MULTI-UNIT AUCTIONS AND COMPETITION STRUCTURE[*]

Raphaële PRÉGET (INRA)[§]

Sophie THOYER (Montpellier SupAgro)[¥]

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ABSTRACT. Is it better for a seller who wants to auction multiple units to face many small bidders or few large bidders? Since multi-unit auction models usually have many equilibria, there are no theoretical predictions on the impact of the competition structure on the performance of a multi-unit auction (in terms of expected revenue and allocation efficiency). Our experimental results with uniform-price auctions support that with a constant degree of rationing, when the number of bidders increases while individual demand decreases, there is less strategic bidding (demand reduction). It leads to higher expected revenue with a lower variance but allocation efficiency is not significantly different.

KEY WORDS: experimental auction; multi-unit auction; demand reduction; competition structure
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[§] INRA, UMR 1135 LAMETA, F-34000 Montpellier, France; email: preget@supagro.inra.fr

[¥] Montpellier SupAgro, UMR 1135 LAMETA, F-34000 Montpellier, France; email: thoyer@supagro.inra.fr

1. INTRODUCTION

Multi-unit auctions (in which bidders are allowed to submit multiple price-quantity bids) are used in an increasing number of domains : the purchase of foreign exchange (Tenorio, 1999), electricity markets (Wolfram 1998), the sale of treasury securities (Binmore and Swierzbinski 2000), the sale of agricultural goods, etc. They could also be mobilized in other areas such as water buybacks in drought-threatened basins (Hailu and Thoyer, 2005). In a multi-unit auction, many items or units are put on sale simultaneously in a single round. If bidders have multi-unit demands (*i.e.* bidders have positive valuations for many units and thus are interested in winning more than one unit), they may submit many bids which indicate the price they propose for each unit.[1] Multi-unit auctions are more flexible than single bid auctions that limit bids to single quantity-price pair bids and thus help to avoid the 'lumpy bid' problem inherent in single-bid auctions (Chan et al, 2003).

The design of an auction seeks to maximize the competition between bidders in order to gain efficiency (both in terms of sale revenue and allocative efficiency). In auctions where each bidder is only interested in one unit, the competition degree can be easily measured by the ratio between the number of units to be sold and the number of bidders: this is the degree of rationing. In multi-unit auctions where bidders want more than one unit, the competition is not so easily defined. Although the degree of rationing can be measured as total supply divided by total demand, the competition structure will depend on the number of bidders and the structure of their individual demand. As a consequence and more generally, for a same degree of rationing we may have different competition structures.

In practice, it may happen that the auctioneer has to select a target group to which the items will be sold (or purchased if it is a procurement auction). Of course, depending on the size of the group (the number of bidders) and the demand of each potential bidder in the group, the

[1] In the auction literature, multi-unit auctions are sometimes formalized as a share auction in which the good is assumed perfectly divisible and bidders submit demand functions. See Wilson (1979), Wang and Zender (2002). A multi-unit auction is a discrete version of a share auction.

degree of competition will differ and it is expected that the auctioneer will choose the target group where the competition is greatest.

Similarly, when the seller defines the size of a unit (the minimal indivisible quantity one can bid on and buy) she may also have an impact on the competition structure. Indeed, if the size of a unit is small enough, most potential buyers can participate on their own to the auction and in this case the bigger buyers will want many units. On the opposite, if the size of a unit is important, only the larger bidders may be able to participate to the auction, unless the smaller buyers cooperate to bid together and then share their purchase.

What is at stake for the auctioneer is to choose the market place and the size of the elementary unit sold, which will ensure the highest expected revenue (and/or potentially the highest allocative efficiency if the auctioneer is a public agency).

The objective in this paper is to compare the performance of multi-unit auction under different structures of competition when bidders have private independent values. More precisely, for a same level of rationing, is it better for the seller to face a group of numerous bidders who have a small individual demand or a smaller group of bidders who have larger individual demand? What is best in terms of auction performance, *i.e.* seller's revenue and allocation efficiency?

We will focus on the case of uniform-price auction in which the stop-out price, *i.e.* the price paid by the winner(s), is the first rejected bid (or the highest losing bid).[2] Although structural properties of equilibrium bidding strategies can be deducted from simple examples, the existing literature offers little insights into their analytical characterization, made intractable by the existence of multiple equilibria. Since theoretical analysis cannot provide a clear-cut answer, we turn to experimental data in order to characterize better the outcomes of multi-unit auctions.

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[2] In a uniform-price auction, the stop-out price is sometimes defined as the lowest accepted bid. This different pricing rule can have substantial impacts on the bidders' strategies and thus on the result of the auction. In this paper, we will consider only the case where the market price is the highest rejected bid.

We therefore analyse a multi-unit auction run into two different "competitive environment" scenarios: the first one includes six bidders with small individual demands, the second is characterized by two bidders with large individual demands. Aggregate demand and supply are the same in both scenarios. We first identify potential Nash equilibria in bidding behaviour and deduct that the auction revenue could potentially fall to zero, were bidders able to coordinate on such equilibria. We then run controlled laboratory experiments to compare the efficiency outcomes in both scenarios. The main conclusion is that with constant degree of rationing, when the number of bidders increases while individual demand decreases, there is less strategic bidding (demand reduction), therefore leading to higher expected revenue (with a lower variance). However, allocation efficiency is not significantly different.

The paper is structured as follows. Next section summarises intuitions and literature findings based on a specific example which will be mobilized in the experiments. The third section describes the experimental protocol. The fourth section presents the experimental results on auction performance. The fifth section analyses strategies in experiments. Last section concludes.

2. LITERATURE AND HYPOTHESES

2.1. Literature findings

In the literature on auctions with multi-unit demand, most articles aim to compare the performance of different auction formats for a given structure of competition. We focus here only on the case of the uniform-price auction. The uniform-price auction has first been considered falsely as the generalisation of the second-price auction in the case of a multi-unit auction. This incorrect analogy led some prominent economists to confusions regarding the incentive effects of uniform-price auctions (Binmore and Swierzbinski, 2000). Indeed, it is a dominant bidding strategy to submit one's true value in the single-unit demand case, but when bidders have non-increasing multi-unit demand they have an incentive to reduce their demand so as to obtain units at a more favourable price (see, for example, Noussair, 1995; Engelbrecht-Wiggans and Kahn, 1998). If we consider the discrete case where many indivisible units are put on sale and bidders submit a bid for each unit they want, a bidder who desires more than one unit in a uniform-price auction has an incentive to shade her bid (bid under her true valuation, *i.e.* underbidding). There is no bid shading on the first unit demanded, but increasing amounts of bid shading occur on subsequent units. The intuition behind the incentive to reduce demand is that bidders have market power in the uniform-price auction since their bid potentially determines the price they will have to pay. Demand reduction reduces seller's revenue and introduces inefficiencies as buyers with lower valued units may earn items in place of higher valued buyers (Ausubel and Cramton, 2002).

Many works study this demand reduction phenomenon. There are in particular many experimental auctions that are conducted to compare demand reduction in different multi-unit auction formats, usually for two identical units: Alsemgeest, Noussair and Olson, 1998; List and Lucking-Reiley, 2000; Kagel and Levin, 2001; Engelmann and Grimm, 2006; Porter and Vragov, 2006. As the theory predicts, clear demand reductions is observed in the uniform-price auction (even when bidders have flat demands). In addition, Engelbrecht-Wiggans, List and Lucking-Reiley (2006) found that the demand reduction diminishes when the number of bidders increases, but does not vanish.

From a theoretical view point, it is difficult to obtain results in a general discrete case (N bidders with multi-unit demand competing for K units). Most papers consider special cases. The most common studied example is when two units are put on sale and bidders have a demand for two units. Engelbrecht-Wiggans and Kahn (1998) and Krishna (2002) provide characterizations of equilibria in such auction.

If there are two units for sale and two bidders with value vectors $\mathbf{X}$ that are identically and independently distributed according to the density function $f(x) = 2$ on $\chi = \{\mathbf{x} \in [0, 1]^2 : x_1 \geq x_2\}$, Krishna (2002, p. 189) proves that $\beta_1(x_1, x_2) = x_1$, and $\beta_2(x_1, x_2) = 0$, *i.e.* bidding one's true value for the first unit and zero for the second, is a symmetric equilibrium of the uniform-price auction. Thus, in every realization, each bidder wins one unit and the price is zero! In such case, allocative efficiency is not guaranteed (since the private value of the first unit of one of the two bidders can be lower than the second private value of the other bidder and is won however).

Can such equilibrium be extended to a more complicated case where the number of bidders or the number of units is strictly greater than two?

2.2. *Theoretical predictions for two competition structure scenarios*

To simplify, we restrict the analysis to symmetric bidders. Bidders who participate in the auction have the same structure of individual demand, *i.e.* they want the same number of units and have decreasing demand values (downward sloping demands). More precisely, each bidder's private values are independently drawn from the same distribution and are ordered from the highest to the lowest. This distribution and ordering rule is common knowledge. In the following experiment we consider a uniform distribution on [0, 100]. In addition, we assume that there is neither communication nor any possibility for explicit collusion.

To fix ideas assume a situation with a total supply of $K = 6$ units. We study two scenarios. In scenario 1, there is a group of six identical bidders with a non increasing demand for two units each. In scenario 2, we consider, two players with a demand for six units each. In short there are six small bidders in scenario 1 and two large bidders in scenario 2. Note that the aggregate demand in both scenarios is identical and is equal to 12 units. Since there are only 6 units put on sale, the degree of rationing is 0.5 in both scenarios. However, the structure of competition differs because the size and numbers of bidders are different. Table 1 summarizes information on both scenarios.

Table 1 : Summary of the two scenarios

	Scenario 1 « 6 small bidders »	Scenario 2 « 2 large bidders »
Number of bidders	6	2
Individual demand	2	6
Total demand	12	12
Degree of rationing	0.5	0.5

Our main question is: which scenario is best from the auctioneer's viewpoint? Which scenario leads to the highest seller's revenue? Which scenario gives the best allocation efficiency? In order to answer these questions from a theoretical point of view we first need to determine optimal bidding strategies and auction equilibria.[3]

[3] Note that when bidders have single unit demand, imagine 12 bidders with a demand for only one unit, their optimal bidding strategy is to bid their true value (no demand reduction). Thus the expected revenue corresponds to the seventh highest value out of twelve and there is allocation efficiency.

Let's first consider scenario 1 with 6 small bidders. This scenario bears similarities with the Krishna case described above. We also have $N = K$ (N is the number of bidders) and bidders still have a demand for two units. Despite the similarities of this game with Krishna's example, we show in Appendix 1 that we cannot obtain the same type of equilibrium (bid sincere on the first unit and bid zero on the second unit) as soon as $N = K > 2$. Actually, bidding one's true value for the first unit remains an optimal strategy, but if the private value of the second unit is high enough, it is profitable for the bidder to attempt to win a second unit by bidding more than zero for this second unit. As a consequence, bidding zero for the second unit is not an equilibrium strategy in our scenario with 6 small bidders when they bid their true value for the first unit. Nevertheless, note that bidding 100 (or any value above) for the first unit and zero for the second unit is a symmetric Nash equilibrium strategy in scenario 1. In this case, no bidder has a profitable deviation whatever their private values are. As a result, extreme demand reduction may occur and bidding above one's highest value for the first unit can be justified here.

Let's now consider scenario 2 with two large bidders having an individual demand of six units each, and a total supply of $K = 6$ units put on sale. Each bidder submits six bids. In this case, bidding one's true values for the first three units and then bidding zero for the last three units, by analogy with Krishna's example, is a Nash equilibrium strategy. Indeed, even in the most favourable realization (the bidder receives the highest possible value, *i.e.* 100 in our experiment, for all six units) there is no profitable deviation. In particular, we show in Appendix 2 that it is not in the bidder's interest to deviate and to bid a positive value for the fourth bid in the attempt to win a fourth unit when the other bidder bids truthfully on the first three units and zero on the other last three units. Moreover, it is also an equilibrium strategy to bid 100 (or any value above) for the first three units and to bid zero for the last three units. In those type of equilibria, the expected price is zero.

To summarize, there are many equilibria in both scenarios. Some equilibria correspond to extreme demand reduction and lead to very low prices and potential allocative efficiency losses. Nevertheless, those equilibria are not undominated equilibria and require some tacit coordination among the bidders to be effective. It is therefore not guaranteed that bidders will reach them. The problem is that we cannot measure the theoretical degree of optimal demand reduction of undominated equilibria due to the impossibility to describe the equilibrium bidding strategies as closed form expressions (Chakraborty, 2006). Thus, it is impossible to give theoretical predictions on performance or to have a clear benchmark equilibrium to compare our results with.

2.3. Intuitions and hypotheses

Intuitively we may expect a greater expected revenue in scenario 1 than in scenario 2. There are two arguments to justify our intuition. First, in scenario 1 with six small bidders, we expect demand reduction only on 6 units since bidders should submit their true value (or above their true value) for the first unit; whereas in scenario 2, with two large bidders we may observe demand reduction for up to ten units. In other words, we expect less underbidding in scenario 1 than in scenario 2. Thus, on average and if we ignore the amount of bid shading in each case, we could expect higher revenue in scenario 1. Second, if we consider the two large bidders in scenario 2 as two "cartels" (collusion among three bidders), it reinforces expectations for lower competitive pressure in scenario 2 and therefore lower revenue for the seller compared to scenario 1 (without collusion). These arguments lead us to hypothesis 1.

Hypothesis 1: For a given degree of rationing, the expected price (and therefore revenue efficiency) of a uniform-price auction with symmetric bidders is greater when the number of bidders increases or equivalently when individual demands decrease.

The outcome in terms of allocative efficiency is less predictable. Since, there is demand reduction on more units a priori in scenario 2, we may think that the risk of misallocation is higher in this scenario. However, the risk of misallocation is a priori lower when there are fewer bidders and the "cartel effect" may suggest the opposite. Indeed, we may think of a better allocation efficiency in scenario 2 since the two "cartels" can be expected to allocate the units won among their members efficiently. Since arguments run in opposite directions, we are not able to offer intuitions on which competition structure is better in terms of allocation efficiency and we may speculate that both effects compensate each other.

Hypothesis 2: For a given degree of rationing, allocative efficiency is not significantly affected by the competition structure, *i.e.* by the number of bidders or equivalently by the size of individual demand as long as bidders have multi-unit demand.

Intuition also suggests that the tacit cooperative outcome (due to strategic bidding and leading to a price of zero) is more straightforward and easier to reach in scenario 2 with two large bidders than in scenario 1 with six small bidders, in particular when bidders cannot communicate but when the game is repeated many times as it is the case in our experiments.
Indeed, when there are only two bidders, they can easily anticipate that one of their bids may determine the equilibrium price and that they could cooperate with their rival by sharing equally the markets between them by bidding a very low price on the fourth unit and the following units. There may be no significant demand reduction for the second and the third bid, but we might observe a big drop-off in the bid for the fourth unit. In scenario 1 with six small bidders, the likelihood for a bidder that his second bid determines the auction price is much lower (one chance out of six a priori) and we may expect less strategic bidding.
In other words, we expect intuitively that when the number of bidders increases, bidders act more as price takers than as price makers, *i.e.* they act more according to decision theory than according to game theory. Since optimal bidding strategies are complex to compute and tacit cooperation is more difficult to reach, bidders may presume that the equilibrium price will be around the seventh highest value out of twelve, so somewhat lower than 50, whatever they bid. From order statistic and our assumptions on the distribution of bidders' private values, the expected value of the seventh highest value out of twelve is:

$$E\left[Y_7^{(12)}\right]=\int_0^{100} y\frac{12!}{5!6!}\left(\frac{y}{100}\right)^5\left(1-\frac{y}{100}\right)^6 dy \cong 46{,}15\,.$$

Finally, according to those intuitions on the bidders' behaviour, we may expect more truthful bidding on average in scenario 1 with six small bidders with an auction price around the seventh highest value. In scenario 2 with two large bidders, depending on whether the pair of bidders gets to a cooperative equilibrium or not, we may observe more diversified outcomes. As a result, our third hypothesis is that the auction competition structure leads to different bidding strategies.

Hypothesis 3: For a given degree of rationing, bidders bid more strategically, *i.e.* they bid high on first unit(s) and low on last unit(s), when the number of bidders is lower. Thus auction results are less predictable (higher standard deviation on auction prices) when the number of bidders is low. On the opposite, bidders tend to bid closer to their true values when the number of bidders increases.

These three hypotheses are tested with an experiment based on the two scenarios.

3. EXPERIMENTS

As presented previously, we set-up experimental uniform-price auctions in which six identical units are put on sale simultaneously. The six highest bids are the winning bids. The bidders pay a price equal to the seventh highest bid (the first rejected bid) for each unit won. The implicit reserve price is zero: bidders can only submit positive bids. This auction game is repeated 13 times in a session with the same set of players for different value sets. The equilibrium price is announced to all the bidders before the next auction starts, so they know how many units they have won in the period and what is their profit. If there are ties, the unit is allocated randomly among the tied bidders.

The first two periods are trial periods to make sure the players understand the game[4]. Thus only the last 11 periods are paid and used to analyse the results of the experiments. Participants received their accumulated round payoffs (plus a show-up fee of €3.00) at the end of the experiment. Excluding the show-up payment, the average earnings per subject were about €18.10. Communication among the players is made impossible during all the experiment.

A total of 40 students from Montpellier University have participated in the experiments in 2004 and 2006. Let's call G1, G2 and G3 the three groups of bidders who played scenario 1 and P1, P2 to P11 the eleven pairs of bidders who played scenario 2 (see Table 2).

All private values are randomly drawn from a uniform distribution between 0 and 100. For each of the six players in scenario 1, two values are independently drawn. The highest one is for the first unit, the lowest value is attributed to the second unit. In scenario 2, the 6 values given to each bidder are ordered from the highest value for the first unit to the lowest for the

[4] The instructions were read aloud to the whole group and included examples. The subjects' understanding of the game was tested through a short questionnaire and was followed by a time allocated to group questions and responses. Protocols and questionnaires are available on request (Evrard, 2004).

last unit wanted. In both scenarios 156 values are needed for one session (13 periods * 12 privates values).

Table 2 : Experimental setting

	Scenario 1 « 6 small bidders »	Scenario 2 « 2 large bidders »	Total
Number of students	18	22	40
Number of sessions	3 (G1, G2, G3)	11 (P1, P2, …, P11)	14
Number of auctions	33	121	154
Number of bids	396	1452	1848

4. RESULTS ON PERFORMANCES

We first analysed results to detect learning effects across periods and potential end-of-game impacts. There is no significant trend indicating such effects (see Figures A1 and A2 in appendix 3). Even, in scenario 2, tacit cooperative outcomes, due to severe demand reduction (strategic bidding), are no more frequent in the last periods than in the first ones. Kruskal-Wallis tests[5] on auction prices and on all other performance indicators described in the following do not allow us to reject the null hypothesis of equality of population across periods at the 5% level of confidence, even when we include the two trial periods. In other words, we cannot reject the hypothesis that the 13 period bids are drawn from the same distribution. As a consequence, we do not distinguish periods in the following analysis. Thus, results presented in next sections rely on all periods (except the two trial periods).

4.1. Performance indicators

Seller's revenue

The seller's revenue is six times the price obtained in the auction. Thus, to measure the revenue efficiency of an auction, we simply compare the auction price to the seventh highest

[5] Kruskal-Wallis test is an extension of the Mann Whitney (rank-sum) test for more than two sub-samples.

value, *i.e.* the price the seller would have received, had all players bid their true values for every units.

$$\text{REseventh} = \frac{\text{auction price}}{\text{seventh highest value}} \times 100$$

However to measure the revenue efficiency, we also compare the revenue obtained to the revenue that the seller would get in perfect information and discriminatory pricing:

$$\text{REinfo} = \frac{\text{auction price} \times 6}{\text{sum of the six highest values}} \times 100$$

Allocation efficiency

Different criteria may be used to measure the allocation efficiency. A perfect efficiency would require that the 6 units are attributed to the bidder(s) who has (have) the six highest values. First, we can measure the efficiency by the ratio: number of units correctly attributed over the number of units to allocate (here 6).

$$\text{AEnum} = \frac{\text{number of unit correctly attributed}}{6} \times 100$$

However we may want a more accurate indicator, since a wrong attribution may be more or less harmful depending on the difference of the bidders' values. So we also define another allocation efficiency indicator: the ratio of the sum of the six values which correspond to the units that have been won over the sum of the six highest values. The closer to 100%, the higher the allocation efficiency.

$$\text{AEvalue} = \frac{\sum \text{values of the 6 won units}}{\sum \text{6 highest values}} \times 100$$

Experimental auction performances are summarized in Table 3 for the two scenarios. Since scenario 1 requires 3 times more bidders, we only have 33 observations when there are six small bidders, but we have 121 auctions with two large bidders.

Table 3 : Global performance in each scenario

	scenario 1					scenario 2				
Variable	Obs	Mean	Std. Dev.	Min	Max	Obs	Mean	Std. Dev.	Min	Max
price	33	39.79	14.16	17	73	121	34.58	17.81	1	90
seventh	33	41.33	12.07	23	61	121	42.61	13.03	20	84
REseventh	33	96.15	19.48	60	152	121	83.28	40.60	2	346
REinfo	33	55.58	16.59	21	86	121	47.26	22.10	1	138
AEnum	33	90.40	11.81	66.67	100	121	91.40	10.49	62.5	100
AEvalue	33	98.12	3.05	89	100	121	98.30	3.10	86	100

In the following comments, tests for normality lead us to use non parametric tests to compare Table 3 results. Significance is given at the 5% level.

4.2. Test of hypothesis 1: revenue efficiency

As predicted, the expected revenue for the seller is higher in scenario 1 (six small bidders) than in scenario 2 (two large bidders). The average auction price is 39.79 in scenario 1 and, as expected, is really close to the average seventh highest value. A Wilcoxon signed-rank test does not allow us to reject the null hypothesis that prices are equal to the corresponding seventh highest value in scenario 1 ($z = -0.989$). Thus, revenue efficiency is relatively high (REseventh = 96.15%) when there are six small bidders. In scenario 2, the average auction price is only 34.58 and is significantly lower than the seventh highest value ($z = -6.876$). Revenue efficiency is significantly lower (REseventh = 83.28%) than the one in scenario 1 with a two-sample Wilcoxon rank-sum (Mann-Whitney) test ($z = 2.724$). The revenue efficiency indicator REinfo is also significantly lower in scenario 2 ($z = 2.361$). In addition, standard deviations for the two revenue efficiency criteria are much higher in scenario 2. Support for hypothesis 3 will be developed in next section, however, this result is a first argument that confirms this hypothesis : auction results are less predictable with two large bidders. They depend on the bidders' ability to reach a tacit cooperative outcome. Indeed, we observe that very low prices have been reached in scenario 2, leading to very low minima for both revenue efficiency criteria (2% and 1%), whereas the minima in scenario 1 are 60% and 21%. We also observe an important difference in maximum values which are much higher in scenario 2 and prove some aggressive bidding when there are only two bidders. Actually, overbidding (at least from some bidders) is found in both scenarios since we observe revenue efficiency higher than 100%. This point will be detailed in section 5 on bidders strategies.

4.3. Test of hypothesis 2: allocation efficiency

For the allocation performance, results are relatively good in both scenarios. There is no significant difference among scenarios. Two-sample Wilcoxon rank-sum (Mann-Whitney) tests give z = -0.270 and z = -0.515 for AEnum and AEvalue respectively. On average, less than 10% of the units are attributed to bidders who have lower values. However, the allocation efficiency in value is still quite high (more than 98% on average) which means that the valuation of wrongly won units is not much lower (less than 2% on average) than the valuation of the bidders who should have won the 10% of misallocated units.

Therefore, we have seen that revenue efficiency is lower when small bidders are aggregated (or “collude”) to form two large bidders (or “cartels”), however this “cartel effect” does not significantly increase allocation efficiency. In this experimental framework, allocation is good and does not depend much on the competition structure. This confirms hypothesis 2.

Before analysing bidding strategies, a quick analysis of performances by groups or pairs for the 11 periods shows two very different outcomes.[6] For scenario 1, based on Kruskal-Wallis tests, we can not reject the null hypothesis of equality among the 3 groups composed of six subjects at the 5% level of confidence except for the variable REseventh. On the opposite, for scenario 2, we reject the null hypothesis of equality among the 11 pairs at the 5% level of confidence for every variable (price, REseventh, REinfo, AEnum and AEvalue). In particular, one pair (P08) presents very low performances (the average price reached by this pair is 7.64). As we are going to see now, this pair managed to reach a tacit cooperative equilibrium using very strategic bidding in most of the periods.

5. ANALYSIS OF BIDDING STRATEGIES

The objective of this section is to test hypothesis 3. We first present global results on bidding strategies; we then analyse individual bidding strategies and identify three types of bidders in scenario 2; we finally conduct regression analysis to explain bidders’ strategies.

[6] Result tables are not reported here, but are available upon request.

5.1. Underbidding, sincere bidding and overbidding

We examine all bids relative to the true value. In scenario 1, we have 396 bids, 198 on the first unit and 198 on the second unit. In scenario 2, we have 1452 observations, 242 bids for each of the six ordered units. Each pair (value, bid) is plotted in Figure 1 and Figure 2 (except 16 pairs in scenario 2 because corresponding bids are above 160). Figure 1 shows that in scenario 1, bids are correlated to values whereas Figure 2 is more fuzzy, revealing many bids at 100 and many at zero. The level of underbidding is limited by zero, but overbidding is not limited and we observe some very unreasonable high bids (maximum is 5000). We count 8 (2%) bids strictly above 100 in scenario 1 and 73 (5%) in scenario 2.

Figure 1 : Scenario 1

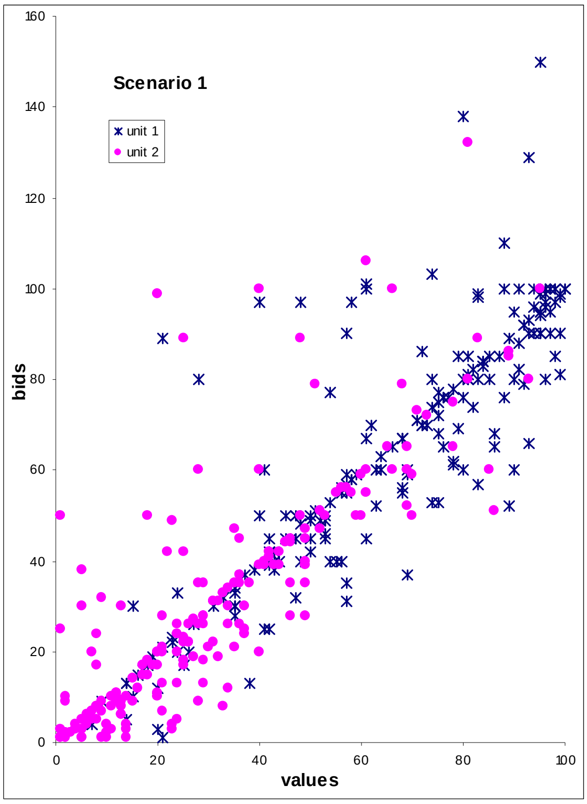
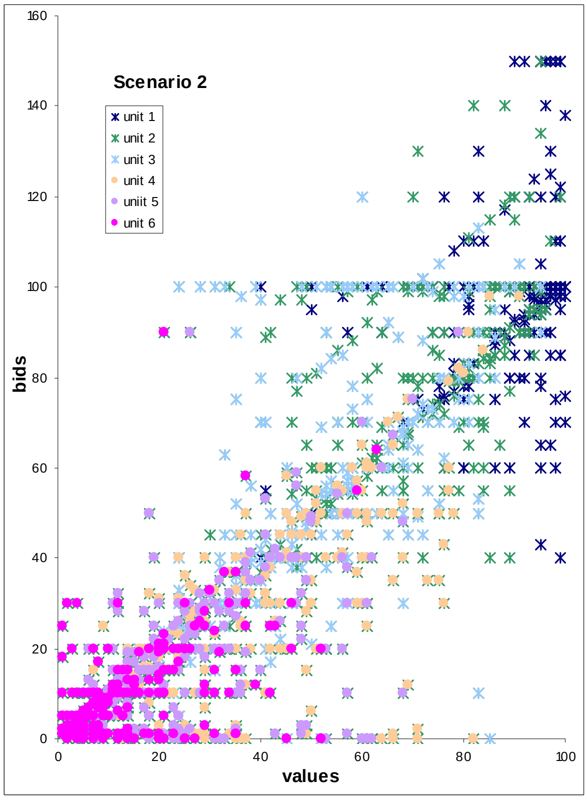


Figure 2: Scenario 2



Since figures 1 and 2 do not unveil any single clear bidding strategy, we first consider in Table 4 the proportion of bidders who underbid (bid < value), bid sincere (bid = value) and overbid (bid > value) by scenario and by ordered units. Naturally this results just give preliminaries indications on bidders' strategies since they do not reflect the intensity of underbidding or overbidding.

Table 4 : Proportion of underbids, sincere bids and overbids

Scenario 1	obs.	underbids	sincere bids	overbids
Unit 1	198	0.5707	0.2020	0.2273
Unit 2	198	0.6566	0.1515	0.1919
All units	396	0.6136	0.1768	0.2096
Scenario 2	obs.	underbids	sincere bids	overbids
Unit 1	242	0.2975	0.0537	0.6488
Unit 2	242	0.3512	0.0413	0.6074
Unit 3	242	0.4959	0.0331	0.4711
Unit 4	242	0.7521	0.0165	0.2314
Unit 5	242	0.7438	0.0372	0.2190
Unit 6	242	0.6942	0.0992	0.2066
All units	1452	0.5558	0.0468	0.3974

Contrary to our intuitions, bidders underbid on more units in scenario 1 (61,36%, we expected 50%: 0% on the first unit and 100% on the second unit) than in scenario 2 (55.58%). However, in scenario 1, results are quite similar for the first and the second units. Surprisingly, 57% of the bids for the first unit are below the corresponding value, although it is a dominant strategy to bid at least one's value for the first unit. Moreover, almost 20% of the bidders bid above their value for the second unit although overbidding for the second unit goes against theoretical results. Nevertheless, two third of bidders shade their second bid as predicted by the theory.

In scenario 2, we clearly see a different strategy for the first three units and the last three units. As predicted, the proportion of bidders who underbid on the fourth unit is the most important (75%). But, as in scenario 1, we still observe many bidders (more than 20%) who bid above their values for the last three units. Nevertheless, this proportion is much lower than for the first three units. Actually we observe quite aggressive bidding for the first units when there are only two bidders, as if bidders absolutely wanted to win at least some of the offered units. These results confirm hypothesis 3 that bidders are more strategic in scenario 2 than in scenario 1 where the proportion of sincere bids is much higher.

To gain more insights into bidders' behaviour we analyse the difference between bids and true values: $BminusV = B - V$ measures the degree of over or underbidding. To prevent bias from observations with extremely high bids, all bids above 100 are set to 100. Indeed, underbidding is limited by zero, since negative bids are not allowed; so in order to analyse underbidding and overbidding on the same scale we do as if bidders were not allowed to bid above 100.

First, results on *BminusV* from Table 5 confirm previous findings. In scenario 1, bidders are relatively sincere, the means of *BminusV* are close to zero and are not significantly different for units 1 and unit 2[7]. In scenario 2, on average, subjects overbid on units 1, 2 and 3 and underbid on units 4, 5 and 6. With a two sample Wilcoxon rank-sum (Mann-Whitney) test, we reject the hypothesis that *BminusV* are the same for the first three and the last three units. In addition, the mean of the amount underbid is the highest for unit 4 (-10.68). Again, these results support hypothesis 3.

Table 5 : Statistics on *BminusV* with max bid = 100

scenario 1	obs.	Mean	St. Dev.	Min	Max
unit 1	198	-1.26	13.72	-37	68
unit 2	198	-0.27	14.43	-35	79
scenario 2	obs.	Mean	St. Dev.	Mean	St. Dev.
unit 1	242	3.78	15.06	-59	60
unit 2	242	7.71	17.84	-49	66
unit 3	242	3.86	21.85	-85	76
unit 4	242	-10.68	17.41	-82	64
unit 5	242	-7.80	15.22	-62	64
unit 6	242	-4.16	11.06	-52	69

5.2. Types of bidders

When examining bidding strategies in scenario 1, we observe that bidders have the same behaviour for the two units in general: bidders who underbid (overbid) on the first unit also underbid (overbid) on the second, and sincere bidders bid their true values (or very close) on both units. Only 5 players (28%) have a more erratic behaviour which is difficult to categorize.

On the opposite, we can roughly distinguish three types of bidders in scenario 2. First, there is a majority of sincere bidders (45%) who bid close to their true values on the first units then bid sincere or underbid a little on the last units (see Figure 3). Second, there are bidders, we call them “strategic bidders”, who significantly overbid on the first units (many of them bid

[7] Skewness/Kurtosis test for Normality lead to reject the null hypothesis: *BminusV* is not Normally distributed, so we use non parametric tests to make comparisons across scenarios and across first and last units within a scenario.

100 or even above) and underbid on the last units (many of them bidding zero). We count 9 such strategic bidders, *i.e.* 41% of the 22 subjects in scenario 2 (see Figure 4). Finally, as in scenario 1, we also observe few bidders (14% in scenario 2) whose behaviour is difficult to categorize (see Figure 5).

Figure 3 : Sincere bidder

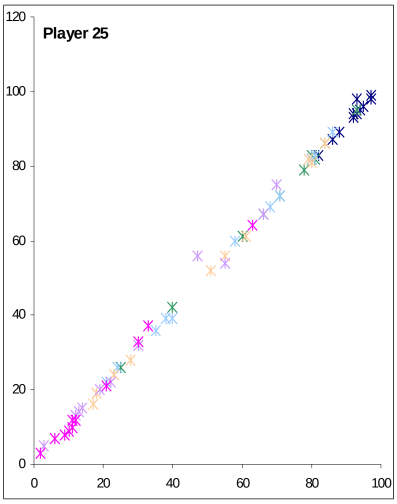


Figure 4 : Strategic bidder

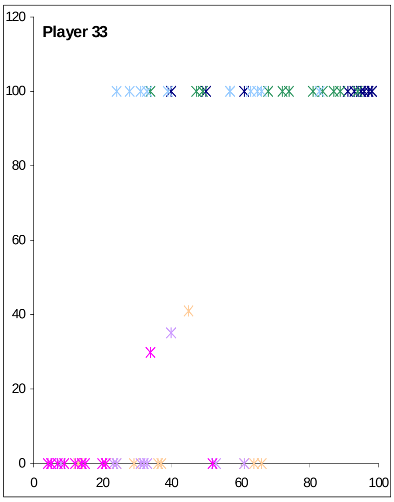
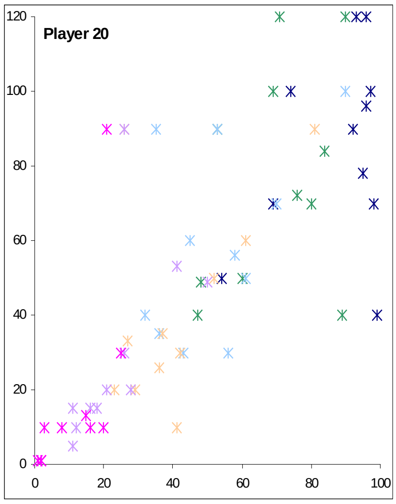


Figure 5: Erratic bidder



5.3. *Regressions on bidding strategies*

Finally we conduct ordinary least square regressions to explain the bidders' bids. First, we propose a simple linear model (1) which seeks to explain the bids by the values of corresponding units.

$$B_i = \alpha + \beta V_i + \varepsilon_i \qquad (1)$$

Then, to compare bidding behaviour in both scenarios, we introduce in model (2) a dummy variable '*last*' (L_i).

$$B_i = \alpha + \beta V_i + \gamma L_i + \varepsilon_i \qquad (2)$$

with, in scenario 1: $L_i = 0$ for unit 1, $L_i = 1$ for unit 2,

in scenario 2: $L_i = 0$ for units 1, 2 and 3, $L_i = 1$ for units 4, 5 and 6.

Results are presented in Table 6.

Table 6 : OLS Regression results

	Scenario 1		Scenario 2	
	(1)	(2)	(1)	(2)
value (V_i)	0.9601***	0.9479***	1.7548***	1.2999***
	(37.57)	(30.30)	(9.49)	(4.32)
last (L_i)		-1.2478		-36.9446**
		(-0.68)		(-2.08)
_cons	1.5568	2.7613	-19.2743	21.28
	(1.09)	(1.21)	(-1.64)	(0.94)
Number of obs.	396	396	1452	1452
Adj R-squared	0.7813	0.7810	0.0467	0.0489

t statistics in parentheses * p < 0.10, ** p < 0.05, *** p<0.01

We obtain a coefficient of determination of 78% in scenario 1, but less than 5% in scenario 2. However, the variable '*value*' is highly significant in both scenarios. Adding the dummy variable '*last*' in model (2) confirms previous findings. In scenario 1, variable '*last*' is not significant, thus the bidding strategy for unit one is not significantly different from the bidding strategy for the second unit. In scenario 2, the estimated coefficient associated to the variable '*last*' is significantly negative at the 5% level of confidence. Thus, bidding strategies for units 1, 2 and 3 display a significant mark-up compared to bidding stratgies for units 4, 5 and 6.

We conduct separate regressions when *last* = 0 and *last* = 1 in both scenarios. Results are reported in Table 7.

Table 7 : OLS Regression results

	Scenario 1		Scenario 2	
	last = 0	*last* = 1	*last* = 0	*last* = 1
	B_1	B_2	B_1, B_2, B_3	B_4, B_5, B_6
value	0.9581***	0.9356***	1.8421***	0.6472***
	(22.45)	(20.26)	(3.21)	(25.14)
_cons	2.1044	1.8913	-16.6883	2.0040**
	(0.71)	(1.07)	(-0.40)	(2.34)
Number of obs.	198	198	726	726
Adj R-squared	0.7186	0.6751	0.0127	0.4654

t statistics in parentheses * p < 0.10, ** p < 0.05, *** p<0.01

Even if α and β are slightly lower for the second unit, as expected in scenario 1, coefficients are not significantly different than the ones estimated for the bids on the first unit. Moreover, coefficients of determination stay relatively high in both cases. On the opposite, in scenario 2, coefficients are much different and coefficient of determination is over 46% when *last* = 1, but is only 1.4% when *last* = 0. Indeed, when we run linear regressions on the 1st, the 2nd and the 3rd bids separately, bids do not significantly depend on corresponding values in scenario 2. These regression results also indicate that the underbidding is more important on last units in scenario 2 than the underbidding on the second unit in scenario 1 (0.6472 versus 0.9356) even if coefficients associated to the constant are not rigorously the same.

There is no improvement in the models when we add the values of the other units as explanatory variables:

$$B_{ki}=\alpha+\beta_1 V_{1i}+\beta_2 V_{2i}+\varepsilon_i \quad \text{in scenario 1}$$

$$B_{ki}=\alpha+\beta_1 V_{1i}+\beta_2 V_{2i}++\beta_3 V_{3i}+\beta_4 V_{4i}+\beta_5 V_{5i}+\beta_6 V_{6i}+\varepsilon_i \quad \text{in scenario 2}$$

with k = 1,2 in scenario 1 and k = 1 to 6 in scenario 2.

Results are not reported here, but as obtained in previous regressions, only the value for the unit corresponding to the bid is significant, except for the first three bids in scenario 2. This last result is due to the strategic bidders who overbid systematically on their first units.

6. CONCLUSION

Although multi-unit auctions are widely used in practice and while the multi-unit auction literature is being rapidly enriched (theoretical models, empirical studies, experimental auctions), no study analyses explicitly the impact of the competition structure on multi-unit auction performances. We show with this paper that the competition structure has an impact on bidding strategies even when the degree of rationing is the same. Our experiments indicate that, with constant degree of rationing, when the number of bidder decreases while individual demand increases, there is more strategic bidding leading to lower expected revenue with a higher variance. Performance in terms of allocation efficiency is less sensitive to the competition structure.

Therefore, when designing a multi-unit auction, it is important to consider the question of the competition structure. Whereas the seller is usually not in capacity to increase the degree of rationing (unless renouncing to sell all his units), we show that if he can determine who is eligible to bid and what is the size of the unit, *i.e.* the minimum amount one can bid on, he can improve his expected revenue.

Naturally, in the absence of theoretical predictions from auction models, more experimental works need to be done to confirm our findings. Moreover, many other questions remain unanswered and deserve attention. For example, what would be the results with heterogeneous bidders, *i.e.* when bidders do not have the same individual demands? An extension of this work could include a third scenario with one large bidder wanting six units facing three smaller bidders each wanting two units.

REFERENCES

Alsemgeest, P., Noussair, C. and Olson M. (1998), Experimental Comparisons of Auctions under Single and Multi-unit Demand, *Economic Inquiry* XXXVI, 87-97.

Ausubel, L. and Cramton, P. (2002), Demand Reduction and Inefficiency in Multi-Unit Auctions, *miméo*. University of Maryland, Department of Economics.

Binmore K. and Swierzbinski, J. (2000), Treasury auctions: uniform or discriminatory?, *Review of Economic Design* 5, 387-410.

Chakraborty, I. (2006), Bundle and separate sales in auctions with entry, *Economic Theory* 29, 197-211.

Chan C., P. Laplagne, and Appels D. (2003), The role of auctions in allocating public resources, *Staff Research Paper, Productivity Commission*, Melbourne, 144 pages.

Engelbrecht-Wiggans, R. and Kahn, C. (1998), Multi-Unit Auctions with Uniform Prices, *Economic theory* 12, 227-58.

Engelbrecht-Wiggans, R., List, J. and Lucking-Reiley, D. (2006), Demand Reduction in Multi-Unit Auctions with Varying Numbers of Bidders : Theory and Evidence from a Field Experiment, *International Economic Review*, 47, 203-31.

Engelmann, D and Grimm, V (2006), Bidding Behavior in Multi-Unit Auctions - An Experimental Investigation, *miméo*.

Evrard, M. (2006), L'intérêt des enchères multi-unitaires pour la gestion environnementale: une approche expérimentale sur les relations concurrence-performances, *Master thesis*, DEA EDAAR, Montpellier Supagro, 95 pages.

Hailu, A. et Thoyer, S. (2005), Multi-Unit Auctions to Allocate Water Scarcity Simulating bidding behaviour with agent based models, *Lameta working paper*, 2005-1, 32 pages (http://www.lameta.univ-montp1.fr/Fr/Productions/Documents/WP2005-01.pdf)

Kagel, J. and Levin D. (2001), Behavior in Multi-Unit Demand Auctions: Experiments with Uniform Price and Dynamic Auctions, *Econometrica* 69, 413-54.

Krishna, V. (2002), *Auction theory.* Academic Press, Elsevier.

List, J. and Lucking-Reiley D. (2000), Demand Reduction in Multi-Unit Auctions: Evidence from a Sportscard Field Experiments, *American Economic Review* 90, 961-72.

Noussair, C. (1995), Equilibria in a Multi-Object Uniform Price Sealed Bid Auction with Multi-Unit Demands, *Economic Theory* 5, 337-51.

Porter, D. and Vragov R. (2006), An experimental examination of demand reduction in multi-unit versions of the Uniform-price, Vickrey, and English auctions, *Managerial and Decision Economics* 27, 445-58.

Tenorio, R. (1999), Multiple unit auctions with strategic price-quantity decisions, *Economic Theory*, 13, 247-60.

Wang, J. and Zender J. (2002), Auctioning Divisible Goods, *Economic Theory* 19, 673-705.

Wilson, R. (1979), Auctions of Shares, *Quarterly Journal of Economics* 93, 675-698.

Wolfram, C. (1998), Strategic bidding in a multiunit auction: An empirical analysis of bids to supply electricity in England and Wales, *Rand Journal of Economics* 29, 703-25.

APPENDIX 1

We consider scenario 1: six bidders have a decreasing demand for two units only.

To prove that bidding sincere on the first unit and zero for the second unit is not a Nash equilibrium in this scenario, we propose a deviation strategy:

$\beta_1(x_1, x_2) = x_1$ and $\hat{\beta}_2(x_1, x_2) > 0$ for a sufficiently high value of x_2,

with x_1 and x_2 the values of unit 1 and 2 respectively, and βi the bidding strategy on unit i, i = 1, 2.

We look for the optimal $\hat{\beta}_2$ when all other bidders follow the strategy:

$\beta_1(x_1, x_2) = x_1$ and $\beta_2(x_1, x_2) = 0 \ \forall \ x_1$ and x_2.

Thus, the deviating bidder's objective function is to maximize the following expected profit:

$$\underset{\hat{\beta}_2}{\text{Max}}\ \mathrm{E}[\pi] = \left[1-\Pr\left(X<\hat{\beta}_2\right)\right](x_1-\hat{\beta}_2)+\Pr(X\le\hat{\beta}_2)\left(x_1+x_2-2\mathrm{E}\left[X\middle|X<\hat{\beta}_2\right]\right)$$

X is the fifth highest bid of his five opponents who bid sincere on their first unit and zero on their second unit. X is also the smallest non zero bid of his five opponents. Therefore X is the unknown price the bidder will have to pay if he wins two units, *i.e.* if $X \le \hat{\beta}_2$.

Each bidder receives two values from a uniform distribution on [0; 100]. From the order statistics, the density and cumulative functions of x_1 (the highest of the 2 received values) are:

$$f_1^{(2)}(x) = 2\frac{x}{100^2} \text{ and } F_1^{(2)}(x) = \frac{x^2}{100^2}$$

The density function of X (the fifth highest value from 5 independent random variables with density function $f_1^{(2)}(x)$) is:

$$\varphi_5^{(5)}(X) = \frac{5!}{0!4!}\left(F_1^{(2)}(X)\right)^0\left(1-F_1^{(2)}(X)\right)^4 f_1^{(2)}(X) = \left(1-\frac{x^2}{100^2}\right)^4\frac{x}{1000}$$

The probability the deviating bidder wins two units is:

$$\Pr\left(X<\hat{\beta}_2\right) = \int_0^{\hat{\beta}_2}\varphi_5^{(5)}(y)dy = 10\left(\frac{\hat{\beta}_2^{\,2}}{2\times100^2}-\frac{\hat{\beta}_2^{\,4}}{100^4}+\frac{\hat{\beta}_2^{\,7}}{100^6}-\frac{\hat{\beta}_2^{\,8}}{2\times100^8}+\frac{\hat{\beta}_2^{\,10}}{10\times100^{10}}\right)$$

The expected price if he wins two units is:

$$\mathrm{E}\left[X\middle|X<\hat{\beta}_2\right] = \int_0^{\hat{\beta}_2}y\varphi_5^{(5)}(y)dy = 10\left(\frac{\hat{\beta}_2^{\,3}}{3\times100^2}-\frac{4\hat{\beta}_2^{\,5}}{5\times100^4}+\frac{6\hat{\beta}_2^{\,7}}{7\times100^6}-\frac{4\hat{\beta}_2^{\,9}}{9\times100^8}+\frac{\hat{\beta}_2^{\,11}}{11\times100^{10}}\right)$$

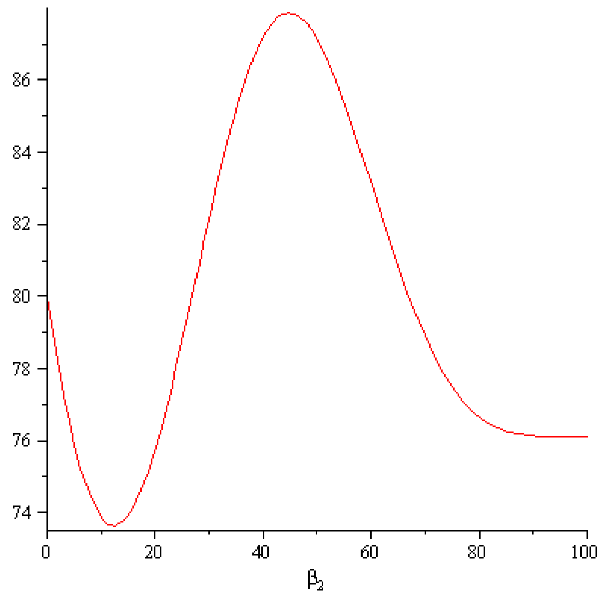
The expected profit is then:

$$\mathrm{E}[\pi] = \left[1-\int_0^{\hat{\beta}_2}\varphi_5^{(5)}(y)dy\right](x_1-\hat{\beta}_2)+\left[\int_0^{\hat{\beta}_2}\varphi_5^{(5)}(y)dy\right]\left(x_1+x_2-2\mathrm{E}\left[X\middle|X<\hat{\beta}_2\right]\right)$$

Using Maple software we plot $\mathrm{E}[\pi]$ with $\hat{\beta}_2$ on the x-axis going from 0 to 100, for many given values x_1 and x_2. We observe that if x_2 is high enough, the expected profit can be higher for some $\hat{\beta}_2 > 0$ than with $\beta_2 = 0$. Therefore, bidding $\beta_1(x_1, x_2) = x_1$ and $\beta_2(x_1, x_2) = 0$ $\forall$ x_1 and x_2 is not a Nash equilibrium.

For example, if a bidder receives $x_1 = 80$ and $x_2 = 70$, his profit will be 80 if he follows the strategy $\beta_1(x_1, x_2) = x_1$ and $\beta_2(x_1, x_2) = 0$ as the other five players do. However, in the following Graph 1, we see that his expected profit will be a little bit higher than 87 if be bids around 47 for the second unit.

Graph 1 : Expected revenue when $x_1 = 80$ and $x_2 = 70$



Actually, the bidding strategy $\beta_1(x_1, x_2) = x_1$ and $\beta_2(x_1, x_2) = 0 \ \forall \ x_1$ and x_2 is a Nash equilibrium when $N = K = 2$. However, as soon as $N = K > 2$, there are profitable deviations from this type of strategy if x_2 is high enough.

APPENDIX 2

We consider, scenario 2: two bidders (1 and 2) have a decreasing demand for six units.

We look for a profitable deviation for bidder 1 when bidder 2 plays:

$\beta_k(x_1, x_2, x_3, x_4, x_5, x_6) = x_k$ when $k = 1, 2, 3$;

$\beta_k(x_1, x_2, x_3, x_4, x_5, x_6) = 0$ when $k = 4, 5, 6$.

If bidder 2 adopts the same "collusive" strategy, his profit is then $x_1 + x_2 + x_3$ since he wins three units and the price is zero. We wonder if it is in bidder's 1 interest to win a fourth unit.

Let's assume that bidder 1 submits $\hat{\beta}_4 > 0$. His objective is to maximize his expected profit.

$$\underset{\hat{\beta}_4}{\text{Max}} \ \mathrm{E}[\pi] = \left[1 - \Pr\left(c_3 < \hat{\beta}_4\right)\right](x_1 + x_2 + x_3 - 3\hat{\beta}_4) + \Pr(c_3 \leq \hat{\beta}_4)\left(x_1 + x_2 + x_3 + x_4 - 4\mathrm{E}\left[c_3 \middle| c_3 < \hat{\beta}_4\right]\right)$$

c_3 is the third highest value out the six values of bidder 2. The density and the cumulative functions of c_3 are:

$$f_3^{(6)}(x) = 0.6\left(\frac{x}{100}\right)^3\left(1 - \frac{x}{100}\right)^2$$

$$F_3^{(6)}(x) = \int_0^x f_3^{(6)}(y)dy = \frac{60}{100^4}\left(\frac{x^4}{4} - \frac{2x^5}{500} + \frac{x^6}{6\times 100^2}\right)$$

The expected value of c_3 (the auction price) when bidder 1 wins four units is:

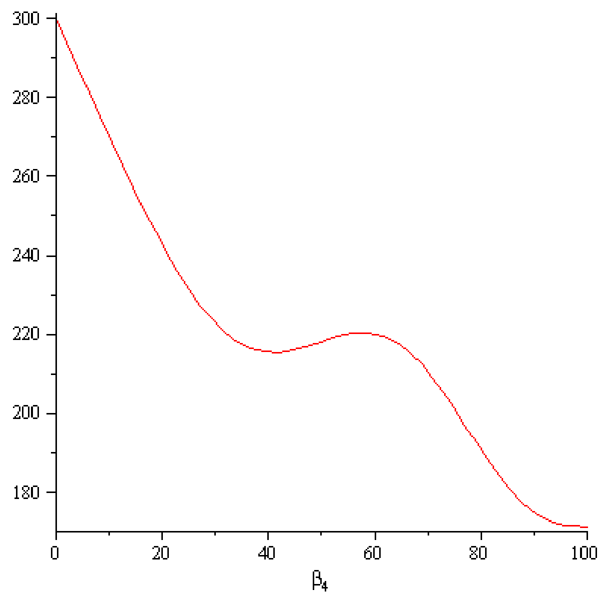
$$\mathrm{E}\left[c_3 \middle| c_3 < \hat{\beta}_4\right] = \int_0^{\hat{\beta}_4} y f_3^{(6)}(y)dy = \frac{60}{100^4}\left(\frac{\hat{\beta}_4^{\,5}}{5} - \frac{\hat{\beta}_4^{\,6}}{300} + \frac{\hat{\beta}_4^{\,7}}{7\times 100^2}\right)$$

Bidder's 1 expected profit is then:

$$\mathrm{E}[\pi] = \left[1 - F_3^{(6)}(x)\right](x_1 + x_2 + x_3 - 3\hat{\beta}_4) + F_3^{(6)}(x)\left(x_1 + x_2 + x_3 + x_4 - 4\mathrm{E}\left[c_3 \middle| c_3 < \hat{\beta}_4\right]\right)$$

Using Maple software we plot $\mathrm{E}[\pi]$ with $\hat{\beta}_4$ on the x-axis going from 0 to 100, with values $x_1 = x_2 = x_3 = x_4 = 100$. From Graph 2, we see that it is not possible to increase the expected profit of bidder 1 with $\hat{\beta}_4 > 0$. As a result, since there is no profitable deviation even in the most favourable case (*i.e.* when $x_1 = x_2 = x_3 = x_4 = 100$), bidding one's true values for the first three units and zero for the last three units is a Nash equilibrium in scenario 2.

Graph 2 : Expected revenue when $x_1 = x_2 = x_3 = x_4 = 100$



APPENDIX 3

Figure A1 : Scenario 1

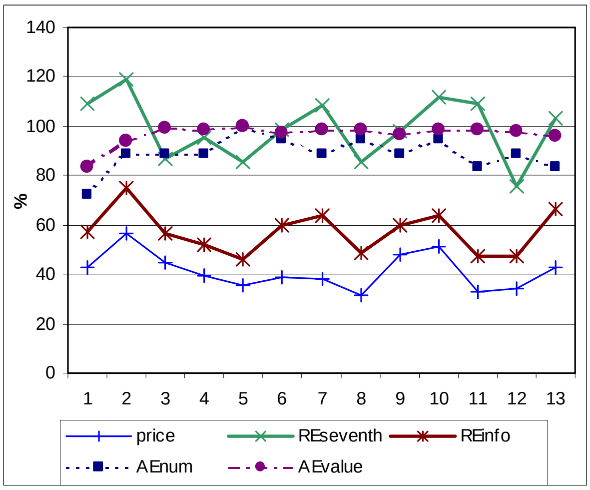


Figure A2 : Scenario 2