Bid Behavior in the Uniform Price and Vickrey Auctions on a General Preference Domain

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Abstract

Why are Vickrey auctions so widely praised by economic theorists, yet so rarely used in practice? I address this question by comparing bid behavior in the Vickrey auction with the more commonly used uniform price auction. I study the case where bidders have private values and multiunit demands, but I remove the standard quasilinearity restriction on bidder preferences. Instead, I allow for a more general preference domain that nests quasilinearity, but also allows budget constraints, financial constraints, risk aversion, and/or wealth effects.

I show that truth-telling is not a dominant strategy in the Vickrey auction. Instead bidders truthfully report demand for their first unit and overstate demands for all other units. This result mirrors the incentive for demand *reduction* in uniform price auctions shown by Ausubel and Cramton (2002). While both auctions are generally inefficient, I show that when the auction is large, both give approximately equal allocations and revenues, and both are approximately ex-post efficient.

Introduction

> "Despite the enthusiasm that the Vickrey mechanism and its extensions generate among economists, practical applications of Vickrey's design are rare at best.... The most novel version of Vickrey's design which applies to sales in which different bidders may want multiple units of homogenous goods - or packages of heterogenous goods - remain largely unused." (Ausubel and Milgrom, 2006)

Vickrey auctions possess desirable properties sought out by market designers; namely, strategyproofness and efficiency. In fact, the results of Holmström (1979) and Williams (1999) show that any dominant strategy mechanism that does not run a deficit, gives losers zero payoffs and implements an efficient allocation is a Vickrey auction. The above quote then poses a natural question: why are Vickrey auction so rare, especially in multiunit settings?

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In their paper, Ausubel and Milgrom go on to describe four weakness of the Vickrey auction that limit its practical applicability. However, their critiques apply only to cases where the objects being sold are heterogenous goods and at least one buyer has complements preferences. Their critiques do not apply to the sale of homogenous goods when bidders have (weakly) declining demands. This leaves half of their original question unanswered: why are Vickrey auctions rarely used to sell multiple homogenous goods? In this paper, I compare the Vickrey auction with the more frequently used uniform price auction. I show that when we remove the quasilinearity restriction, and allow for a more general preference domain, bid behavior in Vickrey auctions mirrors that of the uniform price auction. Indeed, many commonly cited deficiencies of the uniform price auction are present in the Vickrey auction.

More specifically, I drop the quasilinearity restriction and assume only that bidders have weakly positive wealth effects. Thus the goods being sold are (weakly) normal. My specification allows for multidimensional heterogeneity in bidder demands, risk preferences, budgets, financing constraints, etc. It does not make function form restrictions on bidder preferences and it nests the benchmark quasilinear environment. I show that without the quasilinearity restriction, truthful reporting is not a dominant strategy. Instead, bidders have an incentive to truthfully report their demand for their first unit and overreport their demand for all other units. The incentive to overreport demands leads to inefficient outcomes. This mirrors Ausubel and Cramton's (2002) famous results on demand reduction and inefficiency in uniform price auctions.

This result yields an interesting testable implication. A (weakly) greater number of bidders will win at least one object in the uniform price auction. In other words, a bidder has a higher chance of walking away empty-handed in the Vickrey auction versus the uniform price auction. However, bidders who do win objects in the Vickrey auction win a greater number of objects than they would in the Vickrey auction.

While Ausubel and Cramton show the uniform price auction to be inefficient, the large auctions literature shows that these inefficiencies vanish when there are many bidders. In particular, Jackson and Kremer (2006) and Swinkles (2001) both show that uniform price auctions are approximately efficient with many bidders. I show analogous results for uniform price and Vickrey auctions without the quasilinearity restriction. In particular, I show that with large number of bidders, truthfully reporting demand is an ϵ best reply to any undominated play of other bidders. I use this to show that many bidders, both auctions give approximately equal revenues and allocations, and are approximately ex-post efficient. This result does not require that bidder types are independent or single dimensional as is typical in the large auctions literature. Instead, I place few restrictions on the distribution of types and allow for cases where bidder types are multidimensional and correlated.

With these results we can revisit the above quote. When we restrict ourselves to studying quasilinear environments, it is well known that the Vickrey auction implements an efficient allocation in dominant strategies. However, when we expand our analysis to a more general preference domain, this result disappears. Bidders overreport their demand in the Vickrey auction much like they underreport their demand in the uniform price auction. While both auctions are inefficient, in a large auction setting, the two produce nearly identical allocations that are approximately efficient. Thus, without the quasilinearity restriction, it is unclear whether the Vickrey auction has any distinct advantages over the uniform price auction. This can help to explain why the uniform price auction is more frequently used in practice.