The Structure of Negotiations: Bargaining and the Focusing Effect.

Very preliminary. Do not circulate or distribute without permission.

Andrea Canidio[*]*and Heiko Karle*[†]

This Version: April 15, 2014.

Abstract

We provide a theory of incomplete agreements within negotiations. If preferences are distorted by the focusing effect, the negotiating players may negotiate in stages: first discussing a partial agreement and then finalizing the bargaining outcome. The first bargaining stage can be used to eliminate extreme outcomes from the possible bargaining solutions, hence increasing the value of the agreement for the player whose preferences are distorted by the focusing effect. With respect to the existing literature, we provide a justification for the existence of incomplete agreements that does not rely on some uncertainty being resolved between bargaining rounds. We also show that players may endogenously decide to be held up. By first paying the fixed cost of production and then bargaining on the price dimension, a seller may be able to manipulate the preferences of a focused buyer and extract higher profits compared with the case in which quality and price and jointly determined.

JEL classification: C78, D03, D86, F51.

Keywords: Incomplete contracts, Salience, Focusing Effect, Bargaining, Negotiations.

[*]Department of Economics, Central European University, Nádor u. 9, 1051 Budapest, Hungary; email: canidioa@ceu.hu.

[†]Center for Law and Economics, ETH Zurich, Haldeneggsteig 4 8092 Zurich, Switzerland; email: hkarle@ethz.ch.

1 Introduction.

Most agreements are partial agreements, i.e. they specify only some aspects of the final outcome, and rely on a future bargaining round to define the missing provisions. One context where incomplete agreements are frequently used is negotiations. Complex negotiations often happen in stages, with interim, partial agreements preceding a final, comprehensive agreement. For example, the current Doha round of trade negotiations is organized under a set of principles,[1] the first of which is:

> Single Undertaking: Virtually every item of the negotiation is part of a whole and indivisible package and cannot be agreed separately. "Nothing is agreed until everything is agreed".

The Doha round is divided into several tables, and any agreement reached at a specific table is neither final nor binding but provides the framework for a later stage of negotiation. Another case in point is the current round of peace talks between the FARC and the Colombian government. In a preliminary, secret negotiation round, the two parties decided to split the peace talks in 6 parts. In a final bargaining round, the 6 interim agreements will be merged into one comprehensive agreement that will be signed by the parties and will become binding.[2] In procurement, the term *framework agreement* is used to designate the rules under which one or several contracts will be negotiated. For example, a framework agreement may specify a price and a quality at which a transaction may occur, without obligation for the parties to transact, or to transact at the pre-specified price and quality. Hence, framework agreements are partial agreements, in the sense that a new negotiation round is needed to define the terms and nature of the procurement contract. More in general, in many situations the principle of "nothing is agreed until everything is agreed" is not explicitly stated. However in presence of sequential agreements among the same parties, courts typically enforce the latest. Hence, previous agreements provide a framework for future negotiations but are usually not binding.

The existence of partial agreements in the context of negotiations is relevant for economic theory because, typically, no new information is expected to emerge between the different negotiation rounds. The existing theories on incomplete contracts usually rely on new information being generated after the agreement is signed to explain why some provisions of a contract may be left unspecified (see the literature review for more details). The goal of this paper is to provide a behavioral justification to the existence of incomplete contracts in a negotiation setting, in which no new information is generated between bargaining rounds.

[1] See http://www.wto.org/english/tratop_e/dda_e/work_organi_e.html (accessed on the 1st of December 2013).

[2] See http://www.economist.com/news/americas/21584384-hiccup-serves-confirm-government-and-farc-are-making-progress-edge-and (accessed on the 11th of November 2013).

We introduce the *focusing effect* into a simple bargaining problem, and we show that the outcome of the bargaining depends on how the negotiation is structured over time. The focusing effect (or focusing illusion) occurs whenever an agent places too much importance on certain aspects of her choice set or on certain pieces of information (i.e. certain elements are more salient than others). Intuitively, by changing the set of outcomes available to the parties, interim agreements affect the salience of different components of the agreement and the final outcome of the negotiation. Our main result is that the bargaining parties may use partial agreements to eliminate extremely bad outcomes from the set of possible bargaining outcomes. These outcomes will not be reached anyway, but by eliminating them as possible solutions to the bargaining problem, the parties increase the value of the agreement during the last stage of negotiations.

The fact that people's preferences may depend on the choice set available is well documented. For example, when choosing between two vectors of goods x and y, a person may pick x when the choice set is $\{x, y\}$ and may pick y when the choice set expands to $\{x, y, z\}$. The psychological literature identified the focusing effect as one of the reasons why preferences may change with the choice set, arguing that the introduction of a new element in the choice set may affect the salience of the various attributes of the elements of the choice set. Kőszegi and Szeidl (2013) formalize this concept by assuming that agents maximize a *focus-weighted utility*

$$\tilde{U}(x_1, x_2, .., x_n) = \sum_{s=1}^{n} g_s u_s(x_s)$$

where $\{x_1, x_2, ...x_n\}$ is a given good with n attributes, and the *focus weights* g_s are defined as:

$$g_s = g\left(\max_{x\in C} u_s\left(x_s\right) - \min_{x\in C} u_s\left(x_s\right)\right)$$

where C is the choice set and $g()$ is the *focusing function,* assumed strictly increasing. In this formalization, agents overweight the utility of goods in which their options differ more, when these differences are measured in utility terms.[3] It follows that adding a new element to the choice set will change the evaluation of the other elements if this new option is better or worse than the existing options in any dimension.

[3] Schkade and Kahneman (1998) show that, when asked about comparing life in California and in the Midwest, most people report California as the best place to live and cite the weather - i.e. the dimension in which the two choices differ the most - as the main reason. Despite this, actual measures of life satisfactions in the two regions are similar. Similarly, Kahneman, Krueger, Schkade, Schwarz, and Stone (2006) show that people place too much weight on differences in monetary compensation when asked to compare job offers.

In a negotiation context, the focusing effect plays a role because previous interim agreements may constrain the possible outcomes of future bargaining rounds.[4] In the simplest example we present, we consider the case of a buyer and a seller exchanging a good of known quality. We assume that the seller is rational, but the buyer's utility is distorted by the focusing effect. We show that the final transaction price depends on whether the parties bargain in one period, or whether the parties agree first on the maximum price to be charged, and then about the final price. When the bargaining happen in two steps, in the last period the set of possible transaction prices is bounded by the previous agreement. This implies that the buyer will consider the price dimension as less salient, and is willing to accept a final price that is higher then in the one-shot bargaining.

The same logic applies when analyzing pre-bargaining rounds that are not bargaining round themselves. For example, we allow the seller to announce a maximum price before the negotiation begins (or announcing a price, with the understanding that this price can be negotiated downward). The effect of this announcement is to reduce the salience of the price dimension, to decrease the buyer's price sensitivity, and increase the equilibrium transaction price. Therefore, players may engage in pre-bargaining actions that apparently constrain their bargaining position, to manipulate the other player's preferences.

We also address the issue of renegotiation: to what extent partial agreements signed in period 1 bind the bargaining outcome reachable in period 2. We assume that, in period 2, the parties can either choose one of the bargaining outcomes that are allowed by the previous agreement, choose any braining outcome that Pareto improves over period-1 agreement, or wait one period (at no cost) and bargain over the entire bargaining set. Hence, period-1 agreements can completely ignored, but not in period 2. Under this assumption, the period 1 agreement affects the focusing weights in period 2 because it affects the bargaining options that are available in period 2. Also in this case, the way the negotiation is structured affects the final bargaining outcome also in this case.

Finally, we consider the example of a focused buyer and a rational seller deciding on the price and quality of a good. We show that the way the negotiation is structured is welfare relevant, in the sense that the final quality agreed upon depends on whether the negotiation happen in one stage or through an interim agreement. We also show that rather than bargaining in one step, the seller may enter the negotiation having already invested in quality and simultaneously announced a maximum price that will be charged. Doing so, the seller can manipulate the focusing weights of the buyer. If the focusing effect is particularly severe, the seller may prefer to be held up rather than bargain simultaneously over price

[4] Kőszegi and Szeidl (2013) theory is about how the specific choice set affects the agent's preferences. Here we use the same theory to show that, in a bargaining game, the bargaining set affects the player's preferences. There is a continuity argument in support of this assumption: a choice set is a bargaining set in which the other player has no bargaining power.

and quality. In addition, the maximum price announced will be equal to the final price. In practice, the seller prefers to invest in quality and announce a price (which will not be renegotiated) rather than bargain with the buyer.

Relevant Literature.

In their seminal work, Grossman and Hart (1986) define the concept of ownership as the residual right of control: the right to dispose of an object in case a contingency that was not specified in a previous agreement occurs. Hence, ownership is well defined only if contracts are incomplete: two parties cannot specify all details of a contract in one period, but need to negotiate in two stages. Grossman and Hart (1986) justify the assumption of contract incompleteness with uncertainty: it is not possible to write agreements contingent on all possible states of the world, and the parties have to wait for the uncertainty to be resolved to complete the contract.

Several authors argued that contract incompleteness may arise because of cognitive limitations when dealing with uncertainty. The contracting parties may leave some contingencies unspecified and rely on future renegotiation when the ex-ante contracting environment is complex (see Segal, 1999), when becoming aware or thinking about future contingencies is costly (see Tirole, 2009, and Bolton and Faure-Grimaud, 2010), or when there is a cost of specifying contingencies in a contract (see Battigalli and Maggi, 2002). More recently, Hart and Moore (2008) showed that, if the parties cannot specify the outcome of the bargaining in some states of the world, they may decide to rely on ex-post renegotiation also in states of the world that could have been specified in the ex-ante contract. When a state of the world that is not specified in a contract occurs, the two parties will renegotiate their agreement. Hart and Moore (2008) crucially assume that, in the renegotiation, each party takes the ex-ante contract as reference point, and evaluates the outcome of the bargaining with respect to the best possible outcome specified in the contract. In addition, if any player is dissatisfied with the outcome of the bargaining, she can impose a cost on the other player. It follows that, when renegotiation happens with positive probability, the two parties may prefer to leave some provision of the contract unspecified to avoid creating a reference point in future renegotiations.[5]

Our paper is related to the above literature because, also here, agreements are reached in steps. The parties sign an incomplete agreement in period 1, and rely on future renegotiation to reach a complete agreement. However, all the papers mentioned above require that some uncertainty is resolved between the two bargaining rounds, otherwise there is no scope for contract incompleteness. In the context of negotiations, agreements are reached in steps also

[5] For a similar argument, but based on the agent's loss aversion, see Herweg, Karle, and Müller (2013).

when no new information is expected to arrive. Our paper considers exactly these situations. On the other hand, we do not discuss here the issues of ownership and the determinants of ownership, which are at the core of the papers mentioned above. Hence, our paper is complementary to the existing literature on bargaining with behavioral types.

The literature on agenda setting in negotiation has long argued that, when players bargain over multiple issues, the order in which agreements are reached matters for the outcome of the bargaining process (see Lang and Rosenthal, 2001, Bac and Raff, 1996, Inderst, 2000, Busch and Horstmann, 1999b). Extending Rubinstein (1982) model of bargaining as a game of alternating offers, these papers assume that each player can make offers about one or more issues on the table. Once an agreement is reached on one issue, this agreement is binding for both parties. Hence the parties strategically choose whether to make offers about all the issues on the table, or only on some of them. In particular, Busch and Horstmann (1999a) and Flamini (2007) argue that the players may agree on the order in which the issues are resolved (i.e. the offers are made). Here, we are interested in bargaining processes that entail a unique final agreement reached via several interim agreements, rather than several issue-specific agreements.

We employ here the model of salience in economic choice proposed by Kőszegi and Szeidl (2013). Bordalo, Gennaioli, and Shleifer (2012) also develop a model of salience in which preferences depend on features of the choice set. They assume that agents overvalue the goods' attributes that differ the most with respect to a reference point. Also in their approach, as the choice set expands, the consumer will put more weight on the attributes that change the most. Hence, the basic mechanism that underlies our results is present also in Bordalo et al. (2012). We expect our results to be robust to the specific way of modeling the focusing effect.

Finally, we solve each bargaining round using the Nash bargaining solution. This implies that, for *given preferences*, irrelevant alternatives do not affect the bargaining outcome. However, in our model, preferences are derived endogenously and depend on the entire bargaining set. Ponsati and Watson (1997) show that, for given preferences, the Nash bargaining solution is the only solution that satisfies *agenda independence*: the solution is the same whether multiple issues are discussed all at once, or discussed separately (but implemented simultaneously). Hence, our approach isolates a single channel through which the bargaining agenda affects the bargaining outcome: by affecting the players' preferences.

2 A General Framework

A buyer and a seller bargain over the characteristics and the price of a good. We call the different attributes of the good q_i, and the price of the good p. The reservation utility (i.e.

the utility in case of no trade) of both players is zero. The payoff of the seller is

$$U^s(x) = p - c(q_1, ..., q_n)$$

with $c(q_1, ..., q_n)$ positive, continuous, increasing and symmetric. The payoff of the buyer is

$$U^b(x) = \sum_i^n h(\bar{q}_i - \underline{q_i})q_i - h(\bar{p} - \underline{p})p$$

where $h(.)$ is the focusing function, assumed strictly increasing with $h(0) = 1$. For every i, $\bar{q}_i$ and $\underline{q_i}$ are the highest and lowest value of the attribute q_i in the *consideration set*: the set of outcomes that are considered possible by the buyer. Similarly, $\bar{p}$ and $\underline{p}$ are the highest and lowest price in the consideration set. The above formulation implies that the buyer overweights the component of the utility function that varies the most within the consideration set.

Definition 1 (Feasibility)**.** Consider the outcome $x \equiv (q_1, ..., q_n, p)$. The outcome x is feasible if it can be implemented as a solution to the bargaining problem in case the two parties agree on it.

For example, if there is a technological limit to the level of a given attribute, every x that violates this technological limit is not feasible.

Assumption 2 (Consideration Set)**.** *The outcome x is in the consideration set if it is feasible and if both players satisfy their rationality constraint at x.*

The above assumption implies that a vector x is in the consideration set if both utility and profits are greater or equal than zero at x. Note that finding the boundaries of the consideration set is a fixed-point problem: the utility of the buyer depends on the boundary of the consideration set, at the same time the boundaries of the consideration set depend on the utility of the buyer.

The two players bargain over two periods. Neither buyer nor seller discount the future. Call the set of feasible outcomes at the beginning of the game X. A *bargaining tool* is a vector of $l \geq 1$ functions $k(x) \equiv \{k_1(x), k_2(x), ..., k_l(x)\}$ such that:

- $k_i(.) : X \to \mathbf{R}$ for $i \in \{1, ...l\}$,
- each $k_i(.)$ is a continuous function,
- define the *posterior consideration set* $s(y) \equiv \{z \in X | k_i(z) \leq y_i\}$ for all $i \in \{1, ..., l\}$ There exists a $\bar{y}$ such that $s(\bar{y}) \equiv X$.

A *bargaining protocol* specifies a bargaining tool and a $\tau \in \{b, s, j\}$. In period 1, if the bargaining protocol specifies $\tau = b$, then the buyer announces y; if the bargaining protocol specifies $\tau = s$, then the seller announces y; if the bargaining protocol specifies j, then the two parties bargain over y. In the last case, if the parties do not reach an agreement, no trade occurs. In period 2, the two parties bargain over the final outcome x and all transactions occur.

3 A simple problem: the sale of a single-attribute good

Let us start by considering the simplest possible bargaining problem: the sale of a single attribute good. The seller's utility is:

$$U^s = \begin{cases} p - c & \text{if he sells the widget} \\ 0 & \text{otherwise} \end{cases}$$

And the buyer's utility function is:

$$U^b = \begin{cases} h(\bar{q} - \underline{q})q - h(\bar{p} - \underline{p})p & \text{if he buys the good} \\ 0 & \text{otherwise} \end{cases}$$

Solving this problem under different bargaining protocols allows us to easily illustrate the main point of the paper: that with a focused buyer, the particular bargaining protocol affects the focusing weight and the final outcome of the bargaining. In this case, trade occurs or not independently on the bargaining protocol. However, if trade occurs, the bargaining protocol determines the price paid by the buyer, which is never lower than the price paid by a rational buyer. For example, the seller can announce a maximum price she will charge in the future (or just a price, if both parties know that renegotiation over the price is possible) and by doing so she can extract a final price that is greater than if the bargaining happened in one shot. Finally, because the outcome of the bargaining in period 1 can be renegotiated in period 2, we consider only bargaining protocols that are renegotiation proof.

3.1 One-shot bargaining problem.

The first bargaining protocol we consider is a constant bargaining tool such as, for example, $k(x) = 1 \ \forall x$. Under this bargaining protocol, the two parties do not decide anything in period 1, which is equivalent to assuming that the two parties bargain in one shot. In this case, the outside option of not buying is the lower bound of the consideration set, implying that $\underline{q} = \underline{p} = 0$. The only alternative to not buying is to buy q, which is in the consideration set if there is a price p such that both utility and profits are positive at (q, p).

Let us assume that this price exists, in this case $\bar{q} = q$ and the maximum price that can be charged is $\bar{p} : \bar{p}h(\bar{p}) = qh(q)$, so that $\bar{p} = q$. At price equal to q a transaction will occur only if profits are positive, or $q \geq c$. On the other hand, if $q \leq c$ there is no positive price that satisfies both rationality constraints. Hence $\bar{q} = \bar{p} = 0$.

Lemma 3. *When bargaining occurs in one period, the focused buyer puts equal weights on prices and quality and behaves like a rational buyer. The two parties trade at price $p = \frac{q+c}{2}$ if $q \geq c$, and do not trade if $q \leq c$.*

3.2 Two Steps Bargaining Protocol

Suppose now that the bargaining happens in two periods. In the first period, the parties bargain over $\hat{p}$: a maximum price the seller can charge. In the second period, the two parties agree on whether to trade and at what exact price. Formally, the bargaining protocol in this case is a scalar function and $k(x) = p$.

Consider period 2. In this case the boundaries of the consideration set depend on the agreement reached in period 1. Not trading is still an option, so that $\underline{p} = \underline{q} = 0$. If the two parties do trade, the maximum price that can be charged is $\min\{q, \hat{p}\}$. Hence, trade occurs if and only if $\min\{q, \hat{p}\} \geq c$, yielding to the buyer's utility:

$$u^b = h(q)q - h(\min\{q, \hat{p}\})p$$

and the Nash-Bargaining solution:

$$p(\hat{p}) = \min\{\frac{h(q)q + h(\min\{q, \hat{p}\})c}{2h(\min\{q, \hat{p}\})}, \hat{p}\} \tag{1}$$

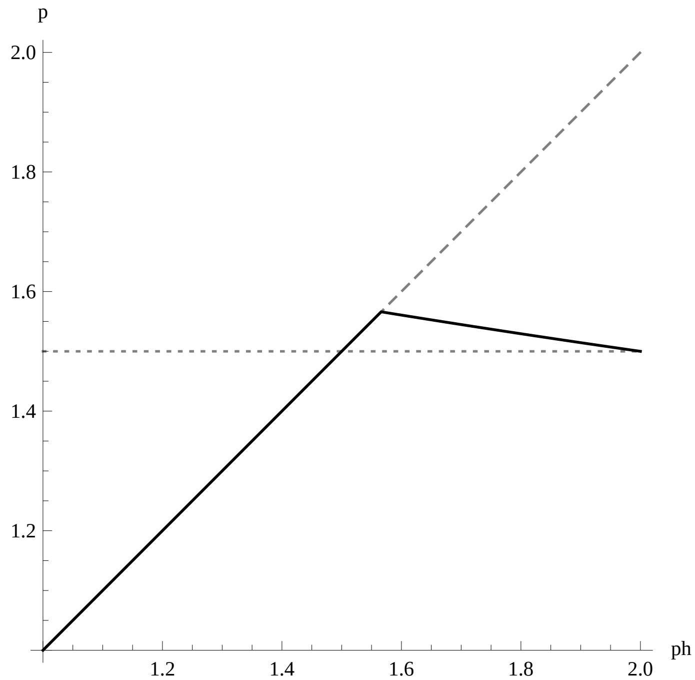
Depending on $\hat{p}$, several outcomes are possible in period 2. If $\hat{p} < c$ there will be no trade tomorrow, while if $\hat{p} > q$ the solution is the same as in the one-shot bargaining problem, which implies that when $q < c$ there is no trade. Note also that

$$\frac{h(q)q + h(\min\{q, \hat{p}\})c}{2h(\min\{q, \hat{p}\})}$$

is decreasing in $\hat{p}$. As a consequence, the highest possible price achievable in period 2 is

$$\hat{p}^\star : \frac{h(q)q + h(\hat{p}^\star)c}{2h(\hat{p}^\star)} = \hat{p}^\star. \tag{2}$$

This is also illustrated in Figure 1. Assume that trade can occur in period 2 for some $\hat{p}$ (i.e $q > c$). From period-1 point of view, when the two parties bargain over $\hat{p}$, the set of possible



$p(\hat{p})$ (solid black line), and equilibrium price in the one-shot bargaining case (dotted gray line). Parameter values are $q = 2$, $c = 1$ and $h(x) = x/5 + 1$; $p^{\star} = \hat{p}^{\star} = 1.5661$.

Fig. 1: Period–2 Price as a Function of the Maximum Price

prices achievable in period 2 is bounded above by $\hat{p}^{\star} < q$. The consideration set, i.e. the set of possible *final outcomes* achievable in period-1 by bargaining over $\hat{p}$, is $\underline{p} = \underline{q} = 0$, $\bar{q} = q$ and $\bar{p} = \hat{p}^{\star}$. Therefore, the buyer puts less weight on prices than in the one-shot bargaining case.

By spanning all the possible $\hat{p}$, all final prices between c and $\hat{p}^{\star}$ can be achieved. Bargaining in period 1 over $\hat{p}$ is equivalent to bargaining over p with the restriction that $p < \hat{p}^{\star}$. The solution is:

$$p^{\star} = \min\{\frac{h(q)q + h(\hat{p}^{\star})c}{2h(\hat{p}^{\star})}, \hat{p}^{\star}\}$$

by definition of $\hat{p}^{\star}$, the above expression implies that $p^{\star} = \hat{p}^{\star}$.

Lemma 4. *Assume that the parties bargain first over the maximum price and then agree on whether to trade and at what price. Whenever $q > c$, the final price $\hat{p}^{\star}$ is such that $q > \hat{p}^{\star} > c$. The seller's profits are greater than in the one-shot bargaining case. If $q < c$ there is no trade.*

Proof. To conclude the proof of lemma 4, we need to show that $q > \hat{p}^\star > c$ whenever $q > c$ and that this implies a greater profit for the seller than in the one-shot bargaining case. It is easy to see that $\hat{p}^\star = q$ whenever $q = c$. By simple calculus, $p(\hat{p} = q)$ in (1) is equal to $(q + c)/2$. Assuming $q > c$, this implies that $p(\hat{p} = q) > c$. Moreover, since $p'(\hat{p}) < 0$ for all $\hat{p} \in [\hat{p}^\star, q]$, it must be that $p(\hat{p}) \geq c$ for all $\hat{p} \in [\hat{p}^\star, q]$. Since $p(\hat{p}^\star) = \hat{p}^\star$, we conclude that $\hat{p}^\star > c$.

In addition, $\hat{p}^\star \leq q$ follows directly from $p'(\hat{p}) < 0$ for $\hat{p} \in [\hat{p}^\star, q]$. For $q > c$, $\hat{p}^\star = q$ leads to a contradiction in (2). Hence, we receive that, for $q > c$, $\hat{p}^\star < q$. Finally, as $p(\hat{p} = q) = (q + c)/2$ is equal to the price in the one-shot bargaining case, and $p'(\hat{p}) < 0$ for $\hat{p} \in [\hat{p}^\star, q]$ we conclude that $\hat{p}^\star > (q + c)/2$.

□

3.3 What Bargaining Protocol?

In the previous sections, we derived the solution to the bargaining problem under 2 bargaining protocols. In certain cases, the bargaining protocol available to the two parties will be determined by exogenous circumstances, such as whether the product they are exchanging is perishable (so that the parties have no time to bargain in two steps) and whether the seller can commit to a maximum price. However, in other cases the two parties may have the option to bargain over the bargaining protocol to use. In this section, we analyze this last case by assuming that, in period 0, buyer and sellers agree on the bargaining protocol to use.

We showed that, if trade is materially efficient (i.e. $q > c$) when the two parties bargain over the maximum price in period 1 the final price will be higher than when the bargaining happens in one step (see sections 3.1 and 3.2).[6] The boundary of the bargaining set depends on the disagreement outcome in period 0. If, when the parties disagree in period 0, there is no trade at all, the lower bounds of the consideration set are again $\underline{p} = \underline{q} = 0$. If instead the disagreement outcome is one of the bargaining protocol, then $\underline{q} = q$ and $\underline{p} = \frac{q+c}{2}$. In both cases the upper bounds of the consideration sets are $\bar{q} = q$ and $\bar{p} = \hat{p}^\star$.

The two parties can make monetary transfers to each other. Because money is fungible, the buyer evaluates each dollar given or received in stage 0 using the same focusing weight used for the price (which could be either $h(\hat{p}^\star)$ or $h(\hat{p}^\star - \frac{q+c}{2})$, depending on the disagreement outcome.) Hence, the buyer is indifferent between a bargaining protocol leading to price prices p and a bargaining protocol leading to price $p' > p$, provided that the seller transfers to the buyer $m = p' - p$. Note that $m = p' - p$ corresponds to the buyer's gain of setting p' instead of p. The period-0 bargaining problem is a transferable utility problem. The lemma

[6] If $q < c$, there is no trade under any bargaining protocol.

follows immediately.

Lemma 5. *If monetary transfers can be used, the two parties are indifferent between the two bargaining protocols. If they implement the one-shot bargaining protocol, the buyer makes a monetary transfer to the seller. If they decide to first bargain over the maximum price and then bargain over the price, the seller makes a positive monetary transfer to the buyer.*

As a corollary, the seller can extract a higher overall payment from the buyer whenever the two parties bargain over the bargaining protocol, compared to the one-shot bargaining case (and the rational-buyer case).

Assumption 6 (Non-binding agreements)**.** *A bargaining outcome $x \in X$ is an element of the period-2 consideration set if and only if at least one of the following conditions holds:*

- $x \in s(y) \cup \underline{0}$,
- $U^b(x) \geq U^b(x')$ *and* $U^s(x) \geq U^s(x')$ *for some* x' *element of the Pareto frontier of* $s(y)$.
- x *is the solution of the bargaining problem over the entire* X

Hence, period 1 agreements are non-binding and can be violated. Players can implement any Pareto improvement over a period-1 agreement during period 2, but need to trigger a fresh round of bargaining to implement any outcome that is not allowed by period-1 agreement and is not a Pareto improvement over period-1 agreement. Note that, in this model, agents are infinitely patients, and are indifferent between reaching the same agreement in period 1, 2 or 3. However, under assumption 6, period-1 agreements affects the set of outcomes that can be reached in period 2 and the focusing weights in period 2.

Going back to the previous example, under assumption 6 for any possible $\bar{p}$ announced in period 1, in period 2 the buyer's utility is:

$$u^b = h(q)q - h(\max\{\min\{q, \hat{p}\}, \frac{q+c}{2}\})p$$

because $\frac{q+c}{2}$ is the price that will be reached in period 3. In other words, the buyer anticipates that going to a new bargaining round would deliver a price equal to $\frac{q+c}{2}$. Hence this price is a possible price achievable in period 2, and $\max\{\min\{q, \hat{p}\}, \frac{q+c}{2}\}$ is the highest possible transaction price achievable in period 2. The Nash-Bargaining solution is:

$$p(\hat{p}) = \min\{\frac{h(q)q + h(\max\{\min\{q, \hat{p}\}, \frac{q+c}{2}\})c}{2h(\max\{\min\{q, \hat{p}\}, \frac{q+c}{2}\})}, \hat{p}\} \tag{3}$$

It is easy to see that the maximum price that can be reached in period 2 by manipulating $\bar{p}$ is again the one derived in the previous section (see equation 2). The period-1 problem is therefore identical to the problem discussed above, and the solution is the same.

Note that, in the previous example, the possibility of Pareto improving over the period-1 agreement plays no role, as the players have opposite preferences over the price. In the next section, we consider a bargaining problem in which price and quality are jointly determined, and the players may renegotiate in period 2 a period-1 agreement. For every point on the Pareto frontier of the set of outcomes allowed by period-1 agreement, we can compute the set of violations from the previous agreement that Pareto dominate a specific boundary point. The unions of these deviations, together with the outcomes allowed by the period-1 agreement constitutes the set of feasible set in period 2.

4 Example 2: endogenous quality dimension

Assume that the buyer and the seller bargain over the quality of the good and on the price to pay. Utilities are:

$$U^b = \begin{cases} h(\bar{q} - \underline{q})q - h(\bar{p} - \underline{p})p & \text{if the buyer buys the good} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$U^s = \begin{cases} p - \frac{1}{2}q^2 & \text{if the seller sells the good} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

for $q \geq 0$.

Similarly to the previous case, we first derive the bargaining solution in case the two players bargain in one shot, then consider the two-steps bargaining solution. Our main result is that, in this set-up, the timing of bargaining is welfare relevant, as it affects the quality of the widget exchanged.

4.1 One shot bargaining

Similarly to the previous example, also here $\underline{q} = \underline{p} = 0$: not trading satisfies the players' outside options. The upper bounds of the consideration set needs to satisfy the two rationality constraints:

$$h(\bar{q})\bar{q} = h(\bar{p})\bar{p} \tag{6}$$

$$\bar{q}^2 = 2\bar{p} \tag{7}$$

which has a unique solution $\bar{q} = \bar{p} = 2$. Also here, the one shot bargaining outcome is equivalent to the outcome when the buyer is rational: $p^\star = \frac{3}{4}$, $q^\star = 1$.

4.2 Two period bargaining.

Contrary to the previous example, here renegotiation may happen. Suppose that in period 1 the two parties agree on a $\hat{p}$, and that in period 2 $\hat{p}$ is binding. The fact that the constraint is binding may imply that there exist a q and a $p > \hat{p}$, preferred by both parties. In other words, by offering a better deal on the quality dimension, the seller may convince the buyer to accept a price higher than the maximum price agreed upon in period 1. A similar argument holds for any period-1 agreement putting bounds on the quality dimension, and any period-1 agreement putting bounds on both the quality dimension and the price dimension.

Consider a period-1 agreement such that $\forall \{p, q\} \in s(y)$

$$h(\max\{\hat{q}, 1\})q - h(\max\{\hat{p}, \frac{3}{4}\})p \geq \underline{u}^b$$

$$p - \frac{1}{2}q^2 \geq \underline{u}^s$$

$$\hat{p} = \max\{p | \{p, q\} \in s(y) \text{ for some } q\}$$

$$\hat{q} = \max\{q | \{p, q\} \in s(y) \text{ for some } p\}$$

for some $\underline{u}^b, \underline{u}^s \geq 0$. All bargaining outcomes in $s(y)$ give at least utility $\underline{u}^b$ to the buyer and utility $\underline{u}^s$ to the seller. An agreement of that form will not be renegotiated if the points on the Pareto frontier of X giving utility at least $\underline{u}^s$ to the seller and $\underline{u}^b$ to the buyer are elements of $s(y)$. More formally, the agreement $s(y)$ will not be renegotiated if all $\{p', q'\}$ such that

$$\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}, 1\})} = \frac{1}{\sqrt{2\,(p' - u^s)}} \tag{8}$$

for

$$\underline{u}^s \leq u^s \leq p' - \frac{1}{2}\left(\frac{h(\max\{\hat{p}, \frac{3}{4}\})p' + \underline{u}^b}{h(\max\{\hat{q}, 1\})}\right)^2 \tag{9}$$

where

$$q' = \sqrt{2(p' - u^s)} \tag{10}$$

are elements of $s(y)$. In other words: all tangency points between indifference curves corresponding to utility between 0 and $\underline{u}^b$, and iso profit lines corresponding to profits between 0 and $\underline{u}^s$ are elements of $s(y)$.

Conditions 8, 9, and 10 impose restrictions on the $\underline{u}^b$, $\underline{u}^s$, $\hat{p}$, $\hat{q}$ of a renegotiation-proof agreements. In particular, only $\underline{u}^b$, $\underline{u}^s$, $\hat{p}$, $\hat{q}$ satisfying:

$$\hat{p} + \frac{\underline{u}^b}{h(\max\{\hat{p}, \frac{3}{4}\})} \geq \left(\frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})}\right)^2 \tag{11}$$

$$\hat{q} \geq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})} \tag{12}$$

$$\hat{p} + \frac{\underline{u}^b}{h(\max\{\hat{p}, \frac{3}{4}\})} \leq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})}\hat{q} \tag{13}$$

$$\hat{p} - \frac{1}{2}\hat{q}^2 \geq \underline{u}^s \tag{14}$$

can be implemented as renegotiation-proof period-1 agreement. The first two conditions state that $\hat{p}, \hat{q}$ should be larger than the largest p, q on the Pareto frontier of the set $s(y)$. The last two constraints are the rationality constraints of the players.

We derive the solution to the game under 3 scenarios, depending on the type of renegotiation-proof agreement that can be reached in period 1. In order to simplify our derivations, we make the following functional-form assumption

Assumption 7. *The focusing function has a quadratic form: $h(x) = x^\gamma$ for some $\gamma > 0$.*

Lemma 8. *Suppose that, in period-1, the players can sign any renegotiation-proof agreement with $\underline{u}^b = \underline{u}^s = 0$. The solution to the bargaining problem is*

$$q^\star = \max\left\{\left(\frac{2}{1 + 2^{-\frac{2\gamma^2+1}{2\gamma^2+\gamma+1}}}\right)^\gamma, 2^{\frac{\gamma}{2\gamma^2+\gamma+1}}\right\}$$

$$p^\star = \frac{1}{2}q^\star + \frac{1}{4}(q^\star)^2$$

Proof. If $\underline{u}^s = \underline{u}^b = 0$, $\hat{p}, \hat{q}$ are renegotiation proof if

$$\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}, 1\})} \leq \frac{\hat{q}}{\hat{p}} \tag{15}$$

$$\frac{2}{\hat{q}} \geq \frac{\hat{q}}{\hat{p}} \tag{16}$$

$$\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}, 1\})} \geq \frac{1}{\hat{q}} \tag{17}$$

$$\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}, 1\})} \geq \frac{1}{\sqrt{\hat{p}}} \tag{18}$$

By Nash bargaining, for every $\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q},1\})}$ that satisfy conditions 15, 16, 17 and 18 the final outcome is

$$q^\star = \left(\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}, 1\})}\right)^{-1}$$

$$p^\star = \frac{1}{2}\left(\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}, 1\})}\right)^{-1} + \frac{1}{4}\left(\frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})}\right)^{-2}$$

Hence, the set of possible bargaining outcomes achievable in period-2 depends on the set of $\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q},1\})}$ that satisfy conditions 15, 16, 17 and 18.

It is easy to see that the maximum $\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q},1\})}$ is reached when both conditions 15, 16 are binding, so that

$$\max\left\{\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}', 1\})}\right\} = 1$$

Consider a given $\hat{q}' \geq 1$. Condition 18 can be written as

$$\left(\max\{\hat{p}, \frac{3}{4}\}\right)\sqrt{\hat{p}} \geq \hat{q}'^{\gamma}$$

which implies that $\hat{p} > 1$. Hence, the smallest $\hat{p}$ that satisfies conditions 16, 17, and 18 is

$$\underline{\hat{p}} = \max\left\{q^{\frac{\gamma-1}{\gamma}}, q^{\frac{2\gamma}{2\gamma+1}}, \frac{q^2}{2}\right\}$$

so that

$$\min\left\{\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}', 1\})}\right\} = \min\left\{\max\left\{\hat{q}'^{-1}, \hat{q}'^{-\frac{\gamma}{2\gamma+1}}, \left(\frac{\hat{q}'}{2}\right)^{\gamma}\right\}\right\},$$

Also, by varying $\hat{q}'$, $\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}',1\})}$ reaches its minimum for

$$\hat{q}' : \hat{q}'^{-\frac{\gamma}{2\gamma+1}} = \left(\frac{\hat{q}'}{2}\right)^{\gamma}$$

or

$$\hat{q}' = 2^{\frac{2\gamma^2+\gamma}{2\gamma^2+\gamma+1}}$$

which implies

$$\min\left\{\frac{h(\max\{\hat{p},\frac{3}{4}\})}{h(\max\{\hat{q},1\})}\right\}=2^{-\frac{\gamma}{2\gamma^2+\gamma+1}}$$

It follows that the highest quality and the highest price achievable in period 2 are $2^{\frac{\gamma}{2\gamma^2+\gamma+1}}$ and $\left(\frac{1}{2}\right)2^{\frac{\gamma}{2\gamma^2+\gamma+1}}+\left(\frac{1}{4}\right)2^{\frac{2\gamma}{2\gamma^2+\gamma+1}}$ respectively. As a consequence, the period-1 bargaining problem is:

$$\max\left\{\left(h\left(2^{\frac{\gamma}{2\gamma^2+\gamma+1}}\right)\alpha-h\left(\left(\frac{1}{2}\right)2^{\frac{\gamma}{2\gamma^2+\gamma+1}}+\left(\frac{1}{4}\right)2^{\frac{2\gamma}{2\gamma^2+\gamma+1}}\right)\right)\left(\left(\frac{1}{2}\alpha+\frac{1}{4}\alpha^2\right)-\frac{1}{2}\alpha^2\right)\text{ s.t } 1\leq\alpha\leq 2^{\frac{\gamma}{2\gamma^2+\gamma+1}}\right\}$$

Note that the quality dimension is more salient than the price dimension. Relative to the one-shot case (when price and quality have equal weights), here the agent will prefer a higher quality and a higher price. The solution is

$$q^\star=\max\left\{\left(\frac{2}{1+2^{-\frac{2\gamma^2+1}{2\gamma^2+\gamma+1}}}\right)^\gamma,2^{\frac{\gamma}{2\gamma^2+\gamma+1}}\right\}$$

$$p^\star=\frac{1}{2}q^\star+\frac{1}{4}(q^\star)^2$$

□

In this case transaction price and quality are greater than in the one-shot case. The presence of the interim agreement increases the salience of the quality dimension. Therefore, the way the negotiation is structured is welfare relevant, as it affects the quality of the item exchanged.

4.3 Two-periods bargaining with hold up.

Suppose that the players can only bargain in one step. However, the seller can decide to bargain having already set the quality and announced a maximum price, or to bargain over price and quality simultaneously. In this section we show that there may be endogenous hold up: by fixing the quality in advance and bargaining from a worse position, the seller can manipulate the buyer's focusing weights. Whenever the focusing effect is strong (high γ), profits are greater in case the seller is held up compared with the case in which price and quality are jointly determined.

Assume that, in period 2, quality q has already been decided, and a maximum transaction price $\hat{p}\leq q$ has been established. By Nash bargaining solution, the final transaction price is

$$p^\star=\max\left\{\left(\frac{q}{\hat{p}}\right)^\gamma\frac{q}{2},\hat{p}\right\}$$

which, as a function of q reaches its maximum at

$$p^\star = \hat{p} = \frac{q}{2^{\frac{1}{1+\gamma}}}$$

Suppose that the seller chooses q and $\hat{p}$ in period 1. The seller solves

$$\max\left\{\frac{q}{2^{\frac{1}{1+\gamma}}} - \frac{1}{2}\left(\frac{q}{2^{\frac{1}{1+\gamma}}}\right)^2\right\}$$

which is maximized at $q = 2^{-\frac{1}{1+\gamma}}$, yielding profits equal to $\pi = 2^{-\frac{3+\gamma}{1+\gamma}}$. Profits in case price and quality are jointly determined are equal to $\frac{1}{4}$. Simple algebra shows that whenever $\gamma > 1$ the seller chooses to fix quality before bargaining: the possibility of manipulating the buyer's focusing weights outweighs the cost of being held up.

References

Bac, M. and H. Raff (1996). Issue-by-issue negotiations: the role of information and time preference. *Games and Economic Behavior 13*(1), 125–134.

Battigalli, P. and G. Maggi (2002). Rigidity, discretion, and the costs of writing contracts. *American Economic Review*, 798–817.

Bolton, P. and A. Faure-Grimaud (2010). Satisficing contracts. *The Review of Economic Studies 77*(3), 937–971.

Bordalo, P., N. Gennaioli, and A. Shleifer (2012). Salience and consumer choice. Technical report, National Bureau of Economic Research.

Busch, L.-A. and I. J. Horstmann (1999a). Endogenous incomplete contracts: a bargaining approach. *Canadian Journal of Economics*, 956–975.

Busch, L.-A. and I. J. Horstmann (1999b). Signaling via an agenda in multi-issue bargaining with incomplete information. *Economic Theory 13*(3), 561–575.

Flamini, F. (2007). First things first? the agenda formation problem for multi-issue committees. *Journal of Economic Behavior & Organization 63*(1), 138–157.

Grossman, S. J. and O. D. Hart (1986). The costs and benefits of ownership: A theory of vertical and lateral integration. *The Journal of Political Economy*, 691–719.

Hart, O. and J. Moore (2008). Contracts as reference points. *The Quarterly Journal of Economics 123*(1), 1–48.

Herweg, F., H. Karle, and D. Müller (2013). Incomplete contracting, renegotiation, and loss aversion. *working paper*.

Inderst, R. (2000). Multi-issue bargaining with endogenous agenda. *Games and Economic Behavior 30*(1), 64–82.

Kahneman, D., A. B. Krueger, D. Schkade, N. Schwarz, and A. A. Stone (2006). Would you be happier if you were richer? a focusing illusion. *Science 312*(5782), 1908–1910.

Kőszegi, B. and A. Szeidl (2013). A model of focusing in economic choice. *The Quarterly Journal of Economics 128*(1), 53–104.

Lang, K. and R. W. Rosenthal (2001). Bargaining piecemeal or all at once? *The Economic Journal 111*(473), 526–540.

Ponsati, C. and J. Watson (1997). Multiple-issue bargaining and axiomatic solutions. *International Journal of Game Theory 26*(4), 501–524.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica: Journal of the Econometric Society*, 97–109.

Schkade, D. A. and D. Kahneman (1998). Does living in california make people happy? a focusing illusion in judgments of life satisfaction. *Psychological Science 9*(5), 340–346.

Segal, I. (1999). Complexity and renegotiation: A foundation for incomplete contracts. *The Review of Economic Studies 66*(1), 57–82.

Tirole, J. (2009). Cognition and incomplete contracts. *The American Economic Review 99*(1), 265–294.