# GROUP SIZE EFFECT ON COOPERATION IN SOCIAL DILEMMAS

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Abstract. Social dilemmas are central to human society. Depletion of natural resources, climate protection, security of energy supply, and workplace collaborations are all issues that give rise to social dilemmas. Since cooperative behaviour in a social dilemma is individually costly, Nash equilibrium predicts that humans should not cooperate. Yet experimental studies show that people do cooperate even in anonymous one-shot situations. However, in spite of the large number of participants in many modern social dilemmas, little is known about the effect of group size on cooperation. Does larger group size favour or prevent cooperation? We address this problem both experimentally and theoretically. Experimentally, we have found that there is no general answer: it depends on the strategic situation. Specifically, we have conducted two experiments, one on a one-shot Public Goods Game (PGG) and one on a one-shot $N$-person Prisoner's Dilemma (NPD). We have found that larger group size favours the emergence of cooperation in the PGG, but prevents it in the NPD. On the theoretical side, we have shown that this behaviour is not consistent with either the Fehr & Schmidt model or (a one-parameter version of) the Charness & Rabin model. Looking for models explaining our findings, we have extended the cooperative equilibrium model from two-player social dilemmas to *some* $N$-person social dilemmas and we have shown that it indeed predicts the above mentioned regularities. Since the cooperative equilibrium is parameter-free, we have also made a direct comparison between its predictions and experimental data. We have found that the predictions are neither strikingly close nor dramatically far from the experimental data.

## 1. Introduction

Social dilemmas are situations in which selfish interest collides with collective interest. Every individual has an incentive to deviate from the common good, but if all subjects acted selfishly they would all be worse off. Depletion of natural resources, intergroup conflicts, climate protection, security of basic social systems, workplace collaborations, and price competition in markets are just some of the fundamental situations that can be modelled by means of a social dilemma. Consequently, understanding how and why cooperation can evolve is of primary importance [1–9].

In modern society, many social dilemmas involve a large number of players: firms trying to sell (approximately) the same product, countries involved in reducing the greenhouse

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gas emissions, taxpayers, work groups, etc. Still, little is known about the effect of group size on cooperation. Does group size influence cooperation and, if so, how?

A classical point of view states that larger group size should reduce cooperation: since the increase in the number of people *inevitably* leads to decreasing individual gains relative to the cost of cooperation, free-riding would be much more pervasive in larger groups: large groups will fail; small groups may succeed [2, 5]. However, others pointed out that the increase in the number of people does not necessarily lead to lower individual gains: the incentive to cooperate can even increase and lead to a positive relation between the size of the group and the level of collective actions [10, 11].

These two possibilities can be captured using two well-known economic games.

**Public Goods Game (PGG).** $N \geq 2$ contributors are endowed with $y$ dollars and must simultaneously decide how much, if any, to contribute to a public pool. The total amount in the pot is then multiplied by a constant and evenly redistributed among all players. So the monetary payoff of player $i$ is $u_i(x_1, \ldots, x_N) = y - x_i + \gamma(x_1 + \ldots + x_N)$, where $x_i$ denotes $i$'s contribution, and the 'marginal return' $\gamma$ is assumed to belong to the open interval $\left(\frac{1}{N}, 1\right)$.

**$N$-person Prisoner's Dilemma (NPD).** $N \geq 2$ agents have the choice of either cooperate (C) or defect (D). To defect means doing nothing, while to cooperate means paying a cost $c > 0$ to generate a benefit $b > c$ that gets shared by all other players. So, if $C_{-i}$ denotes the number of agents other than $i$ who cooperate, then agent $i$'s payoff is $\frac{bC_{-i}}{N-1} - c$ , if $i$ cooperates, and $\frac{bC_{-i}}{N-1}$, if $i$ defects.

The benefit for full cooperation in the PGG increases linearly with the number of players, while individual cost for cooperation remains constant. On the other hand, both the benefit and the cost for cooperation remain constant in the NPD, but, in order to reach the benefit, one needs more people to cooperate. With the above discussion in mind, we should then observe a positive effect of group size on cooperation in the PGG and a negative effect of group size on cooperation in the NPD.

Experimental research has not helped much so far, though it has partially confirmed this prediction. Some experimental results suggest that contributions to the public good are slowly increasing with the number of players [12, 13], while others suggest no group size effect [14] or even a negative effect [15]. Overall, the picture that emerges is that group size has a moderate, positive effect on cooperation in PGG: in her meta-analysis, Zelmer [16] uses data from 27 PGG experiments, conducted using different parametrisations and procedures, and finds a positive, moderate (at the 10% level) effect of group size on contributions. However, all these experimental studies have been conducted on iterated games, suggesting that group structure and long-term strategies may have played a role to shape the results.

The effect of group size on cooperation in the NPD is also unclear, but slightly in the opposite direction than the PGG: some studies report no effect [17], and others report a negative effect [18]. While these studies concern one-shot games, their main issue is that, together with the group size, they also vary either $b$ or $c$. Since these latter parameters are known to influence the rate of cooperation [19–21], it is then difficult to say which parameter has caused which effect.

The experimental contribution of this paper is to clarify this point: Does group size have an effect on cooperation in the PGG and the NPD and, if so, how? We have conducted two experiments using the online labour market Amazon Mechanical Turk (AMT) [22–24], one with the PGG and the other one with the NPD. In each of these studies we have separated the participants in two conditions. In Condition S participants played in small groups and in Condition L they played the same one-shot social dilemma but in large groups. Our results are in line with the ones discussed above, but much cleaner. We report a significant negative effect of group size on cooperation in the NPD and a significant positive effect of group size on cooperation in the PGG.

Our paper also provides a theoretical contribution. Since cooperation is typically enforced by means of external controls, such as punishment for defectors or reputation for cooperators [25–32], predicting the expected rate of cooperation without forms of external controls would allow to optimise the use of these techniques, besides shedding light on the cognitive basis of human decision-making. Are there theoretical models predicting our findings? We show that our results are inconsistent with both Fehr & Schmidt's [33] and Charness & Rabin's [34] models. This motivates us to look for other explanatory models. With this in mind, we extend the cooperative equilibrium model [35] from two-player social dilemmas to *some* $N$-person social dilemmas and we show that it predicts both regularities observed. Moreover, since the cooperative equilibrium is a parameter-free model, we can make a direct comparison between experimental data and predictions. The results are not discouraging: predictions are neither strikingly close nor dramatically far from the experimental data.

The structure of the paper is as follows. Section 2 and Section 3 report the experimental results we have collected on the PGG and the NPD, respectively. Section 4 discusses how Fehr & Schmidt's and Charness & Rabin's model perform in this particular situations and extends the cooperative equilibrium model from two-player to *some* N-player social dilemmas. This section also shows that the cooperative equilibrium model predicts both the regularities observed in the lab, making also a quantitative comparison between predictions and experimental data. Section 5 discusses the results and proposes some directions for further research.

## 2. Study A. Group size effect on cooperation in the Public Goods Game

2.1. **Experimental setup.** We recruited US subjects using the online labour market AMT. As in classical lab experiments, AMT workers receive a baseline payment and can earn an additional bonus depending on how they perform in the game. AMT experiments are easy to implement and cheap to realise, since AMT workers are paid a substantially smaller amount of money than people participating in physical lab experiments. Nevertheless, it has been shown that data gathered using AMT agree both qualitatively and quantitatively with those collected in physical labs [23,24,36,37].

In our Study A, participants earned \$0.30 for participation and were randomly assigned to either of two conditions. In Condition S (as in small), they were asked to play a four-player PGG with endowment $y = \$0.10$ and constant marginal return $\gamma = 0.5$; in Condition L (as in large), they were asked to play a forty-player PGG with the same endowment and marginal return. After entering the survey, AMT workers were asked to type their Turk ID and a long CAPTCHA-like code. Specifically, in order to filter out lazy participants, we asked them to write the following neutral sentence (taken from Wikipedia) in reverse order.

*Morocco has a coast on the Atlantic Ocean that reaches past the Strait of Gibraltar into the Mediterranean Sea. It is bordered by Spain to the north, Algeria to the east, and Western Sahara to the south. Since Morocco controls most of Western Sahara, its de facto southern boundary is with Mauritania.*

Participants who passed this first test were presented the rules of the game. In Condition S they were presented as follows:

*You are part of a group of four participants. The amount of money you can earn depends on each of the four participants' decisions.*

*All participants are given 10¢ and each one has to decide how much, if any, to contribute to a pool. Your gain will be what you keep plus half of the total amount contributed by all players.*

*So, for instance:*

- *If everyone contributes all of their money, then you end the game with 20¢.*
- *If you do not contribute anything and everyone else contributes all of their money, then you end the game with 25¢.*
- *If no one contributes anything, then you end the game with 10¢.*

Instructions for Condition L were exactly the same, a part from obvious changes.

In order to have good quality results, after explaining the rules of the game, we asked four comprehension questions. Participants who failed any of the comprehension questions were automatically excluded from the game. We could do this very easily using the Survey builder Qualtrics, which allows us to use skip logics: programs that automatically end the survey if the correct answer is not selected. Questions were formulated in such a way to make clear the duality between maximising one's own payoff and maximising the others' payoff. Specifically, we asked the following questions.

(1) *What is the choice you should make to maximise your gain?*

(2) *What is the choice you should make to maximise the other participants' gains?*

(3) *What choice should the other participants make to maximise their own gains?*

(4) *What choice should the other participants make to maximise your gain?*

Subjects who passed the comprehension questions were then asked to make their decision. After playing, subjects were asked a few basic demographic questions (gender, age, and level of education) and the reason why they made their decision. After this, the survey ended providing the code to claim for the bonus.

2.2. **Results.** 62 subjects (31% female, average age 29.1) passed the comprehension questions in Condition S and 66 subjects (51% female, mean age 28.9) passed the comprehension questions in Condition L.

In AMT experiments with comprehension questions it is virtually impossible to stop the experiment when a precise number of participants is reached. In order to compute the payoffs (for instance) in Condition L, we randomly created two groups of forty people each and we computed the payoffs. Clearly some of the participants belonged to both groups. For these participants we randomly selected either of the two payoffs to be their final bonus.

Table 1 reports all relevant statistics. SEM denotes the standard error of the mean. It is clear that increasing the group size has the effect that more and more people contribute to the public good. The visual impression is confirmed by the Wilcoxon rank-sum test ($P = 0.0002$).

| Condition | % free-riders | % contributors | Mean contribution | SEM |
|---|---|---|---|---|
| S | 48.38 | 30.64 | 3.92 | 0.56 |
| L | 21.21 | 60.60 | 6.91 | 0.51 |

TABLE 1. Descriptive statistics of Study 1. Public Goods Game with 4 players (Condition S) versus PGG with 40 players (Condition L). In both conditions, the maximum possible contribution was \$0.10 and the marginal return was $\gamma = 0.5$.

## 3. Study B. Group size effect on cooperation in the $N$-person Prisoner's Dilemma

3.1. **Experimental setup.** As in our first study, also Study B uses US subjects recruited through the online labour market AMT. Participants earned \$0.30 for participation and were randomly assigned to either of two conditions. In Condition S, they were asked to play a two-player PD with benefit $b = \$0.30$ and cost $c = \$0.10$. in Condition L, they were asked to play an eleven-player PD with the same benefit and cost. After entering the survey, AMT workers were asked to type their Turk ID and the same long CAPTCHA-like code as in our Study A. Participants who passed this first test were presented the rules of the game. In Condition S they were presented as follows:

*You have been paired with another participant. The amount of money you can earn depends on your and the other participant's decision.*

*You are both given 10¢ and each of you must decide whether to keep it or give it away. Each time a participant gives away their 10¢, the other participant earns 30¢.*

*So:*

- *If you both decide to give the 10¢, you end the game with 30¢.*
- *If you keep it and the other participant gives it away, you end the game with 40¢.*
- *If you give it away and the other participant keeps it, you end the game with 0¢.*
- *If you both keep it, then you end the game with 10¢.*

Instructions for Condition L were exactly the same, a part from obvious changes.

In order to have good quality results, after explaining the rules of the game, we asked the participants four comprehension questions. Participants who failed any of the comprehension questions were automatically excluded from the game. Questions were formulated in such a way to make clear the duality between maximising one's own payoff and maximising the others' payoff. Specifically, we asked the following questions.

(1) *What is the choice you should make to maximise your gain?*

(2) *What is the choice you should make to maximise the other participants' gains?*

(3) *What choice should the other participant make to maximise their own gains?*

(4) *What choice should the other participant make to maximise your gain?*

Subjects who passed the comprehension questions were then asked to make their decision. After playing, subjects were asked a few basic demographic questions (gender, age, and level of education) and the reason why they made their decision. After this, the survey ended providing the code to claim for the bonus.

3.2. **Results.** 75 subjects (28% female, average age 28.8) passed the comprehension questions in Condition S and 78 subjects (32% female, mean age 29.6) passed the comprehension questions in Condition L. Table 2 reports all relevant statistics. SEM denotes the standard error of the mean. It is clear that increasing the group size has the effect that fewer and fewer people cooperate. The visual impression is confirmed by Wilcoxon rank-sum test ($P = 0.0404$).

| Condition | % cooperators | SEM |
|---|---|---|
| S | 41.33 | 4.87 |
| L | 25.64 | 4.97 |

TABLE 2. Descriptive statistics of Study 2. Prisoner's Dilemma with 2 players (Condition S) versus Prisoner's Dilemma with 11 players (Condition L). In both conditions, $b = \$0.30$ and $c = \$0.10$.

## 4. WHICH MATHEMATICAL MODELS PREDICT THESE REGULARITIES?

Previous sections reported experimental results in support of the following two hypotheses: (i) Group size has a positive effect on cooperation in the Public Goods Game; (ii) Group size has a negative effect on cooperation in the $N$-person Prisoner's Dilemma. Are there mathematical models of human behaviour predicting both these regularities?

Here we first consider two of the most widely used models of human behaviour, the Fehr & Schmidt [33] and (a one-parameter version of) the Charness & Rabin's [34] models, and we show that none of them predict the above regularities. Then we extend the cooperative equilibrium model [35] from two-person social dilemmas to *some* social dilemmas and we show that it in fact predicts both of them.

Before starting, let us fix the following notation. Given a game $\mathcal{G}$, let $P$ denote the set of players, each of which has pure strategy set $S_i$ and monetary payoff function $u_i$.

4.1. **Fehr & Shmidt model.** Fehr & Schmidt model [33] assumes that, given the strategy profile $\sigma$, the utility of player $i$ is

$$U_i(\sigma) = u_i(\sigma) - \frac{\alpha_i}{N-1}\sum_{j\neq i}\max(u_j(\sigma) - u_i(\sigma), 0) - \frac{\beta_i}{N-1}\sum_{j\neq i}\max(u_i(\sigma) - u_j(\sigma), 0), \tag{1}$$

where $0 \leq \beta_i \leq \alpha_i$ are individual parameters. Specifically, $\alpha_i$ represents the extent to which player $i$ is averse to inequity in the favour of others, and $\beta_i$ represents the extent to which player $i$ is averse to inequity in his favour.

We show a qualitative result as follows. Given a population $\mathcal{P}$ of players, each of which with parameters $(\alpha_i, \beta_i)$, we denote by $\mu(\mathcal{P}, \mathrm{NPD}(b,c))$ the percentage of people $i \in \mathcal{P}$ such that $U_i(C, \ldots, C, D, C, \ldots, C) \leq U_i(D, \ldots, D)$ in the NPD.

**Proposition 4.1.** *For fixed $b$ and $c$, the function $\mu(\mathcal{P}, \mathrm{NPD}(b,c))$ is decreasing with $N$.*

*Proof.* Fix, for notational simplicity, $i = 1$. We have $U_1(C, \ldots, C) = b - c$ and

$$\begin{aligned} U_1(D, C, \ldots, C) &= b - \beta_1\left(b - b\frac{N-2}{N-1} + c\right) \\ &= b - \frac{b\beta_1}{(N-1)} - \frac{c\beta_1}{N-1}. \end{aligned}$$

It is clear that if $N$ is large enough (depending on the player and on $b$ and $c$), one has $U_1(D, C, \ldots, C) > U_1(C, \ldots, C)$. □

Similarly, consider the PGG with total endowment normalised to be $y = 1$, so as full cooperation means to contribute 1 and defection corresponds to contribute 0. Let $\mu(\mathcal{P}, \mathrm{PGG}(\gamma, N))$ denote the percentage of people $i \in \mathcal{P}$ such that $U_i(1, \ldots, 1, 0, 1, \ldots, 1) \leq U_i(1, \ldots, 1)$.

**Proposition 4.2.** *For fixed $\gamma$, the function $\mu(\mathcal{P}, \mathrm{PGG}(\gamma, N))$ is independent of $N$.*

*Proof.* Fix player $i = 1$. We have $U_1(1, \ldots, 1) = \gamma N$ and

$$\begin{aligned} U_1(0, 1, \ldots, 1) &= \gamma(N-1) + 1 - \beta_1\left(\gamma(N-1) + 1 - \gamma(N-1)\right) \\ &= \gamma(N-1) + 1 - \beta_1. \end{aligned}$$

It is clear then clear that the condition $U_1(0, 1, \ldots, 1) > U_1(1, \ldots, 1)$ is independent of $N$. □

In conclusion, while the results of our Study B are consistent with the Fehr-Schmidt model, those of our Study A are not.

4.2. **Charness & Rabin's model.** We consider the following simple form of the Charness & Rabin model [34]. Given a strategy profile $\sigma$, we assume that player $i$ experiences a tension between self interest and common interest and so he or she has utility

$$U_i(\sigma) = \alpha_i u_i(\sigma) + (1 - \alpha_i)\sum_{j=1}^{N} u_j(\sigma), \tag{2}$$

where $\alpha_i \in [0,1]$ is an individual parameter describing how much player $i$ cares about the total welfare.

As in the previous section, we consider a population $\mathcal{P}$ and we denote $\mu(\mathcal{P}, \mathrm{NPD}(b,c))$ the percentage of people $i \in P$ such that $U_i(C,\dots,C,D,C,\dots,C) \leq U_i(D,\dots,D)$ in the NPD.

**Proposition 4.3.** *For fixed $b$ and $c$ the function $\mu(\mathcal{P}, \mathrm{NPD}(b,c))$ is independent of $N$.*

*Proof.* Fix $i=1$. We have $U_1(C,\dots,C) = \alpha_1(b-c) + (1-\alpha_1)(b-c)(N-1)$ and

$$\begin{aligned} U_1(D,C,\dots,C) &= \alpha_1 b + (1-\alpha_1)b + (1-\alpha_1)(N-1)\left(\frac{b(N-2)}{N-1} - c\right) \\ &= b + (1-\alpha_1)(bN - 2b - cN + c). \end{aligned}$$

Observe that the condition $U_1(D,C,\dots C) > U_1(C,\dots,C)$ reduces to $\alpha > 1 - \frac{c}{b}$ and so it does not depend on $N$. ☐

**Proposition 4.4.** *For fixed $\gamma$, the function $\mu(\mathrm{PGG}(\gamma,N))$ is increasing with $N$.*

*Proof.* We have $U_1(1,\dots,1) = \alpha_1\gamma N + \gamma N^2 - \alpha_1\gamma N^2$ and

$$\begin{aligned} U_1(0,1,\dots,1) &= \alpha_1(1+\gamma(N-1)) + (1-\alpha_1)(1+\gamma(N-1)) + \gamma(1-\alpha_1)(N-1)^2 \\ &= 1 - \gamma N + \gamma N^2 + 2\alpha_1\gamma N - \alpha_1\gamma N^2 - \alpha_1\gamma. \end{aligned}$$

It is then clear that the condition $U_1(0,1,\dots,1) < U_1(1,\dots,1)$ reduces to

$$1 - \alpha_1\gamma + \gamma N(\alpha_1 - 1) < 0,$$

which is always verified if $N$ is large enough. ☐

In sum, this one-parametric version of the Charness & Rabin model makes qualitatively the right prediction in the PGG, but not in the NPD.

We mention that the most general form of the Charness & Rabin model takes also into account inequity aversion and uses the utility function

$$V_i(x_1,\dots,x_N) = (1-\alpha_i)x_i + \alpha_i(\delta_i \min(x_1,\dots,x_n) + (1-\delta_i)(x_1+\ldots+x_N)). \quad (3)$$

Not surprisingly, this general version makes the right prediction in both the PGG and the NPD (we leave the tedious computation to the reader). What we find surprising, however, is that the ultimate reason for the different predictions is caused by two different effects: the tendency to maximise group welfare favours cooperation in large PGG and inequity aversion prevent cooperation in the NPD.

We now pass to the description of a model which predicts both regularities by appealing to the same effect (tendency to maximise the group welfare) and without using any free parameter. The price to pay is a major technical difficulty and the fact that the model, at the moment, is definable only for what we call *highly symmetric* games.

4.3. **Cooperative equilibrium.** The cooperative equilibrium is a new parameter-free solution concept for two player social dilemmas, which has been shown to make reasonably accurate predictions of average behaviour in one-shot [35], [38] and iterated [39] social dilemmas. It is typically a mixed strategy depending on the payoffs, which organises all classical regularities: The level of cooperation in the Prisoner's Dilemma increases as the cost/benefit ratio decreases; The level of cooperation in the Traveler's dilemma increases as the bonus/penalty decreases; The level of cooperation in the Public Goods Game increases as the constant marginal return increases. We now extend this model to *some* N-player social dilemmas and show that it predicts also the regularities reported in Section 2 and 3.

The key idea behind the cooperative equilibrium is the assumption that humans do not act a priori as single agents, but they forecast how the game would be played if they formed coalitions and then act so as to maximise their forecast.

A general theory for every normal form game would require to consider all possible coalition structures and deal with the fact that different players may have different forecasts about the same coalition structure. Here we eliminate this problem by restricting to only *highly symmetric* social dilemmas.

- **Symmetry.** All players have the same set of strategies $S$ and for each player $i$, for each permutation $\pi$ of the set of players, and for each strategy profile $(s_1, \dots, s_N) \in S^N$ one has
$$u_i(s_1, \dots, s_N) = u_{\pi(i)}(s_{\pi(1)}, \dots, s_{\pi(N)}). \tag{4}$$
- **High symmetry.** The game is symmetric and has a unique (and symmetric) Nash equilibrium and a unique (and symmetric) profile of strategy maximizing the total welfare.

Given the high level of symmetry of the games under consideration, we consider only the two extremal scenarios: the selfish coalition structure and the fully cooperative coalition structure.

Let $P$ denote the set of players, each of which has pure strategy set $S$, mixed strategy set $\mathcal{P}(S)$, and utility function $u_i$. Coalition structures are just partitions of the player set. We denote $p_s$ the selfish coalition structure and $p_c$ the fully cooperative coalition structure. Every coalition structure $p \in \{p_s, p_c\}$ gives rise to a new game $\mathcal{G}_p$, where players in the same coalition play as a single player aiming to maximise the sum of the payoffs of the players belonging to that coalition. For every highly symmetric social dilemma, $\mathcal{G}_p$ has a unique Nash equilibrium, which we denote $\sigma^p$. Fix $i \in P$ and let $j \in P \setminus \{i\}$ be another player. We denote $I_j(p)$ the maximum payoff that player $j$ can obtain by leaving the coalition structure $p$. Formally,
$$I_j(p) := \max\{u_j(\sigma^p_{-j}, \sigma_j) - u_j(\sigma^p_{-j}, \sigma^p_j) : \sigma_j \in \mathcal{P}(S)\}. \tag{5}$$

$I_j(p)$ will be called the *incentive* of player $j$ to abandon the coalition structure $p$.

Given a profile of strategies $(\sigma_1, \dots, \sigma_N)$, a strategy $\sigma_i' \in \mathcal{P}(S)$ is called an *i-deviation* from $(\sigma_1, \dots, \sigma_N)$ if $u_i(\sigma_i', \sigma_{-i}) \geq u_i(\sigma_1, \dots, \sigma_N)$.

We denote $D_j(p)$ the maximal loss that players $j$ can incur if he decides to leave the coalition structure $p$ to try to achieve his maximal possible gain, but also other players deviate from the coalition structure $p$ to either follow their selfish interests or anticipate player $j$'s deviation. Formally,

$$D_j(p) := \max\{u_j(\sigma_i^p, \sigma_{-i}^p) - u_j(\sigma_j, \sigma_{-j})\}, \tag{6}$$

where $\sigma_j$ runs over the set of strategies such that $u_j(\sigma_{-j}^p, \sigma_j)$ is maximized and $\sigma_{-j}$ runs over the set of profiles of strategies $(\sigma_k)_{k\neq j}$ for which there is $h$ such that $\sigma_h$ is an $h$-deviation from either $\sigma^p$ or $(\sigma_j^p, \sigma_{-j})$. $D_j(p)$ is called the *disincentive* for player $j$ in abandoning the coalition structure $p$. The number

$$\tau_{i,j}(p) := \frac{I_j(p)}{I_j(p) + D_j(p)}$$

will be informally interpreted as the probability that player $i$ assigns to the event "*player $j$, knowing that all other players are thinking about playing according to $p$, abandons the coalition structure $p$*". In the context of anonymous games, where the reasoning of a player cannot affect the reasoning of another player, we define, for $J \neq \emptyset$,

$$\tau_{i,J}(p) := \prod_{j\in J} \tau_{i,j}(p).$$

Now to define $\tau_{i,\emptyset}(p)$, which is the probability that nobody abandons the coalition structure, we use the law of total probabilities. Assume, for simplicity, that $\tau_{i,j}(p)$ does not depend on $i$ and $j$, as is the case in symmetric games. We find

$$\begin{aligned}
\tau_{i,\emptyset}(p) &= 1 - \sum_{k=1}^{N-1} (-1)^{k+1} \binom{N-1}{k} \tau_{i,j}(p)^k \\
&= 1 + \sum_{k=1}^{N-1} (-1)^{k} \binom{N-1}{k} \tau_{i,j}(p)^k \\
&= \sum_{k=0}^{N-1} (-1)^{k} \binom{N-1}{k} \tau_{i,j}(p)^k \\
&= \sum_{k=0}^{N-1} \binom{N-1}{k} (-\tau_{i,j}(p))^k.
\end{aligned}$$

Using Newton's binomial law we finally find

$$\tau_{i,\emptyset}(p) = (1 - \tau_{i,j}(p))^{N-1}.$$

Now, let $e_{i,\emptyset}(p)$ be the minimum payoff for player $i$ if nobody abandons the coalition structure $p$, which is just $u_i(\sigma^p)$, and, finally, let $e_{i,J}(p)$ be the infimum of payoffs of player $i$ when she plays $\sigma_i^p$ but at least one player $j \in J \setminus \{i\}$ plays a $j$-deviation from $\sigma^p$. The *forecast* of player $i$ associated to the coalition structure $p$ is defined as

$$v_i(p) := \sum_{J\subseteq P\setminus\{i\}} e_{i,J}(p)\tau_{i,J}(p). \tag{7}$$

Observe that symmetry implies that the forecast $v_i(p)$, if $p \in \{p_s, p_c\}$, actually does not depend on $i$ and so there is a coalition structure $\overline{p}$ (independent of $i$) which maximises the forecast for all players. Moreover, $v_i(\overline{p}) = v_j(\overline{p})$, for all $i, j \in P$. We denote this number $v(\overline{p})$ and we use it to define common beliefs or, in other words, to make a tacit binding among the players.

**Definition 4.5.** The induced game $\mathrm{Ind}(\mathcal{G}, \overline{p})$ is the same game as $\mathcal{G}$ except for the set of profiles of strategies: the induced game contains only those strategy profiles $\sigma$ such that $u_i(\sigma) \geq v(\overline{p})$, for all $i \in P$.

The induced game does not depend on the maximizing coalition structure, that is, in case of multiple coalition structures maximising the forecast, one can choose one of them casually to define the induced game and this game does not depend on such a choice.

Since the set of strategy profiles in the induced game is convex and compact (and non-empty) one can compute Nash equilibria of the induced game.

**Definition 4.6.** A cooperative equilibrium for $\mathcal{G}$ is a Nash equilibrium of the game $\mathrm{Ind}(\mathcal{G}, \overline{p})$.

It has been proven in [35] that this model organises all classical observations on two-person social dilemmas: The rate of cooperation in the Prisoner's Dilemma increases as the cost/benefit ratio decreases; the rate of cooperation in the Traveler's dilemma increases as the bonus/penalty decreases; the rate of cooperation in the Public Goods Game increases as the constant marginal return increases. We now show that it organises also the regularities observed in our experiments.

Let $\mathrm{PGG}(N, \gamma)$ denote the Public Goods Game with $N$ players and constant marginal return $\gamma$. To simplify the formulas, we assume that the total endowment of each player is normalized to $y = 1$. Denote

$$v(\gamma, N) = \gamma N \left( \frac{\gamma N - 1}{\gamma(N-1)} \right)^{N-1} + \gamma \left( 1 - \left( \frac{\gamma N - 1}{\gamma(N-1)} \right)^{N-1} \right). \tag{8}$$

**Theorem 4.7.** The only (pure) cooperative equilibrium of the $\mathrm{PGG}(N, \gamma)$ is to contribute

$$\max \left( 0, \frac{v(\gamma, N) - 1}{\gamma N - 1} \right). \tag{9}$$

In particular, it is increasing with the variable $N$.

*Proof.* Since $\mathcal{G}_{p_s} = \mathcal{G}$, then $\sigma^{p_s}$ is the Nash equilibrium of the original game. Since there is no incentive to deviate from a Nash equilibrium, the $\tau$ measure is the Dirac measure concentrated on $J = \emptyset$. Therefore $v_i(p_s)$ coincides with the payoff in equilibrium; that is, $v_i(p_s) = 1$.

Let now $p_c$ be the fully cooperative coalition structure and observe that, for all $j \in P$, one has $I_j(p_c) = 1 - \gamma$ and $D_j(p_c) = \gamma N - 1$. Consequently, $\tau_{i,j}(p_c) = \frac{1-\gamma}{\gamma(N-1)}$, for all

$i, j \in P$, $i \neq j$. Now, $e_{i,J}(p_c) = \gamma$, for all $J \neq \emptyset$, and $e_{i,\emptyset}(p_c) = \gamma N$. Therefore,

$$\begin{aligned} v_i(p_c) &= \gamma N \left(1 - \frac{1-\gamma}{\gamma(N-1)}\right)^{N-1} + \gamma \left(1 - \left(1 - \frac{1-\gamma}{\gamma(N-1)}\right)^{N-1}\right) \\ &= \gamma N \left(\frac{\gamma N - 1}{\gamma(N-1)}\right)^{N-1} + \gamma \left(1 - \left(\frac{\gamma N - 1}{\gamma(N-1)}\right)^{N-1}\right). \end{aligned}$$

To compute the cooperative equilibrium, we observe that this would be the lowest contribution among the ones which, if contributed by all players, would give to all players a payoff of at least $v_i(p_c)$. To compute this contribution it is enough to solve the equation

$$1 - \lambda + \gamma N \lambda = v_i(p_c),$$

whose solution is indeed $\lambda = \frac{v_i(p_c)-1}{\gamma N - 1}$, as stated.

It remains to show that the cooperative equilibrium is increasing with $N$. To this end, we replace $N$ by a continuous variable $x \geq 2$ and denote $v(x) := v_i(p_c)$, $f(x) = \frac{v(x)-1}{\gamma x - 1}$, and $r(x) = \left(\frac{\gamma x - 1}{\gamma(x-1)}\right)^{x-1}$. Observe that all these functions are differentiable in our domain of interest $x \geq 2$. Our aim is to show that $f(x)$ is increasing, that is, $f'(x) > 0$. We start by observing that $r(x)$ is increasing. This can be seen essentially in the same way as one sees the standard fact that $\left(1 + \frac{1}{n}\right)^n$ is increasing in $n$, by using Bernoulli's inequality. Hence, we have

$$v'(x) = \gamma r(x) + \gamma r'(x)(x-1) > \gamma r(x).$$

Consequently, using also the fact that $\gamma x r(x) = v(x) - \gamma(1 - r(x))$, we conclude

$$\begin{aligned} f'(x) &= \frac{v'(x)(\gamma x - 1) - \gamma(v(x) - 1)}{(\gamma x - 1)^2} \\ &> \frac{\gamma(\gamma x r(x) - v(x) + 1)}{(\gamma x - 1)^2} \\ &= \frac{\gamma(1 - \gamma(1 - r(x)))}{(\gamma x - 1)^2} \\ &> 0, \end{aligned}$$

where, the last inequality follows from the fact that both $\gamma$ and $r(x)$ are strictly smaller than 1. $\square$

Since the cooperative equilibrium is parameter-free, we can make a direct comparison between its predictions and experimental data. Results are summarised in Table 3. While the qualitative behaviour is well-captured, the quantitative prediction is neither strikingly close nor dramatically far.

Let $\mathrm{PD}(b, c, N)$ denote the $N$-person Prisoner's Dilemma with cost $c$ and benefit $b$. Denote

$$v(b, c, N) = (b - c)\left(1 - \frac{c}{b}\right)^{N-1} - c\left(1 - \left(1 - \frac{c}{b}\right)^{N-1}\right).$$

| Condition | Mean contribution | CE prediction |
|---|---|---|
| S | 3.92 | 0 |
| L | 6.91 | 3.46 |

TABLE 3. Comparison between predictions of the cooperative equilibrium and experimental data in the PGG. In Condition S, the cooperative equilibrium coincides with the Nash equilibrium. In Condition L, it predicts that players should contribute about 35% of their endowment. CE correctly predicts a positive effect of group size on cooperation. The quantitative predictions are neither strikingly close nor drammatically far.

**Theorem 4.8.** The only cooperative equilibrium of $\mathrm{PD}(b, c, N)$ predicts cooperation with probability

$$\lambda = \max\left(0, \frac{v(b,c,N)}{b-c}\right).$$

So cooperation is predicted to decrease with the number of agents.

*Proof.* The forecast associated to the selfish coalition structure is $v_i(p_s) = 0$, for all players, corresponding to the payoff in (Nash) equilibrium. To compute the forecast associated to the fully cooperative coalition structure, observe that $e_{i,\emptyset}(p_c) = b - c$, corresponding to Pareto optimum where all players cooperate. The incentive to deviate from the cooperative strategy is $I_j(p_c) = c$, while the disincentive is $D_j(p_c) = b - c$, corresponding to the loss incurred in case all other players anticipate player j's defection and decide to defect as well. Finally, $e_{i,J}(p_c) = -c$, for all $J \neq \emptyset$, corresponding to the strategy profile where only player $i$ cooperates and all other players defect. Hence we have

$$v_i(p_c) = (b-c)\left(1-\frac{c}{b}\right)^{N-1} - c\left(1-\left(1-\frac{c}{b}\right)^{N-1}\right).$$

Of course, if $v_i(p_c) \leq 0$, then the cooperative equilibrium coincides with the Nash equilibrium. Otherwise, by symmetry, it is the only strategy $\sigma$ such that

$$u_i(\sigma, \ldots, \sigma) = v_i(p_c), \tag{10}$$

for all $i \in P$. Setting $\sigma = \lambda C + (1-\lambda)D$, we obtain

$$\begin{aligned}
u_i(\sigma, \ldots, \sigma) &= \lambda \sum_{k=0}^{N-1} \lambda^{N-1-k}(1-\lambda)^k \binom{N-1}{k}\left(\frac{b(N-1-k)}{N-1} - c\right) \\
&\quad + (1-\lambda)\sum_{k=0}^{N-1} \lambda^{N-1-k}(1-\lambda)^k \binom{N-1}{k}\left(\frac{b(N-1-k)}{N-1}\right) \\
&= \sum_{k=0}^{N-1} \lambda^{N-1-k}(1-\lambda)^k \binom{N-1}{k}\frac{b(N-1-k)}{N-1} - c\lambda \\
&= b - c\lambda - \frac{b}{N-1}\sum_{k=0}^{N-1} \lambda^{N-1-k}(1-\lambda)^k \binom{N-1}{k} k.
\end{aligned}$$

Now we use the fact that

$$\sum_{k=0}^{N-1} \lambda^{N-1-k}(1-\lambda)^k \binom{N-1}{k} k = (1-\lambda)(N-1),$$

to reduce Equation (10) to

$$\lambda(b-c) = v_i(p_c), \tag{11}$$

which concludes the proof. □

Also in this case we can make a direct comparison between predictions and experimental data. Results are summarised in Table 4. Again, the quantitative prediction is neither strikingly close nor dramatically far.

| Condition | % cooperators | CE prediction |
|---|---|---|
| S | 41.33 | 50 |
| L | 25.64 | 0 |

Table 4. Comparison between the prediction of the cooperative equilibrium and experimental data. The cooperative equilibrium coincides with the Nash equilibrium in Condition L and predicts that half of the people cooperate in Condition S. The cooperative equilibrium correctly predicts a negative effect of group size on cooperation in the NPD. Quantitative predictions are neither strikingly close, nor dramatically far.

## 5. Discussion

We have studied how the size of a group influences cooperation in two one-shot social dilemmas, the Public Goods Game and the $N$-person Prisoner's Dilemma. The reason why we have considered these two games is that we expected opposite results. In the PGG the benefit for cooperating increases linearly with the size of the group, while the cost of cooperation remains constant; in the NPD both the benefit and the cost remain constant, but, in order to reach the benefit, one needs more people to cooperate. This difference suggests that we should see a positive effect of group size on cooperation in the PGG and a negative effect of group size on cooperation in the NPD.

To test this prediction we have conducted two experiments using the online labour market Amazon Mechanical Turk. Our results confirmed it showing that forty players are significantly more efficient in providing the public good than only four players (P=0.0002), and that two players are significantly more cooperative than eleven in a Prisoner's Dilemma (P=0.04).

We have then studied which mathematical models of human behaviour predict these findings. Surprisingly, we have found that neither the Fehr & Schmidt model nor (a one-parameter version of) the Charness & Rabin model predict it. However, the general form of Charness & Rabin's utility makes the right prediction, at the price of using two free parameters. Moreover, different predictions are due to different causes: tendency to maximise the group welfare favours cooperation in large PGGs and inequity aversion prevents cooperation in large NPDs.

Looking for a more predictive model explaining the results by appealing to only one cause, we have extended the cooperative equilibrium from two-player social dilemmas to *some* N-player symmetric games, that we have called *highly symmetric*. Despite the fact that this model does not use any free parameter, it is able to predict both the above mentioned regularities by appealing to a single cause: tendency to maximise total welfare. Moreover, the quantitative predictions of the model are not dramatically far from the data we gathered. On the negative side, the cooperative equilibrium model undoubtedly uses too many hypotheses on the structure of the game on which it can be defined and an extension to every normal form game is currently still beyond the horizon.

This paper generates many research questions in both the experimental and theoretical domains.

On the experimental side, the cooperative equilibrium model makes also other predictions that we have not tested.

- For fixed $N$ and $c$, the benefit $b$ has a positive effect on cooperation in the NPD.
- For fixed $N$ and $b$, the cost $c$ has a negative effect on cooperation in the NPD.

Confirmation of these two predictions have been recently found in the case of two players [19–21] and we cannot find any reasonable motivation why they should fail in larger groups.

The model also predicts that, with $N$ fixed, $\gamma$ has a positive effect on cooperation in the PGG. This fact has been confirmed by several experimental studies on both one-shot and iterated PGGs [16, 40–43].

Another highly symmetric social dilemma to which the cooperative equilibrium model can be applied is the Bertrand Competition (BC). In the BC, $N \geq 2$ firms compete to sell their identical product. Each of the firms can choose a price between the 'price floor' $L$ and the 'reservation value' $H > L$. The firm that chooses the lowest price, say $s$, sells the product at that price, getting a payoff of $s$; all other firms get nothing. Ties get split among all firms that made the corresponding price. One easily sees that the only cooperative equilibrium of this game is to set the price

$$\max\left(L, H \cdot \left(\frac{H}{(H-1)N}\right)^{N-1}\right).$$

So the cooperative equilibrium makes the prediction that the size of the group has a strong (more than exponential) negative effect on cooperation in the BC. This prediction has been partially confirmed by other experimental studies [44–46], which have shown that four people are already enough to completely destroy cooperative behaviour. However, these studies concern iterated games and so more experimental evidence would be needed. We mention that we tried to address this problem, but our experiment was unsuccessful, probably due to a mistake in the design. We recruited subjects on AMT and we assigned them to either Condition S ($N = 2$, $L = \$0.02$, $H = \$1$) or Condition L ($N = 10$, $L = \$0.10$, $H = \$1$). We have not found any difference in behaviour depending on the size of the group and we have also observed unusually high bids (average close to \$0.70 in both conditions). We believe that this happened because the Nash equilibrium guarantees a win of only 1¢ and so it is perceived as being the same as *losing*. This may have stimulated people to risk more and thus playing unusually high numbers. This also puts in evidence the danger of making experiments with AMT. Many papers have demonstrated that results gathered using AMT agrees both qualitatively and quantitatively with those collected on the physical labs. However, one has to keep in mind that some effects may disappear when using too low stakes, due to the fact that behaviours profoundly different from the theoretical point of view, may become equivalent due to the negligible difference of the expected payoffs.

On the theoretical side, many questions concerning the model should be addressed, starting from the broad question of extending the cooperative equilibrium model as far as possible, at least to include other relevant social dilemmas such as the Volunteer dilemma [47] and the collective-risk social dilemma [48]. Unfortunately, these social dilemmas possess one of the general characteristics preventing the development of a general theory: in case the game $\mathcal{G}_p$ has many (possibly infinite) equilibria, which one should be used as a reference strategy profiles $\sigma^p$ to compute incentives and disincentives? Other theoretical questions include: Can the cooperative equilibrium be expressed in terms of classical utility theory through a utility function to be maximised? Is there any 'bridge' between the cooperative equilibrium and standard cooperative game theory?

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