# Commitment or No-Commitment to Monitoring in Emission Tax Systems?

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Abstract

This paper analyzes and compares behavior of the regulator and polluting firms in emission tax systems with and without commitment to monitoring. In the commitment case, firms are found noncompliant at all equilibria. It means that there exists no paradox of *ex ante* commitment to monitoring as shown in principal-agent models. We also discover that the commitment to monitoring system is at least as efficient as the no-commitment to monitoring system. It implies that the regulator may face efficiency loss when she can commit but chooses not to. Accordingly, the regulator has stronger incentive to adopt the commitment system. Finally, relative magnitudes of firms' optimal emissions as well as equilibrium monitoring probabilities in the two systems are uncertain unless firms' weight in the social cost function is no less than one.

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# 1. Introduction

In the literature of principals and agents, monitoring (or inspection) is an important method to mitigate principals' efficiency loss caused by asymmetric information. Many relevant studies assume that principals can make *ex ante* commitment to monitoring, i.e., a preannounced probability of monitoring,(e.g., Baron and Besanko,1984; Reinganum and Wilde, 1985; Graetz et al., 1986; Kofman and Lawarrée, 1993; and Khalil and Lawarrée, 2001). However, two shortcomings arise under this presumption. First, if agent's misreport incentive is totally negated by principal's *ex ante* commitment, auditing itself will become unnecessary *ex post*. This is the paradox of *ex ante* commitment to monitoring. Second, principals' monitoring efforts are usually difficult to verify. Thus, the literature of no-commitment to monitoring ensures (e.g., Reinganum and Wilde, 1986; Khalil, 1997; Chen and Liu, 2005a, 2007; Chen, 2006).

In the field of environmental economics, most theoretical works also assume that regulators can make *ex ante* commitment to monitoring (e.g., Harford, 1978; Beavis and Walker, 1983; Garvie and Keeler, 1994; Stranlund and Dhanda,1999; Macho-Stadler and Perez-Castrillo, 2006). Nevertheless, regulators in the real world may have difficulties in making commitment to monitoring due to its high-cost nature and complicated prosecution procedures. In other words, no-commitment to monitoring could give regulators more flexibility while performing their duties. Moreover, regulators' monitoring commitments are not that credible to firms from time to time for unworthy records of the authorities or the impossibility to verify whether the commitments are kept. Thus, no-commitment to monitoring could be a better choice for regulators. To satisfy researchers' curiosity like ours as well as to provide regulators a criterion when choosing between a commitment and a no-commitment mechanism, this paper would first investigate existence of the paradox of *ex ante* commitment to monitoring in emission tax systems. If the paradox shows, regulators may prefer no-commitment to monitoring. Oppositely, their inclination to a no-commitment setting will be weakened. The second purpose of this paper is to explore behavior of regulators and polluting firms in emission tax systems with and without commitment to monitoring. Then, we can compare both systems and examine whether regulators are better off under a no-commitment mechanism.

An emission tax system with regulator's commitment to monitoring is first built. Under the circumstance, polluting firms' optimal abatements and reported emissions, including both interior and boundary solutions, are derived. Moreover, regulator's emission tax and monitoring probability are endogenous in this model. Then, an emission tax system without regulator's commitment to monitoring is constructed. Based on the two systems, we can inspect how players behave differently, and relative efficiency of the two setups at their associated equilibria. In the commitment system, firms are found noncompliant at all equilibria. This implies that the paradox of *ex ante* commitment to monitoring does not appear in our model. Moreover, the commitment system is shown at least as efficient as the no-commitment system. It suggests that the regulator may face efficiency loss when she is able to commit but chooses not to. These outcomes give the regulator fewer incentives to adopt the no-commitment mechanism in emission tax systems. We also discover that firms' weight in the social cost function is conspicuous in determining relative magnitudes of optimal monitoring probabilities as well as pollutant emissions in the two systems. Basically, relative sizes of firms' optimal emissions and equilibrium monitoring probabilities in the two systems are uncertain unless firms' weight in the social cost function is no less than one.

Our commitment game differs from previous settings as follows. Harford (1978) focuses on firms' interior solutions only, while we consider both interior and boundary solutions. Harford adopts exogenous monitoring probability, while we use endogenous monitoring probability. The regulator minimize firms' total emissions in Macho-Stadler and Perez-Castrillo (2006), while our regulator minimizes the expected weighted social cost. Our model differs from Beavis and Walker's (1983) in terms of regulator's objective function. Garvie and Keeler (1994) and Stranlund and Dhanda (1999) analyze tradeable permit systems considering the commitment case only, while we focus on emission tax systems deliberating both the commitment and no-commitment cases. On the other hand, some environmental protection research assumes regulator's no-commitment to monitoring, such as Grieson and Singh (1990), Bose (1995), Franckx (2002), Chen and Liu (2005b), and Friesen (2006). Players move simultaneously in the first three studies, while they move sequentially in this paper. We (Chen and Liu, 2005b) compare the commitment and no-commitment cases in tradeable-permit pollution control systems, focusing on firms' interior solutions only. Since some equilibria are missed, whether the paradox of *ex ante* commitment to monitoring exists is not addressed, and firms' weight in the social cost function is set equal to one in that paper. Although Friesen's (2006) model allows players to move sequentially, the emphasis is how firms' self-auditing policy affects the social welfare. Finally, the robustness of our findings is enhanced by assuming multiple firms and various firm-specific monitoring probabilities in our setups, or by allowing asymmetric information between players about firm's abatement cost function.

The rest of this paper is organized as follows. Models are introduced in Section 2. Equilibria derived from the commitment and no-commitment systems are presented in Sections 3 and 4, respectively. Comparisons between the two systems are made in Section 5. Robustness of our findings is displayed in Section 6. And conclusions are drawn in Section 7.

## 2. Models

One firm and one regulator are considered in our models.[1] Let $e$ represent the firm's pollutant emission level. Pollution can be reduced if emission abatement technologies

[1] In Section 6.1, the models would be extended to include one regulator and $n$ firms. Same results would still hold.

or equipments are employed. Abatement level $a$ would cost the firm $C(a)$, and make its discharge level become $e = e(a)$, where $e(\cdot)$ is a function describing the relation between pollution abatement and emission levels.[2] Assume function $C(\cdot)$ is strictly increasing and strictly convex, i.e., $C'(a) > 0$ and $C''(a) > 0$ for all $a \geq 0$. And $e(\cdot)$ is strictly decreasing and strictly convex, i.e., $e'(a) < 0$ and $e''(a) > 0$ for all $a \geq 0$. Given the firm's emission $e$, environmental damage $D(e)$ is the monetary loss corresponding to this pollution level with $D'(e) > 0$ and $D''(e) > 0$ for all $e \geq 0$. Function forms of $C(\cdot)$, $e(\cdot)$, and $D(\cdot)$ are known to the regulator and the firm.

In the setup, the regulator is equipped with policy instruments of emission tax and monitoring, and he could commit to monitoring or not.[3] If a precommitment is made, the game is as follows. The regulator first announces emission tax $t^*$ and monitoring probability $\alpha^*$ to minimize the expected weighted social cost. Given $(t^*, \ \alpha^*)$, the firm decides its optimal pollution abatement level $a^*$ and reports emission level $s^*$ to minimize its expected total cost.[4] Then, monitoring is conducted based on the regulator's *ex ante* announcement. Thus, $\{t^*, \ \alpha^*, \ a^*, \ s^*\}$ constitutes a subgame perfect equilibrium (hereafter SPE) of the emission tax system under commitment to monitoring. Alternatively, if no monitoring commitment is made, we have the ensuing game. The regulator first announces emission tax $\hat{t}$ to minimize the expected weighted social cost. Given $\hat{t}$, the firm selects its optimal pollution abatement level $\hat{a}$ and reports emission level $\hat{s}$ to minimize its expected total cost. After knowing the firm's reported emission, the regulator determines an optimal monitoring probability, $\hat{\alpha}$, to minimize the expected weighted social cost. Monitoring is then conducted according

[2]For instance, $e(a) = \bar{e} - a$, where $\bar{e}$ is the firm's maximal emission level without pollution abatement. Alternatively, $e(a) = \bar{e} - a^{\alpha}$ with $0 < \alpha < 1$ also satisfies the required conditions.

[3]The environmental protection agent (EPA) in the States cannot determine emission tax levels, while the EPAs in Hong Kong and Taiwan can. If the regulator is not allowed to choose emission tax in our models, all the outcomes except those of Lemma 2(iic) and Lemma 3 would remain true.

[4]Malik (1993) and Macho-Stadler and Perez-Castrillo (2006) also consider the self-reporting mechanism. Our design is the same as Macho-Stadler and Perez-Castrillo's (2006).

to probability $\hat{\alpha}$. Thus, $\{\hat{t},\ \hat{a},\ \hat{s},\ \hat{\alpha}\}$ is a SPE of the emission tax system under no-commitment to monitoring.

Let $h(\alpha)$ be the regulator's enforcement cost corresponding to monitoring probability $\alpha$. The cost includes all the expenses incurred during the monitoring and prosecuting process. As in Faure-Grimaud, Laffont, and Martimort (1999), $h(\alpha)$ is assumed to be a monotonically increasing and strictly convex function with $h(0) = h_0 \geq 0$, i.e., $h'(\alpha) > 0$ and $h''(\alpha) > 0$ for all $\alpha \geq 0$. Nonzero $h_0$ could represent fixed cost of monitoring activities. As monitoring probability increases, the regulator needs to inspect the firm more frequently. Hence the expenses of monitoring and prosecuting violated firms will expand accordingly. Define $v = e - s$. While $v = 0$ means that the firm is compliant, $v > 0$ indicates that the firm is noncompliant and will be fined $f(v)$. Assume $f(\cdot)$ is strictly increasing and strictly convex with $f(0) = 0$, i.e., $f'(v) > 0$ and $f''(v) > 0$ for all $v \geq 0$. For simplicity, the third and higher-order derivatives of $h(\alpha)$ and $f(v)$ are presumed to be zero.[5] Since the firm's abatement cost function is common knowledge, the regulator can infer the firm's cost-minimizing optimal abatement level from it. Thus, as in Beavis and Walker (1983) and Malik (1993), monitoring is to verify the firm's reported emissions to serve as the official emission readings for later prosecuting usage perhaps. In Section 6.2, the complete information assumption about $C(\cdot)$ would be relaxed, and the firm is presumed to have better information on $C(\cdot)$ than the regulator. Under the circumstance, the regulator would not know the firm's emissions. Therefore, monitoring is necessary to uncover the firm's true emissions and verify its reported emissions. And our outcomes are not affected by this relaxation.

Finally, let us depict the firm's and the regulator's objective functions in the two games. The firm's expected total cost is composed of the expenses of pollution abatement, emission tax levied based on its reported emissions, and expected penalty for

[5] If the third and higher-order derivatives of $h(\alpha)$ and $f(v)$ are nonzero, our outcomes still hold as the derivatives' magnitudes are small.

noncompliance,[6] i.e.,

$$FC(a,\ s) \equiv C(a) + ts + \alpha f(e - s). \tag{1}$$

The regulator's expected weighted social cost is composed of the pollution damage, net monitoring cost, and weighted firm's expected total cost. That is,

$$SC(\alpha,\ t) \ \equiv\ D(e) + [h(\alpha) - ts - \alpha f(e - s)] + \beta FC(a,\ s),$$

where $\beta > 0$ is the firm's weight given in the social cost function.[7] The weight parameter could represent the tradeoff between economic growth and environmental quality. When $\beta > 1$, the environmental damage caused by pollutant emissions is regarded less important than polluting firm's expected total cost. This may describe the situation faced by many developing countries, which put a larger weight on economic growth driven by firm's production. In contrast, when $\beta < 1$, the environmental damage from pollutant emissions is treated more seriously than firm's expected total cost. It could reflect the situation faced by developed countries, which put a bigger weight on environmental quality. When $\beta = 1$, the regulator considers environmental quality and economic growth equally significant. By (1), we have

$$SC(\alpha,\ t) \ \equiv\ D(e) + h(\alpha) + \beta C(a) - (1 - \beta)[ts + \alpha f(e - s)]. \tag{2}$$

The SPEs of our models are derived in the following two sections.

## 3. Commitment to Monitoring

In this section, the SPEs, $(t^*,\ \alpha^*,\ a^*,\ s^*)$, of the emission tax system with commitment to monitoring are derived by the backward induction method.

[6]For simplicity, we do not consider the evaded tax, $\alpha t[e - s]$, for noncompliant firms here. Our results would remain true qualitatively even the evaded tax is counted, and are available on request.

[7]This kind of objective function appears often in the regulatory (e.g., Baron and Myerson, 1982; Baron and Besanko, 1984, 1987) and environmental protection literature (e.g., Friesen, 2006). Firms' weight is usually assumed no greater than one. However, we consider a more general situation here.

First, given emission tax $t$ and monitoring probability $\alpha$, the firm will choose abatement level $a^*$ and report emission level $s^*$, which are the solution of

$$\begin{aligned} &\min_{a,\ s}\ FC(a,\ s) = C(a) + ts + \alpha f(e-s) \\ &\text{s.t. } 0 \le e \le \bar{e} \text{ and } 0 \le s \le e. \end{aligned} \tag{3}$$

The first-order conditions for interior solutions are

$$\frac{\partial FC}{\partial a} = \frac{dC(a)}{da} + \alpha f'(e-s) \cdot \frac{de(a)}{da} = 0, \text{ and} \tag{4}$$

$$\frac{\partial FC}{\partial s} = t - \alpha f'(e-s) = 0. \tag{5}$$

From (4)-(5), we know that the firm's optimal choices vary with monitoring probability and emission tax. To better observe the details, we decompose all value combinations of $\alpha$ and $t$ into four disjoint regions by the increasing order of monitoring probability.

$$\begin{aligned} \text{Region 1} &\equiv \{(\alpha,\ t) \mid \alpha = 0 \text{ and } \underline{t} \le t \le \bar{t}\}, \\ \text{Region 2} &\equiv \{(\alpha,\ t) \mid 0 < \alpha < \frac{t}{f'(\tilde{e}(t))} \text{ and } \underline{t} \le t \le \bar{t}\}, \\ \text{Region 3} &\equiv \{(\alpha,\ t) \mid \frac{t}{f'(\tilde{e}(t))} \le \alpha < \frac{t}{f'(0)} \text{ and } \underline{t} \le t \le \bar{t}\}, \text{ and} \\ \text{Region 4} &\equiv \{(\alpha,\ t) \mid \frac{t}{f'(0)} \le \alpha \le 1 \text{ and } \underline{t} \le t \le \bar{t}\}. \end{aligned}$$

Here $\underline{t}$ and $\bar{t}$ are the lower and upper bounds of emission tax, respectively, with $0 < \underline{t} < \bar{t} < \infty$. The former could represent the minimum emission tax to maintain environmental quality, while the latter is the maximum emission tax under which the firm could still produce. And $\tilde{e}(t) = e(\tilde{a}(t))$ is the firm's emission level associated with abatement amount $\tilde{a}(t)$ given emission tax $t$, which satisfies equation (7) below.

To have non-empty four regions, we make the following assumption.

*Assumption A1:* $f'(0)$ is large enough with $\frac{\bar{t}}{f'(0)} < 1$.

Accordingly, we can obtain the firm's optimal behavior in all regions. Denote $a_i^*$ and $s_i^*$ respectively the firm's optimal abatement and reported emission levels in region $i$, $i \in \{1,\ 2,\ 3,\ 4\}$. They are characterized below.

**Proposition 1**. *Suppose that assumption A1 holds. In the emission tax system with commitment to monitoring, we have the following.*

(i) *In region 1,* $a_1^* = 0,\ e_1^* = \bar{e},$ *and* $s_1^* = 0.$

(ii) *In region 2,* $a_2^* = a^*(\alpha)$ *satisfying*

$$\frac{dC(a_2^*)}{da} = -\alpha f'(e(a_2^*)) \cdot \frac{de(a_2^*)}{da}, \tag{6}$$

$e_2^*(\alpha) = e(a^*(\alpha)),$ *and* $s_2^* = 0$*. Moreover,* $\frac{da_2^*}{d\alpha} > 0$ *and* $\frac{de_2^*}{d\alpha} < 0.$

(iii) *In region 3,* $a_3^* = \tilde{a}(t)$ *satisfying*

$$\frac{dC(a_3^*)}{da} = -t \cdot \frac{de(a_3^*)}{da}, \tag{7}$$

$e_3^*(t) = e(\tilde{a}(t)),$ *and* $s_3^* = \tilde{s}(\alpha,\ t)$ *satisfying (5) and* $0 \leq s_3^* < e_3^*(t)$*. Note that* $s_3^* = 0$ *occurs only when* $\alpha = \frac{t}{f'(\tilde{e}(t))}$*. Moreover,* $\frac{da_3^*}{dt} > 0,\ \frac{de_3^*}{dt} < 0,\ \frac{\partial s_3^*}{\partial \alpha} > 0,$ *and* $\frac{\partial s_3^*}{\partial t} < 0.$

(v) *In region 4,* $a_4^* = \tilde{a}(t)$ *satisfying (7),* $e_4^*(t) = e(\tilde{a}(t)),$ *and* $s_4^*(t) = e_4^*(t)$*. Moreover,* $\frac{da_4^*}{dt} > 0$ *and* $\frac{ds_4^*}{dt} = \frac{de_4^*}{dt} < 0.$

*Proof.* See the Appendix.

Proposition 1 demonstrates how firm's optimal emissions and reported emissions vary with values of monitoring probability and emission tax. Proposition 1(i)-(iii) imply that the firm will not comply when monitoring probability is small, while Proposition 1(iv) shows that the firm complies for large enough monitoring probability. The explanations are given below.

If the regulator does not monitor, the firm will emit as many pollutants as possible and report zero emission due to no punishment. This is what Proposition 1(i) says. To demonstrate cases with nonzero monitoring probability, let us rewrite equations (4) and (5) as follows.

$$-\frac{dC(a)}{da} \cdot [\frac{de(a)}{da}]^{-1} = \alpha f'(e-s) \tag{4'}$$

$$t = \alpha f'(e-s) \tag{5'}$$

Higher emissions will lower the firm's abatement cost, but increase its expected fine. Thus, the LHS and RHS of $(4)'$ are the marginal benefit and marginal cost of the firm's emission, respectively. Similarly, larger reported emissions will reduce the firm's expected fine but raise its tax payment. Thus, the LHS and RHS of $(5)'$ are the marginal cost and marginal benefit of the firm's reported emission, respectively. Equations $(4)'$ and $(5)'$ imply that the marginal cost of the firm's emission equals its marginal benefit of reported emission. The impact of nonzero monitoring probability ($\alpha$) on the firm's optimal emissions is composed of a direct and an indirect effects. As $\alpha$ increases, the marginal cost of emission increases, hence the firm will discharge fewer emissions. This is the direct effect. On the other hand, rising $\alpha$ will enhance the marginal benefit of reported emission, which would lead to larger reported emissions. However, larger reported emissions will lower the marginal cost of emission, hence the firm will emit more. This is the indirect effect. In region 2, the indirect effect is nil because monitoring probability is so small that the increased marginal benefit of reported emission due to rising $\alpha$ is always less than its marginal cost ($t$). Hence, the firm will report zero emission. Consequently, only the direct effect occurs in region 2, and the firm's optimal emissions will decrease with rising $\alpha$. That is what Proposition 1(ii) claims. In regions 3-4, the indirect effect exists because the firm's optimal reported emissions are nonzero. However, the direct and indirect effects will offset each other because the marginal cost of the firm's emission equals its marginal benefit of reported emission. Under the circumstance, the firm's optimal emissions are not affected by monitoring probability, and depend on emission tax only. As to the firm's optimal reported emission in region 3, it will meet (5), hence depends on both emission tax and monitoring probability. And the firm will report more as emission tax decreases or monitoring probability increases. That is what Proposition 1(iii) states. In region 4, since monitoring probability is large enough, the increased marginal benefit of reported emission due to rising $\alpha$ is always larger than its marginal cost ($t$). Consequently, the firm will report as many emissions as possible.

Next, we compare Proposition 1 with relevant literature. Proposition 1 suggests that both the firm's optimal abatement and reported emission levels are interior solutions in region 3 only, i.e., $(a_3^*,\ s_3^*)$, which is also uncovered by Harford (1978). Nevertheless, Harford neglects boundary solution $(a_2^*,\ s_2^*)$, at which firm's optimal emissions depend on monitoring probability. The outcomes of Proposition 1 are similar to those obtained by Macho-Stadler and Perez-Castrillo (2006). Nevertheless, since regulators' objective functions are different in the two models, it will be shown later that solutions in region 4, i.e., $(a_4^*,\ s_4^*)$, cannot be part of our equilibrium, although the firm could comply at some equilibria of Macho-Stadler and Perez-Castrillo (2006).

Before presenting our equilibria, let us first summarize the impact of monitoring probability and emission tax on the firm's optimal behavior below.

**Corollary 1**. *Suppose that assumption A1 holds. In the emission tax system with commitment to monitoring, we have the following.*
(i) $a^*$ *is non-decreasing in* $\alpha$ *and* $t$, *i.e.,* $\frac{\partial a^*}{\partial \alpha} \geq 0$ *and* $\frac{\partial a^*}{\partial t} \geq 0$.
(ii) $e^*$ *is non-increasing in* $\alpha$ *and* $t$, *i.e.,* $\frac{\partial e^*}{\partial \alpha} \leq 0$ *and* $\frac{\partial e^*}{\partial t} \leq 0$.
(iii) $s^*$ *is non-decreasing in* $\alpha$ *and non-increasing in* $t$, *i.e.,* $\frac{\partial s^*}{\partial \alpha} \geq 0$ *and* $\frac{\partial s^*}{\partial t} \leq 0$.
*Proof.* These come from Proposition 1 directly.

The relations between $(e^*,\ s^*)$ and $\alpha$ are graphed in Figure 1 for a given emission tax $t$.

Given the firm's best choices $(a^*,\ s^*)$ as in Proposition 1, the regulator is assumed to pick monitoring probability $\alpha^*$ and emission tax $t^*$ to solve the problem of

$$\begin{array}{ll} \min_{\alpha,\, t} & SC(\alpha,\ t) \equiv D(e^*) + h(\alpha) + \beta C(a^*) - (1-\beta)[ts^* + \alpha f(e^* - s^*)] \\ \text{s.t.} & 0 \leq \alpha \leq 1 \text{ and } 0 < \underline{t} \leq t \leq \bar{t}. \end{array} \tag{8}$$

Recall that $\underline{t}$ and $\bar{t}$ are the lower and upper bounds of emission tax, respectively. The first-order conditions for interior solutions are

$$\begin{aligned} \frac{\partial SC}{\partial \alpha} &= [D_e(e^*)e_a(a^*) + \beta C'(a^*) - (1-\beta)\alpha f'(e^* - s^*)e_a(a^*)]\frac{\partial a^*}{\partial \alpha} + h'(\alpha) \\ &\quad -(1-\beta)[(t - \alpha f'(e^* - s^*))\frac{\partial s^*}{\partial \alpha} + f(e^* - s^*)] = 0, \text{ and} \end{aligned} \tag{9}$$

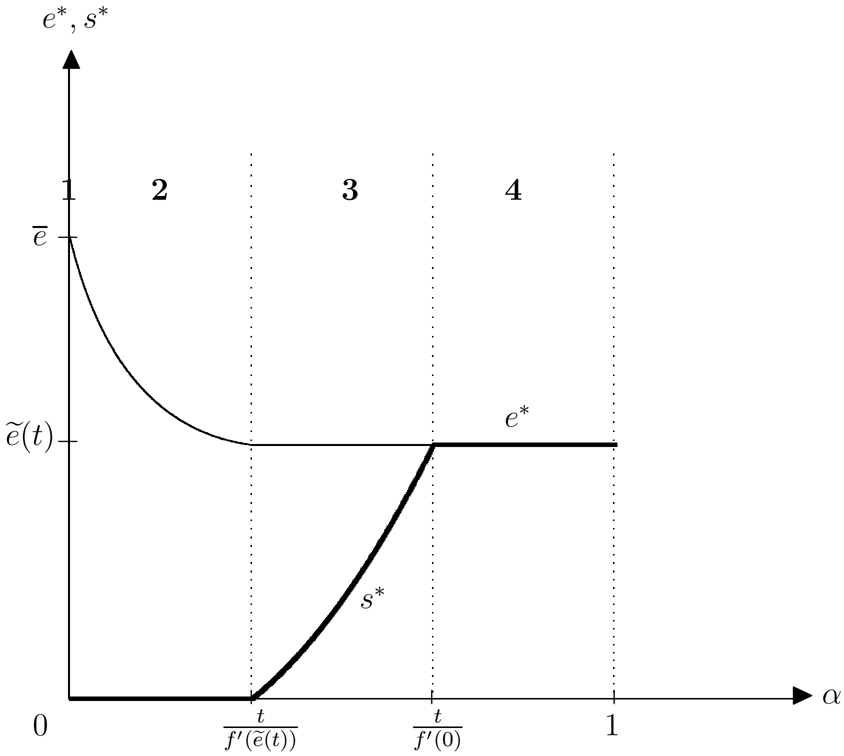


**Figure 1: The Firm's optimal Emissions and Reported Emissions in Regions 1-4**

$$\frac{\partial SC}{\partial t} = [D_e(e^*)e_a(a^*) + \beta C'(a^*) - (1-\beta)\alpha f'(e^* - s^*)e_a(a^*)]\frac{\partial a^*}{\partial t}$$
$$-(1-\beta)[s^* + (t - \alpha f'(e^* - s^*))\frac{\partial s^*}{\partial t}] = 0. \tag{10}$$

We derive $(\alpha^*,\ t^*)$ by the following two steps. First, given the firm's optimal choice in region $i,\ i \in \{1,\ 2,\ 3,\ 4\}$, we try to find $(\alpha_i^*,\ t_i^*)$ in each region to minimize the regulator's expected weighted social cost. Denote $SC_i(\alpha_i^*,\ t_i^*)$ the associated minimum expected weighted social cost in region $i,\ i \in \{1,\ 2,\ 3,\ 4\}$. Second, the optimal solution $(\alpha^*,\ t^*)$ is the one minimizing $SC_i(\alpha_i^*,\ t_i^*)$ over $i \in \{1,\ 2,\ 3,\ 4\}$. That is,

$$(\alpha^*,\ t^*) \in \arg\min_{i\in\{1,\ 2,\ 3,\ 4\}} SC_i(\alpha_i^*,\ t_i^*).$$

By carrying out the two steps, $(\alpha^*,\ t^*)$ could be delineated below.

**Proposition 2**. *Suppose that assumption A1 holds. In the emission tax system with commitment to monitoring, we have the following.*
(i) *Let* $\beta \geq 1$. *If* $(\alpha^*,\ t^*)$ *exist, they could be in region 1, and region 2 if* $C'(a_2^*) < |D_e(e_2^*)e_a(a_2^*)|$. *That is, we have* $0 \leq \alpha^* < \frac{t^*}{f'(\tilde{e}(t^*))}$ *if* $(\alpha^*,\ t^*)$ *exist.*
(ii) *Let* $\beta < 1$. *If* $(\alpha^*,\ t^*)$ *exist, they could be in regions 2 and 3. That is, we have* $0 < \alpha^* < \frac{t^*}{f'(0)}$ *if* $(\alpha^*,\ t^*)$ *exist.*
*Proof.* See the Appendix.

Proposition 2 shows that firm's weight in the social cost function is important in determining optimal monitoring probability and emission tax. The reasons are given below.

If the firm's weight is greater than or equal to one, Proposition 2(i) shows that optimal emission tax and monitoring probability would not appear in regions 3 and 4. That is because the firm's optimal emissions are unaffected by monitoring policy in the regions. Under the circumstance, equation (9) implies that the marginal benefit of monitoring $((1-\beta)f(e^* - s^*))$ is nonpositive, while the marginal cost of monitoring

$(h'(\alpha))$ is positive. Accordingly, no monitoring is optimal. It contradicts regions 3 and 4's positive monitoring probability requirement. Thus, no equilibrium monitoring probability will occur in regions 3 and 4. Also, equilibrium monitoring probability and emission tax will not appear in region 2 when the firm's marginal cost of abatement is large enough. Again, no monitoring is optimal since it will lead to firm's zero abatement, and lower abatement and social cost. This is inconsistent with region 2's positive monitoring probability requirement. In sum, optimal monitoring probability and emission tax will be in region 1 or 2 when the firm's marginal abatement cost is small.

If the firm's weight is less than one, the emission damage will become more significant. Thus, it is optimal for the regulator to suppress firm's emissions through monitoring with positive probability. However, optimal monitoring probability cannot appear in region 4. If it is in region 4, the firm will comply by Proposition 1(iv). And equation (9) implies that the marginal benefit of monitoring the firm $((1-\beta)f(e^*-s^*))$ is zero, but monitoring is costly. Therefore, no monitoring is optimal and the contradiction occurs. By contrast, optimal monitoring probability and emission tax could appear in region 2 or 3. Under the circumstance, the marginal benefit of monitoring $((1-\beta)f(e^*-s^*))$ is positive, so is the marginal cost of monitoring. Thus, equilibrium monitoring probability is positive.

By combining outcomes of Propositions 1 and 2, we obtain Corollary 2.

**Corollary 2**. *Suppose that assumption A1 holds. At equilibria* $(t^*,\ \alpha^*,\ a^*,\ s^*)$ *of the emission tax system with commitment to monitoring, we have the following.*
(i) *When* $\beta \geq 1$, *we have* $0 \leq \alpha^* < \frac{t^*}{f'(\tilde{e}(t^*))}$. *And* $(a^*,\ s^*) = (a_1^*,\ s_1^*)$ *if* $\alpha^* = 0$, *and* $(a^*,\ s^*) = (a_2^*,\ s_2^*)$ *if* $\alpha^* > 0$ *and* $C'(a) < |D_e(e)e_a(a)|$ *for all* $a \geq 0$.
(ii) *When* $\beta < 1$, *we have* $0 < \alpha^* < \frac{t^*}{f'(0)}$. *And* $(a^*,\ s^*)$ *could be* $(a_2^*,\ s_2^*)$ *or* $(a_3^*,\ s_3^*)$.
*Proof.* These come from Propositions 1 and 2 directly.

The first implication of Corollary 2 is that the firm will not comply at all equilibria under the commitment setting. This means that the paradox of *ex ante* commitment to monitoring does not exist here. In contrast, some equilibria with compliant firms appear in Macho-Stadler and Perez-Castrillo's (2006) study. Second, the effectiveness of monitoring in reducing firm's optimal emissions depends on the firm's weight in the social cost function. If the firm's weight is greater than or equal to one, monitoring can effectively reduce pollutant emissions as the regulator monitors with positive probability. This outcome is different from Harford's (1978) finding. However, if the firm's weight is less than one, monitoring may be ineffective even the regulator monitors with positive probability.

## 4. No-Commitment to Monitoring

In this section, the SPEs of the emission tax system with no-commitment to monitoring will be derived by the backward induction method.

First, given emission tax $t$ and the firm's abatement and reported emission level $(a,\ s)$, the regulator is assumed to choose monitoring probability $\hat{\alpha}$ to minimize the expected weighted social cost. That is, $\hat{\alpha}$ solves the problem of

$$\begin{array}{ll} \min_{\alpha} & SC(\alpha \mid a,\ s,\ t) = D(e) + h(\alpha) + \beta C(a) - (1-\beta)[ts + \alpha f(e-s)] \\ \text{s.t.} & 0 \leq \alpha \leq 1. \end{array} \tag{11}$$

At this stage, the regulator behaves exactly like minimizing the net monitoring cost since the firm's choice and emission tax are fixed. Thus, the first- and second-order conditions for interior solutions are

$$\begin{aligned} \frac{dSC(\alpha \mid a,\ s,\ t)}{d\alpha} &= h'(\alpha) - (1-\beta)f(e-s) = 0, \text{ and} \\ \frac{d^2SC(\alpha \mid a,\ s,\ t)}{d\alpha^2} &= h''(\alpha) > 0\ \forall \alpha, \text{ respectively.} \end{aligned} \tag{12}$$

If the firm's weight in social cost function is greater than or equal to one ($\beta \geq 1$) or the firm is compliant ($e = s$), we have $\frac{dSC(\alpha \mid a,\ s,\ t)}{d\alpha} > 0$ by (12). Thus, no monitoring is optimal for the following reasons. When $\beta > 1$, the marginal benefit of monitoring, $(1-\beta)f(e-s)$, is negative because the firm's expected penalty $(\beta\alpha f(e-s))$ is counted more than the regulator's expected revenue of fine $(\alpha f(e-s))$ in the social cost function. Therefore, it is optimal for the regulator not to monitor. When $\beta = 1$, the firm's expected penalty and the regulator's expected revenue of fine offset each other in the social cost function. Hence, the marginal benefit of monitoring is zero, while the marginal cost of monitoring, $h'(\alpha)$, is positive. No monitoring is still the regulator's best choice. Similarly, when the firm is compliant $(e = s)$, it is optimal for the regulator not to monitor because of zero marginal benefit and positive marginal cost of monitoring.

In opposite, when the firm's weight is less than one, the marginal benefit and marginal cost of monitoring are both positive. Therefore, it is optimal for the regulator to monitor with probability $\hat{\alpha} > 0$ satisfying the condition of

$$h'(\hat{\alpha}) = (1-\beta)f(e-s). \tag{13}$$

Equation (13) sets the marginal cost equal to the marginal benefit of monitoring. Consequently, optimal monitoring probability is affected by the firm's abatements and reported emissions. That is, $\hat{\alpha} = \hat{\alpha}(a,\ s)$ exists with[8]

$$\frac{\partial \hat{\alpha}}{\partial a} = \frac{(1-\beta)f'(e-s)\cdot \frac{de(a)}{da}}{h''(\hat{\alpha})} < 0, \text{ and} \tag{14}$$

$$\frac{\partial \hat{\alpha}}{\partial s} = \frac{-(1-\beta)f'(e-s)}{h''(\hat{\alpha})} < 0. \tag{15}$$

It means that the regulator will monitor more (less) *ex post* when the firm abates or reports fewer (more) emissions. Moreover, we have $\frac{\partial \hat{\alpha}}{\partial a} = -\frac{\partial \hat{\alpha}}{\partial s}\frac{\partial e}{\partial a}$. These results are summarized below.

**Lemma 1**. (i) *If* $\beta \geq 1$ *or* $e = s$, *then* $\hat{\alpha} = 0$.
(ii) *If* $\beta < 1$ *and* $e \neq s$, *then* $\hat{\alpha} = \hat{\alpha}(a,\ s) > 0$ *exists with* $\frac{\partial \hat{\alpha}}{\partial a} < 0$ *and* $\frac{\partial \hat{\alpha}}{\partial s} < 0$.

[8]By the implicit function theorem, given $(a,\ s)$, $\hat{\alpha}(a,\ s)$ exists due to $h'' > 0$ for all $\alpha$.

Second, given emission tax $t$ and monitoring probability $\hat{\alpha} = \hat{\alpha}(a,\ s)$, the firm will adopt abatement level $\hat{a}$ and report emission level $\hat{s}$ to the regulator, which solve (3) with $\alpha$ replaced by $\hat{\alpha}$. The first-order conditions for interior solutions are

$$\frac{\partial FC}{\partial a} = \frac{dC}{da} + \hat{\alpha} f'(e-s)\frac{de}{da} + f(e-s)\frac{\partial \hat{\alpha}}{\partial a} = 0, \text{ and} \tag{16}$$

$$\frac{\partial FC}{\partial s} = t - \hat{\alpha} f'(e-s) + f(e-s)\frac{\partial \hat{\alpha}}{\partial s} = 0. \tag{17}$$

Equation (16) suggests that, at the firm's optimal abatement levels, the marginal cost and the marginal benefit of abatement are equal. The marginal benefit of abatement is composed of the changes of fine $(-\hat{\alpha} f'(e-s)\frac{\partial e}{\partial a})$ and monitoring probability $(-f(e-s)\frac{\partial \hat{\alpha}}{\partial a})$ caused by an abatement shift. The latter effect is not present in the commitment case. Similarly, at the firm's equilibrium reported emission levels, emission tax equals the marginal benefit of emission report, which consists of the changes of fine $(\hat{\alpha} f'(e-s))$ and monitoring probability $(-f(e-s)\frac{\partial \hat{\alpha}}{\partial s})$ due to a reported emission shift. Again, the latter effect does not appear in the commitment case.

As in the commitment case, if the regulator will not monitor *ex post*, the firm will abate and report zero emission by (16)-(17). However, unlike the commitment case, only interior solutions, $(\hat{a},\ \hat{s})$, will exist when the regulator monitors with positive probability.[9] Interior solution $\hat{a} = \hat{a}(t)$ meets the condition of

$$\frac{dC(\hat{a})}{da} = -t \cdot \frac{de(\hat{a})}{da} \tag{18}$$

by (16)-(17) and $\frac{\partial \hat{\alpha}}{\partial a} = -\frac{\partial \hat{\alpha}}{\partial s}\frac{\partial e}{\partial a}$. And interior solution $\hat{s} = \hat{s}(t)$ satisfies (17). Next, we derive their properties. Denote $\hat{H}$ the Hessian matrix associated with (16)-(17), where

$$\hat{H} \equiv \begin{bmatrix} \hat{H}_{11} & \hat{H}_{12} \\ \hat{H}_{12} & \hat{H}_{22} \end{bmatrix}$$

[9]For a given $s \geq 0$, if $\frac{dC(0)}{da} \geq -\hat{\alpha} f'(\bar{e}-s)\frac{de(\bar{e})}{da} - f(\bar{e}-s)\frac{\partial \hat{\alpha}(0,\ s)}{\partial a}$, there exists no solution. In contrast, if $\frac{dC(0)}{da} < -\hat{\alpha} f'(\bar{e}-s)\frac{de(\bar{e})}{da} - f(\bar{e}-s)\frac{\partial \hat{\alpha}(0,\ s)}{\partial a}$, there exists a unique interior solution because $\frac{dC(a)}{da}$ is a strictly increasing function of $a$ and $-\hat{\alpha} f'(e-s)\frac{de(a)}{da} - f(e-s)\frac{\partial \hat{\alpha}(a,\ s)}{\partial a}$ is a strictly decreasing function of $a$. Similar arguments can be applied to show the existence of $\hat{s}$. Throughout this paper, the conditions guaranteeing the existence of interior solutions are assumed.

with $\hat{H}_{ij} > 0,\ i,\ j \in \{1,\ 2\}$. For brevity, the definitions of $\hat{H}_{ij},\ i,\ j \in \{1,\ 2\}$, are provided in the Appendix. To make (16)-(17) sufficient conditions as well, the following assumption is required.

*Assumption A2:* $|\hat{H}| = \hat{H}_{11}\hat{H}_{22} - (\hat{H}_{12})^2 > 0$ at $(\hat{a},\ \hat{s})$.

Under assumption A2, the properties of $(\hat{a},\ \hat{s})$ can be derived.

**Lemma 2**. *Suppose that assumption A2 holds. Then we have the following.*
(i) *If* $\beta \geq 1$, *then* $\hat{a} = \hat{s} = 0$ *and* $\hat{e} = \bar{e}$.
(ii) *If* $\beta < 1$, *then* $\hat{a} = \hat{a}(t)$ *and* $\hat{s} = \hat{s}(t)$ *exist with*
(iia) $\frac{d\hat{a}}{dt} = \frac{\hat{H}_{12}}{|\hat{H}|} > 0$,
(iib) $\frac{d\hat{s}}{dt} = \frac{-\hat{H}_{11}}{|\hat{H}|} < 0$, *and*
(iic) $\frac{d\hat{\alpha}}{dt} \geq\ (\leq)\ 0$ *iff* $\frac{de}{da}\hat{H}_{12} \geq\ (\leq)\ -\hat{H}_{11}$.
*Proof*. See the Appendix.

When the firm's weight is greater than or equal to one, the regulator will not monitor by Lemma 1(i). Thus, the firm will abate and report zero emission, and discharge as many pollutants as possible. This is what Lemma 2(i) claims. When the firm's weight is less than one, the regulator will monitor with positive probability by Lemma 1(ii). Hence, the firm's optimal choices are interior solutions and depend on emission tax only. Moreover, the firm will abate more (fewer) and report fewer (more) emissions with rising (falling) emission tax. The latter result is different from that under the commitment case, in which the firm's optimal reported emissions depend on both monitoring probability and emission tax.

Furthermore, emission tax could affect equilibrium monitoring probability positively or negatively under the no-commitment case, while they are independent in the commitment case. As emission tax increases, the firm will abate more emissions by Lemma 2(iia), hence the regulator will monitor less *ex post* by (14). Nevertheless, the firm will report fewer emissions due to higher emission tax by Lemma 2(iib). Thus, more *ex post* monitoring is induced by (15). If the former effect dominates, optimal monitoring probability would decrease with rising emission tax, and monitoring is a substitute for emission tax. In contrast, if the latter effect dominates, optimal monitoring probability would increase with rising emission tax, and monitoring is a complement to emission tax. When the two effects are equal, optimal monitoring probability and emission tax are independent as in the commitment case.

Third, given optimal monitoring probability $\hat{\alpha} = \hat{\alpha}(\hat{a},\ \hat{s})$ and the firm's optimal abatement and reported emission $(\hat{a}(t),\ \hat{s}(t))$, the regulator selects optimal emission tax $\hat{t}$ to solve the problem of

$$\min_t SC(\hat{\alpha}(t),\ t) = D(\hat{e}) + h(\hat{\alpha}) + \beta C(\hat{a}) - (1-\beta)[t\hat{s} + \hat{\alpha} f(\hat{e} - \hat{s})]$$
$$\text{subject to}\ \ 0 < \underline{t} \leq t \leq \bar{t}.$$

The first-order condition for interior solutions is

$$\begin{aligned}\frac{dSC}{dt} &= \left\{D_e(\hat{e})e_a(\hat{a}) + \beta C'(\hat{a}) - (1-\beta)\left[\tfrac{\partial\hat{\alpha}}{\partial a} f(\hat{e}-\hat{s}) + \hat{\alpha} f'(\hat{e}-\hat{s})e_a(\hat{a})\right]\right\}\tfrac{d\hat{a}}{dt} \\ &\quad -(1-\beta)\left[t - \hat{\alpha} f'(\hat{e}-\hat{s}) + \tfrac{\partial\hat{\alpha}}{\partial s} f(\hat{e}-\hat{s})\right]\tfrac{d\hat{s}}{dt} + h'(\hat{\alpha})\tfrac{d\hat{\alpha}}{dt} - (1-\beta)\hat{s} = 0, \quad (19)\end{aligned}$$

where $\frac{d\hat{\alpha}}{dt} = [\frac{\partial\hat{\alpha}}{\partial a}\frac{d\hat{a}}{dt} + \frac{\partial\hat{\alpha}}{\partial s}\frac{d\hat{s}}{dt}]$. By the first-order conditions of (16)-(17), we can simplify (19) to

$$\frac{dSC}{dt} = [D_e(\hat{e})e_a(\hat{a}) + C'(\hat{a})]\,\frac{d\hat{a}}{dt} + h'(\hat{\alpha})\frac{d\hat{\alpha}}{dt} - (1-\beta)\hat{s}(t) = 0. \quad (20)$$

The following assumption makes (20) a sufficient condition for interior $\hat{t}$ as well.

*Assumption A3:* $\frac{d^2SC}{dt^2}(\hat{t}) > 0.$[10]

Then, we can characterize optimal emission tax $\hat{t}$ below.

**Lemma 3**. *Suppose that assumptions A2-A3 hold. We have the following.*
(i) *If* $\beta \geq 1$, *then* $\hat{t}$ *could be any number in* $[\underline{t},\ \bar{t}]$.

[10]This assumption holds if $D''(\hat{e})$ or $C''(\hat{a})$ is large enough.

(ii) *If* $\beta < 1$, *then*

$$\begin{cases} \hat{t} = \bar{t} & \text{if } D_e(\hat{e})e_a(\hat{a}) + C'(\hat{a}) \leq 0 \text{ and } \frac{d\hat{\alpha}}{dt} \leq 0, \\ \hat{t} \in [\underline{t},\ \bar{t}] & \text{otherwise.} \end{cases}$$

*Proof.* See the Appendix.

As in the commitment case, optimal emission tax depends on the firm's weight in the social cost function.

## 5. Comparing the Commitment and No-Commitment Systems

The SPEs of the commitment and no-commitment to monitoring games are compared in terms of the firm's emission level, monitoring probability, emission tax, and the expected weighted social cost.

Given equilibria $(t^*,\ \alpha^*,\ a^*,\ s^*)$ and $(\hat{t},\ \hat{a},\ \hat{s},\ \hat{\alpha})$ at the commitment and no-commitment systems, respectively, we first compare $\alpha^*$ and $\hat{\alpha}$ below.

**Lemma 4**. *Suppose that assumptions A1-A3 hold. Then we have the following.*
(i) *If* $\beta \geq 1$, *then* $\alpha^* \geq \hat{\alpha}$. *The equality holds when* $C'(a) \geq |D_e(e)e_a(a)|$ *for all* $a \geq 0$.
(ii) *If* $\beta < 1$, *then relative sizes of* $\alpha^*$ *and* $\hat{\alpha}$ *are ambiguous.*
*Proof.* See the Appendix.

When the firm's weight is greater than or equal to one, the regulator will not monitor in the no-commitment case, while he may monitor with positive probability in the commitment case by Proposition 2(i). Thus, we have $\alpha^* \geq \hat{\alpha}$ as claimed by Lemma 4(i). In contrast, the regulator will monitor with positive probability in both systems by Proposition 2(ii) and Lemma 1(ii) when the firm's weight is less than one. Accordingly, relative sizes of optimal monitoring probabilities in the two systems are unsure as claimed by Lemma 4(ii). Actually, relative magnitudes of optimal emission taxes in the two systems are also unclear, which is shown after Lemma B in the Appendix.

Next, by combining the outcomes of Lemmas 1, 2, 4, and Proposition 2, we can reach conclusions about the firm's optimal emission and abatement levels.

**Lemma 5**. *Suppose that assumptions A1-A3 hold. Then we have the following.*
(i) *If* $\beta \geq 1$, *then* $a^* \geq \hat{a}$, $s^* \geq \hat{s}$, *and* $e^* \leq \hat{e}$. *The equality holds when* $C'(a) \geq |D_e(e)e_a(a)|$ *for all* $a \geq 0$.
(ii) *If* $\beta < 1$, *then relative sizes of* $a^*$ *and* $\hat{a}$ *are ambiguous, and so are those of* $e^*$ *and* $\hat{e}$.
*Proof.* See the Appendix.

Lemma 5 shows that for $\beta \geq 1$ the firm will abate fewer and discharge more emissions in the no-commitment than in the commitment system. That is because the regulator does not monitor in the no-commitment system. In contrast, as $\beta < 1$, relative sizes of the firm's optimal abatements and emissions in the two systems are unpredictable due to uncertain relative magnitudes of optimal monitoring probabilities and emission taxes in the two systems.

Finally, contrary to previous unjudgeable situations, the expected weighted social costs at equilibria of the two systems can be compared in general.

**Proposition 3**. *Under assumptions A1-A3, we have* $SC(\hat{\alpha},\ \hat{t}) \geq SC(\alpha^*,\ t^*)$.
*Proof.* In the commitment system, the regulator choose $(\alpha^*,\ t^*)$ to solve the problem of

$$\begin{aligned} & \min\nolimits_{\alpha,\,t} SC(\alpha,\ t) = D(e^*) + h(\alpha) + \beta C(a^*) - (1-\beta)[ts^* + \alpha f(e^* - s^*)] \\ \text{s.t.}\quad & (a^*,\ s^*) \in \arg\min\nolimits_{a,\,s} C(a) + ts + \alpha f(e - s), \\ & 0 \leq e \leq \bar{e},\ 0 \leq s \leq e,\ 0 \leq \alpha \leq 1,\ \text{and } 0 < \underline{t} \leq t \leq \bar{t}. \end{aligned} \tag{21}$$

In the no-commitment system, the regulator chooses $(\hat{\alpha},\ \hat{t})$ to solve the problem of

$$\begin{aligned} &\min_{\alpha,\ t} SC(\hat{\alpha}(t),\ t) = D(\hat{e}) + h(\hat{\alpha}) + \beta C(\hat{a}) - (1-\beta)[t\hat{s} + \hat{\alpha} f(\hat{e}-\hat{s})] \\ \text{s.t.}\quad & (\hat{a},\ \hat{s}) \in \arg\min_{a,\ s} C(a) + ts + \hat{\alpha} f(e-s), \\ & \hat{\alpha} \in \arg\min_{\alpha} SC(\alpha \mid \hat{a},\ \hat{s},\ t) = D(\hat{e}) + h(\alpha) + \beta C(\hat{a}) - (1-\beta)[t\hat{s} + \alpha f(\hat{e}-\hat{s})], \\ & 0 \le e \le \bar{e},\ 0 \le s \le e,\ 0 \le \alpha \le 1,\ \text{and } 0 < \underline{t} \le t \le \bar{t}. \end{aligned} \tag{22}$$

Let $S_c$ be the set of $(a,\ s,\ \alpha,\ t)$ satisfying all constraints listed in (21), and $S_{nc}$ be the set of $(a,\ s,\ \alpha,\ t)$ satisfying all constraints listed in (22). It is obvious that $S_{nc} \subseteq S_c$. Consequently, we have $SC(\hat{\alpha},\ \hat{t}) \ge SC(\alpha^*,\ t^*)$. □

Proposition 3 suggests that the commitment system is at least as efficient as the no-commitment system. This means that the regulator may face efficiency loss when she can commit but chooses not to. The intuition behind Proposition 3 is simple. The regulator has the first-mover advantage in the commitment system since she decides monitoring probability *before* the firm's optimal actions, but this advantage disappears in the no-commitment system. Thus, efficiency loss could happen in the latter case. Accordingly, the result of Proposition 3 will weaken regulator's incentive to adopt the no-commitment design.

## 6. Extensions

For brevity, all proofs in this section are omitted and available on request.

### 6.1. One Regulator and $n$ Firms

If damage and monitoring-cost functions over firms are additively separable, the results in Sections 3-5 would hold when our models are extended to include one regulator and $n,\ n \ge 2$, firms. Let $C_i(a_i)$ be firm $i$'s cost at abatement level $a_i$ with $C_i'(a_i) > 0,\ C_i''(a_i) > 0$ for all $a_i \ge 0,\ i = 1,\ 2, \ldots, n$. Moreover, let $s_i$ and $e_i = e_i(a_i)$

represent firm $i$'s reported emission and emission levels with $e_i(0) = \bar{e}_i,\ e_i'(\cdot) < 0$, and $e_i''(\cdot) > 0$. The regulator's monitoring probability for firm $i$ is denoted by $\alpha_i$. Then, in the commitment case, the regulator will choose $\{\alpha_i^*\}_{i=1}^n$ and $t^*$ to solve the problem of

$$\begin{array}{ll} \min_{\alpha_1,\ldots,\alpha_n,\, t} & \sum_{i=1}^n \{D(e_i^*) + h(\alpha_i) + \beta C_i(a_i^*) - (1-\beta)[ts_i + \alpha_i f(e_i^* - s_i^*)]\} \\ \text{s.t.} & (a_i^*,\ s_i^*) \in \arg\min_{a_i,\, s_i} C_i(a_i) + ts_i + \alpha_i f(e_i - s_i), \\ & 0 \le e_i \le \bar{e}_i,\ 0 \le s_i \le e_i,\ 0 \le \alpha_i \le 1,\ 0 < \underline{t} \le t \le \bar{t},\ i = 1,\ 2, \ldots, n. \end{array}$$

In the no-commitment case, the regulator will choose $(\hat{\alpha}_1,\ \hat{\alpha}_1, \ldots, \hat{\alpha}_n, \hat{t})$ to solve the problem of

$$\begin{array}{ll} \min_{\alpha,\, t} & \sum_{i=1}^n \{D(\hat{e}_i) + h(\hat{\alpha}_i) + \beta C_i(\hat{a}_i) - (1-\beta)[t\hat{s}_i + \hat{\alpha}_i f(\hat{e}_i - \hat{s}_i)]\} \\ \text{s.t.} & (\hat{a}_i,\ \hat{s}_i) \in \arg\min_{a_i,\, s_i} C_i(a_i) + ts_i + \hat{\alpha}_i f(e_i - s_i), \\ & \hat{\alpha}_i \in \arg\min_{\alpha_i} \sum_{i=1}^n \{D(\hat{e}_i) + h(\alpha_i) + \beta C_i(\hat{a}_i) - (1-\beta)[t\hat{s}_i + \alpha_i f(\hat{e}_i - \hat{s}_i)]\}, \\ & 0 \le e_i \le \bar{e}_i,\ 0 \le s_i \le e_i,\ 0 \le \alpha_i \le 1,\ 0 < \underline{t} \le t \le \bar{t},\ i = 1,\ 2, \ldots, n. \end{array}$$

Since firms' behavior is pairwisely independent, it is easy to see that the outcomes in Sections 3-5 remain true qualitatively if $\alpha^*,\ \hat{\alpha},\ a^*,\ \hat{a},\ s^*,\ \hat{s},\ C(\cdot)$ and $e(\cdot)$ are replaced by $\alpha_i^*,\ \hat{\alpha}_i,\ a_i^*,\ \hat{a}_i,\ s_i^*,\ \hat{s}_i,\ C_i(\cdot)$ and $e_i(\cdot)$, respectively. Also, equations (10) and (19) should be replaced by

$$\begin{aligned} \frac{\partial SC}{\partial t} &= \sum_{i=1}^n \{[D_e(e_i^*)\frac{de_i(a_i^*)}{da_i} + \beta C_i'(a_i^*) - (1-\beta)\alpha_i f'(e_i^* - s_i^*)\frac{de_i}{da_i}]\frac{\partial a_i^*}{\partial t} - (1-\beta)[s_i^* \\ &\quad + (t - \alpha_i f'(e_i^* - s_i^*))\frac{\partial s_i^*}{\partial t}]\} = 0, \text{ and} \\ \frac{\partial SC}{\partial t} &= \sum_{i=1}^n \{[D_e(\hat{e}_i)\frac{de_i(\hat{a}_i)}{da_i} + \beta C_i'(\hat{a}_i) - (1-\beta)(\frac{\partial \hat{\alpha}_i}{\partial a_i} f(\hat{e}_i - \hat{s}_i) + \hat{\alpha}_i f'(\hat{e}_i - \hat{s}_i)\frac{de_i(\hat{a}_i)}{da_i})]\frac{d\hat{a}_i}{dt} \\ &\quad - (1-\beta)[t - \hat{\alpha}_i f'(\hat{e}_i - \hat{s}_i) + \frac{\partial \hat{\alpha}_i}{\partial s_i} f(\hat{e}_i - \hat{s}_i)]\frac{d\hat{s}_i}{dt} + h'(\hat{\alpha}_i)\frac{d\hat{\alpha}_i}{dt} - (1-\beta)\hat{s}_i\} = 0, \end{aligned}$$

respectively. Here $\frac{d\hat{\alpha}_i}{dt} = \frac{\partial \hat{\alpha}_i}{\partial a_i}\frac{\partial \hat{a}_i}{\partial t} + \frac{\partial \hat{\alpha}_i}{\partial s_i}\frac{\partial \hat{s}_i}{\partial t}$ for $i = 1,\ 2, \ldots, n$.

### 6.2. Incomplete Information on Firm's Abatement Cost Function

In this section, we relax the complete information assumption about the firm's abatement cost function. Our conclusions obtained in Sections 3-5 are shown to remain true.

As in Yates and Cronshaw (2001), it is presumed that there exists a parameter $\epsilon$ in the firm's abatement cost function, i.e., $C(a,\ \epsilon)$. The true value of $\epsilon$ is only known to the firm, and the regulator perceives the distribution of $\epsilon$. In other words, the firm's optimal emission abatement levels, $a^*(\cdot,\ \epsilon)$ and $\hat{a}(\cdot,\ \epsilon)$, in the two systems are stochastic for the regulator, and so are the firm's reported emissions, $s^*(\cdot,\ \epsilon)$ and $\hat{s}(\cdot,\ \epsilon)$. Thus, monitoring is conducted to both discover the firm's true emissions and verify its reported emissions. In the commitment system, the regulator chooses $(\alpha^*,\ t^*)$ to solve the problem of

$$\begin{aligned} & \min_{\alpha,\, t} \bar{SC}(\alpha,\ t) \equiv ED(e^*) + h(\alpha) + \beta EC(a^*) - (1-\beta)[tEs^* + \alpha Ef(e^* - s^*)] \\ \text{s.t.}\quad & (a^*,\ s^*) \in \arg\min_{a,\, s} C(a,\ \epsilon) + ts + \alpha f(e-s), \\ & 0 \le e \le \bar{e},\ 0 \le s \le e,\ 0 \le \alpha \le 1,\ 0 < \underline{t} \le t \le \bar{t}, \end{aligned}$$

where $E$ is the expectation operator taken with respect to $\epsilon$. Accordingly, equations (9)-(10) are replaced by

$$\begin{aligned} \frac{\partial \bar{SC}}{\partial \alpha} &= \int [D_e(e^*)e_a(a^*) + \beta C'(a^*) - (1-\beta)\alpha f'(e^* - s^*)e_a(a^*)]\frac{\partial a^*}{\partial \alpha} dg(\epsilon) + h'(\alpha) \\ & \quad -(1-\beta)\int \{[t - \alpha f'(e^* - s^*)]\frac{\partial s^*}{\partial \alpha} + f(e^* - s^*)\} dg(\epsilon) = 0, \text{ and} \\ \frac{\partial \bar{SC}}{\partial t} &= \int [D_e(e^*)e_a(a^*) + \beta C'(a^*) - (1-\beta)\alpha f'(e^* - s^*)e_a(a^*)]\frac{\partial a^*}{\partial t} dg(\epsilon) \\ & \quad -(1-\beta)\int \{s^* + (t - \alpha f'(e^* - s^*))\frac{\partial s^*}{\partial t}\} dg(\epsilon) = 0, \end{aligned}$$

respectively, where $g(\epsilon)$ is the CDF of random variable $\epsilon$. It is easy to see that Proposition 1 remains true for all $\epsilon$. Consequently, Proposition 2 will also hold. Similarly, in the no-commitment system, the regulator chooses $(\hat{\alpha},\ \hat{t})$ to solve the problem of

$$\begin{aligned} & \min_{\alpha,\, t} \bar{SC}(\hat{\alpha}(t),\ t) \equiv ED(\hat{e}) + h(\hat{\alpha}) + \beta EC(\hat{a}) - (1-\beta)[tE\hat{s} + \hat{\alpha} Ef(\hat{e} - \hat{s})] \\ \text{s.t.} \quad & (\hat{a},\ \hat{s}) \in \arg\min_{a,\, s} C(a,\ \epsilon) + ts + \hat{\alpha} f(e - s), \\ & \hat{\alpha} \in \arg\min_{\alpha} \bar{SC}(\alpha \mid \hat{a},\ \hat{s},\ t) \equiv D(\hat{e}) + h(\alpha) + \beta C(\hat{a}) - (1-\beta)[t\hat{s} + \alpha f(\hat{e} - \hat{s})], \\ & 0 \le e \le \bar{e},\ 0 \le s \le e,\ 0 \le \alpha \le 1,\ 0 < \underline{t} \le t \le \bar{t}. \end{aligned}$$

Accordingly, equation (20) is replaced by

$$\begin{aligned} \frac{d\bar{SC}}{dt} \quad = \quad & \int [D_e(\hat{e}) e_a(\hat{a}) + C'(\hat{a})] \frac{d\hat{a}}{dt} dg(\epsilon) + h'(\hat{\alpha}) \int \frac{d\hat{\alpha}}{dt} dg(\epsilon) \\ & -(1-\beta) \int \hat{s}(t,\ \epsilon) dg(\epsilon) = 0. \end{aligned}$$

It is easy to see that Lemmas 1-3 still hold. Consequently, the outcomes in Section 5 remain true.

## 7. Conclusions

In this paper, we analyze and compare the behavior of one regulator and one polluting firm in emission tax systems with and without commitment to monitoring. The main findings are as follows. First, the firm is noncompliant at all equilibria of the commitment system. This implies non-existence of the paradox of *ex ante* commitment to monitoring. Second, in the commitment system, the effectiveness of monitoring policy in reducing firm's optimal emissions would depend on firm's weight in the social cost function. Third, relative sizes of firm's optimal emissions and reported emissions cannot be sure in the two systems unless firm's weight in the social cost function is no less than one. Relative magnitudes of regulator's optimal monitoring probabilities in the two systems are also uncertain. Fourth, optimal monitoring probability and emission tax are independent in the commitment system, while monitoring probability could be a substitute or complement to emission tax in the no-commitment system. Fifth, the commitment to monitoring system is at least as efficient as the no-commitment system.

Finally, all the above results would hold if our models are generalized to include one regulator and $n$, $n \geq 2$, firms, or if players have asymmetric information about firm's abatement cost function. In the future, it will be interesting to examine whether these findings still hold when the regulator minimizes firms' total emissions as in Macho-Stadler and Perez-Castrillo (2006) or minimizes firms' violation levels as in Stranlund and Dhanda (1999).

## Appendix

*Proof of Proposition 1:* (i) If $\alpha = 0$, then $\frac{\partial FC}{\partial a} = C'(a) > 0$ for all $a \geq 0$, and $\frac{\partial FC}{\partial s} = t > 0$ for all $s \geq 0$. Thus, $a_1^* = 0$, $e_1^* = \bar{e}$ and $s_1^* = 0$.

(ii) For $\alpha \in (0, \ \frac{t}{f'(\tilde{e}(t))})$, two subintervals, $(0, \ \frac{t}{f'(\bar{e})}]$ and $(\frac{t}{f'(\bar{e})}, \ \frac{t}{f'(\tilde{e}(t))})$, should be considered. That is because, for a given $t$, we have $\bar{e} > \tilde{e}(t) > 0$. Hence, $f'(\bar{e}) > f'(\tilde{e}(t)) > f'(0) > 0$, and $0 < \frac{t}{f'(\bar{e})} < \frac{t}{f'(\tilde{e}(t))} < \frac{t}{f'(0)} < 1$ by $f'' > 0$.

For $\alpha \in (0, \ \frac{t}{f'(\bar{e})}]$, we have $t - \alpha f'(e-s) \geq t - \alpha f'(\bar{e}) \geq 0$ for all $s$ because $e - s \leq e \leq \bar{e}$ for all $s$, $f'' > 0$, and $\alpha \leq \frac{t}{f'(\bar{e})}$. Accordingly, $\frac{\partial FC}{\partial s} = t - \alpha f'(e-s) \geq 0$ for all $e$ and $s$. Thus, $s_2^* = 0$. On the other hand, since $\alpha > 0$, $a_2^*$ must satisfy (6).

For $\alpha \in (\frac{t}{f'(\bar{e})}, \ \frac{t}{f'(\tilde{e}(t))})$, firm's optimal choices are still $(a^*(\alpha), \ s^* = 0)$, which is shown below. For $e \in (0, \ \tilde{e}(t)]$, we have $e - s \leq e \leq \tilde{e}(t)$ for all $e \geq 0$ and $s \geq 0$. This in turn implies that $0 < t - \alpha f'(\tilde{e}(t)) \leq t - \alpha f'(e) \leq t - \alpha f'(e-s)$ for all $e \geq 0$ and $s \geq 0$ by $\alpha < \frac{t}{f'(\tilde{e}(t))}$. Accordingly, $\frac{\partial FC}{\partial s} = t - \alpha f'(e-s) > 0$ for all $e \geq 0$ and $s \geq 0$. Thus, $s^* = 0$ and $a^* = a^*(\alpha)$ satisfying (6). For $e \in (\tilde{e}(t), \ \bar{e}]$, relative sizes of $\alpha f'(e-s)$ and $t$ are uncertain because unsure relative sizes of $(e-s)$ and $\tilde{e}(t)$. Thus, the firm's optimal solutions $(a^*, \ s^*)$ must satisfy (4)-(5). However, since $\alpha < \frac{t}{f'(\tilde{e}(t))}$, we have $t > \alpha f'(\tilde{e}(t)) \geq \alpha f'(\tilde{e}(t) - s)$ for all $s \geq 0$. Thus, $\frac{\partial FC}{\partial s} = t - \alpha f'(\tilde{e}(t) - s) > 0$ for all $s \geq 0$. It contradicts (5). Thus, there exists no solution for $e \in (\tilde{e}(t), \ \bar{e}]$. Thus, for $\alpha \in (\frac{t}{f'(\bar{e})}, \ \frac{t}{f'(\tilde{e}(t))})$, the firm's optimal solutions are $(a^*(\alpha), \ s^* = 0)$ with

$e^*(\alpha) = e(a^*(\alpha)) \in (0,\ \tilde{e}(t)]$. Combined with the outcome in subinterval $(0,\ \frac{t}{f'(\bar{e})}]$, we have $(a_2^*,\ s_2^*) = (a^*(\alpha),\ 0)$ with

$$\frac{da_2^*}{d\alpha} = \frac{-f'(e_2^*)e_a(a_2^*)}{C''(a_2^*) + \alpha f''(e_2^*)e_a^2(a_2^*) + \alpha f'(e_2^*)e_{aa}(a_2^*)} > 0 \text{ and } \frac{de_2^*}{d\alpha} = \frac{de(a_2^*)}{d\alpha} \cdot \frac{da_2^*}{d\alpha} < 0.$$

(iii) For $\alpha \in [\frac{t}{f'(\tilde{e}(t))},\ \frac{t}{f'(0)})$, relative sizes of $t$ and $\alpha f'(e-s)$ are unpredictable because $t - \alpha f'(0) \geq t - \alpha f'(e-s) \geq t - \alpha f'(e) \geq t - \alpha f'(\bar{e})$ for all $e$ and $s$, $t - \alpha f'(0) > 0$, and $t - \alpha f'(\bar{e}) < 0$. Thus, the firm's optimal solutions $(a_3^*,\ s_3^*)$ should satisfy (4)-(5), and equal $(\tilde{a}(t),\ \tilde{s}(\alpha,\ t))$ with $0 \leq \tilde{s}(\alpha,\ t) < \tilde{e}(t) = e(\tilde{a}(t))$. The case of $\tilde{s}(\alpha,\ t) = 0$ will occur only when $\alpha = \frac{t}{f'(\tilde{e}(t))}$. On the other hand, if $\tilde{s}(\alpha,\ t) = \tilde{e}(t)$, then $t = \alpha f'(0)$ by (5). It contradicts with $\alpha < \frac{t}{f'(0)}$. Moreover, by Cramer's rule, we can obtain

$$\begin{aligned}
\frac{da_3^*}{dt} &= \frac{-e_a(a_3^*)}{C''(a_3^*) + te_{aa}(a_3^*)} > 0,\ \frac{de_3^*}{dt} = \frac{de(a_3^*)}{da} \cdot \frac{da_3^*}{dt} < 0, \\
\frac{\partial s_3^*}{\partial t} &= \frac{-1 + \alpha f''(e_3^* - s_3^*)e_a \frac{da_3^*}{dt}}{\alpha f''(e_3^* - s_3^*)} < 0,\ \text{and}\ \frac{\partial s_3^*}{\partial \alpha} = \frac{f'(e_3^* - s_3^*)}{\alpha f''(e_3^* - s_3^*)} > 0.
\end{aligned}$$

(iv) For $\alpha \in [\frac{t}{f'(0)},\ 1]$, we have $t - \alpha f'(e-s) \leq t - \alpha f'(0) \leq 0$ for all $e$ and $s$ because $e - s \geq 0$. Thus, $\frac{\partial FC}{\partial s} \leq 0$, which suggests $s_4^* = e_4^*$. Accordingly, $a_4^* = \tilde{a}(t)$ satisfying (7), and $e_4^*(t) = e(\tilde{a}(t)) = s_4^*(t)$. As in part (iii), we have $\frac{da_4^*}{dt} > 0$ and $\frac{de_4^*}{dt} = \frac{ds_4^*}{dt} < 0$.

Finally, the second-order conditions for solutions $(a_i^*,\ s_i^*)$, $i = 1,\ 2,\ 3,\ 4$, hold because for all $(a,\ s)$

$$\begin{aligned}
\frac{\partial^2 FC}{\partial a^2} &= \frac{d^2C(a)}{da^2} + \alpha f''(e-s)(\frac{de}{da})^2 + \alpha f'(e-s)\frac{d^2e}{da^2} > 0, \\
\frac{\partial^2 FC}{\partial s^2} &= \alpha f''(e-s) > 0, \\
\frac{\partial^2 FC}{\partial a \partial s} &= -\alpha f''(e-s)\frac{de}{da} > 0,\ \text{and} \\
\frac{\partial^2 FC}{\partial a^2}\frac{\partial^2 FC}{\partial s^2} - [\frac{\partial^2 FC}{\partial a \partial s}]^2 &= \alpha f''(e-s)[\frac{d^2C(a)}{da^2} + \alpha f'(e-s)\frac{d^2e}{da^2}] > 0.
\end{aligned}$$

*Proof of Proposition 2:* In the first step, we derive the regulator's best choices in all regions if they exist, given the firm's optimal choices. Second, by comparing the associated expected social costs at all possible candidates, we can find the optimal solutions.

Step 1:

(i) In region 1, we have $a_1^* = 0$, $e_1^* = \bar{e}$, and $s_1^* = 0$ by Proposition 1(i). Thus, $\frac{\partial e_1^*}{\partial \alpha} = \frac{\partial a_1^*}{\partial \alpha} = \frac{\partial s_1^*}{\partial \alpha} = \frac{\partial e_1^*}{\partial t} = \frac{\partial a_1^*}{\partial t} = \frac{\partial s_1^*}{\partial t} = 0$. Then we have

$$\begin{aligned} \frac{\partial SC}{\partial \alpha} &= h'(\alpha) - (1-\beta) f(\bar{e}), \text{ and} \\ \frac{\partial SC}{\partial t} &= -(1-\beta) s_1^* = 0 \; \forall t. \end{aligned}$$

Accordingly, $t_1^*$ could be any number in $[\underline{t},\ \bar{t}]$. As to $\alpha_1^*$, it depends on the values of $\beta$. If $\beta < 1$, we have $\alpha_1^* > 0$ satisfying the condition of $h'(\alpha) = (1-\beta) f(\bar{e})$. However, it contradicts the definition of region 1. Thus, there exists no optimal solution in region 1. If $\beta \geq 1$, we have $\frac{\partial SC}{\partial \alpha} > 0$. Thus, $\alpha_1^* = 0$. And the associated minimum expected weighted social cost is

$$SC_1 = D(\bar{e}) + h_0. \tag{23}$$

(ii) In region 2, by Proposition 1(ii), we have $s_2^* = 0$, $e_2^*(\alpha) = e(a_2^*)$, and $a_2^* = a^*(\alpha)$ satisfying $C'(a_2^*) = -\alpha f'(e_2^*) e_a(a_2^*)$. Moreover, $\frac{\partial a_2^*}{\partial \alpha} > 0$ and $\frac{\partial a_2^*}{\partial t} = \frac{\partial s_2^*}{\partial \alpha} = \frac{\partial s_2^*}{\partial t} = 0$. Thus, we have

$$\begin{aligned} \frac{\partial SC}{\partial \alpha} &= [D_e(e_2^*) e_a + C'(a_2^*)] \frac{\partial a_2^*}{\partial \alpha} + h'(\alpha) - (1-\beta) f(e_2^*), \text{ and} \\ \frac{\partial SC}{\partial t} &= -(1-\beta) s_2^* = 0 \; \forall t. \end{aligned}$$

As in part (i), $t_2^*$ could be any number in $[\underline{t},\ \bar{t}]$. If $\beta < 1$, $\alpha_2^*$ satisfying $\frac{\partial SC}{\partial \alpha} = 0$ could be optimal if $\alpha_2^*$ lies in the interval of $(0,\ \frac{t_2^*}{f'(\tilde{e}(t_2^*))})$. Similar situation occurs when $\beta \geq 1$ and $D_e(e_2^*) e_a(a_2^*) + C'(a_2^*) < 0$. When $\beta \geq 1$ and $D_e(e_2^*) e_a(a_2^*) + C'(a_2^*) \geq 0$, $\alpha_2^* = 0$ because $\frac{\partial SC}{\partial \alpha} > 0$. Nevertheless, it contradicts the definition of region 2. Accordingly, the associated expected social cost is

$$SC_2 = D(e_2^*) + h(\alpha_2^*) + \beta C(a_2^*) - (1-\beta) \alpha_2^* f(e_2^*). \tag{24}$$

(iii) In region 3, by Proposition 1(iii), we have $a_3^* = \tilde{a}(t)$ and $s_3^* = \tilde{s}(\alpha,\ t)$ satisfying

(4)-(5) with $\frac{\partial a_3^*}{\partial t} > 0$, $\frac{\partial s_3^*}{\partial \alpha} > 0$, and $\frac{\partial s_3^*}{\partial t} < 0$. Therefore, we have

$$\begin{aligned} \frac{\partial SC}{\partial \alpha} &= h'(\alpha) - (1-\beta) f(e_3^* - s_3^*), \text{ and} \\ \frac{\partial SC}{\partial t} &= [D_e(e_3^*) e_a(a_3^*) + C'(a_3^*)] \frac{\partial a_3^*}{\partial t} - (1-\beta) s_3^*. \end{aligned}$$

If $\beta \geq 1$, then $\frac{\partial SC}{\partial \alpha} > 0$. Thus, $\alpha_3^* = 0$. Again, this is a contradiction. If $\beta < 1$ and $D_e(e_3^*) e_a(a_3^*) + C'(a_3^*) \geq 0$, then $t_3^*$ must satisfy $\frac{\partial SC}{\partial t} = 0$, and $\alpha_3^*$ exists as it satisfies the condition of $\frac{\partial SC}{\partial \alpha} = 0$ and lies in the interval of $[\frac{t_3^*}{f'(\tilde{e}(t_3^*))}, \ \frac{t_3^*}{f'(0)})$. If $\beta < 1$ and $D_e(e_3^*) e_a(a_3^*) + C'(a_3^*) < 0$, then $t_3^* = \bar{t}$, and $\alpha_3^*$ exists as it satisfies the condition of $\frac{\partial SC}{\partial \alpha} = 0$ and lies in the interval of $[\frac{\bar{t}}{f'(\tilde{e}(\bar{t}))}, \ \frac{\bar{t}}{f'(0)})$. Accordingly, the associated expected social cost is

$$SC_3 = D(e_3^*) + h(\alpha_3^*) + \beta C(a_3^*) - (1-\beta)[t_3^* s_3^* + \alpha_3^* f(e_3^* - s_3^*)]. \tag{25}$$

(iv) In region 4, by Proposition 1(iv), we have $a_4^* = \tilde{a}(t)$ satisfying (7), and $e_4^*(t) = e(\tilde{a}(t)) = s_4^*(t)$ with $t - \alpha f'(0) \leq 0$, $\frac{\partial a_4^*}{\partial t} > 0$, and $\frac{\partial s_4^*}{\partial t} < 0$. Consequently, we have

$$\begin{aligned} \frac{\partial SC}{\partial \alpha} &= h'(\alpha) \text{ and} \\ \frac{\partial SC}{\partial t} &= \frac{\partial a_4^*}{\partial t} e_a(a_4^*)[D_e(e_4^*) - \beta t - (1-\beta)\alpha f'(0)] - (1-\beta)[e_4^* + (t - \alpha f'(0)) \frac{ds_4^*}{dt}]. \end{aligned}$$

Since $h'(\alpha) > 0$, we have $\frac{\partial SC}{\partial \alpha} > 0$ for all $\alpha$. Thus, $\alpha_4^* = 0$. It contradicts the definition of region 4. Thus, there exists no solution in region 4.

<u>*Step 2*</u>: (i) Suppose $\beta \geq 1$. By step 1, we know that $(\alpha^*, \ t^*)$ are not in regions 3 and 4. Thus, we have $a_1^* < a_2^*$, $e_1^* > e_2^*$, and $\alpha_1^* < \alpha_2^*$. Then, we have $D(e_1^*) > D(e_2^*)$, $C(a_1^*) < C(a_2^*)$, and $h(\alpha_2^*) > h(\alpha_1^*)$. Under the circumstances, equations (23) and (24) imply that relative sizes of $SC_1$ and $SC_2$ are unpredictable. It means that pair $(\alpha_i^*, \ t_i^*)$, $i = 1, \ 2$, could be optimal solutions. For instance, if $C(a_2^*)$ is large enough and $\beta = 1$, $(\alpha^*, \ t^*)$ could be in region 1. In contrast, if $D(\bar{e})$ is large enough, $(\alpha^*, \ t^*)$ could occur in region 2.

(ii) Suppose $\beta < 1$. By step 1, we know that $(\alpha^*, \ t^*)$ could lie in regions 2-3. Similarly, since optimal emission tax in all regions could be distinct, we can only conclude $s_2^* \leq s_3^*$.

Under the situation, equations (24)-(25) suggest that relative sizes of $SC_2$ and $SC_3$ are unclear. In particular, if $e_2^*$ is large enough, we could have $(\alpha^*,\ t^*)$ in region 2. If $t_3^* s_3^*$ is large enough, we could have $(\alpha^*,\ t^*)$ in region 3. □

*The definition of Hessian matrix H*:

$$\begin{aligned}
\hat{H}_{11} &= \frac{d^2C(a)}{da^2} + \hat{\alpha} f''(e-s)(\frac{de(a)}{da})^2 + \hat{\alpha} f'(e-s)\frac{d^2e(a)}{da^2} + 2f'(e-s)\frac{\partial\hat{\alpha}}{\partial a}\cdot\frac{de(a)}{da} \\
&\quad + f(e-s)\frac{\partial^2\hat{\alpha}}{\partial a^2} > 0, \\
\hat{H}_{12} &= \hat{H}_{21} = f'(e-s)\frac{\partial\hat{\alpha}}{\partial s}\frac{de(a)}{da} - \hat{\alpha} f''(e-s)\frac{de(a)}{da} + f(e-s)\frac{\partial^2\hat{\alpha}}{\partial s\partial a} \\
&\quad - f'(e-s)\frac{\partial\hat{\alpha}}{\partial a} > 0, \\
\hat{H}_{22} &= \hat{\alpha} f''(e-s) + f(e-s)\frac{\partial^2\hat{\alpha}}{\partial s^2} - 2f'(e-s)\frac{\partial\hat{\alpha}}{\partial s} > 0, \\
\frac{\partial^2\hat{\alpha}}{\partial a^2} &= \frac{(1-\beta)}{h''(\hat{\alpha})}[f''(e-s)(\frac{de(a)}{da})^2 + f''(e-s)\frac{d^2e(a)}{da^2}] > 0, \\
\frac{\partial^2\hat{\alpha}}{\partial a\partial s} &= \frac{-(1-\beta)}{h''(\hat{\alpha})} f''(e-s)\frac{de(a)}{da} > 0, \text{ and} \\
\frac{\partial^2\hat{\alpha}}{\partial s^2} &= \frac{(1-\beta)}{h''(\hat{\alpha})} f''(e-s) > 0.
\end{aligned}$$

*Proof of Lemma 2:* Substituting $a$ and $s$ by $\hat{a}(t)$ and $\hat{s}(t)$ respectively in (16)-(17), and taking the derivatives of (16) and (17) with respect to $t$ yield

$$\hat{H}\cdot\begin{bmatrix}\frac{d\hat{a}}{dt}\\ \frac{d\hat{s}}{dt}\end{bmatrix} = \begin{bmatrix}0\\ -1\end{bmatrix} dt.$$

Applying Cramer's rule yields $\frac{d\hat{a}}{dt} = \frac{\hat{H}_{12}}{|\hat{H}|} > 0$ and $\frac{d\hat{s}}{dt} = \frac{-\hat{H}_{11}}{|\hat{H}|} < 0$. It in turn implies that

$$\frac{d\hat{\alpha}}{dt} = \frac{\partial\hat{\alpha}}{\partial a}\frac{d\hat{a}}{dt} + \frac{\partial\hat{\alpha}}{\partial s}\frac{d\hat{s}}{dt} = \frac{(1-\beta)f'}{h''(\hat{\alpha})|\hat{H}|}\left[\frac{de}{da}\hat{H}_{12} + \hat{H}_{11}\right].$$

So, we have $\frac{d\hat{\alpha}}{dt} \geq\ (\leq)\ 0$ iff $\frac{de}{da}\hat{H}_{12} \geq\ (\leq)\ -\hat{H}_{11}$. □

*Proof of Lemma 3:* (i) If $\beta \geq 1$, we have $\hat{\alpha} = 0$. Accordingly, $\hat{a} = \hat{s} = 0$, and $\hat{e} = \bar{e}$. Then, $SC(t) = D(\bar{e}) + h_0$, which is independent of $t$. Thus, $\hat{t}$ can be any number in

$[\underline{t},\ \bar{t}]$.

(ii) For $\beta < 1$, we have $\hat{\alpha} > 0$. Suppose $\frac{d\hat{\alpha}}{dt} \leq 0$ and $D_e(\hat{e})e_a(\hat{a}) + C'(\hat{a}) \leq 0$. Then $\frac{dSC}{dt} \leq 0$, hence $\hat{t} = \bar{t}$. In other situations, we could have $\hat{t} \in [\underline{t},\ \bar{t}]$ satisfying the condition of $\frac{dSC}{dt} = 0$, $\hat{t} = \underline{t}$ if $\frac{dSC}{dt} > 0$, or $\hat{t} = \bar{t}$ if $\frac{dSC}{dt} < 0$. □

*Proof of Lemma 4:* (i) For $\beta \geq 1$, $\alpha^*$ is non-negative by Proposition 2(i), while $\hat{\alpha} = 0$ by Lemma 1(i). Thus, we have $\alpha^* \geq \hat{\alpha}$. In particular, if $C'(a) \geq |D_e(e)e_a(a)|$ for all $a$, then we have $\alpha^* = 0$ by Proposition 2(i). Thus, $\alpha^* = \hat{\alpha}$.

(ii) For $\beta < 1$, $\hat{\alpha} > 0$ satisfies the condition of $h'(\hat{\alpha}) = (1-\beta)f(\hat{e}-\hat{s})$ by (13). However, which conditions $\alpha^* > 0$ would satisfy depends on the region it is in. For instance, in region 3, $\alpha^*$ needs to meet the condition of $h'(\alpha^*) = (1-\beta)f(e_3^* - s_3^*)$. Under the circumstance, we have

$$\alpha^* \geq\ (\leq)\ \hat{\alpha} \text{ iff } \ f(e_3^* - s_3^*) \geq\ (\leq)\ f(\hat{e} - \hat{s})$$

by $h''(\alpha) > 0$ for all $\alpha$. It means that relative magnitudes of $\alpha^*$ and $\hat{\alpha}$ are uncertain. □

*Proof of Lemma 5:* (i) Since $\beta \geq 1$, we have $\hat{\alpha} = 0$ by Lemma 1(i). Hence, $\hat{a} = \hat{s} = 0$ and $\hat{e} = \bar{e}$ by Lemma 2(i). We must have $a^* \geq \hat{a}$, $s^* \geq \hat{s}$, and $e^* \leq \hat{e}$ due to $\alpha^* \geq 0$. The equality will hold when $\alpha^* = 0$, which is implied by $C'(a) \geq |D_e(e)e_a(a)|$ for all $a \geq 0$ and Proposition 2(i).

(ii) If $\beta < 1$, relative sizes of $\alpha^*$ and $\hat{\alpha}$ are unsure by Lemma 4(ii). Accordingly, relative sizes of $\hat{a}$ and $a^*(\alpha^*)$ in region 2 under commitment to monitoring cannot be judged. However, for interior solutions $\hat{a}(t)$ and $a^*(t)$ in region 3 under the commitment case, we have $a^*(t) = \hat{a}(t)$ for a given $t$ as shown below.

**Lemma B**. *Suppose that assumptions A1-A3 hold. For a given* $t$, *we have* $a^*(t) = \hat{a}(t)$, *where* $a^*(t) = a_3^*(t)$ *in the commitment case. However, interior solutions* $\tilde{s}(\alpha,\ t)$ *in region 3 and* $\hat{s}(t)$ *are different functions of* $t$.

*Proof.* For the commitment solutions in region 3, we have $a^*(t)$ satisfying (7), and $\hat{a}(t)$ meeting (18). By (7) and (18), we know that $a^*(t) = \hat{a}(t)$ for a given $t$. Moreover, equations (5) and (17) imply that $\tilde{s}(\alpha,\ t)$ and $\hat{s}(t)$ are different functions of $t$. $\square$

Lemma B suggests that the firm could abate the same amount of emissions in the two systems if two emission taxes are equal. The equal-emission-tax condition could hold in the exogenous-tax setups, but may fail in the endogenous-tax models, such as ours. It is shown below that relative magnitudes of $t^*$ and $\hat{t}$ are ambiguous, which in turn implies uncertain relative values of $a^*$ and $\hat{a}$. Let us compare $t^*$ in region 3 under commitment to monitoring with $\hat{t}$ under no-commitment to monitoring. Thus, for $\beta < 1,\ t^*$ must meet the condition of

$$(1-\beta)s_3^*(\alpha^*,\ t^*) = [\frac{dD(e_3^*(t^*))}{de}\frac{de(a_3^*(t^*))}{da} + \frac{dC(a_3^*(t^*))}{da}]\frac{da_3^*(t^*)}{dt}$$

by (10), and $\hat{t}$ satisfies the requirement of

$$(1-\beta)\hat{s}(\hat{t}) = [\frac{dD(\hat{e}(\hat{t}))}{de}\frac{de(\hat{a}(\hat{t}))}{da} + \frac{dC(\hat{a}(\hat{t}))}{da}]\frac{d\hat{a}}{dt} + h'(\hat{\alpha})\frac{d\hat{\alpha}}{dt}$$

by (20). Since $s_3^*(\alpha,\ t)$ and $\hat{s}(t)$ are different functions of $t$ and the sign of $\frac{d\hat{\alpha}}{dt}$ is uncertain, we cannot tell relative magnitudes of $t^*$ and $\hat{t}$ unless all functions forms in the models are provided. Accordingly, relative sizes of $\hat{a}(\hat{t})$ and $a^*(t^*)$ in region 3 under the commitment case are unsure due to uncertain magnitudes of $\hat{t}$ and $t^*$. $\square$

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