Do I Want It All?

A Simple Model of Satiation in Contests

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Abstract

We present a formal analysis of a contest with possible satiation of the contestants in the prize they might obtain. We consider contests, given a ratio form of the contest success function and risk neutral, possibly asymmetric players. After laying out the Nash-equilibria and the solution in a sequential move game, we will extend the analysis so as to endogenize the order of moves. We find that if the players are sufficiently asymmetric, the effort level will be zero in equilibrium, although players' aggregate demand exceeds the value of the prize.

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1 Introduction

The economic analysis of contests is a flourishing field, as can be seen for example from the surveys by Konrad (2007) and Garfinkel and Skaperdas (2007). However, all the literature we are aware of has an underlying assumption of non-satiation of the contestants in the prize they might obtain. Though natural in many cases, this assumption is not always reasonable.

A contestant might not be in the position to exhaust all the benefits of the prize. Capacity constrained firms do not want a larger market than they can serve, at least not in the short run.[1] Therefore, for example, the relative advertising effort of a firm, also may reflect the ability to serve the underlying market.

More generally, if we look at the way human beings set objectives, we do not see unconditional "the more the better". We are happy when successful, and we define success exactly in relation to a benchmark level of performance. In other words, not achieving a goal (certain level of performance) makes us disappointed and unhappy, whereas achieving it with some "extra" does not make us happier.

In psychology literature this phenomenon is well recognized as a notion of aspiration level (see e.g. Heath et al. (1998)). In decision making theory the tradition goes back to Simon (1959) claiming that people tend to find a satisfactory alternative rather than a utility maximizing one. According to Payne et al. (1980), what is called a reference point in prospect theory is a special case of aspiration level.

Examples of aspiration level relevance abound also in economic literature. To mention a number of recent studies, Karlsson et al. (2004) study empirically its effect on household consumption. Huck et al. (2007) in an experimental study explain the post-merger behavior by aspiration levels. They also help explaining another experimental outcome by Huck and Wallace (2002), whereby Cournot and Stackelberg competition are studied. Page et al. (2007) show that the aspiration levels help to explain the educational choices in an experiment.

The examples considered strongly suggest that the payoff functions of the contestants must be kinked at some satiation (threshold, reference) points. For simplicity and illustration, we take this notion to extreme and assume in our model that anything above such a reference point does not bring additional utility. Thus, the payoff function has a local satiation property.

Turning back to contests, one of the main concerns in the literature is: How much effort is wasted in the process of winning the prize? This topic is highly related to question whether players' are assumed to move in a simultaneous or sequential manner and the type of the assumed contest success function (CSF). One of the commonly used CSF is the so called ratio-form CSF. Here, the share of the prize that a player may win is equal to his *relative* effort.

Given this CSF, Dixit (1987) found that in the case of perfect symmetry between all players, no player has strategic incentive to commit herself to an effort level which

[1] See, for example Levitan and Shubik (1972).

is unequal to the level of effort in the simultaneous move game.[2] Therefore, any precommitment will not alter the strategic position of the players' and, thus, this will also not change the payoffs in the subgame perfect equilibrium of the game (Baik et al. (1999)).

This changes, once we allow for unevenly matched players. Here, according to Dixit (1987), the favourite will always overcommit effort compared to the simultaneous move game, where the favourite is the player whose winning probability (or share of the prize) in the Nash equilibrium exceeds $\frac{1}{2}$. Her contestant (the underdog), on the other hand, has the incentive to undercommit effort, if she has the chance to commit herself.[3] As it turns out, the order of moves has a direct impact, not only on the effort of each single player, but also on the sum of effort exerted in pursuit of the prize. Thus, strategic behavior on the side of the favourite *in*creases the social costs, strategic behavior on the side of the underdog *de*creases the social costs, compared to the Nash-equilibrium.

Assuming that players can simultaneously and independently decide - in a stage zero - whether they want to exert effort in stage one or two, Baik and Shogren (1992) showed that the favourite will never, and the underdog will always move first.[4] Hence the only subgame perfect equilibrium of the game shows commitment by the weaker player. The reason for this is that - given the ratio form of the CSF - the strategic variable of the favourite is a strategic substitute for the underdog, or, in other words, the best response function of the underdog is downward sloping in the Nash-equilibrium of the simultaneous move game. The opposite is true for the favourite: His best response function is upward sloping in the Nash-equilibrium of the simultaneous move game, which indicates, that the underdog's effort is a strategic complement for the favourite.[5]

The restriction to exert effort once and forall was dropped by Yildirim (2005). Assuming that players can decide whether they like to invest in effort in multiple periods or not, he showed that, in a contest environment similar to Baik and Shogren (1992), the Stackelberg scenario, where the underdog leads and the favourite follows never occurs as a subgame perfect equilibrium of the game.[6]

The paper presented here is closely related to Baik and Shogren (1992). We also assume that players can exert effort once and forall. In our basic setting, two risk neutral players act in a simultaneous move Tullock contest over a prize of value R.

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[2]In fact, Dixit explained this phenomena for a more general version of a CSF. Baye and Shin (1999) showed that the incentive to commit in a symmetric contest depends on the magnitude of the third derivative of the contest success function. See also Dixit (1999).

[3]Similar results can be found in Linster (1993), who also allows for incomplete information with respect to the type of the second mover. Glazer and Hassin (2000) considers an n player sequential contest.

[4]The same result can be found in Leiniger (1993), who assumes a less general CSF (see Nitzan (1994) for a proof).

[5]For a general treatment of this issue see Thompson and Faith (1981), Gal-or (1985) and Bulow et al. (1985).

[6]See also Romano and Yildirim (2005) for more general treatment.

Unlike the lottery-like contest model of Tullock (1980) the agents will win a share of the whole prize.[7] This share may be equal to the whole prize, which resembles a winning probability of 1 in the imperfectly discriminating contest, or it may be less. The important fact is that the players' can only benefit from a share of the prize up to their exogenously given demand. Everything above it can not be consumed by the agent and therefore has no additional value, so that players marginal willingness to pay drops to zero. At first we find and characterize the Nash equilibrium of the game. After laying out the findings of the simultaneous move game, we, analogous to Dixit (1987), examine how the equilibrium effort and payoffs will change in a sequential move game. Following this, we, analogous to Baik and Shogren (1992), allow the players to decide about the timing of their once and forall decision with respect to investment in effort.

2 The Model

2.1 Setup

There are 2 contestants competing for a divisible prize. A contestant $i = 1, 2$ exerts costly effort and enjoys a part of the prize R up to the amount $z_i\, R$, where $z_i \in (0, 1]$. There is free-disposal of the prize, so that any share of the prize obtained by the player creates the same gross payoff, as long as this share is equal to or larger than z_i. There is no information imperfection, and the contestants obtain a part of the prize $p_i\,(e_i, e_j)\,R$ in case she makes an effort e_i and her combatant j makes an effort e_j, with $i, j = 1, 2$ and $i \neq j$. Given the linear cost function of agent i, $C_i(e_i) = k_i\, e_i$, the objective function of each contestant becomes

$$u_i(e_i, e_j) = \min\left\{p_i(e_i, e_j),\ z_i\right\}\ R - k_i\, e_i. \tag{1}$$

Accordingly, we assume the following variant of the Tullock contest success function:

$$p_i(e_i, e_j) = \begin{cases} \frac{z_i}{z_i + z_j} & \text{for } e_i = e_j = 0, \\ \frac{e_i}{e_i + e_j} & \text{else.} \end{cases} \tag{2}$$

Thus, if $z_i < 1 - z_j$, there is no need to invest any effort for agent i, since $p_i(e_i = e_j = 0) \geq z_i$ for $i, j = 1, 2$. The corresponding best response function for agent i becomes

$$BR_i(e_j) = \begin{cases} \frac{z_i}{1 - z_i}\, e_j, & \text{for } e_j \in \left(0, \frac{R}{k_i}\ (1 - z_i)^2\right]; \\ \sqrt{\frac{e_j R}{k_i}} - e_j & \text{for } e_j \in \left(\frac{R}{k_i}\ (1 - z_i)^2\,,\ \frac{R}{k_i}\right); \\ 0 & \text{for } e_j \geq \frac{R}{k_i}. \end{cases} \tag{3}$$

[7] Given the risk neutrality of the players, this assumption has no impact on the incentive to exert effort in the contest, compared to the lottery case.

As long as the effort of player j is not too high ($e_j \leq \frac{R}{k}(1-z_1)^2$) agent 1 is willing to invest as much effort as is necessary to guarantee him a share which is equal to his demand, since in this case $p_i(BR_i(e_j), e_j) = z_i$. For higher values of e_j ($e_j \geq \frac{R}{k_i}(1-z_1)^2$) this is no longer possible. Here, agent i's share will be smaller than his demand ($p_i(BR_i(e_j), e_j) < z_i$). It is worth noting that the threshold value of e_j between these two scenarios, decreases in the marginal costs (k_i) and in the demand (z_i) of agent i. Thus, the higher the demand of agent i and the less efficient agent i, the lower the value of the other agent's effort (e_j) which yields $p_i(e_i, e_j) < z_i$. For $e_j \geq \frac{R}{k_i}$ agent i's effort will be even zero, so that his share will be equal to what agent j will left over ($p_i(0, e_j) = 1 - z_j$).
From now on we will assume that $k_1 = k\, k_2 = k$, so that k represents the relative efficiency of player 2. Without loss of generality, we can take a look at figure 1, which represents the best response function of player 1 contingent on various levels of k and z_1. $BR_1(\bar{z}_1, \bar{k})$ and $BR_1(\underline{z_1}, \bar{k})$, for example, represent two best response functions with unique marginal costs and different demand ($\bar{z}_1 > \underline{z_1}$). Comparing $BR_1(\bar{z}_1, \bar{k})$ and $BR_1(\underline{z_1}, \bar{k})$ shows that a greater demand (and unique marginal costs) turns the linear part of the best response function outwards: Only small values of e_2 can guarantee player 1 a share which is as great as his demand. For $z_1 \to 1$ the reaction function gets the typical hump-shaped curve for a best response function in a Tullock-contest.[8]
Comparing $BR_1(\underline{z_1}, \bar{k})$ and $BR_1(\underline{z_1}, \underline{k})$, shows the difference between two reaction functions of unique demand but different efficiency ($\bar{k} > \underline{k}$). The more efficient player 1 (the lower k) the higher the necessary effort of player 2 which leads to $p_1(e_1, e_2) < z_1$, thus the linear part of $BR_1(\cdot)$ extends. Furthermore, the more efficient agent 1 the higher the effort of player 2 which leads to zero effort of player 1.
We will now turn to the analysis of the simultaneous move game.

2.2 Simultaneous game

Consider a simple Tullock formulation outlined above with two players and different cost functions, $c_1(e) = k\, c_2(e) = k$. Introducing the threshold values $\bar{z}_1 \equiv \frac{1}{1+k}$ and $\bar{z}_2 \equiv \frac{k}{1+k}$, we will now describe the possible equilibria in a simultaneous move game, contingent on the relative efficiency (k) and demand parameters (z_1 and z_2).

Proposition 1 (*Equilibria in the Simultaneous Move Game*)
In a two player simultaneous Tullock contest with satiation, there exists a unique pure strategy equilibrium, which can be characterized by the following strategy profiles (e_1^*, e_2^*):

(i) $(0,0)$ for $(z_1, z_2) \in \mathcal{A} = \{z_1, z_2 \,|\, z_1 + z_2 \leq 1\}$. Clearly, $p_1^* \equiv p_1(e_1^*, e_2^*) \geq z_1$ and $p_2^* \equiv p_2(e_1^*, e_2^*) \geq z_2$. The payoffs are thus $u_1^* \equiv u_1(e_1^*, e_2^*) = z_1 R$ and $u_2^* \equiv u_2(e_1^*, e_2^*) = z_2 R$.

[8] See, for example, Grossman and Kim (1995).

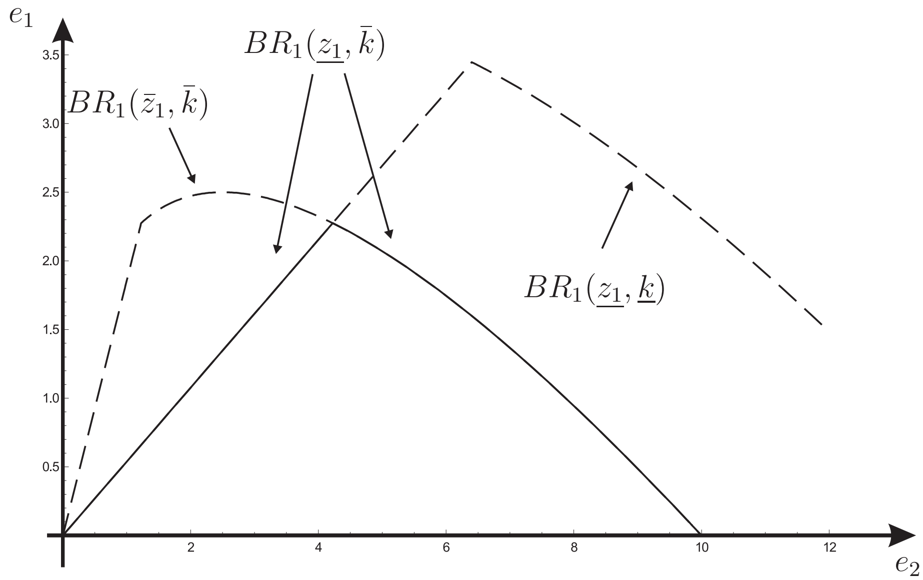


Figure 1: BR_2 for various level of k and z_2, with $\underline{k} < \hat{k} < \overline{k}$ and $\hat{z}_1 > \underline{z_1}$.

(ii) $\left(\frac{R}{(1+k)^2}, \frac{k\,R}{(1+k)^2}\right)$, for $(z_1, z_2) \in \mathcal{B} = \{z_1, z_2 \,|\, z_1 > \bar{z}_1, z_2 > \bar{z}_2\}$. In this case agent 1's and agent 2's demand is larger than their equilibrium share: $z_1 > p_1^*$ and $z_2 > p_2^*$. The resulting payoffs are $u_1^* = \frac{R}{(1+k)^2}$ and $u_2^* = \left(\frac{k}{1+k}\right)^2 R$.

(iii) $\left(z_2\,(1-z_2)\,\frac{R}{k}, z_2^2\,\frac{R}{k}\right)$, for $(z_1, z_2) \in \mathcal{C} = \{z_1, z_2 \,|\, z_1 \geq \bar{z}_1, z_2 < \bar{z}_2, z_1 + z_2 > 1\}$. Here, $p_2^* = z_2$ and $p_1^* = 1 - z_2 < z_1$. The payoffs are $u_1^* = (1-z_2)^2\,R$ and $u_2^* = (k - z_2)z_2\,\frac{R}{k}$.

(iv) $(z_1^2\,R, z_1(1-z_1)\,R)$, for $(z_1, z_2) \in \mathcal{D} = \{z_1, z_2 \,|\, z_1 < \bar{z}_1, z_2 \geq \bar{z}_2, z_1 + z_2 > 1\}$. Here, $p_1^* = z_1$ and $p_2^* = 1 - z_1 < z_2$, and the resulting payoffs are $u_1^* = z_1(1 - k\,z_1)R$ and $u_2^* = (1-z_1)^2\,R$.

Proof. The proof can be found in an appendix. ■

For description of these equilibria we will use figure 2 which presents the possible equilibria contingent on the level of demand of both agents (z_1 and z_2). Without loss of generality, we will assume that $k < 1$.

The trivial case is when $(z_1, z_2) \in \mathcal{A}$, since there is no reason to exert any effort for both agents. Thus, the payoff of both agents in equilibrium is equal to each agent's demand times the value of the resource ($u_i^* = z_i\,R$).

If the demand of both agents is higher than the threshold value $\bar{z}_i$, then both will exert an effort which is equal to the one obtained in the standard Tullock-contest. The important insight for us is that the scope for applicability of the standard theory depends on the relative efficiency of the firms. Namely, the threshold values after which $p_i(e_i, e_j) < z_i$ in equilibrium for both agents, increase in the relative efficiency of the players ($\frac{d\bar{z}_1}{dk} < 0, \frac{d\bar{z}_2}{dk} > 0$). Note, that the payoffs of agents' in equilibrium do not depend on the demand of the players, as long as $(z_1, z_2) \in \mathcal{B}$. Moreover, the

more efficient firm gets higher payoff, i.e. $u_1^* > u_2^*$, if and only if $k < 1$, which goes in line with the argument of Dixit (1987). Further, the payoffs of both agents in equilibrium are increasing in the players' relative efficiency ($\frac{du_1}{dk} < 0, \frac{du_2}{dk} > 0$).
For the third and forth case we will make use of the parameter $\chi \equiv \frac{z_2}{z_1}$, which represents the relative demand of player 2. In the third case, when $(z_1, z_2) \in \mathcal{C}$, agent 2's relative demand is small compared to his relative efficiency ($\chi < k$) and agent 1's demand is equal to or above the threshold value ($z_1 \geq \bar{z}_1$). Hence, agent 2 is able to guarantee himself a share of $p_2^* = z_2$, while agent 1 will get the rest of the resource. The opposite occurs in the forth case, when $(z_1, z_2) \in \mathcal{D}$. Here, agent 2's (relative) demand is higher than his (relative) efficiency ($\chi > k$) and $z_2 \geq \bar{z}_2$. In this case agent 1 will gain a share in equilibrium that is equal to his demand, i.e., $p_1^* = z_1$ and agent 2 will get the rest, which is smaller than his demand, $p_2 = 1 - z_1 < z_2$. Notice that the payoff of the satisfied player increases in her relative efficiency, i.e.

$$\frac{d\,u_1}{d\,k} < 0, \text{ for } (z_1, z_2) \in \mathcal{D} \text{ and } \frac{d\,u_2}{d\,k} > 0 \text{ for } (z_1, z_2) \in \mathcal{C}. \tag{4}$$

The reason for this is that an increase in the relative efficiency of the unsatisfied player, does not alter the equilibrium share of both player in the third and forth scenario, since the relative effort stays unchanged. But increasing relative efficiency reduces the costs of effort for the satisfied agent. On the other hand, the relative efficiency plays no role for the payoff of the non-satisfied player

$$\frac{d\,u_2}{d\,k} = 0, \text{ for } (z_1, z_2) \in \mathcal{D} \text{ and } \frac{d\,u_1}{d\,k} = 0 \text{ for } (z_1, z_2) \in \mathcal{C}. \tag{5}$$

The reason for this is that while the equilibrium share remains unchanged, the increase (decrease) in effort due to the increase (decrease) in the relative efficiency is totally outweight by the decrease (increase) in the costs of effort in equilibrium.

2.2.1 Example: symmetric players

Our satiation property adds an additional source of asymmetry compared to the conventional case. Namely, our contestants may differ not only in their costs or winning technologies, but also in the satiation points. Borrowing an example from IO, two firms possessing the same production technology may have different capacities or historical market shares. Therefore it makes sense to consider the contestants that only differ in their desired shares of the prize, which in terms of our model means $k = 1$.
It can be seen that in this case $\bar{z}_1 = \bar{z}_2 = 1/2$. Proposition 1 then implies that there may be a symmetric equilibrium with both players exerting zero effort ($z_1 + z_2 \leq 1$), or both players choosing $R/4$ ($z_1 > 1/2, z_2 > 1/2$). There may be an asymmetric equilibrium as well, in which the firm i is satiated and exerts effort $z_i^2 R$, and the firm j gets share $1 - z_i$, exerting effort $z_i\,(1 - z_i)\,R$ ($z_i + z_j > 1, z_i < 1/2$).

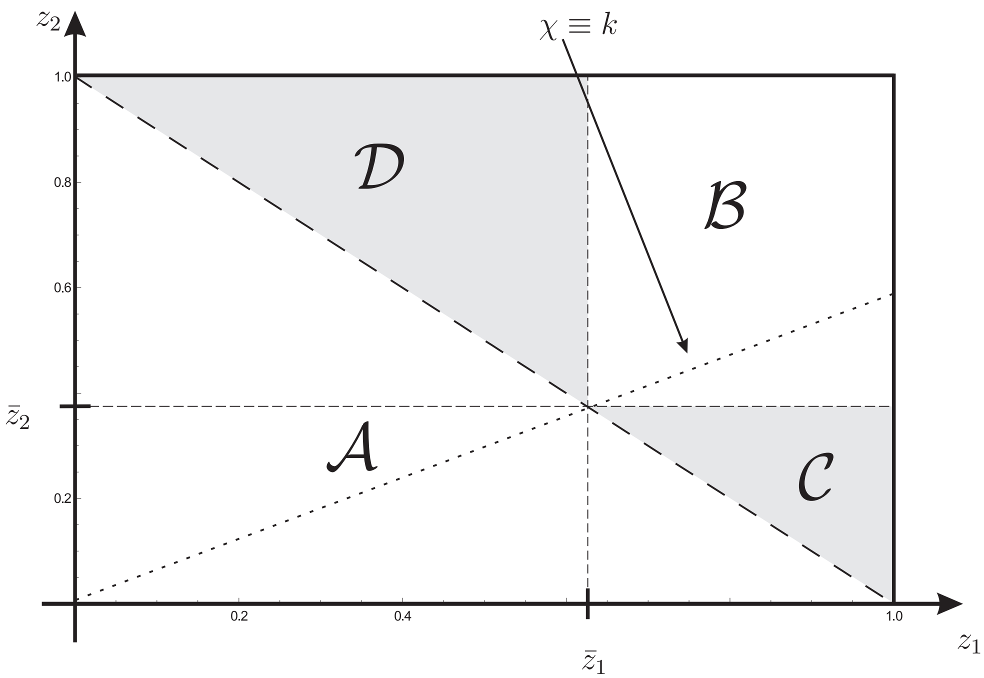


Figure 2: The four different regimes $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and $\mathcal{D}$ for $k < 1$.

We notice that the effort in the asymmetric equilibrium is lower than in the conventional competition for the whole prize:

$$z_i^2 R < z_i (1 - z_i) R < R/4.$$

We can also see that the effort of each contestant in asymmetric equilibrium is increasing in the satiation point of the modest player, $\frac{\partial e_i^*}{\partial z_i} > 0$, $\frac{\partial e_j^*}{\partial z_i} > 0$, $\frac{\partial e_{i,j}^*}{\partial z_j} = 0$, unlike in the conventional case

With symmetric players the intuition is clear-cut: there is a symmetric equilibrium, if both players have small appetite or both players have large appetite. If one of the players has small appetite, and the other big appetite, then the modest player gets what it wants, leaving the more ambitious player with the rest. A contest designer maximizing effort would be better off offering lower prize (and increasing z_i by that). The payoffs in the asymmetric equilibrium are $u_j = (1 - z_i)^2 R, u_i = z_i (1 - z_i) R$, so that the more ambitious guy gets higher payoff. What matters in our formulation is not utility though (the ambitious guy has higher potential utility), but the dissatisfaction level. These are $\tilde{u}_j = \left(z_j - (1 - z_i)^2\right) R, \tilde{u}_i = z_i^2 R$, so that the satiated player is more satisfied whenever it has low enough aspiration level: $2z_i (z_i - 1) < z_j - 1$. Finally, when the two players are identical, $z_1 = z_2 = z$, we only have symmetric equilibria with

$$e_{i,j}^* = \begin{cases} R/4, & z \geq 1/2; \\ 0, & z \leq 1/2. \end{cases}$$

If the desired shares sum up to something smaller than the price available, there will be no effort exerted. We shall see this result persisting through all the contest setups we consider here. What is less intuitive that when they together demand more than the prize available, the effort exerted is exactly the same as they would exert in the case of demanding the whole prize each. In other words, desiring a fraction of prize does not help to reduce the effort - there is no coordination.

From this simple result we already can see that it is not necessarily optimal for a designer to tailor the prize in the way that it just satisfies the need of the winner. Indeed, since in equilibrium higher prize increases the effort $\frac{de^*}{dR} > 0$ even if it is not really demanded by the contestants $z < 0$, it may be beneficial (in terms of getting extra effort) to award "excessive prizes".

2.3 Sequential Game

Dixit (1987) analyzes the strategic behavior with logit and probit contest functions. His results are that (i) the leader-favourite would exert higher effort, (ii) the leader-underdog will exert lower effort (iii) and that with perfect symmetry, none will explore commitment possibility. These findings are grounded in the shape of the best response functions in the simultaneous move game.[9] Thus, if the effort of agent i is a strategic complement for player j (player j's best response function is *in*creasing in e_i in equilibrium) than player j has a second-mover advantage. If the effort of the other player is a strategic substitute for player j (player j's best response function is *de*creasing in e_i in equilibrium) than player j has a first-mover advantage.

Without loss of generality, we will assume that player 1 is the first and player 2 the second mover in the Stackelberg scenario. Given the best response function of agent 2 (cf. equation 3), the payoff function of the first-mover becomes:

$$u_1(e_1) = \begin{cases} (1-z_2)\,R - e_1\,k & \text{for } e_1 \in (0, R(1-z_2)^2]\,, \\ \sqrt{e_1\,R} - k\,e_1 & \text{for } e_1 \in (R(1-z_2)^2, R\,z_1^2)\,, \\ z_1\,R - k\,e_1 & \text{for } e_1 \geq z_1^2\,R. \end{cases} \tag{6}$$

Notice that the second interval is only not non-empty if $\sum_i z_i \leq 1$. Hence, if the sum of demands exceed 1, there will be three non-degenerate intervals of the rival's effort. Thus, four different solutions are possible: Three corner and one interior solution. The interior solution is $\tilde{e}_1 = \frac{R}{4\,k^2}$, where

$$\tilde{e}_1 = \arg\max_{e_1} \sqrt{e_1\,R} - k\,e_1.$$

[9]The reader is referred to the seminal contributions of Bulow et al. (1985) and Gal-or (1985).

Including the payoffs of the three corner solutions we get:

$$u_1\left(e_1=\frac{R}{4\,k^2}\right)=\frac{R}{4\,k} \tag{7a}$$

$$u_1(e_1=0)=R\,(1-z_2) \tag{7b}$$

$$u_1\left(e_1=R(1-z_2)^2\right)=R(1-k(1-z_2))(1-z_2) \tag{7c}$$

$$u_1\left(e_1=z_1^2\,R\right)=R\,z_1(1-k\,z_1). \tag{7d}$$

Comparing the payoffs leads to the following statements: First,

$$(1-z_2)>\big(1-k(1-z_2)\big)(1-z_2),$$

$\forall k>0$ and therefore $e_1=R\,(1-z_2)^2$ can never be a solution. Second, we have to find out, whether the interior solution is in the 'right' interval:

$$\frac{R}{4\,k^2}\in\left[R\,(1-z_2)^2,R\,z_1^2\right]\text{ iff }z_1\geq\frac{1}{2\,k}\text{ and }z_2\geq 1-\frac{1}{2\,k}.$$

Comparing the payoffs of the interior solution (equation (7a)) and of the first corner solution (equation (7b)) we get

$$\frac{1}{4\,k}>(1-z_2)\text{ if }z_2\geq 1-\frac{1}{4\,k}.$$

Moreover, comparing the payoffs of the first and third corner solution (equations (7b) and (7d)), we get that

$$z_1(1-k\,z_1)\geq(1-z_2)\text{ if }z_2\geq z_2^{\beta}\equiv 1-z_1(1-k\,z_1).$$

By introducing the threshold values $\tilde{z}_1\equiv\frac{1}{2\,k}$, $\tilde{z}_2\equiv 1-\frac{1}{4\,k}$ and $z_2^{\beta}\equiv 1-z_1(1-z_1)$, we will now pose the following proposition:

Proposition 2 (*Equilibria in the Sequential Game*)
In a two player sequential move Tullock contest with satiation, with player 1 being the first mover (leader) and player 2 being the second mover (follower), there exists a unique pure strategy Nash equilibrium, contingent on players demand (z_1,z_2) and on the relative efficiency parameter (k), given by the following strategy profile (e_1^L,e_2^F):

1. $(0,0)$ for $(z_1,z_2)\in\mathcal{A}$. In this case $u_1^L\equiv u_1(e_1^L,e_2^F)=z_1\,R$ and $u_2^F\equiv u_2(e_1^L,e_2^F)=z_2\,R$.
2. $\left(\frac{R}{4\,k^2},\frac{R}{2\,k}\left(1-\frac{1}{2\,k}\right)\right)$ for $(z_1,z_2)\in\Phi=\{z_1,z_2\,|\,z_1>\tilde{z}_1,z_2>\tilde{z}_2\}$. In this case agents' equilibrium share is smaller than their demand, $p_1^L=p_1(e_1^L,e_2^F)=\frac{1}{2\,k}<z_1$ and $p_2^F=p_2(e_1^L,e_2^F)=\frac{2\,k-1}{2\,k}<z_2$ and the payoffs are $u_1^L=\frac{R}{4\,k}$ and $u_2^F=\frac{R\,(1-2\,k)^2}{4\,k^2}$. This case only emerges for $k>\frac{1}{2}$.

3. $(z_1^2\, R, z_1(1-z_1)\, R)$, for $(z_1, z_2) \in \Psi = \left\{ z_1, z_2 \,|\, z_2 \geq z_2^{\beta}, z_1 \leq \tilde{z}_1 \right\}$. Agent 1's equilibrium share is equal to his demand, $p_1^L = z_1$, while agent 2's share is smaller than his demand, $p_2^F = 1 - z_1 < z_2$. The payoffs are $u_1^L = R(1 - k\, z_1) z_1$ and $u_2^F = (1 - z_1)^2\, R$.
4. $\left(\varepsilon, \frac{z_2}{1-z_2}\, \varepsilon\right)$ for $(z_1, z_2) \in \Omega = \{z_1, z_2 \,|(z_1, z_2) \notin \{A \cup \Psi \cup \Phi\}\}$. The payoffs are $u_1^L = (1 - z_2)\, R - k\, \varepsilon$ and $u_2^F = z_2\, R - \varepsilon$.

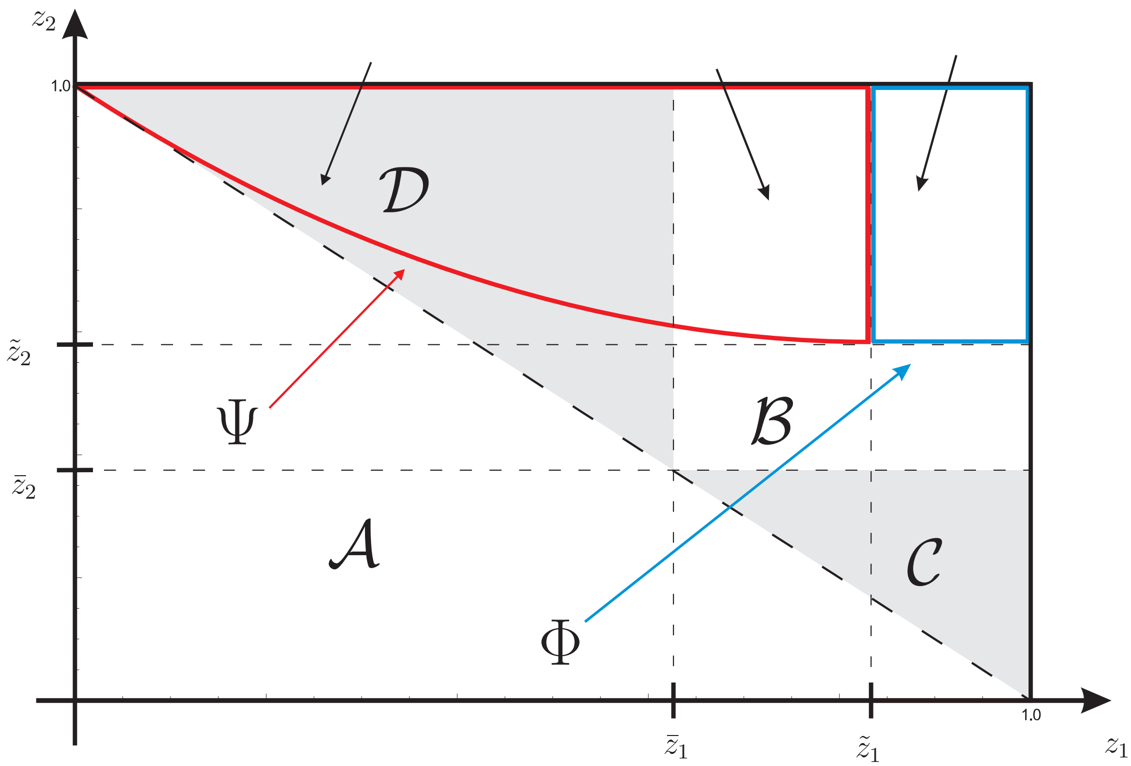


Figure 3: Equilibria in the sequential game contingent on z_1 and z_2 for $k < 1$.

The findings for the first case do not surprise: Since $\sum_i z_i \leq 1$ the first mover has no incentive to invest into effort. Given this commitment, the second mover neither has an incentive to invest into effort.
For $z_1 > \tilde{z}_1$ and $z_2 > \tilde{z}_2$ both agents occupy less than their demand in equilibrium, i.e., $p_1^L < z_1$ and $p_2^F < z_2$. This regime vanishes for $k \leq \frac{1}{2}$, since then $\tilde{z}_1 \geq 1$. The intuition for this is that for sufficiently low values of k, the first mover is so relatively efficient that he will always prefer to invest as much effort as is necessary to satisfy his demand, $p_1^L = z_1$. This is the case in the next regime, when $(z_1, z_2) \in \Psi$: If the demand of the first-mover, relative to his efficiency, is small enough $(z_1 < \tilde{z}_1)$, and the second mover's demand, relative to his efficiency and the other agent's demand,

is high enough ($z_2 > z_2^\beta$) than the first mover will be able to satisfy his demand in equilibrium ($p_1^L = z_1$). Further, the second mover will get the rest of the resource ($p_2^F = 1 - z_1 < z_2$).
The opposite is true, if $(z_1, z_2) \in \Omega$. Here, the second mover's demand, relative to his efficiency and the first mover's demand, is so low, that agent 1 will invest as less effort as possible, which is equal to ε, a strictly positive, but small number. The corresponding level of effort of player 2 is given by his best response function, for $e_1 \leq R(1-z_2)^2$. Thus, player 2 will obtain a share equal to his demand ($p_2^F = z_2$) and player 1 will get $p_2^L = 1 - z_2$. The loss in the share of the resource is than overcompensated by the (close to) zero effort costs.

2.4 Endogenous order of moves

We will now assume that players decide simoultanously and independently the stage in which they will exert effort (announcement stage). This decision is observed by the other contestant. Subsequently the players choose their level of effort in the period to which they were commited in the announcement stage. To keep the analysis simple we will assume that players are equally efficient, i.e., $mc_1 = mc_2 = 1$. Defining $\hat{z}_1 \equiv \frac{3}{4}$, $\hat{z}_2 = \frac{1}{2}$ and $z_1^\beta \equiv 1 - z_2(1-z_2)$, we will now describe the possible equilibria in the sequential move game if player 2 is the first mover:[10]

Proposition 3 (*Equilibria in the Sequential Game II*)
In a two player sequential move Tullock contest with satiation and unit marginal costs, with player 2 being the first mover (leader) and player 1 being the second mover (follower), there exists a unique pure strategy Nash equilibrium, contingent on players demand (z_1, z_2), given by the following strategy profile (e_1^F, e_2^L):

1. $(0, 0)$ for $(z_1, z_2) \in \mathcal{A}$. In this case $u_2^L \equiv u_2(e_2^L, e_1^F) = z_2\, R$ and $u_1^F \equiv u_1(e_2^L, e_1^F) = z_1\, R$.
2. $\left(\frac{R\,(2-k)}{4}, \frac{R\,k}{4}\right)$ for $(z_1, z_2) \in \Theta = \{z_1, z_2 \,|\, z_1 > \hat{z}_1, z_2 > \hat{z}_2\}$. In this case agents' equilibrium share is smaller than their demand, $p_1^F = p_1(e_1^F, e_2^L) = \frac{1}{2} < z_1$ and $p_2^L = p_2(e_1^F, e_2^L) = \frac{1}{2} < z_2$ and the payoffs are $u_1^L = \frac{R\,(2-k)^2}{4}$ and $u_2^F = \frac{k\,R}{4}$.
3. $(R(1-z_2)z_2, R\, z_2^2)$, for $(z_1, z_2) \in \Lambda = \left\{z_1, z_2 \,|\, z_2 \leq \hat{z}_2, z_1 \geq z_1^\beta\right\}$. Agent 2's equilibrium share is equal to his demand, $p_2^L = z_2$, while agent 1's share is smaller than his demand, $p_1^F = 1 - z_2 < z_1$. The payoffs are $u_1^F = R(1-z_2)^2$ and $u_2^L = (1-z_2)\, z_2\, R$.
4. $\left(\varepsilon, \frac{z_2}{1-z_2}\, \varepsilon\right)$ for $(z_1, z_2) \in \Xi = \{z_1, z_2 \,|(z_1, z_2) \notin \{A \cup \Theta \cup \Lambda\}\}$. The payoffs are $u_1^F = z_1\, R - \varepsilon$ and $u_2^L = (1-z_1)\, R - \varepsilon$.

[10]The corresponding equilibria when player 1 is the first mover can be obtained by implementing $k = 1$ in proposition 2.

Here, again, we have four different regimes, contingent on z_1 and z_2, respectively. One trivial ($(z_1, z_2) \in \mathcal{A}$), one interior ($(z_1, z_2) \in \Theta$) and two corner solutions ($(z_1, z_2) \in \Lambda$ and $(z_1, z_2) \in \Xi$).
Given these payoffs we can now turn to the case where players decide about when to exert effort. If both players announce different stages, then, whoever anounces the first stage becomes the Stackelberg-leader, the other player the Stackelberg-follower. Consequently the effort exerted in the game (and therefore also the payoffs) are equal to the one obtained in proposition 2 or 3, respectively. If both players announce the same stage then the subsequent subgame is the simultanous move game and player exert effort equal to the one obtained in proposition 1.

Lemma 4 (*Endogenous order of moves*)
In a two player Tullock contest with satiation and unit margianl costs, where players first decide and announce the stage in which they exert effort, the following order of moves will emerge in equilibrium, contingent on the players' demand (z_1, z_2):

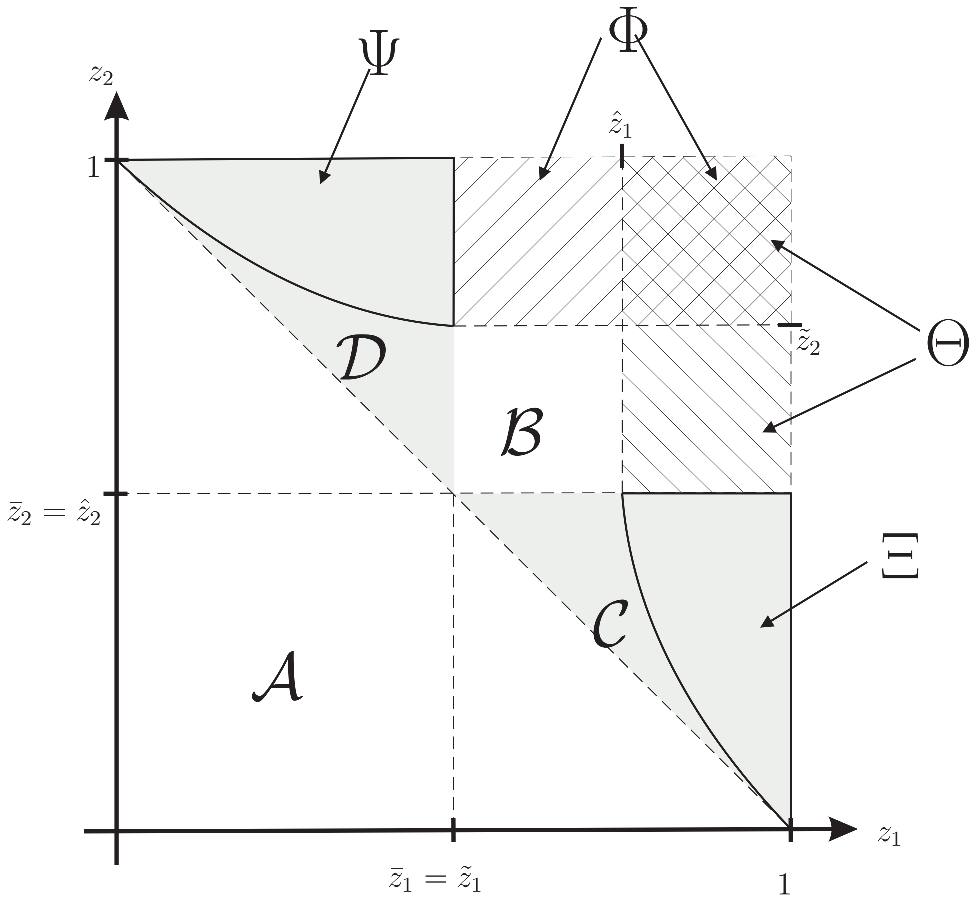
1. For $(z_1, z_2) \in \Psi$ and $(z_1, z_2) \in \Phi \setminus \Theta$ player 2 will move first and player 1 will follow. The effort exterted in equilbrium is equal to the effort for $(z_1, z_2) \in \Xi$ (Proposition 4).

2. For $(z_1, z_2) \in \Lambda$ and $(z_1, z_2) \in \Theta \setminus \Phi$ player 1 will move first and player 2 will follow. The effort exterted in equilbrium is equal to the effort for $z_1, z_2) \in \Omega$ (Proposition 3).

3. For $(z_1, z_2) \in \Phi \cup \Theta$ players are indifferent between all possible order of moves, i.e. the equilbrium efforts and consequently the payoffs, are the same in each case.

4. In all other cases the players will move simoultanously, the exerted effort and the the payoffs are therefor equal to the one outlined in proposition 1.

In the first two cases one of the players (i) is best off choosing an ε-level as a Stackelberg-leader ($u_i^L > u_i^F > u_i^*$), while the other player is best off choosing the follower position and exerting the corresponding Stackelberg-follower effort ($u_j^F > u_j^* > u_j^L$). Necessary conditions for this scenario are sufficiently asymmetric demands.
In the third case, the demands are sufficiently high, so that there is no incentive for the weaker player (the one who's demand is smaller than the deamnd of his competitor) to overcommit effort. Hence, analogous to Dixit (1987) and Baik and Shogren (1992) we find that there is no incentive to commit to effort, since the effort exerted and the payoffs are the same in each scenario ($u_i^F = u_i^L = u_i^*$).
In the forth case both players would be better off in the follower position ($u_i^F > \max\{u_i^L, u_i^*\}$). Thus, given the rules of the game, both end up in a simultanous move game.



3 Conclusion

We developed a model of contest with possible satiation. Analogous to Dixit (1987) and Baik and Shogren (1992) we examined the Cournot-Nash equilibrium and the equilibrium in the sequential game, followed by a game with endogenous order of moves. We found in the simultaneous move game that if players' ability to satisfy their demand depends crucially on the ratio of relative efficiency and relative demand. For example, if the demand of player 1 exceeds that of player 2, it is still possible that player 1 satisfies her demand, if she is sufficiently more efficient.
In the sequential move game, we found that if the player with the higher demand is forced to commit himself, he will exert zero effort, given that his opponent's demand is sufficiently small. The reason for this is that the loss in the share is overcompensated by (close to) zero effort costs.
Finally, we examined the case of endogenous order of moves. Here, we found that zero demand is exerted in equilbrium if players are sufficiently asymmetric.

Appendix

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