Names for Games: A Binomial Nomenclature for 2x2 Ordinal Games

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A binomial nomenclature identifies any two-person, two-move (2x2) ordinal game as a combination of symmetric game payoffs, based on the topology of payoff swaps that arranges 2x2 ordinal games in a natural order. Preference orderings categorize 2x2 ordinal games according to type of ties formed by transformations of strict games. Location of best payoffs defines orientations for games equivalent by interchanging rows or columns. Two-letter abbreviations for symmetric game names provide a compact notation. A systematic and efficient nomenclature identifying equivalent and similar 2x2 games helps locate interesting games; aids in understanding the diversity of elementary models of strategic situations available for experimentation, simulation, and analysis; and facilitates comparative and cumulative research in game theory.

INTRODUCTION

This paper presents a binomial nomenclature that that efficiently identifies the complete set of two-person, two-move (2x2) ordinal games, including asymmetric games and games with ties. The nomenclature helps identify games that are similar or ordinally equivalent, conveniently locates games within the diversity of elementary models of strategic situations, and facilitates comparative and cumulative research in game theory.

The large number of different payoff structures, and differences in how payoffs are shown can make it hard to identify games that are similar or equivalent. There are are 78 strategically distinct strict 2x2 ordinal games, where each player has four differently ranked payoffs (Rapoport and Guyer 1966). If ties are allowed, then there are 726 strategically distinct possibilities (Guyer and Hamburger 1968). Interchanging rows or columns, or switching positions (as Row or Column player) creates many more versions, 576 and 5,625 respectively, which are usually treated as strategically equivalent. For payoffs measured on an interval (ratio) or real scale, each ordinal game represents variants with ordinally equivalent payoff structures. Chicken, Hawk-Dove, and Snowdrift are different names for the same, ordinally-equivalent, game. Conversely, even for strict symmetric ordinal games, there are a variety of coordination games with two Nash equilibria, including several stag hunts (also known as assurance games) and battles, symmetric and asymmetric. Building on earlier taxonomies, the nomenclature proposed here is a tool for showing how particular games, and games that are ordinally equivalent or similar, can be located within this diversity of payoff structures and representations.

The nomenclature is based on Robinson and Goforth's topology of payoff swaps in 2x2 games, which reveals a natural order in the payoff space of 2x2 games (Robinson and Goforth 2005; Robinson, Goforth, and Cargill 2007). Payoffs from strict games combine to form asymmetric games, so a binomial nomenclature can specify any asymmetric game as a combination of two symmetric games. Games with ties can be categorized according to the number of ties in payoffs into eight preference orderings (Guyer and Hamburger 1968; Fraser and Kilgour 1986; Kilgour and Fraser 1988). Games with ties can be treated as transformations of games without ties, so names for twelve strict games and seven transformations suffice to name all the ordinal 2x2 games.

Binomial game names can be linked to existing common names, as well as with numbers assigned to 2x2 games in Rapoport and Guyer's taxonomy (1966; Rapoport, Guyer, and Gordon 1976) ; Brams' typology (1994), and Robinson and Goforth's topology. While previous numbering schemes primarily or exclusively focused on strict games (without ties) the binomial nomenclature uses the eight preference orderings (types of ties) to include the complete set of 2x2 ordinal games, and so should also be compatible with Fraser and Kilgour's (1988) numbering scheme for 2x2 games with ties.

The next section begins by briefly explaining how the topology of 2x2 games provides a natural order for 2x2 games. It presents conventions for displaying payoffs, using numerals from one to four and orienting matrices according to the locations of best payoffs for Row and Column players. Names for the twelve strict symmetric games and eight preference orderings are explained, which then suffice to identify the symmetric ordinal games that combine to form asymmetric games. Abbreviations provide a compact notation and can be used as tags or unique identifiers. A procedure for finding a binomial name for any 2x2 payoff matrix is presented. The results section summarizes the binomial nomenclature and discusses some implications and applications.

METHODS

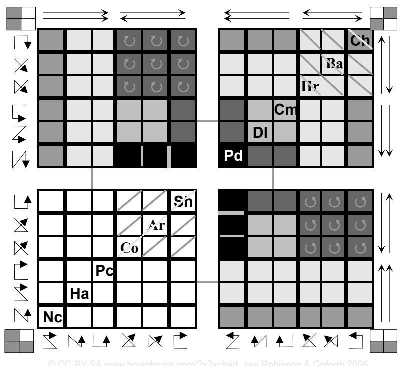
Natural Order in the Topology of 2x2 Games

The topology of payoff swaps provides a natural ordering for arranging the 2x2 games, assuming that games linked by swaps in the lowest payoffs are nearest neighbors (Robinson and Goforth 2005). While the full topology is a three-dimensional torus with 37 holes, it can be conveniently displayed on a two-dimensional surface divided into four "layers," distinguished by the alignment of best payoffs, as shown in Tables 1 and 2.[1] The twelve strict symmetric games form a diagonal axis from lower left to upper right. Games on Layer 1 have best payoffs in diagonally opposed cells, while those on Layer 3 have win-win outcomes with the best payoffs in the same cell. Each layer is a torus, and scrolling Prisoner's Dilemma next to the center elegantly arranges games according to the number of dominant strategies and Nash Equilibria, and other properties. A numeric version of the Robinson-Goforth periodic table of 2x2 games, as in Table 2, illustrates how payoffs from symmetric games combine to form asymmetric games.

The 2x2 ordinal games can be categorized according to the number and type of ties (Guyer and Hamburger 1968; Fraser and Kilgour 1986; Kilgour and Fraser 1988). Games with ties can be linked by half-swaps that make or break ties, forming an expanded topology (Robinson, Goforth, and Cargill 2007). Therefore, symmetric games with ties can be identified as transformations from the twelve strict symmetric games. Conversely, breaking ties differentiates the null game of complete indifference into games with two or three ties, and then then strict games. However, formation of ties from strict games provides a more convenient starting point for a nomenclature. An expanded display of the topology of 2x2 games, as in Figure 3, can show the complete set of 2x2 games, again with symmetric games on the diagonal and asymmetric games formed by combining payoffs from symmetric games (Bruns 2012). In this "checkerboard" display, games with ties on low or middle payoffs are located between the strict games (Robinson, Goforth, and Cargill 2007; Heilig 2012; Hopkins, Brian 2011). A nomenclature based on the symmetric ordinal games then requires coming up with distinctive names for all the symmetric ordinal games, and for the types of ties. Before discussing names for symmetric games and ties, it is useful to discuss previous systems for numbering 2x2 games, and conventions for displaying payoff values.

1 Color versions of 2x2 game charts are available at 2x2atlas.org

Table 1. Twelve Strict Symmetric 2x2 Games. Symmetric games form a diagonal axis in this small schematic diagram of the Periodic Table of 2x2 games. Payoffs from symmetric games combine to form asymmetric games. For topology and periodic table structure, see Robinson and Goforth 2005.



Chicken/Hawk-Dove/Snowdrift
Battle/Battle of the Sexes/Leader
Hero
Compromise/Anti-Chicken
Deadlock/Anti-Prisoner's Dilemma
Prisoner's Dilemma

Stag Hunt
Assurance
Coordination
Peace
Harmony
Concord/No Conflict

Table 3. Eight Preference Orderings. Types of ties categorize the complete set of 1,413 2x2 ordinal games, with and without ties. Adapted from Robinson, Goforth and Cargill 2006. For preference orderings A-H, see Fraser and Kilgour 1986, Kilgour and Fraser 1988.

Strict	STRICT	1,2,3,4	H	6	24	24	36	72	72	72	**144**
Low Tie		1,1,3,4	D	3	12	12	18	36	36	**36**	72
Middle	EDGE	1,3,3,4	F	3	12	12	18	36	**36**	36	72
High Tie		1,2,4,4	G	3	12	12	18	**36**	36	36	72
Double		1,1,4,4	C	3	6	6	**12**	18	18	18	36
Triple	VERTEX	1,4,4,4	E	1	4	**4**	6	12	12	12	24
Basic		1,1,1,4	B	1	**4**	4	6	12	12	12	24
Zero	ORIGIN	0,0,0,0	A	**1**	1	1	3	3	3	3	6
				Zero	Basic	Triple	Double	High	Middle	Low	Strict
	Total	**1,413**		A	B	E	C	G	F	D	H

Table 5. Game Numbers. Binomial names can be matched to earlier game numbers.

a. Rapoport & Guyer Taxonomy

4	Nc	Ha	Pc	Co	As	Sh	Pd	Dl	Cm	Hr	Ba	Ch	1
Ch	55	50	49	70	78	72	39	35	36	65	67	66	
Ba	56	52	51	74	76	71	37	31	32	64	68	67	
Hr	44	41	40	73	75	77	38	33	34	69	64	65	
Cm	18	16	15	53	42	45	10	8	7	34	32	36	
Dl	17	14	13	54	43	46	11	9	8	33	31	35	
Pd	21	19	20	57	47	48	12	11	10	38	37	39	
Sh	26	22	23	58	62	61	48	46	45	77	71	72	
As	27	24	25	59	63	62	47	43	42	75	76	78	
Co	30	28	29	60	59	58	57	54	53	73	74	70	
Pc	2	4	5	29	25	23	20	13	15	40	51	49	
Ha	1	3	4	28	24	22	19	14	16	41	52	50	
Nc	6	1	2	30	27	26	21	17	18	44	56	55	
3													2

b. Brams Typology

4	Nc	Ha	Pc	Co	As	Sh	Pd	Dl	Cm	Hr	Ba	Ch	1
Ch	50	37	36	46	31	29	22	18	19	52	53	57	
Ba	56	39	38	43	45	47	20	14	15	51	54	53	
Hr	49	13	12	42	44	30	21	16	17	55	51	52	
Cm	6	4	3	40	23	25	10	8	7	17	15	19	
Dl	5	2	1	41	24	26	11	9	8	16	14	18	
Pd	35	33	34	48	27	28	32	11	10	21	20	22	
Sh							28	26	25	30	47	29	
As							27	24	23	44	45	31	
Co							48	41	40	42	43	46	
Pc							34	1	3	12	38	36	
Ha							33	2	4	13	39	37	
Nc							35	5	6	49	56	50	
3													2

c. Robinson-Goforth Topology

4	Nc	Ha	Pc	Co	As	Sh	Pd	Dl	Cm	Hr	Ba	Ch	1
Ch	421	426	425	424	423	422	121	126	125	124	123	122	2
Ba	431	436	435	434	433	432	31	136	135	134	133	132	3
Hr	441	446	445	444	443	442	141	146	145	144	143	142	4
Cm	451	456	455	454	453	452	151	156	155	154	153	152	5
Dl	461	466	465	464	463	462	161	166	165	164	163	162	6
Pd	411	416	415	414	413	412	111	116	115	114	113	112	1
Sh	321	326	325	324	323	322	221	226	225	224	223	222	2
As	331	336	335	334	333	332	231	236	235	234	233	232	3
Co	341	346	345	344	343	343	241	246	245	244	243	242	4
Pc	351	356	355	354	353	35	251	256	255	254	253	252	5
Ha	361	366	365	364	363	362	261	266	265	264	263	262	6
Nc	311	316	315	314	313	312	211	216	215	214	213	212	1
3	1	6	5	4	3	2	1	6	5	4	3	2	2

Table 2. Periodic Table of 2x2 Games: Grayscale

Strict ordinal games
on diagonal axis from lower left to upper right
Row & Column Payoffs
Nash equilbrium
Pareto-inferior

1 4	3 3
2 2	4 1

Prisoner's Dilemma

Payoffs from symmetric 2x2 games form asymmetric games
Payoff swaps change a game into a neighboring game
1↔2 Low swaps form tiles of 4 games
2↔3 Middle swaps join tiles into 4 layers
3↔4 High swaps link layers
Layers differ by alignment of best payoffs
Layers scrolled to center Prisoner's Dilemma
Layers and table wrap side-to-side & top-to-bottom

Payoff Families
- **Win-win** 4,4
- **Biased** 4,3
- **Second Best** 3,3
- **Unfair** 4,2
- **Inferior** — Sad 3,2; Dilemma 2,2; Alibi 3,2
- **Cyclic or Indeterminate**

Layer 1, with Pd, upper right. Right-Up Orientation: Row's 4 right, Column's 4 up.

L4 (upper left) · L1 (upper right) · L3 (lower left) · L2 (lower right)

	Nc	Ha	Pc	Co	As	Sh	Pd	Dl	Cm	Hr	Ba	Ch
Ch	2 3 3 4 1 1 4 2 ChNc	2 2 3 4 1 1 4 3 ChHa	2 1 3 4 1 2 4 3 ChPc	2 1 3 4 1 3 4 2 ChCo	2 2 3 4 1 3 4 1 ChAs	2 3 3 4 1 2 4 1 ChSh	2 4 3 3 1 2 4 1 ChPd	2 4 3 2 1 3 4 1 ChDl	2 4 3 1 1 3 4 2 ChCm	2 4 3 1 1 2 4 3 ChHr	2 4 3 2 1 1 4 3 ChBa	2 4 3 3 1 1 4 2 **Chicken**
Ba	3 3 2 4 1 1 4 2 BaNc	3 2 2 4 1 1 4 3 BaHa	3 1 2 4 1 2 4 3 BaPc	3 1 2 4 1 3 4 2 BaCo	3 2 2 4 1 3 4 1 BaAs	3 3 2 4 1 2 4 1 BaSh	3 4 2 3 1 2 4 1 Patron	3 4 2 2 1 3 4 1 BaDl	3 4 2 1 1 3 4 2 BaCm	3 4 2 1 1 2 4 3 BaHr	3 4 2 2 1 1 4 3 **Battle**	3 4 2 3 1 1 4 2 BaCh
Hr	3 3 1 4 2 1 4 2 HrNc	3 2 1 4 2 1 4 3 HrHa	3 1 1 4 2 2 4 3 HrPc	3 1 1 4 2 3 4 2 HrCo	3 2 1 4 2 3 4 1 HrAs	3 3 1 4 2 2 4 1 HrSh	3 4 1 3 2 2 4 1 HrPd	3 4 1 2 2 3 4 1 HrDl	3 4 1 1 2 3 4 2 HrCm	3 4 1 1 2 2 4 3 **Hero**	3 4 1 2 2 1 4 3 HrBa	3 4 1 3 2 1 4 2 HrCh
Cm	2 3 1 4 3 1 4 2 CmNc	2 2 1 4 3 1 4 3 CmHa	2 1 1 4 3 2 4 3 CmPc	2 1 1 4 3 3 4 2 CmCo	2 2 1 4 3 3 4 1 CmAs	2 3 1 4 3 2 4 1 CmSh	2 4 1 3 3 2 4 1 CmPd	2 4 1 2 3 3 4 1 CmDl	2 4 1 1 3 3 4 2 **Compromise**	2 4 1 1 3 2 4 3 CmHr	2 4 1 2 3 1 4 3 CmBa	2 4 1 3 3 1 4 2 CmCh
Dl	1 3 2 4 3 1 4 2 DlNc	1 2 2 4 3 1 4 3 DlHa	1 1 2 4 3 2 4 3 DlPc	1 1 2 4 3 3 4 2 DlCo	1 2 2 4 3 3 4 1 DlAs	1 3 2 4 3 2 4 1 DlSh	1 4 2 3 3 2 4 1 DlPd	1 4 2 2 3 3 4 1 **Deadlock**	1 4 2 1 3 3 4 2 DlCm	1 4 2 1 3 2 4 3 DlHr	1 4 2 2 3 1 4 3 DlBa	1 4 2 3 3 1 4 2 DlCh
Pd	1 3 3 4 2 1 4 2 PdNc	1 2 3 4 2 1 4 3 PdHa	1 1 3 4 2 2 4 3 PdPc	1 1 3 4 2 3 4 2 PdCo	1 2 3 4 2 3 4 1 PdAs	1 3 3 4 2 2 4 1 PdSh	1 4 3 3 2 2 4 1 **Prisoner D**	1 4 3 2 2 3 4 1 PdDl	1 4 3 1 2 3 4 2 PdCm	1 4 3 1 2 2 4 3 PdHr	1 4 3 2 2 1 4 3 PdBa	1 4 3 3 2 1 4 2 PdCh
Sh	1 3 4 4 2 1 3 2 ShNc	1 2 4 4 2 1 3 3 ShHa	1 1 4 4 2 2 3 3 ShPc	1 1 4 4 2 3 3 2 ShCo	1 2 4 4 2 3 3 1 ShAs	1 3 4 4 2 2 3 1 **Stag Hunt**	1 4 4 3 2 2 3 1 ShPd	1 4 4 2 2 3 3 1 ShDl	1 4 4 1 2 3 3 2 ShCm	1 4 4 1 2 2 3 3 ShHr	1 4 4 2 2 1 3 3 ShBa	1 4 4 3 2 1 3 2 ShCh
As	1 3 4 4 3 1 2 2 AsNc	1 2 4 4 3 1 2 3 AsHa	1 1 4 4 3 2 2 3 AsPc	1 1 4 4 3 3 2 2 AsCo	1 2 4 4 3 3 2 1 **Assurance**	1 3 4 4 3 2 2 1 AsSh	1 4 4 3 3 2 2 1 AsPd	1 4 4 2 3 3 2 1 AsDl	1 4 4 1 3 3 2 2 AsCm	1 4 4 1 3 2 2 3 AsHr	1 4 4 2 3 1 2 3 AsBa	1 4 4 3 3 1 2 2 AsCh
Co	2 3 4 4 3 1 1 2 CoNc	2 2 4 4 3 1 1 3 CoHa	2 1 4 4 3 2 1 3 CoPc	2 1 4 4 3 3 1 2 **Coordination**	2 2 4 4 3 3 1 1 CoAs	2 3 4 4 3 2 1 1 CoSh	2 4 4 3 3 2 1 1 CoPd	2 4 4 2 3 3 1 1 CoDl	2 4 4 1 3 3 1 2 CoCm	2 4 4 1 3 2 1 3 CoHr	2 4 4 2 3 1 1 3 CoBa	2 4 4 3 3 1 1 2 CoCh
Pc	3 3 4 4 2 1 1 2 PcNc	3 2 4 4 2 1 1 3 PcHa	3 1 4 4 2 2 1 3 **Peace**	3 1 4 4 2 3 1 2 PcCo	3 2 4 4 2 3 1 1 PcAs	3 3 4 4 2 2 1 1 PcSh	3 4 4 3 2 2 1 1 PcPd	3 4 4 2 2 3 1 1 PcDl	3 4 4 1 2 3 1 2 PcCm	3 4 4 1 2 2 1 3 PcHr	3 4 4 2 2 1 1 3 PcBa	3 4 4 3 2 1 1 2 PcCh
Ha	3 3 4 4 1 1 2 2 HaNc	3 2 4 4 1 1 2 3 **Harmony**	3 1 4 4 1 2 2 3 HaPc	3 1 4 4 1 3 2 2 HaCo	3 2 4 4 1 3 2 1 HaAs	3 3 4 4 1 2 2 1 HaSh	3 4 4 3 1 2 2 1 HaPd	3 4 4 2 1 3 2 1 HaDl	3 4 4 1 1 3 2 2 HaCm	3 4 4 1 1 2 2 3 HaHr	3 4 4 2 1 1 2 3 HaBa	3 4 4 3 1 1 2 2 HaCh
Nc	2 3 4 4 1 1 3 2 **No Conflict**	2 2 4 4 1 1 3 3 NcHa	2 1 4 4 1 2 3 3 NcPc	2 1 4 4 1 3 3 2 NcCo	2 2 4 4 1 3 3 1 NcAs	2 3 4 4 1 2 3 1 NcSh	2 4 4 3 1 2 3 1 NcPd	2 4 4 2 1 3 3 1 NcDl	2 4 4 1 1 3 3 2 NcCm	2 4 4 1 1 2 3 3 NcHr	2 4 4 2 1 1 3 3 NcBa	2 4 4 3 1 1 3 2 NcCh

 Based on Robinson & Goforth 2005 *The Topology of the 2x2 Games: A New Periodic Table* www.cs.laurentian.ca/dgoforth/home.html

For more diagrams, explanations, and references, see *Changing Games: An Atlas of Conflict and Cooperation in 2x2 Games* www.2x2atlas.org

To find a game: Make ordinal 1<2<3<4. Put column with Row's 4 right; row with Column's 4 up; find layer by alignment of 4s; find symmetric games with Row & Column payoffs.

Symmetric Games with Ties

Games with ties lie between strict ordinal games, linked by half-swaps that make or break ties. For example, Low Battle is between Battle and Hero, and Mid Battle (Volunteer's Dilemma) is between Chicken and Battle

Low Ties

ln	lh	lo	ld	lk	lb
1 3 4 4 1 1 3 1 Low Concord	3 1 4 4 1 1 1 3 Low Harmony	1 1 4 4 3 3 1 1 Low Coordinat	1 4 3 3 1 1 4 1 Low Dilemma	1 4 1 1 3 3 4 1 Low Lock	3 4 1 1 1 1 4 3 Low Battle

Middle Ties

mh	mp	mu	mk	ms	mb
3 3 4 4 1 1 3 3 Mid Harmony	3 1 4 4 3 3 1 3 Mid Peace	1 3 4 4 3 3 3 1 Mid Hunt	1 4 3 3 3 3 4 1 Midlock	3 4 1 1 3 3 4 3 MidCompromis	3 4 3 3 1 1 4 3 Mid Battle

High Ties

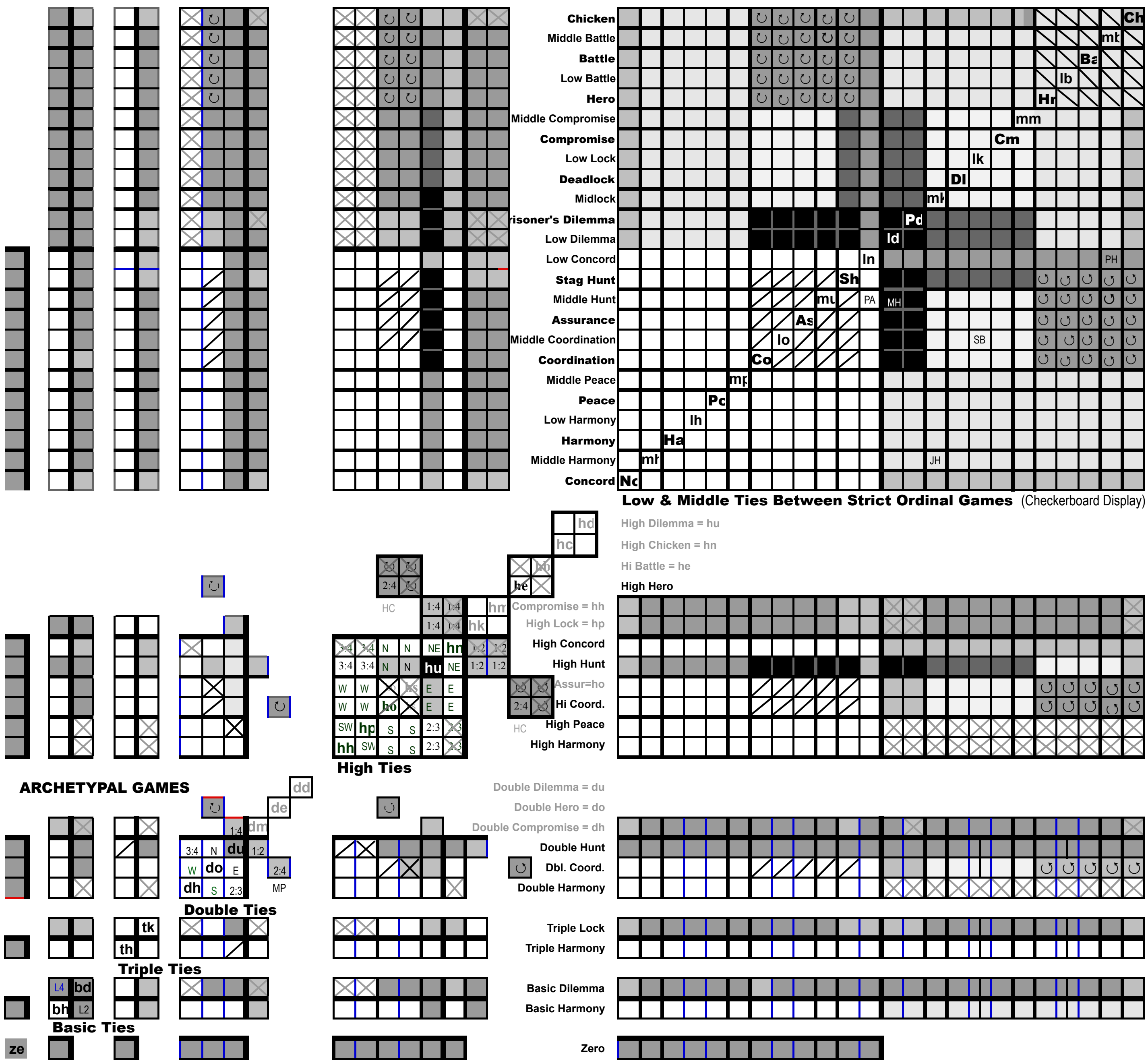
Making high ties (and double ties) creates duplicate games, identical or equivalent by switching rows or columns

hn	hc	hh	hs	hp	hk	ho	hr	hu	hd	he	hb
2 4 4 4 1 1 4 2 High Concord	2 4 4 4 1 1 4 2 =High Chicken	4 2 4 4 1 1 2 4 High Harmony	2 4 1 1 4 4 4 2 ≈HiCompromise	4 1 4 4 2 2 1 4 High Peace	1 4 2 2 4 4 4 1 ≈High Lock	2 1 4 4 4 4 1 2 High Coord.	1 2 4 4 4 4 2 1 ≈High Assurance	1 4 4 4 2 2 4 1 High Hunt	1 4 4 4 2 2 4 1 =High Dilemma	4 4 1 1 2 2 4 4 High Hero	4 4 2 2 1 1 4 4 ≈High Battle

Zero · Basic · Triple Ties · Double Ties

ze	bh	bd	th	tk	dh	dp	do	de	du	dn
0 0 0 0 0 0 0 0 Zero	1 1 4 4 1 1 1 1 Basic Harmony	1 4 1 1 1 1 4 1 Basic Dilemma	4 4 4 4 1 1 4 4 Triple Harmony	4 1 4 4 4 4 1 4 Triple Lock	4 1 4 4 1 1 1 4 DoubleHarmony	4 1 4 4 1 1 1 4 =Double Peace	1 1 4 4 4 4 1 1 Double Coord.	4 4 1 1 1 1 4 4 ≈Double Hero	1 4 4 4 1 1 4 1 Double Hunt	1 4 4 4 1 1 4 1 =DoubleConcord

Table 4. The Complete Set of 2x2 Ordinal Games. Payoffs from symmetric games combine to form asymmetric games. Low and middle ties are between strict games. Games are linked by half-swap transformations that make or break ties. See Robinson, Goforth, and Cargill 2007. Alternate versions of symmetric games, equivalent by row or column swaps, are shaded in gray.



e. To find a game: Make ordinal: Lowest = 1 Highest = 4 Middle ties = 3. Find class by type of ties, for each player. Put column with Row's 4 right, row with Column's 4 up.
Find Layer by alignment of 4s, then intersection of Row and Column payoffs. For high, and double ties, prefer Layer 3 (win-win cell upper right), interchange rows and columns if necessary.

Game Numbers

Rapoport and Guyer (1966; Rapoport, Guyer, and Gordon 1976) showed that there were only 78 strategically distinct strict games, if games equivalent by switching row, column, or position are considered to be the same game. They listed the 78 games with numbers (but no names) in an appendix to their book on 2x2 games. Their numbers are shown in Table 5a. However, their numbering scheme seems to have seen little subsequent use. For his typology of games and Theory of Moves, Steven Brams (1994) assigned a different set of numbers to strict ordinal 2x2 games, shown in Table 5b. No numbers were assigned to “no conflict” games, those with win-win outcomes, since they were not of interest for his analysis. Again, the numbering scheme has not been widely adopted.

As part of their topology of 2x2 games, Robinson and Goforth assigned three-digit index numbers, with the first digit based on the layer, and the second and third on the row and column within the layer, shown in Table 5c. In the topology, games related by switching positions of players are treated as different games, creating pairs of games reflected around the diagonal axis of symmetry. Thus, numbers are needed for 144 games created by combining 12 different payoff patterns. Twelve of these are strict symmetric games, on the diagonal axis, while there are 66 pairs of asymmetric games, equivalent by switching row or column positions. So, 12 symmetric and 66 asymmetric games make up the total of 78 strategically distinct 2x2 strict ordinal games, if positions are not considered relevant. If position as Row or Column is important, then 66 reflected pairs of asymmetric games plus 12 symmetric games compose a total of 144 strict ordinal games.

Robinson and Goforth chose to start their numbering with the most famous game, Prisoner’s Dilemma, a reasonable but somewhat arbitrary choice. In hindsight, this is comparable to starting the periodic table of the elements with element 92, Uranium, an element that is interesting, dangerous, and complex. Scrolling the layers to move Prisoner’s Dilemma next to the center elegantly arranges games according to their properties, but means that their game numbers end up in the sequence 1 6 5 4 3 2, making the numbering scheme more complicated to learn and use.

Consistent with starting with Prisoner's Dilemma as game 111, Robinson and Goforth put Prisoner's Dilemma, and its layer of discordant games with highest payoffs in diagonally opposite cells, in the lower left of their table. An arguably more logical arrangement, analogous to Cartesian coordinates increasing up and to the right, is to put the layer of simpler win-win games in the lower left, and the more complex discordant games in the upper right. If games with no dominant strategies and either two Nash Equilbria (stag hunts and battles) or none (cyclic) are placed in the upper right quadrant of each layer, then there is also a rough trend toward increasing complexity within layers.

Binomial names are easier to remember than arbitrary numbers, if the number of names can be kept small. Names can be linked with numbers where needed, as in Figure 5. Binomial names are consistent across different ways of arranging layers and sequencing symmetric games within layers. In comparison with Robinson-Goforth index numbers, a naming scheme based on symmetric games also turns out to be easier to extend to include games with ties.

Payoff Values

Ordinal payoffs are defined only by their relative ranks, and may be given in terms of algebraic inequalities, for example d<c<b<a. However, if different authors define the inequalities using different symbols, this makes it harder to recognize games that are similar or ordinally equivalent. It is easier and more intuitive to show simple numeric payoffs. While some authors start with zero, this may be confusing, especially if payoff values are transformed. The nomenclature proposed here follows Rapoport, Guyer, and Gordon (1976); Robinson and Goforth (2005); and others in showing payoff values ranging from one to four: 1<2<3<4.

For showing ties on this 1-4 scale, low ties can be treated as setting the two lowest values to 1 and high ties setting the two highest values to 4. This makes it easier to follow the half-swap transformations that form games with ties. Ties for middle payoffs can be conveniently shown as 3, which takes up less space than 2.5, and since the decimal is not meaningful for ordinal ranks. Because the null "game" of complete indifference is unique, it may sometimes be appropriate to show it with zero values for payoffs, all equally good, equally bad, or equally undifferentiated. Following a standard convention for displaying numeric payoff values from one to four makes it easier to identify equivalent and similar games.

Row and Column Orientation

Interchanging rows or columns or both allows allows a game to be arranged in as many as four different ways,[2] which are usually considered to be equivalent (Rapoport, Guyer, and Gordon 1976). The different ways of arranging payoffs is another reason it may be difficult to identify and compare games that are the same or ordinally equivalent. Rapoport, Guyer, and Gordon (2005, 17, 32) define a "natural outcome" and put that in the upper left corner (with some exceptions) which makes the arrangement dependent on understanding and applying their criteria for natural outcomes. Robinson and Goforth rely on graphs, which are the same for any of the possible versions of a game.

It is also possible to specify the arrangement of payoffs based on the location of best payoffs, and to choose one arrangement as a default. For numeric payoff matrices, Robinson and Goforth use a convention of putting Row's highest payoff (4) in the right column, and Column's highest payoff in the upper row, which can be summarized as: Row's 4 right, Column's 4 up, or Right-Up. They justify this as being consistent with the convention in Cartesian graphs of putting higher values up and to the right.[3] Using a particular convention, such as Right-Up, makes it easier to compare games. Discussions of symmetric games conventionally place the cooperate-cooperate (CC) outcome in the upper left cell, a Left-Up orientation. The concept of a cooperate-cooperate outcome is problematic for Battle of the Sexes Games, and for many asymmetric games, making this questionable as a basis for orienting cells.

Subscripts provide a convenient way to identify different orientations of the same game, equivalent by interchanging rows or columns. Thus, Robinson and Goforth's version of Prisoner's Dilemma would be Right-Up: Pd_{RU} while the format used by Axelrod and many others would be Left-Up: Pd_{LU}. The discussion here will follow Robinson and Goforth's choice of a Right-Up, "Cartesian" display as the default arrangement, which is

2 Some games with ties end up with identical patterns of payoffs, and so have fewer than four alternatives.

3 Robinson and Goforth make an exception for games with second-best equilibria, but it keeps things simpler to omit their exception. For a table showing numeric payoffs, as in Table 2, this keeps the Nash Equilibria aligned, making the table easier to read and use.

conveniently consistent with graphical displays of game payoffs. As with using numeric values from one to four, a default arrangement with Row's highest payoff in the right column and Column's best payoff in the upper row makes it easier to identify equivalent and similar games.

Strict Symmetric Games

There are only twelve strict ordinal 2x2 games. Transformations of these form the remaining 2x2 games with ties, and combinations of payoffs from symmetric games form asymmetric games. Thus names for the 12 strict ordinal 2x2 games form the basic elements of the nomenclature. Most but not all of the twelve have established names. The nomenclature proposed here tries to follow established names where appropriate, particularly those in Robinson and Goforth's Periodic Table of 2x2 Ordinal Games (2009), while also seeking names that are distinctive and will yield different abbreviations for a compact notation.[4]

Layer One contains six strict ordinal symmetric games, with best payoffs in diagonally opposite cells, including those that have been the subject of most game theory analysis. In three, both have dominant strategies leading to a single Nash-equilibrium. Three others have no dominant strategies and two Nash Equilibria where one gets best and the other second-best.

- **Prisoner's Dilemma**. With its combination of dominant strategies leading to a Pareto-inferior Nash equilibrium, Prisoner's Dilemma is the most unique strict ordinal game and already has a well established name. Where a shorter name is needed for naming games resulting from tie transformations, these may be labelled just using the word dilemma, for example the Low Dilemma game between Prisoner's Dilemma and Chicken, formed by ties in the lowest two payoffs.
- **Deadlock.** Swaps in middle payoffs turn Prisoner's Dilemma into the game known as Deadlock.[5] Robinson and Goforth call this game Anti-Prisoner's Dilemma, based on the similarity in the payoff graph. In this game, following dominant strategies

4 For additional discussion of relationships between symmetric 2x2 ordinal games, see D. Goforth and Robinson 2010; Huertas-Rosero 2003.

5 see http://www.gametheory.net/dictionary/games/Deadlock.html

means that neither gets their best payoff, and instead at the Nash Equilibrium both get second-best. For a nomenclature, positive names are preferable to ones that define a game in terms of another game. Avoiding “anti” names also makes for shorter names and more convenient abbreviations, so Deadlock is proposed as the standard name for this game, shortened to Lock for corresponding games with ties.

- **Compromise**. Switching lowest payoffs in Deadlock creates another second-best game, which Robinson and Goforth refer to as Anti-Chicken. The name proposed here is Compromise. This avoids defining the game in terms of another game, abbreviates more distinctly, and also, compared to its neighbor Deadlock, reflects a less grim view of the not-so-bad result where dominant strategies lead both players to get second-best.

- **Hero**. Rapoport (1967) distinguishes the two strict Battle of the Sexes type games as Hero and Leader based on the payoff to the player moving away from the “natural” maximin outcome when both avoid the worst payoff and instead both get second-worst. In Hero, the player who changes to the other move, making it possible to reach a Nash Equilibrium, gets second-best as a result, making a kind of heroic sacrifice.

- **Battle**. In Leader, the one who moves from the maximin outcome of both getting second-worst gets the best payoff, while the other gets second-best. Robinson and Goforth use the original name Battle of the Sexes for this game (Luce and Raiffa 1957, 90–92). Concern about gender stereotypes has led to suggestions for alternative names, such as Bach or Stravinsky (Osborne and Rubinstein 1994, 15) (allowing the same abbreviation, BoS). The simpler name Battle is proposed here, to reduce concerns about sexism or gender stereotyping, and because the initial “B” provides a more distinctive abbreviation than the letter “L,” especially since lowercase “l” can sometimes be confused with the number 1. Leader, Battle of the Sexes, and Bach or Stravinsky would then be common names for this game. As with scientific names for species in Linnaean taxonomy, it may be convenient to follow the common name with the binomial name in parentheses, in italic font, for example: Leader (*Battle*).

- **Chicken**. The second-most famous game has two unequal Nash equilibria, where one or the other gets their best result while the other gets second-worst. Both are tempted to defect from the cooperative second-best outcome that would result if both play a dove strategy. However, if both try to get their best result, a Hawk strategy, they instead both end up at the worst outcome. Chicken is also known as Hawk-Dove (Osborne and Rubinstein 1994, 16–17). Chicken is ordinally equivalent to the game of Snowdrift (Kümmerli et al. 2007), for which payoffs have usually been defined in algebraic terms.

The six symmetric strict ordinal games on Layer Three, the win-win layer, include three stag hunts which have two Nash Equilibria, one of which is Pareto-inferior and one win-win. In three more games, dominant strategies for both lead to a single Nash Equilibrium with a win-win outcome.

- **Stag Hunt.** Swapping the top two payoffs for both players turns Prisoner's Dilemma into Stag Hunt, one of three strict symmetric games with a second, Pareto-inferior Nash equilibrium. For the game where the inferior equilibrium is second worst, Robinson and Goforth's name seems well-suited, reflecting Rousseau's (2004, 85–86; and see Skyrms 2004) story about the hunter preferring the safer but much less desirable choice of a rabbit rather than a stag that might be gained if others could be trusted to cooperate.

- **Assurance.** Robinson and Goforth named both the other two symmetric ordinal stag hunts as Coordination. However, for the nomenclature there is a need to distinguish between them. The game next to Stag Hunt, resulting from swapping middle payoffs, represents a severe form of an assurance problem as defined by Sen (1967). This occurs if there are two equilibria, one Pareto-inferior, and choosing the move with the best payoff risks getting the worst payoff if the other does not cooperate. Thus the assurance problem is a conflict between getting the best, win-win outcome, if the other can be trusted to cooperate, versus avoiding the worst outcome.

- **Coordination**. By contrast, in the third of the three strict symmetric stag hunts, the move that avoids the worst payoff also makes it possible to achieve the best, so there is no conflict between getting the win-win outcome and minimizing the risk of getting the worst payoff. It should be noted that the term coordination game can also be used in a more general sense that includes games requiring coordination on one of two or more equilibria, including the strict symmetric Stag Hunt, Assurance, and Coordination games discussed here, the strict games of Hero, Battle, and Chicken, and simpler games with ties, including the simplest coordination game (Double Coordination) discussed below. This more general meaning of the term coordination games is also a reason to prefer the term stag hunts to identify the games with two Nash Equilibria, one win-win and one Pareto-inferior.

- **Peace**. This was the only one of the twelve strict symmetric games left nameless by Robinson and Goforth. It is a game of mixed motives or mixed interests. Its symmetric neighbors, Coordination and Harmony, are games of pure cooperation where one player's incentives always lead to moves that also raise the other player's payoff, positive externalities or, in Greenberg's (1990) terminology, positive inducement correspondence. In Peace, there is an underlying conflict which is overcome. As long as the other player chooses the move that includes win-win, the first players's incentives lead to a move with that raises payoffs for both, a positive externality. However, if the other player did choose the alternate move that does not lead to win-win, then the first player's incentives would encourage a move that would make things worse for the other, imposing a negative externality. Thus in this situation, there is a degree of underlying conflict, even if dominant strategies mean that incentives should lead both to the win-win outcome, suggesting Peace as an appropriate name.

- **Harmony**. Incentives are strongly aligned in Harmony, where moves following dominant strategies raise payoffs by two ranks, from worst to second best or second-worst to best. Robinson and Goforth do not cite a source for this name, but it seems appropriate.

- **Concord.** Moves following dominant strategies only raise payoffs by one rank, but still lead both to win-win, so the incentives are in the same direction as Harmony, although not as strong. Robinson and Goforth call this game No Conflict. However, for games with ties, names based on the tie transformations would lead to awkward terminology, such as Low No Conflict or High No Conflict. Therefore the name Concord, with a similar meaning, is proposed, which conveniently also allows Nc and N as workable abbreviations to distinguish it from Coordination, Compromise, and Chicken, which also begin with the letter C.

Symmetric Games with Ties

Payoffs in ordinal games, with and without ties, may be categorized into eight preference orderings based on the number and type of ties (Guyer and Hamburger 1968; Fraser and Kilgour 1986; Kilgour and Fraser 1988; Robinson, Goforth, and Cargill 2007). Strict games have no ties. There may be a single tie, for the lowest, middle, or highest payoffs, pairs of ties for highest and lowest payoffs (double ties), or three ties on either the lowest or highest payoffs. The zero or null game of complete indifference has all ties. Combinations of the eight preference orderings divide 2x2 ordinal games into 64 preference classes, as shown in Table 3. Making ties in a strict game converts it into a different preference ordering, so names for preference orderings also represent the possible transformations. It may be noted that the term non-strict is sometimes used to refer to ordinal games that are not strict, the ones with ties, and in other cases to refer to the larger set of games with and without ties. To avoid confusion the discussion here will avoid the term non-strict and instead use the phrase “games with ties” for the games that do not have four strictly ranked preferences, and Rapoport, Guyer, and Gordon's term, “the complete set of 2x2 games” for all the 2x2 ordinal games, with and without ties.

- **Low Ties.** These games usually form ideal types for the neighboring four strict games. In the expanded topology, the tile of games is linked by low half-swaps that form ties for the lowest two payoffs. Names can be assigned based on the adjoining strict games, although this requires a somewhat arbitrary choice between the two possibilities.[6] Thus Low Battle lies between Hero and Leader (*Battle*), Low Lock between Deadlock and Compromise, Low Coordination between Assurance and Coordination, and Low Harmony between Harmony and Peace. Low Concord lies between Concord and Stag Hunt, but has weakly

6 Ideally, it would be preferable to have a strict rule that determines a unique name, rather than two options. In general, the approach here favors the “lower left” game in the respective tile, however applying this rule too strictly would generate misnomers, misleading names, such as Middle Dilemma, rather than Midlock, which actually has a single, second-best, equilibrium. Therefore, in order to have more meaningful names, a somewhat less strict approach to naming the 38 symmetric ordinal games is applied here.

dominant strategies leading to a single win-win Nash equilibrium, making it more like Concord. In Low Dilemma, between Prisoner's Dilemma and Chicken, weakly dominant strategies would lead to a Pareto-inferior outcome where both get the worst payoff. This illustrates the limitations of dominant strategies as a solution concept, although the Pareto-superior outcome is still vulnerable to temptation to defect. Having a specific name might be helpful in directing attention to the interesting issues raised by this game.

- **Middle Ties.** Volunteer's Dilemma (*Middle Battle*) (Diekmann 1985; Archetti 2009) is the most well-known game with ties for middle payoffs. It is formed by making ties in Chicken or Battle. Middle Compromise is a second best game, between Hero and Compromise. Middle Lock, between Deadlock and Prisoner's Dilemma, also has a second-best equilibrium. Middle Lock is interesting and unique as the only symmetrical zero-sum (or zero rank-sum) ordinal game, and so an ideal type or exemplar of zero-sum games, although it does not seem to have received much recognition for its uniqueness. It deserves Midlock as a short name. Middle Hunt lies between Stag Hunt and Assurance. The usual story of Rousseau's Stag Hunt makes no mention of concern about whether or not the other hunter might also safely get a hare, suggesting indifference, in which case a game with middle ties would most accurately model the story, suggesting Rousseau's Hunt as a common name. Middle Peace is another harmonious game where dominant strategies lead to win-win. This is also the case for Middle Harmony, between Harmony and Concord. Middle Harmony can be seen as an ideal type for Adam Smith's invisible hand situation, where individual incentives lead to the best outcome with no need for coordination or strategic thinking, suggesting Invisible Hand as a common name.

- **High Ties**. High Hunt lies between Stag Hunt and Prisoner's Dilemma, and is interesting since it shares the problems of both: weakly dominant strategies lead to a Pareto-inferior Nash equilibrium, while both can get their best payoff at a second Nash equilibrium, if they each trust the other to cooperate, but risk getting the worst payoff if the other does not cooperate. The symmetric high ties games all come in two versions, depending on the starting point for the tie transformation. High Hunt ends up with the same arrangement of payoffs as High

Dilemma, as do High Chicken and High Concord. However, the other high ties have multiple versions which differ by the orientation of rows and columns. One can be designated as the default version, and it seems suitable to prefer the version on Layer Three.[7] For a display of the complete set of 2x2 ordinal games, as in Figure 4, the alnternate version (colored gray) is still needed to form some asymmetric games with ties. High Coordination (and High Assurance) and High Hero (and High Battle) both have two Nash equilibria, in one of which both get the best payoff. High Concord (and High Chicken), High Harmony (and High Compromise) and High Peace (and High Lock) all have two dominant strategies leading to win-win at a single equilibrium.

- **Double Ties**. These games have ties for both the two highest and two lowest payoffs. The Avatamsaka Game (Double Hunt) was named after a Buddhist scripture about two people chained in place, each with a spoon too long to feed themselves but able to feed the other, showing pure interdependence (Aruka 2001; Aruka 2011). In this degenerate game, neither's move can directly affect their own payoff, and instead each must depend on the other's moves. Rapoport, Guyer, and Gordon discussed this as game #79 (1976), but this earlier research is not cited in Aruka's work on the Avatamsaka Game, an example of how the lack of standard identifiers hinders cumulative research. Double Coordination is the simplest coordination game, requiring a choice between two equally attractive options, as in the example of driving on the left or right hand side of the road. The Double Ties symmetric games also come in alternate versions, equivalent by interchanging rows and columns.[8]

7 Robinson and Goforth (2005) discovered two forms of linkage between layers, where high swaps connect equivalently located four-game tiles. In *pipes*, high swaps link four tiles on four layers, while in *hotspots*, high swaps link two tiles. This interesting emergent property of the topology affects the structure of high ties and double ties games. The strict games in pipes would have a full set of transformed high ties games on Layer 3. For hotspots, the lower left game in the tile can be preferred as the default for the 1:3 and 2:4 hotspots, and the game with High Hunt payoffs preferred as the default for the 1:2, 1:4, 2:3, and 3:4 hotspots, as shown in Figure 4. Transformations that break ties differentiate Double Ties games into pipes and hotspots.

8 Again, the Layer 3 versions can be preferred as the default. Matching Pennies is asymmetric, but is unique in that it is its own reflection, switching row and column positions to create the same set of payoffs for Double Hero-Double Coordination and Double Coordination-Double Hero. The right-hand version, *dode,* can be preferred as the default.

- **Triple Ties**. In these games, each dislikes one outcome. In both Triple Harmony and Triple Lock, weakly dominant strategies lead to win-win, but Triple Lock also has a second Nash equilibrium.
- **Basic Ties**. These games are archetypal version of Layers One and Three, with best payoffs harmoniously located in the same cell in Basic Harmony, as in Layer Three; and discordantly aligned in diagonally opposed cells in Basic Dilemma (where synchronizing to take turns could be a solution in repeated play).
- **Zero.** All ties, complete indifference, characterizes the null game where players have no preferences between different outcomes.

In total, there are 38 unique 2x2 symmetric ordinal games. High ties and double ties games have alternate versions, equivalent by interchanging rows or columns, some of which are needed to generate asymmetric games outside Layer 3, so Figure 2 shows 47 symmetric games, including the 9 alternates.

Asymmetric Games

Names for asymmetric games can be formed from the two symmetric games that have the same ordinal payoff structure. For example, Samaritan's Dilemma (Buchanan 1977; Schmidtchen 2002) combines payoffs from Harmony and Chicken. Asymmetric games come in two chiral forms, equivalent by switching row and column positions of the players. As mentioned, for the topology, both reflections are needed. For convenience, these can be labelled as right-hand forms, below the diagonal line of symmetric games, and left-hand forms, above the diagonal.[9] Where position does not matter, the right-hand form could be considered the default. The payoffs shown by Buchanan for Samaritan's Dilemma have Row's highest payoff in the lower row, and Column's highest payoff in the right column (Down-Right), and the highest payoffs in the same row (Layer 2), (to the right of the axis of symmetry), and so would be Harmony-Chicken$_{DR}$.

An advantage of the binomial nomenclature is that is makes the reflected pairs of games obvious. By contrast, the Rapoport-Guyer taxonomy and Brams typology do not distinguish between reflections or provide a way to identify which is being shown.

9 As it happens, this right-left division matches the "right-hand rule" often used in science and engineering. The right-hand version of the cyclic games encourages movement counter-clockwise, while the left-hand games cycle clockwise.

Robinson-Goforth index numbers do show the two reflections resulting from switching position of row and column. However, the index numbers require understanding the structure of layers and arrangement of games in each layer in order to recognize the reflected equivalents.

For the asymmetric games, most preference classes create compact square matrices, as shown in Table 4. However, games formed from symmetric games with high ties, double ties, or all ties produce multiple versions equivalent by interchanging rows or columns. As mentioned above, to facilitate identification of equivalent games, it is useful to designate one as a preferred version. This can mostly be done by preferring the version in Layer Three, or, if necessary in Layer Two and Four, but not Layer one, as shown in Figure 3. Using this approach, any asymmetric ordinal game can be identified as the combination of payoffs from two symmetric games.

Abbreviations and Tags

Two-letter abbreviations provide a convenient way to refer to games, following the example of abbreviations for elements. The twelve strict symmetric games can each have their own two letter abbreviation, for the strict game, and a shorter single letter abbreviation used to indicate the related games formed by ties. Games with ties can be identified with a first letter based on the type of tie, and the second letter based on a strict game from which it is created by a forming a tie. Names for the types of ties, the different Fraser-Kilgour preference orderings, have been chosen to have different initial letters. Lowercase letters help to distinguish games with ties from the strict games: Asymmetric games would have a four-letter abbreviation. Abbreviations would be as shown in Table 6.

Examples of abbreviations would be as follows:

Pd Prisoner’s Dilemma

HaCh Harmony-Chicken, common name: Samaritan’s Dilemma

ld Low Dilemma

dode Double Coordination-Double Hero, common name: Matching Pennies (right-hand, counter-clockwise version.

Abbreviations can also be used as tags for games, making it easier to label and find studies of the same game, even when these use different payoff values and orientations. These could be simple hashtags, like #2x2game:pdpd for Prisoner's Dilemma. The topology of 2x2 games can satisfy the requirements of an ontology, and so provide Universal Resource Identifiers (URIs) for the semantic web (Berners-Lee et al. 2001). A systematic way of identifying a preferred default version for games with high, double, or all ties for one or both players is necessary to establish unique URIs for all the 2x2 ordinal games.

Table 6. Abbreviations for a Compact Notation for 2x2 Games

Strict Games		
Ch	c	**Chicken**/Hawk-Dove/Snowdrift
Ba	b	**Battle**/Leader
Hr	e	**Hero**
Dl	k	**Deadlock**/Lock/Anti-Prisoner's Dilemma
Cm	m	**Compromise**/Anti-Chicken
Pd	d	**Prisoner's Dilemma**
Hu	u	**Stag Hunt**
As	s	**Assurance**
Co	o	**Coordination**
Pc	p	**Peace**
Ha	h	**Harmony**
Nc	n	**Concord**/No Conflict
Types of Ties (Preference Orderings)		
1,2,3,4	s	**Strict**
1,1,3,4	l	**Low**
1,3,3,4	m	**Middle**
1,2,4,4	h	**High**
1,1,4,4	d	**Double**
1,4,4,4	t	**Triple**
1,1,1,4	b	**Basic**
0,0,0,0 Ze	z	**Zero**/All ties

Finding a Game

Starting with a matrix of payoff values, tables 2 and 4 can be used to find the name, based on the following steps:

Make ordinal: Rank payoffs from 1 to 4. In case of ties, low ties are 1, high ties are 4, middle ties are 3.

Orient Right-Up. Put Row's best payoff in the right-hand column, and Column's best payoff in the upper row.

Categorize by type of ties: Determine the preference ordering for each player's payoffs.

Inspect preference class: Within class formed by the two preference orderings, find the symmetric game with the same payoff pattern by inspection of Table 2. For strict games, remember that layers differ by the alignment of best payoffs, those with win-win outcomes in the upper left cell are on Layer 3, and those with best payoffs diagonally opposed are on Layer 1.

Check for alternate versions: For high ties, double ties, and all ties, check alternate versions formed by interchanging rows and columns to identify the preferred, default, version in Table 4. Layer 3 versions, with win-win outcomes in the upper left corner of the payoff matrix, are preferred where available. For high ties, prefer games formed by payoffs from High Coordination, High Hero, and High Hunt. For double ties, prefer games formed by payoffs from Double Hunt, and prefer the right-hand, counter-clockwise, versions to the right and below the axis of symmetry, such as the right-hand version of Matching Pennies (Double Coordination-Double Hero).

RESULTS AND DISCUSSION

Names based on the twelve strict symmetric games and transformations creating ties identify all the symmetric ordinal games. Asymmetric ordinal games can be formed by combining payoffs from symmetric games. Therefore, names for symmetric ordinal games provide the basis for a binomial nomenclature to efficiently identify all 2x2 ordinal games.

Payoff values from 1 to 4 indicate the ordinal ranks, making it easier to identify ordinally equivalent games. The location of best payoffs defines four possible orientations, with

Row’s best payoff (4) left or right, and Column’s best payoff up or down. A Right-Up convention can be used as the default, to further facilitate comparison and identification of similar or ordinally-equivalent games.

For High Ties and Double Ties symmetric games that have two versions, equivalent by interchanging rows or columns, the version on Layer Three, with win-win outcomes in the upper right cell, can be preferred as the default version. A few additional specifications, as discussed above and shown on Table 3, then make it possible to uniquely identify the complete set of 2x2 ordinal games. Abbreviations for the strict symmetric games and tie transformations provide a compact notation for identifying 2x2 ordinal games.

Binomial names, and the topology of 2x2 games on which they are based, help to understand similarities and differences between 2x2 games, which form elementary models of strategic situations where one person's choices may depend on what someone else does. The nomenclature can be used to identify equivalent and similar games, and so contribute to cumulative and comparative research. This can help communication, where the same, ordinally equivalent game is known by different names or identifiers, as with Chicken, Hawk-Dove and Snowdrift. The nomenclature can help link older and newer research on interesting games, such as the Avatamsaka (*Double Hunt*) Game of interdependence (Y. Aruka 2001), which Rapoport, Gordon, and Guyer discussed as game number 79. Various authors have discussed the game between Prisoner’s Dilemma and Chicken, including Rapoport, Guyer, and Gordon; and Fraser and Kilgour (Fraser and Kilgour 1986) but it lacks an established name, number, tag or other unique identifier that could help contribute to cumulative research.

The nomenclature distinguishes between similar games, such as Hero, Leader and other Battle of the Sexes-type games with two Nash Equilibria where only one gets the highest payoff, including asymmetric battles and battles with ties. For stag hunt games with two equilibria, in one of which both can get their best outcome, the nomenclature distinguishes between those with and without assurance problems where obtaining the best payoffs conflicts with risk minimization. Understanding the diversity of stag hunts, with a clear way to distinguish between similar but distinct games, may facilitate experimental research and comparison to look at the relationship that different payoff structures have with risk avoidance and maximin strategies, and how this may contribute to a deeper understanding of trust and related issues, including stag hunts with asymmetric payoffs.

The names, and the topology of payoff swaps and half swaps, help to understand the relationship between close neighbors, such as Volunteer's Dilemma (Middle Battle) between Chicken and Leader (Battle), and Low Dilemma between Chicken and Prisoner's Dilemma. While game theory has tended to concentrate on the most difficult situations, names may help direct more attention to situations, such as Deadlock and Compromise, which are not as grim, but which nevertheless may represent empirically important phenomena.

A nomenclature that includes games with ties may help direct more attention to interesting games, such as Midlock (Middle Lock), which exemplifies zero rank-sum situations. Problems with how weak dominance could lead both to get their worst outcome in Low Dilemma show the limits of relying on dominant strategies as a solution concept. The High Hunt games combine the risk avoidance problems of stag hunts with the Prisoner's Dilemma's temptations to defect. Avatamsaka (Double Hunt) shows the relevance of focal points (Schelling 1960) and other solution concepts for this game and the other degenerate games (Rapoport, Guyer, and Gordon 1976) it helps form

Cyclic games and other asymmetric games also deserve more attention. To the extent that payoffs are generated randomly and are not limited to a small number of integer values, games would be expected to occur in the proportions shown in the periodic table of 2x2 games (Simpson 2010). One out of every eight strict 2x2 games is cyclic, with no Nash Equilibrium. Games with real or ratio value payoffs normalized on a 1-4 scale can be mapped onto the Periodic Table, meaning that it also can be used as a chart of the normalized space of 2x2 games (Bruns 2010; Bruns 2012). When symmetric and asymmetric games are categorized according to equilibrium payoffs, biased games where one gets best and the other second-best, as in Samaritan's Dilemma and battles, make up the largest payoff family, even though they are a much smaller proportion, 1/6, of the strict symmetric games.

Prisoner's Dilemma, Chicken, and other 2x2 games are only a few of the many elementary models of strategic interactions that combine incentives for conflict and cooperation. Theoretical research on 2x2 games has concentrated on a small number of well-known games, mostly symmetric and mostly strict games without ties, although there are many more asymmetric games and far more games with ties. A nomenclature that includes asymmetric games and games with ties could facilitate understanding of the diversity of models which may be used for analysis, experimentation, simulation, and other research.

CONCLUSIONS

Payoffs from symmetric games combine to form asymmetric games, and games with different kinds of ties can be formed by making ties in strict games. Therefore a binomial nomenclature based on names for the twelve strict symmetric games and eight types of ties identifies the complete set of 2x2 ordinal games. Default conventions for numeric payoff values and locations of best payoffs make it easier to recognize similar and equivalent games. A binomial nomenclature for the 2x2 ordinal games can help to locate interesting games, understand and apply the diversity of elementary models of strategic situations available for use in analysis, experiments, and simulations, and contribute to cumulative and comparative research on social conflict and cooperation.

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