Multi-unit Auctions with Resale

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Extended Abstract
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This project investigates how the presence of after markets (resale markets) affects the bids and generated revenues in multi-unit-auctions. Understanding bidding strategies in auctions where resale possibilities exist is important to achieve the best auction design given the objective of the auctioneer.

It is not uncommon for the bidders of an auction to trade the auctioned units among each other in post auction markets. These after auction resale markets are difficult to prohibit and even if they are considered to be disadvantageous, the auctioneer often cannot commit not to allow post-auction trades among bidders. If bidders look for such ex-post trade opportunities, it means that the winner of the good is not the one which values the auctioned good at most. One may think of that in many high-stake auctions, such as the Federal Communications Commission (FCC) auctions where personal communications services licenses are auctioned, or the Treasury auctions, the aim is to allocate the units to the highest valuation bidders. If the auction fails to allocate the units efficiently, then, after the auction, the bidders who valued the goods more than the winner but lost the auction are going to approach to the winner and try to buy the goods from her. In such a situation, there is a price that they can agree on and trade between each other. If some resale transactions among bidders take place, it means efficiency improves by the presence of those markets. Therefore, if the ultimate purpose of the auction is efficiency, one may argue that the resale markets are good. However, from a different perspective it may not be so good because if the auction does not allocate the units efficiently by itself and bidders reallocate them later with some profitable transactions, then it means that some revenue is left on the table during the auction stage. This paper provides an analysis of all these issues under three well known auction design: First-price, second-price, and generalized Vickrey auctions.

When multi-unit goods are auctioned, the bidders' demand differs depending on how large or small they are. For example, in the FCC auctions, some bidders are smaller than the others because of their geographical restrictions or financial constraints and therefore they would like to bid on only small number of the licenses.[2] Such asymmetry in bidders' demand

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[2] The asymmetry of bidders can be due to their financial restrictions, geographical location, endowment or having different use of the good. The literature on asymmetric auctions (e.g. Hafalir and Krishna, 2008; Maskin and Riley,

makes it hard to study how the anticipation of resale markets affects the bids and hence the generated revenue of the seller. Two dynamics are effective in determining the answer to this question and the tradeoff between these two effects will determine the expected revenue from the auction:

- Revenue increasing effect: Some bidders, the ones who do not value the auctioned goods so much, may want to bid high and win the auction by hoping that they will sell the good to some other bidders later at a higher price. This kind of speculative bids will increase the revenue of the seller.

- Revenue decreasing effect: There is a possibility that bidders, who know that they can get the goods they want in a post-auction trade, may not bid in the auction as aggressively as they would if no resale was allowed. Such a behavior would decrease the revenue that the auctioneer collects in the auction.

Based on a model developed by Krishna and Rosenthal (1996)[3], we model a situation where there are large bidders (who would like to buy multi units of the auctioned good) and small bidders (who would like to buy a single unit). There are complementarities (synergies) between goods such that for large bidders having more than one good is more valuable than the sum of values of individual goods. For example, in spectrum auctions, some big telecommunication companies might value multiple licenses to serve large geographical locations more than the sum of the values of each license because marginal cost of serving to larger area can be decreasing. In order to highlight the main points of the argument, we simplify the model so that there is only one large bidder and two symmetric small bidders. The large bidder is interested in two units of the good and each small one is interested in a single unit. There are two units of the good to be sold. Private signals regarding the valuation of each type of bidder is independently distributed by the same distribution function. Three auction mechanisms are studied:

- *First Price Sealed Bid Auctions* where each unit of the good is auctioned separately and the winner is the one who bids the highest and she pays what she bids.

- *Second Price Sealed Bid Auctions* where again each unit of the good is auctioned separately and the winner is the highest bidder but this time she pays the second highest bid.

- *Generalized Vickrey Auctions* where all the units are sold in a single auction. Each bidder submits bids for each package of units that she is interested and the units are allocated to the bidders who have the highest total bids for two objects. Each winner pays the price that is equivalent to the externality she exerts on other competing bidders.

The resale markets are designed so that the winners of the auction can make take-it-or-leave-it offer to the unsuccessful bidders who lost the auction.

2000; Krishna and Rosenthal, 1996; Chernomez and Levin, 2008) highlights the possibility of inefficient allocation of the good after the auction in such situations.

[3] Krishna and Rosenthal (1996) develop this model in order to study the FCC auctions of licenses for the radio-frequency spectrum. Their model does not allow for post-auction resale.

We compare the revenues generated by each type of auctions when resale is allowed and when it is not. Some striking theoretical results have been derived:

- For the generalized Vickrey auction case, the presence of resale markets does not make any difference. Bidding truthfully is equilibrium with or without resale markets and the units are allocated efficiently.

- Although the generalized Vickrey auction achieves efficient allocation even in the presence of resale markets, it does not maximize the revenue.

- In Second Price Auction without resale, truth-telling is equilibrium. However, it is not the case when the resale markets exist. Any symmetric and *weakly information invariant equilibrium*[4] of a second price auction with resale generates higher revenue than second price auction without resale and generalized Vickrey auction.

- In First Price Auction with resale, the large and small bidders use the same bidding strategies and for particular distributions the revenue generated by first price auction with resale exceeds the revenue of first price auction without resale.

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[4] An equilibrium notion developed by Borgers and McQuade (2007) and it satisfies a robustness property. In our setting it means the equilibrium strategies are independent of the distribution of valuations.