Moral Hazard: Deterministic Indirect Mechanisms and Efficiency

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Abstract

In this paper we examine strategic interactions between a principal and several agents under moral hazard. We show how (messages) communication may improve on efficiency even in models of complete information. Messages are useful two main reasons. First, if the principal cannot use stochastic mechanisms, mechanisms with messages can sustain mixed strategies and hence indirectly a stochastic outcome. Second, even if stochastic mechanisms are allowed, messages can be used to induce correlation between efforts and outcome. Finally, we provide sufficient conditions under which an equilibrium allocation supported by a stochastic direct mechanism, can be sustained by a deterministic indirect mechanism.

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1 Introduction

In games with a single principal and several agents, the revelation principle is a well-established result. Stated in terms of payoffs, it guarantees that every equilibrium outcome of any communication mechanism can be also supported by an equivalent incentive-compatible mechanism. Individuals' rational behaviors in any coordination mechanism can therefore be characterized without any loss of generality by restricting attention to the set of incentive-compatible mechanisms. Stochastic mechanisms and private recommendations used by the principal with each agent are necessary features for the revelation principle to go through.[1]

This paper analyzes the situation where a single principal strategically interacts with many agents in a scenario of pure moral hazard. That is, there is complete information about agents' types but every agent can take some non-contractible action affecting all other players' payoffs.

Most applications of mechanism design to moral hazard problems with many agents restrict the analysis to take-it or leave-it offers. Research work has shown that allowing for randomization improves on players payoffs.[2]

It is also well-known that if communication is allowed, players can achieve even higher equilibrium outcomes.[3] This depends on the fact that the set of correlated equilibria is larger than the set of Nash equilibria. Nonetheless, these mechanisms are in general quite complex due to their intrinsic stochastic nature.

We propose an alternative method to support correlated outcome in games of moral hazard with multiple agents, which relies on the use of deterministic mechanisms with messages on the part of the principal.

These are standard mechanisms in the literature on incomplete information. We find it interesting that they can be useful even in the context of pure moral hazard.

In the following, we briefly sketch the main features of the general framework we examine, then develop our argument by means of two examples. The first example shows that relative to take-it or leave-it offers, it is possible to introduce a communication mechanism which supports additional equilibrium outcomes. In particular, the example is casted in a framework where the principal can be interpreted as a utilitarian government. We show that the explicit introduction of communication has a welfare enhancing effect, and allows to achieve the first best outcome.

[1] See Gibbard (1973), Green and Laffont (1977), Dasgupta, Hammond, and Maskin (1979), Myerson (1979) and, for generalized principal-agent games, see Myerson (1982).

[2] See Arnott and Stiglitz (1993), Prescott (1999), Gjesdal (1982) for pure moral hazard, and Strausz (2003) for a discussion on the role of lotteries in the context of incomplete information.

[3] See Aumann (1974), Myerson (1982) among others. For the analysis of multi-principal multi-agent games with moral hazard, see Attar, Campioni, Piaser, and Rajan (2007).

In a second example, we show that in a standard model of stochastic production with moral hazard, communication through messages can be welfare improving. This example has a flavor of *delegation*; by letting one agent choose the compensation scheme, the principal gives to the other agent more incentives to choose the right level of effort.

We provide sufficient conditions for the equilibria supported by stochastic direct mechanisms, to be sustained by means of deterministic indirect mechanisms. Then, we discuss the work by Rahman and Obara (2008), which deals with moral hazard in teams and has the principal contracting secretly with some agents to achieve almost efficient allocations. In relating our work to theirs, we show that their equilibrium allocation cannot be decentralized with our deterministic indirect mechanisms.

2 The Model

There is one principal dealing with $n \geq 2$ agents. There is complete information about agent types, so for each agent i the type space Θ^i may be taken to be a singleton. Each agent i chooses an unobservable effort $e^i \in E^i$. Therefore, the model is one of pure moral hazard. We denote the vector of efforts as $e = \left(e^1, e^2, \dots, e^n\right) \in E = \times_{i=1}^n E^i$.

Let Y be a set of allocation rules available to the principal, with generic element $y \in Y$. An allocation can be, for example, monetary transfers, tax rates, prices, or quantities, depending on the interpretation of the model. For convenience, we will consider allocations that lie in $\mathbb{R}^n$. An allocation rule y is an incentive scheme. Typically, in a moral hazard framework, agents' efforts lead to a probability distribution over output or profit. Let $\Delta(\mathbb{R}^n)$ denote the space of probability distributions over $\mathbb{R}^n$, and let Z denote the set of feasible outputs. An allocation rule $y_j : Z \to \Delta(\mathbb{R}^\ell)$ is then a function of the realized output, and the set Y_j is the set of functions from the output space to the space of allocations.

We use the general communication structure for generalized principal-agent models introduced by Myerson (1982). We take M^i and R^i to be the set of messages that the principal can receive from agent i and the set of recommendations he is allowed to send him, respectively. We also denote $M = \times_i M^i$ and $R = \times_i R^i$. As in Myerson (1982), the principal's behavior is described by the choice rule $\pi : M \to \Delta(Y \times R)$. That is, the principal chooses a realization from the lottery π, and communicates the realized recommendations r to the agents. Conditional on observing r^i, agent i updates her belief about the stochastic allocation rule y, but need not know the actual realization of the rule. Since recommendations are private, two agents i and i' may have different posterior beliefs on the chosen allocation rule.

A mechanism offered by principal j is thus given by $\gamma_j = (M_j, R_j, \pi_j)$. Mechanisms are publicly observed, but the message from agent i to the principal, and the recommendation from the principal to agent i, are observed only by the principal and by agent i.

There are two stages at which agent i moves in the game. First, she sends the message array m^i to the principal. Second, after observing her private recommendation r^i, she chooses the effort level $e^i \in E^i$. Agents' and principal's payoffs are evaluated by von Neumann-Morgenstern utilities.

In this complete information framework, a *direct* mechanism is a probability distribution over allocations and efforts, i.e. $\tilde{\pi} \in \Delta(Y \times E)$. The principal does not solicit messages from the agents, and directly suggests the actions they should take. That is, $M^i = \Theta^i$ and $R^i = E^i$ for every $i \in n$.

A *deterministic* mechanism is a mechanism in which the allocation rule is any choice rule $\pi : M^1 \times \cdots \times M^n \to Y \times \left(R^1 \times \cdots \times R^n\right)$. A deterministic direct mechanism allocations is hence defined by a lottery $\tilde{\pi} \in Y \times E$.

As already mentioned, most applications of multi-agent models of moral hazard do not consider any form of communication. In addition, the analysis is typically restricted to equilibria supported by deterministic mechanisms. Let us call those ones as equilibria in *take-it or leave-it offers*. In these cases the mechanism design ability of the principal is (arbitrarily) limited, and this restriction may have relevant positive and normative implications.

3 Communication under Moral Hazard

Take one principal and two agents. Let $Y = \{y_1, y_2, y_3, y_4\}$, $E^1 = \{a_1, a_2\}$ and $E^2 = \{b_1, b_2\}$ be the relevant decision sets. The corresponding payoffs are represented in the following matrices, where the first payoff is that of the principal who chooses the table, the second and third payoffs are those of agent 1 and 2.

$y = y_1$

	b_1	b_2
a_1	$(2,1,1)$	$(10,8,2)$
a_2	$(4,2,2)$	$(15,5,10)$

$y = y_2$

	b_1	b_2
a_1	$(4,2,2)$	$(2,1,1)$
a_2	$(15,10,5)$	$(10,2,8)$

$y = y_3$

	b_1	b_2
a_1	$(10,2,8)$	$(15,10,5)$
a_2	$(2,1,1)$	$(4,2,2)$

$y = y_4$

	b_1	b_2
a_1	$(15,5,10)$	$(4,2,2)$
a_2	$(10,8,2)$	$(2,1,1)$

Let us first consider the take-it or leave-it offer game. One should observe that every continuation game induced by the choice of some y_i admits the unique equilibrium outcome $(10,2,8)$.It is also immediate to check that the principal cannot achieve a payoff of 15 by offering lotteries over allocations. Suppose now that the principal can communicate with agents through the message spaces $M_1 = M_2 = \{m_1, m_2\}$. Consider the following mechanism:

- If both agents send the message m_1, the principal selects y_1.
- If agent 1 sends m_1 and agent 2 m_2, the principal selects y_2.
- If agent 1 sends m_2 and agent 2 m_1, the principal selects y_3.
- If both agents send m_2, the principal selects y_4.

This strategy induces a simultaneous-move game among agents:

	(m_1,b_1)	(m_1,b_2)	(m_2,b_1)	(m_2,b_2)
(m_1,a_1)	$(2,1,1)$	$(10,8,2)$	$(4,2,2)$	$(2,1,1)$
(m_1,a_2)	$(4,2,2)$	$(15,5,10)$	$(15,10,5)$	$(10,2,8)$
(m_2,a_1)	$(10,2,8)$	$(15,10,5)$	$(15,5,10)$	$(4,2,2)$
(m_2,a_2)	$(2,1,1)$	$(4,2,2)$	$(10,8,2)$	$(2,1,1)$

In this game, there is no equilibrium in pure strategies. The unique mixed-strategy Nash equilibrium is:

- Agent 1 mixes between (m_1,a_2) and (m_2,a_1), with probabilities $1/2$ and $1/2$.
- Agent 2 mixes between (m_1,b_2) and (m_2,b_1), with probabilities $1/2$ and $1/2$

At the equilibrium, the principal achieves a payoff of 15. That is, the principal has an incentive to use communication to extract some private information from agents, thereby sustaining a correlated outcome over allocations and efforts.

The example delivers several intuitions.

As a matter of fact, simple take-it or leave-it offers schemes fail to generate a correlation between principal's decisions and agents' efforts. This correlation can only be reproduced letting the principal send recommendations. To derive such recommendations it is sufficient to observe that the distribution over efforts and allocations that is induced at the equilibrium of the indirect mechanism above is: $\tilde{\pi}(y_4,a_1,b_1) = \tilde{\pi}(y_1,a_2,b_2) = \tilde{\pi}(y_3,a_1,b_2) = \tilde{\pi}(y_2,a_2,b_1) = 1/4$.

Suppose that the principal commits to this strategy, choosing allocations and efforts according to $\tilde{\pi}(\cdot)$, and announces the resulting recommendations to the agents. Neither agent has an incentive to deviate, and the mechanism is incentive compatible. As an example, consider the case in which agent 2 is (privately) told "b_2", his posterior beliefs place probability $1/2$ on (y_1, a_2) and $1/2$ on (y_3, a_1). Given these beliefs, b_2 is a best response. The principal reaches a payoff of 15.

This example hence proves that two features of the construction of Myerson (1982), stochastic mechanisms and recommendations, are necessary to establish the Revelation Principle in complete information environments. In other words, communication plays an important role in multi-agent games of moral hazard. This complements the result of Strausz (2003), who argued that in a setting of pure adverse selection with one principal and two agents it is no longer true that any payoff implementable by a deterministic indirect mechanism can be replicated by a deterministic direct mechanism. The example can be also interpreted in welfare terms. Observe first that, for every array of efforts (a,b), the principal's payoff has been designed to be the sum of the agents' payoffs. It follows that recommendations are useful to achieve a first best allocation.

The latter example shows that we cannot get rid of recommendations in problems of moral hazard with several agents. Moreover, even if we want to restrain attention to deterministic mechanisms, indirect mechanisms can outperform the direct ones. In the rest of the section we give conditions under which one can construct a deterministic indirect mechanism that approximately implements a given allocation.

Definition 1 *Let us consider two distributions over allocation D and D', $(D, D' \in \Delta(Y \times E))$. These two distributions are ε-different if $|E_D V - E_{D'} V| < \varepsilon$.*

Let us consider the allocation $A = (y_1, e_1, \ldots, e_n) \in Y \times E$. A distribution of allocation D, ε-different from A, is implementable if it exists an agent i a decision $y_2 \in Y$ and an effort e'_i such that

1. $e_i \in \arg\max_{\hat{e}_i \in E_i} U_i(y_1, \hat{e}_i, \ldots, e_{-i})$,
2. $e'_i \in \arg\max_{\hat{e}_i \in E_i} U_i\left(y_2, \hat{e}_i, e'_{-i}\right)$,
3. $U_i(y_1, e_i, e_{-i}) = U_i(y_2, e'_i, e_{-i})$,
4. $\forall j \neq i$, $\min_{\hat{e}_i \in E_i} [U_j(y_1, \hat{e}_j, e_i, e_{-ij}) - U_j(y_2, \hat{e}_j, e'_i, e_{-ij})] < 0$,
5. $-\min_{e_i \in E_i} [U_j(y_1, \hat{e}_j, e_i, e_{-ij}) - U_j(y_2, e_j, e'_i, e_{-ij})] > \max_{\hat{e}_i \in E_i} [U_j(y_1, \hat{e}_j, e_i, e_{-ij}) - U_j(y_2, \hat{e}_j, e'_i, e_{-ij})]$.

Theorem 1 *If these five conditions are satisfied, even if the allocation A is not implementable, it exists a* ε *such that an* ε*-different distribution of allocations is implementable by a indirect deterministic mechanism.*

Proof. Les us consider the following deterministic mechanism: the principal takes the decision y_1 (resp. y_2) if the agent i sends the message m_1 (resp. m_2).

Conditions 1, 2 and 3 ensure that for agent i it is a best reply to mix between (m_1, e_i) and (m_2, e'_i). Next, if agent i chooses (m_2, e'_i) with a sufficiently hight probability, condition 4 and 5 ensure that for each agent $j \neq i$, it is optimal to chooses the effort $e_j \in E_j$.

Finally, if agent i chooses the couple (m_1, e_i) with probability p, then the implemented allocation is p-different from A. ■

In the following section we consider an economic example of a Principal-Agents model in which indirect deterministic mechanisms are welfare improving compare to take-it or leave-it offers.

4 Communication and Efficiency

This section argues that communication mechanisms can be welfare enhancing in moral hazard frameworks of economic interest. Consider a two-state production economy, with $y > 0$ being the output in the case of success and 0 that corresponding to a failure. The probability of success depends on the combined efforts made by the two agents. We denote $p\left(e^1, e^2\right)$ the probability of success, where e^1 (resp. e^2) is the effort taken by agent 1 (agent 2). In addition, we let $e^i \in E = \{e_L, e_H\}$ with $e_L < e_H$. The probability of success is increasing in both its arguments and it is such that $p\left(e^1, e^2\right) = p\left(e^2, e^1\right)$. We take T^1 and T^2 to be the compensations that the agents receive from the principal in case of success. Agents are protected by limited liability, hence the principal cannot ask them any contribution in case of failure of the project.

All players have linear preferences. The principal's payoff is :

$$V = \left(y - T^1 - T^2\right) \, p\left(e^1, e^2\right). \tag{1}$$

Agents' utilities are given by:

$$U^1 = p\left(e^1, e^2\right) T^1 - e^1, \qquad U^2 = p\left(e^1, e^2\right) T^2 - e^2. \tag{2}$$

We say that the transfers (T^*, T^*) to the two agents induce a first best allocation if given the compensation T^*, each agent has an incentive to select the effort e_H and earns her reservation utility of zero. That is:

$$p(e_H, e_H)\, T^* - e_H = 0, \tag{3}$$

which implies

$$T^* = \frac{e_H}{p(e_H, e_H)}. \tag{4}$$

The relevant incentive constraints can be represented in a simple way. We take $\tilde{T}$ to be the smallest transfer to Agent 1 which induces her to select the effort e_H provided that Agent 2 has also chosen e_H. That is:

$$p(e_H, e_H)\, \tilde{T} - e_H = p(e_L, e_H)\, \tilde{T} - e_L. \tag{5}$$

Hence,

$$\tilde{T} = \frac{e_H - e_L}{p(e_H, e_H) - p(e_L, e_H)}. \tag{6}$$

In a similar way, one can define $\check{T}$ as the smallest transfer such that e_H is incentive compatible for Agent 1 when Agent 2 has chosen e_L:

$$p(e_H, e_L)\, \check{T} - e_H = p(e_L, e_L)\, \check{T} - e_L, \tag{7}$$

which gives:

$$\check{T} = \frac{e_H - e_L}{p(e_H, e_L) - p(e_L, e_L)}. \tag{8}$$

Since the probability of success is symmetric across agents, the transfers $\tilde{T}$ and $\check{T}$ also characterize the strategic behavior of Agent 2.

To make the incentive problem meaningful, we consider the situation where the first best allocation cannot be supported at equilibrium. That is, each agent has an incentive to select e_L whenever she is offered T^* and the second agent has chosen e_H:

$$p(e_L, e_H)\, T^* - e_L > p(e_H, e_H)\, T^* - e_H. \tag{9}$$

In addition, we refer to the standard situation where the single principal finds optimal to induce both agents to select the high level of effort e_H. This corresponds to:

$$\left(y-2\tilde{T}\right)\,p\left(e_H,e_H\right) > \left[y-\check{T}-\frac{e_L}{p\left(e_L,e_H\right)}\right]p\left(e_L,e_H\right) > \left(y-2e_L\right)p\left(e_L,e_L\right). \tag{10}$$

We argue that in this standard setting there always exists an open set of parameters satisfying (9) and (10) such that it is optimal for the principal to make use of an indirect communication mechanism.

Consider, as an example, the following probability distributions:

$$p(e_L,e_L)=0,\ \ p(e_L,e_H)=p(e_H,e_L)=1/4,\ \ p(e_H,e_H)=2/3. \tag{11}$$

It follows that $p(e_H,e_H)-p(e_L,e_H)=5/12$, and the corresponding transfers are:

$$T^*=\frac{3}{2}e_H,\ \ \tilde{T}=\frac{12}{5}(e_H-e_L),\ \ \check{T}=4(e_H-e_L).$$

It is also immediate to check that (9) can be rewritten as

$$e_H>\frac{8}{3}e_L, \tag{12}$$

while (10) is satisfied whenever

$$\frac{5}{12}y>\frac{11}{5}e_H-\frac{16}{5}e_L, \tag{13}$$

where the right-hand side is positive given (12). If (13) is satisfied, then the array of transfers $(\tilde{T},\tilde{T})$ gives the highest utility to the principal in the class of simple deterministic mechanisms in which communication is not allowed.[4]

Let us now explicitly construct an indirect communication mechanism that dominates the array of take-it or leave-it offers $(\tilde{T},\tilde{T})$ from the principal's point of view.

Consider first the transfer $\hat{T}$ such that:

$$\begin{aligned}&\pi\,p\left(e_L,e_L\right)\hat{T}-\pi e_L+(1-\pi)\,p\left(e_L,e_H\right)T^*-(1-\pi)\,e_L\\&=\pi\hat{T}\,p\left(e_L,e_H\right)-\pi e_H+(1-\pi)\,T^*p\left(e_H,e_H\right)-(1-\pi)\,e_H,\end{aligned} \tag{14}$$

[4] Since in our setting $\check{T}=4(e_H-e_L)>\tilde{T}=\frac{12}{5}(e_H-e_L)$, one should observe that the principal cannot get a payoff greater than $(y-2\tilde{T})p(e_H,e_H)$ by inducing some mixed strategy equilibrium in the effort game played among agents.

with $\pi \in (0,1)$. $\hat{T}$ can be interpreted as the transfer that makes Agent 2 indifferent between e_L and e_H when she faces the following uncertainty:

- with probability π she receives $\hat{T}$ and she believes that the Agent 1 will choose e_L
- with probability $(1-\pi)$ she receives T^* and she believes that the Agent 1 will choose e_H.

Given our parameters, the corresponding value of $\hat{T}$ will be:

$$\hat{T} = \frac{4}{\pi}(\frac{3}{8}e_H - e_L) + \frac{5}{2}e_H. \tag{15}$$

Suppose now that the principal can communicate with the agents through the message spaces $M^1 = M^2 = \{m_1, m_2\}$, and consider the indirect communication mechanism:

$$\begin{cases} (m_1,m_1) \longrightarrow, \left(\tilde{T}, T^*\right) \\ (m_1,m_2) \longrightarrow \left(\tilde{T}, \hat{T}\right), \\ (m_2,m_1) \longrightarrow \left(\tilde{T}, \hat{T}\right), \\ (m_2,m_2) \longrightarrow \left(\tilde{T}, T^*\right), \end{cases} \tag{16}$$

that is, when Agent 1 and Agent 2 coordinate on the same message they receive $\tilde{T}$ and T^*, respectively. If messages are different, the array $\left(\tilde{T}, \hat{T}\right)$ is proposed. Consider the agents' continuation game induced by this mechanism. The reader can check that it admits a continuum of equilibria where Agent 2 plays (e_H, m_1) and Agent 1 selects any probability distribution over (e_H, m_1) and (e_L, m_2).[5]

Let π be the probability that Agent 1 assigns to the choice (e_L, m_2), the corresponding principal's payoff is:

$$p(e_H,e_H)\left(y - T^* - \tilde{T}\right)(1-\pi) + \pi\, p(e_L,e_H)\left(y - \hat{T} - \tilde{T}\right), \tag{17}$$

which is monotonically decreasing in π. Given our parameters, there always exists a $\pi^* \in (0,1)$ which makes the principal indifferent between playing the direct mechanism $(\tilde{T}, \tilde{T})$ and the indirect mechanism described before. Plugging the parameters defined in (4) into 17, one gets:

$$\pi^*\left[\frac{5}{12}y + \frac{11}{8}e_H - e_L\right] = \frac{39}{40}e_H - 13/5e_L. \tag{18}$$

One should observe that the right-hand side of (18) is always (strictly) greater than zero, since $e_H > 8/3e_L$ by (12). To show that $\pi^* \in (0,1)$, it is enough to remark that $\frac{5}{12}y + \frac{11}{8}e_H - e_L >$

[5] One should observe the definition of $\hat{T}$ guarantees that Agent 2 has no incentive to deviate towards (e_L, m_1) Agent 1 mixes over (e_H, m_1) and (e_L, m_2) with probabilities $(1-\pi)$ and π.

$\frac{39}{40}e_H - 13/5e_L$. It follows that there is a continuum of equilibrium allocations, such that the corresponding π is smaller than π^*, which provide the single principal with a profit strictly greater than what is available through simple take-it or leave-it offers.

In order to implement the indirect mechanism that we describe in the example, the principal must be able to pay $\hat{T}$. This requires y to be large enough.

5 Discussion

Following the hints that we gain from the previous examples, we provide now sufficient conditions to decentralize equilibria in direct mechanisms by means of deterministic indirect mechanisms.

Recall that $\tilde{\pi}(y, e^i, e^{-i})$ is the probability distribution induced by a direct mechanism. Abusing notation we can also denote $\tilde{\pi}(y_i, e^i, e^{-i})$ the probability with which the principal chooses allocation y_i and sends recommendations e^i and e^{-i} to the agents, respectively.

Let us introduce some useful properties which can be satisfied by a direct mechanism, and will allows us to support the corresponding equilibrium allocations by means of a deterministic indirect mechanism.

Definition 2 *We say that a direct mechanism satisfies the condition of* independence*, if for every agent $i \in I$, for every effort $e^i \in E^i$ and $e^{-i} \in E^{-i}$, $\pi(e^i|e^{-i}) = \pi(e^i)$.*

In addition, we want that for every given vector of efforts, the principal chooses one allocation. In other words, that there exists a mapping from the efforts' space E to the space of principal's decisions Y.

Definition 3 *We say that a direct mechanism satisfies the condition of* identification*, if for every $y \in Y$ and for all $e \in E$, either $\pi(y|e) = 1$ or $\pi(y|e) = 0$.*

Finally, we also require an *indifference* condition on the part of every agent i:

Definition 4 *We say that a direct mechanism satisfies the condition of* indifference*, if for every $i \in I$ and for every pair $(e^i, \tilde{e}^i) \in E^i$,*

$$\Sigma_{e^{-i}}\Sigma_y\, u^i(y, e^i, e^{-i})\pi(y, e^{-i}|e^i) = \Sigma_{e^{-i}}\Sigma_y\, u^i(y, \tilde{e}^i, e^{-i})\pi(y, e^{-i}|\tilde{e}^i)$$

Given these properties, we can state the following proposition:

Proposition 1 *An equilibrium allocation rule $\tilde{\pi}(.)$ induced by a stochastic direct mechanism, can be supported by a deterministic indirect mechanism if the direct mechanism satisfies the conditions of independence, identification and indifference.*

Proof. Let us consider agent i. Since the space of messages M^i is sufficiently large, we can construct a mapping μ^i from the set of effort E^i to the set of messages M^i such that if $e^i \neq \tilde{e}^i$, then $\mu^i(e^i) \neq \mu^i(\tilde{e}^i)$.

Using the *identification* condition we can construct a deterministic indirect mechanism γ^i in the following way. Whenever $\tilde{\pi}(y|e^i, e^{-i}) = 1$, then $\gamma\left(\mu^i(e^i), \mu^{-i}(e^{-i})\right) = y$.

Consider the subgame induced by the mechanism γ, we require that for each agent i it is a mixed strategy equilibrium to play the strategy $(e^i, \mu^i(e^i))$ with probability π^i. This is directly implied by the conditions of *independence* and *indifference*. ■

Consider that not all correlated equilibria can be decentralized using deterministic mechanisms with messages. To see this, let us consider an example in the spirit of the partnership game with a secret principal, as developed by Rahman and Obara (2008).

Assume there are n individuals and, of course, one principal. Each agent i can either work ($e^i = 1$) or shirk ($e^i = 0$). The cost of working is denoted by c, and effort is not observable. The output realization can be good (g) or bad (b) depending on the efforts chosen by the agents. The probability of the g outcome is $P(\Sigma_i e^i)$, where $P(.)$ is a strictly increasing function. The principal offers to the agents transfers that are contingent on the realization of the output, under the constraint of balancing the budget.

This is a standard framework used to analyze moral hazard in team. It is known that take-it or leave-it offers cannot implement an efficient allocation, not even approximately (Radner, Myerson, and Maskin (1986)).

Rahman and Obara (2008) propose a mechanism that supports an approximately efficient allocation. In terms of a stochastic direct mechanism, their mechanism corresponds to the following one. The principal uses private recommendations to communicate with the agents, and an associated scheme of transfers. In particular,

- with probability $(1-\varepsilon)$, he recommends to every agent to work ($e^i = 1$); and transfers are $T^i = 0$ for every i whatever the output realization;
- with probability $\frac{\varepsilon}{n}$, he recommends to agent i to shirk ($e^i = 0$) and to the other agents to work; offering $T^{-i}(g|e^i = 0) = \overline{T}$ and $T^{-i}(b|e^i = 0) = 0$. To balance the budget, the principal must propose to agent i, $T^i = -\Sigma_{-i} T^{-i}$.

To ensure that incentive compatibility is satisfied, it must be true that whenever an agent receives the recommendation to work, he must weakly prefer to work rather than to shirk:

$$\frac{\varepsilon}{n}(n-1)P(n-1)\overline{T}-c \geq \frac{\varepsilon}{n}(n-1)P(n-2)\overline{T}$$

For an agent that receives the recommendation to shirk, since his payoff is negative, he has no incentive to work.

For every $\varepsilon > 0$, if $\overline{T}$ is sufficiently high the incentive compatibility conditions are satisfied.

Can we implement this outcome by means of a deterministic indirect mechanism?

The answer is no. Let the principal now offer a deterministic mechanism with messages. In this case, a pure strategy of every agent i is a pair (effort, message). In order to implement the previous allocation, each agent must put some non-zero probability on shirking. Therefore, if this is the case there is a non-zero probability that two agents shirk simultaneously which is impossible under the Rahman and Obara (2008) mechanism. This corresponds to a violation of our *independence* property.

6 Conclusion

With this research note, we wanted to emphasize that communication plays an important role in multi-agent models of moral hazard. By means of two examples, we showed that equilibrium outcomes in take-it or leave-it offers can be improved upon. Then, we argue that deterministic mechanisms with messages can decentralize correlated equilibria, and we provide sufficient conditions for this result to go through.

References

ARNOTT, R., AND J. STIGLITZ (1993): "Equilibrium in Competitive Insurance Markets with Moral Hazard," mimeo, Boston College.

ATTAR, A., E. CAMPIONI, G. PIASER, AND U. RAJAN (2007): "On multiple-principal multiple-agent models of moral hazard," mimeo IDEI-Luiss.

AUMANN, R. J. (1974): "Subjectivity and correlation in randomized strategies," *Journal of Mathematical Economics*, 1, 67–95.

DASGUPTA, P., P. HAMMOND, AND E. MASKIN (1979): "The implementation of social choice rules: some results on incentive compatibility," *Review of Economic Studies*, 46, 185–216.

GIBBARD, A. (1973): "Manipulation of voting schemes: A general result," *Econometrica*, 41(4), 587–601.

GJESDAL, F. (1982): “Information and Incentives: The Agency Information Problem,” *Review of Economic Studies*, 49(3), 373–390.

GREEN, J., AND J.-J. LAFFONT (1977): “Characterization of satisfactory mechanisms for the revelation of preferences for public goods,” *Econometrica*, 45(2), 427–438.

MYERSON, R. B. (1979): “Incentive compatibility and the bargaining problem,” *Econometrica*, 47, 61–73.

——— (1982): “Optimal coordination mechanisms in generalized principal-agent problems,” *Journal of Mathematical Economics*, 10, 67–81.

PRESCOTT, E. (1999): “A Primer on Moral Hazard Models,” *Federal Reserve Bank of Richmond Economic Quarterly*, 85(1), 47–77.

RADNER, R., R. MYERSON, AND E. MASKIN (1986): “An Example of a Repeated Partnership Game with Discounting and with Uniformly Inefficient Equilibria,” *Review of Economic Studies*, 53(1), 59–69.

RAHMAN, D., AND I. OBARA (2008): “Secret Contracts for Efficient Partnerships,” UCLA Department of Economics.

STRAUSZ, R. (2003): “Deterministic Mechanisms and the Revelation Principle,” *Economic Letters*, 79, 333–337.