Collaborate or Consolidate: Assessing the Competitive Effects of Production Joint Ventures*

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Abstract

We analyze a symmetric joint venture in which two firms facing external competition collaborate in input production. Under certain regularity conditions, such a collaboration leads to higher profits than a horizontal merger between these two firms, whereas the effect on prices and quantities depends on the form of downstream competition. When firms compete in prices, downstream prices for all firms are higher following a joint venture than those following a horizontal merger. The reverse result may obtain when firms compete in quantities. Nevertheless, prices and profits remain higher in a Cournot equilibrium than in a Bertrand equilibrium. We use our methodology to compare counterfactual joint ventures and horizontal mergers in the U.S. mobile wireless industry.

Keywords: Production Joint Venture; Collaboration Among Competitors; Horizontal Merger; Bertrand Oligopoly; Cournot Oligopoly

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1 Introduction

Collaboration via joint production can present an attractive alternative for competing firms contemplating a horizontal merger, particularly for large firms concerned with being challenged by the antitrust agencies. U.S. antitrust guidelines distinguish competitor collaborations from mergers, stating that in contrast to mergers, collaborations generally preserve some form of competition among participants.[1] Production collaboration involves agreements where parties produce through common production facilities or a jointly controlled company while remaining separated in other facets of operation.[2]

It is by now well established in the economics literature that production collaboration may engender anti-competitive effects as great as those of a horizontal merger (Bresnahan and Salop, 1986; Reynolds and Snapp, 1986; O'Brien and Salop, 2000; and Chen and Ross, 2003). Antitrust agencies also recognize that such collaborations can have competitive effects identical to those that would arise if the participants merged and delineate the circumstances under which competitor collaborations should be treated as mergers. A prevalent view, however, is that a production joint venture that is not found to be per se illegal, should almost surely be allowed if the participants would be permitted to merge.[3] Historically, collaborations that are not treated by the agencies as mergers have been considered to be pro-competitive and have faced relatively few legal challenges. Notably, Werden (1998) could not identify a single case in which a joint venture not treated as a cartel or merger was dissolved by court order following an antitrust challenge.

[1]See the U.S. Antitrust Guidelines for Collaborations Among Competitors (2000).

[2]The more recent collaboration guidelines issued by Canada (2009) and the European Commission (2011) suggest that production collaborations may vary in form and scope and include among them subcontracting arrangements where one party retains another to produce products on its behalf.

[3]See, for instance, Shapiro and Willig (1990). The U.S. Guidelines for Collaborations define agreements of a type that always or almost always tends to raise price or reduce output as per se illegal. Werden (1998) has observed that the only per se illegal joint ventures are those that are merely cartels which involve no efficiency-enhancing integration.

In this manuscript, we make a positive comparison of the potential competitive impact of production collaboration with that of a horizontal merger between two firms in an oligopoly setting. In seeking a better understanding of production joint ventures, we illustrate under reasonably general conditions, that the treatment of production joint ventures as mergers does *not* suffice to identify a production collaboration that antitrust practitioners would find detrimental to competition. Implicit in our analysis is the idea that the mechanics behind a production collaboration and the potential anti-competitive harms they entail can be markedly different from those underlying a horizontal merger and may call for a significantly different antitrust treatment. This point has been recognized by antitrust authorities around the world notwithstanding the more lenient treatment of joint ventures, and explains the development of antitrust guidelines to deal with collaborations which are separate from those used to evaluate horizontal mergers. At the same time, in order to facilitate a meaningful comparison, we are careful to set up a model of collaboration that preserves the same product space, costs and timing as that observed in the horizontal merger alternative.

To place structure on our analysis, we focus on production joint ventures in which the outcome of collaboration is a product that is transferred to participants for independent marketing or used by them as an input in the autonomous production and retail of downstream products. Such "input" collaboration is an exceedingly common method of organization observed in various industries. Examples of input joint ventures include collaborations between automobile manufacturers who set up joint manufacturing facilities to produce automobile components or complete automobiles branded separately with each partners' marque; mobile wireless communication providers who jointly operate communications networks and share wireless spectrum, but offer distinct service bundles; and petroleum companies that share crude oil refining facilities but separately market and distribute fuel.[4]

In the model below, we show that two firms competing in differentiated substitute products who also face an additional oligopolistic competitor would prefer to collaborate via a symmetric input joint venture and continue to compete downstream than to merge completely.[5] Moreover, when downstream competition is differentiated Bertrand, such joint ventures lead to higher prices (and hence lower consumer and total welfare) for all firms in the market than a horizontal merger. The input joint venture achieves higher industry profits and prices via two effects, one internal to the collaboration, and one due to the presence of external competition. Internally, upstream collaboration allows firms to profit by raising the input price paid by their joint venture partners, which leads to higher prices downstream. Externally, by setting input prices other than at marginal cost, the joint venture facilitates collusion with outside competitors in a way that the merger of two vertically integrated competitors cannot. The latter effect is reminiscent of the price increasing influence that vertical separation has on rival firms (e.g., see Bonanno and Vickers, 1988). However, crucially, vertical separation is absent in our model—the input pricing decision is made directly by the joint venture partners, not delegated to an upstream input producer.

The situation differs in the case of Cournot competition, where downstream actions are strategic substitutes. In equilibrium, the quantities produced by the joint venture partners

[4]For instance, consider the AutoAlliance joint ventures between Ford and Mazda Motor Companies, the Everything Everywhere mobile network operated by Deutsche Telekom and Orange S.A. (which may alternatively be argued to be a merger), and Singapore Refining Company, which is shared by Chevron Corporation and Singapore Petroleum Company. Numerous additional examples are provided by Morasch (2000b), Chen and Ross (2003), and Rossini and Vergari (2011).

[5]Product differentiation serves two important purposes: (i) it allows us to accommodate evolving trends in antitrust policy away from a focus on market concentration and toward more direct indicators of the consequences of firm interactions (see for instance, the U.S. Horizontal Merger Guidelines, 2010, §6.1; Baker and Shapiro, 2008; and Farrell and Shapiro, 2010) and (ii) it avoids the paradoxical results present in homogeneous good oligopoly—marginal cost pricing in Bertrand; unprofitable mergers in Cournot (see Shapiro, 1989 and Salant et al., 1983).

are higher than they would be in a horizontal merger whereas the quantity produced by an outside oligopolist is lower. Although the joint venture remains more profitable than a horizontal merger, as we show in the case of linear demand, the prices for all firms can be lower in the joint venture scenario and total welfare higher. Surprisingly, nevertheless, in such a production joint venture, Cournot prices remain higher for all firms than Bertrand prices. That is, the pricing results obtained by Singh and Vives (1984) and Vives (1985) persist in spite of the direction of prices in a joint venture relative to a horizontal merger under price and quantity competition.

Much of the literature on production collaboration has focused on the study of output production. The most frequently adopted approach to modeling output joint ventures treats them as partial equity interests in existing producers (e.g., see Farrell and Shapiro, 1990) or newly formed production units (Kwoka, 1992). Partial equity interests may be silent, but can also entitle owners to partial or even complete control over other producers. Reynolds and Snapp (1986) have shown that silent equity interests can align firm incentives to such an extent that they may achieve the same effect as a horizontal merger. It has also been shown that when partial equity interest entitles a firm with full operational control of a competing producer, prices and possibly joint profits may exceed those of a horizontal merger between the two competitors (Foros et al. 2011). However, a full control scenario does not strictly fall under the standard definition of competitor collaboration because like a horizontal merger, it eliminates *all* competition between competitors, and is therefore more likely to draw the ire of antitrust agencies.[6] Bresnahan and Salop (1986) and O'Brien and Salop (2000) explore the various scenarios involving partial control and generally find that the effects on concentration fall below those of a horizontal merger.

[6]The U.S. Horizontal Merger Guidelines (2010) state that when the agencies determine that a partial acquisition results in effective control of the target firm, or involves substantially all of the relevant assets of the target firm, they analyze the transaction much as they do a merger.

The competitive implications of input joint ventures have garnered less attention in the study of industrial organization. The most closely related paper to ours is by Chen and Ross (2003), who show that a symmetric input joint venture that includes all participants in the market can perfectly replicate the profits, prices, and output in a merger to monopoly. Unlike this manuscript, Chen and Ross only introduce firms outside the collaboration in a non-strategic way (by varying the elasticity of demand), so there is no opportunity for joint venture partners to earn greater than merger profit. Morasch (2000a) considers endogenous joint venture formation with multiple firms, but his focus is on determining the ideal size of a collaboration.[7] The complications that arise in the joint venture formation stage of his game restrict him to an analysis of homogenous product producers facing linear demand. In more recent articles, Cooper and Ross (2009) show that joint ventures among firms engaged in multimarket competition may facilitate collusion across unconnected markets while Rossini and Vergari (2011) examine the competitive implications of vertical separation via input joint venture.

The remainder of this manuscript is organized as follows. In Section 2, we lay out assumptions and set up our general model. Section 3 derives our main results. Section 4 presents a linear demand example in which we are able to compare additional implications of this model. Section 5 explores the robustness of our model in a setting with imperfect information regarding the price of the input. Section 6 applies our methodology to compare counterfactual joint ventures and horizontal mergers in the U.S. mobile wireless industry. Section 7 concludes. All proofs can be found in the Appendix.

[7] Input joint ventures have also been analyzed in the context of international trade. In a companion article, Morasch (2000b) explores conditions under which an input joint venture can replicate a strategic trade policy intended to increase the profits of oligopolistic firms in international markets. Spencer and Raubitschek (1996) show that high-cost input joint ventures may be profitable because they can reduce the import prices paid for key components.

2 A general model

Three firms indexed 1, 2, and 3 produce imperfectly substitutable goods. In a baseline scenario without collaboration, every firm consists of a separate upstream and downstream division. Each upstream division can produce a unit of an intermediate good at the same constant marginal cost and with no constraints on capacity. Downstream divisions require one unit of the intermediate good as an input for each unit of output that they produce. We assume that downstream divisions have no other input requirements.[8] Moreover, firms are vertically integrated and there is no market for inputs across competitors. Let w_i denote the input price charged by each upstream division to its downstream division. Let θ_i denote the action of the downstream division and let $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ be the profile of all downstream actions. Downstream actions may represent prices, p_i or quantities, x_i.

On the other side of the market,[9] we have a representative consumer who maximizes $\left\{U(\mathbf{x}) - \mathbf{p} \cdot \mathbf{x} : \mathbf{x} \in \mathbf{R}^3_+\right\}$, where $U(\cdot)$ is a symmetric, $\mathbf{C}^3$ (differentially) strictly concave utility on $\mathbf{R}^3_+$, which is (differentially) strictly increasing in a non-empty, bounded set $X \subset \mathbf{R}^3_+$. The utility maximizing consumer gives rise to an inverse demand function f_i for each good i, which is $\mathbf{C}^2$ on the interior of X and decreasing in all its arguments ($\partial f_i/\partial x_j < 0$ for all j). The system of inverse demands can be inverted to yield direct demand functions $x_i = h_i(\mathbf{p})$ which are $\mathbf{C}^2$ in the interior of the region of price space for which demands are positive (denote the region P). When positive, direct demands are downward sloping ($\partial h_i/\partial p_i < 0$ for all i) and yield positive cross effects ($\partial h_i/\partial p_j > 0$ for $i \neq j$). Additionally, we assume that own effects are larger than cross effects: that is, for $i \neq j$, $|\partial f_i/\partial x_i| > |\partial f_i/\partial x_j|$ and $|\partial h_i/\partial p_i| > \partial h_i/\partial p_j$.

Firms 1 and 2 may be assumed to be parties to a horizontal agreement: either a merger

[8] We note that even if other inputs are required for downstream production, as long as the intermediate good produced by upstream divisions cannot be substituted, our general setup is without loss of generality.

[9] Our formulation of demand follows Vives (1985).

or a symmetric input joint venture. The horizontal merger preserves both downstream products, but consolidates all decisions. A joint venture produces and prices the requisite input to be used by its owners, who evenly split the profits of the collaboration, but continue to compete against each other downstream. It is assumed that the firm outside a joint venture is aware of the ownership and financial division between the joint venture partners.[10] Within the joint venture, the input is presumed to be homogenous. It is further assumed that the input is bought from the joint venture if and only if a firm is a party to the joint venture, that parties to the joint venture must procure their input from the collaboration, and that buying from or selling to outside parties is ruled out by the collaboration contract (e.g., see Morasch, 2000a). Alternatively, following Choi and Yi (2000), we could suppose that inputs are (symmetrically) specialized, so that it becomes prohibitively costly for joint venture partners to interact in the input market with non-partners. In Section 7, we briefly contemplate the possibility that the outside oligopolist may be able to supply the joint venture input to its partners (a la Ordover et al., 1990).

To keep the analysis simple, we assume that there is no efficiency gain from making a horizontal agreement. Thus, the marginal cost c of producing the intermediate good does not change in the event of a merger or joint venture. Although efficiencies stemming from horizontal collaboration are a major component of antitrust review, our primary focus is on comparison of competitive ramifications of a joint venture relative to a horizontal merger, and not on whether the ultimate agreement turns out to be anti-competitive. Likewise, we abstract from fixed costs by supposing that additional entry into the market is not permitted.

For notational ease, firm profits are written as π_i whether firms compete in prices or quantities downstream. Henceforth, the arguments of the profit function will be used

[10]Firms frequently announce the details of joint ventures and other collaborations to the public.

to denote the appropriate competitive scenario: $\pi_i(\mathbf{p})$ for Bertrand, $\pi_i(\mathbf{x})$ for Cournot, and $\pi_i(\boldsymbol{\theta})$ when an expression might apply to either. Moreover, the arguments will be suppressed wherever they becomes self-evident. Regardless of whether we analyze the baseline or a scenario with a horizontal agreement, we make the following additional assumptions on firm profits, which should be taken to apply to all $\mathbf{p}$ in the interior of P or all $\mathbf{x}$ in the interior of X as appropriate:

Assumption 1. *Firm profits are concave in downstream actions:* $\partial^2\pi_i/\partial\theta_i^2 < 0$ *for* $i = 1, 2, 3$.

Assumption 2. *Downstream, prices are strategic complements and quantities are strategic substitutes (a la Bulow et al., 1985). That is, for* $i, j = 1, 2, 3, i \neq j$:

$$\frac{\partial^2\pi_i}{\partial p_i \partial p_j} > 0 \ , \qquad\qquad \frac{\partial^2\pi_i}{\partial x_i \partial x_j} < 0 \ .$$

Consider the Jacobian matrix of the vector of own partials of firm profits:

$$\mathbf{J}_{\boldsymbol{\theta}} = \begin{pmatrix} \frac{\partial^2\pi_1}{\partial\theta_1^2} & \frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2} & \frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_3} \\ \frac{\partial^2\pi_2}{\partial\theta_2\partial\theta_1} & \frac{\partial^2\pi_2}{\partial\theta_2^2} & \frac{\partial^2\pi_2}{\partial\theta_2\partial\theta_3} \\ \frac{\partial^2\pi_3}{\partial\theta_3\partial\theta_1} & \frac{\partial^2\pi_3}{\partial\theta_3\partial\theta_2} & \frac{\partial^2\pi_3}{\partial\theta_3^2} \end{pmatrix}$$

Assumption 3. *The following stability relationships hold:*

1. *The determinant of* $\mathbf{J}_{\boldsymbol{\theta}}$, $|\mathbf{J}_{\boldsymbol{\theta}}|$, *is negative,*
2. $\dfrac{\partial^2\pi_1}{\partial\theta_1^2}\dfrac{\partial^2\pi_2}{\partial\theta_2^2} > \dfrac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2}\dfrac{\partial^2\pi_2}{\partial\theta_2\partial\theta_1}$.

Observe that the first item in Assumption 3 is necessary for the existence of a locally strictly stable equilibrium while the second item allows us to preserve this stability in the absence of firm 3. Together, Assumptions 1 and 3 imply that $\mathbf{J}_{\boldsymbol{\theta}}$ is negative definite. The second item in Assumption 3 is used to derive certain comparative static results which enable us to compare input joint ventures with horizontal mergers. As an alternative, we could instead rely on the more familiar condition that equal increases in θ_1 and θ_2 are less profitable for firm $i = 1, 2$ the higher its initial action: $\partial^2\pi_i/\partial\theta_i^2 + |\partial^2\pi_i/\partial\theta_i\partial\theta_j| < 0$.

Firms are assumed to play the following two-stage game. In the first stage, firms choose input prices. In the event that firms 1 and 2 are parties to an input joint venture, the joint venture chooses a price w that meets the approval of both owners. In the symmetric context discussed here, this is a price that maximizes each owner's total profit.[11] Note that at this stage, it may be assumed that any firm that is not party to a joint venture agreement (including a multi-product firm) employs marginal cost input pricing. This is because in the absence of capacity constraints, the optimal downstream price of a firm with complete ownership and control over its upstream production facility is invariant to the input price set by that facility (as long as inputs are not substitutable across firms). At stage two, after learning the input prices, all downstream firms compete against each other by choosing their actions θ_i.

The equilibrium concept used to solve the two-stage game is subgame perfect Nash equilibrium. When firms 1 and 2 form a joint venture, absent a market for inputs, the assumption that the outside firm learns the input price set by the joint venture may be too strong. In Section 5, we will explore the robustness of our main result in the imperfect information scenario where firm 3 does not learn the input price before stage two competition ensues.

[11] This is in contrast to upstream profit only, which the joint venture would maximize if the collaborators delegated the input pricing decision to it a la Bonanno and Vickers (1988). We assume this is not the case in our model.

3 Bertrand and Cournot equilibria

3.1 Downstream competition

We begin by analyzing the stage two equilibrium when firms 1 and 2 form a joint venture. Given an input price w, firms choose their actions to maximize profits. When firms compete in prices downstream, the profits of firm $i = 1, 2$ are:

$$\pi_i(\mathbf{p}) = (p_i - w)\, h_i(\mathbf{p}) + \frac{w-c}{2}\, [h_1(\mathbf{p}) + h_2(\mathbf{p})] \tag{1}$$

Observe that firm i derives profits from selling its output downstream as well as from its share of the joint venture (though we do not assume that $w \geq c$). Firm 3's profit equation is given by $\pi_3(\mathbf{p}) = (p_3 - c)h_3(\mathbf{p})$, where it becomes clear that the input price set by its upstream division falls out. Similarly, when firms compete in quantities downstream, the profits of firm $i = 1, 2$ are:

$$\pi_i(\mathbf{x}) = [f_i(\mathbf{x}) - w]\, x_i + \frac{w-c}{2}(x_1 + x_2) \tag{2}$$

while firm 3's profit becomes $\pi_3(\mathbf{x}) = [f_3(\mathbf{x}) - c]\, x_3$.

Let $\mathbf{g}_{\boldsymbol{\theta}} = (\partial\pi_1/\partial\theta_1,\, \partial\pi_2/\partial\theta_2,\, \partial\pi_3/\partial\theta_3)$. The first-order conditions to firms' profit maximization problems in, respectively, the Bertrand and Cournot competitive scenarios become:

$$\mathbf{g}_{\mathbf{p}}(\mathbf{p},\, w) = \begin{pmatrix} h_1 + (p_1 - w)\,\partial h_1/\partial p_1 + (w-c)\,(\partial h_1/\partial p_1 + \partial h_2/\partial p_1)/2 \\ h_2 + (p_2 - w)\,\partial h_2/\partial p_2 + (w-c)\,(\partial h_1/\partial p_2 + \partial h_2/\partial p_2)/2 \\ h_3 + (p_3 - c)\partial h_3/\partial p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{g}_{\mathbf{x}}(\mathbf{x},\, w) = \begin{pmatrix} (\partial f_1/\partial x_1)\, x_1 + f_1 - w + (w-c)/2 \\ (\partial f_2/\partial x_2)\, x_2 + f_2 - w + (w-c)/2 \\ (\partial f_3/\partial x_3)\, x_3 + f_3 - c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

From this point forward, we restrict w to an open, bounded set, W_p or W_x in $\mathbf{R}$, such that Assumptions 1, 2, and 3 apply to Bertrand or Cournot competition, respectively. Thus, firm reaction functions as specified by Expressions (3) or (4) lead to a strictly stable equilibrium in prices or quantities, respectively. For a given $w \in W_\theta$, we denote the equilibrium action of firm i as a function of w, $\theta_i(w)$. The following Lemma provides the comparative statics of firm actions with respect to w.

Lemma 1. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture. If Assumptions 1, 2, and 3 hold, then:*

1. *Under downstream Bertrand competition, equilibrium prices increase in w.*

2. *Under downstream Cournot competition, the equilibrium quantities of firms 1 and 2 decrease in w and the equilibrium quantity of firm 3 increases in w.*

Although the proof of this lemma is somewhat involved, the intuition is straightforward. Higher input prices make it more costly to produce downstream. When downstream competition is Bertrand, this causes the joint venture partners' equilibrium prices to rise and because prices are strategic complements, the outside firm responds in kind. When downstream competition is Cournot, this causes the joint venture partners' equilibrium quantities to fall while the outside firm, whose production costs are effectively unchanged, takes advantage of the opportunity by producing more.

Before we move on to analyze the first stage, it is worthwhile to set up the downstream game in the event of a horizontal merger between firms 1 and 2. The merged firm's Bertrand profit equation is $\pi_M(\mathbf{p}) = (p_1 - c)h_1(\mathbf{p}) + (p_2 - c)h_2(\mathbf{p})$ and its Cournot profit equation is $\pi_M(\mathbf{x}) = [f_1(\mathbf{x}) - c]\,x_1 + [f_2(\mathbf{x}) - c]\,x_2$. The profit functions for firm 3 remain the same as in the joint venture scenario.

Let $\mathbf{g}_{\boldsymbol{\theta}}^{\mathbf{M}}$ represent the vector of own partials of firm profits in the horizontal merger scenario. Then, the first-order conditions in, respectively, the Bertrand and Cournot competitive scenarios become:

$$\mathbf{g}_{\mathbf{p}}^{\mathbf{M}}(\mathbf{p},\, w) = \begin{pmatrix} h_1 + (p_1 - c)\partial h_1/\partial p_1 + (p_2 - c)\partial h_2/\partial p_1 \\ (p_1 - c)\partial h_1/\partial p_2 + h_2 + (p_2 - c)\partial h_2/\partial p_2 \\ h_3 + (p_3 - c)\partial h_3/\partial p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

$$\mathbf{g}_{\mathbf{x}}^{\mathbf{M}}(\mathbf{x},\, w) = \begin{pmatrix} (\partial f_1/\partial x_1)\, x_1 + f_1 - c + (\partial f_2/\partial x_1)\, x_2 \\ (\partial f_1/\partial x_2)\, x_1 + (\partial f_2/\partial x_2)\, x_2 + f_2 - c \\ (\partial f_3/\partial x_3)\, x_3 + f_3 - c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

Observe that the $\mathbf{g}_{\boldsymbol{\theta}}^{\mathbf{M}}$ are only artificially functions of w, which in this case could be interpreted as the input price paid by the downstream divisions of the horizontally merged firm. As is always the case for firm 3, the merged firm's input price(s) cancels out of the profit equation such that this scenario could properly be analyzed as a single-stage game with marginal cost input pricing. The two-stage setup preserves the timing of the game across the joint venture and horizontal merger scenarios.[12]

Assuming that $c \in W_\theta$, Assumptions 1, 2, and 3 apply, such that the firm reaction functions specified by Expressions (5) or (6) lead to a strictly stable equilibrium in prices or quantities, respectively. We denote the equilibrium action with regard to product i (where the merged firm controls products 1 and 2), θ_i^M.

[12]In principal, our two-stage setup in the horizontal merger and baseline scenarios result in a multiplicity of equilibria with respect to the input prices across which all firms are indifferent. As a tie-breaking rule, we assume marginal cost input pricing, which is implicit in the single-stage game.

3.2 Setting the input price

In the joint venture scenario, substituting $\mathbf{p}(w)$ into Equation (1) and $\mathbf{x}(w)$ into Equation (2) yields firm i's ($i = 1, 2$) stage one Bertrand and Cournot profit functions, denoted $\pi_i(\mathbf{p}(w))$ and $\pi_i(\mathbf{x}(w))$, respectively. Our symmetry assumptions imply that were the joint venture under the complete operational control of one of the firms (with profits split exactly as before), assuming price discrimination across downstream divisions is not allowed, that firm's profit function would be precisely the same as the profit function of its silent joint venture partner. Consequently, both firms would agree to the same input price, such that either first-order condition $\mathrm{d}\pi_i(\boldsymbol{\theta}(w))/\mathrm{d}w = 0$, $i = 1, 2$, suffices to determine the equilibrium input price, w^*.

Because our objective is to assess the competitive effects of a production joint venture relative to those of a horizontal merger, before examining the solution for w^*, it will aid the exposition to consider the joint venture's best response when firm 3 fixes its action to one which would prevail in the horizontal merger scenario. We define the equilibrium input price that prevails in this situation as $\bar{w}$.

Proposition 1. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture and suppose that firm 3's action is fixed at θ_3^M. Then, the equilibrium input price, $\bar{w}$, is such that $\theta_i(\bar{w}) = \theta_i^M$ for $i = 1, 2, 3$ and $\pi_1(\boldsymbol{\theta}(\bar{w})) + \pi_2(\boldsymbol{\theta}(\bar{w})) = \pi_M(\boldsymbol{\theta}^{\mathbf{M}})$.*

Proposition 1 states that when firm 3's action is fixed as if it were in the horizontal merger scenario, the joint venture optimally prices the input to replicate the horizontal merger outcome. As shown in the Appendix, mathematically this results because our symmetry assumptions imply that when $\mathrm{d}\theta_3(w)/\mathrm{d}w = 0$, the first order condition in the joint venture scenario boils down to the one following a horizontal merger. This is very similar to the main result obtained by Chen and Ross (2003), who show that an industry wide joint venture whose partners compete in prices downstream allows the partners to achieve the monopoly outcome by committing to an input price above marginal cost. As can be seen when substituting $\mathbf{p}(w)$ into Equation (1), the commitment is facilitated by each collaborator's ability to directly profit from increases in their joint venture partner's input prices, coupled with the need to pay a higher w. As Chen and Ross point out, when w is increased above c, the optimal prices charged by both firms rise. Even taking into account the fact that half the joint venture's profits will be returned to it, firm i still pays more for its inputs when w increases, and so it buys less. This is precisely the joint venture's "internal effect" discussed in Section 1. Note, however, that the monopoly outcome is not obtained in Proposition 1 because even though firm 3 does not behave "strategically," it remains outside of the horizontal agreement. As a result, there may be room for improvement.

When firm 3 acts like a standard oligopoly competitor, the input price paid by the joint venture partners (or in the case of imperfect information, firm 3's beliefs regarding that price) influences its action. Firms 1 and 2 recognize this effect and consider it when setting w. From Lemma 1 we know that firm 3's equilibrium action always rises in w. Because this increase affects the joint venture partners differently when actions are strategic complements from when they are strategic substitutes, for the remainder of this section it will aid clarity to consider downstream Bertrand and Cournot competition in turn.

Proposition 2. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture and firms compete in prices downstream. In equilibrium,* $w^* > \bar{w}$ *and* $p_i(w^*) > p_i^M$*,* $i = 1, 2, 3$*. Additionally,* $\pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*)) > \pi_M(\mathbf{p^M})$ *and* $\pi_3(\mathbf{p}(w^*)) > \pi_3(\mathbf{p^M})$*.*

From Proposition 1, we know that collaborators can exploit the commitment power inherent in a higher input price to achieve the same effect attained by complete consolidation of downstream pricing decisions. Moreover, partners to a joint venture also understand that the input price matters to an outside oligopolist as well, via its affect on joint venture partners' downstream prices. Because downstream prices are strategic complements, the joint venture partners realize that firm 3 responds to a higher input price (which leads the collaborators to set higher downstream prices) with a higher downstream price. Therefore, although setting the input price above one which causes the joint venture to replicate a horizontal merger would lead to an unprofitable decline in the quantities of products 1 and 2 demanded absent an outside oligopolist, the effect that an increase in firm 3's price has on the demand for products 1 and 2 makes an input price higher than $\bar{w}$ worthwhile. Unlike the joint venture, a horizontal merger cannot use the input price to facilitate collusion between the partner firms and firm 3 because as we have discussed following Equations (5) and (6), the merger's downstream prices are invariant to w.

An informative means to capture the impact that firm 3's reaction has on the equilibrium prices of joint venture partners 1 and 2 is by writing firm i's stage one first-order condition in terms of a price-cost margin. In particular, by employing our symmetry assumptions, we can write the first-order condition for firm $i \neq j = 1, 2$ as:

$$\frac{p_i(w^*) - c}{p_i(w^*)} = \frac{1}{\varepsilon_i|_{\mathbf{p}(w^*)} - \varepsilon_{ij}|_{\mathbf{p}(w^*)} - \varepsilon_{i3}|_{\mathbf{p}(w^*)} \dfrac{p_i(w^*)}{p_3(w^*)} \left[\dfrac{\mathrm{d}p_3(w)}{\mathrm{d}w} \Big/ \dfrac{\mathrm{d}p_i(w)}{\mathrm{d}w} \right] \Big|_{w^*}} \tag{7}$$

where firm i's own-price elasticity of demand is $\varepsilon_i = -\left(\partial h_i / \partial p_i\right)\left(p_i / h_i\right)$ and firm i's cross-price elasticity with respect to the price of firm $j \neq i$ is $\varepsilon_{ij} = \left(\partial h_i / \partial p_j\right)\left(p_j / h_i\right)$.

The horizontal merger counterpart to Equation (7) is $(p_i^M - c)/p_i^M = \left(\varepsilon_i|_{\mathbf{p^M}} - \varepsilon_{ij}|_{\mathbf{p^M}}\right)^{-1}$. It can now be observed that whereas the input price is present in Equation (7), it does not affect equilibrium downstream prices in the horizontal merger scenario. From Lemma 1, we know that $\mathrm{d}p_i/\mathrm{d}w > 0$ for all i and that consequently,

$$\frac{p_i^M - c}{p_i^M} < \frac{1}{\varepsilon_i|_{\mathbf{p}(\bar{w})} - \varepsilon_{ij}|_{\mathbf{p}(\bar{w})} - \varepsilon_{i3}|_{\mathbf{p}(\bar{w})} \dfrac{p_i(\bar{w})}{p_3(\bar{w})} \left[\dfrac{\mathrm{d}p_3(w)}{\mathrm{d}w} \Big/ \dfrac{\mathrm{d}p_i(w)}{\mathrm{d}w} \right] \Big|_{\bar{w}}} \tag{8}$$

where we use $p_i^M = p_i(\bar{w})$ on the right-hand side. The crux of the proof of Proposition 2 (see Appendix) is in showing that the right-hand side in Equation (7) is greater than the right-hand side in Inequality (8), such that the joint venture equilibrium price-cost margin is higher than that of a horizontal merger. The incremental difference in profits and prices from the horizontal merger scenario to the joint venture scenario represents the "external effect" of an input joint venture as referred to in Section 1. Equation (7) shows that the relative difference in the price-cost margins between the two scenarios increases when firms produce close substitutes (i.e., large ε_{i3}) and when firm 3's downstream price is more responsive to w relative to the downstream price of firm i.

Proposition 3. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture and firms compete in quantities downstream. In equilibrium,* $\bar{w} > w^*$ *and* $x_i(w^*) > x_i^M$*,* $i = 1, 2$ *whereas* $x_3^M > x_3(w^*)$*. Additionally,* $\pi_1(\mathbf{x}(w^*)) + \pi_2(\mathbf{x}(w^*)) > \pi_M(\mathbf{x}^{\mathbf{M}})$ *whereas* $\pi_3(\mathbf{x}^{\mathbf{M}}) > \pi_3(\mathbf{x}(w^*))$.

As is the case with Bertrand competition, the joint venture partners know that the input price affects firm 3's downstream action. However, because quantities are strategic substitutes, firm 3 responds to the higher quantities produced by the collaborators when the joint venture lowers the input price by lowering its own quantity. Thus, under Cournot competition, the joint venture profitably sets its input price below $\bar{w}$ in order to induce the outside oligopolist to produce less.

In contrast to the Bertrand outcome, where higher prices lead to diminished consumer and total welfare in the joint venture scenario relative to the horizontal merger, the welfare consequences are ambiguous under Cournot competition downstream. Because the joint venture increases its own output relative to the horizontal merger while decreasing that of the outside competitor, total welfare depends on the curvature of demand. In order that we may calculate explicit solutions and fully characterize our equilibria, we next present a fully specified model of competition using linear demand.

4 A linear example

Consider our general model with the following quadratic utility specification:

$$U(\mathbf{x}) = \alpha(x_1 + x_2 + x_3) - \kappa\left(x_1^2 + x_2^2 + x_3^2\right)/2 - \beta(x_1x_2 + x_1x_3 + x_3x_2) \tag{9}$$

where α, κ, and β are positive and $\kappa > \beta$. This utility function gives rise to a linear demand structure with the inverse demand for product i given by:

$$p_i = \alpha - \kappa x_i - \beta \sum_{j \neq i} x_j \tag{10}$$

in the region of X where prices are positive. Solving the system of 3 inverse demand equations for $i = 1, 2, 3$ yields the direct demand for product i in the region of P over which quantities are positive:

$$x_i = a - kp_i + b \sum_{j \neq i} p_j \tag{11}$$

where we write $\alpha = a/(k-2b)$, $\kappa = (k-b)/\left[(k+b)(k-2b)\right]$, and $\beta = b/\left[(k+b)(k-2b)\right]$, and where a, k, and b are positive and $k > 2b$. In addition to our utility assumptions, without loss of generality, suppose that the marginal cost c is zero.

Working backwards, given an input price $w_p \in W_p$ or $w_x \in W_x$, we can solve firms' first-order conditions under Bertrand (Expression (3)) or Cournot (Expression (4)) competition, respectively to yield firms' conditional equilibrium actions. Specifically, for $i = 1, 2$ these are

$$\begin{aligned} p_i(w_p) &= \frac{a}{2(k-b)} + \frac{k(k+b)w_p}{2(2k+b)(k-b)}\ , \\ p_3(w_p) &= \frac{a}{2(k-b)} + \frac{b(k+b)w_p}{2(2k+b)(k-b)} \end{aligned} \tag{12}$$

under Bertrand competition and

$$
\begin{aligned}
x_i(w_x) &= \frac{\alpha}{2(\kappa+\beta)} - \frac{\kappa w_x}{2(2\kappa-\beta)(\kappa+\beta)} \ , \\
x_3(w_x) &= \frac{\alpha}{2(\kappa+\beta)} + \frac{\beta w_x}{2(2\kappa-\beta)(\kappa+\beta)}
\end{aligned}
\tag{13}
$$

under Cournot competition. Because firms 1 and 2 can profit from their participation in the joint venture as well as from sales downstream whereas firm 3 only profits from the latter, $p_i(w_p) > p_3(w_p)$ for any $w_p > 0$ and conversely, $x_i(w_x) < x_3(w_x)$ for any $w_x > 0$.

We can now substitute $\mathbf{p}(w_p)$ into Equation (1) and $\mathbf{x}(w_x)$ into Equation (2) to solve for the equilibrium input prices:

$$
w_p^* = \frac{a(2k+b)b}{2(k^2-bk-b^2)k} \ , \qquad\qquad w_x^* = \frac{\alpha(\kappa-\beta)(2\kappa-\beta)\beta}{2(\kappa^2+\kappa\beta-\beta^2)}
\tag{14}
$$

which are both positive given our assumptions on utility. Substituting w_p^* and w_x^* into Equations (12) and (13), respectively, we can obtain the joint venture equilibrium prices, quantities, and profits under Bertrand and Cournot competition.

In Table 1, we compute prices, quantities, and profits for firms $i = 1, 2$ and 3 in the joint venture scenario and compare these to the corresponding variables had firms 1 and 2 merged instead when all firms compete in prices downstream. The superscript M_p represents the merger scenario with downstream Bertrand competition. It is now easily confirmed that under Bertrand competition downstream, the combined profits of the joint venture partners as well as the profits of firm 3 are higher than they would be in the case of a horizontal merger. Moreover, as stated in Proposition 2, every price is higher in the joint venture scenario. However, when demand is linear, it turns out that in spite of its higher price, firm 3 produces more in the event of a joint venture between firms 1 and 2 than in the case of a merger. The joint venture partners set the input price so far above the price that would replicate a horizontal merger that even though the ensuing equilibrium is more profitable to them than a horizontal merger (because of strategic complementarity),

Table 1: Bertrand Equilibrium: Joint Venture vs. Horizontal Merger

	Joint Venture	Horizontal Merger
Firm i	$p_i(w_p^*) = \dfrac{a(2k+b)}{4(k^2-kb-b^2)}$	$p_i^{M_p} = \dfrac{a(2k+b)}{2(2k^2-2kb-b^2)}$
	$x_i(w_p^*) = \dfrac{a(2k+b)}{4k}$	$x_i^{M_p} = \dfrac{a(2k+b)(k-b)}{2(2k^2-2kb-b^2)}$
	$\pi_i(\mathbf{p}(w_p^*)) = \dfrac{a^2(2k+b)^2}{16k(k^2-kb-b^2)}$	$\pi_M(\mathbf{p}^{\mathbf{M}_p}) = \dfrac{a^2(2k+b)^2(k-b)}{2(2k^2-2kb-b^2)^2}$
Firm 3	$p_3(w_p^*) = \dfrac{a(2k^2-b^2)}{4k(k^2-kb-b^2)}$	$p_3^{M_p} = \dfrac{ak}{2k^2-2kb-b^2}$
	$x_3(w_p^*) = \dfrac{a(2k^2-b^2)}{4(k^2-kb-b^2)}$	$x_3^{M_p} = \dfrac{ak^2}{2k^2-2kb-b^2}$
	$\pi_3(\mathbf{p}(w_p^*)) = \dfrac{a^2(2k^2-b^2)^2}{16k(k^2-kb-b^2)^2}$	$\pi_3(\mathbf{p}^{\mathbf{M}_p}) = \dfrac{a^2k^3}{(2k^2-2kb-b^2)^2}$

the downstream prices of firms 1 and 2 become sufficiently high to permit firm 3 to gain customers at its own higher price.

Higher prices together with symmetry culminate in lower consumer and total welfare in the joint venture scenario than in the horizontal merger scenario. This can be observed in our linear example by substituting the equilibrium quantities in Table 1 into Equation (9) and rewriting α, κ, and β in terms of a, k, and b. Total welfare is lower in the joint venture scenario than in the horizontal merger scenario if $U\left(\mathbf{x}(w_p^*)\right) < U\left(\mathbf{x}^{\mathbf{M}_p}\right)$. After some straightforward algebraic manipulation, this inequality reduces to:

$$\frac{-a^2b^2(2k+b)\left(16k^5-32k^4b-20k^3b^2+30k^2b^3+24kb^4+5b^5\right)}{32k\left(2k^2-2kb-b^2\right)^2\left(k^2-bk-b^2\right)^2} < 0 \tag{15}$$

Similarly, consumer welfare is lower in the joint venture scenario than in the horizontal merger scenario if $U\left(\mathbf{x}(w_p^*)\right) - \mathbf{p}(w_p^*)\cdot\mathbf{x}(w_p^*) < U\left(\mathbf{x}^{\mathbf{M}_p}\right) - \mathbf{p}^{\mathbf{M}_p}\cdot\mathbf{x}^{\mathbf{M}_p}$, which may be rewritten as:

$$\frac{-a^2b^2(2k+b)\left(16k^5-16k^4b-28k^3b^2+10k^2b^3+16kb^4+3b^5\right)}{32k\left(2k^2-2kb-b^2\right)^2\left(k^2-bk-b^2\right)^2}<0 \tag{16}$$

Without loss of generality, we may normalize k to 1 in Inequalities (15) and (16) to see that under our assumptions (in particular, $b < k/2$), total and consumer welfare decline when firms 1 and 2 form a joint venture instead of merging horizontally and firms compete in prices downstream.

Table 2: Cournot Equilibrium: Joint Venture vs. Horizontal Merger

	Joint Venture	Horizontal Merger
Firm i	$p_i(w_x^*) = \dfrac{\alpha(2\kappa-\beta)}{4\kappa}$	$p_i^{M_x} = \dfrac{\alpha(2\kappa-\beta)(\kappa+\beta)}{2(2\kappa^2+2\kappa\beta-\beta^2)}$
	$x_i(w_x^*) = \dfrac{\alpha(2\kappa-\beta)}{4(\kappa^2+\kappa\beta-\beta^2)}$	$x_i^{M_x} = \dfrac{\alpha(2\kappa-\beta)}{2(2\kappa^2+2\kappa\beta-\beta^2)}$
	$\pi_i(\mathbf{x}(w_x^*)) = \dfrac{\alpha^2(2\kappa-\beta)^2}{16\kappa(\kappa^2+\kappa\beta-\beta^2)}$	$\pi_M(\mathbf{x}^{\mathbf{M}_x}) = \dfrac{\alpha^2(2\kappa-\beta)^2(\kappa+\beta)}{2(2\kappa^2+2\kappa\beta-\beta^2)^2}$
Firm 3	$p_3(w_x^*) = \dfrac{\alpha(2\kappa^2-\beta^2)}{4(\kappa^2+\kappa\beta-\beta^2)}$	$p_3^{M_x} = \dfrac{\alpha\kappa^2}{2\kappa^2+2\kappa\beta-\beta^2}$
	$x_3(w_x^*) = \dfrac{\alpha(2\kappa^2-\beta^2)}{4k(\kappa^2+\kappa\beta-\beta^2)}$	$x_3^{M_x} = \dfrac{\alpha\kappa}{2\kappa^2+2\kappa\beta-\beta^2}$
	$\pi_3(\mathbf{x}(w_x^*)) = \dfrac{\alpha^2(2\kappa^2-\beta^2)^2}{16\kappa(\kappa^2+\kappa\beta-\beta^2)^2}$	$\pi_3(\mathbf{x}^{\mathbf{M}_x}) = \dfrac{\alpha^2\kappa^3}{(2\kappa^2+2\kappa\beta-\beta^2)^2}$

Table 2 presents the analogous price, quantity, and profit comparison to Table 1 in the event of quantity competition downstream. The superscript M_x represents the merger scenario with downstream Cournot competition. As was the case under Bertrand competition, the profit comparison shows that the joint venture partners are better off than they would be had they merged, whereas firm 3 is now worse off. Firms 1 and 2 sell more than they would had they merged at a lower downstream price. Firm 3 sells less (because of strategic substitutability), but also at a lower price. The input price set by the joint venture raises the quantities sold by firms 1 and 2 to such an extent that firm 3's quantity decline is insufficient to offset the price decline precipitated by its competitors.

In contrast to the Bertrand case, the joint venture scenario results in higher consumer and total welfare than the horizontal merger scenario. Total welfare is higher in the joint venture scenario than in the horizontal merger scenario if the following inequality holds:

$$\frac{\alpha^2\beta^2(2\kappa-\beta)\left(16\kappa^5+16\kappa^4\beta-28\kappa^3\beta^2-10\beta^3\kappa^2+16\kappa\beta^4-3\beta^5\right)}{32\kappa\left(\kappa^2+\kappa\beta-\beta^2\right)^2\left(2\kappa^2+2\beta\kappa-\beta^2\right)^2}>0 \tag{17}$$

Likewise, consumer welfare is higher in the joint venture scenario than in the horizontal merger scenario if:

$$\frac{\alpha^2\beta^2(2\kappa-\beta)\left(16\kappa^5+32\kappa^4\beta-20\kappa^3\beta^2-30\beta^3\kappa^2+24\kappa\beta^4-5\beta^5\right)}{32\kappa\left(\kappa^2+\kappa\beta-\beta^2\right)^2\left(2\kappa^2+2\beta\kappa-\beta^2\right)^2}>0 \tag{18}$$

Without loss of generality, we may normalize κ to 1 in Inequalities (17) and (18) to see that total and consumer welfare increase when firms 1 and 2 form a joint venture instead of merging horizontally and firms compete in quantities downstream.

The next proposition summarizes the results of this section up to this point.

Proposition 4. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture and that firms face linear demand. In equilibrium, the combined profits of firms 1 and 2 are higher than the profits of a horizontal merger between firms 1 and 2. Additionally:*

1. *Under downstream Bertrand competition, the equilibrium profit and quantity of firm 3 and all prices are higher than in the horizontal merger scenario. The quantities of products 1 and 2 and total and consumer welfare are lower.*

2. *Under downstream Cournot competition, the equilibrium quantities of firms 1 and 2 and total and consumer welfare are higher than in the horizontal merger scenario. All prices, as well as the profit and quantity of firm 3 are lower.*

A well known result in oligopoly theory is that for a given utility representation conforming to certain regularity conditions, Cournot competition among sellers of imperfect substitutes leads to higher firm profits and prices than differentiated Bertrand competition (e.g., Singh and Vives, 1984). This occurs because Cournot competitors perceive demand to be less elastic than Bertrand competitors given the actions of their rival firms. The result is not robust across all competitive scenarios (for instance, Arya et al., 2008, show that the results may be reversed when the production of key inputs is outsourced to a vertically integrated retail competitor) and in light of our findings in Propositions 4, one might expect the pricing result to fail in the event of the joint venture scenario analyzed in this manuscript. When demand is linear, this turns out not to be the case.

Table 3: Joint Venture: Bertrand Equilibrium vs. Cournot Equilibrium

Bertrand Equilibrium	Cournot Equilibrium
$p_i(w_p^*) = \dfrac{a(2k+b)}{4(k^2-kb-b^2)}$	$p_i(w_x^*) = \dfrac{a(2k-3b)}{4\,(k-b)\,(k-2b)}$
$x_i(w_p^*) = \dfrac{a(2k+b)}{4k}$	$x_i(w_x^*) = \dfrac{a\,(2k-3b)\,(k+b)}{4\,(k^2-kb-b^2)}$
$\pi_i(\mathbf{p}(w_p^*)) = \dfrac{a^2(2k+b)^2}{16k(k^2-kb-b^2)}$	$\pi_i(\mathbf{x}(w_x^*)) = \dfrac{a^2\,(2k-3b)^2\,(k+b)}{16\,(k-b)\,(k-2b)\,(k^2-kb-b^2)}$
$p_3(w_p^*) = \dfrac{a(2k^2-b^2)}{4k(k^2-kb-b^2)}$	$p_3(w_x^*) = \dfrac{a(2k^2-4kb+b^2)}{4\,(k-2b)\,(k^2-kb-b^2)}$
$x_3(w_p^*) = \dfrac{a(2k^2-b^2)}{4(k^2-kb-b^2)}$	$x_3(w_x^*) = \dfrac{a\,(2k^2-4kb+b^2)\,(k+b)}{4\,(k-b)\,(k^2-kb-b^2)}$
$\pi_3(\mathbf{p}(w_p^*)) = \dfrac{a^2(2k^2-b^2)^2}{16k(k^2-kb-b^2)^2}$	$\pi_3(\mathbf{x}(w_x^*)) = \dfrac{a^2\,(2k^2-4kb+b^2)^2\,(k+b)}{16\,(k-2b)\,(k-b)\,(k^2-kb-b^2)^2}$

In Table 3 we compare the joint venture column from Bertrand Table 1 with the joint venture column from Cournot Table 2 rewritten in terms of a, k, and b. Comparing profits and prices across Bertrand and Cournot joint venture scenarios, it becomes apparent that the result of Singh and Vives (1984) persists (for all firms). Conversely, contrary to the linear result of Singh and Vives, the quantities produced by the joint venture partners are greater under Cournot competition. The next proposition summarizes our findings.

Proposition 5. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture and that firms face linear demand. Then in equilibrium, all firm profits and prices are greater under Cournot competition than under Bertrand competition. The quantities produced by firms 1 and 2 are greater under Cournot competition and the quantity of firm 3 is lower.*

5 Imperfect information

Unless the joint venture announces the price of its input—as it might if it were to actively sell the input to its competitors—the assumption that the outside firm learns the input price set by the joint venture prior to downstream competition may be unreasonable. The benefit of this assumption was that it allowed us to refine the equilibrium of our game using subgame perfection. Without this assumption, the game can no longer be characterized as a continuous game of almost perfect information (as defined by Harris et al., 1995), and every Nash equilibrium turns out to be subgame perfect. As such, certain equilibria which are undesirable to the the joint venture partners (because they are Pareto dominated by the equilibrium that prevails in the game of almost perfect information) are nevertheless subgame perfect.

For example, consider the Nash equilibrium where the joint venture sets the input price to c and each firm plays $p_i(c)$, $i = 1, 2, 3$, for every value of w set in the first stage. This equilibrium leads to the outcome that prevails in the absence of any collaboration—

that is, the standard differentiated Bertrand outcome in a game with three vertically integrated firms. The equilibrium is indeed Nash—when firms play $p_i(c)$ regardless of w, the joint venture has no alternative better than to play c, and by definition $p_i(c)$ is a best downstream response to c. However, in the game of almost perfect information, this equilbrium is not subgame perfect because $p_i(c)$ is not a best response to any subgame off the equilibrium path. In a game where the outside firm does not learn the joint venture input price, henceforth referred to as a game of imperfect information, we would like to be able to rule out such "undesirable" equilibria without needing to appeal to Pareto dominance.

Although further refinement is one sensible approach to dealing with imperfect information, a rigorous treatment of equilibrium refinement in continuous action games such as this one is beyond the scope of this manuscript.[13] Instead, we consider a reasonable, simple extension of the two stage game to rule out equilibria where the joint venture partners would be worse off than they would be were they to merge horizontally instead. Because we are primarily concerned with collaborations which are more profitable than horizontal mergers, but also socially, less desirable, for the remainder of this manuscript we will focus on downstream competition in prices. In particular, let us modify the general setup from Section 2 in two ways, (i) by assuming imperfect information regarding the input price and (ii) by adding a preliminary stage in which firm 1 determines whether to collaborate in a joint venture with firm 2 or to consolidate in a horizontal merger with firm 2 and evenly split the resulting profit. Suppose that all firms know firm 1's stage one decision prior to stage two, at which point they proceed with either the imperfect information variant of the original joint venture or horizontal merger game. This extension endogenizes the

[13]In the Appendix, we loosely outline a set of strategies and beliefs that lead to a sequential equilibrium that induces the outcome that prevailed in the game of almost perfect information, but we do not attempt to determine whether this sequential equilibrium is unique.

collaborative decision made by firms 1 and 2. Because of our assumptions on firms and consumers, the extension turns out to be without loss of generality with regard to whether firm 1 or 2 determines the method of cooperation and to a variety of decisions made by the initial decision making firm's partner. For instance, the result that follows is substantially unaltered by giving firm 2 the option to publicly reject firm 1's decision.

Proposition 6. *Suppose that firms compete in prices downstream. In a pure strategy equilibrium of the extended game of imperfect information, whenever firms 1 and 2 collaborate in a symmetric input joint venture on the equilibrium path,* $w^* \geq \bar{w}$ *and* $p_i(w^*) \geq p_i^M$*,* $i = 1, 2, 3$*. Additionally,* $\pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*)) \geq \pi_M(\mathbf{p^M})$ *and* $\pi_3(\mathbf{p}(w^*)) \geq \pi_3(\mathbf{p^M})$*.*

Even though the extended game is one of imperfect information regarding the input price, the extension allows us to rely on subgame perfection to compare a joint venture with a horizontal merger. The result is a weakening of Proposition 2, in which the horizontal merger nonetheless, never outperforms the joint venture in instances where a joint venture is formed.

6 A simulation of the U.S. wireless industry

In order to make our general model mathematically tractable, we previously imposed a number of simplifying assumptions. In particular, we supposed an ex-ante symmetric oligopoly with three firms. We further assumed that ex-post, neither a joint venture, nor a horizontal merger leads to any efficiency gains. In order to show that our methodology has practical application beyond the three firm symmetric case, we simulate here a counterfactual 50-50 input joint venture between AT&T and Verizon Wireless, the two largest competitors in the U.S. mobile wireless communications industry. We compare our results to an alternative counterfactual horizontal merger that leads to the same level of efficiencies to show that the input joint venture may lead to an anti-competitive outcome, even when the horizontal merger does not.

Providers of mobile wireless services offer an array of mobile voice and data services, including interconnected mobile voice services, text and multimedia messaging, and mobile broadband Internet access services.[14] As of year-end 2011, there were four facilities-based mobile wireless service providers in the United States that industry observers typically described as "nationwide", four multi-regional and multi-metro service providers, and dozens of regional and local providers.[15] Service providers rely on inputs such as spectrum, towers, network equipment, and backhaul facilities to transmit voice and/or data via mobile devices to consumers.

In order to simulate counterfactual scenarios for this industry, we first calibrate demand using the simple approach applied by the Federal Communications Commission in its staff analysis of AT&T's unsuccessful attempt to acquire rival service provider, T-Mobile.[16] We then use the calibrated demand parameters to simulate a counterfactual horizontal merger and alternative input joint venture between AT&T and Verizon Wireless. The joint venture counterfactual supposes that the two service providers combine their spectrum and network inputs, but continue to compete separately in all their downstream segments.[17] For simplicity, we suppose that all variable costs are borne by joint venture partners' upstream segment. As we note below, the relaxation of this assumption would

[14]See the Federal Communications Commission's Sixteenth Annual Mobile Wireless Competition Report ("Sixteenth Report"), ¶ 19. Mobile wireless services also include machine-to-machine connections for fleet management systems, smart grid devices, vehicle tracking, home security systems, and other telematics services.

[15]See Sixteenth Report, ¶¶ 26-28 and Tables 11-13. As of March 2014, the only multi-regional provider that had not been acquired by a nationwide provider was US Cellular. See Baker et al. (2014).

[16]See Federal Communications Commission Staff Analysis and Findings (2011). AT&T formally ended its bid to acquire T-Mobile in December 2011, following findings by the Department of Justice and Federal Communications Commission that the merger would likely result in significant harms to competition.

[17]The Commission approved a similar, albeit asymmetric joint venture between two Alaskan service providers in 2013. See Baker et al. (2014).

only strengthen the implications of our simulation.

The FCC's calibration assumes Bertrand differentiated products competition where each of five firms facing linear demand is assumed to produce a single good in each period at constant marginal cost.[18] The five firms consist of the four nationwide service providers—AT&T, Sprint, T-Mobile, and Verizon Wireless–along with a firm composed of all other firms and denoted as "Other".[19] In order to calibrate the demand parameters we require data on firm prices, quantities of output and price-cost margins. Furthermore, to calculate cross-price elasticities, we either need data on customer diversion (see Werden, 1996) or a suitable proxy (see Farrell and Shapiro, 2010). Our data was primarily obtained from service provider SEC filings and is described in more detail in the Appendix.[20] We can solve for b_{ij}, the slope parameter on the price of good j in the demand equation of firm i using the relationship $b_{ij} = \varepsilon_{ij} x_i / p_j$ and the first order condition for each service provider's profit maximization. The intercepts then obtain directly from providers' demand equations.

After calibrating demand parameters we simulate a horizontal merger or joint venture between AT&T and Verizon Wireless using essentially the approach applied in Section 4, but with the demand and marginal cost symmetry assumptions relaxed and with three outside oligopoly competitors in place of one. Table 4 displays the percent price changes following a number of collaboration/consolidation scenarios as a proportion of the

[18]Competition among various input segments (e.g., among owners of towers or backhaul) remains unmodeled, as is downstream market segmentation (e.g., between retail subscribers and business subscribers). Similarly, we treat AT&T's and Verizon's wireline segments as independent from wireless and do not consider them here. Moreover, our data is insufficient for us to consider regional or local variation in competition.

[19]We believe that the composite of all other firms to be more appropriate than the independent treatment of non-nationwide providers. Mobile wireless consumers search for providers in local areas where they live, work and travel. The total number of providers in the United States far exceeds the number of providers that compete in any single local area and most non-nationwide providers do not compete with each other in the majority of local geographic markets. For instance, as of December 2011, only 19 percent of Cellular Market Areas in the United States contained five or more providers with at least five percent market share. See Sixteenth Report, ¶ 58 and Table 10.

[20]We corroborate and supplement some of the data using the Sixteenth Report.

Table 4: Post-Transaction Price Changes[1,2]

Transaction	MC[3]	AT&T	VZW	Sprint	TM	Other	Profits
Merger	$\times 1$	23.3%	23.4%	7.1%	11.2%	10.2%	33.5%
JV	$\times 1$	46.4%	47.0%	14.1%	22.4%	20.4%	60.9%
Merger	$\times 2/5$	-0.2%	0.2%	0.0%	0.0%	0.0%	96.8%
JV	$\times 2/5$	28.0%	28.8%	8.6%	13.6%	12.4%	135.5%
Merger	$\times 0$	-15.9%	-15.2%	-4.7%	-7.4%	-6.8%	145.8%
JV	$\times 0$	15.7%	16.6%	4.9%	7.8%	7.1%	185.5%

[1]Percentages represent price or profit changes in proportion to pre-transaction prices or profits, respectively.

[2]Verizon Wireless (VZW); T-Mobile (TM).

[3]Marginal Cost (MC) adjustments are applied only to AT&T and Verizon Wireless.

pre-transaction prices. It also displays the percent joint profit changes for the collaborating/consolidating firms. Not surprisingly, in our model, the merger of two service providers that together comprise almost two-thirds of the market leads to significant price increases absent any reductions in marginal costs. For instance, as shown in Table 4, absent efficiencies, the merger simulation predicts price increases of approximately 23 percent for both AT&T and Verizon Wireless. What is striking is the sizable increase in prices following the joint venture: approximately 46 and 47 percent for AT&T and Verizon Wireless, respectively. Moreover, even if we were to reduce the collaborator partners' marginal costs to approximately two-fifths of their original level, such that a horizontal merger would not entail an increase in prices, a joint venture would nevertheless lead to significant price gains for all firms. In fact, as seen in the last row in Table 4, price gains persist even when we reduce the marginal costs of AT&T and Verizon Wireless to zero, in which case a merger would lower prices. This finding is particularly stark because we have not assumed any downstream marginal costs for the joint venture partners. The relaxation of this assumption could lead to an even greater difference between the competitive effects following a horizontal merger and those following a joint venture.

7 Conclusion

We have shown that, contrary to conventional wisdom, production joint ventures that preserve some competition between collaborators may nevertheless be more profitable than a complete consolidation of all decision making authority. This is achieved through the joint venture's ability to commit to an input price that not only influences the downstream actions of the collaborators, but also those of outside oligopolists. As is true with many other phenomena in oligopoly theory, the relative welfare consequences depend on whether downstream actions are strategic complements or strategic substitutes. This suggests that antitrust agencies concerned about a joint venture need to take into account their beliefs about the type of competition that occurs downstream. The agencies should be particularly attentive if they believe that downstream competition is characterized by price-setting for imperfect substitute products because in this case consumer welfare can decline more than if the competitors merged.

In our analysis, we largely abstracted from an investigation of the potential efficiency-enhancing effects of consolidation or collaboration. If we believe that a horizontal merger is able to better integrate its costly processes than a production joint venture, then the collaboration's profitability advantage diminishes, but its potentially detrimental welfare impact rises relative to that of a merger. On the other hand, if the lack of integration is a sign that the joint venture has a termination point in the foreseeable future, it should be considered less of a concern than a horizontal merger.

Our comparison of input production joint ventures with horizontal mergers is by no means exhaustive. For instance, throughout we have supposed that inputs are purchased from the joint venture if and only if a firm is a party to the joint venture. We believe that this is a reasonable assumption that holds under many circumstances, even when inputs are effectively homogeneous—such as in our automobile assembly and wireless network sharing examples. Nonetheless, suppose to the contrary that outside firm 3 were to stand ready to supply the input to firms 1 and 2 at some input price w_3 below $w > 0$. For w_3 sufficiently low, we would expect that a joint venture partner firm might wish to cheat on its arrangement by purchasing from firm 3 and it is not immediately clear whether an equilibrium would entail firm 3 supplying all inputs or whether firms 1 and 2 would play mixed strategies in input purchasing. For higher w_3, there may be no incentive to buy from firm 3. In this setup, pinning down which combination(s) of w and w_3 are subgame perfect appears to us a difficult exercise, though one of potential interest to antitrust practitioners contemplating conditions on joint venture contracts which could hinder collusion.

We have also generally assumed that collaborators are identical and face symmetric demands. These assumptions allowed us to rely on an equal partnership to avoid potential input pricing disagreements by the collaborators. Although 50-50 production joint venture partnerships are a popular form of collaboration, in the real world, we also observe unequal partnerships. In separate, ongoing research, we have found that asymmetries in the joint venture structure (which may stem from underlying differences in the firms) can have important implications to the profitability and welfare consequences of collaboration, suggesting that an investigation of the ownership arrangement should be a critical part of any antitrust analysis.

Appendix

Proof of Lemma 1.

Proof. Let $\mathbf{p}^*$ and $\mathbf{x}^*$, represent the values of $\mathbf{p}$ and $\mathbf{x}$ such that $\mathbf{g_p}(\mathbf{p}^*, w) = 0$ and $\mathbf{g_x}(\mathbf{x}^*, w) = 0$. Note that $\mathbf{g_p}$ and $\mathbf{g_x}$ map from, respectively, int $P \times W_p$ and int $X \times W_x$

into $\mathbf{R}^3_+$. Additionally, define $D_w \mathbf{g}_{\boldsymbol{\theta}}$ as the column vector of own partials differentiated with respect to w. That is,

$$D_w \mathbf{g}_{\boldsymbol{\theta}} = \begin{pmatrix} \frac{\partial^2 \pi_1}{\partial \theta_1 \partial w} & \frac{\partial^2 \pi_2}{\partial \theta_2 \partial w} & \frac{\partial^2 \pi_3}{\partial \theta_3 \partial w} \end{pmatrix}^{\mathsf{T}}$$

From our assumptions on utility along with Assumption 3, we know that we can apply the implicit function theorem to obtain the derivative of firm actions with respect to w. In particular, $\boldsymbol{\theta}^* = \boldsymbol{\theta}(w)$ and $\boldsymbol{\theta}'(w) = -\left(\mathbf{J}_{\boldsymbol{\theta}}\right)^{-1} D_w \mathbf{g}_{\boldsymbol{\theta}}$. Observe that $\left(\mathbf{J}_{\boldsymbol{\theta}}\right)^{-1} = \left(\mathbf{C}_{\boldsymbol{\theta}}\right)^{\mathsf{T}} / \left|\mathbf{J}_{\boldsymbol{\theta}}\right|$ where $\mathbf{C}_{\boldsymbol{\theta}}$ is the following cofactor matrix:

$$\begin{pmatrix} \frac{\partial^2 \pi_2}{\partial \theta_2^2}\frac{\partial^2 \pi_3}{\partial \theta_3^2} - \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_3}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_2} & \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_3}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} - \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_1}\frac{\partial^2 \pi_3}{\partial \theta_3^2} & \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_1}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_2} - \frac{\partial^2 \pi_2}{\partial \theta_2^2}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} \\ \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_2} - \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\frac{\partial^2 \pi_3}{\partial \theta_3^2} & \frac{\partial^2 \pi_1}{\partial \theta_1^2}\frac{\partial^2 \pi_3}{\partial \theta_3^2} - \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} & \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} - \frac{\partial^2 \pi_1}{\partial \theta_1^2}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_2} \\ \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_3} - \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\frac{\partial^2 \pi_2}{\partial \theta_2^2} & \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_1} - \frac{\partial^2 \pi_1}{\partial \theta_1^2}\frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_3} & \frac{\partial^2 \pi_1}{\partial \theta_1^2}\frac{\partial^2 \pi_2}{\partial \theta_2^2} - \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_1} \end{pmatrix}$$

Our symmetry assumptions on utility, marginal costs of production, and the division of joint venture profits imply that $\theta_1^* = \theta_2^*$ along with the following equilibrium relationships on demand and inverse demand:

$$\frac{\partial h_1}{\partial p_1} = \frac{\partial h_2}{\partial p_2}, \quad \frac{\partial h_1}{\partial p_2} = \frac{\partial h_2}{\partial p_1}, \quad \frac{\partial h_1}{\partial p_3} = \frac{\partial h_2}{\partial p_3}, \quad \frac{\partial h_3}{\partial p_1} = \frac{\partial h_3}{\partial p_2}$$

$$\frac{\partial f_1}{\partial x_1} = \frac{\partial f_2}{\partial x_2}, \quad \frac{\partial f_1}{\partial x_2} = \frac{\partial f_2}{\partial x_1}, \quad \frac{\partial f_1}{\partial x_3} = \frac{\partial f_2}{\partial x_3}, \quad \frac{\partial f_3}{\partial x_1} = \frac{\partial f_3}{\partial x_2}$$

$$\frac{\partial^2 h_1}{\partial p_1^2} = \frac{\partial^2 h_2}{\partial p_2^2}, \quad \frac{\partial^2 h_1}{\partial p_2^2} = \frac{\partial^2 h_2}{\partial p_1^2}, \quad \frac{\partial^2 f_1}{\partial x_1^2} = \frac{\partial^2 f_2}{\partial x_2^2}, \quad \frac{\partial^2 f_1}{\partial x_2^2} = \frac{\partial^2 f_2}{\partial x_1^2}$$

$$\frac{\partial^2 h_1}{\partial p_1 \partial p_2} = \frac{\partial^2 h_2}{\partial p_1 \partial p_2}, \quad \frac{\partial^2 h_1}{\partial p_1 \partial p_3} = \frac{\partial^2 h_2}{\partial p_2 \partial p_3}, \quad \frac{\partial^2 h_1}{\partial p_2 \partial p_3} = \frac{\partial^2 h_2}{\partial p_1 \partial p_3}, \quad \frac{\partial^2 h_3}{\partial p_3 \partial p_1} = \frac{\partial^2 h_3}{\partial p_3 \partial p_2}$$

$$\frac{\partial^2 f_1}{\partial x_1 \partial x_2} = \frac{\partial^2 f_2}{\partial x_1 \partial x_2}, \quad \frac{\partial^2 f_1}{\partial x_1 \partial x_3} = \frac{\partial^2 f_2}{\partial x_2 \partial x_3}, \quad \frac{\partial^2 f_1}{\partial x_2 \partial x_3} = \frac{\partial^2 f_2}{\partial x_1 \partial x_3}, \quad \frac{\partial^2 f_3}{\partial x_3 \partial x_1} = \frac{\partial^2 f_3}{\partial x_3 \partial x_2}$$

Our symmetry assumptions also imply the following profit relationships:

$$\frac{\partial^2\pi_1}{\partial\theta_1^2} = \frac{\partial^2\pi_2}{\partial\theta_2^2}, \quad \frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2} = \frac{\partial^2\pi_2}{\partial\theta_2\partial\theta_1}, \quad \frac{\partial^2\pi_1}{\partial\theta_1\partial w} = \frac{\partial^2\pi_2}{\partial\theta_2\partial w}$$
$$\frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_3} = \frac{\partial^2\pi_2}{\partial\theta_2\partial\theta_3}, \quad \frac{\partial^2\pi_3}{\partial\theta_3\partial\theta_1} = \frac{\partial^2\pi_3}{\partial\theta_3\partial\theta_2}$$

Applying the profit relationships above to $\mathbf{J}_{\boldsymbol{\theta}}$ and $\mathbf{C}_{\boldsymbol{\theta}}$, $\boldsymbol{\theta}'(w)$ reduces to the following:

$$\boldsymbol{\theta}'(w) = \begin{pmatrix} \frac{\partial^2\pi_1}{\partial\theta_1\partial w}\frac{\partial^2\pi_3}{\partial\theta_3^2} \Big/ \left[2\frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_3}\frac{\partial^2\pi_3}{\partial\theta_3\partial\theta_1} - \frac{\partial^2\pi_3}{\partial\theta_3^2}\left(\frac{\partial^2\pi_1}{\partial\theta_1^2} + \frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2}\right)\right] \\ \frac{\partial^2\pi_1}{\partial\theta_1\partial w}\frac{\partial^2\pi_3}{\partial\theta_3^2} \Big/ \left[2\frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_3}\frac{\partial^2\pi_3}{\partial\theta_3\partial\theta_1} - \frac{\partial^2\pi_3}{\partial\theta_3^2}\left(\frac{\partial^2\pi_1}{\partial\theta_1^2} + \frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2}\right)\right] \\ 2\frac{\partial^2\pi_1}{\partial\theta_1\partial w}\frac{\partial^2\pi_3}{\partial\theta_3\partial\theta_1} \Big/ \left[\frac{\partial^2\pi_3}{\partial\theta_3^2}\left(\frac{\partial^2\pi_1}{\partial\theta_1^2} + \frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2}\right) - 2\frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_3}\frac{\partial^2\pi_3}{\partial\theta_3\partial\theta_1}\right] \end{pmatrix}$$

while $|\mathbf{J}_{\boldsymbol{\theta}}|$ reduces to:

$$|\mathbf{J}_{\boldsymbol{\theta}}| = \left[\frac{\partial^2\pi_3}{\partial\theta_3^2}\left(\frac{\partial^2\pi_1}{\partial\theta_1^2} + \frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2}\right) - 2\frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_3}\frac{\partial^2\pi_3}{\partial\theta_3\partial\theta_1}\right]\left(\frac{\partial^2\pi_1}{\partial\theta_1^2} - \frac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2}\right) \tag{19}$$

Bertrand: The expression for $\partial^2\pi_1/\partial\theta_1\partial w$ reduces to:

$$\frac{\partial^2\pi_1}{\partial p_1\partial w} = \frac{1}{2}\left(\frac{\partial h_2}{\partial p_1} - \frac{\partial h_1}{\partial p_1}\right),$$

which is clearly positive on P. As a result, from Assumption 1 we know that the numerator in $p_1'(w) = p_2'(w)$ is negative whereas from Assumption 2 for Bertrand competition (strategic complementarity), we know the numerator in $p_3'(w)$ is positive. Moreover, Assumptions 1 and 2 imply that the rightmost parenthetical expression on the right-hand side of Equation (19) is negative so that by Assumption 3, the denominator in $p_1'(w) = p_2'(w)$ is negative and the denominator in $p_3'(w)$ is positive.

Cournot: The expression for $\partial^2\pi_1/\partial\theta_1\partial w$ now becomes simply $\partial^2\pi_1/\partial x_1\partial w = -(1/2)$. Therefore, from Assumption 1 and Assumption 2 for Cournot competition (strategic substitutability), we know that all the numerators in $\mathbf{x}'(w)$ are positive. Applying our symmetric profit relationships, we can rewrite the inequality found in the second item of

Assumption 3 as:

$$\left(\frac{\partial^2 \pi_1}{\partial x_1^2}+\frac{\partial^2 \pi_1}{\partial x_1 \partial x_2}\right)\left(\frac{\partial^2 \pi_1}{\partial x_1^2}-\frac{\partial^2 \pi_1}{\partial x_1 \partial x_2}\right)>0 \tag{20}$$

Assumptions 1 and 2 imply that the leftmost parenthetical expression on the left-hand side of Inequality (20) is negative, which implies the same for the remaining parenthetical expression in the inequality. Observe that the latter parenthetical expression is the Cournot variant of the rightmost parenthetical expression on the right-hand side of Equation (19), so that according to the first item of Assumption 3, the denominator in $x_1'(w) = x_2'(w)$ is negative and the denominator in $x_3'(w)$ is positive. □

Proof of Proposition 1.

Proof. We approach the proofs for the Bertrand and Cournot scenarios in turn:

Bertrand: When firm 3's price is constant at p_3^M, firm i's, $i \neq j = 1, 2$, first-order condition becomes:

$$\begin{aligned}\frac{\mathrm{d}\pi_i(\mathbf{p}(w))}{\mathrm{d}w} &= \frac{\mathrm{d}p_i}{\mathrm{d}w}h_i + \frac{1}{2}(h_j - h_i) + (p_i - w)\left(\frac{\partial h_i}{\partial p_i}\frac{\mathrm{d}p_i}{\mathrm{d}w} + \frac{\partial h_i}{\partial p_j}\frac{\mathrm{d}p_j}{\mathrm{d}w}\right) \\ &+ \frac{w-c}{2}\left[\left(\frac{\partial h_i}{\partial p_i}+\frac{\partial h_j}{\partial p_i}\right)\frac{\mathrm{d}p_i}{\mathrm{d}w} + \left(\frac{\partial h_i}{\partial p_j}+\frac{\partial h_j}{\partial p_j}\right)\frac{\mathrm{d}p_j}{\mathrm{d}w}\right] = 0\end{aligned} \tag{21}$$

Symmetry implies that in equilibrium, $h_1 = h_2$, $\mathrm{d}p_1/\mathrm{d}w = \mathrm{d}p_2/\mathrm{d}w$, $\partial h_1/\partial p_1 = \partial h_2/\partial p_2$, and $\partial h_2/\partial p_1 = \partial h_1/\partial p_2$. As a result, equation (21) reduces to:

$$h_i + (p_i - c)\left(\frac{\partial h_i}{\partial p_i}+\frac{\partial h_j}{\partial p_i}\right) = 0 \tag{22}$$

Referring back to Expression (5) and noting that symmetry also implies that $p_1^M = p_2^M$ (or alternatively, that $p_1(\bar{w}) = p_2(\bar{w})$), we see that Equation (22) is equivalent to the first-order condition for product i in the horizontal merger scenario. Because firm 3's price is p_3^M by assumption, it follows that $p_i(\bar{w}) = p_i^M$ for $i = 1, 2$ as well. Furthermore, because $p_i(\bar{w}) = p_i^M$ for $i = 1, 2$, p_3^M turns out to be firm 3's best response when the joint venture sets input price $\bar{w}$, so that we may write $p_3(\bar{w}) = p_3^M$. Consequently,

$\pi_1(\mathbf{p}(\bar{w})) + \pi_2(\mathbf{p}(\bar{w})) = (p_1 - c)h_1(\mathbf{p}(\bar{w})) + (p_2 - c)h_2(\mathbf{p}(\bar{w})) = \pi_M(\mathbf{p^M})$.

Cournot: The Cournot proof is analogous to its Bertrand counterpart. That is, when firm 3's quantity is constant at x_3^M, firm i's, $i \neq j = 1, 2$, first-order condition becomes:

$$\begin{aligned}\frac{\mathrm{d}\pi_i(\mathbf{x}(w))}{\mathrm{d}w} &= \left(\frac{\partial f_i}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{\partial f_i}{\partial x_j}\frac{\mathrm{d}x_j}{\mathrm{d}w}\right)x_i + (f_i - w)\frac{\mathrm{d}x_i}{\mathrm{d}w} \\ &+ \frac{1}{2}(x_j - x_i) + \frac{w-c}{2}\left(\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{\mathrm{d}x_j}{\mathrm{d}w}\right) = 0\end{aligned} \tag{23}$$

Symmetry implies that in equilibrium, $x_1(\bar{w}) = x_2(\bar{w})$ and $\mathrm{d}x_1/\mathrm{d}w = \mathrm{d}x_2/\mathrm{d}w$. As a result, equation (23) reduces to:

$$f_i - c + x_i\left(\frac{\partial f_i}{\partial x_i} + \frac{\partial f_i}{\partial x_j}\right) = 0 \tag{24}$$

Referring back to Expression (6) and noting that symmetry also implies that $\partial f_2/\partial x_1 = \partial f_1/\partial x_2$ and $x_1^M = x_2^M$, we see that Equation (24) is equivalent to the first-order condition for product i in the horizontal merger scenario. Because firm 3's quantity is x_3^M by assumption, it follows that $x_i(\bar{w}) = x_i^M$ for $i = 1, 2$ as well. Furthermore, because $x_i(\bar{w}) = x_i^M$ for $i = 1, 2$, x_3^M turns out to be firm 3's best response when the joint venture sets input price $\bar{w}$, so that we may write $x_3(\bar{w}) = x_3^M$. Consequently, $\pi_1(\mathbf{x}(\bar{w})) + \pi_2(\mathbf{x}(\bar{w})) = [f_1(\mathbf{x}(\bar{w})) - c]x_1 + [f_2(\mathbf{x}(\bar{w})) - c]x_2 = \pi_M(\mathbf{x^M})$. □

Proof of Proposition 2.

Proof. The change in firm i's, $i \neq j = 1, 2$, profit with respect to w can be written:

$$\begin{aligned}\frac{\mathrm{d}\pi_i(\mathbf{p}(w))}{\mathrm{d}w} &= \frac{\mathrm{d}p_i}{\mathrm{d}w}h_i + \frac{1}{2}(h_j - h_i) + (p_i - w)\left(\frac{\partial h_i}{\partial p_i}\frac{\mathrm{d}p_i}{\mathrm{d}w} + \frac{\partial h_i}{\partial p_j}\frac{\mathrm{d}p_j}{\mathrm{d}w}\right) \\ &+ \frac{w-c}{2}\left[\left(\frac{\partial h_i}{\partial p_i} + \frac{\partial h_j}{\partial p_i}\right)\frac{\mathrm{d}p_i}{\mathrm{d}w} + \left(\frac{\partial h_i}{\partial p_j} + \frac{\partial h_j}{\partial p_j}\right)\frac{\mathrm{d}p_j}{\mathrm{d}w}\right] \\ &+ (p_i - w)\frac{\partial h_i}{\partial p_3}\frac{\mathrm{d}p_3}{\mathrm{d}w} + \frac{w-c}{2}\left(\frac{\partial h_i}{\partial p_3} + \frac{\partial h_j}{\partial p_3}\right)\frac{\mathrm{d}p_3}{\mathrm{d}w}\end{aligned} \tag{25}$$

Symmetry implies that in equilibrium, $h_1 = h_2$, $\mathrm{d}p_1/\mathrm{d}w = \mathrm{d}p_2/\mathrm{d}w$, $\partial h_1/\partial p_1 = \partial h_2/\partial p_2$, $\partial h_2/\partial p_1 = \partial h_1/\partial p_2$, and $\partial h_1/\partial p_3 = \partial h_2/\partial p_3$. As a result, equation (25) reduces to:

$$\frac{\mathrm{d}\pi_i(\mathbf{p}(w))}{\mathrm{d}w} = \left[h_i + (p_i - c)\left(\frac{\partial h_i}{\partial p_i} + \frac{\partial h_j}{\partial p_i}\right)\right]\frac{\mathrm{d}p_i}{\mathrm{d}w} + (p_i - c)\frac{\partial h_i}{\partial p_3}\frac{\mathrm{d}p_3}{\mathrm{d}w} \tag{26}$$

Substituting $\bar{w}$ into Equation (26) and applying Proposition 1 yields:

$$\left.\frac{\mathrm{d}\pi_i(\mathbf{p}(w))}{\mathrm{d}w}\right|_{\bar{w}} = (p_i - c)\left.\frac{\partial h_i}{\partial p_3}\frac{\mathrm{d}p_3}{\mathrm{d}w}\right|_{\bar{w}} > 0 \tag{27}$$

where the inequality follows by our assumption that products are gross substitutes and from the first item in Lemma 1. The inequality in Expression 27 tells us that $\bar{w}$ does not lead to an optimum in the complete game so that by definition, $\pi_i(\mathbf{p}(w^*)) > \pi_i(\mathbf{p}(\bar{w}))$ for $i = 1, 2$ and by Proposition 1, $\pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*)) > \pi_M(\mathbf{p}^\mathbf{M})$.

Now suppose that contrary to the statement of the Proposition, $w^* < \bar{w}$. This leads to the following contradiction:

$$\begin{aligned}\pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*)) &= \pi_M(\mathbf{p}(w^*)) \\ &< \pi_M(p_1(w^*), p_2(w^*), p_3(\bar{w})) \\ &< \pi_M(\mathbf{p}(\bar{w})) \\ &= \pi_M(\mathbf{p}^\mathbf{M}) < \pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*))\end{aligned}$$

The initial equality follows from symmetry. The first inequality follows from Lemma 1 (whereby $w^* < \bar{w}$ implies that $p_3(w^*) < p_3(\bar{w})$) together with gross substitutability. The remaining relations follow from Proposition 1. We have thus proven that $w^* > \bar{w}$. From Lemma 1 it immediately follows that $p_i(w^*) > p_i^M$, $i = 1, 2, 3$.

It remains to show that $\pi_3(\mathbf{p}(w^*)) > \pi_3(\mathbf{p}^\mathbf{M})$. The change in firm 3's profit with respect to w is given by:

$$\begin{aligned}\frac{\mathrm{d}\pi_3(\mathbf{p}(w))}{\mathrm{d}w} &= \frac{\mathrm{d}p_3}{\mathrm{d}w}h_3 + (p_3 - c)\left(\frac{\partial h_3}{\partial p_1}\frac{\mathrm{d}p_1}{\mathrm{d}w} + \frac{\partial h_3}{\partial p_2}\frac{\mathrm{d}p_2}{\mathrm{d}w} + \frac{\partial h_3}{\partial p_3}\frac{\mathrm{d}p_3}{\mathrm{d}w}\right) \\ &= (p_3 - c)\left(\frac{\partial h_3}{\partial p_1}\frac{\mathrm{d}p_1}{\mathrm{d}w} + \frac{\partial h_3}{\partial p_2}\frac{\mathrm{d}p_2}{\mathrm{d}w}\right) > 0\end{aligned}$$

The second equality follows from firm 3's second stage first-order condition (see Expression (3)) and the inequality follows from Lemma 1 together with gross substitutability. The proof follows from Proposition 1 because $w^* > \bar{w}$. □

Proof of Proposition 3.

Proof. The change in firm i's, $i \neq j = 1, 2$, profit with respect to w can be written:

$$\begin{aligned}\frac{\mathrm{d}\pi_i(\mathbf{x}(w))}{\mathrm{d}w} &= \left(\frac{\partial f_i}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{\partial f_i}{\partial x_j}\frac{\mathrm{d}x_j}{\mathrm{d}w} + \frac{\partial f_i}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}w}\right) x_i \\ &+ (f_i - w)\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{1}{2}(x_j - x_i) + \frac{w-c}{2}\left(\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{\mathrm{d}x_j}{\mathrm{d}w}\right)\end{aligned} \tag{28}$$

Symmetry implies that in equilibrium, $x_1(w^*) = x_2(w^*)$ and $\mathrm{d}x_1/\mathrm{d}w = \mathrm{d}x_2/\mathrm{d}w$. As a result, equation (28) reduces to:

$$\frac{\mathrm{d}\pi_i(\mathbf{x}(w))}{\mathrm{d}w} = \left[f_i - c + x_i\left(\frac{\partial f_i}{\partial x_i} + \frac{\partial f_i}{\partial x_j}\right)\right]\frac{\mathrm{d}x_i}{\mathrm{d}w} + x_i\frac{\partial f_i}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}w} \tag{29}$$

Substituting $\bar{w}$ into Equation (29) and applying Proposition 1 yields:

$$\left.\frac{\mathrm{d}\pi_i(\mathbf{x}(w))}{\mathrm{d}w}\right|_{\bar{w}} = x_i\left.\frac{\partial f_i}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}w}\right|_{\bar{w}} < 0 \tag{30}$$

where the inequality follows by our assumption that products are substitutes and from the second item in Lemma 1. The inequality in Expression 30 tells us that $\bar{w}$ does not lead to an optimum in the complete game so that by definition, $\pi_i(\mathbf{x}(w^*)) > \pi_i(\mathbf{x}(\bar{w}))$ for $i = 1, 2$ and by Proposition 1, $\pi_1(\mathbf{x}(w^*)) + \pi_2(\mathbf{x}(w^*)) > \pi_M(\mathbf{x}^{\mathbf{M}})$.

Now suppose that contrary to the statement of the Proposition, $\bar{w} < w^*$. This leads to the following contradiction:

$$\begin{aligned}\pi_1(\mathbf{x}(w^*)) + \pi_2(\mathbf{x}(w^*)) &= \pi_M(\mathbf{x}(w^*)) \\ &< \pi_M(x_1(w^*), x_2(w^*), x_3(\bar{w})) \\ &< \pi_M(\mathbf{x}(\bar{w})) \\ &= \pi_M(\mathbf{x}^{\mathbf{M}}) < \pi_1(\mathbf{x}(w^*)) + \pi_2(\mathbf{x}(w^*))\end{aligned}$$

The initial equality follows from symmetry. The first inequality follows from Lemma 1 (whereby $\bar{w} < w^*$ implies that $x_3(\bar{w}) < x_3(w^*)$) together with substitutability. The remaining relations follow from Proposition 1. We have thus proven that $\bar{w} > w^*$. From Lemma 1 it immediately follows that $x_i(w^*) > x_i^M$, $i = 1, 2$ and $x_3^M > x_3(w^*)$.

It remains to show that $\pi_3(\mathbf{x^M}) > \pi_3(\mathbf{x}(w^*))$. The change in firm 3's profit with respect to w is given by:

$$\begin{aligned}\frac{\mathrm{d}\pi_3(\mathbf{x}(w))}{\mathrm{d}w} &= x_3\left(\frac{\partial f_3}{\partial x_1}\frac{\mathrm{d}x_1}{\mathrm{d}w} + \frac{\partial f_3}{\partial x_2}\frac{\mathrm{d}x_2}{\mathrm{d}w} + \frac{\partial f_3}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}w}\right) + (f_3 - c)\frac{\mathrm{d}x_3}{\mathrm{d}w} \\ &= x_3\left(\frac{\partial f_3}{\partial x_1}\frac{\mathrm{d}x_1}{\mathrm{d}w} + \frac{\partial f_3}{\partial x_2}\frac{\mathrm{d}x_2}{\mathrm{d}w}\right) > 0\end{aligned}$$

The second equality follows from firm 3's second stage first-order condition (see Expression (4)) and the inequality follows from Lemma 1 together with substitutability. The proof follows from Proposition 1 because $\bar{w} > w^*$. □

Proof of Proposition 6.

Proof. Assumptions 1 and 3 together with our symmetry assumptions on utility, marginal costs of production, and the division of joint venture profits, imply that for any w chosen by the joint venture, there is a unique equilibrium in downstream prices in which firms 1 and 2 set the same price and earn the same profit. Moreover, if the joint venture is played on the equilibrium path, the profits earned by the joint venture partners must be no lower than the profits earned by a horizontal merger between them (we have assumed that horizontal merger profits would be equitably divided among the merging firms). Now suppose that the joint venture is played on the equilbrium path, but that contrary to the statement of the Proposition, $w^* < \bar{w}$. This leads to the following set of inequalities:

$$\begin{aligned}\pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*)) &= \pi_M(\mathbf{p}(w^*)) \\ &< \pi_M(p_1(w^*), p_2(w^*), p_3(\bar{w})) \\ &< \pi_M(\mathbf{p}(\bar{w})) = \pi_M(\mathbf{p^M})\end{aligned}$$

which would contradict the joint venture having been selected in place of the horizontal merger in the first stage. The remainder of the proof follows precisely the proof of Proposition 2. □

Sequential equilibrium in the imperfect information game.

Here, we show that the (Pareto dominant) assessment consisting of the joint venture playing w^*, followed by firm $i = 1, 2$ playing $p_i(w)$ for any $w \in W_p$ and firm 3 playing $p_3(w^*)$ accompanied by the belief that w^* was played with probability one, constitutes a sequential equilibrium of the two stage joint venture pricing game of imperfect information. To simplify the exposition, let us proceed with the extensive form transformation of the second simultaneous move stage in which firm 1's move is followed by that of firm 2, which is followed by that of firm 3 and in which subsequent movers are not made aware of the previous history of the stage. This extensive form specification requires us to additionally specify beliefs about prior pricing moves for firms 2 and 3. Let us suppose that in equilibrium, firm 2 believes that firm 1 plays $p_1(w)$ with probability one contingent on w having been played in stage one and that firm 3 believes that firm $i = 1, 2$ plays $p_i(w^*)$ with probability one.

The sequential rationality of the assessment above follows immediately from the definitions of w^* and $p_i(w)$ provided in Section 3. In order to show that the assessment is consistent, we first define the following density functions, each of which is positive on the interior of their supports: $\varphi^{\epsilon}_{JV} : W_p \to [0, 1]$, $\varphi^{\epsilon}_3 : P \to [0, 1]$ and conditional density $\varphi^{\epsilon}_i : P \times W_p \to [0, 1]$, $i = 1, 2$, which is conditional on $w \in W_p$, and where the superscript ϵ

represents a positive integer. Further, suppose that $\lim_{\epsilon\to\infty} \varphi^{\epsilon}_{JV}(w^*) = 1$, $\lim_{\epsilon\to\infty} \varphi^{\epsilon}_{3}(p_3(w^*)) = 1$, and $\lim_{\epsilon\to\infty} \varphi^{\epsilon}_{i}(p_i(w)|w) = 1$.

To show consistency, we may now define a sequence of assessments consisting of completely mixed strategies σ^{ϵ} and Bayes' rule derived beliefs μ^{ϵ} which converge to the assessment above. For each ϵ, define the strategy of the joint venture as $\sigma^{\epsilon}_{JV}(\varnothing)(w) = \varphi^{\epsilon}_{JV}(w)$, where the first set of parenthesis denotes each player's information set. Likewise, define the strategy of firm $i = 1, 2$ conditional on w as $\sigma^{\epsilon}_{i}(w)(p_i) = \varphi^{\epsilon}_{i}(p_i|w)$ and the strategy of firm 3 as $\sigma^{\epsilon}_{3}(W_p)(p_3) = \varphi^{\epsilon}_{3}(p_3)$. Proceeding according to the extensive form transformation above, for each ϵ, we may define the beliefs of firm 1 as $\mu^{\epsilon}_{1}(w)(w) = \varphi^{\epsilon}_{JV}(w)$, the beliefs of firm 2 as $\mu^{\epsilon}_{2}(w \times P)(w, p_1) = \varphi^{\epsilon}_{JV}(w)\varphi^{\epsilon}_{1}(p_1|w)$, and the beliefs of firm 3 as $\mu^{\epsilon}_{3}(W_p \times P \times P)(w, p_1, p_2) = \varphi^{\epsilon}_{JV}(w)\varphi^{\epsilon}_{1}(p_1|w)\varphi^{\epsilon}_{2}(p_2|w)$. It becomes immediately apparent that the sequence of strategies and beliefs converges to the assessment above and that for each ϵ, beliefs are defined from strategies according to Bayes' rule, such that the assessment is indeed consistent.

Wireless simulation data.

Prices: Average revenue per user (ARPU) is used as a measure of price. Within our simple model, ARPU has the advantage over service providers' posted prices for monthly service plans in that it aggregates across all services in proportion to each customer segment. Moreover, ARPU as obtained from service provider SEC filings averages out any regional or local variation in prices and accounts for any device subsidies or other discounts. For consistency across service providers, ARPU is calculated as the total of 2011 wireless operating revenue excluding equipment divided by twelve times the 2011 average number of subscribers.[21] ARPU for "Other" service providers is calculated as a weighted

[21] The average number of subscribers was either reported in Form 10-K or determined using an average of quarterly midpoints, depending on data availability.

average of the ARPUs of MetroPCS, US Cellular, and Leap Wireless, three of the four multi-regional providers. Clearwire Communications, the fourth multi-regional provider, is a 51.5% owned investee company of Sprint, whose results of operations are included in Sprint's Form 10-K.[22]

Quantities: Market share as determined by a provider's share of nationwide wireless subscribers is used as the measure of service provider output. The subscribership of the eleven largest service providers as reported by number of connections in the FCC's Sixteenth Wireless Competition Report is used to approximate nationwide subscribers, with 2011 subscribers ranging from 107.8 million for Verizon Wireless[23] to 414.5 thousand for NTELOS.[24] The share of Other service providers is assumed to be the difference of nationwide subscribership and that of the four nationwide providers.

Margins: Price-cost margins are obtained by subtracting a proxy for variable cost from ARPU. The variable cost proxy equals the 2011 cost of operations net of depreciation and amortization as reported in Form 10-K multiplied by one minus the ratio of one quarter lagged depreciation and amortization to the costs of operations in 2011. Our proxy supposes that a firm uses the previous quarter's depreciation and amortization results to determine spending on capital replacement (fixed costs) in the current quarter. The proxy cannot account for the possibility that a firm may want to expand or contract its network. Margins for Other service providers is calculated as a weighted average of the margins of MetroPCS, US Cellular, and Leap Wireless.

Diversion Ratios: Ideally, in order to calibrate demand, we would use data on the

[22]Additionally, although Clearwire offers mobile broadband data services, it does not offer circuit-switched mobile voice services and most of its wholesale subscribers are also Sprint retail subscribers.

[23]The Sixteenth Report, which only reports 92.2 million Verizon Wireless subscribers only includes retail subscribers.

[24]In addition to the nationwide and multi-regional providers, this includes regional/local providers C Spire Wireless, Atlantic Tele-Network, Cincinnati Bell Wireless, and NTELOS. We exclude Clearwire Communications. See Sixteenth Report Table 13.

Table 5: Wireless Simulation Data

Data	AT&T	VZW	Sprint	TM	Other
ARPU	\$47.7	\$46.6	\$43.7	\$45.8	\$45.6
Shares	32.0%	33.4%	17.0%	10.3%	7.3%
Margins	38.4%	39.4%	21.8%	33.1%	29.7%

degree of substitutability between service providers to calculate cross-price elasticities, or alternatively, diversion ratios (the diversion ratio d_{ji} to product j from product i is defined as $d_{ji} = \varepsilon_{ji}x_j/\varepsilon_{ii}x_i$). Absent such data, we proxy for diversion ratios based on wireless service providers' market shares, s_i: $d_{ji} = s_j/(1 - s_i)$.[25] These proxies are reported in Table 6.

Table 6: Wireless Diversion Ratios

		To Provider				
From Provider		AT&T	VZW	Sprint	TM	Other
	AT&T		49.1%	25.1%	15.1%	10.8%
	VZW	48.0%		25.6%	15.4%	11.0%
	Sprint	38.5%	40.2%		12.4%	8.8%
	TM	35.6%	37.2%	19.0%		8.2%
	Other	34.4%	35.9%	18.3%	11.0%	0.4%

[25] Ordinarily, we would multiply market share based diversion ratios by the "market recapture ratio," the fraction of sales lost by one service provider from a small increase in its price that is gained by the remaining service providers (Farrell and Shapiro, 2010). Because we do not observe the market recapture ratio, we assume full recapture.

References

[1] Arya, A., Mittendorf, B., Sappington, D.E.M., 2008. "Outsourcing, Vertical Integration, and Price vs. Quantity Competition." *International Journal of Industrial Organization* 26, 1-16.

[2] Baker, A., Brennan, T., Erb, J., Nayeem, O., Yankelevich, A., 2014. "Economics at the FCC, 2013-2014." Mimeo. Federal Communications Commission, Washington, DC.

[3] Baker, J., Shapiro, C., 2008. "Reinvigorating Horizontal Merger Enforcement," in *How the Chicago School Overshot the Mark: The Effect of Conservative Economic Analysis on Antitrust*, Robert Pitofsky, ed., Oxford University Press.

[4] Bonanno, G., Vickers, J., 1988. "Vertical Separation." *The Journal of Industrial Economics* 3, 257-265.

[5] Bresnahan, T.F., Salop, S.C., 1986. "Quantifying the Competitive Effects of Production Joint Ventures." *International Journal of Industrial Organization* 4, 155-175.

[6] Bulow, J.I., Geanakoplos, J.D., Klemperer, P.D., 1985. "Multimarket Oligopoly: Strategic Substitutes and Complements." *Journal of Political Economy* 93, 488-511.

[7] Canada, Competition Bureau, 2009. "Competitor Collaboration Guidelines." Ottawa, December.

[8] Chen, Z., Ross, T.W., 2003. "Cooperating Upstream While Competing Downstream: A Theory of Input Joint Ventures." *International Journal of Industrial Organization* 21, 381-397.

[9] Choi, J.P., Yi, S., 2000. "Vertical Foreclosure with the Choice of Input Specifications." *The RAND Journal of Economics* 31, 717-743.

[10] Cooper, R.W., Ross, T.W., 2009. "Sustaining Cooperation with Joint Ventures." *The Journal of Law, Economics, and Organization* 25, 31-54.

[11] European Commission, 2011. "Guidelines on the Applicability of Article 101 of the Treaty on the Functioning of the European Union to Horizontal Cooperation Agreements." *Official Journal of the European Union* 2011/C, 11/01-11/72.

[12] Farrell, J., Shapiro, C., 1990. "Asset Ownership and Market Structure in Oligopoly." *The RAND Journal of Economics* 21, 275-292.

[13] Farrell, J., Shapiro, C., 2010. "Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition." *The B.E. Journal of Theoretical Economics* 10, Article 9.

[14] Foros, Ø., Kind, H.J., Shaffer, G., 2011. "Mergers and Partial Ownership." *European Economic Review* 55, 916-926.

[15] Harris, C., Reny, P., Robson, A., 1995. "The Existence of Subgame-Perfect Equilibrium in Continuous Games with Almost Perfect Information: A Case for Public Randomization." *Econometrica* 63, 507-544.

[16] Kwoka, J.E., Jr., 1992. "The Output and Profit Effects of Horizontal Joint Ventures." *The Journal of Industrial Economics* 40, 325-338.

[17] Morasch, K., 2000a. "Strategic Alliances as Stackelberg Cartels—Concepts and Equilibrium Alliance Structure." *International Journal of Industrial Organization* 18, 257-282.

[18] Morasch, K., 2000b. "Strategic Alliances: a Substitute for Strategic Trade Policy?" *Journal of International Economics* 52, 37-67.

[19] O'Brien, D.P., Salop, S.C., 2000. "Competitive Effects of Partial Ownership: Financial Interest and Corporate Control." *Antitrust Law Journal* 66, 559-614.

[20] Ordover, J.A., Saloner, G., Salop, S.C., 1990. "Equilibrium Vertical Foreclosure." *The American Economic Review* 80, 127-142.

[21] Reynolds, R.J., Snapp, B.R., 1986. "The Competitive Effects of Partial Equity Interests and Joint Ventures." *International Journal of Industrial Organization* 4, 141-153.

[22] Rossini, G., Vergari, C., 2011. "Input Production Joint Venture." *The B.E. Journal of Theoretical Economics* 11, Article 5.

[23] Salant, S.W., Switzer, S., Reynolds, R.J., 1983. "Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium." *The Quarterly Journal of Economics* 98, 185-199.

[24] Shapiro, C., 1989. "Theories of Oligopoly Behavior." in *Handbook of Industrial Organization*, Richard Schmalensee and Robert Willig, eds., North Holland.

[25] Shapiro, C., Willig, R.D., 1990. "On the Antitrust Treatment of Production Joint Ventures." *The Journal of Economic Perspectives* 4, 113-130.

[26] Singh, N. and Vives, X., 1984. "Price and Quantity Competition in a Differentiated Duopoly." *The RAND Journal of Economics* 15, 546-554.

[27] Spencer, B.J. and Raubitschek, R.S., 1996. "High-Cost Domestic Joint Ventures and International Competition: Do Domestic Firms Gain?" *International Economic Review* 2, 315-340.

[28] United States of America, Department of Justice and Federal Trade Commission, 2010. "Horizontal Merger Guidelines." Washington, DC, August.

[29] Unites States of America, Federal Communications Commission, 2011. "Applications of AT&T Inc. and Deutsche Telekom AG for Consent to Assign or Transfer Control of Licenses and Authorizations, Staff Analysis and Findings." WT Docket No. 11-65. Washington, DC, December.

[30] Unites States of America, Federal Communications Commission, 2013. "Sixteenth Report. Implementation of Section 6002(b) of the Omnibus Budget Reconciliation Act of 1993. Annual Report and Analysis of Competitive Market Conditions with Respect to Mobile Wireless, Including Commercial Mobile Services." WT Docket No. 11-186. Washington, DC, March.

[31] United States of America, Federal Trade Commission and Department of Justice, 2000. "Antitrust Guidelines for Collaborations Among Competitors." Washington, DC, April.

[32] Vives, X., 1985. "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation." *Journal of Economic Theory* 36, 166-175.

[33] Werden, G.J., 1996. "A Robust Test for Consumer Welfare Enhancing Mergers Among Sellers of Differentiated Products." *The Journal of Industrial Economics* 44, 409-413.

[34] Werden, G.J., 1998. "Antitrust Analysis of Joint Ventures: An Overview." *Antitrust Law Journal* 66, 702-735.