# Core-stable bidding rings

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Auctions are typically modeled as strategic, non-cooperative games (see, e.g., Krishna (2002)). If, before the bidding stage, the agents can form cartels, a new game between coalitions emerges, which can be studied by cooperative tools. The strategic interaction of the agents generates indirect externalities (in the sense that the outcome of the auction for a player depends on the bids of the others). As in Jehiel and Moldovanu (1996), we assume that, in addition to the indirect externalities, the bidders also face direct external effects: their final utility depends on the winner's identity. For instance, they are not indifferent between the outcome in which a given bidder gets the object and the outcome in which no sale occurs. In the presence of externalities, the natural cooperative model is the partition form game (Lucas and Thrall (1963)).

Let us fix a non-cooperative game $\Gamma$ modeling a (first price or second price) sealed bid auction. We assume independent, not necessarily identically distributed, private values and deterministic, identity dependent, direct external effects. Let $P$ be a partition of the players. A new, non-cooperative, game $\Gamma(P)$ can be defined as follows: the players are the coalitions $S$ in $P$; a strategy for $S$ is an incentive compatible mechanism which determines a bid for every member of $S$ together with balanced transfers between the members of $S$, as a function of the information of the members of $S$; given a strategy for every coalition in $P$, the payoff of $S$ in $\Gamma(P)$ is the sum of the payoffs (in $\Gamma$) of the members of $S$. For every partition P, let $\sigma(P)$ be an incentive compatible Nash equilibrium of $\Gamma(P)$, namely an incentive compatible coalitional equilibrium (see Ray (2007)). We prove that, under our assumptions, incentive compatibility is without loss of generality. We generate a partition form game by defining $v_\sigma(S;P)$ as the sum of the expected payoffs of the players in $S$ at the equilibrium $\sigma(P)$. Observe that, if $\sigma(P)$ is not unique, the equilibrium behavior in $\Gamma(P)$ does not depend on the negotiation process leading to $P$, but only on $P$ itself.

As is well-known, several definitions of the core of a partition form game are conceivable (see, e.g., Hafalir (2007)). Let $N$ be the set of all players. For every coalition $S$, let $B(S)$ be a partition of the players which contains $S$ as a cell. We interpret $B(S)$ as a belief of $S$ on the way the players outside $S$ partition themselves if $S$ secedes from the grand coalition $N$. The $B$-core of a partition form game $v$ is just the core of the game in characteristic function form $f_B$ defined by $f_B(S) = v(S; B(S))$. For instance, in the core with "singleton expectations", or $s$-core, every coalition $S$ believes that the players not in $S$ act individually (namely, $B(S) = \{S; \{j\}, j \in N\backslash S\}$); in the core with "merging expectations", or $m$-core, every coalition $S$ believes that the players not in $S$ form the single coalition $N\backslash S$ (namely, $B(S) = \{S, N\backslash S\}$).

We adopt the following definition: the grand coalition $N$ is core-stable with respect to $\sigma$ if, for some specification of the beliefs $B$, the $B$-core of $v_\sigma$ is not empty. Equivalently, let $w_\sigma(S)$ be defined as the minimum, over all partitions $P$ of $N\backslash S$, of $v_\sigma(S; \{S, P\})$; $N$ is core-stable with respect to $\sigma$ if the core of $w_\sigma$ is not empty. The core underlying this definition is called "core with cautious expectations" in Hafalir (2007) and can be viewed as an $\alpha$-core (Aumann (1961)) that would be consistent with a weak, natural requirement of sequential rationality. More precisely, under complete information, our core is included in the $\alpha$-core of the original game $\Gamma$. Indeed, the $\alpha$-core reflects the "careful collusion" of coalitions which expect the strongest punishment from the complementary coalition in case of secession. Under incomplete information, our solution concept can be interpreted as an incentive compatible *ex ante* (sequentially rational) $\alpha$-core (Forges and Minelli (2001), Forges et al. (2002)).

If a second price auction takes place in the absence of direct externalities, the grand coalition is always core-stable (see Mailath and Zemsky (1991) and Barbar and Forges (2007)). By relying on results in Lebrun (1999) and Waehrer (1999), we establish a similar property in the case of first price auctions. We propose an example of a first (or second) price auction game with direct externalities in which the grand coalition is not core-stable. Our example confirms that direct externalities make cooperative behavior difficult, as already suggested in the literature, but gives a more precise content to this phenomenon. Jehiel and Moldovanu (1996) concentrate on "negative externalities" (i.e., a bidder suffers more if a competitor wins the auction than if the object is not sold at all) and show that, under reasonable assumptions, no agreement between (some of) the buyers and/or the seller can be stable. They thus depart from collusion of the bidders in the original auction game $\Gamma$. Our core, which only captures the latter form of collusion, is never empty in Jehiel and Moldovanu (1996)'s framework. Caillaud and Jehiel (1998) point out that direct externalities can prevent the grand coalition from being ex post efficient but do not address the question of its ex ante stability.

We extend our definition of core-stability to the case of a single bidding ring $R$, which does not gather all the bidders (e.g., $R$ consists of "incumbents", as opposed to "new comers"). In other words, we assume that the bidders outside $R$ act as singletons, which is customary in the literature (see, e.g., Marshall and Marx (2007)). Let $S$ be a subset of $R$; the secession of $S$ from $R$ cannot modify the absence of cooperation outside $R$; however it affects the players who are in $R$ but not in $S$. Hence, $R$'s beliefs $B(R)$ take the form of a partition of the players in $R$ but not in $S$, together with $S$ itself and singletons for the players not in $R$. We define the core stability of a bidding $R$ in the same way as for the grand coalition $N$, by considering, for every equilibrium specification $\sigma$, the restriction of $v_\sigma$ to $R$, with the previous restriction on beliefs. According to this definition, single bidders are core-stable. In the case of second price auctions with no direct externalities, all rings are core-stable. Simulation results of Marshall et al. (1994) illustrate the same property for first price auctions. In the previously mentioned example (where, due to externalities, the grand coalition is not core stable), there exist non-singleton rings which are core-stable. In another, three person, auction game, with negative externalities, a natural two bidder cartel is not stable. Direct externalities thus offer a possible explanation for collusion patterns observed in practice.

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