Not So Cheap Talk:
A Model of Advice with Communication Costs

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Abstract

We model a game similar to the interaction between an academic advisor and advisee. Like the classic cheap talk setup, an informed player sends information to an uninformed receiver who is to take an action which affects the payoffs of both sender and receiver. However, unlike the classic cheap talk setup, the preferences regarding the receiver's actions are identical for both sender and receiver. Additionally, the sender incurs a communication cost which is increasing in the complexity of the message sent. We characterize the resulting equilibria. We show that if communication is costly then there is no equilibrium in which communication is complete. Under one out-of-equilibrium condition, our equilibrium is analogous to that found in Crawford and Sobel (1982). Under a more restrictive out-of-equilibrium condition, our equilibrium is analogous to that under the No Incentive to Separate (NITS) condition as discussed in Chen, Kartik and Sobel (2008). Finally, we model the competency of the advisee by the probability that the action is selected by mistake. We show that the informativeness of the sender is decreasing in the likelihood of the mistake. Therefore, we expect the informativeness of the relationship to be increasing in the competency of the advisee.

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1 Introduction

Consider the interaction between advisor and advisee in the preparation of a job market paper. The advisor takes a look at the current state of the paper and has a significantly better idea of its shortcomings than does the advisee. Further, the advisor prefers that the advisee correct these shortcomings so that the paper is successful on the job market. The advisor prefers success because either this will reflect well on the advisor or perhaps the advisor might have an intrinsic preference for the success of the advisee. The advisee obviously prefers to correct those shortcomings in order to secure employment, however the nature of the necessary corrections are not known to the advisee. Although there are material incentives for both advisee and advisor to correct the shortcomings, accomplishing this requires the advisor to take time to communicate the nature of these shortcomings. Such communication is costly for the advisor as it would take time out of her busy schedule. Therefore, the advisor decides on the optimal level of detail to communicate to the advisee: more detail increases the quality of the paper but also implies greater communication costs borne by the advisor.

In this paper we analyze the strategic interaction between an informed sender and an uninformed receiver. In our model, the sender learns the state of the world and transmits a message to the receiver. Based on the message, the receiver is to take an action which affects the payoffs of both sender and receiver. We deviate from the classic cheap talk setup in that the sender and the receiver have identical preferences regarding the action of the receiver. However, the sender faces communication costs which are increasing in the complexity of the message.

We assume that the sender has an infinite number of messages m_i where $i \in \{0, 1, 2, ...\}$ available for transmission. We assume that the complexity of the message is increasing in the subscript and that a more complex message is more costly to send. Specifically, we assume that the cost of transmitting message m_i is equal to ci, where $c > 0$. For instance, one can interpret the costless message m_0 as the empty message, $\varnothing$., the message m_1 as a single element, $\{0\}$, the message m_2 as two elements, $\{0, 0\}$ and so on. We view this as the simplest way to model complex communication.

Analogous to the cheap talk literature, equilibrium is partitional: a unique action is induced on connected intervals of the state space. We show that the equilibrium is efficient in that no signal is used in equilibrium when there is an unused, cheaper one available. We show that under a relatively permissive out-of-equilibrium condition (Condition M) the equilibrium is analogous to the multiplicity found in the original cheap talk model. We show that under a relatively restrictive out-of-equilibrium condition (Condition L), only the most informative class of equilibria survives. This result is analogous to No Incentive to Seperate ($NITS$) refinement of the cheap talk model. Finally, we model the competence of the advisee by the probability that the advisee makes a mistake in selecting an action. We show that the informativeness of the sender is decreasing in the probability of a mistake. We interpret this result as suggesting that the informativeness of the advising relationship is increasing in the competency of the advisee.

2 Related Literature

Despite that every economist has negotiated a relationship with their advisor in graduate school and that many continue to perform the complementary role of advisor, this relationship has garnered relatively little attention in the literature. There are however three related strands of literature, each of which focuses on different issues than we do here. For instance the cheap talk literature examines settings in which communication is costless and the players have different preferences over the action taken. However, we focus on a setting in which preferences over the action taken are identical and communication is costly. The existing costly communication literature tends to focus on cases where information is either understood or not. However, in our model there can be shades of understanding. Finally, we discuss the empirical literature on the academic advising relationship.

2.1 Cheap Talk and Related Models

The large strand of cheap talk literature was initiated by Crawford and Sobel (1982), hereafter referred to as CS. In the CS model, an informed sender learns the state of the world and decides to communicate some information to an uninformed receiver where the receiver is to take an action which affects the payoffs of both sender and receiver. However, given any state of the world, the sender and receiver have different preferences over the action of the receiver. The authors show that for mild differences in the preferences of receiver and sender, meaningful communication can occur. Additionally, the authors show that there is no equilibrium in which communication is complete. CS shows that equilibrium always takes the form that the state space is partitioned and the messages are sent such that a unique action is induced within each element of the partition. Our equilibrium is analogous in that a unique message is sent on an interval. We also find that for any nonzero communication costs, the communication is never complete.

A number of papers have extended the original CS model. Morgan and Stocken (2003) extend the CS model to the case where there is uncertainty regarding the degree of divergence between the preferences of the sender and receiver. Fischer and Stocken (2001) model a situation where receiver has imperfect information about the state. Blume, Board and Kawamura (2007) modify the CS setup where communication errors (or noise) can occur. We view our paper in the spirit of these papers, as we wish to learn the importance of a particular assumption: the presence of communication costs which are increasing in the complexity of the message sent.

The original CS model exhibits a large number of possible equilibria. Specifically, CS shows that for a given difference in the preferences of the sender and receiver, if there is an equilibrium where the state space is partitioned into a finite number of partitions (say n) then there are equilibria which partition the state space into 1, 2,.. and $n-1$ elements. Our out-of-equilibrium Condition M leads to a similar result in that, for a given set of parameter values, there exists a maximum number of messages (again say n) which could constitute an equilibria. Additionally under Condition M, there are equilibria where the number of messages equals 1, 2,... and $n-1$.

As is often the case for multiple equilibria, researchers have sought to reduce the number of cheap talk equilibria through refinements.[1] A recent innovation in this regard is the

[1] For instance, see Farrell (1993), Banks and Sobel (1987), Kohlberg and Mertens (1987), Matthews, Okuno-Fujiwara and Postlewaite (1991).

Condition *No Incentive to Seperate* ($NITS$) as discussed in Chen, Kartik and Sobel (2008). This condition restricts attention to equilibria in which it is not the case that the sender type who receives the lowest possible state ($s = 0$) does not prefer to perfectly reveal the state. In their Proposition 3, the authors show that if the monotonicity condition holds in the CS model (as it does in the commonly used "uniform-quadratic" case) then $NITS$ selects a unique equilibrium which is the most informative, i.e. contains the largest possible number of partitions. In our paper, the equilibria under Condition L is similar to $NITS$ in that if an out of equilibrium message is observed then the beliefs of the receiver are such that that the state which is *worst off* in the equilibrium. In our model, for a given equilibrium, several states qualify as the worst and so as a matter of convention we select the smallest of these, state $s = 0$. In other words, Condition L specifies that if the receiver sees an out-of-equilibrium message then the receiver believes that the state is certain to be $s = 0$. And similar to $NITS$, Condition L rules out each equilibria except the most informative class of equilibria.

Morris (2001) presents a model where an informed sender and uninformed receiver have identical preferences over the action of the receiver but due to reputation effects, the sender might not truthfully reveal the state of the world. To our knowledge, Morris is the only other communication paper to assume that the sender and receiver have identical preferences over the receiver's action.

2.2 Costly Communication

Dewatripont and Tirole (2005) present a communication model where the sender incurs costs of effectively communicating the information and the receiver incurs costs in better absorbing the information. In Dewatripont and Tirole information is either understood or not, by contrast the states in our model are better characterized by the degree to which they are learned.

In Austin-Smith (1994), information acquisition comes at a cost to the sender. Although the receiver cannot verify that the sender is uninformed, the receiver can verify that sender is informed. Austin-Smith shows that the ex-ante uncertainty about the receiver being informed enlarges the set of parameters in which there is an informative equilibrium in the CS model. However, by contrast to our model the sender is completely informed or completely uninformed.

To our knowledge, Calvo-Armengol et. al. (2009) is the only example of a costly communication paper in which with there can be shades of understanding. There the sender transmits a necessarily noisy signal but can affect the precision of the communication by a incurring larger communication cost. In our view this assumption is less appropriate when modeling complex communication as the signal actually sent is not necessarily less complex than the sender's most preferred signal.

2.3 Empirical Literature

Relevant aspects of our model appear in the academic advising literature. For instance, Knox et. al. (2006) discuss the costs and benefits of being an academic advisor.[2] The benefits of advising include the personal satisfaction involved in guiding a student. Hence, we find support for our assumption that advisor and advisee have identical preferences over the action of the advisee. The costs of advising are primarily composed of the time and energy required by the relationship. Therefore, we regard these as supporting our specification of the payoffs of the advisor.

Schlosser and Kahn (2007) find that advisor and advisee often share the same impression of quality of relationship and of the advisee's competency. We interpret this as confirming the appropriateness of our information assumptions. Additionally, Green and Bauer (1995) find that more capable students receive more supervisory attention than less capable students. Corcoran and Clark (1984) find that more successful researchers received better sponsorship from graduate school advisors than less successful researchers.[3] These findings are in line with Proposition 5, which shows that the informativeness of the sender is decreasing in the probability that the receiver makes a mistake in selecting an action.

[2] See Schlosser et. al. (2003) and Schlosser and Gelso (2001) for more on the measurement of the advisee's preferences.

[3] Also see Hollingsworth and Fassinger (2002).

3 Model

A sender S and receiver R play a communication game in a single period. Payoffs for both players depend on the receiver's action a, as well as the state of the world s. A state is an element of the closed interval $\mathcal{S}$ =$[0, 1]$. The receiver's action space $\mathcal{A} = [0, 1]$ is equal to the state space $\mathcal{S}$. The receiver's utility from action a when the state is s is:

$$u^R(a, s) = s - (a - s)^2.$$

States are not payoff equivalent: it is better to be accurate with state s than state $s' < s$.

The receiver has ex-ante beliefs that the state is uniformly distributed on $\mathcal{S}$. The sender, observes the state and can communicate some information about the state to S, by sending a message m where $m \in \mathcal{M} = [m_0, m_1, ...]$. We interpret message m_i as more *complex*, and therefore more costly send, than m_j if $i > j$. Specifically, communication costs ($c : \mathcal{M} \Rightarrow\mathbb{R}$) are such that $c(m_j) = jc$. The sender has the same preferences over actions as R, however incurs a cost of communication. Therefore, her utility is:

$$u^S(a, m, s) = s - (a - s)^2 - c(m)$$

Note that unlike the cheap talk literature, both S and R have identical preferences over the action of R: both prefer $a = s$.

The sender's strategy is then $\mu : \mathcal{S} \to \mathcal{M}$. The receiver's strategy is then $\alpha : \mathcal{M} \to \mathcal{A}$. We seek an equilibrium where S chooses the optimal action, given beliefs R chooses the optimal action and R's beliefs are derived from Bayes' Rule wherever possible. R's beliefs are denoted $\beta(s|m)$. Specifically, we require that

$$\mu \text{ such that for each } s \in [0, 1],\ m \text{ solves } \max_m u^S(a, m, s) \tag{1}$$

$$\alpha \text{ such that for each } m \in M,\ a(m) \text{ solves } \max_a \int u^R(a, s)\beta(s|m)ds \tag{2}$$

and R's beliefs are derived from S's strategy.

As stated earlier, R uses Bayes' Rule whenever possible, however we have yet to specify the out-of-equilibrium beliefs. We will use one of the following two out-of-equilibrium specifications of beliefs. The first, Condition L, specifies that if an out-of-equilibrium message is observed then R believes that among the states which are farthest from the ideal (s which has the largest $(a-s)^2+c(m)$), the state is the smallest of these. Effectively, if an out-of-equilibrium message is observed then under Contidion L the receiver believes that the state is $s=0$.

Condition L: Given $(\mu,\alpha;c)$, if there does not exist an $\widehat{s}$ such that $\mu(\widehat{s})=\widehat{m}$ and R observes $\widehat{m}$ then R believes that S is certain to be the smallest state s' among those states where $s=\arg\max_s(a-s')^2+c(m')$.

For a given equilibrium with n messages there will be $n+1$ states[4] which will satisfy $\arg\max_s(a(m')-s')^2+c(m')$, so as a matter of convention, R has beliefs that the state is the smallest of these. In practice, this means that Contition L specifies that after observing an out-of-equilibruim message, R believes the state is certain to be $s=0$. Therefore, to check for a deviation using an out-of-equilibrium signal, Condition L specifies that it suffices to check for $s=0$. Note that Condition L is very close in spirit to $NITS$.

The second condition which we consider, Condition M, specifies that if an out-of-equilibrium message is observed then R believes that the state is, among those which are identical to an action induced, has the largest communication cost.

Condition M: Given $(\mu,\alpha;c)$, if there does not exist an $\widehat{s}$ such that $\mu(\widehat{s})=\widehat{m}$ and R observes $\widehat{m}$ then R believes with certainty that the state s' with the largest $c(m')$ among those states where $a(m')=s'$.

Condition M supports "more" equilibria and Condition L supports "less." This is because under Condition M an out-of-equilibrium message does not induce an a which is not used in equilibrium and it is therefore relatively difficult to find a deviation from an equilibrium. However, under Condition L an out-of-equilibrium message does induce an a which is not used in equilibrium and so it is relatively easy to find a deviation.

[4] See Lemma 5.

Before we proceed to the results, we briefly discuss some of our modeling choices. First, we designed the model in order to avoid the issue of misrepresentation therefore we model the message space and state space as distinct ($\mathcal{M} \neq \mathcal{S}$). There exists evidence that in experimental settings, meaningful communicion can occur even when there is no a priori meaningful language.[5] Second, the state space is designed to be more rich than the message space. Our state space is uncountably infinite and our message space is countably infinite. In fact, when communication is costly the only equilibria which exist involve a finite number of messages. We believe that this captures an important aspect of reality: it is impossible to completely communicate the complexity of the real world, one may only increase the precision of communication by expending more costly effort.

As mentioned earlier, one could imagine that our messages m_0, m_1, m_2, ...correspond to $\varnothing$, $\{0\}$, $\{0,0\}$, ... An alternate formulation of the message space would be $\varnothing \times \{0,1\}^n$ where the cost of communication $c(m) = cn$. For instance, the message $(0,1,1)$ would cost $3c$ as would every other message with three digits. However, our formulation is a reduced form of this specification. The equilibria in this alternate formulation would be more complicated but would not provide more insight.

4 Results

Although our equilibria share some of the familiar characteristics of the cheap talk literature, the additional results which emerge require the flexibility provided by the notation which we now define. Like the CS equilibria, messages are sent on disjoint intervals. Therefore, we may characterize an equilibrium by a set of cutoff states where we denote the number of messages used in equilibrium as $n+1$: m_0, ..., m_n. When listing the cutoff states, a superscript indicates the rank of the cutoff state. Specifically, s^i indicates the state which is the ith cutoff, where

$$0 = s^0 \leq s^1 \leq s^2 \leq ... \leq s^i \leq ... \leq s^n \leq 1 = s^{n+1} \tag{3}$$

When denoting a cutoff state, a subscript indicates the smallest state associated with the message. Specifically, state s_i indicates $\mu(s) = m_i$ for $s \in [s_i^{g(i)}, s^{g(i)+1})$. The function g is a

[5] See Blume et. al. (1998) and Blume et. al. (2001).

one-to-one mapping from messages used into the the rank of the smallest state associated with that message ($g : \{0, .., n\} \to \{0, .., n\}$). In other words, g relates each message to elements of expression (3).

Equilibrium is such that S's messages are sent as intervals on the state space:[6]

$$\mu(s) = m_i \text{ for } s \in [s_i^{g(i)}, s^{g(i)+1}) \tag{4}$$

and R best responds in a straightforward manner:

$$\alpha(m_i) = \overline{a}(s_i^{g(i)}, s^{g(i)+1}) = \arg\max_a \int_{s_i^{g(i)}}^{s^{g(i)+1}} u^R(a, s)\beta(s|m)ds$$

where $\overline{a}(\underline{s}, \overline{s})$ is the best response of R if the state is known to be between $\underline{s}$ and $\overline{s}$.

The arbitrage equation, also standard in the cheap talk literature, characterizes the equilibrium set of cutoff states:

$$u^S(\overline{a}(s_i^{g(i)}, s^{g(i)+1}), m_i, s) = u^S(\overline{a}(s_j^{g(j)}, s^{g(j)+1}), m_j, s) \text{ for } i \text{ and } j \text{ where } s^{g(i)+1} = s_j^{g(j)} \tag{A}$$

We define λ_i to be the mass of states such that $\mu(s) = m_i$. Since the messages are sent on an interval of the state space and the states are distributed uniformly, $\lambda_i = s^{g(i)+1} - s_i^{g(i)}$ when $\mu(s) = m_i$ for $s \in [s_i^{g(i)}, s^{g(i)+1}]$ and $\mu(s) \neq m_i$ for $s \notin [s_i^{g(i)}, s^{g(i)+1}]$.

All equilibria are efficient in the sense that there are no unused, cheaper signals available. This is the content of our "no holes" result. We demonstrate that, in equilibrium, there is no unused message which is cheaper than any of the used messages.

Proposition 1 *Consider m_i and m_j where $i > j$. Under Condition M, there is no equilibrium where there is an s_i such that $\mu(s_i) = m_i$ but there is no s_j such that $\mu(s_j) = m_j$.*

Proof: Suppose that there is a equilibrium $(\mu, \alpha; c)$ where there exists s_i such that $\mu(s_i) = m_i$ and there does not exist an s_j such that $\mu(s_j) = m_j$. We denote the largest signal

[6] See the appendix for proof of the results that only one message gets sent for any particular state and the proof of the result that the equilibrium strategy for S entails sending a message m' for states which are conntected intervals.

used in equilibrium as m_v such that $\mu(s) = m_v$ for $s \in [s_v^{g(v)}, s^{g(v)+1})$. When the signal m_j is observed, R believes that the state is certain to be state $\frac{s_v^{g(v)}+s^{g(v)+1}}{2}$. A profitable deviation for $s \in [s_v^{g(v)}, s^{g(v)+1})$ is $\mu(\frac{s_v^{g(v)}+s^{g(v)+1}}{2}) = m_j$, therefore $(\mu, \alpha; c)$ cannot constitute an equilibrium.

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The above proposition suggests that each of our equilibria will have no holes: if message m_i is used in equilibrium then so is every m_j such that $i > j$. Also note that we only required Condition M in the proof and therefore the result also applies to Condition L. As a result of Proposition 1, we can denote an equilibrium by the most complex message used. Therefore, if m_n is the most complex message used in equilibrium then we will say that we have an n-equilibrium.

Definition 1 *An n-**equilibrium** is one in which messages m_0, m_1, ..., m_n are used.*

We will use λ to rewrite expression (A). Consider an n-equilibrium. Message m_0 is associated with a mass of $\lambda_0 = s^{g(0)+1} - s_0^{g(0)}$, message m_1 is associated with a mass of $\lambda_1 = s^{g(1)+1} - s_1^{g(1)}$,..., message m_n is associated with a mass $\lambda_n = s^{g(n)+1} - s_n^{g(n)}$. We require that

$$\lambda_i \geq 0 \text{ for every } i \in \{0, ..., n\}$$

and

$$\lambda_1 + \lambda_2 + ... + \lambda_n = 1 \tag{5}$$

In our notation, a monotonic n-equilibrium would be such that $g(n') = n'$ for $n' \in \{0, ..., n\}$. In other words, a monotonic n-equilibrium would be characterized by:

$$\begin{aligned}
\lambda_0 &= s^1 - 0 \\
\lambda_1 &= s^2 - s^1 \\
&\ldots \\
\lambda_{n-1} &= s^n - s^{n-1} \\
\lambda_n &= 1 - s^n.
\end{aligned}$$

This monotonic n-equilibrium is a special case of the more general notation developed above.

Lemma 1 *The necessary conditions for an n-equilibrium are:*

$$\lambda_j^2 - \lambda_i^2 = 4c(i-j)$$

where $n \geq i > j \geq 0$.

Proof: As there are $n+1$ messages used in equilibrium (m_0, m_1, ...,m_n), it must be that there are n equations in expression (A). A typical such expression would be the cutoff state between message m_i and m_j where $\mu(s') = m_i$ for $s' \in [s_i^{g(i)}, s^{g(i)+1})$, $\mu(s'') = m_j$ for $s'' \in [s_j^{g(j)}, s^{g(j)+1})$ and $s^{g(i)+1} = s_j^{g(j)}$:

$$-\left(\frac{s_i^{g(i)} + s^{g(i)+1}}{2} - s^{g(i)+1}\right)^2 - c(i) = -\left(\frac{s_j^{g(j)} + s^{g(j)+1}}{2} - s_j^{g(j)}\right)^2 - c(j).$$

Without loss of generality, we can write

$$\begin{aligned} -\left(\frac{s_i^{g(i)} - s^{g(i)+1}}{2}\right)^2 - c(i) &= -\left(\frac{s^{g(j)+1} - s_j^{g(j)}}{2}\right)^2 - c(j) \\ -\left(\frac{-\lambda_i}{2}\right)^2 &= -\left(\frac{\lambda_j}{2}\right)^2 + c(i-j) \\ \lambda_j^2 - \lambda_i^2 &= 4c(i-j) \end{aligned}$$

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The Lemma above establishes the relationship between the mass of states for any two signals which are adjacent in the state space. If adjacent messages are also adjacent in terms of complexity, then the less complex message has a mass of $4c$ larger than the more complex message.

If an n-equilibrium is to exist then it must be that expression (A) is satisfied for each of the n cutoff states. We define the highest possible value of c such that it is still possible, to satisfy expression (A) in an n-equilibrium as $c^*(n)$.

Lemma 2 *The cutoff cost* $c^*(n) = \left(\frac{1}{2\sum_{i=0}^{n}\sqrt{i}}\right)^2$ *for* $n \geq 1$.

Proof: At the largest c such that signal n is feasible, it must be that $\lambda_n^2 = 0$. By Lemma 1 it must be that, $\lambda_{n-1}^2 = 4c$, $\lambda_{n-2}^2 = 8c$, ..., $\lambda_1^2 = 4(n-1)c$, $\lambda_0^2 = 4nc$. Therefore, we may write expression (5) as

$$2\sqrt{nc} + 2\sqrt{(n-1)c} + ... + 2\sqrt{2c} + 2\sqrt{c} = 1$$

and so the lemma is proved.

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As the result of Lemma 2, if $c < c^*(n)$ we will describe an n-equilibrium as *feasible*. If $c > c^*(q)$ then we will describe an n-equilibrium as not feasible. Therefore, for $c \in (c^*(n+1), c^*(n))$ an n-equilibrium is feasible but it an $n+1$-equilibrium is not feasible. As is apparent from the expression for $c^*(n)$, there is a decreasing sequence of cutoffs costs: the cheaper the communication costs, n-equilibria with a larger n become feasible.

As a result of Lemma 2 we have the following corollary.

Corollary 1 *For any* $c > 0$ *there exists a finite* n^* *such that* $c^*(n^*) \geq c > c^*(n^* + 1)$

Corollary 1 implies that for any nonzero communication costs, there will be at most n^* signals used in equilibrium and so communication is never complete.

We now present two positive results. The first is that under Condition L, we are guaranteed at least one equilibrium. The proof consists in demonstrating that in any candidate n-equilibrium it is not profitable for any $s \in [0, 1]$ to send message $n+1$.

Proposition 2 *If* $c \in (c^*(n+1), c^*(n))$ *then under Condition* L *there exists an* n*-equilibrium.*

Proof: Suppose that $c \in (c^*(n+1), c^*(n))$. We need to check that it is not profitable for the sender who received $s = 0$, to send message m_{n+1}. Because $c < c^*(n)$ it must be that

$$\lambda_i^2 - \lambda_{i+1}^2 = 4c \text{ for every } i \in \{0, n\}$$

and

$$\lambda_n > 0.$$

Therefore, $\lambda_0 = \sqrt{4nc + \lambda_n^2}$. And so the equilibrium payoffs for the S who received signal $s = 0$ is:

$$-\left(\frac{\lambda_0}{2} - 0\right)^2 = -\left(\frac{\lambda_1}{2} - 0\right)^2 - c = ... = -\left(\frac{\lambda_n}{2} - 0\right)^2 - nc$$

All of the messages used in equilibrium will not provide a profitable deviation, therefore we must use an out-of-equilibrium message to find a deviation. Any deviation accomplished by message $n + z$ where $z > 1$ can be accomplished with a lower communication cost by sending message m_{n+1}. Therefore, the cheapest (and therefore best candidate) out-of-equilibruim message is then the message m_{n+1}. If m_{n+1} is sent, R would have beliefs that the message was sent by state $s = 0$. Sending this signal yields a payoff of $-(n + 1)c$. Therefore, the signal m_{n+1} will be profitable when $\lambda_n^2 > 4c$. At $c = c^*(n + 1)$ it must be that $\lambda_{n+1} = 0$ and so

$$\lambda_0 + \lambda_1 + ... + \lambda_n + \lambda_{n+1} = 1$$

$$\sqrt{4nc + \lambda_n^2} + \sqrt{4(n-1)c + \lambda_n^2} + ... + \sqrt{\lambda_n^2} + 0 = 1$$

Therefore, $\lambda_n^2 = 4c$. However at $c > c^*(n + 1)$, it must be that $\lambda_n^2 < 4c$ and there is no profitable deviation.

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Therefore, Proposition 2 suggests that under Condition L there is no profitable deviation from an equilibrium which uses the largest number of signals which are feasible. Since Proposition 2 holds under Condition L, the result will also hold under Condition M. To see this, note that under Condition L an out-of-equilibrium message induces an action which was not induced in equilibrium, therefore a deviation possibly benefits the sender at $s = 0$. Condition M does not induce a new action, therefore a deviation is not profitable and the above result also holds under Condition M.

The second positive result is that under Condition L we are guaranteed a unique profile of λ for any c. Since Proposition 2 demonstrated that there was no profitable deviation from an equilibrium involving the most complex, feasible equilibrium, we now show that there is not an equilibrium using any profile of signals less than the most complex, feasible one.

Proposition 3 *Under Condition L, if $c \in (c^*(n+1), c^*(n))$ then $\lambda = (\lambda_0, \lambda_1, ..., \lambda_n)$ which constitute an n-equilibrium is unique.*

Proof: Suppose that $c \in (c^*(n+1), c^*(n))$. Consider a candidate equilibrium where only $n-1$ messages are sent. This candidate $n-1$-equilibrium is characterized by:

$$\begin{aligned} \widetilde{\lambda}_j^2 - \widetilde{\lambda}_i^2 &= 4(i-j)c \text{ for } n-1 \geq i > j \geq 0 \\ \widetilde{\lambda}_{n-1}^2 &> 0 \\ \widetilde{\lambda}_0 + \widetilde{\lambda}_1 + ... + \widetilde{\lambda}_{n-1} &= 1 \end{aligned}$$

At $c = c^*(n)$ an n-equilibrium would require:

$$\lambda_0 + \lambda_1 + ... + \lambda_n = 1.$$

where $\lambda_n = 0$ and $\lambda_{n-1} = 4c$. At $c < c^*(n)$ an n-equilibrium would require:

$$\lambda_0 + \lambda_1 + ... + \lambda_n = 1.$$

where $\lambda_n > 0$ and $\lambda_{n-1} = 4c + \lambda_n^2 > 4c$. Therefore, it must be that $\widetilde{\lambda}_{n-1}^2 > 4c$ and that $\widetilde{\lambda}_0^2 > 4nc$. So we can write the equilibrium payoffs as:

$$U^S = -\left(\frac{\widetilde{\lambda}_0^2}{2} - 0\right)^2 < -nc$$

Deviation payoffs are $-nc$, therefore equilibrium payoffs are less than deviation payoffs and so an $n-1$-equilibrium cannot exist.

To see that each n-equilibria uniquely determines the values of λ, we can rewrite

$$\lambda_0 + \lambda_1 + ... + \lambda_{n-1} + \lambda_n = 1$$

as

$$\sqrt{4nc + \lambda_n^2} + ... + \sqrt{4c + \lambda_n^2} + \lambda_n = 1 \tag{6}$$

Expression (6) is strictly increasing in λ_n and therefore must only hold for a single value of λ_n. And so the proposition is proved.

■

Together Propositions 2 and 3 demonstrate that under Condition L there is only one class of equilibria: only the most complex, feasible equilibria does not have a profitable deviation. Additionally, Proposition 3 shows that, although monotonicity of the equilibrium as found in CS does not hold in our setting, the equilibrium is unique in the sense that in each equilibrium, signals of a given complexity are sent on identical mass. These Propositions are reminiscent of Proposition 3 in Chen, Kartik and Sobel (2008), as they show that in the CS model where monotonicity holds, $NITS$ admits only the most informative equilibrium.

To provide an antidote for the abstract nature of the above discussion, we now provide an example of the possible equilibria under Condition M and Condition L.

Example 1 *Consider the case where $c(m_i) = 0.01i$. Note that:*

$$c^*(4) = 0.00662 < 0.01 < c^*(3) = 0.0428.$$

Under Condition M, there are four classes of equilibria, $n \in \{0, 1, 2, 3\}$. For the $n = 0$ equilibrium, m_0 gets sent for all states. For the $n = 1$ case, there are two equilibria. There is an equilibrium where m_0 is sent for states $[0, 0.52)$ and m_1 for states $[0.52, 1]$. There is another equilibrium where m_1 is sent for states $[0, 0.48)$ and m_0 for states $[0.48, 1]$. Note that in each of the $n = 1$ equilibria $\lambda_0 = 0.52$ and $\lambda_1 = 0.48$. For the $n = 2$ case, there are six equilibria. There is a monotonic equilibria where m_0 is sent for states $[0, 0.392)$, m_1 for states $[0.392, 0.729)$ and m_2 for $[0.729, 1]$. The remaining 5 equilibria require that $\lambda_0 = 0.392$, $\lambda_1 = 0.337$, and $\lambda_2 = 0.271$. For the $n = 3$ case, there are 24 equilibria. There is a monotonic equilibria where m_0 is sent for states $[0, 0.363)$, m_1 for states $[0.363, 0.665)$, m_2 for $[0.665, 0.892)$ and m_3 for $[0.892, 1]$. The remaining 23 equilibria require that $\lambda_0 = 0.363$, $\lambda_1 = 0.302$, $\lambda_2 = 0.227$ and $\lambda_3 = 0.108$. For Condition L, only the 24, $n = 3$ equilibria are *possible.*

Note that we have identified equilibria which the values of λ_i are neither increasing nor decreasing. In other words, Monotonicity (Condition M in CS) does not hold in our setting (unlike the quadratic preferences, uniform state case in CS.) Also since monotonicity fails in this model we should not be surprised that Condition L does not lead to a unique equilibrium as Proposition 1 in Chen, Kartik and Sobel demonstrates that when monotonicity fails in their model, $NITS$ fails to lead to a unique equilibrium.

Proposition 4 *In each n-equilibrium the expected utility of R and S is the same.*

Proof: For an equilibrium $(s^1, ..., s^n)$ the ex-ante expected utility of R can be written as

$$EU^R(s^1, ..., s^n) = \int_0^{s^1} (s - (\overline{a} - s)^2)ds + \int_{s^1}^{s^2} (s - (\overline{a} - s)^2)ds + ... + \int_{s^n}^{1} (s - (\overline{a} - s)^2)ds$$

$$= 0.5 - \int_0^{s^1} (\frac{s^1}{2} - s)^2 ds - \int_{s^1}^{s^2} (\frac{s^1 + s^2}{2} - s)^2 ds - ... - \int_{s^n}^{1} (\frac{s^n + 1}{2} - s)^2 ds \tag{7}$$

By Proposition 3 each equilibria with $n+1$ signals has an identical profile of $\lambda = (\lambda_1, ..., \lambda_n)$ and so we may rewrite expression (7) as

$$EU^R =$$

$$0.5 - \int_{s_0^{g(0)}}^{s^{g(0)+1}} (\frac{s_0^{g(0)} + s^{g(0)+1}}{2} - s)^2 ds - \int_{s_1^{g(1)}}^{s^{g(1)+1}} (\frac{s_1^{g(1)} + s^{g(1)+1}}{2} - s)^2 ds$$

$$-... - \int_{s_n^{g(n)}}^{s^{g(n)+1}} (\frac{s_n^{g(n)} + s^{g(n)+1}}{2} - s)^2) ds$$

and so for any equilibria with $n + 1$ signals it must be that

$$EU^R(s^1, ..., s^n) = EU^R(\widehat{s^1}, ..., \widehat{s^n}) \text{ for any equilibria } (s^1, ..., s^n) \text{ and } (\widehat{s^1}, ..., \widehat{s^n})$$

Therefore, we may write $EU^R(n)$ for the expected utility of R in an equilibrium when the message n is used. Since we can write $EU^R(n)$, and by Proposition 3 each of the equilibria with n signals has an identical profile of $\lambda = (\lambda_1, ..., \lambda_n)$, we can also write $EU^S(n)$ for as:

$$EU^S(n) = EU^R(n) - \lambda_1 - 2\lambda_2 - 3\lambda_3 - ... - n\lambda_n.$$

and so the proposition is proved.

■

4.1 Competency of the Advisee

In any relationship involving advice, the advisor has beliefs regarding ability of the advisee to execute the advice. It would seem that this competency would influence effort supplied by the advisor. To analyze these issues, we supplement the model to allow for the possibility that the sender might make a mistake in the execution of the action. Specifically, with probability p the receiver perfectly executes the most preferred action:

$$\alpha(m_i) = \overline{a}(s_i^{g(i)}, s^{g(i)+1}) = \arg\max_a \int_{s_i^{g(i)}}^{s^{g(i)+1}} u^R(a, s)\beta(s|m)ds.$$

With probability $1-p$, the action a is distributed uniformly on the action space ($U[0,1]$).[7] We interpret p as the competency of the advisee. We now state our result, which demonstrates that the informativeness of the relationship is increasing in the competency of the advisee.

Proposition 5 *The informativeness of the sender is increasing in* p

Proof: In the presence of the possibility of mistakes, the new arbitrage expression can be written:

$$\begin{aligned} & -p\left(\frac{s_i^{g(i)} - s^{g(i)+1}}{2}\right)^2 - (1-p)\int_0^1 (s-(x-s)^2)dx - c(m_i) \\ = \quad & -p\left(\frac{s_j^{g(j)} - s^{g(j)+1}}{2}\right)^2 - (1-p)\int_0^1 (s-(y-s)^2)dy - c(m_j) \end{aligned}$$

[7]Note that we assume that p is unrelated to the message. We could have allowed p to be decreasing in the comlexity of the message, however this would only strenghten our result below.

Therefore, the necessary conditions for equilibrium can be written:

$$\lambda_j^2 - \lambda_i^2 = \frac{4c}{p}(i-j).$$

So for any n-equilibrium, a decrease in p will lead to a decrease in $E[(a-s)^2]$. Additionally a decrease in p can lead to an equilibrium with a smaller number of signals, as the cutoff costs can be written:

$$c^*(n;p) = \frac{p}{\left(2\sum_{i=0}^{n}\sqrt{i}\right)^2}.$$

■

Proposition 5 suggests that in equilibrium S will expend less effort on communication when R exhibits a larger probability of making a mistake in executing the action. We interpret this result as indicating that in equilibrium, advisors will provide more attention to capable advisees.

5 Conclusions

We have modeled an interaction between an informed sender and uninformed receiver, as in relationship between academic advisor and advisee. Advisor and advisee have identical preferences over the action of the advisee, however the advisor faces a cost of communication which is increasing in the complexity of the message sent. We have characterized the equilibria where a unique message is sent on an interval of the state space. There exists a cutoff cost, which determines the number of messages sent in the most informative equilibria. We have demonstrated that under Condition L only the most informative class of equilibria exists. This result is analogous to the findings of the No Incentive to Seperate ($NITS$). Finally, we have also provided a result which demonstrates that the more competent advisee will enjoy a more informative advising relationship.

There however remain important questions which are unanswered. For instance, it is not known what happens when sender and receiver have different preferences over actions of the receiver. For instance, it is possible to imagine a case where the advisor and advisee have different preferences over the content of the paper. Also, we have modeled the interaction as a single repetition. However, there might be interesting to model the relationship when the interaction is repeated. There are three possibilities as perhaps the relationship is finitely repeated, infinitely repeated or is repeated until the project attains some threshold. An additional issue which arises only in the repeated version of the game relates to learning on the part of the advisee. Presumably there is a relationship between some publicly observable signal and the optimal action for the advisee and also that the advisor wishes to teach the advisee this relationship.

We are also eager to learn the validity of the results in an experimental setting. Like most communication games, the equilibria here is rather complicated and this fact makes experimental investigation rather difficult. On the other hand, experimental papers (for instance Cai and Wang (2006) and Kawagoe and Takizawa (2008)) have found suitable simplifications of the theoretical papers which they test. We are confident that a similar such a simplification can be found in our setting.

6 Appendix

Together the below two propositions demonstrate that equilibrium messages are sent only on connected disjoint intervals.

Lemma 3 *For any state $\widehat{s}$, there can only be one message $\widehat{m}$ sent in equilibrium.*

Proof: Suppose that $\mu((s_1, s_2)) = m'$ and $\mu((s_3, s_4)) = m''$ where $(s_1, s_2) \neq (s_3, s_4)$ and $(s_1, s_2) \cap (s_3, s_4) \neq \emptyset$. The sender S can transmit the same information by sending only the least costly of the two, $argmin(c(m'), c(m''))$ and so $\mu((s_4, s_2)) = \{m', m''\}$ cannot be equilibrium.

■

Lemma 4 *The equilibria must be connected intervals.*

Proof: Now suppose there exists m such that $\mu^{-1}(m) = [s_1, s_2) \cup [s_3, s_4)$ with $[s_1, s_2) \neq [s_3, s_4)$, $[s_1, s_2) \cap [s_3, s_4) = \emptyset$ where $s_2 < s_3$.

Case 1: Suppose $\overline{a}(m) \in (s_2, s_3)$. Define $\mu(\overline{a}(m)) = m'$, where $m \neq m'$. If $\overline{a}(m) = \overline{a}(m')$, either $c(m) \neq c(m')$ and there exists a profitable deviation for S in choosing the cheaper message, or $c(m) = c(m')$, and there exists a payoff-equivalent equilibrium in which we send the same message at $\mu^{-1}(m), \mu^{-1}(m')$. Therefore, suppose $\overline{a}(m) \neq \overline{a}(m')$. If $c(m) \leq c(m')$, the sender strictly prefers to send m on $(\overline{a}(m) - \varepsilon, \overline{a}(m) + \varepsilon) \in \mu^{-1}(m')$. If $c(m) > c(m')$ and $\overline{a}(m) < \overline{a}(m')$, the sender strictly prefers to send m' on $[s_3, s_4)$. If $c(m) > c(m')$ and $\overline{a}(m) > \overline{a}(m')$, the sender strictly prefers to send m' on $[s_1, s_2)$.

Case 2: Suppose $\overline{a}(m) \in [s_1, s_2)$. If there exists $m' \in \mu((s_2, s_3))$ with $c(m') \leq c(m)$, such that $\overline{a}(m') \in (s_2, s_3)$ then the sender strictly prefers to send m' on $[s_3, s_4)$. If $c(m') > c(m)$ for $m' \in \mu((s_2, s_3))$ then the sender strictly prefers to send m on $[s_2, s_3)$.

Case 3: $\overline{a}(m) \in [s_3, s_4)$. The proof is analogous to Case 2 and the lemma is proved.

■

Hence, the inverse of μ is a collection of disjoint intervals with the property that if $s, s' \in \mu^{-1}(m)$ for some m, so is $s'' \in (s, s,)$. Unless S indifferent between sending two signals at state s, then the same message is sent for some $(s - \varepsilon, s + \varepsilon)$ for some $\varepsilon > 0$.

Lemma 5 *There are* $n+1$ *solutions to* $\max_s(\overline{a}(m') - s')^2 + c(m')$.

Proof: Suppose that $U^S(\overline{a}, \widehat{m}, \underline{s}) > U^S(\overline{a}, \widehat{m}, \overline{s})$ where $\mu([\underline{s}, \overline{s})) = \widehat{m}$. As the distribution is uniform, $\overline{a} = \frac{\underline{s}+\overline{s}}{2}$. This implies that $\left(\frac{\underline{s}+\overline{s}}{2} - \underline{s}\right)^2 > \left(\frac{\underline{s}+\overline{s}}{2} - \overline{s}\right)^2$, which cannot be the case. Combined with expression (A), we have $n+1$ such solutions.

■

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