Signaling private choices*

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This version: July 2009

Abstract

For a number of important applications of signaling, it is sometimes more reasonable to assume that the sender rather than nature chooses its unobservable features (e.g. its private choice of quality). In other situations, it makes no sense at all for nature to determine the sender's unobservable features (e.g. its private choice of capacity, investment, contract or price). This paper provides a framework to analyze a wide range of such endogenous signaling problems. An equilibrium concept (*Reordering Invariance*) is proposed which is powerful in eliminating unreasonable equilibria and relatively easy to apply. A class of monotone endogenous signaling games is characterized, in which the sender can influence the receivers' actions to its benefit through signaling. For such games, we show that a sender's private choice can still have some commitment value even though it is not observed, and that in equilibrium, the sender's signals must be exaggerated. These points are illustrated with a simple model of costly announcements that applies to the classic time inconsistency problem of monetary policy. The paper also explains how to apply our framework to more complicated settings, including to situations which have not previously been considered as signaling problems (e.g. to loss leader pricing and to the opportunism problem that arises when a manufacturer sells to competing retailers through secret contracts).

JEL Classification: C72, D82, L10.

Keywords: signaling, invariance, announcements, opportunism.

* We thank Murali Agastya, In-Koo Cho, Guillaume Rocheteau, Larry Samuelson, Roberto Serrano, Satoru Takahashi, Asher Wolinsky, conference participants at Games 2008, SET 2008, the 5th PPCGT, and the 27th AETW and especially John Hillas, for their very helpful comments and discussions.

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1 Introduction

The classic treatment of signaling games assumes that a sender's unobservable features (or types) are exogenously determined by nature so that the game is one of incomplete information. While this makes sense for some applications (for instance, where a player's type is their innate ability), for other applications it may be more reasonable (or just as reasonable) that the sender's unobservable features are actually chosen endogenously by the sender. Examples include limit pricing and predation, where the incumbent's costs are determined through its unobservable investments in cost reducing technologies or capacity, price and advertising to reveal a firm's unobservable choice of product quality, and an entrepreneur's choice of (corporate) financing to reveal its unobservable choice of effort (and so likely project quality). In some other applications it makes no sense at all for nature to determine the sender's unobservable feature. Consider the problem of a multi-product firm choosing prices only some of which are observed (possibly through its advertising). Clearly it makes no sense to think of nature determining the firm's unobservable prices. Other examples where signaling arises include unobserved capacity choices, private contracts, a country's military tactics, or a government's secret inflation target (as opposed to its announced target). Surprisingly, this class of games does not appear to have been studied formally, at least in a systematic way. This paper attempts to fill this gap.

We consider a class of games of imperfect information in which senders take multiple actions, only some of which are observed by receivers. Typically, these games have a multitude of equilibria reflecting that receivers' beliefs are not pinned down off the equilibrium path. We develop a natural equilibrium concept for these games based on a very weak version of the invariance condition of strategic stability. Provided each sender makes its choices of unobservables and observables without gaining any new (payoff-relevant) information in between, the order of these moves should not matter. Specifically, we construct a reordered game in which observable actions are chosen before unobservable actions which shares the same reduced-normal form as the original game. This game often has a unique equilibrium outcome which is relatively simple to characterize.[1] This is taken as the refined equilibrium outcome of the original game. We call the equilibria consistent with an equilibrium of the reordered game *RI-equilibria* (*RI* stands for *Reordering Invariance*), and the associated beliefs *RI-beliefs*. The beliefs can be interpreted as a type of forward induction. Roughly speaking, upon

[1] In contrast, using refinements based on the hypothesis that deviations by a sender are interpreted as mistakes or trembles (either in observable or unobservable actions, or both), does not lead to a unique equilibrium outcome (in settings where we have a unique *RI-equilibrium* outcome) and in general requires a tedious process to characterize equilibria.

seeing a sender's observable actions, the receiver infers the sender's unobservable actions were chosen "optimally" with these observable actions (and other players' equilibrium strategies) in mind. As a result, in single-sender games, the equilibrium outcome selected yields the highest possible payoff to the sender among all the equilibria of the original game.

To capture the signaling aspect of the games we consider, we consider a particular structure on the sender's and receiver's payoffs so as to define a class of monotone endogenous signaling games. In these games, the sender can influence the receivers' actions to its benefit through its choice of signals. Despite the fact that it is unobservable, we establish that the sender's private action still involves a "commitment effect" in that the sender chooses different levels of both the unobservable and observable actions compared to the equilibrium under passive beliefs, to its benefit. This has implications for the large literature on business strategy in which firms also choose observable prices or quantities in the product market, showing that the literature still applies (to some extent) even when the firms' "investments" are unobservable.

Another implication of the monotone endogenous signaling games we study is that a sender will exaggerate its choice of signal in equilibrium, compared to the equivalent choice of signal in a full information game. Such "signal exaggeration" arises in equilibrium to ensure that the sender does not have any incentive to further manipulate beliefs by choosing a different signal. In the case of limit pricing, starting from its normal monopoly price, a small decrease in the monopolist's price does not give rise to any first-order loss. However, since it lowers the rivals' beliefs about the level of costs the monopolist has chosen (and therefore the rival's likelihood of entry in the subsequent period), it does have a first order benefit. In equilibrium, the monopolist's price must be exaggerated (in the downward direction) to ensure that it does not have any incentive to further manipulate beliefs, i.e. so the receiver is not fooled. We illustrate with an application to the exaggeration of announcements.

The setting we consider can be generalized in many ways. We consider some of these, explaining how to apply the *RI-equilibrium* concept to more complicated settings including where there are multiple senders and receivers, where both senders and receivers move at the same time, and where senders are also receivers (as arises in business strategy settings with multiple firms). Where senders can determine which features are observable and which are unobservable (as arises when firms choose which prices to advertise), we show how to use the receiver's information partition to decompose the senders' actions into observable and unobservable actions, so as to construct the relevant reordered game. In other applications, different receivers may have different information partitions in the reaction phase. If the receivers' information partitions are not appropriately ordered, more than one reordering of the original game may be necessary to pin down the

RI-equilibria. We illustrate this with an application to the classic opportunism problem (Hart and Tirole, 1990 and McAfee and Schwartz, 1994) of a supplier that makes secret offers to two downstream retailers. Of the three different types of beliefs considered in the literature, we show symmetry beliefs and passive beliefs are not generally consistent with our *RI-equilibrium*, whereas wary beliefs are.

The rest of the paper proceeds as follows. Section 2 discusses the related literature. Section 3 presents the theory for a class of canonical endogenous signaling games. Section 4 adds a particular monotone structure, explores some interesting properties of these games with an application to monetary policy announcements, and compares our setting with classical signaling games. Generalizations are given in Section 5, which includes an application to the opportunism problem of vertical contracting. Section 6 briefly concludes.

2 Related literature

It may be tempting to call the choice of the unobservable feature in our setting a "moral hazard problem" and relate it to the vast literature analyzing moral hazard problems. Actually, the problem we study is more like a cross between an adverse selection problem and a moral hazard problem. It is distinct from an adverse selection problem (with signaling) based on who determines the unobservable feature(s), i.e. the sender rather than nature. It is distinct from a moral hazard problem based on when the unobservable feature(s) are chosen, i.e. they are chosen before the observable actions (in this context, contracts) are decided, as well as which player offers the contracts, i.e. the informed rather than the uninformed player.

As mentioned in the Introduction, there is surprisingly little existing analysis of the class of games we consider. The one application where a sizeable literature does already exist is the study of how firms can use price to signal their endogenous choice of quality. Prominent examples include Klein and Leffler (1981), Wolinsky (1983), Riordan (1986), Rogerson (1988) and Bester (1998). In contrast to classical signaling models, in this literature, equilibria have generally been pinned down by a seemingly ad-hoc approach to how expectations are formed for out-of-equilibrium price offers.[2] In part, this may have reflected that several of the papers considered competitive firms in a market setting. Typically, authors assumed that although prices and quality are set simultaneously, consumers interpret prices that are different from equilibrium levels as "an indication of high quality if the seller

[2] Perhaps due to this ad-hoc approach, the "endogenous-quality" literature never developed the role of advertising, in addition to price, as a signal of a firm's quality. This contrasts with the "exogenous-quality" literature, which typically allows both price and advertising to be signals. Using our approach, it would be straightforward to consider both price and advertising as signals of unobservable quality.

has no incentive to disappoint this expectation" (Bester, p. 834).[3] Assuming that the relevant constraint is that firms should not have an incentive to cut quality, this amounts to assuming that given a price, a firm would not have an incentive to choose a quality different from expected. Implicitly, in determining consumer expectations, it is as though price is set first and quality expectations are pinned down by the fixed point between firm's choice of quality and consumer expectations.[4] In the simplest setting we consider, this is the approach implied by our *RI-equilibrium.* In more complicated settings, as we show in this paper, the *RI-equilibrium* is more subtle and no longer based on this approach.

Aside from models of unobserved quality choice, our framework applies to some other existing works. In political science, our signal exaggeration result accords with the finding of Bueno de Mesquita and Stephenson (2009) who show a policymaking agent will overinvest in observable effort and underinvest in unobservable effort (to improve the quality of regulation) in the face of an overseer that can veto new regulation. Consistent with our *Reordering Invariance*, they solve the game by first solving for the equilibrium level of unobservable effort given the level of the observable effort, although they do not explain why. Rocheteau (2009) uses our *RI-equilibrium* concept to study the issue of counterfeiting in monetary economics, in which buyers can first choose whether to counterfeit an asset which acts as a medium of exchange and then what price to offer when paying with this asset. Monetary theorists have struggled to come up with the right equilibrium concept for games of counterfeiting (Nosal and Wallace, 2007). As Rocheteau shows, using our *RI-equilibrium* concept allows some of the issues that have been raised to be addressed. Rao and Syam (2001) consider a game of pricing and advertising by competing supermarkets which fits into a generalized version of our framework, in which the supermarkets choose which of two different goods to advertise the price of, at the same time as determining the level of the two prices. For the most part, Rao and Syam solve the game by *assuming* that the unadvertised price is chosen after the other choices are made public, thereby avoiding having to pin down out-of-equilibrium beliefs. In Section 3.7 of their paper they state an equilibrium of the game in which unadvertised prices are set at the same time as advertised prices

[3] Wolinsky (p. 655) requires admissible consumer expectations to be such that for any given price, "the highest quality that consumers can expect the entrant ... to produce without being disappointed." Riordan (p. 272) writes "I assume that consumers expect the maximal quality that is consistent with the firm's incentives."

[4] Rogerson (p. 223) is more explicit in stating that "A very natural method for defining rational beliefs would be to require that they be consistent with the equilibrium quality which actually would be supplied if a price was offered in equilibrium." Similarly, Farrell (1986, p. 444) who considers scale x as a signal of quality q, writes "We are skirting a subtle game-theoretic issue here. The entrant can be regarded as "simultaneously" choosing both x and q. ... It is not clear how buyers "ought" to infer q from x; we assume that they assume that q will be optimal, given x."

but without explaining how they pin down out-of-equilibrium beliefs. We discuss the right way to do so in Section 5.4 of our paper.

One large set of new applications of our framework is to the case of unobservable "investment". Such applications are a cross between the large literature studying business strategy in which firms' endogenous (but observable) "investments" have commitment effects and the classical models of limit pricing and information manipulation in which firms' unobservable (but exogenously determined) costs are signaled through the product market. For instance, consider limit pricing when the incumbent's private cost is determined endogenously by its unobservable investment in cost-reducing R&D. Such limit pricing is a cross between the top-dog entry deterrence strategy of Fudenberg and Tirole (1984) and the classic model of limit pricing studied by Milgrom and Roberts (1982). Table 1 illustrates.

	Observable case	Unobservable case
Nature chooses		**Limit pricing** Milgrom & Roberts (1982) **Information manipulation** Riordan (1985)
Sender chooses	**Business strategy** Fudenberg & Tirole (1984)	**Signaling private choices**

Table 1: Connection with existing literature

One other existing application that fits our framework is Dana's (2001) analysis of competition, in which firms first choose unobservable capacity and then without observing each other's capacity choices, they compete in observable prices. Consumer use prices as a signal of the firms' capacity choices. Dana uses the "Never Weak Best Response" (*NWBR*) property to find a unique equilibrium. In his setting, the implied forward induction reasoning of *NWBR* corresponds to using our *RI-equilibrium.* In other, more complicated settings, the two approaches may differ. Even where the outcomes are the same, a key advantage of the *RI-equilibrium* concept in the class of games we consider is that it is much simpler to apply.[5]

[5] Hillas (1998), Hillas and Kohlberg (2002), and Govindan and Wilson (2009) have investigated the implication of adding invariance to backward induction on forward induction. Our *RI-equilibrium* concept exhibits a flavor of forward induction, but since the invariance that we consider takes a particular form, our refinement is also generally weaker than the forward induction defined by Govindan and Wilson.

3 Canonical endogenous signaling games

In this section, we define a class of games in which senders take multiple actions, only some of which are observed by receivers. Initially we do not impose any restrictions on the payoffs so that there need not be any particular linkage between the players' actions. We propose an equilibrium concept for this class of games, and investigate the optimality of the proposed equilibria.

3.1 Class of games

We consider a class of extensive-form games of imperfect information with perfect recall. There are a finite number of players. Players are either a sender or a receiver or both. There are at least two players: at least one sender (S) and at least one receiver (R).

There are two phases in the game: the signaling phase and the reaction phase. In the signaling phase, a sender or senders take(s) multiple actions of two kinds: an action or actions that are observable and an action or actions that are unobservable to the receiver(s) and the other sender(s). The sender(s) take(s) the actions either sequentially or simultaneously. As the game unfolds during the signaling phase, from a particular sender's perspective, no new payoff-relevant information arrives other than its own moves. The reaction phase begins from the point of time when a receiver or receivers start(s) to move *having observed* the senders' observable actions. The receivers may move *without observing* any of the senders' actions in the signaling phase. The senders may move in the reaction phase as well.

The simplest of such games are two-player (one sender and one receiver) games with the following timing of moves (for example, see panel (a) of Figure 1), where T, A, and B are any sets (for example, $\{\text{Apple}, \text{Beef}, \text{Cereal}\}$, a subset of a multidimensional Euclidean space, etc.):

1. In stage 1, the sender chooses $t \in T$.
2. In stage 2, the sender chooses $a \in A$.
3. In stage 3, having observed the sender's choice of a, but not t, the receiver chooses $b \in B$.

Stages 1 and 2 constitute the signaling phase[6] and stage 3 is the reaction phase.

For example, in a limit-pricing game, t could be the incumbent's unobservable cost-reducing investment, a could be the incumbent's (limit) price, and b could be the rival's decision to enter or not. In the case of product quality signaling, t could be a firm's unobservable quality, a could be the

[6] The analysis in this section continues to apply even if the sender chooses t and a simultaneously.

firm's price and advertising, and b could be the buyer's decision to purchase or not. In a corporate finance setting, t could be the entrepreneur's unobservable effort to determine the likely project quality, a could be the entrepreneur's offer of financing contract, and b could be the investor's decision to invest or not.

The payoffs are $\pi_S(t, a, b)$ and $\pi_R(t, a, b)$ to each player respectively. We call these games "canonical endogenous signaling games," denote this class of games by Γ_o, and its typical element G_o. Note that the classical signaling games in the literature have the same structure as the canonical endogenous signaling games except that the first move is made by nature in stage 1.

3.2 Equilibrium

For extensive-form games of imperfect information, two equilibrium concepts are widely used in the literature: sequential equilibrium (Kreps and Wilson, 1982) and perfect Bayesian equilibrium (Fudenberg and Tirole, 1991). For the purpose of expositional simplicity, however, we start with a less refined equilibrium concept, subgame-perfect equilibrium (SPE). We do so because using the concept of SPE as the basic equilibrium concept is enough to show our results for the class of canonical endogenous signaling games (Γ_o). In a later section, we adopt a version of perfect Bayesian equilibrium (Fudenberg and Tirole) as our basic equilibrium concept for the broader class of games that we consider. When there is no risk of confusion, in this section we also use the term equilibrium to mean SPE. We focus on pure-strategy equilibria except in Section 3.5 where we explain how our framework continues to hold for behavior strategies, and omit "pure strategy" when referring to equilibria throughout the paper wherever possible.

For canonical endogenous signaling games, subgame-perfection does not provide any refinement to the set of the Nash equilibria, and therefore the set of SPE coincide with the set of Nash equilibria. As a result, we typically find many SPE. This phenomenon of multiple equilibria is quite robust. As we will see later, even if we adopt more refined equilibrium concepts such as sequential equilibrium or perfect Bayesian equilibrium, we typically find many equilibria in the class of games we consider. This is due to the indeterminacy of the off-the-equilibrium beliefs.

As one of the requirements for strategic stability (Kohlberg and Mertens, 1986), *Invariance* was proposed, which says that a solution of a game should also be a solution of any equivalent game (i.e. having the same reduced normal form). We consider a particular extensive-form game which shares the same reduced normal form (up to the relabeling of the strategies) as the original game. For any $G_o \in \Gamma_o$, consider the following extensive-form game, where the timing of the sender's moves is now reversed as follows:

1. In stage 1, the sender chooses $a \in A$.

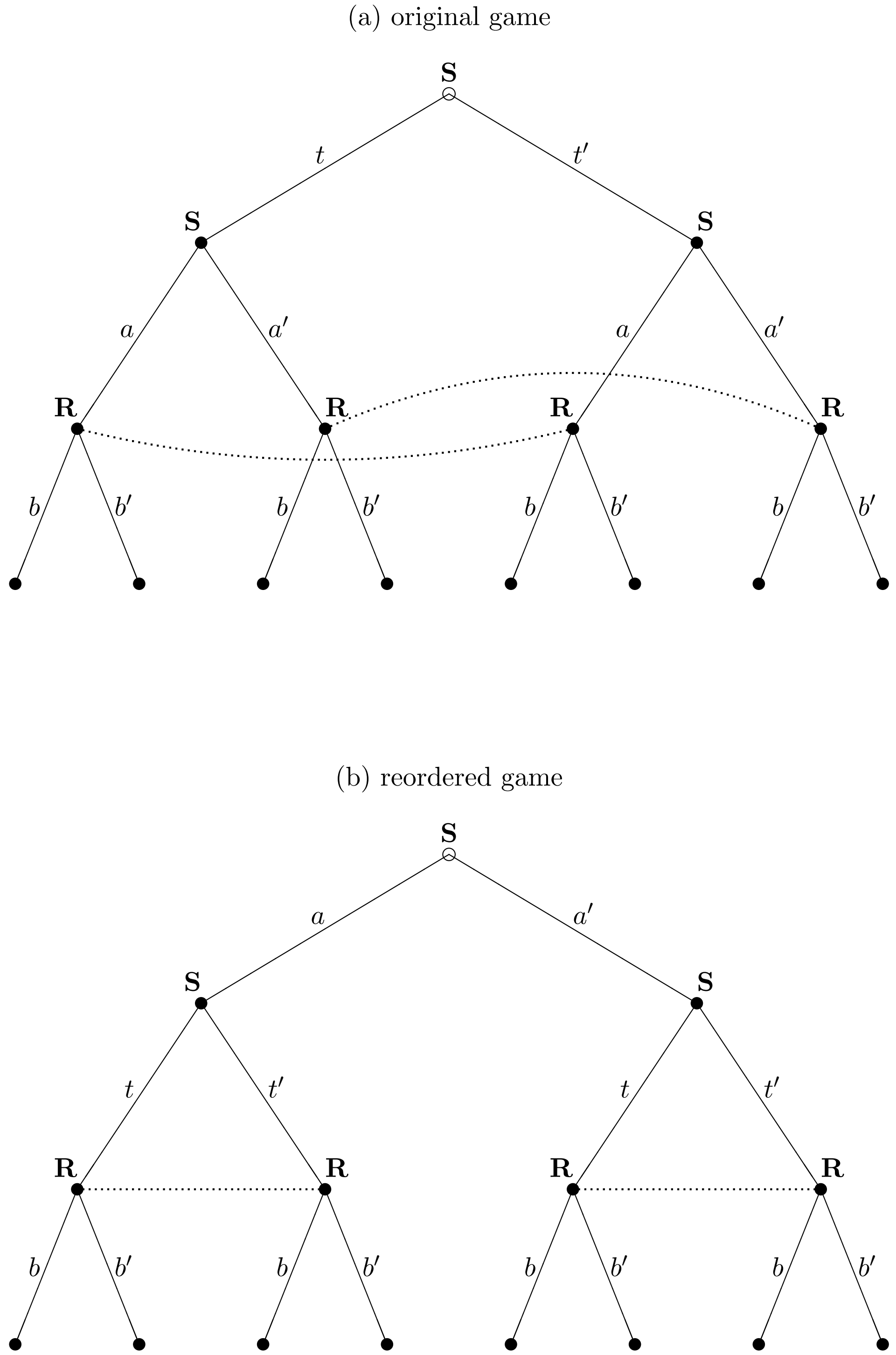

Figure 1: Illustration of original and reordered games

2. In stage 2, the sender chooses $t \in T$.

3. In stage 3, having observed the sender's choice of a, but not t, the receiver chooses $b \in B$.

The payoffs are the same as in the original game. We denote this game by G_r, and the collection of such games Γ_r.

Note that for the games in Γ_r, the requirement for a SPE is the same as that of a perfect Bayesian equilibrium (as defined in Section 5.2)

We call the probability distribution on the terminal nodes that is induced by a strategy profile the outcome of the game associated with the strategy profile. The probability distribution may be degenerate. Note that the probability distribution on the terminal nodes is equivalent to the probability distribution on the paths of the game tree. The following proposition, which provides the foundation for our equilibrium concept for the games $G_o \in \Gamma_o$, then follows fairly directly (all proofs are in the appendix).

Proposition 1 (Reordering Invariance) *Consider a game $G_o \in \Gamma_o$ and its reordered game G_r. If an outcome (a^o, t^o, b^o) is supported by a SPE of G_r, then (t^o, a^o, b^o) is supported by a SPE of G_o.*

This proposition enables us to construct equilibria of the original game based on the equilibria of the reordered game. Specifically, among the SPE of the original game (G_o), we select only those which share the same outcome (up to the reordering of t and a) and the same receiver's strategy with a SPE of the reordered game. We call these equilibria *RI-equilibria* and the associated receiver's beliefs *RI-beliefs* (*RI* stands for *Reordering Invariance*). *Reordering Invariance* is weaker than *Invariance* because it requires only that a solution of a game should also be a solution of a particular equivalent game.

This equilibrium concept has a strong decision-theoretic justification. Given the receiver's beliefs at each of the receiver's information sets are the same across the two games, the sender's optimal choice of multiple actions should not depend on the order of its choices, as far as the sender makes its choices of unobservable and observable actions without gaining any new (payoff-relevant) information in between.

The beliefs that the receiver holds in a *RI-equilibrium* can be interpreted as a type of forward induction in the following sense: when the receiver observes the signal, the receiver infers that the sender has chosen its unobservable action "optimally" with the action to be observed (and other players' equilibrium strategies) in mind. This is in contrast to another common approach to defining beliefs in the applied literature, "passive beliefs," in which the receiver holds the same belief about the sender's choice of unobservable action regardless of the sender's observable choices. In general, passive beliefs will be inconsistent with *RI-equilibria* given the sender's "optimal" private action may depend on its intended choice of signal. Proposition 4 and corollary 2 provide conditions under which one can order the equilibrium outcomes under these two approaches. The applications in Sections 4 and Different further illustrate the differences. Given that in our view the *RI-equilibrium* is the appropriate equilibrium to focus on, these findings can be viewed as a critique of using passive beliefs.

The *RI-equilibrium* provides a way to refine the set of equilibrium outcomes of the original game, which is just the set of Nash equilibrium outcomes of that game. Since the reordered game has proper subgames, perfection is used to reduce the set of equilibria in the reordered game, and so the set of Nash equilibrium outcomes of the original game. Thus, the *RI-equilibrium* concept can be powerful in reducing the number of equilibrium outcomes in the same sense that requiring subgame perfection can be powerful. As we will see in our applications, it often delivers a unique equilibrium outcome of the original game. The key idea is that reordering disentangles information sets while leaving the senders' decision problem essentially unchanged. When we consider more complicated games in Section 4, a similar idea will still apply even though the reordered game will no longer necessarily have any proper subgames.

An important advantage of our *RI-equilibrium* concept in the class of games we consider is that it is relatively easy to use in applications. Solving for the equilibria of the original game is often cumbersome in this class of games, unless ad-hoc belief functions (such as passive beliefs) are adopted. However, in applications, often we only care about equilibrium outcomes. In this respect, the *RI-equilibrium* concept provides an efficient way to find the reasonable equilibrium outcomes of the original game since finding the equilibria of the reordered game is typically a much easier exercise. Similarly, the *RI-equilibrium* concept is generally easier to apply than some other strong equilibrium concepts, such as Myerson's (1978) proper equilibrium, particularly when strategy sets are infinite as they are in most of the applications we have in mind.[7]

3.3 Existence

It is already known that there exists at least one sequential equilibrium and therefore at least one SPE, possibly involving behavior strategies, in any finite extensive-form games (Kreps and Wilson, 1982). Because of Proposition 1 we also know that there exists at least one *RI-equilibrium*, possibly involving behavior strategies, for any finite game $G_o \in \Gamma_o$. We will be clear about what we mean by a behavior-strategy *RI-equilibrium* in Section 3.5.

[7] Simon and Stinchcombe (1995) extend the definition of proper equilibrium to infinite games.

It may be useful to note a sufficient condition for the existence of a pure-strategy SPE for a game $G_r \in \Gamma_r$, which in turn means the existence of a pure-strategy *RI-equilibrium* for a game $G_o \in \Gamma_o$.

Proposition 2 (Existence) *Suppose that a game $G_o \in \Gamma_o$ is such that the sets T, A, and B are nonempty compact convex subsets of a Euclidean space, the sender's payoff function π_S is continuous in (t, a, b) and quasi-concave in t, and the receiver's payoff function π_R is continuous in (t, a, b) and quasi-concave in b. Then there exists a pure-strategy RI-equilibrium of G_o.*

3.4 Optimality

Given the canonical endogenous signaling game we are considering has a single sender, any *RI-equilibrium* has the nice property that no other equilibria of the original game yield better outcomes for the sender. (Obviously this need not be the case when there are multiple senders, for example, if the senders are competitors.)

Proposition 3 (Optimality) *Consider a game $G_o \in \Gamma_o$. Suppose that the set of RI-equilibria is nonempty and the set of their payoffs to the sender admits a maximal element. Then there exists <u>at least</u> <u>one</u> RI-equilibrium that yields the best payoff to the sender among all the SPE. Furthermore, any SPE that is not outcome-equivalent to a RI-equilibrium yields a strictly lower payoff than this best payoff to the sender.*

To give the main idea of the proof, suppose on the contrary there exists a SPE that is not outcome-equivalent to a *RI-equilibrium* but yields the best payoff to the sender. Since the payoff of this SPE corresponds to a Nash equilibrium payoff in one of the proper subgames of the reordered game, the sender could have achieved this payoff by choosing the corresponding observable action in the first stage of the reordered game (i.e. selecting the subgame with the best Nash equilibrium payoff). The fact the sender did not choose this action in any *RI-equilibrium*, means there is another action leading to a better Nash equilbrium payoff to the sender, which is a contradiction. In the following corollary, we provide a sufficient condition for the stronger result that any *RI-equilibrium* yields the best payoff to the sender.

Corollary 1 (Complete Optimality) *Consider a game $G_o \in \Gamma_o$ and its reordered game G_r. Suppose that there is a unique Nash equilibrium payoff to the sender in each of the proper subgames in G_r and that the set of the Nash equilibrium payoffs to the sender admits a maximal element. Then <u>any</u>*

RI-equilibrium yields the best payoff to the sender among all the SPE in G_o. Furthermore, any SPE that is not outcome-equivalent to a RI-equilibrium yields a strictly lower payoff than this best payoff to the sender in G_o.

3.5 Behavior strategies

Up until this point, we have restricted our analysis to pure-strategy equilibria. The idea and optimality of *RI-equilibrium* continues to apply if we allow behavior strategies. Proposition 1 extends trivially. Proposition 3 also extends easily when the action sets are finite. We illustrate how to construct an *RI-equilibrium* by an example. Consider the game in panel (a) of Figure 1. Suppose one finds a behavior-strategy SPE in the reordered game (panel (b)) as follows:

- In stage 1, the sender chooses a with probability $p(a)$ and a' with probability $p(a')$.
- In stage 2, the sender chooses t with probability $q_a(t)$ and t' with probability $q_a(t')$ if it has chosen a in stage 1; the sender chooses t with probability $q_{a'}(t)$ and t' with probability $q_{a'}(t')$ if it has chosen a' in stage 1.
- In stage 3, the receiver chooses b with probability $r_a(b)$ and b' with probability $r_a(b')$ if it has observed a; the receiver chooses b with probability $r_{a'}(b)$ and b' with probability $r_{a'}(b')$ if it has observed a'.

The outcome assigns probabilities $p(a)q_a(t)$, $p(a')q_{a'}(t)$, $p(a)q_a(t')$, and $p(a')q_{a'}(t')$ (followed by $r_a(b)$ and $r_a(b')$) to the terminal nodes ta, ta', $t'a$, and $t'a'$ (followed by b and b') respectively. Then the following is a behavior-strategy *RI-equilibrium* in the original game, which has the same outcome (up to the reordering of t (or t') and a (or a')):

- In stage 1, the sender chooses t with probability $(p(a)q_a(t)+p(a')q_{a'}(t))$ and t' with probability $(p(a)q_a(t') + p(a')q_{a'}(t'))$.
- In stage 2, the sender chooses a with probability $\frac{p(a)q_a(t)}{p(a)q_a(t)+p(a')q_{a'}(t)}$ and a' with probability $\frac{p(a')q_{a'}(t)}{p(a)q_a(t)+p(a')q_{a'}(t)}$ if it has chosen t in stage 1 (in case of $(p(a)q_a(t)+p(a')q_{a'}(t)) = 0$, any probability distribution over a and a' will do); the sender chooses a with probability $\frac{p(a)q_a(t')}{p(a)q_a(t')+p(a')q_{a'}(t')}$ and a' with probability $\frac{p(a')q_{a'}(t')}{p(a)q_a(t')+p(a')q_{a'}(t')}$ if it has chosen t' in stage 1 (in case of $(p(a)q_a(t') + p(a')q_{a'}(t')) = 0$, any probability distribution over a and a' will do).
- In stage 3, the receiver chooses b with probability $r_a(b)$ and b' with probability $r_a(b')$ if it has observed a; the receiver chooses b with probability $r_{a'}(b)$ and b' with probability $r_{a'}(b')$ if it has observed a'.

4 Monotone endogenous signaling games

Thus far, we have not put any particular structure on the players' payoffs. However, typically our interest in these games stems from the fact that the sender can influence the receivers' actions to its benefit through its choice of signals. In this section we explore one particular structure of payoffs which gives rise to this feature and explore some of its economic implications.

Specifically, we consider a setting which has a monotone structure, such that (i) the sender has an incentive to induce the receiver to choose a higher level of action; (ii) in order that the receiver prefers to choose a higher level of action, the sender must convince the receiver that it has chosen a higher level of the unobservable action; and (iii) the sender will want to choose a higher level of the unobservable action when it intends to choose a higher level of the observable action.[8]

We start by assuming that the sets T, A, and B are partially ordered sets and that there exists a unique equilibrium (i.e. SPE) in the reordered game G_r, which we denote by $(a^*, t^*(a), b^*(a))$ such that

$$\begin{aligned} a^* &\in \arg\max_{a\in A} \pi_S(t^*(a), a, b^*(a)), \\ t^*(a) &\in \arg\max_{t\in T} \pi_S(t, a, b^*(a)) \quad \text{for each } a \in A, \quad \text{and} \\ b^*(a) &\in \arg\max_{b\in B} \pi_R(t^*(a), a, b) \quad \text{for each } a \in A. \end{aligned}$$

Corresponding to the informal conditions (i)-(iii) above, suppose (i) the sender's payoff function π_S is increasing in b, (ii) $\arg\max_{b\in B} \pi_R(t, a, b)$ is nonempty and strongly increasing[9] in t for each $a \in A$, and (iii) the function t^* is increasing in a. The conditions (ii) and (iii) are satisfied if the subgames following each $a \in A$ in the reordered game are supermodular games parameterized by $a \in A$ where π_S has increasing differences in (a, t) and π_R has increasing differences in (a, b) and is strictly quasi-concave in b (see Topkis, 1978, p. 317 and Vives, 1999, p. 32 and 35). Since the term "increasing" is based on particular binary relations over the sets, this setting actually captures eight different monotone structures, corresponding to changing "increasing" to "decreasing" anywhere in the conditions (i)-(iii), or equivalently reversing the binary relations over the sets T, A, and B.

[8] Cho and Sobel (1990, pp. 391-392) identified three main conditions (A1$'$, A3, and A4) to give classical signaling games a certain monotone structure. Our first two conditions correspond to their A1$'$ and A3. Our third condition is distinct from their A4, which guarantees that the sender of higher type is more willing to send a higher signal. Instead, in our setting what matters for the *RI-equilibrium* is that the sender will want to choose a higher "type" when it intends to choose a higher signal. This reflects the difference in game structure (i.e. who chooses the "type") rather than any difference in payoff structure, and sometimes leads to qualitatively different results.

[9] A correspondence ϕ from T to S is strongly increasing if $t \leq t', t \neq t'$ implies that for each $s \in \phi(t)$ and $s' \in \phi(t')$, $s \leq s'$. In this case, we write $\phi(t) \overset{s}{\leq} \phi(t')$ or $\phi(t') \overset{s}{\geq} \phi(t)$.

We consider these conditions on the reordered game as the primitives of the problem we are interested in, which involves characterizing properties of the equilibria of the original game. This takes advantage of the fact that analyzing the reordered game is often straightforward. We call original games satisfying the conditions (i)-(iii) on their corresponding reordered games "monotone endogenous signaling games," denote this class of games by Γ_o^+, and its typical element G_o^+.

4.1 Commitment effect

The first property we explore is whether the sender's private choice can still have some commitment effect even though it is not observed. By commitment effect we have in mind that the sender's choice of the unobservable action t can enhance its payoff through its influence on the receiver's choice, relative to what would happen if t were chosen at the same time as the receiver's choice of b. The difficulty with this definition of the commitment effect in our context is that a, which is chosen after (or at the same time as) the choice of t, may have a direct influence on the receiver's choice of b. To allow for this, we instead define the benchmark (without the commitment effect) to be an equilibrium sustained by the receiver's passive beliefs (the receiver ignores any observable actions by the sender when forming its beliefs about the unobservable actions) and compare it with *RI-equilibrium.*

We start by defining the two equilibria that we compare. Given that there exists a unique equilibrium in the reordered game, we know by Proposition 1 that there exists a unique *RI-equilibrium* outcome $(\tilde{t}, \tilde{a}, \tilde{b}) = (t^*, a^*, b^*)$ in the original game G_o,[10] supported by a strategy profile $(\tilde{t}, \tilde{a}(t), \tilde{b}(a))$, where $\tilde{b}(a) = b^*(a)$ for each $a \in A$,

$$\begin{aligned} (\tilde{t}, \tilde{a}(t)) &\in \arg\max_{(t,a(t))\in T\times A^T} \pi_S(t, a(t), \tilde{b}(a(t))), \quad \text{and} \\ \tilde{b} &\in \arg\max_{b\in B} \pi_R(\tilde{t}, \tilde{a}, b). \end{aligned}$$

On the other hand, a passive-belief equilibrium of G_o^+, $(t^{pa}, a^{pa}(t), b^{pa}(a))$ satisfies

$$\begin{aligned} t^{pa} &\in \arg\max_{t\in T} \pi_S(t, a^{pa}(t), b^{pa}(a^{pa}(t))), \\ a^{pa}(t) &\in \arg\max_{a\in A} \pi_S(t, a, b^{pa}(a)) \quad \text{for each } t \in T, \quad \text{and} \\ b^{pa}(a) &\in \arg\max_{b\in B} \pi_R(t^{pa}, a, b) \quad \text{for each } a \in A. \end{aligned}$$

Note that the receiver's belief is passive and fixed at t^{pa} in this equilibrium. To make our exercise meaningful, we assume that the passive-belief equilibrium outcome is in the interior of $T \times A \times B$.

[10] For brevity, we have suppressed the arguments of the equilibrium outcomes, that is, $\tilde{a} \equiv \tilde{a}(\tilde{t})$, $\tilde{b} \equiv \tilde{b}(\tilde{a}(\tilde{t}))$, $t^* \equiv t^*(a^*)$, and $b^* \equiv b^*(a^*)$. We follow this convention hereafter.

Proposition 4 (Weak Commitment Effect) *For any game $G_o^+ \in \Gamma_o^+$, $\tilde{t} \geq t^{pa}$, $\tilde{a} \geq a^{pa}$, and $\pi_S(\tilde{t}, \tilde{a}, \tilde{b}) \geq \pi_S(t^{pa}, a^{pa}, b^{pa})$.*

This proposition shows that in monotone endogenous signaling games, the sender's choices of both observable and unobservable actions at the *RI-equilibrium* outcome are weakly higher than those in the passive-belief benchmark. In the following corollary, we provide conditions under which the sender's choices of both observable and unobservable actions at the *RI-equilibrium* outcome are strictly higher than those in the passive-belief benchmark.

Corollary 2 (Strong Commitment Effect) *Consider a game $G_o^+ \in \Gamma_o^+$. Suppose the function t^* is strictly increasing in a at a^{pa} and there exists an $a \in A$ such that $\pi_S(t^{pa}, a, b^*(a)) > \pi_S(t^{pa}, a^{pa}, b^{pa})$. Then $\tilde{t} > t^{pa}$, $\tilde{a} > a^{pa}$, and $\pi_S(\tilde{t}, \tilde{a}, \tilde{b}) > \pi_S(t^{pa}, a^{pa}, b^{pa})$.*

Starting from the passive-belief equilibrium outcome, an increase in a has three effects on the sender's payoff: it may (1) directly change the sender's payoff (direct effect), (2) directly affect the receiver's choice of action b and so the sender's payoff (strategic effect), and (3) indirectly affect the receiver's choice of action b through the receiver's belief about the sender's choice of t and so the sender's payoff (belief effect). The inequality assumed in the corollary implies that the sum of these three effects is positive for some change in a. This assumption can be motivated as follows. The passive-belief equilibrium outcome is "optimal" with respect to the sum of the first two effects, which means the first-order effect of the sum of the direct and strategic effects is zero in differentiable cases. The positive belief effect that arises from the monotone structure of the games (conditions (i)-(iii) above) therefore explains the inequality assumed in the corollary.

The additional assumptions in the corollary can be satisfied, for example, if the sets T, A, and B are compact intervals in $\mathbb{R}$ and locally (in the neighborhood of the passive-belief equilibrium outcome) π_S is strictly increasing in b and the subgames following each $a \in A$ in the reordered game are smooth strictly supermodular games parameterized by $a \in A$ where π_S has strictly increasing differences in (a, t) and π_R has strictly increasing differences in (a, b) and is strictly concave in b. With these sufficient conditions, the sum of the direct effect and strategic effect is zero at the the passive-belief equilibrium outcome and the belief effect is positive (see the appendix).

Even though the sender's private choice is unobserved, the corollary says it is still chosen as though it is observed, to some extent. That is, the sender's private choice may still have some commitment value in terms of influencing the receivers' choice to its benefit (relative to the outcome under passive beliefs). The result implies that much of the large literature studying business strategy in which firms commit to observable "investments" so as to benefit from strategic effects can be redone using this framework even when investments are not observable, provided they can be signaled through related observable actions such as repeated product market competition.

4.2 Signal exaggeration

Another interesting property of monotone endogenous signaling games is what we refer to as "signal exaggeration," in which the sender's signal is exaggerated in equilibrium, compared to the equivalent choice of signal in a full information game. This is somewhat similar to the well-known property of separating equilibria in classical signaling games, in which a high type sender has to exaggerate its signal to ensure the low type sender does not want to mimic its choice of signal. In our setting, signal exaggeration arises in equilibrium to ensure that the sender itself does not have any incentive to further manipulate beliefs by choosing a different signal.

To formalize what we mean by signal exaggeration, we define an observable benchmark and compare the sender's choice in the benchmark with that in a *RI-equilibrium*. To make our exercise meaningful, we assume that the *RI-equilibrium* outcome is in the interior of $T \times A \times B$.

Consider a variant of the game $G_o^+ \in \Gamma_o^+$ where the action t is now observed by the receiver. We denote it by G^{ob}. Consider an equilibrium of G^{ob}, $(t^{ob}, a^{ob}(t), b^{ob}(t,a))$, where

$$\begin{aligned} t^{ob} &\in \arg\max_{t \in T} \pi_S(t, a^{ob}(t), b^{ob}(t, a^{ob}(t))), \\ a^{ob}(t) &\in \arg\max_{a \in A} \pi_S(t, a, b^{ob}(t,a)) \quad \text{for each } t \in T, \quad \text{and} \\ b^{ob}(t,a) &\in \arg\max_{b \in B} \pi_R(t,a,b) \quad \text{for each } (t,a) \in T \times A. \end{aligned}$$

Under the same conditions as in Proposition 4, we can conclude that the sender would choose a higher observable action at the *RI-equilibrium* outcome than it would choose if the same unobservable action were observed by the receiver.

Proposition 5 (Weak Signal Exaggeration) *For any game $G_o^+ \in \Gamma_o^+$, $\tilde{a}(\tilde{t}) \geq a^{ob}(\tilde{t})$.*

With an additional condition we obtain a strict version of the result.

Corollary 3 (Strong Signal Exaggeration) *Consider a game $G_o^+ \in \Gamma_o^+$ and its variant G^{ob}. Suppose there exists an $a \in A$ such that $\pi_S(\tilde{t}, a, b^{ob}(\tilde{t}, a)) > \pi_S(\tilde{t}, \tilde{a}, \tilde{b})$. Then $\tilde{a}(\tilde{t}) > a^{ob}(\tilde{t})$.*

The assumed inequality means that starting from the *RI-equilibrium* outcome, the sum of the direct and strategic effects (defined earlier) is negative. This assumption is consistent with the fact that the sum of all three effects (direct, strategic, and belief effects) should be zero at the *RI-equilibrium* outcome in differentiable cases and that the belief effect is positive from the monotone structure of the games. An implication is that the sender would prefer to choose a lower action a when $\tilde{t}$ is observed, compared to when it chooses a to manipulate the receiver's belief (as it does in the *RI-equilibrium*).[11]

To illustrate another feature of signal exaggeration, consider the case firms can choose cost-reducing investments which are unobservable to each other but then they compete repeatedly in the product market by choosing observable quantities each period. For a given cost, starting from their corresponding one-shot Nash equilibrium quantities, a small increase in the initial quantity by a firm does not give rise to any first-order loss, but since it lowers the rivals' beliefs about the firm's costs (and therefore the rival's best-response quantity in the subsequent period), it does have a first order benefit. Therefore, in equilibrium, initial quantities must be exaggerated for any given cost to ensure firms do not have any incentive to further manipulate beliefs, i.e. so the receiver is not fooled.

In a similar setting but when firms can deter entry, limit pricing in which prices are set below their full information (i.e. monopoly) level is an obvious application of signal exaggeration. Signal exaggeration also provides a novel explanation of loss-leader pricing, in which multi-product firms set (and advertise) one or more of their products at a low price (possibly below cost). Advertised low prices can signal to potential consumers that a firm's other unadvertised prices are also low, if products are substitutes and consumers prefer one-stop shopping. In this case, advertised prices are exaggerated (in a downward direction) to the point where a firm would not gain by further manipulation of consumer beliefs.[12]

In monotone endogenous signaling games, the sender would naturally want to reveal its true choice of its unobservable feature to the receiver, thereby avoiding the need to engage in costly signal exaggeration. The problem is that in doing so it has an incentive to lie. However, provided there is some cost to lying, the sender's announcement can still work as an inflated signal of its true choice. A costly announcement game is therefore a somewhat generic example of a monotone endogenous signaling game,

[11] The additional assumption in the corollary can be satisfied by the same set of sufficient conditions that were given for the additional assumptions of Corollary 2, except that these sufficient conditions are now assumed to hold in the neighborhood of the *RI-equilibrium* outcome. The proof of this claim is almost identical to before and is given in the appendix.

[12] See Section 5.4 for further details on how to apply our framework to such a case, when the firm can choose which good to advertise.

which is why we turn to it now in order to illustrate the various results of this section.

4.3 Application to costly announcements and inflation

Consider the following simple game. In stage 1, a sender chooses its private effort t which is unobserved by a receiver. In stage 2, the sender makes an announcement on its level of effort a so as to try to make the receiver believe its effort is higher. In stage 3, the receiver reacts by choosing some action b (which could be its belief about the sender's effort, or some action it takes given its beliefs). To capture this situation, we adopt the standard quadratic payoff specification of cheap-talk games as in Crawford and Sobel, 1982 (this is the first term in (1)), and modify it to introduce a cost to lying as in Kartik et al. (2007) (the second term in (1)) and that the sender cares about its choice of effort (the third term in (1)). The payoffs are

$$\pi_S = -(b - (t + \beta))^2 - \kappa(a - t)^2 - \lambda(t - \hat{t}\,)^2 \tag{1}$$

for the sender and $\pi_R = -(b - t)^2$ for the receiver, with $\kappa > 0$ and $\lambda > 0$.

As in cheap-talk games, the parameter β measures a bias between the two players. The sender prefers the receiver to choose $b = t + \beta$ while the receiver would prefer $b = t$ if t were known. Thus, the sender would like to mislead the receiver into choosing a higher action by convincing the receiver it has chosen higher effort than it actually has. However, to the extent the sender's announced effort differs from its true effort, it will suffer a penalty which is captured by the second term in π_S (e.g. this could be the sender's subjective disutility or expected penalty from lying). $\hat{t}$ is the level of effort that the sender would choose in a full information game, i.e. its first-best level.

An application that fits this specification is a variation on the classic time inconsistency problem of monetary policy (Kydland and Prescott, 1977) in which the government privately instructs the non-independent central bank to achieve a target level of inflation t while publicly announcing a target level a, where $\hat{t}$ represents the government's first-best level of inflation. After hearing the government's announcement, the private sector chooses b to maximize π_R, which is equivalent to forming a rational expectation on the level of inflation in Kydland and Prescott. Setting $\beta < 0$ captures the government's incentive to pursue expansionary policy. In the existing literature $\kappa = 0$, implying any announcement is ignored.

In the original game there are a continuum of Nash equilibria, which are difficult to characterize in general. Assuming differentiability of the receiver's strategy $b(a)$ and second-order conditions hold, the equilibria $(\bar{t}, \bar{a}(t), \bar{b}(a))$ are characterized by the first-order conditions $\bar{t} = \bar{b}(\bar{a}) = (\kappa\bar{a} + \lambda\hat{t} - \beta)/(\kappa+\lambda)$ and $\bar{b}'(\bar{a}) = \kappa(\lambda\bar{a} - \lambda\hat{t} + \beta)/(\beta(\kappa+\lambda))$, where $\bar{a} \equiv \bar{a}(\bar{t})$. Without assuming differentiability of the receiver's equilibrium strategy $b(a)$, additional equilibria may arise. Moreover, determining whether second-order conditions hold is not straightforward unless $\bar{b}''(\bar{a}) = 0$.

By setting $\bar{b}'(\bar{a}) = 0$, we obtain the equilibrium with passive beliefs $t^{pa} = a^{pa} = b^{pa} = \hat{t} - \beta/\lambda$. Since the sender's announcement is ignored, it may as well tell the truth. However, in equilibrium it chooses a lower level of effort compared to its ideal level $t^{ob} = \hat{t}$ due to its lack of commitment (if it could commit to $t = \hat{t}$ it would be better off). This corresponds to the discretionary inflation level (higher than first-best) in the time inconsistency example.

To solve the reordered game, we look for an equilibrium in the choice of t and b for a given a. The result is that $t^*(a) = b^*(a) = (a\kappa - \beta + \hat{t}\lambda)/(\lambda + \kappa)$. Given this, solving for the optimal choice of a in the first stage, the unique *RI-equilibrium* outcome of the original game is $\tilde{t} = \tilde{b} = \hat{t} - \beta/(\lambda + \kappa)$ and $\tilde{a} = \hat{t}$. Clearly, if $\beta > 0$ then $\tilde{t} > t^{pa}$ and $\tilde{a} > a^{pa}$ (one can also confirm that $\pi_S(\tilde{t}, \tilde{a}, \tilde{b}) > \pi_S(t^{pa}, a^{pa}, b^{pa})$) so that even though the sender's effort is unobserved, it is still able to obtain a commitment benefit. This corresponds to reducing inflation towards the government's first-best level in the time inconsistency example, i.e. $\tilde{t} < t^{pa}$ and $\tilde{a} < a^{pa}$ reflecting that $\beta < 0$. This is not a trivial result, given that the sender chooses its announcement after (or equivalently, at the same time as) fixing its level of effort, so that in general the receiver need not react to the announcement at all (i.e. passive beliefs) or in any particular way.[13]

Consistent with signal exaggeration, we also find announcements are inflated: $\tilde{a} = \hat{t} > a^{ob}(\tilde{t}) = \tilde{t} = \hat{t} - \beta/(\lambda + \kappa)$ if $\beta > 0$. Although the sender still chooses less effort than its first-best level $\hat{t}$, due to signal exaggeration it turns out that its announced effort level a is exactly equal to its ideal level $\hat{t}$. This surprising result is specific to the quadratic payoff specification (in general it may announce a somewhat higher or lower level of t compared to $\hat{t}$). As a consequence, in this example, while signal exaggeration increases as lying costs get small, the level of signal exaggeration is bounded — the equilibrium level of effort converges towards the passive beliefs level $\hat{t} - \beta/\lambda$, while the announced effort remains at $\hat{t}$. Even with a very small cost of lying, the sender will not want to exaggerate very much.

4.4 Comparison with classical signaling games

The characterization of equilibria in our setting is quite different to that of equilibria in classical signaling games. A general difference arises from the fact that the sender makes an additional choice in our setting (the choice of its unobservable action), meaning there are additional ways it can deviate which must be ruled out in any equilibrium. For example, an interesting

[13] If instead the sender chooses its announcement first, then by announcing a high level of effort it can obviously influence its choice of effort to obtain a commitment benefit.

feature in a limit-pricing setting where the incumbent's private cost is determined endogenously by its unobservable investment in cost-reducing R&D, is that partial entry deterrence may arise. This reflects, that in contrast to standard exogenous-cost limit pricing models, here limit pricing can alter the incumbent's investment incentives in the *RI-equilibrium.* Since investment in cost reduction is generally more profitable under competition than under monopoly, the rival may prefer to stay out when the incumbent expects it to enter (and vice-versa). As a result, the unique *RI-equilibrium* outcome may involve the potential rival randomizing over whether to enter or not, and the incumbent will sometimes regret limit pricing.

In addition, what matters for *RI-equilibria* in monotone endogenous signaling games is that the sender will want to choose a higher "type" when it intends to choose a higher signal, whereas what matters for separating equilibria in classical signaling games is that the sender of higher type is more willing to send a higher signal. This distinction reflects the difference in game structure (i.e. who chooses the "type") rather than any difference in payoff structure, and sometimes leads to qualitatively different results. For example, when quality is determined by nature, a high quality firm may choose a positive level of wasteful advertising (together with a high price) to make it unprofitable for a low quality firm to mimic it, whereas when quality is chosen by the firm, "burning money" will not arise as an equilibrium since such advertising is a sunk cost, which has no bearing on the firm's optimal choice of unobservable quality.

Compared to classical signaling games in which the equilibrium level of the signal is pinned down by a no-mimicking constraint, the factors affecting the *RI-equilibrium* level of the signal in our setting can be very different. Consider for example the standard signaling games with two types of sender, "high" and "low". A factor which increases the payoff to the sender of being believed to be of high type will generally increase the equilibrium level of the signal in a classical setting since mimicking will be more attractive for a low type and so requires more costly signaling to rule it out. The same factor will have no effect on the *RI-equilibrium* level of the signal in our setting when the equilibrium is pinned down by an equivalent of the no mimicking constraint. This is because the relevant comparison is now between the payoffs from choosing high and low types followed by the signal for which the receiver believes the sender is of high type. On the other hand, a factor which increases the payoff to the sender of actually being of high type (for given beliefs) will not affect the equilibrium level of the signal in a classical setting since the no-mimicking constraint (for the low type) is not affected. The same factor will tend to decrease the *RI-equilibrium* level of the signal in our setting since costly signaling is less important when the sender anyway prefers to be of a higher "type" for given beliefs.

This logic can explain why with an otherwise identical specification, comparative results can be reversed depending on whether "types" are determined by nature or chosen by the sender. One can easily construct an example in which an increase in consumers' valuation for the seller of high type would increase the equilibrium level of advertising in a classical quality-signaling game whereas it would decrease the *RI-equilibrium* level of advertising in an endogenous quality-signaling game.[14] One can also construct an example in which an improvement in the high type borrower's prospects would increase the equilibrium level of collateral in the classical signaling model of costly collateral pledging (Tirole, 2006, pp. 251-254) whereas it would decrease the *RI-equilibrium* level of collateral in an endogenous counterpart of this game in which the borrower's quality is determined by its prior efforts. The comparative statics of this *RI-equilibrium* may be easier to reconcile with the empirical evidence (Coco, 2000, p. 191), which suggests borrowers post less collateral when their prospects improve.

5 Generalizations

The concept of *RI-equilibrium* and some of its properties apply to a much broader class of games. In this section, we extend our framework beyond canonical endogenous signaling games, starting in Section 5.1 with relatively straightforward generalizations. In Section 5.2, we propose a stronger version of *RI-equilibrium* to handle more complicated endogenous signaling games. In Section 5.3, we allow for multiple senders. In Section 5.4, we discuss how to decompose sender's actions to deal with general partial observability. In Section 5.5, we discuss how to construct our *RI-equilibrium* when there are multiple receivers with different information partitions, including more complicated cases in which the receivers' information partitions are not appropriately ordered. Finally, in Section 5.6, we illustrate the latter with an application to opportunism in vertical contracting.

5.1 Straightforward generalizations

We start by briefly discussing some straightforward generalizations, to which Proposition 1 extends trivially.

1. Nature's move. We can allow nature to move. The generalization is straightforward if (1) the realization is not observed by any strategic players, i.e. the senders and receivers,[15] or (2) the move of nature is at the start of the game such that the move of nature becomes common knowledge.

[14] Note that an increase in consumers' valuation for the seller of high type would increase the payoff to the seller of being believed to be of high type (consumers are willing to pay more for their initial purchase) and of actually being of high type (consumers will pay more for their repeated purchase).

[15] E.g. this case allows us to capture situations where the realization of a firm's efforts to improve quality are only determined after consumers decide whether to buy.

2. Multiple actions. We have already allowed for the players' action sets to be multi-demenstional. Similarly, we can allow for the sender to choose multiple actions in any sequence, only some of which are observed by receivers. In this case, we rearrange the sequence of actions in the signaling phase so that all the actions (a's) observed by the receivers are set first followed by the actions (t's) unobserved by the receivers. This maintains the same information structure except for the different sequence of the senders' own actions.

3. Extra phase. We can have an extra phase after the reaction phase where all the previous moves become common knowledge. In this case, one can identify the equilibria of the continuation games and embed the equilibrium payoffs in the payoff functions.

4. Simultaneous choices. In this case the sender(s) chooses observable and unobservable actions simultaneously. Note that applying PBE or sequential equilibrium without further refinement usually yields multiple equilibria. For example, if the actions t and a were chosen simultaneously in the canonical endogenous signaling games, the multiplicity problem would still remain. In this case, we can use the same analysis of considering the reordered games in Section 3, as was noted in the example in Section 4.3.

5. Multiple receivers. Having multiple receivers is not really an issue if the information partitions of the receivers are the same.[16] For example, consider a quality game where there are a continuum of consumers, a fraction of which are uninformed. In such a game, the multiplicity problem becomes more severe because we should allow different beliefs for different uninformed consumers at their information set off the equilibrium path. However, the receivers' equilibrium belief at their information set off the equilibrium path can be (often uniquely) determined by the sender's equilibrium strategy in its reordered game.

6. Repeated games. Our framework generalizes to repeated games. For example, if the stage game $t \to a \to b$ repeats finitely many times and has a unique *RI-equilibrium*, our refinement still pins down the equilibrium.

5.2 Equilibrium

For games with more complicated structures, using SPE in the reordered games is usually not powerful enough given the lack of proper subgames. We therefore make use of a more refined equilibrium concept than SPE for our basic equilibrium concept when dealing with the games in the subsequent sections.

Two equilibrium concepts are widely used in the literature: sequential equilibrium and perfect Bayesian equilibrium. The sequential equilibrium

[16] Even though the information partitions of the receivers are different, sometimes the difference can be absorbed by the configuration of the payoff functions. For the general treatment in case of different information partitions, see Section 5.5.

concept involves some technical difficulties in defining consistency for games with infinite action sets, which arise in many applications. Perfect Bayesian equilibrium does not suffer from this problem, but the weak version of perfect Bayesian equilibrium is too weak even for the simplest class of games (Γ_o). For these reasons, we make use of Fudenberg and Tirole's (1991) perfect extended Bayesian equilibrium as our basic equilibrium concept. We will refer to this more restricted version of perfect Bayesian equilibrium simply as perfect Bayesian equilibrium (PBE). When there is no risk of confusion, we also use the term equilibrium to mean PBE for the remainder of Section 5. For games with finite action sets, our results would be unchanged if instead we use sequential equilibrium as our basic equilibrium concept. Note that for the simpler games such as the canonical endogenous signaling games of Section 3, the sets of PBE and SPE coincide for reordered games so that using the PBE concept in Section 3 would be no more restrictive.

Despite a lack of proper subgames, reordering still helps disentangle information sets to a certain extent, so that PBE still has some bite. We interpret Fudenberg and Tirole's definition (p. 254) of general reasonableness more broadly so that we may apply the concept of PBE to infinite-action-set cases.

Definition A profile of strategies together with a conditional probability system (on the set of terminal nodes) is generally reasonable if

1. the conditional belief on the nodes which are successors to a decision node coincides with the probability distribution on the corresponding actions implied by the behavior strategy at the information set to which the decision node belong, and
2. the conditional belief on the nodes which can be reached by the same action from the nodes in an information set should inherit the conditional belief on the nodes in the information set,

where the conditional beliefs are induced by the conditional probability system. ■

Since we focus on pure-strategy equilibria, we provide a definition for a pure-strategy PBE.

Definition A profile of pure strategies together with a conditional probability system (on the set of terminal nodes) is a pure-strategy PBE if

1. it is generally reasonable, and
2. the profile of pure strategies together with the conditional beliefs induced by the conditional probability system is sequentially rational.

■

Sometimes we refer to the profile of strategies only or the profile of strategies together with conditional beliefs only at the receiver's information sets in the reaction phase as a PBE, whenever no confusion arises.

For the remaining games in this section, we first find PBE in the reordered games. Then among the PBE of the original game, we select only those which share the same outcome (up to the reordering) and the same receiver's beliefs and strategy with a PBE of the reordered game. As before, we call these equilibria *RI-equilibria* and the associated receiver's beliefs *RI-beliefs*.

5.3 Multiple senders and simultaneous moves

An important class of endogenous signaling games involve more than one player moving within a phase. We start by considering a setting where there are multiple senders who are uninformed of the other senders' private choices. An example would be competitors' choices of price (and advertising) to signal their choices of quality.

Consider extensive-form games with multiple uninformed senders ($i \in \{1, 2, \ldots, I\}$) and the following timing of moves, where T_i, A_i, and B are (possibly multi-dimensional) sets. Senders choose $t_i \in T_i$ simultaneously in stage 1 and $a_i \in A_i$ simultaneously in stage 2. In stage 2, each sender i knows its own choice of t_i, but not t_{-i}.[17] For this reason we call the senders "uninformed". Having observed only the senders' choices in stage 2, the receiver chooses $b \in B$ in stage 3. Stages 1 and 2 constitute the signaling phase and stage 3 is the reaction phase. The payoffs are $\pi_{S_i}(t, a, b)$ and $\pi_R(t, a, b)$ to each player respectively, where $t \equiv (t_i, t_{-i})$ and $a \equiv (a_i, a_{-i})$. We denote this class of games by Γ_o^m, and its typical element G_o^m (the superscript m is for "multiple" senders).

For any game $G_o^m \in \Gamma_o^m$, we define its reordered game with the following timing of moves. Senders choose $a_i \in A_i$ simultaneously in stage 1 and $t_i \in T_i$ simultaneously in stage 2. In stage 2, each sender i knows its own choice of a_i, but not a_{-i}. Having observed only the senders' choices in stage 1, the receiver chooses $b \in B$ in stage 3. We denote this game by G_r^m, and the collection of such games Γ_r^m.

The following proposition provides the foundation for applying our *RI-equilibrium* concept to the games $G_o^m \in \Gamma_o^m$.

Proposition 6 *Consider a game $G_o^m \in \Gamma_o^m$ and its reordered game G_r^m. Suppose that an outcome is supported by a PBE of G_r^m. Provided a mild regularity condition is satisfied (for each sender i, a best observable action*

[17] As usual, "$-i$" refers to all senders other than i.

a_i exists for any unobservable action t_i it has chosen, given the equilibrium strategies of all other players[18]*), the outcome is supported by a PBE of G_o^m.*

The intuition behind Proposition 6 is similar to that of Proposition 1. Each sender's optimal choice of multiple actions should not depend on the order of its choices, given all other players' choices at each of their information sets are the same across the two games and given the sender makes its choices of unobservable and observable actions without gaining any new (payoff-relevant) information in between. Compared to Proposition 1, it involves an additional step arising from the fact that we need to construct a PBE of the original game, rather than just a Nash equilibrium, which is why we require the mild regularity condition.

We already know from the existing literature (Kohlberg and Mertens, 1986) that if there exists a unique sequential equilibrium outcome, possibly involving behavior strategies, in a finite game $G_r \in \Gamma_r$ there should be a sequential equilibrium in its original game G_o which is outcome-equivalent. Proposition 6 states more than that because it does not require the uniqueness of the equilibrium outcome and it applies to games with infinite strategy sets as well.

Proposition 6 does not extend to games where multiple senders are informed of other senders' private choices, for example, a quality signaling games where the competing firms are informed of other firms' choices of quality before choosing their own prices/advertisments. In this case, from a particular sender's perspective, new payoff-relevant information arrives before its own observable action, making it impossible to preserve the original information structure in the reordered game. These games do not belong to the class of games we analyze.

The analysis of uninformed multiple senders also applies when the senders are receivers as well. An example would be competitors' choices of price or quantity to signal their choice of cost-reducing investment (unobservable to the rivals) in a competition game (two rounds of competition, with the first one in the signaling phase and the second one in the reaction phase). The reordering is done in the same way as above. The timing of the original game is $t_1 \cdots t_I \to a_1 \cdots a_I \to b_1 \cdots b_I$, where "$\to$" distinguishes the stages and "$\cdots$" denotes simultaneous moves (each player i observes only t_i in stage 2). The timing of its reordered game is $a_1 \cdots a_I \to t_1 \cdots t_I \to b_1 \cdots b_I$ (each player i observes only a_i in stage 2). As shown in the supplementary appendix, Proposition 6 continues to hold provided that the regularity condition is modified such that it applies to each sender's choice of a best

[18] The regularity condition holds either (1) if the sender's observable action set is finite, or (2) if the sender's observable action set is infinite but compact and the sender's payoff function is continuous in its observable action when the receiver's best response is taken into account. A supplementary appendix contains a more formal statement of the regularity condition and the proof of the proposition.

observable action taking into account its own equilibrium choice as a receiver in the reaction phase.

A similar analysis applies when the senders and the receivers move simultaneously in the signaling phase and/or the reaction phase (provided that they don't observe each other's choices until the reaction phase). An example would be a predator's choice of price to signal its choice of cost-reducing investment in a predation game, where the rival also sets its price at the same time as the predator prior to deciding whether to exit or not. The reordering is done in the same way as above. The timing of the (three-stage) original game is $t \to a_S \cdots a_R \to b_S \cdots b_R$ (only the sender observes t in stage 2). The timing of its reordered game is $a_S \to t \cdots a_R \to b_S \cdots b_R$ (only the sender observes a_S in stage 2). As shown in the supplementary appendix, Proposition 6 continues to hold provided that the regularity condition is modified such that it applies to each sender's choice of a best observable action taking into account its own equilibrium choice in the reaction phase, if any.

5.4 Decomposing the sender's actions

Sometimes whether an action is observable may depend on the choice of another action. For example, consider a game in which competing supermarkets choose which goods to advertise the price of, at the same time as determining the level of the prices. The decisions on the prices are potentially unobservable and the decision on the advertisement determines which price decision(s) will become observable. In this section, we show how to decompose senders' actions into observable and unobservable actions in such a case.

We consider an example with two competing supermarkets (senders 1 and 2), two different goods (goods x and y), two price levels for each good, high price (h) or low price (l), and advertising decisions, whether to advertise (a) or not (n). In this case, each sender (supermarket) has sixteen choices. For example, if for good x a supermarket chooses a low price which it advertises while for good y it chooses a high price which it does not advertise, then we will denote its choice by *lahn*. We use the receiver's (representative consumer) information partition to decompose the senders' actions into observable and unobservable actions. If there were only one sender, the receiver's (representative consumer) information partition would be

$$\begin{aligned}\{ \quad & \{haha\}, \{hala\}, \{laha\}, \{lala\}, \\ & \{hahn, haln\}, \{lahn, laln\}, \\ & \{hnha, lnha\}, \{hnla, lnla\}, \\ & \{hnhn, hnln, lnhn, lnln\} \qquad \},\end{aligned}$$

with nine elements, each element being one of the receiver's information sets. We denote this partition by P. Since there are two senders, the receiver's information partition will be P^2 with 81 elements (information sets).

In this case, the observable action is the decision on which of the receiver's information set(s) should be reached and the unobservable action is the decision on which decision node(s) of the information set(s) should be reached. From the perspective of one particular sender, the observable action is to choose one of the nine elements of P and the unobservable action is to choose a decision node in the information set chosen if it is not a singleton. As we see here, the choice set of the unobservable action now depends on the choice of observable action. Once we have decomposed the senders' actions into observable and unobservable actions, we construct our reordered game as follows:

1. In stage 1, the two senders choose one of the nine elements of P (observable actions) simultaneously.
2. In stage 2, the two senders choose a decision node in the information set it has chosen (if the information set is not a singleton) simultaneously.
3. In stage 3, having observed only which element of the partition P^2 was reached, the receiver makes a choice.

In stage 2, each sender knows its own choice of the element in P, but not the rival's, as was in the uninformed multiple senders game (Section 5.3).

In general, facing situations with partial observability, we use the receiver's information partition (the partition of the set of receiver's decision nodes) at the beginning of the reaction phase to decompose the senders' actions into observable and unobservable actions.[19]

- Observable action: to choose an information set (or information sets in case of uninformed multiple senders) in the receivers' information partition.
- Unobservable action: to choose a decision node (or decision nodes in case of uninformed multiple senders) in the information set (or information sets in case of uninformed multiple senders) it has chosen if it is not a singleton.

Then the construction of the reordered game should be the same as before: all the observable actions are set first followed by the unobservable actions.

[19] We assume that the receiver's information partition is in the form of a Cartesian product, where each individual set is a partition showing the receiver's information about each sender's actions. Otherwise, it would not be possible to preserve the original information structure in the reordered game.

5.5 Different information partitions

In this section, we show how we construct our *RI-equilibrium* when there are multiple receivers with different information partitions (at the beginning of the reaction phase). For simplicity we only consider the cases with a single sender and multiple receivers at least one of which has a non-singleton information partition (if all the receivers' information partitions are a singleton then no receiver observes any of the sender's actions, which is not an interesting case).

Given the collection of the receivers' information partitions, we add information partitions which are the infimum (according to the usual partial order[20]) of any sub-collection of partitions if they are not in the collection yet. We call these added information partitions "dummy partitions." Provided we can order all the receivers' information partitions together with the dummy partitions linearly at each of the following steps, the reordering of the original game should be as follows (we don't need to reorder the receivers' moves).

1. (Step 1) The sender chooses an information set in the coarsest non-singleton information partition of all the information partitions (including the dummy partitions).

2. (Step 2, ..., Second last step) The sender chooses an information set in the coarsest information partition of the partitions that divide the information set chosen in the previous step into at least two elements, if there are still such partitions left. (Note that this will end in a finite number of steps because there are finite number of receivers.)

3. (Last step) If the finally chosen information set is already a singleton, the sender does not need to make an additional choice. Otherwise, the sender chooses one element of the finally chosen information set.

For example, consider a game with one sender and four receivers. The sender chooses one of eight actions $a_1, \ldots, a_8$. Then the receivers, $R_1, \ldots, R_4$, choose $b_{R_i} \in B_{R_i}$ ($i \in \{1, 2, 3, 4\}$) simultaneously, having observed *partially* the sender's action according to the following information partitions:

$$\begin{aligned}
\text{Receiver 1:}\quad P_1 &= \{\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},\\
\text{Receiver 2:}\quad P_2 &= \{\{a_1, a_2, a_3, a_4\}, \{a_5, a_6\}, \{a_7, a_8\}\},\\
\text{Receiver 3:}\quad P_3 &= \{\{a_1, a_2\}, \{a_3, a_4\}, \{a_5, a_6, a_7, a_8\}\},\\
\text{Receiver 4:}\quad P_4 &= \{\{a_1\}, \{a_2\}, \{a_3, a_4\}, \{a_5, a_6\}, \{a_7, a_8\}\}.
\end{aligned}$$

[20] Given two partitions P and P' of a set, we say that P is finer than P', or equivalently, that P' is coarser than P, if every element of P is a subset of some element of P'. The relation of "being-finer-than" is the partial order we use here.

In this case, we have one dummy partition to add, which is the infimum of the information partitions for receivers 2 and 3.

$$\text{Dummy partition:} \quad \{\{a_1, a_2, a_3, a_4\}, \{a_5, a_6, a_7, a_8\}\}.$$

The reordering of the original game should be as follows:

1. In stage 1, the sender chooses between $\{a_1, a_2, a_3, a_4\}$ and $\{a_5, a_6, a_7, a_8\}$.
2. In stage 2, the sender chooses between $\{a_1, a_2\}$ and $\{a_3, a_4\}$ if it has chosen $\{a_1, a_2, a_3, a_4\}$ in stage 1, and between $\{a_5, a_6\}$ and $\{a_7, a_8\}$ if it has chosen $\{a_5, a_6, a_7, a_8\}$ in stage 1.
3. In stage 3, the sender chooses one element of the information set it has chosen in stage 2.
4. In stage 4, the receivers choose $b_{R_i} \in B_{R_i}$ $(i \in \{1, 2, 3, 4\})$ simultaneously.

Once the reordered game is defined, we can find our *RI-equilibria* as usual.

If we cannot order all the receivers' information partitions, including the dummy partitions, linearly at some of the steps, we need to consider more than one reordering of the original game. We illustrate this with an example motivated by the opportunism problem of a supplier that makes secret offers to two downstream retailers (see also Section 5.6). Consider a game with one sender and two receivers. The sender chooses two actions $a_1 \in A_1$ and $a_2 \in A_2$ either simultaneously or sequentially. Then receivers, R_1 and R_2, choose $b_{R_i} \in B_{R_i}$ $(i \in \{1, 2\})$ simultaneously, having observed *partially* the sender's action according to the following information partitions:

$$\begin{aligned} \text{Receiver 1:} \quad P_1 &= \{\{a_1\} \times A_2 : a_1 \in A_1\}, \\ \text{Receiver 2:} \quad P_2 &= \{A_1 \times \{a_2\} : a_2 \in A_2\}. \end{aligned}$$

That is, receiver 1 only observes a_1 and receiver 2 only observes a_2. In this case, we cannot find a coarsest non-singleton information partition in step 1 above because P_1 and P_2 cannot be ordered according to the partial order and the dummy partition would be a singleton. We consider the following two reordered games (only one of them is the reordered game if the sender chooses the two actions sequentially in the original game). One reordering of the original game is as follows:

1. In stage 1, the sender chooses $a_1 \in A_1$.
2. In stage 2, the sender chooses $a_2 \in A_2$.
3. In stage 3, the receivers choose $b_{R_i} \in B_{R_i}$ $(i \in \{1, 2\})$ simultaneously.

The other reordering of the original game is as follows:

1. In stage 1, the sender chooses $a_2 \in A_2$.
2. In stage 2, the sender chooses $a_1 \in A_1$.
3. In stage 3, the receivers choose $b_{R_i} \in B_{R_i}$ ($i \in \{1,2\}$) simultaneously.

With these two reordered games, we can find our *RI-equilibria* as follows. Among the PBE of the original game, we select only those which share the same outcome (up to the reordering) and the same receiver's beliefs and strategy with the PBE of both reordered games. As before, we refer to these equilibria as *RI-equilibria*. More specifically, if the sender chooses a_1 and a_2 simultaneously in the original game, the strategy profile $(\tilde{a}_1, \tilde{a}_2, \tilde{b}_{R_1}(\tilde{a}_1), \tilde{b}_{R_2}(\tilde{a}_2))$ is a *RI-equilibrium* if

$$\begin{aligned}
\tilde{a}_1 &\in \arg\max_{a_1 \in A_1} \pi_S(a_1, \tilde{a}_2(a_1), \tilde{b}_{R_1}(a_1), \tilde{b}_{R_2}(\tilde{a}_2(a_1))), \\
\tilde{a}_2 &\in \arg\max_{a_2 \in A_2} \pi_S(\tilde{a}_1(a_2), a_2, \tilde{b}_{R_1}(\tilde{a}_1(a_2)), \tilde{b}_{R_2}(a_2)), \\
\tilde{a}_2(a_1) &\in \arg\max_{a_2 \in A_2} \pi_S(a_1, a_2, \tilde{b}_{R_1}(a_1), \tilde{b}_{R_2}(a_2)) \quad \text{for each } a_1 \in A_1, \\
\tilde{b}_{R_1}(a_1) &\in \arg\max_{b_{R_1} \in B_{R_1}} \pi_{R_1}(a_1, \tilde{a}_2(a_1), b_{R_1}, \tilde{b}_{R_2}(\tilde{a}_2(a_1))) \quad \text{for each } a_1 \in A_1, \\
\tilde{a}_1(a_2) &\in \arg\max_{a_1 \in A_1} \pi_S(a_1, a_2, \tilde{b}_{R_1}(a_1), \tilde{b}_{R_2}(a_2)) \quad \text{for each } a_2 \in A_2, \quad \text{and} \\
\tilde{b}_{R_2}(a_2) &\in \arg\max_{b_{R_2} \in B_{R_2}} \pi_{R_2}(\tilde{a}_1(a_2), a_2, \tilde{b}_{R_1}(\tilde{a}_1(a_2)), b_{R_2}) \quad \text{for each } a_2 \in A_2.
\end{aligned}$$

If the sender chooses a_1 and a_2 sequentially in the original game, e.g. a_1 then a_2, the strategy profile $(\tilde{a}_1, \tilde{a}_2(a_1), \tilde{b}_{R_1}(\tilde{a}_1), \tilde{b}_{R_2}(\tilde{a}_2))$ is a *RI-equilibrium* if

$$\begin{aligned}
\tilde{a}_1 &\in \arg\max_{a_1 \in A_1} \pi_S(a_1, \tilde{a}_2(a_1), \tilde{b}_{R_1}(a_1), \tilde{b}_{R_2}(\tilde{a}_2(a_1))), \\
\tilde{a}_2(a_1) &\in \arg\max_{a_2 \in A_2} \pi_S(a_1, a_2, \tilde{b}_{R_1}(a_1), \tilde{b}_{R_2}(a_2)) \quad \text{for each } a_1 \in A_1, \\
\tilde{b}_{R_1}(a_1) &\in \arg\max_{b_{R_1} \in B_{R_1}} \pi_{R_1}(a_1, \tilde{a}_2(a_1), b_{R_1}, \tilde{b}_{R_2}(\tilde{a}_2(a_1))) \quad \text{for each } a_1 \in A_1, \\
\tilde{a}_1(a_2) &\in \arg\max_{a_1 \in A_1} \pi_S(a_1, a_2, \tilde{b}_{R_1}(a_1), \tilde{b}_{R_2}(a_2)) \quad \text{for each } a_2 \in A_2, \quad \text{and} \\
\tilde{b}_{R_2}(a_2) &\in \arg\max_{b_{R_2} \in B_{R_2}} \pi_{R_2}(\tilde{a}_1(a_2), a_2, \tilde{b}_{R_1}(\tilde{a}_1(a_2)), b_{R_2}) \quad \text{for each } a_2 \in A_2.
\end{aligned}$$

5.6 Opportunism and beliefs

One can apply our framework to the opportunism problem of bilateral contracting, where an upstream monopolist supplier behaves opportunistically when it can make secret offers to competing downstream firms (Hart and Tirole, 1990 and McAfee and Schwartz, 1994). Such opportunism prevents the upstream supplier from fully exerting its market power. This motivates the use of exclusive deals, RPM or vertical integration to restore monopoly profits.

A key issue in this literature is how downstream firms react to unexpected (out-of-equilibrium) offers. McAfee and Schwartz propose three types of "beliefs" (i) symmetry beliefs, in which each downstream firm believes the rival has received the same offer as itself, (ii) passive beliefs, in which each downstream firm continues to believe their rival receives its equilibrium offer, and (iii) wary beliefs, in which each downstream firm assumes their rival receives the offer which is optimal for the upstream supplier given their own observed offer. McAfee and Schwartz show that symmetry and passive beliefs have quite different implications, with opportunism not arising under symmetry beliefs. Rey and Vergé (2004) show passive beliefs and wary beliefs also have different implications, once they allow for price competition, or quantity competition but with interim observability.

In general, in case the receivers have different information partitions which cannot be linearly ordered, such as in this example, then we may need to consider more than one reordering of the original game for our *RI-equilibrium*. Specifically, in the case in which there are two downstream firms who only observe their own offers, in one reordering the monopolist offers to retailer 1 first and then to retailer 2, while in the other reordering it offers to retailer 2 first and then to retailer 1. Then among the equilibria of the original game, we select only those which are consistent with the equilibria of both reordered games. Section 5.5 provides the conditions for the *RI-equilibrium*.

Using this approach, we can show that symmetry beliefs and passive beliefs are not generally consistent with our *RI-equilibrium*, whereas wary beliefs are. For example, consider the equilibrium under symmetry beliefs described by McAfee and Schwartz for the case of quantity competition with interim unobservability. In the equilibrium, the monopolist offers two-part tariffs to each retailer which are the same as it would offer under publicly observed offers, thereby obtaining monopoly profits. Call these the "monopoly tariffs". Now consider the reordered game in which the monopolist offers to retailer 1 first. Suppose retailer 1 were offered a slightly lower wholesale price together with a slightly higher fixed fee such that it would be better off accepting the offer when retailer 2 receives the equilibrium offer (i.e. the monopoly tariff) but would be worse off accepting the offer when retailer 2 receives the same off-the-equilibrium offer. In this case the proposed equilibrium prescribes retailer 1 to reject the off-the-equilibrium offer (due to symmetry beliefs) but one can see that accepting this offer is a profitable deviation because the monopolist will still find it optimal to offer the monopoly tariff to retailer 2. Furthermore, given retailer 1 would accept the offer, there always exists such a deviation by the monopolist that also increases its profit, at the expense of retailer 2.[21]

6 Concluding remarks

This paper considered a setting which is widely applicable: a signaling-like environment in which the sender rather than nature chooses its unobservable features. We called these endogenous signaling games. Surprisingly, despite the plethora of potential applications, endogenous signaling games have not been systematically explored.

We proposed an approach to find reasonable equilibria of endogenous signaling games, which involves reordering the moves of the senders in a certain way and solving the reordered game. As well as having a strong game theoretic rationale, our approach (which we called *Reordering Invariance*) is relatively easy to use in applied problems. In particular, it avoids solving directly for the set of equilibria of the original game, which can be troublesome in the settings we consider.

In addition to analyzing some of the key economic properties of canonical endogenous signaling games, the paper offered a guide for how to correctly do the required reordering of moves in a wide range of more complicated situations. These included settings in which the sender can choose which information to make observable (e.g. through advertising) and where different receivers have access to different information (e.g. in the case each is a retailer receiving a private offer from a manufacturer). In the latter case, we showed that applying our framework to the opportunism problem of vertical contracts allows one to settle the issue of how retailers' beliefs should be formed. It implies only wary beliefs (and not passive or symmetric beliefs) should be used.

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[21] Similar arguments can be made to rule out passive beliefs when there is price competition, or quantity competition but with interim observability.

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APPENDIX

A Proofs

Proof of Proposition 1. Suppose an outcome (a^o, t^o, b^o) is supported by a SPE of G_r. It is also a Nash equilibrium of G_r by definition. Then the outcome (t^o, a^o, b^o) is supported by a Nash equilibrium of G_o since G_o and G_r share the same reduced normal form. As the game G_o has no proper subgame, the outcome (t^o, a^o, b^o) is also supported by a SPE of G_o. ■

Proof of Proposition 2. Consider a subgame of G_r following the sender's choice of $a \in A$. In the normal form of this subgame we have payoff functions $\pi_S(t, a, b)$ and $\pi_R(t, a, b)$ for any fixed $a \in A$. Since the sets T and B are compact convex subsets of a Euclidean space and the payoff functions π_S and π_R are continuous and quasi-concave in t and b respectively, there exists a pure-strategy Nash equilibrium in the normal-form game. Consider the pure-strategy Nash equilibrium correspondence $N : A \to T \times B$. This correspondence is upper-hemi continuous, and π'_S such that $\pi'_S(a) \equiv \pi_S(t^N(a), a, b^N(a))$ is also upper-hemi continuous. Since A is a compact convex subset of a Euclidean space, a selection of π'_S can be chosen so that $\max_{a \in A} \pi'_S(a)$ exists. ■

Proof of Proposition 3. Suppose that $(t^o, a^o(t), b^o(a))$ is a SPE in G_o. Then

$$(t^o, a^o(t)) \in \arg \max_{(t,a(t)) \in T \times A^T} \pi_S(t, a(t), b^o(a(t))) \quad \text{and} \tag{2}$$

$$b^o \in \arg \max_{b \in B} \pi_R(t^o, a^o, b). \tag{3}$$

The inclusion (2) implies that there exists a certain function $t^{o*} : A \to T$ such that $t^{o*}(a^o) = t^o$ and

$$(t^{o*}(a), a^o) \in \arg \max_{(t(a),a) \in T^A \times A} \pi_S(t(a), a, b^o(a)). \tag{4}$$

The inclusion (4) implies

$$t^{o*}(a^o) \in \arg\max_{t \in T} \pi_S(t, a^o, b^o). \tag{5}$$

The inclusion (3) can be rewritten as

$$b^o \in \arg\max_{b \in B} \pi_R(t^{o*}(a^o), a^o, b). \tag{6}$$

From the inclusions (5) and (6) we know that $(t^{o*}(a^o), b^o)$ is a Nash equilibrium in the proper subgame of G_r following a^o.

Among the *RI-equilibrium* (SPE) in G_o, choose $(\tilde{t}, \tilde{a}(t), \tilde{b}(a))$ which is outcome-equivalent to a SPE in G_r, $(a^*, t^*(a), b^*(a))$ such that $t^*(a^o) = t^{o*}(a^o)$ and $b^*(a^o) = b^o$. Clearly the payoff to the sender at this SPE cannot be lower than the payoff to the sender at the Nash equilibrium in the proper subgame of G_r following a^o. Since the set of *RI-equilibrium* payoffs to the sender admits a maximal element, there exists at least one *RI-equilibrium* that yields the best payoff to the sender among all the SPE.

Now consider a *RI-equilibrium* (SPE) in G_o that yields the best payoff to the sender among all the SPE. We denote it again by $(\tilde{t}, \tilde{a}(t), \tilde{b}(a))$, which is outcome-equivalent to a SPE in G_r, $(a^*, t^*(a), b^*(a))$. Suppose on the contrary that a SPE in G_o that is not outcome-equivalent to a *RI-equilibrium* (we denote it by $(t^o, a^o(t), b^o(a))$) yields the same payoff as this best payoff to the sender. Then the strategy profile $(a^o, t^{**}(a), b^{**}(a))$ is a SPE in G_r and outcome-equivalent to $(t^o, a^o(t), b^o(a))$, where

$$t^{**}(a) = \begin{cases} t^o & \text{for } a = a^o \\ t^*(a) & \text{for } a \neq a^o, \end{cases} \quad \text{and}$$

$$b^{**}(a) = \begin{cases} b^o & \text{for } a = a^o \\ b^*(a) & \text{for } a \neq a^o. \end{cases}$$

This means that the strategy profile $(t^o, a^o(t), b^o(a))$ is also outcome-equivalent to a *RI-equilibrium* (SPE) in G_o and we obtain the contradiction. ■

Proof of Proposition 4. Given $\arg\max_{b \in B} \pi_R(t, a, b)$ is strongly increasing in t for each $a \in A$, if a is such that

$$\left\{ \begin{array}{l} t^*(a) < t^{pa} \\ t^*(a) = t^{pa} \\ t^*(a) > t^{pa} \end{array} \right\} \text{ then } \left\{ \begin{array}{l} \arg\max_{b \in B} \pi_R(t^*(a), a, b) \overset{\text{s}}{\leq} \arg\max_{b \in B} \pi_R(t^{pa}, a, b) \\ \arg\max_{b \in B} \pi_R(t^*(a), a, b) = \arg\max_{b \in B} \pi_R(t^{pa}, a, b) \\ \arg\max_{b \in B} \pi_R(t^*(a), a, b) \overset{\text{s}}{\geq} \arg\max_{b \in B} \pi_R(t^{pa}, a, b) \end{array} \right\}.$$

Since $\tilde{b}(a)$ is unique for each $a \in A$, $\arg\max_{b \in B} \pi_R(t^*(a), a, b)$ is a singleton. Therefore, $\tilde{b}(a) \leq b^{pa}(a)$, $\tilde{b}(a) = b^{pa}(a)$, and $\tilde{b}(a) \geq b^{pa}(a)$ respectively. Since π_S is increasing in b, this implies that

$$\text{if } a \text{ is such that } \left\{ \begin{array}{l} t^*(a) < t^{pa} \\ t^*(a) = t^{pa} \\ t^*(a) > t^{pa} \end{array} \right\} \text{ then } \left\{ \begin{array}{l} \pi_S(t, a, \tilde{b}(a)) \leq \pi_S(t, a, b^{pa}(a)) \\ \pi_S(t, a, \tilde{b}(a)) = \pi_S(t, a, b^{pa}(a)) \\ \pi_S(t, a, \tilde{b}(a)) \geq \pi_S(t, a, b^{pa}(a)) \end{array} \right\},$$

for each $t \in T$. Note that $t^*(a^{pa}) = t^{pa}$ (the receiver's belief should be correct on the equilibrium path). Since t^* is increasing in a,

$$\text{if } \left\{ \begin{array}{c} a < a^{pa} \\ a = a^{pa} \\ a > a^{pa} \end{array} \right\} \text{ then } \left\{ \begin{array}{c} \pi_S(t, a, \tilde{b}(a)) \leq \pi_S(t, a, b^{pa}(a)) \\ \pi_S(t, a, \tilde{b}(a)) = \pi_S(t, a, b^{pa}(a)) \\ \pi_S(t, a, \tilde{b}(a)) \geq \pi_S(t, a, b^{pa}(a)) \end{array} \right\} \text{ for each } t \in T. \tag{7}$$

Note that $\pi_S(t^{pa}, a^{pa}, b^{pa}) = \max_{(t,a(t)) \in T \times A^T} \pi_S(t, a(t), b^{pa}(a(t)))$. As there exists a unique *RI-equilibrium* outcome, $\tilde{a} \geq a^{pa}$ and $\tilde{t} \geq t^{pa}$. The inequality $\pi_S(\tilde{t}, \tilde{a}, \tilde{b}) \geq \pi_S(t^{pa}, a^{pa}, b^{pa})$ follows directly from Proposition 3. ■

Proof of Corollary 2. Let a^o satisfy $\pi_S(t^{pa}, a^o, b^*(a^o)) > \pi_S(t^{pa}, a^{pa}, b^{pa})$. If $a^o \leq a^{pa}$, then from (7) in the proof of Proposition 4 we know that $\pi_S(t^{pa}, a^o, b^*(a^o)) \leq \pi_S(t^{pa}, a^o, b^{pa}(a^o)) \leq \pi_S(t^{pa}, a^{pa}, b^{pa})$, which contradicts the inequality assumed. Therefore, $a^o > a^{pa}$.

By the definition of the equilibrium in the reordered game, we have

$$\pi_S(t^*(a^*), a^*, b^*(a^*)) \geq \pi_S(t^*(a^o), a^o, b^*(a^o)) \geq \pi_S(t^{pa}, a^o, b^*(a^o)).$$

Combining this with the inequality assumed, we obtain $\pi_S(t^*(a^*), a^*, b^*(a^*)) > \pi_S(t^{pa}, a^{pa}, b^{pa}) = \pi_S(t^*(a^{pa}), a^{pa}, b^*(a^{pa}))$, which implies $a^* \neq a^{pa}$. Therefore, $\pi_S(\tilde{t}, \tilde{a}, \tilde{b}) > \pi_S(t^{pa}, a^{pa}, b^{pa})$, $\tilde{a} > a^{pa}$ (from Proposition 4), and $\tilde{t} > t^{pa}$ (since t^* is increasing in a, and strictly increasing at a^{pa}). ■

Sufficient conditions for additional assumptions of Corollary 2. At the passive-belief equilibrium outcome,

$$\begin{aligned} \frac{d\pi_S(t, a, b^*(a))}{da} &= \frac{d\pi_S(t, a, b^{pa}(a))}{da} + \frac{\partial \pi_S(t, a, b)}{\partial b} \left(\frac{db^*(a)}{da} - \frac{db^{pa}(a)}{da} \right) \\ &= \frac{\partial \pi_S(t, a, b)}{\partial b} \left(- \frac{\frac{\partial^2 \pi_R(t,a,b)}{\partial b \partial t}}{\frac{\partial^2 \pi_R(t,a,b)}{(\partial b)^2}} \cdot \frac{dt^*(a)}{da} \right), \end{aligned}$$

where the second equality follows from the definition of the passive-belief equilibrium ($\frac{d\pi_S(t,a,b^{pa}(a))}{da} = 0$) and the total differentiation of the relevant first-order conditions for the receiver's equilibrium choices of b. By assumption, $\frac{\partial \pi_S(t,a,b)}{\partial b} > 0$ and $\frac{\partial^2 \pi_R(t,a,b)}{(\partial b)^2} < 0$. From the assumptions of strict supermodularity and strict increasing differences, it follows that $\frac{\partial^2 \pi_R(t,a,b)}{\partial b \partial t} > 0$ and $\frac{dt^*(a)}{da} > 0$. Therefore, $\frac{d\pi_S(t,a,b^*(a))}{da} > 0$. ■

Proof of Proposition 5. Note that $\tilde{b}(a) = b^*(a) \in \arg\max_{b \in B} \pi_R(t^*(a), a, b)$ for each $a \in A$, and $b^{ob}(t, a) \in \arg\max_{b \in B} \pi_R(t, a, b)$ for each $(t, a) \in T \times A$. Since $\arg\max_{b \in B} \pi_R(t, a, b)$ is strongly increasing in t for each $a \in A$ and π_S

is increasing in b, following the same argument as in the proof of Proposition 4,

$$\text{if } a \text{ is such that } \left\{ \begin{array}{c} t^*(a) < t \\ t^*(a) = t \\ t^*(a) > t \end{array} \right\} \text{ then } \left\{ \begin{array}{c} \pi_S(t, a, \tilde{b}(a)) \leq \pi_S(t, a, b^{ob}(t, a)) \\ \pi_S(t, a, \tilde{b}(a)) = \pi_S(t, a, b^{ob}(t, a)) \\ \pi_S(t, a, \tilde{b}(a)) \geq \pi_S(t, a, b^{ob}(t, a)) \end{array} \right\},$$

for each $t \in T$. Note that $t^*(\tilde{a}(\tilde{t})) = \tilde{t}$. Since t^* is increasing in a,

$$\text{if } \left\{ \begin{array}{c} a < \tilde{a}(\tilde{t}) \\ a = \tilde{a}(\tilde{t}) \\ a > \tilde{a}(\tilde{t}) \end{array} \right\} \text{ then } \left\{ \begin{array}{c} \pi_S(\tilde{t}, a, \tilde{b}(a)) \leq \pi_S(\tilde{t}, a, b^{ob}(\tilde{t}, a)) \\ \pi_S(\tilde{t}, a, \tilde{b}(a)) = \pi_S(\tilde{t}, a, b^{ob}(\tilde{t}, a)) \\ \pi_S(\tilde{t}, a, \tilde{b}(a)) \geq \pi_S(\tilde{t}, a, b^{ob}(\tilde{t}, a)) \end{array} \right\}.$$

Note that $\pi_S(\tilde{t}, \tilde{a}(\tilde{t}), \tilde{b}(\tilde{a}(\tilde{t}))) = \max_{a \in A} \pi_S(\tilde{t}, a, \tilde{b}(a))$. As there exists a unique *RI-equilibrium* outcome, $\tilde{a}(\tilde{t}) \geq a^{ob}(\tilde{t})$. ■

Sufficient conditions for additional assumptions of Corollary 3. At the *RI-equilibrium* outcome,

$$\begin{aligned} \frac{d\pi_S(\tilde{t}, a, b^{ob}(\tilde{t}, a))}{da} &= \frac{d\pi_S(\tilde{t}, a, \tilde{b}(a))}{da} + \frac{\partial \pi_S(t, a, b)}{\partial b} \left(\frac{db^{ob}(\tilde{t}, a)}{da} - \frac{d\tilde{b}(a)}{da} \right) \\ &= \frac{\partial \pi_S(t, a, b)}{\partial b} \left(\frac{\frac{\partial^2 \pi_R(t,a,b)}{\partial b \partial t}}{\frac{\partial^2 \pi_R(t,a,b)}{(\partial b)^2}} \cdot \frac{d\tilde{t}(a)}{da} \right), \end{aligned}$$

where the second equality follows from the definition of the *RI-equilibrium* ($\frac{d\pi_S(\tilde{t},a,\tilde{b}(a))}{da} = 0$) and the total differentiation of the relevant first-order conditions for the receiver's equilibrium choices of b following $\tilde{t}$. By assumption, $\frac{\partial \pi_S(t,a,b)}{\partial b} > 0$ and $\frac{\partial^2 \pi_R(t,a,b)}{(\partial b)^2} < 0$. From the assumptions of strict supermodularity and strict increasing differences, it follows that $\frac{\partial^2 \pi_R(t,a,b)}{\partial b \partial t} > 0$ and $\frac{d\tilde{t}(a)}{da} > 0$. Therefore, $\frac{d\pi_S(\tilde{t},a,b^{ob}(\tilde{t},a))}{da} < 0$. ■