Gelder (2013) but decisive in Konrad and Kovenock (2009). In Gelder's framework, the probability of an upset is increasing in the relative size of the losing penalty (as is the strength of the last stand). While we cannot support that comparative static, Table 9 confirms that winning the initial contest is a strong correlate of winning the ultimate tournament—or at least being in the lead at the time that the tournament ends (whether by winning or through the random ending rule). Throughout the different treatments, roughly 70% to 80% of all winners at state (4, 4) went on to win the tournament. The one exception being the Win 300 Lose 0 treatment with the Inexperienced group where the percentage dropped to 61.9%.

5. Conclusion

While last stands are found in a number of anecdotal accounts, this paper provides a controlled laboratory framework to actually access this type of behavior. To do so we examine how aggressively players compete at different stages of a best-of-seven tournament. When the cost of losing is high, we find that players tend to bid more

 $^{^{12}}$ For example, Klumpp and Polborn (2006) examine the disproportionately large amount of attention that New Hampshire and Iowa receive as the first states to vote in the US presidential primary elections.

		Win Tournament	End in Lead
Experienced	L285	73.9	72.7
	L150	77.4	77.8
Mixed	L285	80.7	79.7
	L150	80.5	77.8
	L0	74.9	75.8
Inexperienced	L285	70.6	70.6
I	L150	79.7	79.4
	L0	61.9	66.7

Table 9: Percent of Initial Battle Winners to Win the Tournament

aggressively as they fall farther behind in the tournament. This is consistent with theory and embodies the notion of a last stand. Conversely, we find that subjects bid less aggressively as they fall behind in a tournament with no losing penalty. Although not as stark as the theoretical prediction of completely giving up, this pattern of less aggressive bidding is in line with the general prediction of surrendering. Players are cutting their overall losses by keeping their bids to a minimum. While theory suggests that a player will defend his lead if it is threatened, we find little evidence to support this hypothesis. We do find that the propensity to make a last stand or surrender leads to reasonable predictions about winning margins. Just as theory predicts, neck-and-neck tournaments are more likely when the losing penalty is relatively high, and landslide victories are more common with a relatively large prize. Finally, there are relatively few upsets with the winner of the initial battle going on to win the entire tournament roughly three-quarters of the time.

Given that last stands are fundamentally linked with significant loss—be it life, limb, liberty, or property—there is a natural challenge in designing an experiment which exposes subjects to a loss that is in some sense meaningful, yet minor enough to conform with standard institutional review board guidelines. We suggest, therefore, that any evidence of last stand behavior in our low stakes experiment would be magnified in situations that involve more substantive losses.

Appendix: Instructions

Thank you for your willingness to participate in this experiment. You will have the opportunity to earn some money as part of this experiment—the exact amount you earn will be based on both your choices and the choices of the other participants. Funding has been provided by the Economic Science Institute. You will be paid privately at the conclusion of the experiment.

In order to preserve the experimental setting, we ask that you DO NOT talk with the other participants, make loud noises, or otherwise disturb those around you. You will be asked to leave and will not be paid if you violate this rule. Please raise your hand if you have any questions.

There are two parts to this experiment.

Part 1

In the first part of the experiment, you will be given a set of 15 choices. You will be asked to choose between receiving \$1 for sure (Option A) and receiving \$3 with some probability and nothing otherwise (Option B). The probability of winning \$3 in Option B varies across the 15 choices. You will receive payment for one of your choices. The computer will draw a number between 1 and 15 at random, and you will be paid for your choice corresponding to that number. If you chose Option A, you will receive \$1. If you selected Option B, the computer will randomly draw another number between 1 and 20, and the result of that draw will determine whether you are paid \$3 or \$0.

Are there any questions?

Part 2

The second part of the experiment consists of 20 best-of-7 tournaments. In each tournament, you will be paired at random on the computer with another participant. The winner of each tournament will receive a prize and the loser will incur a penalty.

The currency for this part of the experiment is rupees, and the exchange rate is 50 rupees = 1 US dollar. As part of this experiment you have received an account with 850 rupees (equivalent to 17.00). This account is in addition to the 7.00 show up fee. The prize for winning a tournament is 150 rupees, and the penalty for losing is

Round	1	2	3	4	5	6	7
% of Tournaments	100%	90%	81%	73%	66%	59%	53%

150 rupees.

In order to win a tournament you must be the first player to win 4 contests. A contest consists of entering a bid on the computer screen. The computer will allow bids that are either whole numbers or have up to one decimal point that are between 0 and 300 inclusive. You win a contest if your bid is higher than your opponent's (in case a tie occurs, the computer will decide the winner randomly, giving each player a 50% chance of winning). Once both players have entered their bids, the computer will display the two bids and indicate which player is the winner. The computer will also display past bids and the total number of contests that each player has won so far in the tournament.

After each contest, there is a 10% chance that the tournament will suddenly end.

The computer will randomly determine whether or not to end the tournament by selecting an integer between 1 and 10 (each number is equally likely to be drawn). If a 1, 2, 3,..., 9 is drawn, then the tournament will continue, and you will return to the bidding screen to bid in another contest. However, if the computer draws a 10, then the tournament will end early. Numbers that the computer has drawn previously may be drawn again. Given that no player has won 4 contests, there is always a 90% chance of continuing to the next round of the tournament. The following table shows the percent of all tournaments that are expected to reach a given round provided that no player has won 4 contests by that round.

Earnings

Your earnings for each tournament are based on your bids and whether you win or lose the tournament. All of your bids throughout the tournament will be subtracted from your earnings. Please note that each of your bids will be subtracted regardless of whether you win or lose each contest.

The prize of 150 rupees will be added to your earnings if you win the tournament, and the penalty of 150 will be subtracted from your earnings if you lose.

Here are some examples to illustrate how your earnings for a tournament are calculated. If you win a tournament in six rounds, then your earnings are as follows:

150 - (Round 1 bid) - (Round 2 bid) - (Round 3 bid) - (Round 4 bid) - (Round 5 bid) - (Round 6 bid)

Similarly, your earnings for losing a tournament in five rounds are given below:

-150 - (Round 1 bid) - (Round 2 bid) - (Round 3 bid) - (Round 4 bid) - (Round 5 bid)

If the computer does end the tournament before one of the players has won 4 contests, then neither player receives a prize or incurs a penalty. However, your bids will still be subtracted from your earnings. For example, if the computer stops the tournament after three rounds, you earn the following:

- (Round 1 bid) - (Round 2 bid) - (Round 3 bid)

When a tournament ends, either by a player winning 4 contests or by the computer ending it early, the computer will display your earnings for that tournament. You will then be paired at random with another participant for the next tournament.

We ask for your patience as there may be a short pause between tournaments. This may happen, for example, if your tournament ended early, but your next randomly selected partner is still competing in a tournament.

Payment

At the end of the experiment, 2 of the 20 best-of-seven tournaments will be selected at random. Your payment will be based on the average of your earnings in those 2 tournaments. The average will be added to your 850 rupee account and then converted from rupees to dollars (50 rupees = 1 US dollar). Positive earnings will increase the balance in your account, while negative earnings will decrease it. You will be paid the balance of your account.

Quiz #1

Your account initially has 850 rupees. The winning prize is 150, and the losing penalty is -150.

Contest 1: Your bid: 45Your opponent's bid: 73Contest 2: Your bid: 92Your opponent's bid: 100Contest 3: Your bid: 21Your opponent's bid: 21

Tournament randomly terminated after 3rd contest.

How many rupees would you receive from this portion of the experiment if this tournament was selected for payment?

Quiz #2

Your account initially has 850 rupees. The winning prize is 150, and the losing penalty is -150.

Contest 1:	Your bid:	295	Your opponent's bid:	23
Contest 2:	Your bid:	70	Your opponent's bid:	150
Contest 3: ¹	Your bid:	51	Your opponent's bid:	40
Contest 4:	Your bid:	80	Your opponent's bid:	20
Contest 5:	Your bid:	72	Your opponent's bid:	80
Contest 6:	Your bid:	51	Your opponent's bid:	70
Contest 7:	Your bid:	200	Your opponent's bid:	175

How many rupees would you receive from this portion of the experiment if this tournament was selected for payment?

Quiz #3

Your account initially has 850 rupees. The winning prize is 150, and the losing penalty is -150.

Contest 1: Your bid: 27	Your opponent's bid: 295
Contest 2: Your bid: 41	Your opponent's bid: 150
Contest 3: Your bid: 200	Your opponent's bid: 40
Contest 4: Your bid: 20	Your opponent's bid: 78
Contest 5: Your bid: 31	Your opponent's bid: 83

How many rupees would you receive from this portion of the experiment if this tournament was selected for payment?

This is the end of the instructions. If you have any questions, please raise your hand and a monitor will come by to answer them. If you are finished with the instructions, please click the Start button. The instructions will remain on your screen until the experiment begins. We need everyone to click the Start button before we can begin the experiment.

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