Group Evolutionary Stable Strategy

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Abstract

We revisit in this paper the relation between evolution of species and the mathematical tool of evolutionary games which has been used to model and predict it. We indicate known shortcoming of this model that restricts the capacity of evolutionary games to model groups of individuals that share a common gene or a common fitness function. In this paper we provide a new concept to remedy this shortcoming in the standard of evolutionary games in order to cover this kind of behavior. Further, we explore the relationship between this new concept and Nash equilibrium or ESS. We indicate through the study of many examples as Hawk-Dove game, Stag-Hunt Game and Prisoner's Dilemma, that when taking into account a utility that is common to a group of individuals, the equilibrium structure may change dramatically.

I. Introduction

Evolutionary games become a central tool for predicting and even design evolution in many field. Its origins come from biology where it was introduced by Maynard Smith to model conflicts among animals. It differs from classical game theory by its focusing on the evolution dynamics of the fraction of members of the population that use a given strategy, and in the notion of Evolutionary Stable Strategy (ESS) which includes robustness against a deviation of a whole (possibly small) fraction of the population who may wish to deviate (this is in contrast with the standard Nash equilibrium concept that only incorporates robustness against deviation of a single user). It became perhaps the most important mathematical tool for describing and modeling evolution since Darwin. Indeed, on the importance of the ESS for understanding the evolution of species, Dawkins writes in his book "The Selfish Gene" [1]: "we may come to look back on the invention of the ESS concept as one of the most important advances in evolutionary theory since Darwin." He further specifies: "Maynard Smith's concept of the ESS will enable us, for the first time, to see clearly how a collection of independent selfish entities can come to resemble a single organized whole. In this paper, we identify inherent restrictions on the modeling capacity of classical evolutionary games apply.

We observe that since classical EG associates with an individual both the interactions with other individuals as well as the fitness, then it is restricted to describing populations in which the individual is the one that is responsible for the reproduction and where its goal in choosing its own strategies are completely selfish. In the biology, some species like bees or ants, the one who interacts is not the one who reproduces. This implies that the Darwinian fitness is related to the entire swarm and not to a single bee and thus, standard EG models excludes these species in which the single individual which reproduces is not the one that interacts with other individuals. Furthermore, in many species, we find altruistic behaviors, which may hurt the individual adopting it, favouring instead the group he belongs to. Altruistic behaviors are typical of parents toward their children: they may incubate them, feed them or protect them from predator's at a high cost for themselves. Another example is the stinging behavior of bees is an altruistic one: it serves to protect the hive, but its lethal for the bee which strives. In human behavior, many phenomena where individuals do care about other's benefits in their groups or about their intentions can be observed in the real word. Hence the assumption of selfishness becomes inconsistent with the real behavior of individual in a population. This however does not allow us anymore to model behavior at the level of the individual.

In this work we present a new model for evolutionary games, in which we distinguish between actors and players of the game i.e. the individuals which interacts and the group of individuals whose utility is maximized. We thus have two different levels, as we consider pairwise interactions among individuals, which can belong to the same group or to two different ones but we suppose that the utility they maximize is the one of the group they belong to.

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II. New natural concept on evolutionary games

In this section we present a new concept for evolutionary games, in which the idea of the player as a single individual is substituted by that of a player as a whole group of individuals. Let now focus on the case of monomorphic populations in which each individual uses a mixed strategy. We assume that the population is composed of N groups: G_i, $i = 1, 2, .., N$ where the size of G_i is noted by α_i with $\sum_{j=1}^{N} \alpha_i = 1$. For clarity of presentation, we restrict our analysis to pairwise interactions, where each individual can meet a member of its same group or of a different one. It has a finite set of available actions: $\mathbf{A} = \{a_1, a_2, .., a_M\}$. Let p_{ik} be the probability that an individual in the group G_i choses an action $a_k \in \mathbf{A}$ and we associate to each group i the vector $\mathbf{p_i} = (p_{i1}, p_{i2}, .., p_{iM})$ where $\sum_{l=1}^{M} p_{il} = 1$. By assuming that each individual can interact with another individual with equal probability, then the expected utility of a group (player) i is

$$U_i(\mathbf{p}_i, \mathbf{p}_{-i}) = \sum_{j=1}^{N} \alpha_j J(\mathbf{p}_i, \mathbf{p}_j) \tag{1}$$

where $\mathbf{p}_{-i}$ is the profile strategy of other groups and $J(\mathbf{p}_i, \mathbf{p}_j)$ is the immediate expected reward of a player playing $\mathbf{p}_i$ against an opponent playing $\mathbf{p}_j$.

The definition of Group Equilibrium Stable Strategy (GESS) is related to the robustness property inside each group. To be evolutionary stable, the strategy $\mathbf{q}$ must be resistant against mutations in each group. If the group G_i plays according to strategy $\mathbf{q}_i$, the $\epsilon-$deviation, where $\epsilon \in (0, 1)$, consists in a shift to the group's strategy $\bar{\mathbf{p}}_i = \epsilon \mathbf{p}_i + (1 - \epsilon \mathbf{q}_i)$. Thus we get the following equivalent definition of GESS:

Definition 1. *A strategy $\mathbf{q} = (\mathbf{q}_1, \mathbf{q}_2, .., \mathbf{q}_N)$ is a GESS if $\forall i \in \{1, \ldots, N\}$, $\forall \mathbf{p}_i \neq \mathbf{q}_i$, there exists some $\epsilon_{\mathbf{p}_i} \in (0, 1)$, which may depend on $\mathbf{p}_i$, such that for all $\epsilon \in (0, \epsilon_{\mathbf{p}_i})$*

$$U_i(\bar{\mathbf{p}}_i, \mathbf{q}_{-i}) < U_i(\mathbf{q}_i, \mathbf{q}_{-i}) \tag{2}$$

where $\bar{\mathbf{p}}_i = \epsilon \mathbf{p}_i + (1 - \epsilon)\mathbf{q}_i$. Hence from equation (2), the GESS can defined as the following manner

- *Either $\forall \mathbf{p}_i \in [0, 1]^M$*
$$F_i(\mathbf{p}_i, \mathbf{q}) := \alpha_i \Omega(\mathbf{p}_i, \mathbf{q}_i) - U_i(\mathbf{p}_i, \mathbf{q}_{-i}) + U_i(\mathbf{q}_i, \mathbf{q}_{-i}) > 0, \tag{3}$$
where $\Omega(\mathbf{p}_i, \mathbf{q}_i) := J(\mathbf{p}_i, \mathbf{p}_i) - J(\mathbf{p}_i, \mathbf{q}_i) - J(\mathbf{q}_i, \mathbf{p}_i)) + J(\mathbf{q}_i, \mathbf{q}_i)$.
- *or $F_i(\mathbf{p}_i, \mathbf{q}) = 0$ for some $\mathbf{p}_i \neq \mathbf{q}_i$, then*
$$\Omega(\mathbf{p}_i, \mathbf{q}_i) < 0 \tag{4}$$

Proposition 1.

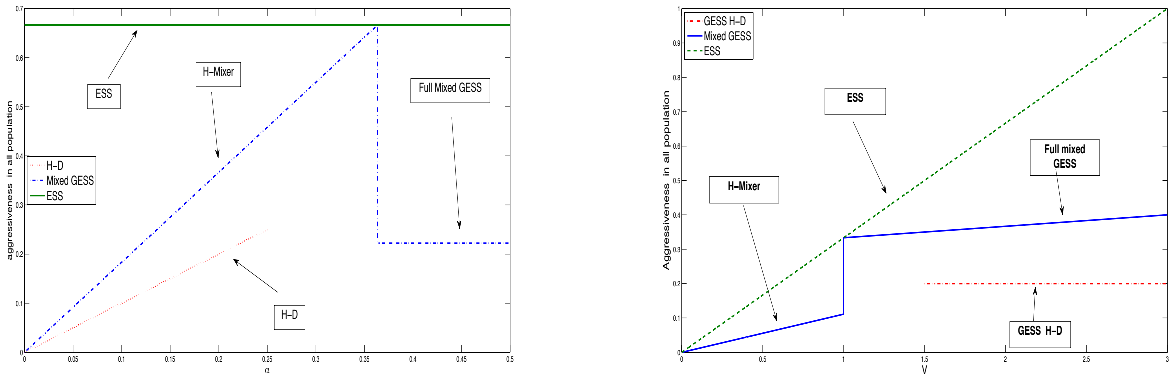
- *The condition (4) can be rewritten as $U_i(\mathbf{q}_i, \mathbf{q}_{-i}) > U_i(\mathbf{p}_i, \mathbf{q}_{-i})$, which is exactly the definition of the strict Nash equilibrium of the game composed by N groups and each group i maximises his utility U_i [2].*
- *Consider games whose payoff is symmetric $J(\mathbf{p}, \mathbf{q}) = J(\mathbf{q}, \mathbf{p})$, then any ESS is a GESS.*

From the definition of the strict Nash equilibrium in population games (see [2]), it is easy to show that any strict Nash equilibrium is a GESS defined in equation (2). But in our context, we address several questions on relationship between the GESS, ESS and the Nash equilibrium. For simplicity of presentation, we restrict to the case of two-group games with two strategies.

III. Some Examples

In this section we analyze a number of examples with two players and two strategies: the Hawk and Dove game, The Stag-Hunt game and the Prisoner's Dilemma.

Hawk and Dove Game The Hawk and Dove Game game allows to study the level of aggressivenes in the population. The two strategies of the game are the aggressive one H and the non-aggressive one D. We study this game in groups framework, considering two groups of size α and $1 - \alpha$. We fix in the payoff matrix $V = 2$ and $C = 3$, where V is the fitness related to the resource and C is the cost of the fight. We study the behavior of GESSs and of NEs as a function of the size of the first group α and we find that the NEs and GESSs always coincide. We also find interesing results when comparing the two-groups game equilibria to the standard Hawk and Dove game. In the latter, the pure (H,D) and (D,H) are NEs but they can't be ESSs, as from the definition of ESS one cannot consider asymmetric strategies. When considering groups game, our definition of GESS allows asymmetric strategies



(a) (b)

Fig. 1. The global level of aggressiveness for the different GESSs in the two-groups Hawk and Dove Game, as a function of α in 1(a) and of V in 1(b) with $C = 3$.

to be stale and in this case we obtain that for small (respectvely hig) values of α, (H, D) (respectively (D, H)) is a GESS. In standard Hawk and Dove, the unique ESS is the symmetric mixed one (q^*, q^*) with $q^* = V/C$; our game, only admits a mixed non-symmetric GESS (q_1^*, q_2^*) in an interval of α values close to $1/2$.

Stag-Hunt Game

This example represents a competition beetween safely and social cooperation. The two available strategies are $\{S, H\}$. In the two groups case we find that the strong GESSs and the NEs don't coincide. The two-groups Stag-Hunt Game only have the pure strict NE (S, S), for all values of α, whereas, we find that (S, S) and (H, H) always are strong GESSs. Strategy (S, H) is a GESS only in an interval of values of α, and we observe that the global rate of collaboration (Stag-strategy) is increasing in this interval. This indicates that our concept discovers many stable equilibria that are missed under Nash equilibrium concept used in population games [2]. Another relevant observation is about the impact of the structure of groups on the rate of collaboration. It seems that the global rate of collaboration (Stag-strategy) is increasing in this interval. It is interesting to remark that (S, S) (payoff dominant equilibrium) and (H, H) (risk dominant equilibrium) are the NEs of the standard Stag-Hunt game, which also admits a third non strict NE in mixed strategies. The Stag-Hunt two groups game never admits mixed GESS.

Prisoner's Dilemma

This examples shows the competition between cooperation and defection; the set of stragegies is thus $\{C, D\}$. As in the previous example, in the two groups game, we find strict GESSs which are not NEs. In particular, we find that strategy (D, D) is always a GESS but not a NE, for all values of α. If we compare our results to the standard evolutionary games, we note that in our game the pareto optimal solution, representing a cooperative behavior (C, C) always is a $GESS$, unlike the standard game whose unique equilibrium is (D, D). Also the partially cooperative behavior (C, D) (conversely (D, C) is a GESS for α sufficiently large (small). We also observe that, as in the previous example, the rate of cooperation in the population is increasing in the size of the cooperative group.

REFERENCES

[1] R Dawkins. *The Selfish Gene*, Oxford University Press, Oxford, UK, (1976)

[2] William H. Sandholm, *Population Games and Evolutionary Dynamics*, MIT Press, 2009.