Freedom to Not Join: A Voluntary Participation Game of a Discrete Public Good

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Abstract

A problem of the provision of a discrete public good is considered. All members in the society are homogeneous and they decide simultaneously whether to contribute to the provision. Contribution cost per person is fixed and non-refundable. Because of the free-rider problem, inefficiency in the provision is inevitable, even in the most efficient symmetric Nash equilibrium. However, when we add a pre-stage game where all members decides simultaneously whether to voluntarily participate to the original contribution game, expected social welfare might be improved in the symmetric subgame perfect equilibrium. It turns out that the improvement is always possible when the cost of contribution is sufficiently high.

1 Introduction

In many societies, a group of individuals faces to a situation where a voluntary contribution to the provision of a public good would benefit the whole members of the society. However, it is broadly known that efficient provision of the public good is unlikely because of the free-rider problem. Since each individual has an incentive to avoid bearing the cost of provision, contribution to the provision by all members does not constitute a Nash equilibrium.

But what is exactly the meaning of 'voluntary contribution'? In the most commonly used models of the voluntary provision of public goods, it is implicitly assumed that all players in the provision game are supposed to be involved in the game itself. In other words, most models assume that the players are forced to play the game, even though it is unclear if the players wanted to 'voluntarily' join the game itself. Because of the nature of the public goods, the benefit from the public good is non-excludable. Therefore, the 'players' of the provision game would have an incentive to refrain from being incorporated to the game itself. Since the focus of the discussion is the voluntary participation, the definition of the word 'voluntary' needs to be clarified in order

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to fully understand the behavior of the individuals facing the 'voluntary' provision of public good. At least, it would be useful to see how the equilibrium behavior would change if we modify the usual models which implicitly assume that all the players have agreed to join the game.

One way to explicitly model the voluntary participation is to consider a two-stage game. At stage one, each member in the society chooses either to join the contributors' group or not. At stage two, the ones who chose to join at stage one jointly produce the public good. We call the stage-one game as the *participation game* and the stage-two game as the *contribution game*.

Indeed, several papers in the literature have employed this approach to model the voluntary public good provision. For example, Dixit and Olson (2000) argue that previous treatments fail to recognize the full meaning of 'voluntary'. They considered a two-stage game where the public good is indivisible and the choice of the individual is binary (either to participate or not). Once the participation decisions are made, ex post efficient decision is made and the cost of the public good is equally divided if it is produced. They showed by numerical examples that in the symmetric mixed-strategy equilibrium, the probability of participation at stage one is generally low, implying inevitable inefficiency in the voluntary provision. Saijo and Yamato (1999) considered a two-stage game where the members simultaneously make a binary decision whether to join the contribution group at stage one, and the Lindahl equilibrium is realized among the participants at stage two. The public good is divisible and they assume Cobb-Douglas utility function. They showed that voluntary participation at stage one does not form a Nash equilibrium unless the Cobb-Douglas coefficient for the public good is sufficiently high. In short, these two papers provided negative results in the two-stage voluntary participation game of the public good provision.

In this paper, we also consider a two-stage participation game. But the motivation to add another stage is slightly different. We do not assume any Pareto efficiency at stage two. In our model, the public good is discrete and the individual contribution cost is fixed and non-refundable. At stage one, each member decides simultaneously whether to join the contribution game. There is no cost directly associated to this decision. At stage two, the participants play the contribution game where they choose whether to contribute or not. All the members are assumed to be homogenous and we characterize the most efficient symmetric subgame perfect equilibrium. Then the expected payoff of this two-stage game equilibrium is compared with the expected payoff of the most efficient symmetric Nash equilibrium in the one-stage game. I believe that this is the most appropriate way to understand the meaning of the word 'voluntary participation.' In this way, we can compare the Nash equilibrium outcome in the game where all the players are 'forced' to play, with the subgame perfect equilibrium outcome where the freedom to not join the contribution game is given to the members before the game is actually played. Adding a voluntary participation game before a Pareto-efficient allocation phase seems to be just pointing out the existence of the free-rider problem, which has been known in the literature for a long time. Instead, adding a voluntary participation game on the top of another voluntary contribution game would be helpful to clarify the difference between two aspects of the expression 'voluntary provision of the public good', that is, voluntary contribution and voluntary participation. These two aspects have been muddled up and this paper gives an attempt to separate them.

Our result is both positive and negative. Expected payoff may increase in the two-stage game than in the one-stage game. More precisely, by adding the freedom to not join the game, expected payoff in the most efficient symmetric subgame perfect Nash equilibrium in the two-stage game may be higher than the expected payoff of the most efficient symmetric Nash equilibrium in the one-shot game. We show that this improvement in expected payoff occurs when the individual contribution cost is sufficiently high. The intuition is the following. In the one-stage game, the non-refundable cost of contribution gives an incentive to free ride on other members' contribution. When the cost is high, all the players in the contribution game are afraid of ending up with the outcome where there are not enough contributors thus the cost is paid in vain. This is what is called 'fear factor' in Palfrey and Rosenthal (1984). In the two-stage game, the fear factor is removed because there is no cost associated to the participation decision at stage one. If there are not enough people participating at stage one, no one contributes in the unique Nash equilibrium at stage two, thus the waste of contribution cost can be avoided in this case. This increases the probability of participation in the symmetric mixed-strategy equilibrium at stage one. On the other hand, obviously, giving a freedom to not join the contribution game would decrease the number of potential contributors at stage two. Therefore adding a participation game would deteriorate the expected payoff. It turns out that the benefit outweighs when the cost of contribution is sufficiently high.

Section 2 decribes the model and Section 3 shows the main results. In Section 4, we discuss how the result depends on the cost structure. Section 5 concludes.

2 The Model

N homogenous individuals play a game of public good provision. The public good is assumed to be indivisible, thus the social choice is either to provide the public good or not. The benefit is normalized as 1, if the public good is provided. The public good is non-excludable. Hence, everyone can enjoy the benefit when it is produced, regardless of the choice of participation. There is a fixed amount of cost, c, associated to the contribution. c is assumed to be homogenous and non-refundable.[1] To provide the public good, a minimum number of contributors, w, is required. w is given exogenously, and we assume $w < N$.[2]

There are two types of games we consider: a one-stage game and a two-stage game. In the one-stage game, all N individuals decide simultaneously whether to contribute or not. In the two-stage game, first, at stage one, all N individuals decide simultaneously whether to participate in the

[1] We assume $c < 1$. Otherwise, no one has an incentive to contribute.

[2] If $w = N$, there is no free-rider problem, because all of the players are pivotal to the public good provision, and thus have enough incentive to contribute. If $w > N$, then no one has an incentive to contribute.

contribution game. Then, at stage two, the ones who chose to participate decide simultaneously whether to contribute or not. The players who chose non-participation stay out and are not allowed to join the contribution game. Only the ones who contribute at stage two pay the cost c, and everybody receives the benefit if the public good is provided.

We focus on the analysis of the most efficient symmetric equilibrium. There are reasons to claim that it is the focal point. The players in our game are completely homogenous. Therefore, it is reasonable to assume that coordination on any asymmetric equilibrium would be extremely difficult. This difficulty of coordination is pointed out also in Bagnoli and Lipman (1988), Holmström and Nalebuff (1992), and Dixit and Olson (2000).[3] Crawford and Haller (1990) showed that the strategy profile would converge to one which gives equal stage-game payoffs when the homogenous players have an identical preference on the choice of multiple equilibria.

3 Equilibrium Analysis

3.1 One-Stage Game

First, let us characterize the properties of the symmetric Nash equilibrium in the one-stage game. Let $V_c(k)$ (resp. $V_{nc}(k)$) be the payoff of a contributor (resp. non-contributor) when there are k contributors in total. Then $V_c(k) = \mathbf{1}(k \geq w) - c$ and $V_{nc}(k) = \mathbf{1}(k \geq w)$.

Since the strategy set is binary (i.e. the players contribute or not), any mixed strategy is characterized by a number between 0 and 1, which represents the probability of choosing the strategy of contribution. Since we assume $w < N$, there is no pure-strategy equilibrium where everyone contributes. Thus we look for a symmetric mixed-strategy equilibrium.

Let q be the probability of contribution in a symmetric mixed-strategy profile. In equilibrium, the expected payoff should be equal for both contributing and not contributing. Therefore,

$$W(c,q) = W(nc,q) \tag{1}$$

where $W(c,q)$ (resp. $W(nc,q)$) is the expected payoff of a contributor (resp. non-contributor) when the other players use the mixed-strategy with probability q. Explicitly,

$$\begin{aligned} W(c,q) &= \sum_{k=0}^{N-1} B(q|N-1,k)\, V_c(k+1) \\ W(nc,q) &= \sum_{k=0}^{N-1} B(q|N-1,k)\, V_{nc}(k), \end{aligned}$$

where $B(q|N-1,k)$ is the binomial distribution function in which k among $N-1$ players contribute

[3] Moreover, the main purpose of this analysis is to understand the trade-off between the benefit from the public good provision and the free-riding on the effort of other players. It seems to be difficult to isolate and describe the pure effect of free-riding when some degree of coordination among the players is present.

with the probability of contribution q.[4]

$$B\left(q|N-1,k\right)=\frac{(N-1)!}{k!\,(N-1-k)!}q^{k}\left(1-q\right)^{N-1-k}.$$

(1) can be explicitly written as:

$$\begin{pmatrix} B\left(q|N-1,0\right) \\ \vdots \\ B\left(q|N-1,w-2\right) \\ B\left(q|N-1,w-1\right) \\ B\left(q|N-1,w\right) \\ \vdots \\ B\left(q|N-1,N-1\right) \end{pmatrix} \cdot \begin{pmatrix} -c \\ \vdots \\ -c \\ 1-c \\ 1-c \\ \vdots \\ 1-c \end{pmatrix} = \begin{pmatrix} B\left(q|N-1,0\right) \\ \vdots \\ B\left(q|N-1,w-2\right) \\ B\left(q|N-1,w-1\right) \\ B\left(q|N-1,w\right) \\ \vdots \\ B\left(q|N-1,N-1\right) \end{pmatrix} \cdot \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ 1 \\ \vdots \\ 1 \end{pmatrix}.$$

Therefore,

$$B\left(q|N-1,w-1\right)=c. \tag{2}$$

Proposition 1 *For any N and w, there exists $\bar{c}$ such that a symmetric mixed-strategy equilibrium with positive probability of contribution exists if and only if $c\leq\bar{c}$. If $w>1$, $\bar{c}=B\left(\frac{w-1}{N-1}\middle|N-1,w-1\right)$. Otherwise, $\bar{c}=1$.*

Proof. $B\left(q|N-1,w-1\right)$ is a single-peaked function, maximized at $q=\frac{w-1}{N-1}$. Therefore, (2) has a solution if $c\leq B\left(\frac{w-1}{N-1}\middle|N-1,w-1\right)$. ■

Now, suppose $c\leq\bar{c}$ and let $q_N\left(c\right)$ be the biggest solution of (2). Let $U_1\left(c\right)$ be the expected payoff in this symmetric mixed-strategy equilibrium. The subscript 1 means that this is the expected payoff of the one-stage game. Then by (1),

$$U_1\left(c\right)=\sum_{k=w-1}^{N-1}B\left(q_N\left(c\right)|N-1,k\right)-c=\sum_{k=w}^{N-1}B\left(q_N\left(c\right)|N-1,k\right). \tag{3}$$

When $c>\bar{c}$, there is no strictly mixed strategy equilibrium because (2) has no solution. In such a case, the unique symmetric equilibrium is the pure-strategy equilibrium where no player contributes. Thus $U_1\left(c\right)=0$ for $c>\bar{c}$.

3.2 Two-Stage Game

Now let us characterize the symmetric subgame perfect equilibrium in the two-stage game. Rememeber that we use the term "participate" for stage one to describe the players' choice of participating in the contribution game in stage two, and the term "contribute" to describe the action of contribution in stage two.

[4]By convension, let $B\left(0|n,0\right)=B\left(1|n,n\right)=1$ for any $n>0$.

Suppose that at stage one, n players choose to participate to the contribution game. The equilibrium strategy in the subgame (i.e. at stage two) depends on the realized value of n. If n is very big, there might be no symmetric mixed-strategy equilibrium where the participants contribute with positive probability, because the free-rider problem is too severe. If n is so small that $n < w$, then of course no one contributes. Let $E_p(n)$ be the expected payoff of a participant and let $E_{np}(n)$ be the expected payoff of a non-participant. The next proposition characterizes the equilibrium strategy and the expected payoffs in the subgames.

Let

$$\overline{N} = \max\left\{ k \geq w \,\middle|\, B\left(\frac{w-1}{k-1}\,\middle|\, k-1, w-1\right) > c \right\}.$$

Proposition 2 *Given w and c, let $\overline{N}$ be as defined above. Let n be the number of players who have chosen participation at stage one. (i) If $n > \overline{N}$, then no one contributes in the unique symmetric equilibrium. Expected payoff is $E_p(n) = E_{np}(n) = 0$. (ii) If $w < n < \overline{N}$, then the probability of participation, q_n, in the most efficient mixed-strategy equilibrium, is characterized as the biggest solution of*

$$B(q_n|n-1, w-1) = c. \tag{4}$$

And

$$\begin{aligned} E_p(n) &= \sum_{k=w}^{n-1} B(q_n|n-1, k), \\ E_{np}(n) &= \sum_{k=w}^{n} B(q_n|n, k). \end{aligned}$$

(iii) If $n = w$, then there is a pure-strategy equilibrium where all the participants contribute. Expected payoff of a participant is $E_p(w) = 1 - c$, and the expected payoff of a non-participant is $E_{np}(w) = 1$. (iv) If $n < w$, then no one contributes. Expected payoff is $E_p(n) = E_{np}(n) = 0$.

The proof is obvious, and therefore omitted.

Now, let r be the probability of participation at stage one in the most efficient symmetric mixed-strategy profile. Let $F(p, r)$ (resp. $F(np, r)$) be the expected payoff of participation (resp. non-participation) when all the other players use the mixed strategy with r. Then

$$\begin{aligned} F(p, r) &= \sum_{n=0}^{N-1} B(r|N-1, n)\, E_p(n+1) \\ F(np, r) &= \sum_{n=0}^{N-1} B(r|N-1, n)\, E_{np}(n). \end{aligned}$$

First, we show that there is no subgame perfect equilibrium where everyone participates at stage one.

Proposition 3 *There is no subgame perfect equilibrium where all players participate at stage one.*

Proof. Suppose $r = 1$. Then

$$\begin{aligned} F(p, 1) &= E_p(N) = \sum_{k=w}^{N-1} B(q_N | N-1, k) \\ F(np, 1) &= E_{np}(N-1) = \sum_{k=w}^{N-1} B(q_{N-1} | N-1, k). \end{aligned}$$

Remember that q_N and q_{N-1} are characterized as the biggest solutions of (4). Hence $q_N < q_{N-1}$. Since $\sum_{k=w}^{N-1} B(q|N-1, k)$ is increasing in q, $F(p, 1) < F(np, 1)$. Participation is not the best response when all the other players are participating. ■

Now, let us consider an equilibrium where the players use strictly mixed strategy at stage one, that is, $r \in (0, 1)$. In such an equilibrium, expected payoff should be equal for both participation and non-participation. Hence

$$F(p, r) = F(np, r). \tag{5}$$

Equivalently,

$$\begin{pmatrix} B(r|N-1,0) \\ \vdots \\ B(r|N-1,w-1) \\ B(r|N-1,w) \\ B(r|N-1,w+1) \\ \vdots \\ B(r|N-1,N-1) \end{pmatrix} \cdot \left(\begin{pmatrix} 0 \\ \vdots \\ 1-c \\ E_p(w+1) \\ E_p(w+2) \\ \vdots \\ E_p(N) \end{pmatrix} - \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ E_{np}(w+1) \\ \vdots \\ E_{np}(N-1) \end{pmatrix} \right) = 0.$$

The following Proposition shows that such an equilibrium always exists.

Proposition 4 *For any N, w and $c\,(< \overline{c})$, there always exists at least one symmetric subgame perfect equilibrium where all the players join with a positive probability at stage one.*

Proof. It suffices to show that there is at least one solution of (5) in the range of $r \in (0, 1)$. Since $B(r|N-1, k) = O\left(r^k\right)$ as $r \to 0$,

$$\begin{aligned} F(p, r) &= (1-c)\, B(r|N-1, w-1) + O(r^w) \\ F(np, r) &= O(r^w). \end{aligned}$$

Therefore, $F(p, r) > F(np, r)$ when r is sufficiently small. On the other hand, when $r = 1$, $F(p, 1) < F(np, 1)$ as we have shown in Proposition 3. Since $F(p, r)$ and $F(np, r)$ are both continuous in r, there is at least one solution $r \in (0, 1)$ in (5). ■

Let $\widehat{r}(c)$ be the solution of (5). If there are multiple equilibria, assume that the most efficient one is chosen. Note that $\widehat{r}$ depends on c, given N and w. Let $U_2(c)$ be the expected payoff of this symmetric mixed-strategy equilibrium in the two-stage game. Then

$$U_2(c) = F(p, \widehat{r}(c)) = F(np, \widehat{r}(c)). \tag{6}$$

Now using (3) and (6), we compare the expected payoff of the one-stage game and that of the two-stage game. It turns out that the expected payoff is higher in the two-stage game when the cost is high enough.

Theorem 1 *For any N and w, there exists $\widetilde{c}\,(<\overline{c})$ such that $U_2(c) > U_1(c)$ for $c > \widetilde{c}$.*

To show the Theorem, we need three Lemmas. Let $q^* = \frac{w-1}{N-1}$.

Lemma 1

$$F(np, q^*) \leq U_1(\overline{c}) < F(p, q^*). \tag{7}$$

Lemma 2 *There exists $\bar{r}\,(> q^*)$ such that $F(p,r)$ is increasing in $r \in (0, \bar{r})$.*

Lemma 3 $F(p, \bar{r}) < F(np, \bar{r})$.

Proofs of these Lemmas are in the Appendix.

Proof of Theorem 1. We show that $U_2(\overline{c}) > U_1(\overline{c})$. Then by continuity, Theorem follows. Suppose the cost is $c = \overline{c}$. Then q^* is the probability of contribution in the equilibrium of the one-stage game. By Lemma 1 and Lemma 3, (5) has a solution $r^* \in (q^*, \bar{r})$. Then $F(p, q^*) < F(p, r^*)$ by step two. Since U_2 is the expected payoff in the most efficient equilibrium, $F(p, r^*) \leq U_2(\overline{c})$. This establishes $U_1(\overline{c}) < F(p, q^*) < F(p, r^*) \leq U_2(\overline{c})$. ■

Moreover, we have the following conjecture:

Conjecture 2 *For any N, w, there exists c^* such that $F_2(c) > F_1(c)$ if and only if $c > c^*$.*

If this conjecture is true, there is a cutoff value where the expected payoff in the two-stage game is higher than that of the one-stage game, if and only if the cost is higher than that threshold.

3.3 An Example

Suppose $N = 10, w = 4$. Then there exists a symmetric mixed-strategy Nash equilibrium with positive probability of contribution in the one-stage game, if the cost c is smaller than

$$\overline{c} = B\left(\frac{w-1}{N-1}\middle| N-1, w-1\right) = 0.273.$$

Figure 1 shows the difference of the payoffs, $U_2(c) - U_1(c)$, as a function of c. The value of c is varied from 0 to $\overline{c}$.

As we can see, the expected payoff is higher in the two-stage game when c is sufficiently high. At the maximum cost where a one-stage symmetric Nash equilibrium can exist, the expected payoffs are $U_1(\overline{c}) = 0.350$ and $U_2(\overline{c}) = 0.529$, showing an improvement of 31.8%. In the worst case, $U_2(c) - U_1(c)$ is minimized at $c = 0.117$. Then $U_2(c) = 0.822$ and $U_2(c) = 0.832$, showing a loss of merely 1.2%. When the cost is higher than $\overline{c}$, $U_2(c) - U_1(c)$ is always positive because $U_2(c)$ is positive, while $U_1(c) = 0$, as no one contributes in the unique symmetric Nash equilibrium in the one-stage game.

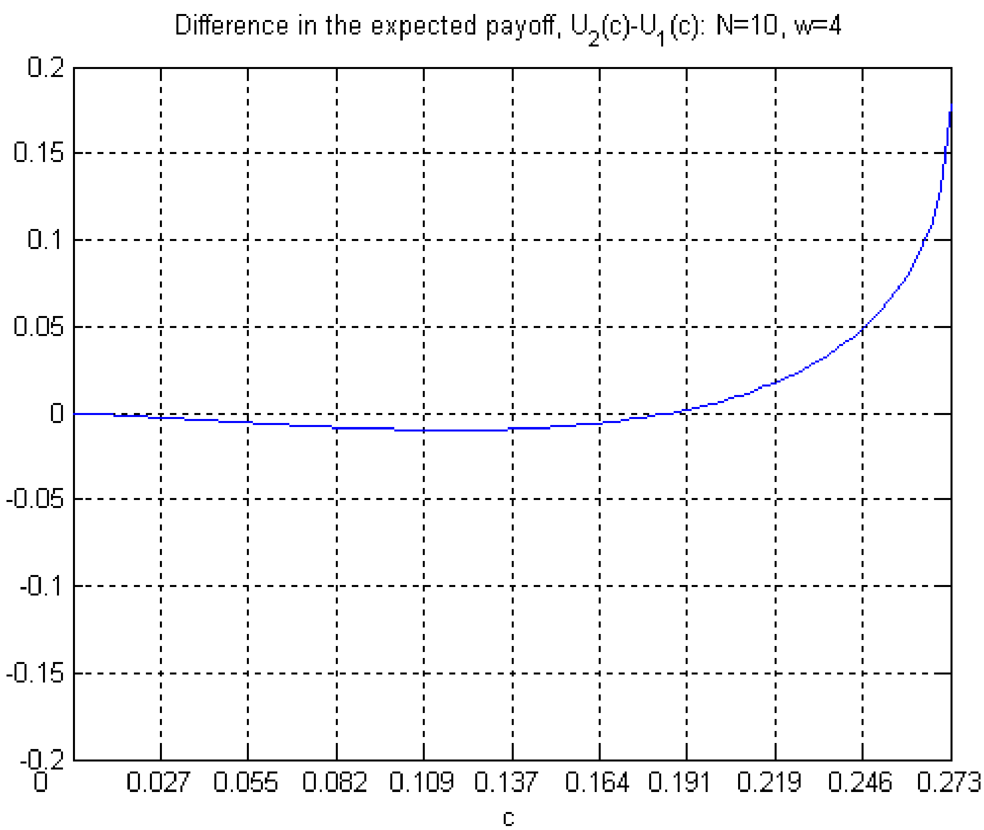

Figure 1: Difference of the expected payoffs

4 Other Cost Structures

In the previous section, we considered the model where the cost of contribution is fixed and non-refundable. In this section, we show the results of numerical computation for different cost structures.

4.1 Individually Fixed, Refundable Cost

Suppose the individual cost of contribution is fixed and refundable. If the number of players who chose the contribution is smaller than the number necessary to produce the public good, then the public good is not produced and the cost is refunded. This cost structure corresponds to the one referred to as "with refund" in Palfrey and Rosenthal (1984). Remember that $V_c(k)$ (resp. $V_{nc}(k)$) is the payoff of a contributor (resp. non-contributor) when there are k contributors. In this case, $V_c(k) = (1-c) \cdot \mathbf{1}(k \geq w)$ and $V_{nc}(k) = \mathbf{1}(k \geq w)$. Then by (1), the probability of contribution in the symmetric mixed-strategy equilibrium should satisfy

$$H(q|N-1, w-1) = c \tag{8}$$

where H is the hazard rate function:

$$H(q|n,k) = \frac{B(q|n,k)}{\sum_{j=k}^{n} B(q|n,j)}.$$

The hazard rate function is strictly decreasing in q. Also, $H(0|n,k) = 1$ and $H(1|n,k) = 0$. Therefore, (8) has a unique solution for any $c \in (0,1)$. As opposed to the case of non-refundable cost, there always exists a unique symmetric mixed-strategy Nash equilibrium with positive probability of contribution in the one-stage game. The expected payoff of the one-stage game, $U_1(c)$, is defined by (3) as above.

For the two-stage game, the symmetric subgame perfect equilibrium is characterized by (5) as before. In a similar way as in the previous section, it is possible to show that a symmetric subgame perfect equilibrium always exists. Then the expected payoff, $U_2(c)$, is defined by (6) as a function of cost c.

Figure 2 is the graph of the difference of the expected payoffs, $U_2(c) - U_1(c)$, between the two-stage game and the one-stage game. An improvement in the expected payoff is always impossible. The intuition is the following: when the cost is refundable, the players in the one-stage game (and in the subgame in the two-stage game) have no fear of the outcome where the number of contributors is less than the number w, which is necessary to produce the public good. The refundable cost removes the fear that the cost is wasted. One advantage of the two-stage game is to reduce the fear factor, but this is invalid in this case. Therefore, giving the players the freedom to voluntarily participate in the game would not improve the expected payoff.

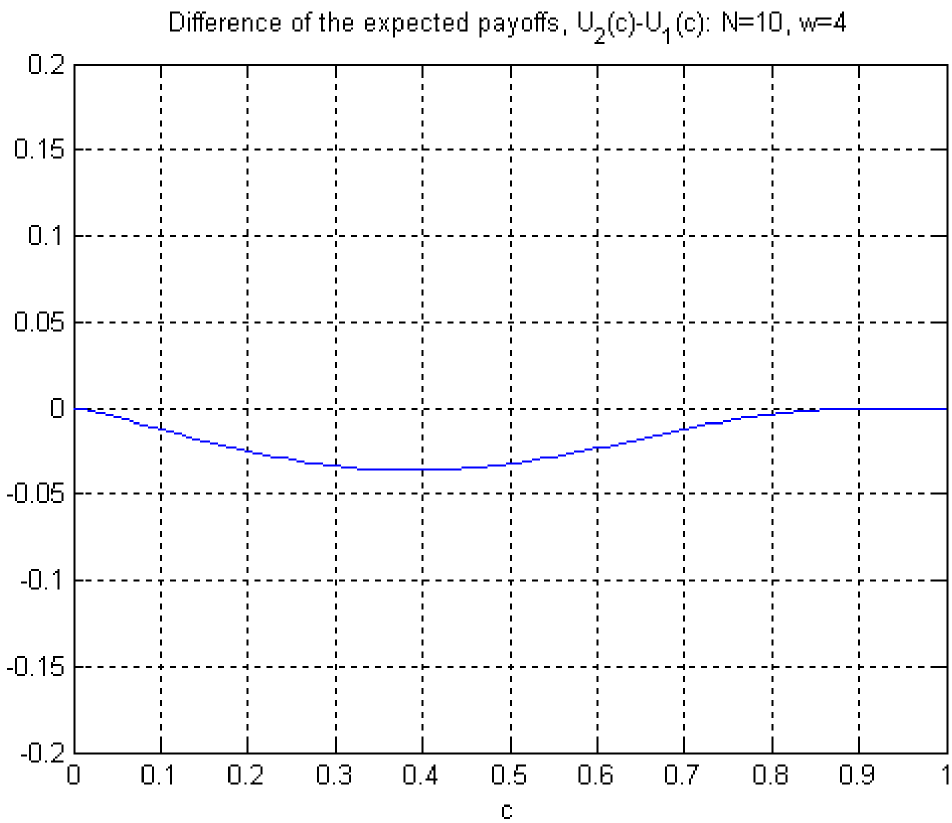

Figure 2: Refundable cost

4.2 Fixed Total Cost

Instead of an individually fixed cost, we consider the case where the total cost of public good is fixed. This cost structure is the same as the one studied in Dixit and Olson (2000).

Suppose C is the total cost of the discrete public good. If $n\,(> C)$ players contribute, the public good is produced, and the cost is equally divided among the contributors. Notice that the minimum number of players necessary to provide the public good, w, is not exogenously given in this case. Instead, w is endogenously determined as $w = \lceil C \rceil$ (i.e. the least integer which is not smaller than C).

Now, let k be the number of contributors. If $k \geq w$, then the public good is provided, and the cost is divided equally. Therefore the expected payoff is $1 - C/k$. If $k < w$, then the public good is not provided. Hence, the payoffs are $V_c\,(k) = (1 - C/k) \cdot \mathbf{1}\,(k \geq w)$ and $V_{nc}\,(k) = \mathbf{1}\,(k \geq w)$. Then (1) is equivalent to

$$H\,(q|N, w) = \frac{C}{w}.$$

Figure 3 is the graph of the difference of the expected payoffs, $U_2\,(c) - U_1\,(c)$, between the two-stage game and the one-stage game.

In this case, it is impossible to obtain a higher expected payoff in the two-stage game as compared to the one-stage game. The intuition is similar to the one described in the case of a refundable cost. There is no fear factor in the one-stage game since the public good is not provided if the number of contributors is less than the minimum number necessary to keep the cost per capita smaller than the benefit.

5 Conclusion

In our simple two-stage game model, two aspects of voluntary provision are separately analyzed. At stage one, all players voluntarily participate to the game, and at stage two, the participants play a game of voluntary contribution.

The results show that a higher expected payoff than the one-stage game can be obtained in the two-stage game, when the fixed individual cost is non-refundable and sufficiently high. The intuition is that the fear of wasting the cost in the one-stage game is removed by adding the pre-game participation stage. When the cost of participation is sufficiently high, the fear factor is so strong that the benefit from removing the fear factor outweighs the loss from losing the potential contributors at stage one.

Numerical computations show that this improvement in expected payoff is not available under two other cost structures: individually fixed refundable cost and fixed total cost. The intuition is that the fear factor is not present in the one-stage game in these two cases. Therefore, the benefit is too small to overcome the potential loss from adding the participation phase.

The result of this paper is potentially valuable for the organization of social decision-making

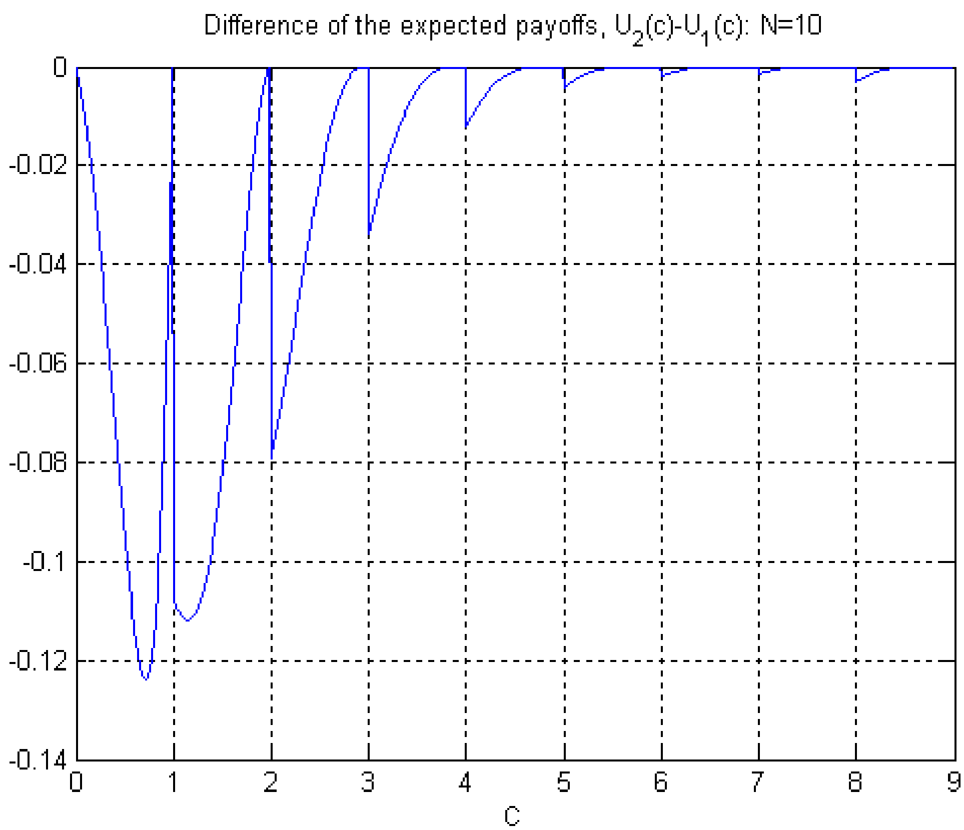

Figure 3: Fixed Total Cost

process. It may be profitable to give players the freedom to choose voluntarily whether to participate in the group before the actual choice of voluntary contribution. For example, in certain situations, holding a meeting before the actual contribution takes place could improve the social outcome. Whether the improvement in the two-stage game is possible or not depends on the cost structure. This will be a future research project to analytically understand more specifically when the improvement is possible.

6 Appendix

Properties of Binomial distribution

Lemma 4

$$\frac{\partial}{\partial r} B\left(r|n,k\right) = n\left\{B\left(r|n-1,k-1\right) - B\left(r|n-1,k\right)\right\}.$$

Proof.

$$\begin{aligned}
\frac{\partial}{\partial r} B\left(r|n,k\right) &= \frac{\partial}{\partial r}\left(\frac{n!}{k!\left(n-k\right)!} r^{k}\left(1-r\right)^{n-k}\right) \\
&= \frac{n!}{\left(k-1\right)!\left(n-k\right)!} r^{k-1}\left(1-r\right)^{n-k} - \frac{n!}{k!\left(n-k-1\right)!} r^{k}\left(1-r\right)^{n-k-1} \\
&= nB\left(r|n-1,k-1\right) - nB\left(r|n-1,k\right)
\end{aligned}$$

■

Lemma 5

$$\frac{B\left(q|n-1,k-1\right)}{B\left(q|n,k\right)} = \frac{k}{nq}, \frac{B\left(q|n-1,k\right)}{B\left(q|n,k\right)} = \frac{n-k}{n\left(1-q\right)}.$$

Proof.

$$\begin{aligned}
\frac{B\left(q|n-1,k-1\right)}{B\left(q|n,k\right)} &= \frac{\left(n-1\right)!}{\left(k-1\right)!\left(n-k\right)!}\frac{k!\left(n-k\right)!}{n!}\frac{q^{k-1}\left(1-q\right)^{n-k}}{q^{k}\left(1-q\right)^{n-k}} = \frac{k}{nq}, \\
\frac{B\left(q|n-1,k\right)}{B\left(q|n,k\right)} &= \frac{\left(n-1\right)!}{k!\left(n-k-1\right)!}\frac{k!\left(n-k\right)!}{n!}\frac{q^{k}\left(1-q\right)^{n-k-1}}{q^{k}\left(1-q\right)^{n-k}} = \frac{n-k}{n\left(1-q\right)}.
\end{aligned}$$

■

Lemma 6 *For $n > w$,*

$$\frac{dq_n}{dc} = \frac{q_n\left(1-q_n\right)}{c\left(w-1-\left(n-1\right)q_n\right)}.$$

Proof. Remind that q_n is the (biggest) solution of $B(q_n|n-1,w-1)=c$ (if exists).

$$\begin{aligned}
\frac{dc}{dq} &= (n-1)\left\{B(q_n|n-2,w-2)-B(q_n|n-2,w-1)\right\} \\
&= (n-1)\,c\left\{\frac{B(q_n|n-2,w-2)-B(q_n|n-2,w-1)}{B(q_n|n-1,w-1)}\right\} \\
&= c\left(\frac{w-1}{q_n}-\frac{n-w}{1-q_n}\right) \\
&= c\frac{(w-1)(1-q_n)-(n-w)\,q_n}{q_n(1-q_n)} \\
&= c\frac{w-1-(n-1)\,q}{q_n(1-q_n)}
\end{aligned}$$

■

Lemma 7 *For $n>w$,*

$$\begin{aligned}
\frac{d}{dc}E_p(n) &= \frac{q_n(n-w)}{w-1-(n-1)\,q_n} \\
\frac{d}{dc}E_{np}(n) &= \frac{nq_n(1-q_n)}{w-1-(n-1)\,q_n}
\end{aligned}$$

Proof.

$$\begin{aligned}
\frac{d}{dc}E_p(n) &= \frac{d}{dc}\sum_{k=w}^{n-1}B(q_n|n-1,k) \\
&= \sum_{k=w}^{n-1}(n-1)\left\{B(q_n|n-2,k-1)-B(q_n|n-2,k)\right\}\frac{dq_n}{dc} \\
&= (n-1)\,\frac{q_n(1-q_n)}{c\,(w-1-(n-1)\,q_n)}B(q_n|n-2,w-1) \\
&= \frac{(n-1)\,q_n(1-q_n)}{w-1-(n-1)\,q_n}\frac{B(q_n|n-2,w-1)}{B(q_n|n-1,w-1)} \\
&= \frac{(n-1)\,q_n(1-q_n)}{w-1-(n-1)\,q_n}\frac{n-w}{(n-1)(1-q_n)} = \frac{q_n(n-w)}{w-1-(n-1)\,q_n}.
\end{aligned}$$

■

$$\begin{aligned}
\frac{d}{dc}E_{np}(n) &= \frac{d}{dc}\sum_{k=w}^{n}B(q_n|n,k) \\
&= \sum_{k=w}^{n}n\left\{B(q_n|n-1,k-1)-B(q_n|n-1,k)\right\}\frac{dq_n}{dc} \\
&= n\frac{q_n(1-q_n)}{c\,(w-1-(n-1)\,q_n)}B(q_n|n-1,w-1) \\
&= \frac{nq_n(1-q_n)}{w-1-(n-1)\,q_n}
\end{aligned}$$

Proposition 5 *Let q_n be the solution of $H(q_n|n-1,w-1)=c$. Then*

$$\frac{dc}{dq_n}=c\left\{\frac{(1-c)(w-1)}{q_n}-\frac{n-w-1}{1-q_n}\right\}.$$

Proof. $H(q_n|n-1,w-1)=c$ is equivalent to

$$B(q_n|n-1,w-1)-c\sum_{j=w-1}^{n-1}B(q_n|n-1,j)=0$$

Hence

$$\begin{aligned}
\frac{dc}{dq_n} &= (n-1)\frac{B(q_n|n-2,w-2)-B(q_n|n-2,w-1)-cB(q_n|n-2,w-2)}{\sum_{j=w-1}^{n-1}B(q_n|n-1,j)} \\
&= (n-1)\,c\frac{(1-c)B(q_n|n-2,w-2)-B(q_n|n-2,w-1)}{B(q_n|n-1,w-1)} \\
&= (n-1)\,c\left\{(1-c)\frac{(w-1)}{(n-1)q_n}-\frac{n-w}{(n-1)(1-q_n)}\right\} \\
&= c\left\{\frac{(1-c)(w-1)}{q_n}-\frac{n-w}{1-q_n}\right\}
\end{aligned}$$

■

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