# Information revelation in markets with pairwise meetings : complete revelation in dynamic analysis

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#### Abstract

We study information revelation in markets with pairwise meetings. We focus on the one-sided case and perform a dynamic analysis of a constant entry flow model. The same question has been studied in an identical framework in Serrano and Yosha (1993) but they limit their analysis to the stationary steady states. Blouin and Serrano (2001) study information revelation in a one-time entry model and obtain results different than Serrano and Yosha (1993). We establish that the main difference is not due to the steady state analysis but is due to the differences concerning the entry assumption.

**Keywords** information revelation, asymmetric information, decentralized trade

JEL Classification Numbers D49, D82, D83

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## Introduction

As far as asymmetric information is concerned, the adverse selection problem has been the subject of an extended literature. In this case, the information asymmetry concerns the *nature* of one of the agents, and the impossibility for the ignorant party to discriminate effectively against certain types of agents: the insurer does not know whether the activities of the contractor are risky per se or not; the buyer does not know whether the car seller will try to swindle him or not. However, a different type of problem can arise when considering the information asymmetry phenomenon, i.e. the information revelation problem. In this last case, the asymmetry of information concerns the state of the world rather than the nature of one of the negotiating parties.

One can actually illustrate the general issue of information revelation in markets with pairwise meetings with a concrete phenomenon, observable in numerous places of interest in Egypt. Egyptian guides try to sell a guided tour of the place, while tourists play the role of potential buyers. There is neither a central institution nor a unique public price. The bargaining phase takes place after a matching has occurred between a seller and a buyer. When an Egyptian reaches an agreement with a tourist, the two quit the market and begin the tour. In the case of a disagreement, the two separate and are matched anew with an agent of the opposite type.

The asymmetric information concerns the interest of the place, i.e. whether it is one of the main highlights of the region, or only a minor site with few tourist attractions. Some tourists can be completely uninformed, while some others (owners of a travel book for instance) had access to some prior information. Of course, all the Egyptian guides on the other hand know the exact interest of the place.

The interest of the place has an influence on the value and the cost of the guided tour. Indeed, it is more interesting to have a guide when the site is rich in interesting anecdotes and historical references. At the same time, one can assume it is more costly for an Egyptian guide to offer a tour when the place is interesting, at least because the visit will be more time-consuming. Hence, we can expect the *appropriate* price of the tour to be higher when the place is actually one of the main cultural highlights of the region. It is also natural for the uninformed tourists to try to extract information from their matches with different partners. This learning is however expensive, because of its time-consuming nature. Naturally, sellers try to exploit their information's advantage by misrepresenting. By misrepresenting, sellers incur also a

cost for the same reason, i.e. the waste of time.

The main issue in this literature is then to determine whether the trading process implies an information revelation or not. Especially when the agents become infinitely patient, i.e. the market becomes approximately frictionless.

In markets with pairwise meetings, the information revelation literature began with the seminal paper by Wolinsky (1990).<sup>1</sup> The model studied in this paper is more general than ours, since it includes also some uninformed sellers. The main result of Wolinsky (1990) is that some trade occurs at a wrong price (i.e. a non-appropriate price considering the interest of the place), even when markets becomes approximately frictionless.

Gale (1989) conjectures the central role played by the assumption of uninformed agents being present on two sides of the market, because of a noise being created if the cost of learning decreases. Indeed, decreasing costs cause the probability -of an uninformed agent to meet another uninformed agentto increase. This requires, however, the informational power of meeting to decrease when the cost of learning declines.

Serrano and Yosha (1993) show that Gale's conjecture is correct. They use the same model than Wolinsky (1990), but they assume that all sellers are informed. The noise force disappears, since uninformed buyers always meet informed sellers. Finally, Serrano and Yosha (1993) establish that all transactions occur at the right price, whenever the market becomes approximately frictionless.

Wolinsky (1990) and Serrano and Yosha (1993) use a constant flow entry model, where a certain number of new agents enter the market at each period. To simplify the analysis, these papers consider only the stationary steady states, i.e. they consider the situations where the number of agreements is exactly equal to the entry flow. Blouin and Serrano (2001) on the other hand study the information revelation question in a one-time entry model, where all the agents are present in the first period and no entrance is allowed in the following periods.<sup>2</sup> They obtain a dramatically different

<sup>&</sup>lt;sup>1</sup>Concerning the market with pairwise meetings with perfect information, there is a significant literature studying following the seminal works of Gale, Rubinstein and Wolinsky. For a review, see Osborne and Rubinstein (2000).

 $<sup>^{2}</sup>$ For a discussion of these two hypotheses (constant entry flow and one time entry) in the perfect information case, see Gale (1987). Generally, the implicit economy in the constant entry flow model is not well defined. Nevertheless, the constant entry flow model remains interesting, at least because it may correspond better to some real markets.

result in the one-sided case. Indeed, they conclude in this case that some transactions occur at wrong prices, even when the market is frictionless. The two-sided analysis provides results similar to Wolinsky (1990).

The question at hand in our paper is to determine whether the differences observed in results are due to the existing differences in the hypotheses, or to the analysis being restricted to the steady states in the case of a constant entry flow model. For the opposite results to be due to differences in the hypotheses, one could conjecture the existence a kind of externality between the different generations of agents in the case of a constant entry flow. On the other hand, concerning the restriction to the steady states, it is not unreasonable to believe that some dynamics are being ignored, which could explain why full revelation is obtained in Serrano and Yosha (1993) but not in Serrano and Blouin (2001).

In order to answer to that question, our paper studies the same model than Serrano and Yosha (1993) but assumes an initial period. The model thereby has a starting point *outside* a steady-state, and can study the robustness of the Serrano and Yosha results when extending the analysis framework to the study of transition dynamics. As common in the existing literature, this paper focuses on markets becoming approximately frictionless.

A first intuition could be that a transition phase will be observed before the steady-states. The first proposition states that such a transition phase does not exist for the steady-states implying a complete information revelation.

This kind of model often presents a multiplicity of equilibria. Another intuition could then be that a steady-state analysis is unable to find all these equilibria. Among these ignored equilibria, one could expect to find some equilibria without complete information revelation. The second proposition shows that this is not the case, at least if uninformed buyers are sufficiently suspicious. In our Egyptian story, "sufficiently suspicious" would mean that the probability for the place to be interesting is not considered to be very high by uninformed tourists. In this case, the first proposition equilibrium is the unique one.

Hence, we conclude that the differences in the results obtained by Serrano and Yosha (1993) and Blouin and Serrano (2001) cannot be completely explained by the restricted analysis framework used by Serrano and Yosha (1993). Let's finally note that surprisingly enough, the dynamic analysis reduces the number of equilibria rather than adds some new ones. Some of the steady-state cannot be reached from our starting point.

In the first section, we present the model. The second section provides some characterizations of the equilibria that are useful in the next sections. The third section introduces the first proposition. The second proposition is presented and proved in the last section.

### 1 The model

We consider the model of Serrano and Yosha (1993) and study it without assuming an  $a \ priori$  stationarity of the equilibrium.

Times runs discretely from 0 to  $\infty$ . Each period is identical. On one side, there are sellers who have one unit of indivisible good to sell. On the other side, there are buyers who want to buy one unit of this good. In each period, a continuum of measure M of new sellers and the same quantity of buyers enter the market. The agents quit the market when they have traded. Hence, the number of sellers is always equal to the number of buyers.

There exist two possible states of the world, which influence the payoff of the agents. If the state is low (L), the cost of production  $(c_L)$  for the sellers but also the utility  $(u_L)$  of the buyers are low. If the state is high (H), the corresponding parameters  $(c_H \text{ and } u_H)$  are high. The state remains identical during all the periods.

All sellers know the state of the world, whereas not all buyers are perfectly informed. Among the newcomers, there is a part  $x_B$  of buyers which are perfectly informed. The remaining buyers are uninformed and possess a common prior belief  $\alpha_H \in [0, 1]$  that the state is H and  $(1 - \alpha_H)$  that the state is L.

At each period, all the agents are randomly matched with an agent of the other type<sup>3</sup>. At each meeting, the agents can announce one of two prices :  $p^{H}$  and  $p^{L}$ . If both agents announce same price, trade occurs at this price. If a seller announces a lower price, trade occur at an intermediate price  $p^{M}$ . If a seller announces a higher price, trade does not occur. The different

 $<sup>^3 \</sup>mathrm{See}$  Duffie and Sun (2007) for a rigorous proof of the existence of independent random matching between two continua.

parameters are assumed to be ordered such that :

$$c_L < p^L < u_L < p^M < c_H < p^H < u_H$$
 (1)

Remaining on the market implies a zero payoff. The instantaneous payoff when a transaction occurs is the price minus the cost for a seller and the utility minus the price for a buyer. All agents discount the future by a constant factor  $\delta$ .

In state H, we call  $p^H$  the good price because trade at other prices implies a loss for the sellers. Similarly, the price  $p^L$  is the good price in state L because trade at other prices involves loss for the buyers.

After each meeting with a seller who announces  $p^H$ , a buyer will update his belief  $\alpha_H$  according to Bayes'rule. If an uniformed buyer meets a seller who announces  $p^L$ , he will know that the state of the world is L, but it does not really matter any more, since this buyer will trade and leave the market.

It is convenient to say that a seller (resp. a buyer) plays *soft* when he announces  $p^{L}$  (resp.  $p^{H}$ ) and *tough* when he announces  $p^{H}$  (resp.  $p^{L}$ ). When an agent plays *soft*, he is ensured to trade and to quit the market. Hence, to describe completely the strategy of an agent, it is sufficient to give the number of periods in which he plays *tough*. The strategy of an agent might depend on the time of entry on the market. We note  $n_{SH}(t)$  the number of periods during which a seller plays *tough* when he enters in time t on a market which is in state H. Similarly, we define  $n_{SL}(t)$ ,  $n_{BH}(t)$ ,  $n_{BL}(t)$ . Finally, we define  $n_{B}(t)$  as the strategy of an uninformed buyer, which is independent of the state of the world.

We define now the proportions of agents who play *tough* when state is L. The proportion of the total number of buyers in the market who at period t announce  $p^L$  is called  $B(t)^4$ . Similarly,  $S(t)^5$  is the proportion of sellers who at period t announce  $p^H$ . These values are known to all agents.

An equilibrium is a profile of strategies where each agent is maximizing his expected payoff, given the strategies of the other agent. All parameters  $(p^H, p^M, p^L, c_H, c_L, u_H, u_L, x_B, \delta, \alpha_H)$  are common knowledge.

<sup>&</sup>lt;sup>4</sup>In Serrano and Yosha (1993), this proportion is noted  $B_L^l$ .

<sup>&</sup>lt;sup>5</sup>This proportion is equivalent to  $S_L^{\hat{h}}$  in Serrano and Yosha (1993)

### 2 Preliminary results

In this section, some definitions and preliminary results are introduced. Those results will be useful in order to prove the two main propositions.

#### 2.1 Trivial strategies

In the following claim, we characterize the equilibrium strategies of sellers in state H and of informed buyers.

**Claim 1** In any equilibrium  $n_{SH}(t) = \infty$ ,  $n_{BL}(t) = \infty$  and  $n_{BH}(t) = 0 \ \forall t$ .

**Proof** An informed seller in state H knows that his payoff will be negative if he trades at an other price than  $p^H$ . Since the payoff of perpetual disagreement is 0, he will always prefer to play *tough* even if it implies a long delay before trading. The reasoning is identical for an informed buyer in state L. An informed buyer in state H will understand that  $n_{SH}(t) = \infty$  and thus that he will never trade while playing *tough*. Since playing *tough* in this case only delays the payoff, it is better for this kind of buyer to play immediately *soft*.

#### 2.2 Strategy of uninformed buyers

We define  $\Delta V_B(t)$  which is the difference of gain between playing *soft* tomorrow and playing *soft* today for an uninformed buyer which enters the market in period t.

$$\Delta V_B(t) = \Delta V_B(S(t), S(t+1))$$
  
=  $\alpha_H(u_H - p^H)\delta$   
+  $(1 - \alpha_H)[(1 - S(t))(u_L - p^L) + \delta S(t)[(u_L - p^M) + S(t+1)(p^M - p^H)]]$   
-  $[\alpha_H(u_H - p^H) + (1 - \alpha_H)[(u_L - p^M) + S(t)(p^M - p^H)]]$  (2)

The last line corresponds to the payoff obtained when playing soft today<sup>6</sup>.

<sup>&</sup>lt;sup>6</sup>The payoff in state H which is equal to  $(u_H - p^H)$  is multiplied by the probability that the state is H. The term in brackets, which is multiplied by the probability that the state is L, is naturally the payoff in state L. This payoff can be written  $(1 - S(t))(u_L - p^M)$ (i.e. the probability to meet a *soft* seller times the payoff involved by this meeting) plus  $S(t)(u_L - p^H)$  (i.e. the probability to meet a *tough* seller times the payoff involved).

The two first lines correspond to playing *tough* today and *soft* tomorrow<sup>7</sup>.

If the difference of gain between playing *soft* tomorrow and playing *soft* today is positive, it is clear that an uninformed buyer will not play *soft* today. So, we can state :

Claim 2 Optimal strategies are such that

$$\Delta V_B(t) > 0 \Longrightarrow n_B(t) \ge 1 \tag{3}$$

#### **2.3** Characterization of S(t) at equilibrium

We define  $\Delta V_{SL}(B(t), B(t+1))$  which is the difference of gain between playing *soft* tomorrow and playing *soft* today for an informed seller in state L. This difference depends on time because B(t) may be non-stationary. Remark that  $\Delta V_{SL}(B(t), B(t+1)) < 0$  does not imply that the best solution is to stop in t.

$$\Delta V_{SL}(B(t), B(t+1)) = (1 - B(t))(p^H - c_L) + B(t)\delta[((1 - B(t+1))(p^M - c_L) + B(t+1)(p^L - c_L)] - [((1 - B(t))(p^M - c_L) + B(t)(p^L - c_L)] = B(t) \Big[ (-p^H + p^M - p^L + c_L) + \delta(p^M - c_L) + \delta B(t+1)(p^L - p^M) \Big] + (p^H - p^M)$$
(4)

In the first equality, the two first lines correspond to playing *tough* today and *soft* tomorrow while the third one corresponds to playing *soft* today.<sup>8</sup>

<sup>&</sup>lt;sup>7</sup>The meaning of the first line is obvious. It is just important not to forget the discount factor  $\delta$ . Indeed, if the state is H, a buyer who announces  $p^L$  does not trade. In the case where the state is L, there is a probability (1 - S(t)) that a buyer meets a *soft* seller and obtains today  $(u_L - p^L)$ . If a buyer does not have this luck, which happens with probability S(t), he will have tomorrow an expected payoff equal to the expression in brackets. Once again, we must not forget the discount factor.

<sup>&</sup>lt;sup>8</sup>If a seller plays *soft* today, he has a probability (1 - B) to meet a *soft* buyer and consequently to obtain a payoff  $(p^M - c_L)$ , otherwise (i.e. with probability B) he will get  $(p^L - c_L)$  due to a meeting with a *tough* buyer. If a seller announces  $p^H$ , he will reach an agreement only if he is matched with a *soft* buyer. It occurs with a probability (1 - B)and the payoff is then  $(p^H - c_L)$ . Otherwise, with a probability B, he will remain in the market. In the next period, if he plays *soft*, he has an expected payoff equal to the expression between brackets which must be multiplied by the discount factor  $\delta$  because trade occurs one period later.

Assume that a seller stops today playing tough.  $\Delta V_{SL}$  is a measure of the gain of a seller that decides to play tough one period more. The measure of the gain of a seller that decides to play tough T periods more is given by the sum of successive  $\Delta V_{SL}$ , balanced in order to take account of the discount factor  $\delta$ . If there exists a T such that this sum is positive, then playing tough T periods more gives a higher expected payoff than playing soft today. If this sum is negative for all T, then the maximum expected payoff is reached by playing soft today. If the sum is null for a given T, then the seller is indifferent between playing soft today or playing tough T periods more.

**Claim 3** Optimal strategies are such that the sequence  $S(t) \in [0, 1]$  satisfies

$$S(t) = 1 \implies \exists T \ s.t. \ \sum_{i=0}^{T} \delta^{i} \Delta V_{SL}(t+i) \ge 0 \quad (5)$$

$$S(t) < 1 \implies \sum_{i=0}^{T} \delta^{i} \Delta V_{SL}(t+i) \le 0 \quad \forall T$$
 (6)

$$\exists T \ s.t. \ \sum_{i=0}^{T} \delta^{i} \Delta V_{SL}(t+i) > 0 \implies S(t) = 1$$
(7)

$$\sum_{i=0}^{T} \delta^{i} \Delta V_{SL}(t+i) < 0 \quad \forall T \implies S(t) = 0$$
(8)

### **3** Complete Information Revelation

The following proposition establishes the existence of a steady state equilibrium, called E1 in Serrano and Yosha (1993), without convergence phase when  $\delta$  is high enough. The novelty compared to Serrano and Yosha (1993) is the dynamic context of the proof.

In other words, there exists, in a dynamic analysis, an equilibrium with full information revelation when market are sufficiently frictionless.  $^9$ 

#### **Proposition 1** If

$$\delta \ge 1 - \frac{1 - \alpha_H}{\alpha_H} \frac{p^M - p^L}{u_H - p^H} \tag{9}$$

then  $n_{SL}(t) = 0$  and  $n_B(t) = 1 \forall t$  imply an equilibrium.

<sup>&</sup>lt;sup>9</sup>Indeed, no buyer plays *soft* in state L at the equilibrium. Henceforth, there is no room for a transaction at a *wrong* price.

**Proof**  $n_{SL}(t) = 0$  implies S = 0. Since no seller misrepresents, once a buyer has met a seller who announces a state H, he knows that it is useless to play *tough*. So,  $n_B$  cannot be higher than one. To see that  $n_B \neq 0$ , it is sufficient to observe that  $\Delta V_B(0,0) > 0$ . Hence,  $n_B(t) = 1$  is an optimal strategy given  $n_{SL}(t) = 0$ .

The proposed strategies imply B = 1. So,  $\Delta V_{SL} < 0$  and the conditions given by claim ?? are fulfilled. Hence, no seller has an incentive to deviate.<sup>10</sup>

### 4 Uniqueness

The next proposition states that provided uninformed buyers are not overly optimistic about the probability for the state of the world to be H, then the complete revelation equilibrium is the unique one when the market becomes approximately frictionless.

**Proposition 2** If  $\alpha_H < \frac{p^H - u_L}{u_H - u_L}$ , there exists a  $\bar{\delta}$  such that for all  $\delta > \bar{\delta}$  there is a unique equilibrium described by proposition ??.

One can distinguish three stages in the proof. First, we introduce a claim necessary to our proof, ensuring that the uninformed buyers are suspicious enough to play *tough* at least *one* period. Second, we show that some sellers necessarily have to say the truth in the first period in order to reach an equilibrium. Finally, we show that no seller can misrepresent at the equilibrium.

In what follows,  $\check{\delta}$  and  $\tilde{\delta}$  will be introduced. The links between them and  $\tilde{\delta}$  is the following:  $\tilde{\delta} = \max(\check{\delta}, \bar{\delta})$ .

#### Stage 1 : Uninformed buyers are suspicious enough

The following claim ensures that uninformed buyers are suspicious enough and prefer to play *tough* during their first period spent on the market, independently of the strategy of the other agents.

**Claim 4** The following condition is sufficient to ensure  $n_B(t) \ge 1 \ \forall t$ .

$$\delta \ge \frac{p^H - p^M - u_L + p^L}{p^H - u_L} = \breve{\delta} \quad and \quad \alpha_H < \frac{p^H - u_L}{u_H - u_L} \tag{10}$$

<sup>&</sup>lt;sup>10</sup>This result depends crucially on the fact that an individual deviation does not affect the value of S and B because agents are negligible.

**Proof** It is obvious that  $\Delta V_B \geq \Delta V_B(S, 1)$ . Clearly,  $\Delta V_B(S, 1)$  is a linear function in S. So, either  $\Delta V_B(0, 1)$  or  $\Delta V_B(1, 1)$  is the minimum value that  $\Delta V_B$  can take. The second inequality of (??) is equivalent to  $\Delta V_B(1, 1) > 0$ . The first inequality of (??) is the condition such that  $\Delta V_B(1, 1)$  is the minimal value of  $\Delta V_B$ .

# Stage 2 : Some sellers must say the truth in the first period

For this second stage, let's assume that all sellers misrepresent in first period. We will proceed in eight steps to prove that in cannot be the case at equilibrium.

**Step 1** We know that all buyers will play *tough* in first period (by claims ?? and ??).

**Step 2** Since the proportion of *tough* sellers is the same in the two states of the world, meeting a *tough* buyer doesn't carry any information with respect to the prevalent state of the world. So, the uninformed buyers from first period will not modify their belief  $\alpha_H$  and will act in second period as the newcomers.

Step 3 Henceforth, all buyers in second period will play tough.

**Step 4** Then, let's observe that playing *tough* in first period is costly for sellers due to the delay<sup>11</sup>.

**Step 5** If sellers from period one play *soft* in second period, they will simply incur a delay cost since the proportions of *soft* buyers are identical in the two periods. Hence, they continue to play *tough* in second period.

**Step 6** If it is optimal for sellers from period 1 to play *tough* in second period, it must also be the case for sellers who enter the market in second period. Indeed, they face the same problem since, for the sellers, there is no updating of any parameter.

Step 7 By recurrence, all agents play *tough* in every period.

**Step 8** It implies an infinite costly delay for the sellers. So, such a strategy cannot be an equilibrium.

 $<sup>^{11}\</sup>mathrm{Given}$  the nil probability of meeting a  $\mathit{soft}$  buyer in first period.

# Stage 3 : All sellers must say the truth at the equilibrium

We proceed similarly in six steps and by contradiction. Let's assume that some but not all the sellers misrepresent in first period.

**Step 1** If  $\delta > \tilde{\delta}^{12}$ , playing *tough* necessarily implies a strictly positive cost of delay. Indeed, a strictly negative *cost* of delay would involve that all sellers misrepresent in first period, such a situation being excluded by the previous stage. It is less obvious that the cost of delay is not null. We prove this fact in the appendix with claim ??.

**Step 2** If some sellers play *tough* in first period, all sellers misrepresent in the second period. The argument is similar to steps 5 and 6 of the previous stage.

**Step 3** If in one period, all the sellers misrepresent then in the following one, all the buyers play *tough*. The argument is a variation around step 2 of the second stage.

**Step 4** By combining the two previous steps, if some sellers misrepresent in first period then all buyers play *tough* in third period.

**Step 5** Playing *tough* during two periods must give a null expected payoff for the sellers from the initial period. If this payoff were positive, the best strategy for sellers in period one would be to misrepresent. Then all the sellers would have to misrepresent in first period and it is a contradiction with the assumption that some sellers say the truth in first period. If the payoff were negative, all the sellers in period four would have to misrepresent and we come back to a situation similar to the previous stage. Indeed, all the sellers and the buyers would play *tough* in period three. By recurrence, it would also be the case in the following periods.

**Step 6** One can show that satisfying step 5 would imply, in period 1, a positive benefit of delay if  $\delta$  is larger than  $\tilde{\delta}$  (see claim ?? in the appendix). We would thus have a contradiction with the first step.

 $<sup>{}^{12}\</sup>tilde{\delta}$  is defined in the appendix

# A Appendix

**Claim 5** If  $\delta > \tilde{\delta}$ , we cannot have an equilibrium with S(0) > 0,  $n_B(0) \ge 1$ and  $\Delta V_{SL}(B(0), B(1)) = 0$ . With

$$\tilde{\delta} = \frac{p^L - c_L}{(p^M - c_L) - (p^M - p^L)x_B} < 1$$
(11)

**Proof** When  $n_B(0) = 1^{13}$ ,

$$B(0) = 1$$
 and  $B(1) = \frac{1 + S(0)x_B}{1 + S(0)}$  (12)

S(0) such that  $\Delta V_{SL}(1, \frac{1+S(0)x_B}{1+S(0)}) = 0$  is given by

$$S(0) = \frac{(1-\delta)(p^L - c_L)}{(p^L - c_L) - \delta(p^M - c_L) + \delta(p^M - p^L)x_B}$$
(13)

The claim is obtained by noting that this expression is negative if  $\delta > \tilde{\delta}$ .

**Claim 6** Assuming that the condition stated in claim ?? is satisfied, we cannot have an equilibrium with 1 > S(0) > 0 and  $n_B(0) = 1$  when  $\delta > \tilde{\delta}$ .  $\tilde{\delta}$  defined as above.

**Proof** Since  $n_B(0) = 1$ , B(1) is given by equation (??). By step 2 and 4, S(1) = 1 and B(2) = 1. S(0) such that  $\Delta V_{SL}(1, \frac{1+S(0)x_B}{1+S(0)}) + \Delta V_{SL}(\frac{1+S(0)x_B}{1+S(0)}, 1) = 0$  is equal to

$$\frac{2(1-\delta)(p^L-c_L)}{p^H-p^L-(1-\delta)(p^M-c_L)+[(1-\delta)(p^M-p^L+c_L)-p^H+\delta p^L]x_B} > 0(14)$$

Clearly, if  $\delta > \tilde{\delta}$ , this expression is larger than the one given by the equation (??). Observe that  $\Delta V_{SL}(1, \frac{1+S(0)x_B}{1+S(0)}) = 0$  is increasing in S(0). Hence,  $\Delta V_{SL}(1, \frac{1+S(0)x_B}{1+S(0)})$  with S(0) given by (??) is positive. It is a contradiction with the fact that S(0) < 1.

<sup>&</sup>lt;sup>13</sup>The case with  $n_B(0) > 1$  is trivial.

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