Like-Biased Experts And Noisy Signals

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Abstract

This paper revisits the literature on strategic information transmission with multiple senders of like biases. We impose restrictions on information transmission by assuming that experts observe noisy signals. The paper compares fully revealing equilibria with partition equilibria, to reach some findings that are of use for the design of practically feasible game forms. We find that fully revealing equilibria do not survive our perturbation, while the class of partition equilibria that we study is robust. In the proposed partition equilibrium with two senders, the decision maker designs a communication protocol that gives the second sender a reduced message space, and after the disclosure the decision maker best responds to his beliefs. The paper so highlights the value of partition strategies for viable game forms of expertise, vis-a-vis fully revealing equilibria, and closes a lacuna that has remained open in the literature on strategic information transmission with like-biased senders.

Preliminary and incomplete. Please do not quote.

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1 Introduction

One may regard it as one of the limiting factors for human success that those who are empowered to make decisions typically are not well informed, while those who are better informed would typically disagree with what should be the best action if the decision maker had access to perfect information – this difference in interest often being referred to as "bias." The inherent trade-off appears to be even more acute in today's world where access to specialized knowledge has become the key for professional and corporate success, left in the hands of typically less than perfectly informed experts. We cannot hold experts accountable for what they observe and what they know, yet we need to believe to some extent what they tell us.

It is easy to illustrate these thoughts in the form of an economically relevant example. Consider an uninformed business CEO who has to decide how to structure communication when consulting experts in a meeting. Can he design the flow of information in a way so as to generally extract additional information from having more than one expert, say by giving the two experts different message spaces? Imagine he can access a less and a more biased expert: given the optimal structure of communication, should he try to only consult the less biased expert, or can he extract more information when a second, more biased player is consulted as well?

1.1 Literature review

While these questions are not entirely new, they have been treated controversially in the literature on strategic information transmission, with results that largely depend on assumptions concerning the type of communication. In particular, there exists little research so far that helps to form a broader concept on how to best design practically feasible modes of expertise with many players. Some intuitive stepping stones can be found in Krishna and Morgan (2004) who expand on the art of conversation between the CEO and *one* expert. Their result is that additional rounds of consultation may refine the classic partition equilibrium of Crawford and Sobel (1982) (CS hereafter), with written reports in addition to verbal disclosure leading to more information.

While there is broad agreement among scholars about the fact that adding a second expert is a natural way to refine cheap talk games,[1] multisender expertise games have, by and large, not been analyzed in order to distill some insights for the design of game forms for real-life expertise with economic relevance.

There are exceptions. Hori (2006), in his analysis of decision and communication procedures stud-

[1] See e.g. Chen, Kartik and Sobel (2007).

ies two stylized forms of communication: hierarchical and horizontal. While horizontal communication bears the advantage to identify a wide range of different messages, hierarchical communication benefits from the fact that information manipulation is reduced. Wolinsky (2002) studies a form of information exchange in which the decision maker may be able to divide experts into groups. When experts within a group can communicate before disclosing their information to the decision maker, more information can be extracted. Gerardi, McLean and Postlewaite (2006) show in a model with noise that communication can be improved through imposing a distortion on decisions. Depending on how important an expert's information is, this distortion can be kept relatively small. Lastly, in a paper in the literature on reputation and forecasting, Ottaviani and Sorensen (2006) show that it may pay for the decision maker to have competition among forecasters.

Models with two experts trace back to Gilligan and Krehbiel (1987 and 1989), who study the impact of legislative rules on information transmission in political organization. In their base model, two experts with *opposing* biases are placed symmetrically around an uninformed decision maker (median voter). Under this particular assumption, full information revelation is obtained – a finding that has lead to the generally accepted view in political science that "two informed opinions are better than one, especially when the informants are natural adversaries."[2]

With Austen-Smith's (1990) focus on *debate* equilibria, this literature has gone beyond the case when expert biases are opposed.

In an early paper on imperfectly informed senders and binary signal spaces, Austen-Smith (1993) has extended and revised several thoughts of the early literature, followed by Krishna and Morgan's (2001a) well-known paper that, by and large, marks the end point of the literature on one-dimensional expertise with opposing biases.

We hereafter do not discuss the case of opposing biases any further, for immediate reasons, but take up the field of like-biased expertise, in particular because of its attractiveness and applicability to many economically relevant settings.[3]

Our paper so resumes a discussion that has been started in Krishna and Morgan (2001b) (KM hereafter). While KM focus on *sequential* disclosure, two of their research questions, in our view, are of particular importance for a treatment of expertise with like-biased senders and sequential

[2]Krehbiel (1991), p. 84.

[3]To give another example: think of a consumer who shops for a plasma TV in a consumer electronics outlet. Would she prefer to consult only one salesperson before making a decision – namely, the one closer to her bliss point (or her preferred budget) – or would she benefit from the additional presence of a more biased salesperson? Since the buyer can ask one of the two salespersons about an outside option, she will do better by consulting both. A related example can be found in Dziuda (2008).

disclosure. While we refer to more recent findings, in particular the work of Battaglini (2002) and Ambrus and Takahashi (2007), we reconsider the issues of existence and robustness of fully revealing equilibria (FRE) under like biases, and, second, whether there exist any partitional perfect Bayesian equilibria following CS, but with two senders. However, our primary focus remains on the unidimensional state space.

Two more observations are in order. First, KM find that "messages of one expert can be used to discipline the other, [...] this disciplining has only the effect of reducing informativeness."[4] Consequentially, other than in Krehbiel's (1991) quote, two senders cannot be any better than one when considering like biases when disclosure is sequential.

Second, note that KM's dismissal of partially revealing equilibria under simultaneous disclosure follows the observation that a receiver, because of the strategic interaction between the senders, cannot combine two CS Best Equilibrium profiles to reach a refinement since a combination of two CS profiles cannot lead to a perfect Bayesian equilibrium.[5] While correct in this form, it does not preclude the existence of other partition equilibria, specifically those that use a different message space for the second sender. We characterize such an equilibrium, endowing the second sender with a message space that is different from the one used in the CS profile. There exists a class of equilibria under simultaneous disclosure in which the receiver best responds to his beliefs in all states. As we show, this equilibrium permits the decision maker to extract more information than the CS equilibrium with consulting the less biased sender only. Specifically, there exists a game form in which the receiver best responds to his beliefs, the less biased sender plays CS "best equilibrium" strategies, and the second sender is given a message space that does not depend directly on the first sender's disclosure. The general idea behind our construction is close to Sobel's (2008) view that in two-sender games "[t]he second sender has preferences that depend on type and the receiver's action, but not directly on the message sent." (p.8). It moreover follows the idea that introducing "novel" messages is a refinement.[6] Our findings are consistent with the view that two experts are better than one, but that full revelation is too much to expect in a game with two experts.

The paper is organized as follows. Section 2 characterizes fully revealing equilibria, introducing the implications of Ambrus and Takahashi's (2007) refinement to diagonally-continuous fully revealing equilibria for the unidimensional state space, and examining the robustness through introducing a perturbation, namely the use of noisy signals. Section 3 introduces a partition equilibrium that is robust under noisy signals and informationally superior to both KM and CS. Section 4 summarizes

[4]KM, p. 759

[5]See KM p. 757.

[6]See Chen, Kartik and Sobel (2008), CKS hereafter.

our preliminary findings. A leading example is given in the appendix.

2 Fully Revealing Equilibria With Two Senders

2.1 Existence

Characterization (Sobel(2008)). With two senders, S_1 and S_2 and biases b^{S_1} and b^{S_2} there exists a fully revealing equilibrium (FRE) iff for all states t and $t' \in T$ the union of the actions

$$B(t' + b^{S_1}, |b^{S_1}|) \cup B(t + b^{S_2}, |b^{S_2}|) \tag{1}$$

does not contain T. [7]

$B(t' + b^{S_1}, |b^{S_1}|)$ here denotes the set of actions that Sender 1 prefers to the true state t. Similarly, $B(t + b^{S_2}, |b^{S_2}|)$ is the set of actions that Sender 2 prefers to the true state. If (1) does not contain T, then there exists an action that R can take to make both senders worse off if they deviate from the equilibrium path. For small biases, there always exists such a state and this condition is fulfilled, both for like and opposing biases.

2.2 Continuity, diagonally-continuous FRE and implications for unidimensional state space

Cheap talk games are characterized by the fact that perfect Bayesian equilibria impose little or no restriction on the receiver's out-of-equilibrium beliefs. While Battaglini (2002) has introduced a robustness criterion that we will apply in the next subsection, it should be mentioned that there exit other refinements to FRE. Ambrus and Takahashi (2007) have developed the concept called diagonal continuity, which translates another robustness criterion, namely Battaglini's (2002) second robustness test, based on the requirement that the receivers out-of-equilibrium beliefs are continuous in the senders' reports. Essentially, Ambrus and Takahashi (2007) found a continuous transformation of Battaglini's continuity criterion. Specifically, they require continuity at the points where both senders observe the same signal.

A brief illustration may read as follows: in the two-dimensional version, take a sequence of pairs of states such that in the limit when the number of observations go to infinity, both states approach

[7] See e.g. Sobel, 2008, Battaglini, 2002.

the true state t. Then, a FRE is continuous on the diagonal if for this sequence, the actions of the receiver lie on the diagonal. (see Definition 2 in Ambrus and Takahashi (2007), p.16).

It is easy to show that in a one-dimensional version of the state space, FRE with opposing biases neither fulfill Ambrus and Takahashi's criterion of diagonal continuity, nor do they fulfill Battaglini's continuity criterion in general, since the receiver's beliefs are discontinuous in the senders' reports.

2.3 Like biases and robustness when signals are noisy

As a special case of (1), consider the FRE with like biases. This FRE can be characterized as follows:

R holds beliefs $P(t = \min(m^{S_1}, m^{S_2})|m^{S_1}, m^{S_2}) = 1$ and consequently chooses action $a(m^{S_1}, m^{S_2}) = \min(m^{S_1}, m^{S_2})$. Assume $\mu^{S_1} = t$. Then, S_2 cannot do better than disclosing t as well. Note that S_2 will never disclose a lower value, and disclosing any higher value will not change the results.
We now introduce noisy observation and check if these beliefs still sustain the FRE.

Claim. There is no robust fully revealing equilibrium when signals are noisy.

Proof. Assume R receives two signals in close vicinity, one sender suggesting the true state to be $\tilde{t}$, the other sender to be $\tilde{t} + \varepsilon$. R, in his posterior beliefs, will now put only a marginally positive probability on the fact that both senders have deviated. However, R will believe that one of the two senders will disclose t truthfully. This implies that the posterior is in close vicinity to the two signals. Specifically, the posterior is a weighted average of the two signals. While the probability that both signals are wrong is marginal, one of the two senders will have an incentive to deviate since any deviation, even if small, will move the receiver's choice of action and thus be profitable for the deviating sender, which implies that the fully revealing equilibrium is not stable.

We conclude this section finding that, although FRE under like biases survive both Battaglini's (2002) continuity criterion and Ambrus and Takahashi's (2007) refinement of diagonal continuity, they are not robust when signals are noisy, and thus not feasible candidates for real-world expertise.

3 A 2-sender model with partial information revelation

In our construction we follow the canonical setting of CS with uniform-quadratic preferences for all players but allow for two senders, with the more biased sender being given a binary message space. Biases b are common knowledge, with $b^R = 0$, $b^{S_1} < b^{S_2} \leq \frac{1}{4}$. Senders S_1 and S_2 observe the state of nature (= their type) t.[8] The receiver, R, discloses a communication protocol announcing that he will play the CS best equilibrium strategy with S_1 and permitting S_2 to suggest a known default action $\tilde{a}$. The value of $\tilde{a}$ is common knowledge.

Each sender discloses a message m^{S_j} according to its message space determined in the communication protocol. R observes the messages and best responds to his beliefs by choosing an action.[9]

3.1 Characterization

With two senders, $S_j, j \in \{1,2\}$ and one-directional biases where $0 < b_1 < b_2$, a pure-strategy PBE consists of

(i) a message strategy μ^{S_j}, with $\mu^{S_1} : [0,1] \rightarrow M = \{m_1, ..., m_{N-1}\}$ and $\mu^{S_2} : [0,1] \rightarrow \{\tilde{a}, \neg\tilde{a}, \emptyset\}$,

(ii) an action strategy $\alpha : M \times \{\tilde{a}, \neg\tilde{a}, \emptyset\} \rightarrow \mathbb{R}$ for the receiver, and

(iii) an interpretation of the message (updating rule) $\beta(t|m^{S_1}, m^{S_2})$ such that

- for each $t \in [0,1], \mu(t)$ solves $\max\limits_{m_i^{S_j}} U^{S_j}(\alpha(m_i^{S_j}, t), t)$,

- for the message pair $(m_i^{S_1}, m^{S_2}), \alpha(m_i^{S_1}, m^{S_2})$ solves $\max\limits_{a} \int\limits_0^1 U^R(a,t)\beta(t|m_i^{S_1}, m^{S_2})\ \mathrm{dt}$,
where $(t|m_i^{S_1}, m^{S_2})$ is derived from μ and F from Bayes' Rule whenever possible.

We extend the notation used in CKS and Sobel (2008) and first define a type $t(\tilde{a})$ to so characterize a two-sender equilibrium by a partition of the set of types, $t(N) = (t_0(N), ..., t_N(N))$ with $0 = t_0(N) < t_1(N), ..., t_k(N) < t(\tilde{a}) < t_{k+1}(N), ..., t_{N-1}(N) < t_N(N) = 1, S_1$'s and messages $m_i^{S_1}, i = 1, ..., N$, and m^{S_2} such that for all $i = 1, ..., N-1$,

$$U^{S_1}(\bar{a}(t_i, t_{i+1}), t_i) - U^{S_1}(\bar{a}(t_{i-1}, t_i), t_i) = 0 \qquad (S_1\text{-types on the boundary are indifferent})$$

$$\mu^{S_1}(t) = m_i^{S_1} \ \textit{for } t \in (t_{i-1}, t_i]. \ (S_1\text{-types in a common element pool and send the same message.})$$

We divide our analysis about R's best response into the following two intervals.

[8] In this section, we characterize the equilibrium, assuming that the two senders observe the state perfectly, while we show later that this assumption can be relaxed and the equilibrium studied here is robust under noisy signals.

[9] We use j to label the senders, with $j \in \{1,2\}$. When speaking of Sender j, the second sender is labeled S_{-j}. Otherwise, our notation follows closely the exposition in CKS.

Case 1: Intervals between $t_k(N)$ and $t_{k+1}(N)$

• R best responds to $m^{S_2} = \tilde{a}$ by either implementing

$$\alpha(m^{S_j}) = \frac{\tilde{a} + \bar{a}(t_k(N))}{2} \; or \;\; \alpha(m^{S_j}) = \frac{\tilde{a} + \bar{a}(t_{k+1}(N))}{2}.$$

• Instead, when $m^{S_2} = \neg\tilde{a}$, R will update his beliefs about t and then holds beliefs of either

$$t \in [t_k(N), t(\frac{\tilde{a} + \bar{a}(t_k(N))}{2})]$$

or

$$t \in [t(\frac{\tilde{a} + \bar{a}(t_{k+1}(N))}{2}), t_{k+1}(N)),$$

to which he best replies by taking action $\frac{1}{2}[t_k(N)+t(\frac{\tilde{a}+\bar{a}(t_k(N))}{2})]$ or action $\frac{1}{2}[t(\frac{\tilde{a}+\bar{a}(t_{k+1}(N))}{2})+t_{k+1}(N)]$, respectively. To summarize this case, R updates his beliefs after a disclosure of $m^{S_2} = \tilde{a}$ that t must be in a subset of $[t_k(N), t_{k+1}(N)]$ while knowing after a disclosure of $m^{S_2} = \neg\tilde{a}$ that t is in the complement of it. In both cases, R will best respond to the messages received by the two senders. No sender deviates and each disclosure of Sender 2 leads to a refinement.

Case 2: Next Adjacent Intervals.
At the left side of the above characterized interval, S_1 is no longer indifferent between triggering action $\bar{a}(t_{k-1}(N))$ and $t(\frac{\tilde{a}+\bar{a}(t_k(N))}{2})$ and deviates to the left. Call this new break point for S_1 $t_k^L(N)$. For the interval $[t_k^L(N),\, t_k(N)]$ S_2 is better off by disclosing $m^{S_2} = \emptyset$ to so signal to R that S_1 has deviated. R best responds by choosing action $\frac{1}{2}[t_k^L(N),\, t_k(N)]$.

By symmetry, the same holds on the right side of the interval characterized in Case 1: S_1 will now deviate to the right until the new break point $t_k^t(N)$ is reached. S_2 again signals this deviation through a disclosure of $m^{S_2} = \emptyset$, and R best replies with action $\frac{1}{2}[t_k^R(N),\, t_k(N)]$. This completes the characterization; for further intervals on the left or on the right, R plays CS Best Equilibrium strategies with S_1, with S_2 disclosing $\neg\tilde{a}$.

A numerical example is given in the appendix.

3.2 Noisy signals

We now check whether partitional equilibria like the one sketched above are stable when signals are noisy. Specifically, we show that the CS profile played with Sender 1 is robust, to reach the same conclusion for partitional equilibria in general.[10]

Sender 1 now observes the true state with probability $1-\varepsilon$, and with probability ε he observes a state $t \in [0,1]$. Since each disclosure is a random signal with probability ε, the receiver will now best respond by implementing action

$$\bar{a}_i = (1-\varepsilon)(\frac{t_{i-1}+t_1}{2}) + \varepsilon\frac{1}{2}.$$

Reducing the observation to this particular interval, the sender's expected utility when disclosing $m^{S_1} \in [t_{i-1}, t_i]$ is

$$E[U^{S_1}(t)] = -(1-\varepsilon)(t+b_1-\bar{a}_i)^2 - \varepsilon\int_0^1 (t'+b_1-\bar{a}_i)^2 \ \ dt'.$$

Because of the CS "no-arbitrage" condition, types t are indifferent between inducing action $\bar{a}_i$ and $\bar{a}_{i-1}$ at any break points, the equilibrium condition

$$U^{S_1}(t,\bar{a}_i) = U^{S_1}(t,\bar{a}_{i-1})$$

leading to

$$t_{i+1} - t_i = t_i - t_{i-1} + \frac{4b}{1-\varepsilon},$$

which reveals that the last interval must be by $\frac{4b}{1-\varepsilon}$ longer than the first.[11]

We conclude that while partitional equilibria are sustained under noisy observation, they however entail an informational loss for all players since the single partitions become less evenly spaced.

4 Summary of the preliminary findings and wrap-up

So far, our findings reveal the following. First, along the spectrum of possible equilibria between full revelation and no communication at all, there exist fully revealing equilibria under like biases. These equilibria survive both Battaglini's (2002) continuity criterion since the receiver's beliefs are continuous in the sender's reports. They also survive Ambrus and Takahashi's (2007) refinement of diagonal continuity. However, as shown in Section 2, they are not robust to noise.

[10]For the sake of generality we assume that both experts are affected by the same noise with the same probability.

[11]I thank Oliver Board for the discussion of this case. A similar argument using sender errors can be found in Blume et al. (2007).

Second, we have characterized a class of partially revealing equilibrium with two like-biased experts and one uninformed decision maker under simultaneous disclosure. In this equilibrium, the receiver makes use of a natural message that is present when two senders are consulted.[12] By using additional messages from the second, more biased expert and giving him a minimal message space, the decision maker can generally extract more information than by consulting the less biased sender alone. While many other equilibria exist,[13] we have also shown that our construction is robust when the expert signals are not perfectly correlated but noisy. Noise, however, entails a loss of communication.

More generally, our preliminary findings fit the intuition that real world expertise with two senders leads to a degree of information transmission that – on one hand – leads to less than full information. Intuitively, it is too much to expect perfect knowledge of the state by adding a second sender. On the other hand, adding a second sender clearly improves information transmission. This is supported by our result on partition equilibria. In the light of KM's paper, which shows that under sequential disclosure, two senders reveal as much as the less biased sender does, this result seems worth mentioning.

More research needs to be done to cover possible generalizations and extensions. While the two-sender case has been the most prominent one discussed in the literature, an extension toward more than two senders seems particularly worthwhile. While trivial under perfect information, noisy signals with more than two senders would be of interest. With three senders the receiver, upon seeing two identical reports, may want to ignore the deviating sender. However, a sender still has an incentive to exaggerate his report since there is a positive probability that at least one of the other senders did not observe the true state. Misreporting will then lead to 3 different messages, and no message can be ignored anymore by the receiver. Such extensions are left for future research.

Results from experiments seem to further confirm the usefulness of partition equilibria for real-world expertise. Cai and Wang's (2006) results support CS's findings in that individuals play equilibrium strategies as in CS. In addition, the authors observe that "overcommunication" occurs in experiments. That is, senders generally transmit more information than assumed under CS, but they do not fully reveal the state. This furthermore underpins the value of partition equilibria as a field of study for multisender expertise.

[12] Note that in CS, once a receiver chooses a sender, he can no longer refuse to become "informed" and to implement the message suggested by the sender.

[13] As always, there exist babbling equilibria.

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6 Appendix: a numeric example

Similar to KM we offer a numerical example with $b_1 = \frac{1}{20}, b_2 = \frac{1}{15}, \tilde{a} = \frac{1}{2}$ to illustrate a particular case for $t_k(N) < \tilde{a} < t_{k+1}(N)$. We start by defining the message spaces for this equilibrium. A bias of $b_1 = \frac{1}{20}$ implies CS Best Equilibrium messages of $M_1 = \{\frac{1}{15}, \frac{3}{10}, \frac{11}{15}\}$. $M_2 = \{\tilde{a}, \neg\tilde{a}, \emptyset\}$. Note that R discloses at the beginning of the game that he will play CS best equilibrium with S_1 and to simultaneously ask S_2 to disclose whether he prefers the action suggested by S_1 or whether he prefers $\tilde{a}$. Note also that R plays a 3-partition best CS equilibrium profile with S_1, with $\tilde{a}$ being located between the last two equilibrium actions of the single sender game. Thus, the characterization of the next adjacent intervals given above only applies to the left side of $\tilde{a}$.

- Receiver's posterior beliefs

With $\mathcal{U}[x,y]$ denoting the uniform distribution over the interval $[x,y]$, R's posterior beliefs are:

$$P(\cdot \mid m^{S_1}, m^{S_2}) = \begin{cases} \mathcal{U}[0, \dfrac{1}{10}] \text{ if } m^{S_1} = \frac{1}{10} \text{ and } m^{S_2} = \neg\tilde{a} \\ \mathcal{U}[\dfrac{1}{10}, \dfrac{2}{15}] \text{ if } m^{S_1} = \frac{1}{10} \text{ and } m^{S_2} \notin \{\neg\tilde{a}, \tilde{a}\} \\ \mathcal{U}[\dfrac{1}{10}, \dfrac{1}{6}] \text{ if } m^{S_1} = \frac{3}{10} \text{ and } m^{S_2} = \neg\tilde{a} \\ \mathcal{U}[\dfrac{1}{6}, \dfrac{7}{15}] \text{ if } m^{S_1} = \frac{3}{10} \text{ and } m^{S_2} = \tilde{a} \\ \mathcal{U}[\dfrac{7}{15}, \dfrac{3}{4}] \text{ if } m^{S_1} = \frac{11}{15} \text{ and } m^{S_2} = \tilde{a} \\ \mathcal{U}[\dfrac{3}{4}, 1] \text{ if } m^{S_1} = \frac{11}{15} \text{ and } m^{S_2} = \neg\tilde{a} \\ \mathcal{U}[0, 1] \text{ else.} \end{cases}$$

- Receiver's strategy profile:

$$a(m^{S_1}, m^{S_2}) = \begin{cases} \dfrac{1}{20} \text{ if } m^{S_1} = \frac{1}{10} \text{ and } m^{S_2} = \neg\tilde{a} \\ \dfrac{7}{60} \text{ if } m^{S_1} = \frac{1}{10} \text{ and } m^{S_2} \notin \{\neg\tilde{a}, \tilde{a}\} \\ \dfrac{7}{30} \text{ if } m^{S_1} = \frac{3}{10} \text{ and } m^{S_2} = \neg\tilde{a} \\ \dfrac{2}{5} \text{ if } m^{S_1} = \frac{3}{10} \text{ and } m^{S_2} = \tilde{a} \\ \dfrac{37}{60} \text{ if } m^{S_1} = \frac{11}{15} \text{ and } m^{S_2} = \tilde{a} \\ \dfrac{49}{60} \text{ if } m^{S_1} = \frac{11}{15} \text{ and } m^{S_2} = \neg\tilde{a} \end{cases}$$

- Strategy Profile of Sender 1:

$$\mu^{S_1} = \begin{cases} \dfrac{1}{15} \text{ if } t \in [0, \frac{2}{15}) \\ \dfrac{3}{10} \text{ if } t \in [\frac{2}{15}, \frac{7}{15}) \\ \dfrac{11}{15} \text{ if } t \in [\frac{7}{15}, 1] \end{cases}$$

- Strategy Profile of Sender 2:

$$\mu^{S_2} = \begin{cases} \tilde{a} \text{ if } t \in [\frac{1}{6}, \frac{3}{4}) \\ \neg\tilde{a} \text{ if } t \in [0, \frac{1}{10}) \text{ or } t \in [\frac{2}{15}, \frac{1}{6}) \text{ or } t \in [\frac{3}{4}, 1] \\ \emptyset \text{ otherwise.} \end{cases}$$

Note first that the CS best equilibrium profile of S_1 leads to break points at $t = \frac{2}{15}$ and $t = \frac{7}{15}$, and the message space for S_1 is $M_1 = \{\frac{1}{15}, \frac{3}{10}, \frac{11}{15}\}$. We start at the break point $t = \frac{7}{15}$. From this value of t to the right, S_1 can break down the state space into one interval, namely $(\frac{7}{15}, 1]$, by disclosing $\frac{11}{15}$. If S_2 suggests $\tilde{a}$, R best replies with $\frac{37}{60}$, while believing that t is between $\frac{11}{20}$ and 1 when S_2 suggests $\neg\tilde{a}$. In the latter case, R consequently implements $\frac{49}{60}$. Consulting the second sender refines the information structure. Discussing disclosures and equilibrium actions left of $t = \frac{7}{15}$ reveals that R best responds to $m^{S_1} = \frac{3}{10}$ and $m^{S_2} = \tilde{a}$ by choosing $\frac{2}{5}$. Note that between $\frac{1}{10}$ and $\frac{2}{5}, S_1$ is better off suggesting $\frac{1}{15}$ while $m^{S_2} = \emptyset$. R best replies by implementing $\frac{7}{60}$. Left of $\frac{1}{10}$, $m^{S_1} = \frac{1}{15}$ and $m^{S_2} = \neg\tilde{a}$. For this 3-partition CS best equilibrium profile for S_1 we thus show that having a second sender permits to break down the state space into a total of 6 partitions. *Fig. 1* below grapically illustrates this equilibrium:

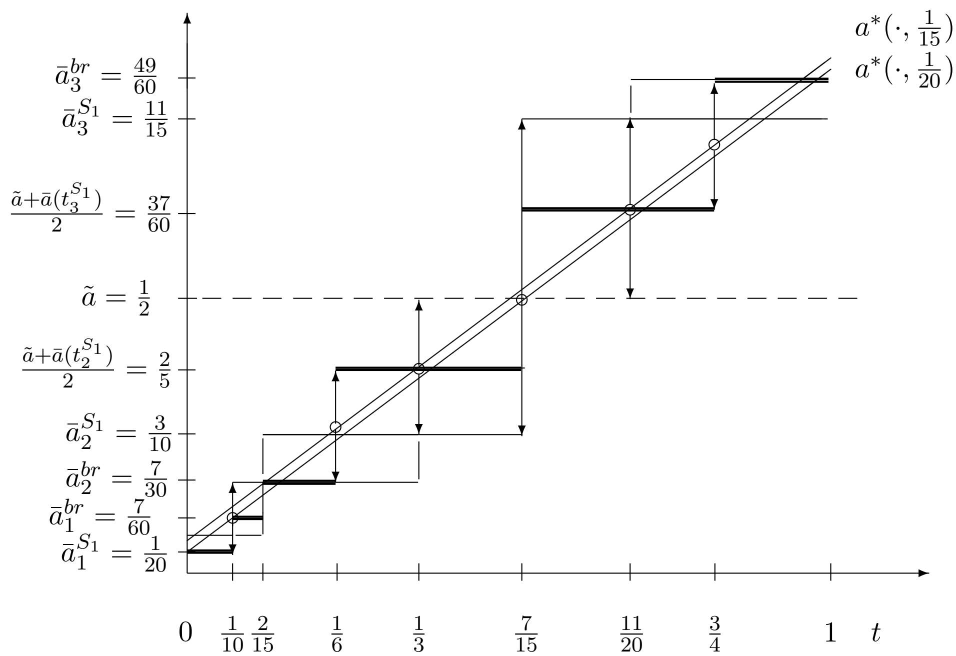

Fig. 1: Numerical example with $b_1 = \frac{1}{20}, b_2 = \frac{1}{15}, \tilde{a} = \frac{1}{2}$

Lastly, we show numerically that this equilibrium indeed performs better than having R playing CS best equilibrium with the less biased sender. The ex-ante utility of R in this two-sender equilibrium is

$$-\left[\int_{0}^{\frac{1}{10}}\left(\frac{\frac{1}{10}}{2}\right)^2+\int_{\frac{1}{10}}^{\frac{2}{15}}\left(\frac{\frac{2}{15}-\frac{1}{10}}{2}\right)^2+\int_{\frac{2}{15}}^{\frac{1}{6}}\left(\frac{\frac{1}{6}-\frac{2}{15}}{2}\right)^2+\int_{\frac{1}{6}}^{\frac{7}{15}}\left(\frac{\frac{7}{15}-\frac{1}{6}}{2}\right)^2+\int_{\frac{7}{15}}^{\frac{3}{4}}\left(\frac{\frac{3}{4}-\frac{7}{15}}{2}\right)^2+\int_{\frac{3}{4}}^{1}\left(\frac{1-\frac{3}{4}}{2}\right)^2\right]=-\frac{589}{108000}$$

or approximately -0.0054537. This shows that R does better by playing the two-sender game instead of playing CS best equilibrium strategy with the less biased sender, which would yield an ex-ante utility of approximately -0.00637.