The Recommendation Effect in the Hotelling Game - How Consumer Learning Leads to Differentiation*

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Abstract

Hotelling's famous 'Principle of Minimum Differentiation' suggests that two firms engaging in spatial competition will decide to locate at the same place. Interpreting spatial competition as modeling product differentiation, firms will thus offer products that are not differentiated and equally share the market demand.

We extend (a fixed price version of) Hotelling's model by introducing sequential consumer purchases and a second dimension of variation of the goods, quality. Consumers have differential information about the qualities of the goods and uninformed consumers observe the decision of their predecessors.

With this extension a rationale for differentiating products emerges: Differentiation makes later consumers' inference from earlier consumers' purchases more informative, so that firms are confronted with two offsetting effects. On the one hand, differentiating one's product decreases the likelihood that it is bought in earlier periods, but on the other hand, by making inference more valuable, it increases the likelihood that later consumers buy the differentiated good. We show that the second effect, the recommendation effect, can dominate, leading to an equilibrium with differentiated products. Our model thus introduces an aspect similar to the herding literature in that consumers might base their decisions on observable actions of others and thus potentially on 'wrong' decisions.

Keywords: Hotelling, Herding, Principle of Minimum Differentiation, Consumer Learning

JEL codes: L13, L15, D83

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1 Introduction

The probably most popular result in spatial product differentiation is the Principle of Minimum Differentiation by Hotelling (1929). In his original model, two firms compete on a linear and bounded market by first choosing their locations and then setting their prices. Hotelling argued that in equilibrium both firms locate at the center and set the same price.

It has later been shown by D'Aspremont et al. (1979) that the celebrated Principle of Minimum Differentiation - in opposition to Hotelling's conjecture - is only valid when prices are exogenously set and fixed. With fixed prices the firms' goals reduce to serving the largest possible market share. To this end, given the other firm's position, a firm locates on the longer side of the market as close as possible to the other firm. Given that firms are located directly next to each other, the only situation without any incentive to relocate is the one where both firms are located at the center. As this equilibrium is the one Hotelling conjectured, we will refer to this setup, with exogeneous prices, as the 'fixed-price Hotelling model'.

We modify this fixed-price Hotelling model in several aspects. Assuming sequential location choice of the firms constitutes the first departure. The second and more important change is the introduction of a second dimension of differentiation, which we interpret as quality. Firms thus offer products that are horizontally and vertically differentiated. Consumers are either completely informed or uninformed about the good's qualities, which is the third modification. Finally, consumers make their purchase and observe the previous consumer's choice behavior.

While we will call the second dimension of differentiation 'quality' in the remainder of the article, other aspects of a good can very well fit this model aspect, too. In general this can be any characteristic of a good, where all consumers agree on the ranking of its different manifestations.

We show that introducing a second dimension of differentiation about which consumers have differential information and the possibility of consumer learning generates a 'recommendation effect' possibly driving firms to offer horizontally differentiated products. This is in contrast to the 'Principle of Minimum Differentiation'.

If later consumers (laggards) are uninformed, they use the observed choices of antecedent consumers (early adopters) to infer information about the good's qualities. This updating process crucially depends on the firms' locations, as an uninformed early adopter will base his decision only on the traveling costs to the two firms (which he perceives as homogeneous). To build intuition suppose that both firms are located at the center and consider the incentives to deviate. A firm which moves away from the center decreases the probability of being chosen by uninformed early adopters, while it increases the probability to be chosen by uninformed laggards. This is because, whenever the deviating firm in this setup is chosen in the first period, it is more likely that this is due to a better quality, which is known to informed early adopters. Hence, uninformed laggards will now tend to follow the early adopters' decisions more often, this is the aforementioned 'recommendation effect'. If the additional demand from laggards outweighs the lost demand from early adopters, the total demand is increased by moving away from the opponent located at the center.

A critical assumption in our (and, as the name suggests, in the fixed-price Hotelling) setup is the one of exogenous prices. Usually prices are considered flexible and are seen as an endogenously chosen component of the market competitors' strategies, but there are nonetheless some examples where the assumption of fixed prices seems plausible. This can be the case either if prices are actually fixed or if price differences among different products are perceived as too small to have an influence on the consumption decision (see for example Courty, 2000). Consider the movie industry: the entrance fees for blockbusters of the same length at cinemas are usually the same.[1] A recent event in this area very well fits our model. The movie 'The Artist', which aired in cinemas in 2011, was a major success of that year and in addition to receiving mainly positive critique it won numerous prizes, including five Oscars.[2] It brought in almost \$ 133.5 Mio. worldwide, while being produced with a \$ 15 Mio. budget.[3] So both - artistically and economically - it was a major success. What makes this movie especially interesting for our case however, is that compared to the advanced techniques commonly used in cinemas nowadays with its 3D-effects and 'Dolby Surround', the means used for the shooting of 'The Artist' were rather unconventional: it was entirely shot in black-and-white and mainly abstracted from dialogues, almost making it a silent movie. Thus, one can say that, compared to the other blockbusters at that time, this movie was rather a 'niche product'. Yet, it may well be that the high popularity of this unconventional movie among the early adopters in the first weeks of broadcasting induced the laggards to attribute the reason for that choice behavior to the high cinematographic quality of 'The Artist'. Probably the producers anticipated just this reasoning and therefore decided to dive into this rather unorthodox project.

Note that we use the term 'niche product' in the sense that such a product is of relatively low appeal to uninformed consumers. As our model shows, and the example of the movie 'The artist' illustrates, a niche product according to this definition can still generate a larger demand than a mainstream product ex-post. In our model this is due to the recommendation effect, which generates a higher demand by laggards for the 'niche product' than for the mass product.

[1] See for instance Orbach and Einav (2007). Many people arguably decide on which movie to watch before seeing the prices. They most probably do not revise their decision when finding out that prices are slightly different than expected.

De Vany (2006) discusses the three different pricing levels of the movie industry (producers, distributors, box offices) extensively and shows that empirically box office prices are fixed - which indeed is an economic puzzle. Additionally it is shown that the producers obtain a contractually regulated share of the revenues generated by the box offices. This implies that the only way how producers can influence their revenue is by generating a large audience.

[2] See http://articles.economictimes.indiatimes.com/2012-02-27/news/31104573_1_oscars-foreign-language-category-actor-race and http://www.theguardian.com/film/2011/dec/08/artist-silent-film-michel-hazanavicius.

[3] See http://www.boxofficemojo.com/movies/?id=artist.htm and http://www.imdb.com/title/tt1655442/business?ref_=ttrel_ql_4.

It seems plausible that enterprises attribute more and more attention to the behavior of early adopters nowadays, as these find a growing number of opportunities to publicly announce their choice behavior and the underlying motivation using internet platforms such as yelp or the recommendation opportunities on the online market place amazon for instance.

The paper at hand is structured as follows. In Section 2 we review the related literature. Section 3 introduces the model setup. The benchmark model with simultaneous consumer choice is discussed in Section 4. In Section 5 we illustrate our main result by the analysis of a model with sequential consumer choice, while Section 6 concludes. The proof of the main theorem can be found in the Appendix.

2 Literature

The seminal paper on **spatial competition** and product differentiation is Hotelling (1929).[4] This early work studies competition between two firms who simultaneously set prices and choose their products' characteristics represented by locations in a bounded, linear market. Consumers are heterogeneous with respect to their preferred product characteristic, i.e. the location of the firm. In Hotelling's model, consumers incur a disutility that increases linearly in the distance between themselves and the firm. He proposed that in an equilibrium of this game, firms set the same price and choose the same location, namely the center.

D'Aspremont et al. (1979, p. 1145) later argued that this "so-called Principle of Minimum Differentiation [...] is invalid". They show that whenever the distance between firms' locations is small, they have an incentive to slightly undercut the rival's price. As in any model of spatial competition, there exist two offsetting effects in Hotelling's setup. On the one hand, firms have an incentive to increase the distance, thereby relaxing competition ('competition effect'). On the other hand, decreasing the distance allows to serve a larger share of the market ('market size effect').[5]

Hotelling seemingly did not consider the 'competition effect' and concentrated only on the 'market size effect'. This is evident in his proposed equilibrium, where both firms choose the same location. The situation is then essentially the same as in a 'Bertrand Model' of price competition. In a Bertrand model each firm would rationally undercut the competitor's price as long as the competitor's price is above marginal costs of production, leading to a situation where the price equals marginal costs and firms make zero profits. D'Aspremont et al. (1979) show that this can not happen in Hotelling's original model, because a firm could relocate further away from its competitor, thereby generating some

[4] Alternatively, models of spatial competition can be interpreted in a political science context, where the analogy to the Principle of Minimum Differentiation is the Median Voter Theorem. It states that two political parties - in order to best please the median voter's crucial taste - will always position in the middle of the political spectrum. The seminal work here is Black (1948). Osborne (1995) offers a more recent survey.

[5] See for example Belleflamme and Peitz (2010, Chapter 5.2).

market power, which would then allow to set a positive price and make profits. Furthermore they show that in Hotelling's setup no Nash equilibrium in pure strategies exists and propose a variation of the original model which entails quadratic 'transport costs'.[6] The result of maximal differentiation is a consequence of the assumed setup.

In contrast to the work of d'Aspremont et al. (1979), De Palma et al. (1985) defend the Principle of Minimum Differentiation using a model where consumers' preferences regarding the firms' products are even more heterogeneous, allowing two consumers with the same location to have different utilities from the same product. This grants some market power to firms at any location making it optimal to cluster around the center.

Empirical evidence concerning spatial differentiation can be found in Borenstein and Netz (1999). Additionally, Lieberman and Asaba (2006) surveys the empirical findings on imitation among firms.

In the literature on **social learning**, Bikhchandani et al. (1992) and Banerjee (1992) were the first to examine the phenomenon of **herding**. They show that in sequential consumer choice rational Bayesian inference from the previous behavior of others may guide consumers to ignore their own (imperfect) private signal on the quality of a firm; a behavior which in the end may result in informational cascades driving all subsequent consumers to buy only from one firm.

Another strand of literature has taken a look at the impact of social learning among consumers on competition among firms producing horizontally differentiated products. In Caminal and Vives (1996)) - as in our model - firms do not know the quality of their product. Consumers are homogeneous but have different information and they observe the history only partially. Given the incomplete observation of the history, consumers are led to believe that a good is of higher quality whenever its market share is high. The authors show that this leads to a strategic incentive for the firms to generate a higher demand in early periods by setting a low price.

The paper at hand combines the literature on social learning and spatial competition. Another paper which combines the ideas of **Hotelling and herding** is Ridley (2008), however his research question is fundamentally different to ours: He models two firms with different information levels about market demand and - as they sequentially decide about entering the market - the second mover can possibly deduce information from the other firm's decision.

Tucker and Zhang (2011) empirically show that - in line with the intuition of our theoretical results - popularity information (indicated by the choice of previous consumers) is especially beneficial for niche products, because for the same popularity, niche products are

[6] The advantage of using quadratic costs is that it eliminates discontinuities that occur in a model with linear transport costs. With linear costs, discontinuities arise because minimal changes of the locations can suddenly make a mass of consumers become indifferent between both products. In the model with quadratic transportation costs firms locate at the extremes of the linear market, thereby maximizing differentiation. For a discussion on discontinuous economic games, see Shaked (1982), Dasgupta and Maskin (1986a) and Dasgupta and Maskin (1986b).

more likely to be of superior quality than mainstream products.

An empirical paper is even more suitable for our analysis - and especially our example on the movie industry with 'The Artist': Moretti (2011) is the first one to analyze real world data on social learning: he investigates in how far it influences movie sales. The results show that social learning indeed matters and that 'surprise' in the early demand increases later demand for a movie. That is, if a movie was seen by surprisingly many consumers (compared to the prior) in the first weeks of airing, this will have the (indirect) effect of a social multiplier: while it also immediately increases profits to the cinemas, it also generates a higher demand in the following periods. We can infer that this yields an incentive for movie producers to create 'surprising' movies in the sense that they are very successful in the first weeks compared to a (common) prior. This may just be the reason to produce a black and white silent movie nowadays, indeed the director of 'The Artist', Michel Hazanavicius, said that when he presented his idea "[he'd] only get an amused reaction - no one took this seriously".[7]

Our contribution to the literature is to show in a theoretical model how the firms' incentives to differentiate are affected by social learning among heterogeneous consumers.

3 Model Setup

The following describes the model setup, which is a generalization of the before mentioned fixed-price Hotelling game.

The model has two firms A and B which produce (potentially) differentiated goods. Both firms produce at zero costs, and the retail price is regulated and set to p. The firms' locations, that describe their products' characteristics, are confined to the unit interval and denoted by a and b for firm A and B respectively.[8] Location choice of the firms occurs sequentially, with firm A choosing its location first, and firm B following. The situation if A chooses $a < 0.5$ is equivalent to a situation where A chooses $\tilde{a} = 1 - a$ instead. Thus it is without loss of generality to restrict $a \in [0.5, 1]$.

A firm's profit simply is the number of consumers served, multiplied with the regulated price p. It should be noted that introducing discounting future profits does not alter the results qualitatively, and it is thus left out for simplicity.

Besides the horizontal differentiation as measured by the firms' locations, the goods are also of different 'value' to consumers. This value can be thought of as representing a good's quality. The value of firm A's product is common knowledge and given by $v_a = v$.

[7] See page 5 of the official press kit at: `http://www.festival-cannes.com/assets/Image/Direct/041859.PDF`. Additionally the success of the movie was called surprising by the media 'surprising', see e.g. `http://www.theguardian.com/film/2012/feb/04/hollywood-nostalgia-chaplin-valentino`.

[8] Note that we use a different notation than Hotelling (1929): we let a and b be the distance from the left end of the unit interval, while Hotelling uses a as the distance to the left and b as the one to the right end.

The second firm's quality v_b is randomly determined after the firm has chosen its location, and is either $v_b = v + \delta$ or $v_b = v - \delta$, both of which occur with probability 0.5.[9]

On the other side of the market, there are 2 consumers with heterogeneous preferences, who sequentially make their purchase decisions in periods $t = 1$ and $t = 2$. Both consumers buy at most one good and will be referred to by the period they have the opportunity to make a purchase. Heterogeneity is modeled by assuming that each consumer t and each product i is described by a location on the unit interval. In every period, the location of consumer t, x_t is drawn from a uniform distribution on $[0, 1]$. It measures the consumer t's preference towards a good located at i. The closer the location of the consumer and the good she buys (holding everything else constant), the higher is the resulting utility.

In each period t, one consumer has the possibility to buy a good from one of the two firms and obtains the following utility when buying a product from firm i:

$$u(x_t, i) = v_i - p - \tau|x_t - i|$$

If a consumer abstains from buying either good, she receives a utility which we normalize to zero. Note that this utility function implies risk-neutrality. As long as preferences are quasilinear and the expected utility of both firms (gross of prices and transportation costs) are the same, the results would not change. Let $C_t \in \{A, B, \emptyset\}$ denote the decision of the consumer in period t, where $C_t = \emptyset$ if the consumers in period t abstained from buying and C_t equals the name of the firm the consumer bought the good from, otherwise.

Additionally to the heterogeneous preferences, consumers differ in the information they possess about firm B's product. Informed consumers observe the realization of v_b, whereas uninformed consumers only have the prior information that $v_b = v \pm \delta$ with probability 0.5. In each period $t \geq 1$ consumer t is informed with probability q and uninformed with probability $(1 - q)$. Consumer t's informational level is independent of his location x_t and of the information other consumers possess.[10]

In the second period, the consumer observes the actions taken by her predecessor, but not her location or whether she was informed.

The following simplifying assumption will be made about consumers behavior.

Assumptions.

1. *Every consumer prefers to buy one good to not buying any good.*

2. *All informed consumers buy from the firm with higher v_i.*

[9] The exogeneity of quality seems plausible in many cases. For instance, concerning the example of the movie 'The Artist' and the movie industry in general, it may well be that producers can not completely influence the (perceived) quality of a movie. See also De Vany (2006) on this aspect of the movie industry. Also note that movies - and many other goods - are experience goods (see e.g. De Vany, 2006), whose value is revealed to consumers only after their consumption. In many cases even firms arguably do not know their product's quality (or the consumers' perceived quality) ex ante.

[10] In our example of the movie 'The Artist' the signal may be a trailer or a review article about the movie in a newspaper.

The first assumption simply guarantees that the market is covered. The second assumptions implies that the choice of informed consumers is independent of their type. It is shown in the Appendix that the second assumption is without loss of generality. The assumption is made only for ease of exposition, in particular it simplifies the updating of an uninformed consumer in period 2. Similar reasoning applies to the assumed information levels of consumers and one could well assume more than two possible levels of information so that consumers would not either be completely informed or completely uninformed. This however would complicate the Bayesian updating in the same way as introducing the fact that not all informed consumers will choose the high quality product. Therefore, we leave out such specifications to focus on the main issue under consideration.

The assumption that firms do not know their (relative) quality ex ante directly implies that the firms can not signal information about the realized quality to consumers. If their location choice would signal information to consumers, i.e. in a separating equilibrium, the information would already be revealed before the first consumer's choice and social learning would not occur. Since the effects of social learning by the consumers on the firm's location choices are exactly what we are interested in, situations with separating equilibria are not of our primary interest. We thus impose the assumption that firms are unaware of their quality in order to make sure that the social learning of interest is possible. Furthermore, with Assumption 2 it is directly clear that no separating equilibrium can exists as this would mean that the 'low-type' firm would be identified by its location and it would make zero profits. Given that the 'high-type' firm makes positive profits, it would always be imitated by the 'low-type' firm, thus ruling out the existence of a separating equilibrium if Assumption 2 is imposed. Nevertheless, we show in the Appendix C that, without Assumption 2, a separating equilibrium exists only for specific parameter constellations and that it can be 'refined away' using the concept of 'Perfect Sequential Equilibrium' (Grossman and Perry, 1986).

We conjecture that our result are also robust against a modification of the model where the quality of the products is revealed publicly after some time, e.g. due to the fact that information has spread among all consumers via communication. If all consumers know the qualities, only the high quality product is consumed. Hence, what matters in the spatial competition among firms are the periods with asymmetric information among the consumers before the qualities become public.

The figure below depicts the timing of the game:

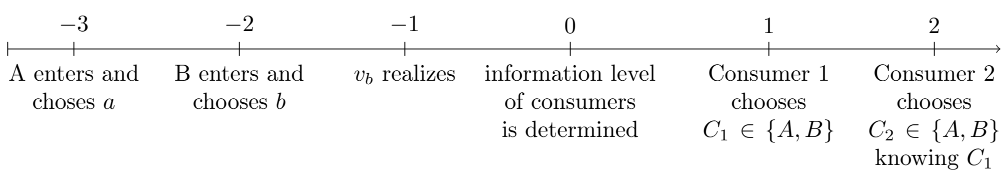


Figure 1: Timing of the Game

As the next sections will show, the sequential structure of consumers' decisions, together with the possibility to learn from previous consumers' actions, can drastically change the outcome of the game compared to the standard Hotelling model with exogeneous prices.

To this end, it will be shown in Section 4, that without this possibility to learn from other consumers' actions, the result will be the usual 'minimum differentiation' result obtained by Hotelling. In the then following section, consumers are given the possibility to learn from their predecessors' actions, and it will be illustrated that this can lead to product differentiation.

4 Benchmark: Simultaneous Consumer Choice

As a benchmark, we first look at a model in which consumers are unable to infer information from other consumers' actions. To take away the learning possibility from consumers in this section, it is assumed that the second period consumer simply does not observe other consumers' actions, which is essentially the same as letting consumers decide simultaneously. In all other aspects, the model remains unchanged.

The model can then easily be solved by backwards induction, which yields the optimal behavior of the agents. Starting with the consumer side, by assumption, all informed consumers buy from the better firm. It remains to characterize the decision of uninformed consumers. Because the products have the same expected quality to uninformed consumers and they are offered at the same price, they make their purchase decision based on which firm's product fits their preference best, i.e. they choose the firm whose product is located closer to their type x_t.

Let $\tilde{x}_t$ be the threshold, such that all types $x_t \leq \tilde{x}_t$ choose the product of firm B.

An uninformed consumer in period $t \geq 1$ is indifferent whenever her type x_t equalizes her (expected) utility for the products of both firms. Two cases can occur. Either - in the first case - all consumers are indifferent, this happens whenever both firms are at the same location ($a = b$). In this case it is assumed that $\tilde{x}_t = b = (a + b)/2$, i.e. all consumer types left of B's location choose B's product.[11]

Or - in the second case - with $b < a$,[12] the (unique) indifferent consumer is located in

[11] Assuming that indifferent consumers follow this strategy is not necessary for the result of 'minimum differentiation', and is only assumed since it guarantees the existence of a best response of firm B to any location of firm A.

[12] Note that $b > a \geq \frac{1}{2}$ can not be optimal, since B's demand equals $2 \cdot (1 - \tilde{x})$ in this case, which is smaller than $2 \cdot (1 - \frac{1}{2})$, as $\tilde{x} > \frac{1}{2}$ due to assuming $b > a \geq \frac{1}{2}$. For any $a \geq \frac{1}{2}$ firm B can guarantee itself $D_B(a, 0.5) \geq 2 \cdot (1 - \frac{1}{2})$.

(a, b). In this case $\tilde{x}_t$ is equal to the type of the indifferent consumer, which means

$$\begin{aligned} & u(A, \tilde{x}_t) = EU(B, \tilde{x}_t) \\ \iff \quad & v - p - \tau(a - \tilde{x}_t) = v - p - \tau(\tilde{x}_t - b) \\ \iff \quad & \tilde{x}_t = \frac{a+b}{2} \end{aligned}$$

Clearly, all consumer types left of $\tilde{x}_t$ prefer firm B's product and all to the right of the indifferent consumers prefer product A, so that (due to the restriction to the unit interval) the probability of an uninformed consumer buying product B equals $\tilde{x}_t$, independently of the realization of v_b.

Anticipating consumers' strategies and observing the location of firm A, firm B will choose its location such that as many consumers as possible are located closer to b than to a. In other words, firm B will choose b in a way that maximizes its (probabilistic) market share (of uninformed consumers). Because the probabilistic market division is the same for uninformed consumers in each period $t > 0$ it suffices to maximize the probability that B is being chosen by uninformed consumers in one period t. This is achieved by locating at the same position firm A is located at, making all uninformed consumers indifferent and dividing the market into two parts, one left of both firms and one on the right. With the assumed tie-breaking rule and locations $b = a$, the probability that firm B is chosen by uninformed consumers is given by a by assumption. B has no incentive to relocate, since for any locations $b < a$, the indifferent consumer type is located at $(a + b)/2$ and then all consumer types left of the indifferent consumer will purchase from B resulting in a probability of $(a + b)/2 < a$ that uninformed consumers choose B. This shows that $b = a$ is firm B's best response to any location a.

Having characterized the strategies of consumers and firm B, it remains to find the optimal location for firm A. Location a is chosen to maximize the probability that uninformed consumers choose firm A, given B's best response of $b = a$. With locations $a = b$, the probability that A is chosen by uninformed consumers is $(1 - a)$ and maximizing it is equivalent to minimizing the probability that B is chosen, a, because by assumption no consumer chooses her outside option. Clearly a is minimized (in the admissible range) by setting $a = 0.5$, leading to Hotelling's familiar 'minimum differentiation' result, which is summarized in the following proposition:

Proposition 1 (Hotelling)**.** *In the model with simultaneous consumer choice, firms do not differentiate their products and equilibrium locations are* $a = b = 0.5$.

5 Sequential Consumer Choice

Adding back in the consumer's ability to observe her predecessor's action, new effects arise in the model.

Certainly informed consumers can not benefit from observing other consumers' actions, and as a result, their strategies remain as in the previous sections. Observing the predecessors' actions becomes useful for uninformed consumers, because of the possibility that the previous consumer's decision was made by an informed consumer. Hence, history C_1 can now contain information that allows an uninformed consumer in period $t = 2$ to update her estimate of which firm produces the good of higher value via Bayes' Rule. All uninformed consumers in period 2 will then use their updated estimates when deciding which good to buy.

5.1 Updated Probabilities and Indifferent Consumers

With the restriction to two periods,[13] the interesting question is what an uninformed consumer in period $t = 2$ (laggard) can infer from the choice of the consumer in $t = 1$ (early adopter) about the value of firm B's good. More precisely, the consumer in period 2 will use history C_1 to calculate the probability that firm B is the higher quality firm, that is $v_b > v_a = v$, using Bayes' Rule as follows:

$$p_b(C_1) := Pr(v_b > v|C_1) = \frac{Pr(C_1|v_b > v) \cdot Pr(v_b > v)}{Pr(C_1)}$$

As the formula shows, the strategy (and thus the probabilities of $C_1 = A$ and $C_1 = B$) of the consumer in $t = 1$ needs to be calculated first.

First Period

By assumption, informed consumers choose firm B if $v_b > v$ and A otherwise. Because both products are equally likely to be superior from an ex-ante perspective, before v_b is realized, the probability that a consumer chooses product B, given she is informed, equals $Pr(v_b > v_a) = 0.5$. From the perspective of the uninformed consumer in period $t = 1$, both firms are symmetric and she behaves as the uninformed consumers in $t > 0$ in the benchmark model, meaning the threshold type is given by:

$$\tilde{x}_1 = \frac{a+b}{2}.$$

This threshold characterizes the behavior of uninformed consumers. To summarize, if the consumer in period 1 is informed, her optimal strategy is given by:

$$C_1 = \begin{cases} B & \text{if} \quad v_b > v \\ A & \text{else} \end{cases}$$

[13] Since the updated probabilities are different for each history of the game, the indifferent consumer type is also potentially different for each history, meaning that in each period $t > 1$, 2^{t-1} indifferent consumers have to be determined, quickly making the model intractable. The effects we wish to characterize are already apparent with one period of updating, i.e. with two consumers, which is why we concentrate on this case.

and if she is is uninformed by

$$C_1 = \begin{cases} B & \text{if} \quad x_1 \leq \tilde{x}_1 \\ A & \text{if} \quad x_1 > \tilde{x}_1. \end{cases}$$

Combining those observations with the probability q that the consumer in period 1 is informed, $Pr(C_1 = B)$ and $Pr(C_1 = B|v_b > v)$ can be calculated as:

$$\begin{aligned} Pr(C_1 = B) &= q \cdot Pr(v_b > v) + (1-q)Pr(x_1 \leq \tilde{x}_1) = \frac{q}{2} + (1-q)\tilde{x}_1 \\ Pr(C_1 = B|v_b > v) &= q + (1-q)Pr(x_1 \leq \tilde{x}_1) = q + (1-q)\tilde{x}_1 \end{aligned}$$

Obviously, the higher quality firm is more likely to be chosen in period 1, as $Pr(C_1 = B|v_b < v) < Pr(C_1 = B) < Pr(C_1 = B|v_b > v)$. We can now combine the previous calculations to get the updated estimate of an uninformed period 2 consumer about the likelihood that firm B's product is superior having observed that the first period consumer bought the product B. Using Bayes' Rule it is given as follows:

$$\begin{aligned} p_b(B) := Pr(v_b > v|C_1 = B) &= \frac{Pr(C_1 = B|v_b > v) \cdot Pr(v_b > v)}{Pr(C_1 = B)} \\ &= \frac{(q + (1-q)\tilde{x}_1) \cdot \frac{1}{2}}{\frac{1}{2}q + (1-q)\tilde{x}_1} = \frac{q + (1-q)\frac{(a+b)}{2}}{q + (1-q)(a+b)} \end{aligned}$$

The updated probability after observing $C_1 = A$ can be calculated similarly, and is given by:

$$p_b(A) = Pr(v_b > v|C_1 = A) = \frac{(1-q)\frac{1}{2}(2-a-b)}{q + (1-q)(2-a-b)} = 1 - p_b(B)$$

As one would expect, $p_b(B) \geq 0.5 \geq p_b(A)$, meaning that observing $C_1 = B$ $(C_1 = A)$ increases (decreases) the probability that B sells the good of higher value. One can observe, that when all consumers are informed ($q = 1$), it is the case that $p_b(A) = 0$ and $p_b(B) = 1$, implying that the choice behavior of consumers is perfectly informative. On the other hand, setting $q = 0$, i.e. no consumer is informed, implies that 'updated' probabilities equal the prior: $p_b(A) = p_b(B) = \frac{1}{2}$.

An interesting observation that can be made with regard to the updated probabilities, is that differentiating has two useful effects for the firm. Suppose that for a fixed location a, firm B considers increasing the differentiation to A's product. With the assumption $b \leq a$ this means that B considers moving its location further to the left, i.e. decreasing b. Increasing the product differentiation, which can be interpreted as producing a 'niche product', makes it less likely that product B is chosen by uninformed consumers in the first period, thus $Pr(C_1 = B)$ and $Pr(C_1 = B|v_b > v)$ both get smaller. But as the effect is more pronounced for the unconditional probability $Pr(C_1 = B)$, the updated probability that B's product is superior given it was chosen in the first period, $p_b(B)$, increases. This mechanism lays the foundation for the 'recommendation effect'. Intuitively, since a niche product is a good match to relatively few consumer types (compared to a mainstream product), if it was chosen in $t = 1$, it is more likely that this was due to superior information about the quality than due to a better match of the product's characteristic and the consumer's taste.

An opposing effect is created regarding the updated probability, $p_b(A)$. With A being the mainstream product (compared to B), an observed choice of it in the first period was more likely induced by a good match (small distance of x_1 and a) than by an informed consumer.

Put differently, the increased confidence that a product is of superior quality after having observed that it was bought in the first period, is higher for niche products than for mainstream products. Those are precisely the effects for which Tucker and Zhang (2011) find empirical evidence by examining the usefulness of popularity information for what they call products of 'narrow' and 'broad appeal'.

Second Period

Because informed consumers already have perfect information about both goods, an informed consumer in $t = 2$, behaves as an informed consumer in $t = 1$. Therefore the following will concentrate on the optimal decision of an uninformed period 2 consumer.

In the first period, the products of both firms have the same expected utility (gross of transportation costs and for uninformed consumers), because the only available information on the relation of the goods' valuations is the prior information $Pr(v_b > v) = 0.5$. That is, in period 1 there is a symmetry in the expected valuations of the products. This however is not the case in the second period. The previous calculation showed how consumers in period 2 rationally update this prior probability as they observe the choice made in period 1. Hence, when comparing the utility of buying good A to the expected utility from purchasing firm B's product, the updated probability $p_b(C_1)$ must be used when calculating this expected utility. The updating introduces an asymmetry in expected valuations of the products, implying that in contrast to period $t = 1$, it is possible that no type of consumer is indifferent between the products.

It is thus necessary to distinguish three cases, either the unique indifferent type is in between (a, b) or the consumer of type $x_2 = a$, respectively $x_2 = b$, is indifferent between products. In the latter cases all types right of a, respectively left of b, are also indifferent; the intuition behind this will be explained in more detail below.

We will start with assuming that there is a single consumer type who is indifferent between both firms and this unique indifferent type is in (a, b). B's expected demand in the second period from uninformed consumers, who observed the choice C_1, is equal to the indifferent consumer in this case, denoted by the threshold $\tilde{x}_2(C_1)$, and characterized as follows:

$$
\begin{aligned}
& & u(A,\tilde{x}_2(C_1)) =& EU(B,\tilde{x}_2(C_1),C_1) \\
\iff & & v-p-\tau(a-\tilde{x}_2(C_1)) =& p_b(C_1)[v+\delta-p-\tau(\tilde{x}_2(C_1)-b)] \\
& & & + (1-p_a(C_1))[v-\delta-p-\tau(\tilde{x}_2(C_1)-b)] \\
\iff & & \tilde{x}_2(C_1) =& \frac{a+b}{2}+\frac{\delta\,(2p_b(C_1)-1)}{2\tau} \\
\iff & & \tilde{x}_2(C_1) =& \tilde{x}_1+\frac{\delta}{\tau}\Big(Pr(v_b>v|C_1)-Pr(v_b>v)\Big)
\end{aligned}
$$

The uninformed indifferent consumer can nicely be interpreted, in that it is the first period's indifferent type, shifted to the left (right) by a term that weighs the product of the additional likeliness that B is the superior firm, if the choice in the first period was firm B (firm A) and the excess utility from choosing the better product against the additional transport costs.

We will now come back to the two cases, where there is no unique indifferent consumer type. This ocurrs, if the consumer of type $x_2 = a$, respectively $x_2 = b$, is indifferent between the products or strictly prefers B's product, respectively A's product. As the calculation of $\tilde{x}_2(C_1)$ showed, the indifferent type in period 2 can be shifted both to the left and to the right of the center between firms' locations.

If, given the choice in period 1, C_1, the type $x_2 = b$ is indifferent, all consumer types left of b are also indifferent between both products. This is the case because compared to a type located at b, consumers left of b have to incur the same additional transport costs when buying from A instead of buying from B as the consumer located at b. This also implies that whenever the consumer type $x_2 = b$ prefers to buy from firm A, all consumers prefer to buy good A. In such a situation, we will set $\tilde{x}_2(C_1) = 0$. It is useful to remember that $\tilde{x}_2(C_1)$ is also equal to firm B's probablistic market share generated in period 2 by uninformed consumers after they observed history C_1. In the literature on social learning 'herding' is defined as a behavior, where an agent's action is independent of his private signal: all information she uses comes from the (possibly updated) public belief derived from the behavior of others. In our setup we can interpret the situation described above as an analogue to herding behavior: independent of her own position (i.e. the taste), an agent chooses to buy from one firm and all information used comes from the observed behavior of other consumers.
Similarly, whenever $x_2 = a$ is indifferent (observing C_1), all consumers to the right of a are also indifferent, and if $x_2 = a$ prefers firm B, all consumers prefer firm B, and we will set $\tilde{x}_2(C_1) = 1$. Thus, the structure of $\tilde{x}_2(C_1)$ is discontinuous.

Given the calculation of $\tilde{x}_2(C_1)$, we can now describe the optimal strategy of a period 2 consumer. If the consumer in period 2 is informed, it is given by

$$
C_2 = \begin{cases} B & \text{if} \quad v_b > v \\ A & \text{else} \end{cases}
$$

and, if she is is uninformed it is (depending on the history) given by

$$C_2 = \begin{cases} B & \text{if} \quad C_1 = B, x_2 \leq \tilde{x}_2(B) \quad \text{or} \quad C_1 = A, x_2 \leq \tilde{x}_2(A) \\ A & \text{if} \quad C_1 = B, x_2 > \tilde{x}_2(B) \quad \text{or} \quad C_1 = A, x_2 > \tilde{x}_2(A) \end{cases}$$

5.2 Firms' Expected Demands

The previous subsection characterized the consumers' optimal strategies depending on their types and level of information. Those can be used to calculate the firms' expected demands, depending on the locations a and b. Firm B's expected demand will be denoted by $D_B(a, b)$. With the assumption that every type of consumers buys one good, A's demand equals $D_A(a, b) = 2 - D_B(a, b)$, hence we only need to consider B's demand in what follows.

The expected demand consists of the probabilistic demand in both periods, thus:

$$\begin{aligned} D_B(a,b) = & \overbrace{\frac{q}{2} + (1-q)\tilde{x}_1}^{\text{demand in first period}} \\ & \underbrace{+ \frac{q}{2} + (1-q)\left[Pr(C_1 = B)\tilde{x}_2(B) + Pr(C_1 = A)\tilde{x}_2(A)\right]}_{\text{demand in second period}} \\ = & \, q + (1-q)\Big[\tilde{x}_1 + Pr(C_1 = B)\tilde{x}_2(B) + Pr(C_1 = A)\tilde{x}_2(A)\Big]. \end{aligned}$$

Because of the discontinuous structure of $\tilde{x}_2(C_1)$ for both possible choices C_1, the expected demand is discontinuous as well. A case distinction with cases where both, one or none of $\tilde{x}_2(A)$ and $\tilde{x}_2(B)$ are at interior or corner values has to be made. As a first period purchase from one firm shifts the second period indifferent consumer closer to this firm, a situation with $\tilde{x}_2(B) = 0$ or $\tilde{x}_2(A) = 1$ can not occur. Therefore, four cases emerge, where each case j gives rise to a certain demand structure for the demand of firm B, which we will call D_B^j. The cases and the related demand parts for firm B have to be distinguished as follows

1) $\tilde{x}_2(B) \in (0, 1)$, $\tilde{x}_2(A) \in (0, 1)$, called D_B^1

2a) $\tilde{x}_2(B) = 1$, $\tilde{x}_2(A) \in (0, 1)$, called D_B^{2a}

2b) $\tilde{x}_2(B) \in (0, 1)$, $\tilde{x}_2(A) = 0$, called D_B^{2b}

3) $\tilde{x}_2(B) = 1$, $\tilde{x}_2(A) = 0$, called D_B^3.

B's demand always consists of three parts characterized by 1) and 3) and, depending on the value of a, either 2a) or 2b). Let us build the intuition behind these demand parts. Suppose that firm A has already chosen a location a, and remember that if firm B was chosen in the first period, uninformed second period consumers always attribute a higher probability to the possibility that B's product is superior than to the possibility that A's product is superior.

Now, if firm B chooses its rightmost location $b = a$, then the choice behavior of the first period consumers gives a clear instruction for second period consumers: if firm B was chosen, then uninformed period 2 consumer should also always (that is independent of her type) choose firm B, $\tilde{x}_2(B) = 1$, as traveling costs to both firms are the same, and because of the updated probability, the good of firm B has a higher expected valuation. The reasoning is analogous for the case where firm A was chosen in the first period, i.e. $\tilde{x}_2(A) = 0$. That is, for high choices of location b, i.e. close to firm A, firm B's demand is characterized by D_B^3.

What happens, if for a given a, firm B chooses its opposite extreme, i.e. positioning as far away from firm A as possible by choosing $b = 0$? Then the previous consumer choice behavior is not giving such clear suggestions: for all possible histories there are some uninformed period 2 consumer types for whom traveling costs are too high compared to the additional expected valuation obtained by buying from the firm that was chosen in period 1. Hence, for low choices of location b, firm B's demand is characterized by D_B^1, where $\tilde{x}_2(C_1) \in (a, b)$ for both choices in the first period.

What happens, if firm B chooses none of its extreme options? Now, depending on the values of a and b, two new cases may arise. If firm A chose a position very much in the right of the unit interval (a is high, say $a = 0.75$), consumption of the product of firm A by first period consumers gives a strong signal for firm A being of higher quality - considering that firm B is neither at the one ($b = a$) nor the other extreme ($b = 0$), but say at the center, i.e. $b = 0.5$. Given this situation, the probability that firm A is chosen by uninformed consumers in period 1 is relatively low ($\tilde{x}_1$ is large), while the probability that informed period-1 consumers choose A remains the same. Therefore, a choice in favor of firm A is more likely to be done by an informed consumer. It happens that B's second period demand by uninformed consumers is zero, $\tilde{x}_2(A) = 0$, while a choice of B in the first period does not entail that strong a suggestion, $\tilde{x}_2(B) \in (a, b)$. This part of B's demand is given by D_B^{2b}.

The most interesting part of B's demand for our purposes is however given by D_B^{2a}, which is the mirror image of part D_B^{2b} from above. This occurs if firm A chose a rather low a, say firm A is positioned at the center with $a = 0.5$ and firm B again chose an intermediate position at neither of the extremes, say $b = 0.4$. Now consumer updating may be advantageous for firm B: as B is positioned further away from the center than A, for certain constellations of a and b, all uninformed second period consumers follow the choice behavior of previous consumers if it was in favor of firm B, $\tilde{x}_2(B) = 1$, while a choice of firm A in period 1 does not lead to such extreme results in the behavior of uninformed second period consumers, i.e. $\tilde{x}_2(A) \in (0, 1)$. Note that firm A can choose, which part of the 'middle' demand occurs for firm B (D_B^{2a} or D_B^{2b}) by its location choice.

In the Appendix we show that D_B^1 and D_B^3 are linear in a and b (with different slopes), while D_B^{2a} and D_B^{2b} are generally quadratic in both arguments. We also show, for fixed a, that B's demand in all parts is strictly increasing in b in the relevant range, so that the maximal demand will always be obtained at the largest b in one of the parts.

The two points of disconintuity in B's demand, that is the two points where, for a given a, the unique indifferent uninformed second-period consumer ceases to exist, have to be calculated. Hence, we need to find the smallest values of b such that $\tilde{x}_2(B) = 1$ and $\tilde{x}_2(A) = 0$. To illustrate the underlying mechanism of the discontinuity in the demand of firm B, suppose that b is chosen such that $\tilde{x}_2(B) = 1$. Then all consumers at and to the right of a are indifferent between both firms. Increasing b would decrease transport costs for the consumers $x_2 \geq a$, so that with this increased b, all uninformed consumers in period $t = 2$, will choose B, if they observe that it was chosen in period 1. Similarly, at $\tilde{x}_2(A) = 0$ all consumers at and to the left of b are indifferent, and with a decreased b all uninformed second-period consumers choose A if it was chosen in $t = 1$.

Those two discontinuity points are implicitly characterized by the following equations where the indifferent type in period 2 after A (B) was chosen in the first period is located at b (a):

$$b \quad \text{s.t.} \quad \tilde{x}_2(A) = b \quad \Leftrightarrow \quad \begin{aligned} b =& \frac{a+b}{2} + \frac{\delta}{2\tau}(2p_b(A) - 1) \\ \frac{\delta q}{\tau} =& (2-q)(a-b-(1-q)\left(a^2-b^2\right) \end{aligned} \tag{1}$$

and

$$b \quad \text{s.t.} \quad \tilde{x}_2(B) = a \quad \Leftrightarrow \quad \begin{aligned} a =& \frac{a+b}{2} + \frac{\delta}{2\tau}(2p_b(B) - 1) \\ \frac{\delta q}{\tau} =& q(a-b) + (1-q)\left(a^2-b^2\right) \end{aligned} \tag{2}$$

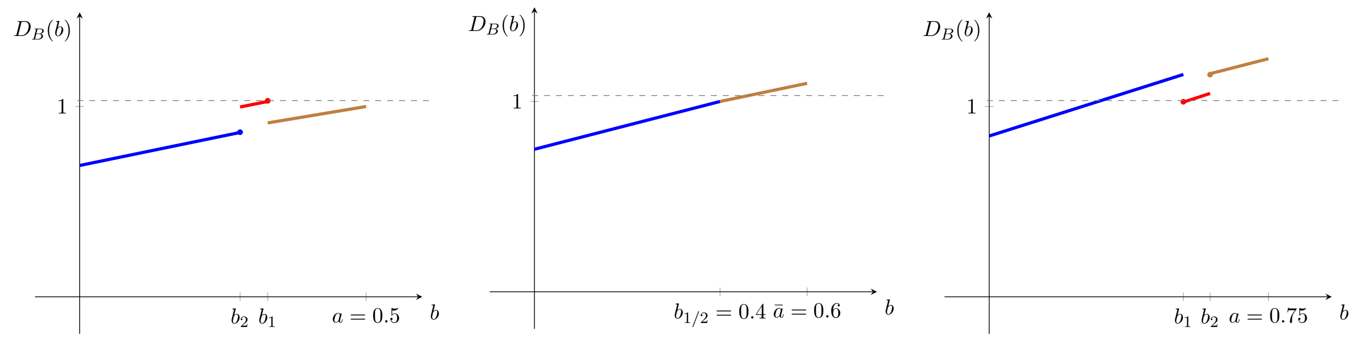
Since both equations are quadratic in b, they both have two solutions. As is shown in the Appendix, only one of those solutions, for each equation, lies in the permissible range of $[0, 1]$. Those permissible solutions will in the following be referred to by $b_1(a)$ and $b_2(a)$ respectively. As $b_1(a)$ and $b_2(a)$ are the discontinuity points in B's demand, they mark the borders between demand parts D_B^1, D_B^{2a} and D_B^3 or between D_B^1,D_B^{2b} and D_B^3.

If, for a fixed a and $C_1 = A$, firm B positions itself at the location $b_2(a)$, all uninformed second-period consumers to the right of a are indifferent between both firms. If B instead chose a location slightly smaller than $b_2(a)$, this would increase the transport costs of all consumers located at or to the right of a, meaning that those (uninformed second-period) consumers would then prefer A's product. On the other hand, a location b slightly larger than $b_2(a)$ would induce all those consumers to buy the product from firm B. Taken together this implies that at $b_2(a)$, B's demand has an upward jump.

Similar reasoning leads to the observation that B's demand jumps downward at $b_1(a)$ as at this point the whole mass of consumers to the left of b switches from preferring B's good to preferring the one of firm A, if $C_1 = B$.

Depending on the value of a, the calculations in the Appendix show that either $b_2(a) < b_1(a)$, $b_2(a) > b_1(a)$ or $b_1(a) = b_2(a)$ may occur. Let $a = \bar{a}$ denote the location of firm A, when the latter possibility realizes. In the case of $b_2(a) < b_1(a)$, B's 'middle' demand is characterized by D_B^{2a} and starts with an 'upward jump'. In the case of $b_2(a) > b_1(a)$, B's 'middle' demand is characterized by D_B^{2b} and starts with a 'downward jump'. Note that by choosing the strategies of indifferent consumers, we can choose to which part of the demand the discontinuity points belong.

With those observations and the above descriptions of the demand parts, we can depict B's demand for a fixed a and varying choices of b, as is done in Figure 2, which shows the three generic cases that may occur in our model.



(a) B's expected demand given $a = 0.5$ (exploiting the recommendation effect is possible)

(b) B's expected demand given $a = \bar{a} = 0.6$

(c) B's expected demand given $a = 0.75$ (exploiting the recommendation effect is not possible)

Figure 2: B's expected demand as a function of the chosen location b, for three different locations of A. In the left panel $b_1 > b_2$ so that part 2 of demand is an upward step. Reversed situation in the right panel and no jump in the middle panel. The parameters yield a type 2 equilibrium with differentiation, which is visualized also by the dashed line. (Parameters: $q = 0.4, \tau = 2, \delta = 1$)

5.3 Firm's Best Responses and Equilibria

Since our model is a generalization of the fixed-price Hotelling game, the result of 'minimum differentiation' from the previous section is obtained for certain parameter constellations. This happens for example, when all consumers are uninformed, i.e. $q = 0$, where only the market size effect exists.

In the following we concentrate on situations where this equilibrium does not exist. The non-existence of 'minimum differentiation' equilibrium is formalized by the first condition in the following proposition, stating that firm B prefers to locate at $b = b_1(0.5)$ instead of $b = 0.5$ given that A locates at the center.[14]

[14] Additionally we need the conditions $b_1(0.5) > 0$ and $D_B^{2a}(b_1(\bar{a}), \bar{a}) > D_B^3(\bar{a}, \bar{a})$ to be fulfilled. The first one is of technical nature an guarantees, that firm B's optimal choice when differentiating lies in the admissible range. If the second one is fulfilled, then firm B prefers to differentiate for all locations a of

Proposition 2. *If*

$$D_B^{2a}(0.5, b_1(0.5)) > D_B^3(0.5, 0.5) \tag{3}$$

the strategies from the benchmark model do not constitute an equilibrium. Furthermore,

1. *equilibrium locations are $a = b = \bar{a}$ (Equilibrium 1) if firm A does not prefer to induce differentiation, that is, if*

$$D_B^{2a}(0.5, b_1(0.5)) > D_B^3(\bar{a}, \bar{a}) \tag{4}$$

2. *equilibrium locations are $a = 0.5$ and $b = b_1(0.5) < 0.5$ (Equilibrium 2) if firm A prefers to induce differentiation, that is, if*

$$D_B^{2a}(0.5, b_1(0.5)) \leq D_B^3(\bar{a}, \bar{a}). \tag{5}$$

In the description of the different parts of the demand it was argued that B's demand is increasing in b in each part. It is then clear that for a given a, the demand of firm B is maximized by setting b equal to one of the points $b_2(a)$, $b_1(a)$ or a. Through its location choice a, firm A can induce an outcome with $b = a$, which would be the case for a sufficiently large a as depicted in the middle and right panel of Figure 2, or an outcome with $b = b_1(a) < a$ which would occur for relatively small values of a as shown in the left panel of the figure mentioned before. Since B's demand at the points $b = a$ and $b = b_1(a)$ is increasing in a, the opposite must be true for A's demand. Hence A will either choose the smallest point that induces $b = a$ or the smallest point inducing $b = b_1(a)$, where - with the restriction of $a \in [0.5, 1]$ - the latter is given by $a = 0.5$. Which of the situations is preferred by firm A depends on whether (4) or (5) is fulfilled. The following figure depicts parameter constellations inducing the two equilibria.

firm A, which allow to exploit the recommendation effect, i.e. which yield demand part D_B^{2a}. Details on these two conditions can be found in the Appendix (for the first condition see Lemma 6 and for the second condition see Lemmata 7 and 8).

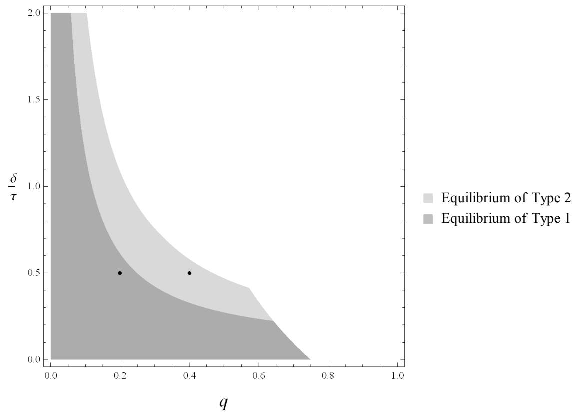


Figure 3: The figure depicts the parameter restrictions of Proposition 2. The black dots show that the parameter combinations used in Figure 2 fulfill all the conditions for a type 2 equilibrium, while the ones of Figure 4 fulfill the conditions of a type 1 equilibrium (See also subsection 'Numerical Examples' on this).

Figure 3 shows that $\frac{\delta q}{\tau}$ has to be sufficiently small for either of the two new equilibria to exist. The reason is that for both equilibria, firm B must want to deviate from the Hotelling equilibrium by differentiating, which is formalized by condition (3). For differentiation to matter, $\frac{\delta q}{\tau}$ must not be too large. A large fraction implies that either the relative gain from choosing the higher quality product, $\frac{\delta}{\tau}$, or the likelihood that the first consumer is informed, q, is high. This makes it especially promising for uninformed consumers to follow the previous consumers' behavior even for large distances between the two firms' locations, making differentiation unattractive for the firms.

Comparing the two new equilibria, $\frac{\delta q}{\tau}$ must not be too small for differentiation to be observed in equilibrium. To build intuition, it is crucial to remember that, in essence, firm A chooses from two different scenarios. It either leaves firm B a relatively high (first period) market share so that B will not differentiate (equilibrium 1) or it serves a high (first period) market share itself, inducing B to differentiate (equilibrium 2). To see how the kind of equilibrium is influenced by the parameters, suppose the situation is such that the equilibrium with differentiation is obtained (Equilibrium 2). If $\frac{\delta q}{\tau}$ decreases, firms' locations matter, even if the distance between them is relatively small meaning if $a = 0.5$, B's optimally chosen location ($b = b_1(0.5)$) gets closer to a. B's market share in period 1 would therefore increase, worsening the situation for firm A. At some point, A will prefer to have the other firm located at the same location (larger than 0.5) rather than locating at $a = 0.5$ and inducing firm B to differentiate.

In contrast to Hotelling's result of 'minimum differentiation' where both firms choose to locate at the center, the firms' positions are not symmetric in the two equilibria from above. The doubly sequential nature of the game clearly makes firm A worse off compared to the situation where consumers decided simultaneously (or were unable to observe others' decisions).

In our model with consumer learning and in contrast to the fixed-price Hotelling setup, firm B would prefer to differentiate if firm A locates at the center. Intuitively, by differentiating from A's product, B decreases the probability that uninformed consumers choose its product in the first period. Since informed consumers are not affected by the location of the firms, given the observation that a first period consumers chose firm B, an uninformed consumer in the second period reasons that the likelihood of consumer 1 being informed is higher than if A was chosen. This makes second period consumers updating favorable for the firm producing the niche product and unfavorable for mainstream firms. If this 'recommendation effect' of an early purchase on later consumers outweighs the decreased demand in early periods, the second mover has an incentive to differentiate. The 'recommendation effect' can turn out to be so strong that firm A prefers to leave a relative high market share to firm B in order to keep it from differentiating and exploiting this effect. This consideration by firm A distinguishes our two equilibria.

Although the 'recommendation effect' has similar implications on the firm's location as the 'competition effect' found in the literature and mentioned before, it is of a different nature. With the 'competition effect', the firm seeks to lessen competition to increase its market power and markup. The 'recommendation effect' in contrast, exploits the way consumers conduct inference after observing earlier choices.

5.4 Numerical Example

In order to get a feeling how the results derived above depend on the parameters, we will present two numerical examples for the two equilibria described in Propoposition 2. For both examples we will set $\delta = 1$ and $\tau = 2$. With $\frac{\delta}{\tau} = 0.5$ it can be seen from Figure 3, that an equilibrium of type-1 is obtained for values of q approximately below 0.25 and an equilibrium of type-2 emerges if q is in the approximate interval of $[0.25, 0.45]$. For both those cases, namely for $q = 0.2$ and for $q = 0.4$, equilibrium locations, as well as the demands and the probabilities resulting from consumer behavior will be presented.

We will start with equilibrium locations and the resulting demands:

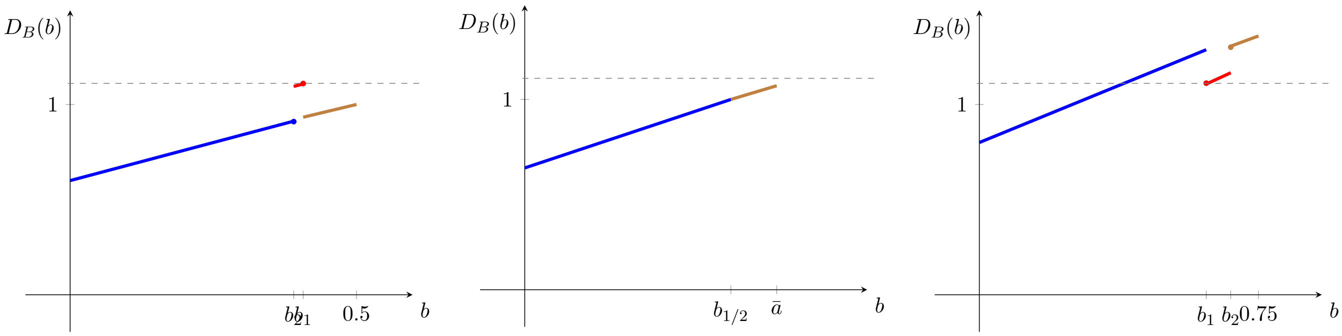
Eqm. type	Type-1 (No Differentiation)	Type-2 (Differentiation)
q	0.2	0.4
Eqm. locations	$a = \bar{a} = 0.55$	$a = 0.5$
	$b = \bar{a} = 0.55$	$b = b_1(0.5) = 0.32$
$D_B(0.5, b_1(0.5))$	1.25	1.02
$D_B(\bar{a}, \bar{a})$	1.07	1.10

It was shown before that Firm A can either induce an equilibrium of Type-1 by choosing

$a = \bar{a}$ or it can induce a type-2 equilibrium by setting $a = 0.5$. As the values show, the former leads to a lower demand for B (and thus a higher demand for A) if $q = 0.2$ and the opposite is true for $q = 0.4$, leading to the two equilibria where either both firms locate at $a = b = 0.55$ ($q = 0.2$) or A locates at the center and firm B chooses a location of 0.32 (q=0.4).

Eqm. type	Type-1 (No Differentiation)	Type-2 (Differentiation)
q	0.2	0.4
$\tilde{x}_1$	0.55	0.41
$Pr(C_1 = B)$	0.54	0.45
$Pr(v_b > v \mid C_1 = B)$	0.59	0.72
$Pr(v_b > v \mid C_1 = A)$	0.39	0.32
$\tilde{x}_2(A)$	0	0.32
$\tilde{x}_2(B)$	1	1
$Pr(C_2 = B)$	0.53	0.57

Since the firms' demands and thus the equilibria presented above, are dictated by the choices of consumers, it is interesting to see their behavior in the two different parameter regimes. Given the equilibrium locations from above, it can be seen that the demand from uninformed consumers in period 1 ($\tilde{x}_1$) is divided in an asymmetric way and that therefore also the total demand in period 1 is not symmetric ($Pr(C_1 = B)$). More interestingly, the table also depicts the probabilities that firm B is offering the product of higher quality, that an uninformed consumer in period 2 calculates when observing the choice made in the first period. Here, the driving force of the recommendation effect can be seen at work. The firm with the lower (probabilistic) demand in the first period (A if $q = 0.2$ and B if $q = 0.4$), benefits more from the updating done by later consumers. Take the type-2 equilibrium. Here, B's (expected) demand in the first period equals 0.41, but whenever an uninformed consumer in the second period observes that B was bought in the first period, she updates her belief that product B is superior to 0.72 from the prior of 0.5. Now if A was bought in the first period, an uninformed second period consumer still assigns a probability of 0.32 to B being the better product. The complementary probabilities, i.e. the ones describing how likely it is that A offers the superior product, are calculated as 0.28 if B was bought and 0.68 if A was bought in the first period, thus showing that the Bayesian updating benefits the firm with the smaller market share in period 1. This Bayesian updating leads to the indifferent uninformed consumers in period 2, $\tilde{x}_2(A)$ and $\tilde{x}_2(B)$. It can be seen that the two threshold types are spread asymmetrically around the $\tilde{x}_1$ in the equilibrium of type-2, where all uninformed period 2 consumers buy B if it was bought before ($\tilde{x}_2(B) = 1$), but not all consumers buy A if this was the case in period 1. $\tilde{x}_2(A) = 0.32$ equals firm B's location, and this reflects the optimal choice of B's location. It was shown before that, given $a = 0.5$, B optimally chooses the maximal b, such that $\tilde{x}_2(B) \in (b, a)$ and this is precisely obtained by choosing b such that $\tilde{x}_2(B) = b$. The asymmetrically shifted consumer threshold types are also reflected in the expected demand of period 2, $Pr(C_2 = B)$.



(a) B's expected demand given $a = 0.5$ (exploiting the recommendation effect is possible)

(b) B's expected demand given $a = \bar{a} = 0.55$

(c) B's expected demand given $a = 0.75$ (exploiting the recommendation effect is not possible)

Figure 4: B's expected demand as a function of the chosen location b, for three different locations of A. In the left panel $b_1 > b_2$ so that part 2 of demand is an upward step. Reversed situation in the right panel and no jump in the middle panel. The parameters yield a type 1 equilibrium without differentiation, which is visualized also by the dashed line. (Parameters: $q = 0.2, \tau = 2, \delta = 1$)

6 Conclusion

This paper has given a reasoning for why a firm producing a product which appeals to relatively few consumers ex-ante, may generate a larger demand ex-post than a product which ex-ante appeals to the mass.

In our extension of the classical model of spatial competition due to Hotelling (1929) (with fixed prices), the effect emerges since consumers, who are heterogeneous with respect to their preferred good and with respect to the level of information they possess, make their purchase decisions sequentially and are able to observe which good previous consumers bought. Having differential information about which good offers the superior quality, uninformed consumers in later periods rationalize the choice of other consumers by considering that earlier consumers possibly made their decision because they were better informed about the quality of the different goods. An uninformed consumer thus updates her estimate about the difference in the good's quality after observing previous consumer choices using Bayes' Rule.

This updating is especially favorable for niche products. Because niche products are not as appealing as mainstream products to a broad range of consumers, later consumer's reasoning after having observed the purchase of a niche product puts more weight on the possibility that this purchase was due to the consumer being informed instead of being due to a good match of the earlier consumer's preference and the good's characteristic.

When deciding about the good's characteristics, i.e. how much to differentiate from the opponent's product, a firm has to take into account two offsetting effects. On the one hand producing a niche product decreases the product's overall appeal to consumers, hence the expected demand in early periods is decreased. But on the other hand, exactly because the overall appeal is decreased, an early purchase of the niche product leads to a higher boost of later uninformed consumers' confidence in the niche products superior quality. As this paper shows, the second effect can dominate, leading to an equilibrium with differentiated goods. This effect, the 'recommendation effect', is different from what is generally called the 'competition effect' which goes into a similar direction as it makes differentiation profitable for firms, but here the driving force is that it relaxes competition, thus increasing possible markups.

Note that in biology there is an effect similar to the one described in this paper: the Handicap Principle (see e.g. Zahavi (1975)) explains why some animals have certain features which at first sight seem to be an evolutionary disadvantage. A popular example is the tail of the peacock. At first sight this tail probably is a huge obstacle when being persuaded by predators. But if one such peacock survives and is chosen by a mate to pass on his genes, then the (probably even bigger) tail of the offspring can work as a strong signal for that peacock of being of high (evolutionary) quality.

The simplifying assumptions made in our model - e.g. concerning the signaling structure, the restriction of the number of consumers to two or the importance of the good's quality / travelling cost - are not neccessary and the results hold in even more general settings. We also conjecture that different cost functions than the linear one applied here would not eliminate the underlying effects of our model. Remember for instance that in the common Hotelling model, quadratic costs only enhance the incentive to differentiate and would thus probably make the detection of the driving forces yielding the differentiation result in our model more complicated.

Regarding future work, an interesting extension would be to model firm's location choice as being simultaneous instead of being sequential. In such a model the existence of a pure strategy equilibrium is not very likely, as the discontinuities play an even more pronounced role than they already do in our setup. We nevertheless conjecture that firms' play a mixed strategy which will lead to an outcome with differentiated products in many occasions.

In this paper we abstracted from the issue of price setting of the firms. Clearly it would be interesting to extend the model with endogenous prices. This would make it necessary to also consider different forms of transport costs, for example quadratic ones, since the linear model with endogenous prices usually does not possess an equilibrium in pure strategies.

Last but not least, it would be worthwhile to relax the symmetry of the model. For example a similar intuition as the one for why early purchases are especially valuable for niche products suggest that an ex-ante inferior firm potentially benefits more from early adopters choosing its product.

A Proof of Proposition 2

The following gives the full proof of the main proposition which is restated here for convenience.

Proposition 2. *If*

$$D_B^{2a}(0.5, b_1(0.5)) > D_B^3(0.5, 0.5) \tag{3}$$

the strategies from the benchmark model do not constitute an equilibrium. Furthermore,

1. *equilibrium locations are $a = b = \bar{a}$ (Equilibrium 1) if firm A does not prefer to induce differentiation, that is, if*

$$D_B^{2a}(0.5, b_1(0.5)) > D_B^3(\bar{a}, \bar{a}) \tag{4}$$

2. *equilibrium locations are $a = 0.5$ and $b = b_1(0.5) < 0.5$ (Equilibrium 2) if firm A prefers to induce differentiation, that is, if*

$$D_B^{2a}(0.5, b_1(0.5)) \leq D_B^3(\bar{a}, \bar{a}). \tag{5}$$

As the firms' best responses are discontinous, we construct an equilibrium rather than using a fixed point analysis. The proposition is proven using a succession of Lemmas that together imply the result.

Lemma 1. *Firm B's expected demand is piecewise defined and strictly increasing in b in each part.*

The general form of B's (expected) demand is given by:

$$D_B(a, b) = q + (1 - q)\Big[\tilde{x}_1 + Pr(C_1 = B)\tilde{x}_2(B) + Pr(C_1 = A)\tilde{x}_2(A)\Big]$$

Case distinctions have to be made regarding the values of $\tilde{x}_2(C1)$. The following reformulation of the second period parts of demand of an uninformed consumer will prove to be useful:

$$\begin{aligned} Pr(C_1)\tilde{x}_2(C_1) &= Pr(C_1)\left(\frac{a+b}{2} + \frac{\delta}{2\tau}\left(2\frac{Pr(C_1|v_b > v) \cdot Pr(v_b > v)}{Pr(C_1)} - 1\right)\right) \\ &= Pr(C_1)\tilde{x}_1 + \frac{\delta}{2\tau}\Big(Pr(C_1|v_b > v) - Pr(C_1)\Big) \end{aligned}$$

Part 1: $\tilde{x}_2(B), \tilde{x}_2(A) \in (a, b)$

If the uninformed indifferent consumer in period $t = 2$ and thus $\tilde{x}_2(C1)$ is always in between the location of both firms, demand can be written as follows:

$$D_B^1(a,b) = q + (1-q)\Big\{\tilde{x}_1 + Pr(C_1 = B)\tilde{x}_1 + \frac{\delta}{2\tau}\Big(Pr(C_1 = B|v_b > v) - Pr(C_1 = B)\Big)$$
$$+Pr(C_1 = A)\tilde{x}_1 + \frac{\delta}{2\tau}\Big(Pr(C_1 = A|v_b > v) - Pr(C_1 = A)\Big)\Big\}$$

with $Pr(C_1 = A) = 1 - Pr(C_1 = B)$ and $Pr(C_1 = A|v_b > v) = 1 - Pr(C_1 = B|v_b > v)$, this simplifies to

$$D_B^1(a,b) = q + (1-q)(2 \cdot \tilde{x}_1) = q + (1-q)(a+b)$$

Hence, for the case where $\tilde{x}_2(C1)$ is between both firms' locations, B's demand increases linearly in b.

Part 2a: $\tilde{x}_2(B) = 1, \tilde{x}_2(A) \in (b, a)$

If a purchase of B in the first period is always followed by an uninformed second period consumers, but not a purchase of A, demand of B is given by

$$\begin{aligned}
D_B^{2a}(a,b) &= q + (1-q)\Big[\tilde{x}_1 + Pr(C_1 = B) + Pr(C_1 = A)\tilde{x}_2(A)\Big] \\
&= q + (1-q)\Big\{\tilde{x}_1 + \frac{q}{2} + (1-q)\tilde{x}_1 + Pr(C_1 = A)\tilde{x}_1 \\
&+ \frac{\delta}{2\tau}\Big(Pr(C_1 = A|v_b > v) - Pr(C_1 = A)\Big)\Big\} \\
&= q + (1-q)\left\{\tilde{x}_1\Big[2 - \tfrac{q}{2} + (1-q)(1-\tilde{x}_1)\Big] - \frac{q\delta}{4\tau} + \frac{q}{2}\right\}
\end{aligned}$$

Which is quadratic in $\tilde{x}_1$ and thus in b. Nevertheless, the derivative:

$$\frac{\partial D_B^{2a}}{\partial \tilde{x}_1} = [3 - \tfrac{3}{2}q - 2(1-q)\tilde{x}_1](1-q)$$

shows that it is strictly increasing in b for the relevant values of a and b.

Part 2b: $\tilde{x}_2(B) \in (b, a), \tilde{x}_2(A) = 0$

In this part, second-period consumers follow the choice if the first period consumer chose A, if $C_1 = B$, the indifferent consumer in period 2 lies between the two firm's locations.

Hence, B's demand calculates as

$$\begin{aligned}D_B^{2b}(a,b) &= q + (1-q)\Big[\tilde{x}_1 + Pr(C_1 = B)\tilde{x}_2(B)\Big] \\ &= q + (1-q)\Big[\tilde{x}_1 + \frac{q + (1-q)(a+b)}{2} + \frac{q\delta}{4\tau}\Big]\end{aligned}$$

and the following derivative shows that $D_B^{2b}(a,b)$ is strictly increasing in b:

$$\frac{\partial D_B^{2b}}{\partial \tilde{x}_1} = (1-q)[\tfrac{1}{2}q + 1 + 2(1-q)\tilde{x}_1]$$

Part 3: $\tilde{x}_2(B) = 1, \tilde{x}_2(A) = 0$

$\tilde{x}_2(B) = 1, \tilde{x}_2(A) = 0$ means that an uninformed second-period consumer always follows the lead of the consumer in period 1. The demand in such a situation is described by

$$\begin{aligned}D_B^{3}(a,b) &= q + (1-q)\Big[\tilde{x}_1 + Pr(C_1 = B)\Big] \\ &= q + (1-q)\Big[(2-q)\Big(\frac{a+b}{2}\Big) + \frac{q}{2}\Big]\end{aligned}$$

As the demand in Part 1), demand in this case is linear, and increasing in b.
Inspection of the different demand parts shows that updating of the second period consumers and thus the shifting of the indifferent consumer types is symmetric in parts 1) and 3) and asymmetric only in parts 2a) and 2b), only in those cases does the demand depend on the parameters δ and τ.

Lemma 2. *Equations (1) and (2), giving the discontinuities of B's demand, have at most one solution in* $[0, 1]$*. Call these solutions* $b_1(a)$ *and* $b_2(a)$ *respectively.*

Equations (1) and (2) are given by

$$\begin{aligned}\min b \quad \text{s.t.} \quad \tilde{x}_2(A) = 0 \quad \Leftrightarrow \quad b =& \frac{a+b}{2} + \frac{\delta}{2\tau}(2p_b(A) - 1) \\ \frac{\delta q}{\tau} =& (2-q)(a-b) - (1-q)\left(a^2 - b^2\right) \end{aligned} \tag{1}$$

and

$$\begin{aligned}\min b \quad \text{s.t.} \quad \tilde{x}_2(B) = 1 \quad \Leftrightarrow \quad a =& \frac{a+b}{2} + \frac{\delta}{2\tau}(2p_b(B) - 1) \\ \frac{\delta q}{\tau} =& q(a-b) + (1-q)\left(a^2 - b^2\right) \end{aligned} \tag{2}$$

The possibly valid solutions are given by

$$b_1(a) = \frac{(2-q) - \sqrt{(2-q)^2 - 4(1-q)\left[(2-q)a - (1-q)a^2 - \frac{\delta q}{\tau}\right]}}{2(1-q)}$$

for (1)

$$b_2(a) = \frac{q - \sqrt{q^2 + 4(1-q)\left[(qa + (1-q)a^2 - \frac{\delta q}{\tau})\right]}}{-2(1-q)}$$

for (2).

A bit of calculation shows that both discontinuities exist for a such that

$$b_1(a) \in [0,1] \;\Leftrightarrow\; a \in \left[\frac{(2-q) - \sqrt{(2-q)^2 - 4(1-q)\frac{\delta q}{\tau}}}{2(1-q)},\; \frac{(2-q) - \sqrt{(2-q)^2 - 4(1-q)\frac{\delta q + \tau}{\tau}}}{2(1-q)}\right]$$

and

$$b_2(a) \in [0,1] \;\Leftrightarrow\; a \in \left[\frac{-q + \sqrt{q^2 + 4(1-q)\frac{\delta q}{\tau}}}{2(1-q)}, \frac{-q + \sqrt{q^2 + 4(1-q)\frac{\delta q + \tau}{\tau}}}{2(1-q)}\right]$$

Lemma 3. $\frac{\partial b_2(a)}{\partial a} > 1 > \frac{\partial b_1(a)}{\partial a} > 0$ *for* $b \leq a$ *and* $a, b \in [0,1]$. *Furthermore* $b_1(a)$ *and* $b_2(a)$ *cross only once at* $\bar{a} = \frac{1}{2}(1 + \frac{\delta q}{\tau})$ *and in addition* $a = 1 - b$. *Hence for* $a < \bar{a}, b_2(a) < b_1(a)$ *and for* $a > \bar{a}, b_2(a) > b_1(a)$.

Derivatives can be directly calculated from Equations (1) and (2) via the Implicit Function Theorem as:

$$\frac{\partial b_1}{\partial a} = \frac{(2-q) - 2(1-q)a}{(2-q) - 2(1-q)b_1}$$

and

$$\frac{\partial b_2}{\partial a} = \frac{q + 2(1-q)a}{q + 2(1-q)b_2}$$

The second part of the lemma is obtained by simply equalizing equations (1) and (2), which gives the condition $a = 1 - b$. Plugging in one of the values of $b_1(a)$ or $b_2(a)$ for b yields the condition stated in the lemma.

Lemma 4. *For any $a \leq \bar{a}$, $b_1(a)$ lies in the interval* $\left[a - \frac{\delta q}{\tau}, \frac{1}{2} - \frac{\delta q}{2\tau}\right]$.

Lemma 3 implies that the distance $a - b_1(a)$ is increasing in a for all $a \leq \bar{a}$, hence

$$a - b_1(a) \leq \bar{a} - b_1(\bar{a}) = 2\bar{a} - 1$$

which, for any $a \leq \bar{a}$ gives a lower bound on $b_1(a)$ as

$$\underline{b_1}(a) := a - \frac{\delta q}{\tau}$$

Since $b_1(a)$ is increasing in a, $\overline{b_1} := b_1(\bar{a})$ is an upper bound on $b_1(a)$ for all $a \leq \bar{a}$ and the Lemma follows.

Lemma 5. *B's demand at the optimum of parts 2a) and 3), $D_B^{2a}(a, b_1(a))$ and $D_B^3(a, a)$ respectively, is increasing in a. Thus, A will either set a equal to the smallest a such that $b = b_1(a)$ or to the smallest a such that $b = a$.*

It is clear that B's demand increases in a if both firms choose the same location, since then the probability that uninformed consumers choose B is given by a.
For B's demand $D_B^{2a}(a, b_1(a))$, we can calculate the derivative w.r.t. a as follows

$$\frac{\partial D_B^{2a}(a, b_1(a))}{\partial a} = (1-q)\left(1 + \frac{\partial b_1(a)}{\partial a}\right)\left[\frac{4-q}{4} + (1-q)\frac{1-a-b_1(a)}{2}\right]$$

This derivative is positive whenever

$$b_1(a) \leq \frac{6-3q}{2-2q} - a$$

which always holds for $a \in [0, 1]$.

Lemma 6. $b_1(0.5) > 0$ *if*

$$1 > q\left(\frac{1}{3} + \frac{4}{3}\frac{\delta}{\tau}\right)$$

which implies the condition $\bar{a} = \frac{1}{2}(1 + \frac{\delta q}{\tau}) < 1$

The restriction in the lemma is directly obtained by plugging in $a = 0.5$ into the formula for b_1 from Lemma 2.

Lemma 7. *If $\bar{a} \leq 1$ and $b_1(0.5) > 0$, a sufficient condition for the Type 1 equilibrium of the proposition to exist is given by*

$$1 \geq \frac{\delta q}{\tau}\left[9 - 3q + \frac{\delta q}{\tau}(1-q)\right].$$

The claim is true if simultaneously the following three inequalities hold

1. $D_B^{2a}(0.5, b_1(0.5)) > D_B^3(0.5, 0.5)$
2. $D_B^3(\bar{a}, \bar{a}) < D_B^{2a}(0.5, b_1(0.5))$
3. $D_B^{2a}(\bar{a}, b_1(\bar{a})) > D_B^3(\bar{a}, \bar{a})$.

The previous Lemmata established that all demand functions are increasing in both arguments, the second equation thus implies the first and the third one. A sufficient condition for the second equation is given by

$$D_B^{2a}(0.5, \underline{b_1}(0.5)) > D_B^3(\bar{a}, \bar{a})$$

Using previous results, this leads to the condition stated in the Lemma.

Lemma 8. *If $\bar{a} \leq 1$ and $b_1(0.5) > 0$, a sufficient condition for the Type 2 equilibrium of the proposition to exist is given by*

$$\frac{\delta q}{\tau}\left[\frac{\delta q}{\tau}(1-q) + 5 - q\right] \leq 1 \leq \frac{\delta q}{2\tau}\left[\frac{\delta q}{2\tau}(1-q) + 14 - 5q\right].$$

The claim is true if simultaneously the following three inequalities hold

1. $D_B^{2a}(0.5, b_1(0.5)) > D_B^3(0.5, 0.5)$
2. $D_B^3(\bar{a}, \bar{a}) \geq D_B^{2a}(0.5, b_1(0.5))$
3. $D_B^{2a}(\bar{a}, b_1(\bar{a})) > D_B^3(\bar{a}, \bar{a})$.

Sufficient conditions for the first two inequalities are given by

1. $D_B^{2a}(0.5, \underline{b_1}(0.5)) \geq D_B^3(0.5, 0.5)$
2. $D_B^3(\bar{a}, \bar{a}) \geq D_B^{2a}(0.5, \overline{b_1}(0.5))$.

Reformulating these inequalities using the previous results gives the condition stated in the lemma.
Reformulating the third inequality using $b_1(\bar{a}) = 1 - \bar{a}$ yields

$$\frac{\delta q}{\tau}(5 - 2q) < 1.$$

This condition is always fulfilled in the parameter space, if the LHS of the lemma holds, and the result follows.

The following figure shows where the conditions from the previous Lemmata are fulfilled.

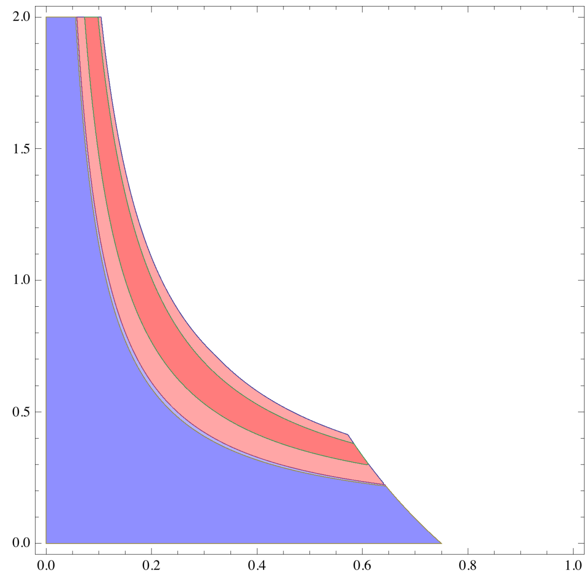


Figure 5: Shown are parameter combinations, with q on the x-axis and $\frac{\delta}{\tau}$ on the y-axis. The light blue (light red) area gives the true conditions for a Type-1 (Type-2) equilibrium. The darker blue and red areas show where the sufficient conditions from Lemmata 7 and 8 are fulfilled.

B Relaxing Assumption 2

Proposition 3. *In the equilibria from above, informed consumers optimally always buy from the better firm, that is, dropping Assumption 2 does not change the results.*

In all of the paper it was simply assumed that informed customers buy the higher quality product. If this assumption is dropped, it could happen that informed consumers choose the lower quality product because the gain from the higher quality product does not offset the additional transport costs. Hence, indifferent types for informed consumers have to be calculated, just as they have been calculated for uninformed consumers before. Informed customers possess all available information, so that there can not be any updating in later periods and the indifferent types are the same in both periods and depend only on which firms' product is of higher quality.

As in the case of the uninformed consumers the indifferent type might either be located in between the firms' locations, or all consumers prefer one of the two goods. If the indifferent types are located in between the firms, and calling the indifferent types of informed consumers $\tilde{x}_I(A)$ if product A is superior and $\tilde{x}_I(B)$ if $v_b > v$, they can easily be calculated as

$$\tilde{x}_1 - \frac{\delta}{2\tau}$$

if the A is the superior product, and

$$\tilde{x}_1 + \frac{\delta}{2\tau}$$

in the case that B's product is of higher quality.
It can directly be seen that the interior solutions for the two indifferent types are symmetrically positioned around the average of the two firms locations, given by $\tilde{x}_1$. Thus $\tilde{x}_1 - \frac{\delta}{2\tau} \leq b$ directly implies $\tilde{x}_1 + \frac{\delta}{2\tau} \geq a$ and vice versa. In such a case all consumers prefer good A and B respectively, and as before $\tilde{x}_I$ is set to 0 and 1 respectively.
Having calculated the indifferent types of informed consumers, B's demand without Assumption 2 is given by

$$D_B(a,b) = q(\tilde{x}_I(A) + \tilde{x}_I(B)) + (1-q)\Big[\tilde{x}_1 + Pr(C_1 = B)\tilde{x}_2(B) + Pr(C_1 = A)\tilde{x}_2(A)\Big]$$

Obviously, the demand is the same as before whenever $\tilde{x}_I(A) + \tilde{x}_I(B) = 1$. The crucial insight is that the indifference type for informed consumers is always shifted further apart from $\tilde{x}_1$ than the one of the uninformed consumers, meaning that $\tilde{x}_2(A) \geq \tilde{x}_I(A)$ and $\tilde{x}_2(B) \leq \tilde{x}_I(B)$. The demand parts of interest in the model are the parts 2 and 3 which are precisely characterized by either $\tilde{x}_2(A) = 0$ and $\tilde{x}_2(B) = 1$ or both. In those parts, the demand is therefore not affected by dropping Assumption 2.
The demand only differs when $\tilde{x}_I(A) + \tilde{x}_I(B) \neq 1$. For a fixed a, $\tilde{x}_I$ is in the interval (b,a) for relatively small values of b (if at all) For larger values of b, both, $\tilde{x}_I(A)$ and $\tilde{x}_I(B)$, switch to zero and one respectively. Dropping the assumption about the behavior of informed consumers thus introduces another part for B's demand function which occurs whenever a and b are relatively far apart. Call this 'new' part $D_B^0(a,b)$. Dropping the assumption is unproblematic if B will never choose b such that this part of the demand is the relevant one.
With the arguments from above, we know that $\tilde{x}_I(A) > 0$ and $\tilde{x}_I(B) < 1$ imply $\tilde{x}_2(A), \tilde{x}_2(B) \in (b,a)$. With all indifferent types being at interior levels, they are symmetrically spread around $\tilde{x}_1$ and $D_B^0(a,b)$ simplifies to:

$$\begin{aligned} D_B^0(a,b) &= q(\tilde{x}_I(A) + \tilde{x}_I(B)) + (1-q)\Big[\tilde{x}_1 + Pr(C_1 = B)\tilde{x}_2(B) + Pr(C_1 = A)\tilde{x}_2(A)\Big] \\ &= q(2\tilde{x}_1) + (1-q)(2\tilde{x}_1) = 2\tilde{x}_1 = a + b \end{aligned}$$

$D_B^0(a,b)$ is clearly increasing in a and b.

If B were to locate in this part of the demand, it would choose the maximal b, that is, it would locate at $b_0(a)$ given by

$$b_0(a) = \max_b \quad \text{s.t.} \quad \tilde{x}_I(B) \leq a$$
$$b_0(a) = a - \frac{\delta}{\tau}$$

Instead of choosing $b = b_0(a)$, B could also locate directly next to A. B will prefer $b = b_0(a)$ to $b = a$ if

$$D_B^0(a, b_0(a)) \geq D_B^3(a, a)$$
$$a + b_0(a) \geq q + (1-q)\left[(2-q)a + \frac{q}{2}\right]$$
$$a \geq \bar{a}_0 := \frac{1}{2} + \frac{\delta}{\tau(3q - q^2)}$$

Note that

$$\bar{a}_0 = \frac{1}{2} + \frac{\delta}{\tau(3q - q^2)} \geq \bar{a} = \frac{1}{2} + \frac{\delta q}{2\tau}$$

for all $q \in [0, 1]$.
Recall that Firm A could always guarantee a demand of $2 - D_B^3(\bar{a}, \bar{a})$ to itself by setting $a = \bar{a}$. We have established that A would have to choose a location further to the right, if it would want to induce $b = b_0(a)$. The smallest a that would induce $b_0(a)$ is given by $\bar{a}_0$, and by construction:

$$D_B^0(\bar{a}_0, b_0(\bar{a}_0)) \geq D_B^3(\bar{a}_0, \bar{a}_0)$$

Since $D_B^3(a, a)$ is increasing in a and $\bar{a}_0 > \bar{a}$:

$$D_B^0(\bar{a}_0, b_0(\bar{a}_0)) \geq D_B^3(\bar{a}_0, \bar{a}_0) \geq D_B^3(\bar{a}, \bar{a})$$

As A's demand is always given by $2 - D_B$, and A can secure a Demand of $2 - D_B^3(\bar{a}, \bar{a})$ to itself, it will never choose its location such that $b = b_0(a)$ is induced. Thus it will always be the case that $\tilde{x}_I(A) = 0$ and $\tilde{x}_I(B) = 1$, and dropping assumption 2 does not change the results.

C Quality and Separating Equilibria

Proposition 4. *There is no perfect sequential separating equilibrium.*

The following will show that even if Assumption 2 is dropped, a separating equilibrium can only exist under specific parameter conditions and even then it is eliminated using the concept of perfect sequential equilibrium (Grossman and Perry, 1986).
In order to establish this claim, we will first construct a perfect Bayesian separating equilibrium and then show that it violates the conditions of a perfect sequential equilibrium as formalized in (Sadanand and Sadanand, 1995).
In a separating equilibrium, firm B chooses its location depending on its realized quality (its type). Call the two types of the second moving firm B_ℓ and B_h, where the former is the one with the lower quality product (i.e. $v_B = v_A - \delta$) and the latter is the one with higher quality (i.e. $v_B = v_A + \delta$). Let b_ℓ and b_h be the respective locations and $\tilde{x}_\ell$ and $\tilde{x}_h$ the corresponding indifferent consumers defined as before.
Now, for the equilibrium to be incentive compatible, the demand of the two types of firms B has to be equal:

$$D_{B_h} = 2\tilde{x}_h := 2 \cdot \left[\frac{a + b_h}{2} + \frac{\delta}{2\tau}\right] \stackrel{!}{=} 2 \cdot \left[\frac{a + b_l}{2} - \frac{\delta}{2\tau}\right] =: 2\tilde{x}_\ell = D_{B_\ell}$$
$$\Leftrightarrow b_l = b_h + \frac{2\delta}{\tau},$$

that is, the lower quality firm has to be positioned to the right of the higher quality firm.
Optimality of the low-type firm implies that, for a given a, b_ℓ will be chosen as far to the right as possible as long as $\tilde{x}_\ell \geq b_\ell$, which implies $b_\ell = a - \frac{\delta}{\tau}$ and $b_h = a - \frac{3\delta}{\tau}$. Furthermore, in this setup the optimal choice of firm A is to position at the center, i.e. $a^* = 0.5$. Thus, a necessary condition for b_h to be above zero is given by $\frac{\delta}{\tau} \leq \frac{1}{6}$. If this is the case, beliefs that put probability one on the low-type firm for any locations but b_h sustain this equilibrium.
The constructed separating equilibrium seems 'implausible' as (both types of) firm B are 'forced' to choose their locations by the off-equilibrium beliefs of the consumers. Both types would prefer to locate at the center directly next to firm A if consumers were to change their beliefs accordingly, and this is precisely what is required in a perfect sequential equilibrium.
More precisely, let $I(b') \in \{B_\ell, B_h, \{B_\ell, B_h\}\}$ denote what is called (uninformed) consumers' interpretation of a deviation of firm B to the location b' with the idea that when uninformed consumers observe that firm B chooses b' instead of b_ℓ or b_h the interpretation gives the type of firm B consumers attribute the deviation to.
An interpretation $I(b')$ of the deviation to b' is consistent if precisely the type(s) consumers attribute the deviation to, prefer this deviation over their equilibrium payoff given consumers update their beliefs according to $I(b')$.
Finally, an equilibrium is perfectly sequential if there is no deviation with a consistent interpretation.

In the separating equilibrium from above, B's profit was $2\tilde{x}_\ell = 2\tilde{x}_h = 1 - 2\delta/\tau < 1$. Now consider firm B changing its strategy to pooling on $b = 0.5$. Given the interpretation $I(0.5) = \{B_\ell, B_h\}$ this leads to an equally shared demand between firm A and B, i.e. firm B's demand now equals 1 which is higher than in the separating equilibrium (for both types). Both types of firm B would therefore prefer to deviate to a pooling strategy given that consumers attribute this deviation to both types, showing that there is a deviation with a consistent interpretation in the separating equilibrium.

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