INEFFICIENCIES IN TRADE NETWORKS

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Abstract. Buyers and sellers make relationship specific investments to enable trade, which is modeled as a network formation problem. Inefficiencies are investigated and depend on bargaining power and the investment protocol: whether buyers and sellers must make fixed non-substitutable exogenous investments, or whether they can endogenously negotiate individual contributions. It is shown that inefficiencies can consume *all* the gains from trade, except when exogenous investment are made in proportion to bargaining power. Inefficiencies are partitioned into three types: over-investment in relationships used only to generate outside options, under-investments in relationships that should be used for trade, and coordination inefficiencies. With exogenous investments, under-investment inefficiency can consume all the gains from trade whenever investment shares are not exactly proportional to bargaining power, whilst over-investment inefficiency is bounded. With endogenous investment, there is no under-investment inefficiency, but over-investment inefficiency can consume all the gains from trade.

Key Words: Trade networks, network formation, bargaining, outside option, inefficiency, relationship specific investment, hold up.

1. Motivation

In many markets costly relationship specific investments are necessary to enable trade: In labour markets relationship specific investments may occur in the form of interviews and other aspects of the recruitment process; a supplier may have to learn the specific requirements of a manufacturer and the manufacture the capabilities of the supplier before they can trade;[1] the same may be true of activities outsourced by a firm; prior to the acquisition of a firm a relationship specific investment may have to be made by the acquirer through the due diligence process whilst the target may have to share sensitive

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[1]See Uzzi (1996) and Nishiguchi (1994).

information;[2] before buying a painting a collector may invest in authenticating it; etc.[3] This paper analyzes inefficiencies in these investment decisions when contracts cannot be written[4].

Relationship specific investments have been extensively analyzed in the literature and it is well understood, in a variety of contexts, when such investments will be made efficiently and what inefficiencies might be present.[5] Even in the case of trade network formation these questions have received attention from Kranton and Minehart (2000b), (2001). This paper provides a more thorough analysis of inefficiencies in the context of trade network formation than has previously been undertaken. It's main contributions[6] are to: (i) investigate how large inefficiencies can get; (ii) determine which inefficiencies are important when; and (iii) utilize these results to consider the impact of parties endogenously negotiating their investments rather than having to make fixed exogenous investments to enable trade.[7]

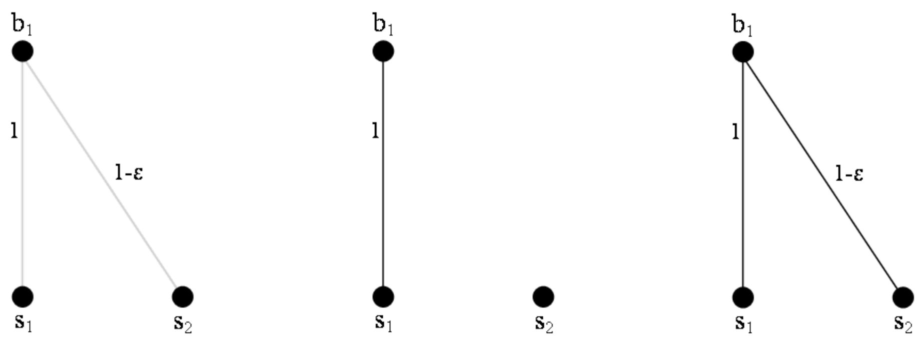
Although a full review of the literature is delayed until Section 6, Kranton and Minehart (2001) provides a useful point of comparison. Kranton and Minehart (2001) consider inefficiency in trade network formation where outcomes are determined by an auction process over the network such that buyers have all the bargaining power, only buyers can form links, sellers are identical and ex-ante buyers are identical. Their main and salient result is that the efficient network is stable. This paper considers a more general environment. There are ex-ante heterogenous gains from trade between buyer-seller pairs, different levels of bargaining power, and alternative investment protocols: buyers and sellers must make fixed non-substitutable exogenous investments, or they can endogenously negotiate individual contributions. Whilst it is well understood that moving away from the special case considered by Kranton and Minehart inefficiencies may be present in the stable networks, it might seem reasonable to conjecture that inefficiencies are initially bounded and

[2]See Brandenburger and Nalebuff (1996) (pg 80).
[3]Although the model presented is stylized and not intended to capture the functioning of any one of these specific markets it is hoped that it will capture features present in these markets and many others.
[4]Whilst this assumption abstracts from the underlying causes of contractual incompleteness, it can be both theoretically and empirically motivated. When complete, state contingent contracts are cannot be written contracting is not necessarily an improvement over ex-post negotiation. See for example Che and Hausch (1999), Segal (1999) and Bernheim and Whinston (1998). Further, Uzzi (1996) and Nishiguchi (1994) provide evidence of the gains from trade being determined after investment decisions have been made.
[5]See the literature review in Section 6.
[6]Other contributions are made in relation to the bargaining model developed to determine how gains from trade are split over formed networks. This model is intuitive, permits heterogenous gains from trade between buyer seller-pairs and has a number of appealing properties. See Section 3.
[7]To the best of my knowledge endogenous investments of this form have not been considered in the context of trade network formation or more generally.

small, at least on the best stable network, and then grow as the environment is changed more. This conjecture is incorrect. Moving only slightly away from the special case, in any one of a number of possible directions, inefficiencies on even the best stable network may consume *all* the net surplus generated by the efficient network (Proposition 1). Inefficiencies are then partitioned into three types: Over-investment in relationships used only to generate outside options, under-investments in relationships that should be used for trade, and coordination inefficiencies. Which inefficiencies are important depends on bargaining power and the investment protocol. With exogenous investments, under-investment inefficiency can consume all the gains from trade, except for when bargaining power is exactly proportional to parties' cost shares, whilst over-investment inefficiency is bounded. It might then be conjectured that permitting endogenous negotiation of investment shares will eliminate under-investment inefficiency thereby bounding overall inefficiency.[8] This conjecture is also incorrect. Although under-investment inefficiency is indeed eliminated, over-investment inefficiency can now consume all the gains from trade (Proposition 2).

To illustrate these results consider Example 1

Example 1. *Figure 1a shows potential gains form trade between buyers and sellers, where* ε *is a small positive number and the cost of link formation is* $c = \frac{1}{2} - \varepsilon$*. Buyers and seller first form links to enable trade and then bargain over the formed network. Each party can ultimately trade with at most one other.*



(A) Potential links (B) Efficient network (C) Complete network

FIGURE 1. Inefficiencies of stable networks

[8]Currarini and Morelli (2000) model non-cooperative network formation where the split of surplus is endogenously determined as part of the formation process. This is a little bit like moving to a contractible environment but is also similar to endogenizing investment contributions. They find that networks are formed efficiently when their condition of size monotonicity is satisfied.

The efficient network maximizes the net gains from trade (the realized gains from trade less costs expended on link formation) and is shown in Figure 1b. The net gains from trade generated by this network are $1 - \left(\frac{1}{2} - \varepsilon\right) = \frac{1}{2} + \varepsilon$*. Consider the different types of inefficiency possible in the context of this example. If the empty network were formed there would be inefficiency due to an under-investment in trade links:* b_1 *and* s_1 *do not trade with anyone and could jointly gain more from trading than it would cost to form a link between them. If the complete network were formed there would be inefficiency due to an over-investment in non-trade links: the link between* b_1 *and* s_2 *would not be traded over and would therefore make no contribution to total surplus but still cost* c *to form. Finally, if only the link between* b_1 *and* s_2 *were formed there would be coordination inefficiency: were the link between* b_1 *and* s_1 *formed instead, gains from trade could be increased without affecting the total amount spend on forming links.*

Let sellers have all the bargaining power such that their trade partners receive only the minimum payoff necessary to prevent them from trading with another seller they are connected to. Consider the case where both the buyer and seller must make equal exogenous investments of $\frac{c}{2} = \frac{1}{4} - \frac{\varepsilon}{2}$ *to form a link between themselves. The efficient network is not stable. Were the efficient network formed* b_1 *would have no alternative but to trade with* s_1 *and as sellers have all the bargaining power* s_1 *would extract all the gains from trade leaving* b_1 *with a negative payoff of* $-\left(\frac{1}{4} - \frac{\varepsilon}{2}\right)$*. The complete network is not stable either. As* b_1 *can trade with only one seller, the seller they do not trade with will receive a payoff of* $-\left(\frac{1}{4} - \frac{\varepsilon}{2}\right)$*. The empty network is the unique stable network.*

Suppose now that buyers and sellers could negotiate over the link formation costs they each pay and that a link between them is formed as long as their combined investment in it is greater than c*. The empty network is no longer stable:* b_1 *and* s_1 *could negotiate to split the cost* c *such that* s_1 *pays the entire cost, leaving* b_1*'s payoff unaffected but strictly increasing* s_1*'s payoff. However, the efficient network is not stable either. Buyer* b_1 *will want to form a link to* s_2*. Once this link is formed* b_1 *can play* s_1 *and* s_2 *off against each other and will be guaranteed a payoff of at least* $1 - \varepsilon$*. Forming a link to* s_2*, which* b_1 *alone pays for, increases* b_1*'s payoff by* $1 - \varepsilon$ *at a cost of* $\frac{1}{2} - \varepsilon$*. The unique stable network is now the complete network.*

When buyers and sellers had to pay an equal share of the costs of link formation the unique stable network was the empty network: b_1 *refused to invest in a link to* s_1 *for fear of hold up. This led to an under-investment in relationships that would have been generated more gains from trade than the investment would have cost. Further, this* *under-investment inefficiency eliminated all the gains from trade that would have been generated by the efficient network. Allowing buyers and sellers to negotiate their cost shares eliminates this under-investment inefficiency. However, there are now more links formed in the stable network than the efficient network:* b_1 *invests in a link they have no intention of ever trading over in order to improve their terms of trade. Over-investment inefficiency now consumes all the gains from trade as* $\varepsilon \to 0$*: Eliminating the under-investment inefficiency by permitting parties to negotiate their investment contributions results in over-investment inefficiency that consumes all the gains from trade.*

The paper proceeds as follows. Section 2 sets up the model before Section 3 analyzes bargaining over a fixed network. This section presents a new cooperative model of bargaining over networks with heterogenous gains from trade filling a gap in the literature and permitting the analysis of later sections. Section 4 then identifies the efficient network(s) providing a useful benchmark for the set of stable networks which are considered in Section 5. Section 5 examines the inefficiency present in stable networks and contains the main results of the paper. Section 6 then places these results in the context of the related literature before Section 7 concludes. Proofs are relegated to the Appendices.

2. Model Set Up

There is a set of m buyers denoted B and a set of n sellers denoted S. The value of trade between a buyer (b_i or i) and a seller (s_j or j) is given by $\alpha_{ij} \geq 0$ where the first subscript always refers to the buyer and the second subscript to the seller. The $m \times n$ dimensional matrix α describes the value of all potential bi-lateral trades.

In stage one pairs of buyers and sellers form links between themselves at a cost of $c > 0$ for each link. Two cases are considered: a buyer and seller each pay some fixed proportion of c or they bargain over how c is split such that the link is formed whenever they jointly benefit from it. The formation of links is binary (they are formed or not formed) and is represented in the $m \times n$ dimensional matrix of ones and zeros L. The formed links determine which buyers can trade with which sellers. It is assumed that these investments are non-contractible.

In stage two buyers and sellers are matched where L determines the possible matches and α determines the gains from trade from these matches. A match $\mu(L)$ is a function from the set of all buyers and sellers into itself, $\mu(L) : B \cup S \to B \cup S$, such that: (i) $\mu(i, L) \in S \cup i$ (a buyer is matched to themselves if they are not matched to a seller); (ii)

$\mu(j,L) \in B \cup j$; (iii) if $\mu(i,L) = j$, then $\mu(j,L) = i$ and $l_{ij} \in L$; and (iv) if $\mu(j,L) = i$, then $\mu(i,L) = j$ and $l_{ij} \in L$. The set of all possible matches for a network L is denoted $M(L)$. Buyer b_i's payoff is denoted by $\pi_i^B(L,\alpha) : \Re^{+^{m\times n}} \to \Re^+$ and seller s_j's payoff by $\pi_j^S(L,\alpha) : \Re^{+^{m\times n}} \to \Re^+$, although, where obvious, notation will be abused and the superscripts S and B dropped.

A coalition of buyers and sellers $\Psi \subseteq B \cup S$, can obtain a maximum surplus (joint payoff) of $V(\Psi, L)$: $V(\Psi, L) = max_{\mu\in M(L)} \left\{\sum_{i\in\Psi} \alpha_{i\mu(i)}\right\} = max_{\mu\in M(L)} \left\{\sum_{j\in\Psi} \alpha_{\mu(j)j}\right\}$, where $\alpha_{ii} = \alpha_{jj} = 0$.

Where there can be no confusion notation will be abused the argument L dropped from the above notations.

The symmetry of buyers and sellers in the model means that for all the results found for buyers there are equivalent results for sellers. To save on notation some results and definitions are stated for just buyers or just sellers.

3. Bargaining over a Fixed Network

To analyze network formation a backward induction approach will be applied and trade over a given network considered first. This section proposes a mapping from the network structure (L,α) into outcomes (matches and payoffs) and identifies how the network structure affects these outcomes.

The following example helps to clarify the bargaining outcomes that might be expected over a network with heterogenous gains from trade:

Example 2. *Consider Figure 2:*

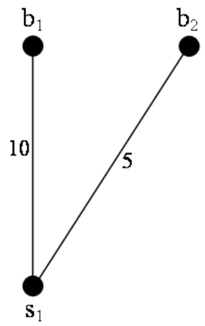


Figure 2. Simple Network

A natural outcome on this network would be for buyer b_1 *to be matched to seller* s_1 *and for* s_1 *to receive a payoff* $\pi_{s_1} \in [5,10]$, *whilst* b_1 *receives a payoff* $\pi_{b_1} = 10 - \pi_{s_1} \in [0,5]$. *In* *this example the network provides s_1 with the outside option of trading with b_2 instead of b_1 and they should be able to use this outside option to guarantee themselves a payoff of at least* 5*. One way to think about this outside option is as the (best) payoff s_1 could achieve if their trade partner (b_1) were removed from the network. Buyer b_1 has no outside option and so the best they can guarantee themselves is* 0*. Equivalently, were s_1 removed from the network b_1 would receive a payoff of* 0*.*

The bargaining outcomes that are considered in this section satisfy the following three condition:

Condition C1: The match generates the maximum possible combined surplus.

Condition C2: Matched buyer-seller pairs split their trade surplus such that each party receives a payoff at least as large as the payoff they could achieve were their trade partner removed from the network.

Condition C3: Additional gains from trade, after each party has received their minimal payoff, are exogenously split in proportion to buyers' bargaining power relative to sellers' (β).

It can immediately be seen that these three conditions are consistent with the bargaining outcomes that would be expected on the simple network presented in Example 2. Applying condition C1 requires that b_1 trades with s_1. Condition C2 restricts b_1 to receive a payoff greater than or equal to 5 and places no lower bound on the payoff that can be received by s_1. Condition C3 generates the following payoffs: $\pi_{b_1} = 5 + \beta 5$, $\pi_{s_1} = (1 - \beta)5$ and $\pi_{s_2} = 0$.

It will be shown that conditions C1, C2 and C3 generate a mapping from each network into a unique vector of payoffs (for a given β). Conditions C1 and C2 will be shown to be relatively minimal requirements.[9] Condition C3 is most restrictive and it is this condition that pins down the unique set of payoffs. To emphasize that many of the results presented in this section do *not* rely on this third condition it will only be imposed at the end of this section.

Assumption: Conditions C1 and C2 are satisfied by bargaining outcomes.

The first condition identifies a generically unique match for any given the network. This match is referred to as the 'optimal match': $\mu(L)^* = argmax_{\mu \in M(L)} \left\{ \sum_{i \in B} \alpha_{i\mu(i)} \right\}$. If $b_{i'}$ is matched to $s_{j'}$ in this optimal match then the following notation will be used:

[9]It will be shown that these conditions are necessary for no connected buyer-seller pair to be able to profitably deviate by breaking their existing matches and instead trading with each other.

$s_{j'} = \mu(i', L)^*$ and $b_{i'} = \mu(i', L)^*$. The second condition is not yet well defined but will place a lower bound on the payoffs that can be received by any party. To determine the payoff a party 'could achieve were their trade partner removed from the network' the negotiated rematching process is considered:

Definition: The *negotiated rematching process* proceeds as follows.

(1) Consider a bargaining outcome (matches and payoffs) such that $b_{i'}$ trades with $s_{j'}$: $s_{j'} = \mu(i')^*$.
(2) Suppose now that $s_{j'}$ is removed from the network and consider this new network.
(3) Sellers payoffs can be thought of as the price they charge. At the current price vector consider which buyers demand which goods over this new network. If it is possible to match each buyer to a seller in their demand set[10] the process stops.
(4) If the process does not end there must be some minimal over demanded set of sellers.[11] The price charged by these sellers is then increased by one unit. Return to step 3.[12]

Although this negotiated rematching process is run by a fictitious auctioneer, it is intended to capture a decentralized renegotiation of matches and terms of trade following the removal of a party from a network:[13] Following the removal of a seller other sellers cannot be expected to lower their prices in response to increased demand and there is no reason for buyers to bid up seller's prices beyond the minimum necessary levels for the market to clear. Whilst the payoff a party achieves in the negotiated rematching process following the removal of their trade partner from the network does depend on the initial prices being charged by sellers:[14]

Lemma 1. *For any initial bargaining outcome in the core, following the removal of $\mu(i')^*$ the negotiated rematching process:*

[10]A buyers demand set is the set of sellers with whom they most like to trade at current prices.
[11]$\Psi_s \subset S$ is minimally over-demanded if no strict subset of Ψ_s is also over-demanded.
[12]This process can also be run following the removal of a buyer from the network with the roles of buyers and sellers reversed.
[13]The negotiated rematching process is adapted from the multi-unit auction mechanism analyzed by Demange, Gale and Sotomayor (1986) and discussed in Roth and Sotomayor (1990). The only differences are that as a rematching process is being modeled the initialization is the old price vector rather than the zero price vector and trade is precluded between buyers and sellers who are not linked, although this can be modeled in the multi-unit auction mechanism by setting the gains from trade to zero for unlinked buyers and sellers.
[14]For example the negotiate rematching process only ever increases the price of sellers so if sellers' payoffs on the initial network are very high, their payoffs following the negotiated rematching process will also be very high.

(i) *maximizes the combined gains from trade realized on the reduced network* $L - \mu(i')^*$*; and*

(ii) *yields a payoff for* $b_{i'}$ *of* $\pi^B_{i'}(L - \mu(i')^*)^{BO}$*, where* $\pi^B_{i'}(L - \mu(i')^*)^{BO}$ *is* $b_{i'}$*'s highest possible core payoff on the reduced network* $L - \mu(i')^*$.

Proof. It is shown in Appendix A that the negotiated rematching process will terminate at a core outcome. Part i) then follows immediately. Lemma 6 in Appendix A shows that for any initial bargaining outcome in the core the negotiated rematching process following the removal of supplier $\mu(i')^*$ gives remaining suppliers a payoff equal to $max\{\pi^S_j(L), \pi^S_j(L - \mu(i')^*)^{BO}\}$. By construction $b_{i'}$ must bid up the payoff of any supplier they ultimately trade with and so can only be matched to a seller receiving a payoff $\pi^S_j(L - j')^{BO}$. If matched $b_{i'}$ must then receive a payoff equal to $\pi^B_{i'}(L - \mu(i')^*)^{BO}$. If unmatched $b_{i'}$ receives a payoff of zero, but this will then be their unique core payoff on the network $L - \mu(i')^*$. Thus regardless of whether $b_{i'}$ is matched or not they receive a payoff $\pi^B_{i'}(L - \mu(i')^*)^{BO}$. □

When referring to the negotiated rematching process in the rest of the paper it will be assumed that the initial outcomes on the network are in the core such that the results of Lemma 1 hold.[15] Although the core is formally defined in Appendix A, for the purpose of interpreting Lemma 1 it will be more helpful to apply the result that a payoff and match are in the core if and only if no buyer-seller pair can form a deviating coalition by, for example, breaking their existing matches and rematching to each other.[16]

The negotiated rematching process itself makes precise what is meant by the payoff a party 'could achieve were their trade partner removed from the network'. Lemma 1 shows that this payoff corresponds to the highest payoff that can be achieved by $b_{i'}$ on $L - \mu(i')^*$ without some buyer-seller pair having a profitable deviation. This can then also be interpreted as the best $b_{i'}$ can hope to do without their trade partner $\mu(i')^*$.

Definition: $b_{i'}$'s *outside option value* ($\underline{\pi}_{i'}$) is the payoff they would receive from the negotiated rematching process were their trade partner $\mu(i')^*$ removed from the network.[17]

So far a generically unique match has been identified for each network and bounds have been placed on possible payoffs. However, nothing has been said about how the network structure affects these bounds. The value of each party's outside option is closely related

[15] This is consistent with the bargaining outcomes proposed in this section, which will be shown to be in the core.

[16] See Shapley and Shubik (1972) and Roth and Sotomayor (1990).

[17] The negotiated rematching process will generate a unique outside option value for each party.

to the rematching that must occur, by condition 1, were their trade partner removed from the network.

Definition: An *optimal rematch* rematches buyers and sellers following the removal of a party from a network such that the combined gains from trade are maximized. $s_{j'}$'s *outside trade partner* ($\nu(j')$) is the buyer they would be matched to in an optimal rematch following the removal of their trade partner from the network: $\nu(j') = \mu(j', L - \mu(j', L)^*)^*$.[18]

The optimal rematch following the removal of a party from a network is generically unique and must occur for the new bargained outcome to be consistent with condition C1. By Lemma 1 the negotiated rematching process implements the optimal rematch and rematches the party who's trade partner is removed from the network to their outside trade partner.[19]

It is shown in Appendix A that the optimal rematch can be represented as a chain (a sequence of links that share one of the parties they connect in common). If $s_{j'}$'s trade partner ($b_{i'}$) is removed from the network and $s_{j'}$ is rematched to $b_{i''}$, $b_{i''}$ might have already been matched to another seller $s_{j''}$. Seller $s_{j''}$ would then be displaced and may be rematched to some other buyer $b_{j'''}$ and so on. This optimal rematch then follows the following chain of links: $l_{i'j''}; l_{i''j''}; l_{i''j'''}; \ldots$. Further, this chain alternates between links utilized for trade in the optimal rematch $l_{i'j''}, l_{i''j'''}$ and links that cease being used for trade in the optimal rematch $l_{i''j''}$. For reasons that will become clear this chain of rematches is referred to as an 'outside option chain'.

Definition: Seller $s_{j'}$'s *outside option chain* is defined as the sequence of links reached by the optimal rematch when represented as a chain, following the removal of their trade partner ($\mu(j', L)^*$) from the network.[20][21] A link is *upstream* in seller $s_{j'}$'s outside option chain of another link if it is earlier in the sequence of links that constitute $s_{j'}$'s outside option chain.

A party's outside option chain determines their outside option value in a relatively simple way. Consider a seller $s_{j'}$ and suppose that their trade partner is removed form the

[18]When $\mu(j', L - \mu(j', L)^*)^* = j'$, j' is said to have no outside trade partner.
[19]A party's outside trade partner is then generically unique. Unlike parties' trade partners, $\nu(j') = b_{i'}$ does *not* imply $\nu(i') = s_{j'}$.
[20]Outside option chains, although derived and motivated in a very different way, are similar to the opportunity paths identified by Kranton and Minehart (2000a) for networks with homogeneous gains form trade. Outside option chains can be viewed as a generalization of opportunity paths.
[21]Generically each party's outside option chain is unique and in this paper each party's outside option chain will be treated as unique. When parties have multiple outside option chains any one of them can be selected and the same results will carry through.

network. Denote the set of newly traded over links in $s_{j'}$ outside option chain by $L^{j'}_{s\to b}$ and the set of links that are ceased trading over by $L^{j'}_{b\to s}$.

Lemma 2.

(i) *Seller $s_{j'}$'s outside option value is $\underline{\pi}^S_{j'} = \sum_{l\in L^{j'}_{s\to b}} \alpha_l - \sum_{l\in L^{j'}_{b\to s}} \alpha_l$;*
(ii) *Buyer $\mu(j',L)^*$'s payoff when their trade partner $s_{j'}$ receives only their outside option is their Vickrey payoff (their marginal contribution to the grand coalition); and*
(iii) *These payoffs correspond to seller $s_{j'}$'s and buyer $\mu(j',L)^*$'s buyer optimal core payoffs.*

Lemma 2 is proved in Appendix A. It uncovers specifically how the network structure determines each party's outside option payoff- the minimal payoff they could receive over the network. It is also motivates the use of the negotiated rematching process to define the payoff a party 'could achieve were their trade partner removed from the network': For seller $s_{j'}$ this payoff is equal to the lowest payoff that they can achieve in any core outcome on the initial network L: $\pi_{j'}(L)^{BO} = \pi_{j'}(L - \mu(j',L)^*)^{SO}$.

For the interested reader a simple process for decomposing any network into a directed network that simultaneously identifies all parties outside option chains is presented in Appendix B. This provides a powerful tool for taking any network and quickly determining which links matter and how for each party's payoff. The process is possible because in every outside option chain parties are only ever rematched to their outside trade partner. It also therefore identifies an interdependence between parties' outside option chains.[22]

Having identified a generically unique match implemented by a network and bounded each party's payoff, additional restrictions can be placed on the bargaining outcomes to identify unique payoffs. To do this condition 3 is imposed: Matched buyer-seller pairs split the remaining gains from trade, after they have each received their outside option payoffs, in proportion to buyers' bargaining power relative to sellers (β) which is exogenously given.

Assumption: Condition 3 holds.

[22]Suppose seller $s_{j'}$ is downstream of seller $s_{j''}$ in $s_{j'''}$'s outside option chain. The links which determine seller $s_{j'}$'s outside option will also affect seller $s_{j''}$'s outside option. In particular, an increase in seller $s_{j'}$'s outside option value will have a 1:1 affect on seller $s_{j''}$'s outside option value.

By construction the role the network plays is in affecting bargaining outcomes is through each party's outside option.[23] Gains from trade remaining after each party has received their outside option are split such that the buyer receives a proportion β and the seller a proportion $1-\beta$ where β is an exogenous parameter. Assuming that buyers' bargaining power relative to sellers (β) is the same for all buyer-seller pairs can be motivated by thinking of the institutional environment of the market determining β.[24][25]

Payoffs can therefore be represented as follows:

$$\pi^B_{i'} = \underline{\pi}^B_{i'} + \beta(\alpha_{i'\mu(i',L)^*} - \underline{\pi}^B_{i'} - \underline{\pi}^S_{\mu(i',L)^*}) \tag{1}$$

$$\pi^S_{j'} = \underline{\pi}^S_{j'} + (1-\beta)(\alpha_{\mu(j',L)^*,j'} - \underline{\pi}^B_{\mu(j',L)^*} - \underline{\pi}^S_{j'}) \tag{2}$$

Remark: Each party's payoff can be represented as a weighted sum of the gains from trade over the link they trade over, the links in their outside option chain and the links in their trade partners outside option chain by applying Lemma 2. These payoffs are still in the core as the core is convex and by Lemma 2 the set of payoffs where buyers receive only their outside option payoff ($\beta = 0$) is the seller optimal point of the core and the set of payoffs where sellers only receive their outside option payoff ($\beta = 1$) is the buyer optimal point of the core.

This representations of payoffs permits, for a given level of bargaining power, each network to be mapped into a unique set of payoffs. More importantly, for the purpose of examining network formation, this process determines how the network structure affects these payoffs. In particular the process identifies which links affect which parties' payoffs and how.

[23]Outside options in this paper play a similar role to outside options in an alternating offer bargaining game where there is some probability that the trading opportunity will be lost each period. Outside options play a different role in an alternating offer bargaining game where the cost of delay is captured in time preference. In that case outside option only have an affect on the split reached when they bind. See Binmore, Rubinstein and Wolinsky (1986). This role of outside options employed here follows much of the search literature (see Rogerson, Shimer and Wright (2005)) but also differs from much of the contract theory literature (see Malcomson (1997)).

[24]This assumption identifies a subset of points in the core corresponding to different value of β. Whilst allowing for heterogeneous bargaining powers for different buyer-seller pairs would generate outcomes that cover the core, it would also included outcomes outside the core. To fully characterize the core heterogeneous bargaining powers could be permitted, but with additional restrictions on the β's.

[25]Restricting attention to only this subset of core outcomes can be motivated by the equilibrium payoffs of a public alternating offer bargaining game over the network. When the gains from trade are homogeneous, a public alternating offer bargaining game, as considered by Corominas-Bosch (2004), identifies the same subset of the core considered in this paper. This alternating public offer bargaining game is defined, and equivalence to the core outcomes considered in this section, proved in Section AM-2 of the additional material.

4. Efficient Network Formation

When considering network formation it is useful to have the benchmark of efficient networks.

Definition: A network will be viewed as efficient (and referred to as efficient) when it maximizes the net gains from trade.[26] The *efficient match* is the optimal match for the efficient network.[27]

Definition: The *net gains from trade* generated by a network L with $Q(L)$ links ($NGT(L)$) is the trade surplus (TS) generated by the network less the costs of forming the links in L:

$$NGT(L) \;=\; TS(L) - cQ(L) = \sum_{i \in B} \alpha_{i\mu(i,L)^*} - cQ(L)$$

where $\alpha_{ii} = 0$, $\forall i \in B$.

The efficient network only identifies a subset of the Pareto frontier. However, consider a network L that is not efficient but is Pareto efficient. There will always exist transfers that could be made on the efficient network that would constitute a Pareto improvement to L.[28] In efficient networks there are no links formed and not traded over.

5. Network formation

The objective of this section will be to compare networks that are likely to be formed (stable networks) to the efficient network. Broadly, the following questions will be considered: When are efficient networks stable? How inefficient can stable networks get in different circumstances? What types of inefficiencies can be present? When are these different types of inefficiencies important? In order to address these questions it will be necessary to define a measure of the size of inefficiencies, identify different types of possible inefficiencies and identify conditions under which networks can be considered stable. The main results of the paper will then be presented.

[26]This notion of efficiency was introduced in Jackson and Wolinsky (1996) and referred to as 'strong efficiency'.

[27]Generically the efficient network is unique and it will be treated as unique for the rest of the paper. However, were there multiple efficiency networks then these would all generate the same net gains from trade (by definition) and any of them could be used as an equally valid point of comparison for stable networks.

[28]Further, it is not necessary for transfers to be possible between all buyers and sellers to achieve this Pareto improvement. For the Pareto improvement to occur transfers only have to be made over the links that are traded over in network L. See Jackson (2003).

5.1. **Measuring the size of inefficiencies.** When the cost of link formation is zero the complete network can be formed and the size of any inefficiency goes to zero. This corresponds to markets that have traditionally been considered where buyers can costlessly transact with any seller. When the cost of link formation is larger than the potential gains from trade between any buyer and any seller the empty network will be the unique stable network and it is also the efficient network. This corresponds to the case of transaction costs being prohibitively high for any trade. Thus there is no inefficiency for both very large costs of link formation and no cost of link formation. This paper is concerned with the case of intermediate costs of link formation.

To consider the size of inefficiency for intermediate costs of link formation it is useful to compare the net gains from trade generated on the efficient network to the net gains form trade generated by stable networks, and in particular the best stable network and the worst stable network. The best and worst stable networks correspond to the stable networks with the highest and lowest net gains from trade respectively:

Definition: The *best case efficient loss* ($BCEL$) is the proportion of the net gains from trade generated by the efficient network that are lost on the best stable network ($L^{\overline{s}}$): $\frac{NGT(L^E)-NGT(L^{\overline{s}})}{NGT(L^E)} \in [0,1]$.[29]

Definition: The *worst case efficient loss* ($WCEL$) is the proportion of the net gains from trade generated by the efficient network that are lost on the worst stable network ($L^{\underline{s}}$): $\frac{NGT(L^E)-NGT(L^{\underline{s}})}{NGT(L^E)} \in [0,1]$.[30]

The best case efficiency loss measures the efficiency lost on the best stable network compared to the efficient network. For example a value of 0.5 implies that 50% of the net gains from trade realized on the efficient network are lost on the best stable network.

5.2. **Different types of inefficiency.** This section will identify different types of inefficiency that can be present in stable networks.

Definition: The inefficiency of a network L ($Inefficiency(L)$) is the difference between the net gains from trade generated by the efficient network (L^E) and the net gains from trade generated by L: $NGT(L^E) - NGT(L)$.

Three types of inefficiency can be identified in networks:

(1) under investment in trade links;

[29] This is the analog of the price of anarchy which is defined as $P_{anarchy} \equiv \frac{NGT(L^E)}{NGT(L^{\overline{s}})} \in [1,\infty)$.

[30] This is the analog of the cost of anarchy which is defined as $C_{anarchy} \equiv \frac{NGT(L^E)}{NGT(L^{\underline{s}})} \in [1,\infty)$.

(2) over investment in non-trade links (outside options); and
(3) coordination inefficiency.[31]

These types of inefficiencies for a network L are defined below:

Definition: Inefficiency due to *over-investment in non-trade links* is the resources allocated to forming links that are not used for trade on the network L. When there are Q links formed on the network L and K matches occur inefficiency due to over-investment in non-trade links is: $(Q(L) - K(L))c$.

Definition: Consider a network L such that a set of m' buyers B' and a set of n' sellers S' are unmatched. Let the $m' \times n'$ matrix $\hat{\alpha}'$ denote the potential net gains from trade between these buyers and sellers: the gains from trade between each possible unmatched buyer-seller pair less the cost of link formation c.[32] Let μ' be a matching on the set $B' \cup S'$ and denote the set of all such possible matchings M'. Inefficiency due to *under-investment in non-trade links* (UI) on the network L is then the maximum net gains from trade that could be obtained by matching unmatched parties on L: $UI(L) = max_{\mu' \in M'} \sum_{i' \in B'} \{\hat{\alpha}'_{i'\mu'(i')}\}$, where $\hat{\alpha}'_{i'i'} = \hat{\alpha}'_{j'j'} = 0$.

Definition: Coordination inefficiency on a network L ($CI(L)$) is the increases in the net gains from trade that could have been obtained by buyers and sellers if they had coordinated on the formation of links even after all over-investment inefficiency and under-investment inefficiency has been removed from L. Coordination inefficiency is given by: $CI(L) = NGT(L^E) - (TS(L) + UI(L) - K(L)c)$

Remark: Each type of inefficiency is greater than or equal to zero and by construction they are mutually exclusive and collectively exhaustive:

$$(Q(L) - K(L))c + UI(L) + CI(L) \quad = \quad NGT(L^E) - NGT(L)$$

It is interesting to note that over-investment in non-trade links will be most damaging when the cost of link formation is large, but this is when incentives for over-investment are lowest. Similarly under-investment in a trade link will be most damaging when the value of this link is very high relative to the costs of link formation but this is when

[31]To be in the core the gains from trade must be maximized by the match implemented on the formed network. Coordination inefficiency captures that the efficient match is not possible on the formed network.
[32]For example an unmatched buyer i' and unmatched seller j' have net potential gains from trade $\hat{\alpha}'_{i'j'} = \alpha_{i'j'} - c$.

the incentives for under-investment are lowest. Nonetheless, it will be shown that these inefficiencies can be important.

5.3. **Exogenous cost shares.** Before considering the different types of inefficiency and the size of inefficiencies in more detail a stable network has to be defined. To do this cost shares will be assumed (at first) to be exogenous:

Assumption: For every link formed the buyer pays γc and the seller pays $(1-\gamma)c$, $\gamma \in [0,1]$.

This assumption can be motivated by thinking of buyers and sellers both having to make separate investments to form a link where these investments are non-substitutable.[33]

A network will be viewed as stable (and referred to as stable) when it meets the requirement of pairwise Nash stability.

Definition: The simultaneous link formation game is a simultaneous move game of complete and perfect information. Buyers and sellers are the players. Their strategy sets consist of the different sets of links they could pay to form. Payoffs are the payoffs generated on the formed network (given β) less the costs paid towards forming links (given γ). When $\gamma = 1$ a link is formed if and only if the buyer chooses to form it, when $\gamma = 0$ a link is formed if and only if the seller chooses to form it and when $\gamma \in (0,1)$ a link is formed if and only if both the buyer and seller choose to form it.

Whilst it is natural to assume that a stable network should be a Nash equilibrium of the simultaneous link formation game this permits networks where a buyer and seller fail to coordinate on forming a link that would benefit them both. In particular the empty network will be stable $\forall \gamma \in (0,1)$. To eliminate these types of networks from the set of stable networks buyers and sellers will be allowed to coordinate on forming a link that benefits them both. This is captured through the requirement of pairwise stability.

Definition: A network L is *pairwise stable* if and only if:[34]

(i) All links formed benefit both connected parties:
- $\pi_i^B(L) \geq \pi_i^B(L - l_{ij}) + \gamma c$; and
- $\pi_j^S(L) \geq \pi_j^S(L - l_{ij}) + (1-\gamma)c$, $\forall l_{ij} \in L$.

(ii) Unformed links would not benefit both connected parties if formed:
- If $\pi_i^B(L + l_{ij}) - \gamma c > \pi_i^B(L)$, then $\pi_j^S(L + l_{ij}) - (1-\gamma)c < \pi_j^S(L)$; and

[33]It is (implicitly) assumed that buyers and sellers cannot make transfers based on these investments or more generally contract over these investments.
[34]This concept is introduced by Jackson and Wolinsky (1996).

- If $\pi_j^S(L+l_{ij}) - (1-\gamma)c > \pi_j^S(L)$, then $\pi_i^B(L+l_{ij}) - \gamma c < \pi_i^B(L)$, $\forall l_{ij} \notin L$.

where $L + l_{ij}$ is the network L with the additional link l_{ij}.

Combining the requirements of pairwise stability and that the links formed are a Nash equilibrium of the simultaneous link formation game, networks will be considered stable when they are pairwise Nash stable.[35]

Definition: A network is pairwise Nash stable if and only if it is both a Nash equilibrium of the simultaneous link formation game and pairwise stable.[36]

Remark: For all potential gains from trade ($\forall\alpha$), for all levels of bargaining power ($\forall\beta$) and for all cost shares where each party makes some contribution towards the cost of link formation ($\forall\gamma \in (0,1)$) there exists a stable network.[37]

The question of when efficient networks will be stable can now be considered. This can be done by simplifying the above conditions for a network to be stable using the properties of efficient networks. Necessary and sufficient conditions for the efficient network to be stable are identified and discusses in the additional material.[38] Of particular interest is the result that the efficient network will be stable when buyers (sellers) have to pay the entire cost of forming a link, $\gamma = 1$ ($\gamma = 0$), but have all the bargaining power, $\beta = 1$ ($\beta = 0$).[39]

Having defined a stable network under-investment and over-investment inefficiencies can be considered more precisely. Consider again the network in Example 1 and the case where both buyers and sellers had to make an equal exogenous investment for links to form. It was shown that the unique stable network was the empty network such that all the gains from trade that would have been generated by the efficient network were lost on the stable network. The best and worst case efficient loss for this example is therefore 100% and it is due to an under-investment in links that should be traded over. In this example there is no inefficiency due to over-investment in non-trade links. Example 3 below considers a network with over-investment inefficiency.

[35]Pairwise stability here refers to network formation. Matches on a given network may also be pairwise stable or not depending on whether two agents can profitably deviate in their trades on the formed network.

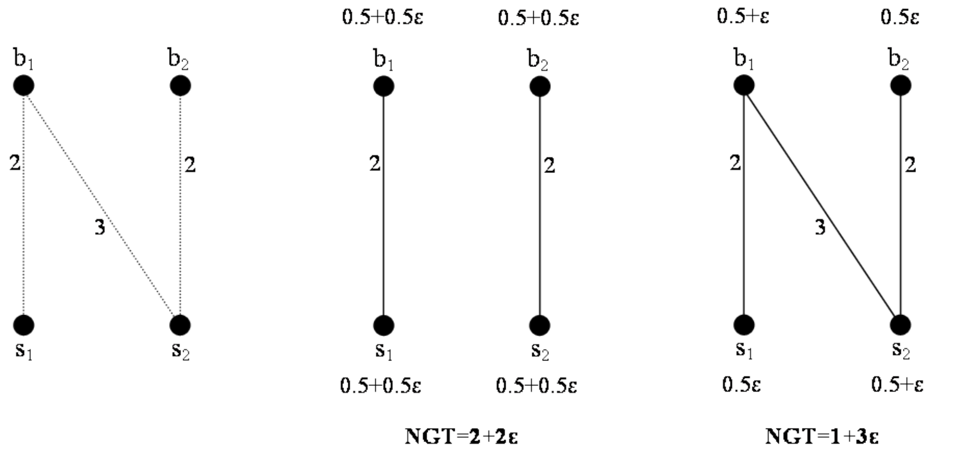
[36]Pairwise Nash stability is a relatively minimal requirement for a network to be stable. In particular a network can be pairwise Nash stable even if a buyer and seller could profit from forming a link between themselves and simultaneously deleting some other link.

[37]This is proved in Section AM-3 of the additional material.

[38]Section AM-4

[39]This corresponds to the main result of Kranton and Minehart (2001).

Example 3. *Assume that buyers bargaining power is $\beta = \frac{1}{2}$, costs are again shared evenly ($\gamma = \frac{1}{2}$) and the cost of link formation is $c = 1 - \varepsilon$.*



(A) Potential links (B) Efficient network (C) Stable network

FIGURE 3. Over-investment inefficiency

The efficient network is again unstable, but now it is because of over-investment in non-trade links. Buyer b_1 and seller s_1 can increase their payoffs by forming a non-trade link between themselves. Again the stable network found is the unique stable network. Here not all the net gains from trade generated by the efficient network are lost. However, half the gains from trade are lost as $\varepsilon \to 0$ (the best case and worst case efficiency loss is 50%).

Under-investment inefficiency in the first example is a direct result of the assumption that cost shares are exogenous. If buyers and sellers could negotiate over the costs of link formation they would always be able to negotiate a cost share that eliminated any under-investment inefficiency. This possibility is considered in the next section.

5.4. **Endogenous cost shares.**

Assumption: Link formation is endogenous to the network such that a link is formed whenever the buyer-seller pair jointly benefit from it.[40]

This assumption can be motivated in a couple of ways. Most simply buyers and sellers just have to make a joint investment where investment by one of them is perfectly substitutable with investment by the other. Alternatively, buyers and sellers may be able to

[40]This is made clearer by the definition of side payment pairwise stable networks given below.

make transfers to one another based on the non-substitutable investments they make.[41] With this new cost sharing arrangement it is necessary to adjust the definition of a stable network.

Definition: Given potential gains from trade α, a network L is *side payment pairwise stable* if and only if:[42]

(i) All links formed jointly benefit the connected parties:
- $\pi_i^B(L) + \pi_j^S(L) \geq \pi^B i(L - l_{ij}) + \pi_j^S(L - l_{ij}) + c$, $\forall l_{ij} \in L$; and

(ii) All links not formed would not jointly benefit both connected parties:
- $\pi_i^B(L) + \pi_j^S(L) \geq \pi_i^B(L + l_{ij}) + \pi_j^S(L + l_{ij}) - c$, $\forall l_{ij} \notin L$

where if an additional link $l' \notin L$ were added to L, the cost shares over each link $l \in L$ would remain the same on the network $L + l'$ as they were on L.[43]

When considering the simultaneous link formation game there are now many possible different cost sharing arrangements. All these possibilities are considered when defining a side payments pairwise Nash stable network.

Definition: A network is now a *Nash equilibrium of the simultaneous link formation game* if the costs of link formation are shared between the buyers and sellers forming each link such that the set of links formed by each party is a best response to the links formed by all other parties.[44]

Definition: A network is side payment pairwise Nash stable if and only if there exists some share of link formation costs such that the set of links formed is both a Nash equilibrium of the simultaneous link formation game and pairwise stable.[45]

[41] Brandenburger and Nalebuff (1996) provide an example of investments being negotiated endogenously in this way. When Craig McCaw wanted to acquire LIN and he was the only credible bidder, LIN paid Bell South's due diligence and other costs so they would enter the bidding process.

[42] This concept of pairwise stability is a natural extension of pairwise stability to environments with transfers and is introduced in Jackson and Wolinsky (1996).

[43] Whilst this condition does restrict the scope of possible profitable deviations (i.e. a link cannot be added and the cost shares of different links simultaneously changed) it is in the same sprit as pairwise stability which considered a network to be stable even if it is profitable for a link to be jointly formed whilst others are simultaneously deleted.

[44] Instead of searching over different possible cost sharing arrangements to identify the set of Nash equilibria, the simultaneous link formation game could be changed. The strategy set of parties in the simultaneous link formation game could be expanded such that players choose not only which links they want to form but also how much they are prepared to pay towards the formation of these links. A link is then formed if and only if the contributions named by the buyer and seller are sufficient to cover the cost of forming it.

[45] A side payment pairwise Nash stable network does not always exist. This is shown in Section AM-3 of the additional material.

Necessary and sufficient conditions for the efficient network to be stable are identified and discussed in the additional material.[46]

Having defined what is meant by a stable network when cost shares are endogenous the size of inefficiencies can be considered.[47] Consider again Example 1. When cost shares are endogenous it was argued that the unique stable network was the complete network and that as $\varepsilon \to 0$ all of the net gains from trade would be lost due to an over-investment in outside option links by b_1. This can now be made precise. In the stable (complete) network b_1 must pay the entire cost of forming the link l_{12} as s_2 does not trade and therefore receives no benefit from the link. However, the exact share of costs for link l_{11} is not pinned down. s_1 would be willing to pay up to ε for it and b_1 must then pay the rest. Thus $\pi_{b_1} \in [\varepsilon, 2\varepsilon]$ and $\pi_{s_1} \in [0, \varepsilon]$ depending on the cost share agreed. Note that both these payoffs go to zero as $\varepsilon \to 0$. Indeed $BCEL = WCEL = \frac{\frac{1}{2}+\varepsilon-2\varepsilon}{\frac{1}{2}+\varepsilon} = \frac{1-2\varepsilon}{1+2\varepsilon}$ which goes to 100% as $\varepsilon \to 0$.

The examples considered so far suggest that under-investment in trade-links may be a more serious problem than inefficiency due to over-investment in non-trade links when cost sharing is exogenous whilst inefficiency due to over-investment in non-trade links will become more problematic when cost shares are endogenous. Proposition 2 in the next section makes this intuition precise.

5.5. **Main Results.** With the definitions of stability, the different types of inefficiency identified and a measure of the size of inefficiencies in hand, the main results of this paper can be presented:

Proposition 1. *When buyers have all the bargaining power $\beta = 1$ and have to pay all the costs of link formation $\gamma = 1$ the efficient network is stable for any potential gains from trade (α) and there does not exist a stable network with any inefficiency due to over-investment in non-trade links or under-investment in trade links. However, if either:*

- *buyers' bargaining power declined ($\beta < 1$); or*
- *sellers had to pay some costs of link formation ($\gamma < 1$); or*

[46] Section AM-4.

[47] It is shown in Section AM-5 of the additional material that when cost sharing is endogenous the absolute size of inefficiency on any stable network with Q links is bounded from above by Qc. It is interesting that as either $c \to 0$ or $Q \to 0$ there can be no inefficiency. That there is no inefficiency as $c \to 0$ is intuitive and consistent with observation that at $c = 0$ the efficient network will be stable. The fact that inefficiency goes to zero for $Q \to 0$ emphasizes that inefficiency due to under-investment has been removed from the network: No longer can there be any inefficiency on the empty network if it is stable.

- *buyers and sellers could negotiate over the costs of link formation*

there would exist potential gains from trade (α) where, on even the most efficient stable network, ALL *the gains from trade are consumed by inefficiencies.*

It is well known that inefficiencies can be present when parties cost shares and bargaining power do not coincide. The contribution of Proposition 1 is identifying the discontinuity in the level of inefficiency present moving away from the special case where the efficient network is always stable: Moving even slightly away from this special case all the gains from trade generated by the efficient network can be lost on even the best stable network.

Proposition 2.

When costs shares are exogenous:

- *inefficiency due to over-investment in non-trade link on even the* worst *stable network is bounded at 50% of the net gains from trade generated by the efficient network, for $\gamma \in (0,1)$;*
- *whilst inefficiency due to under-investment in trade links on even the* best *stable network can be 100% of the net gains from trade generated by the efficient network for $\beta \neq \gamma$.*

Endogenizing cost shares:

- *inefficiency due to under-investment in non-trade links is eliminated;*
- *but inefficiency due to over-investment in non-trade links on even the best stable network is no longer bounded and can be 100% of the net gains from trade generated by the efficient network.*

Proposition 2 emphasizes that inefficiency is not an artifact of the cost sharing assumption, although this does affect which types of inefficiency are likely to be most important and when. Removing under-investment inefficiency problems by permitting buyers and sellers to endogenously negotiate over the cost split exacerbates over-investment inefficiency problems insofar as they are no longer bounded.

An immediate consequence of Proposition 2 is given in Corollary 1:

Corollary 1. *For each change identified in Proposition 1 that moves the environment away from the special case of $\beta = 1$ and $\gamma = 1$, either inefficiency due to over-investment in non-trade links alone or inefficiency due to under-investment in trade links alone can* *account for the existence of potential gains from trade (α) where the inefficiency consumes all the gains from trade:*

Proposition 1 change	*Inefficiency responsible for the result*
Buyers' bargaining power declines ($\beta < 1$)	*under-investment in trade links*
Sellers have to pay some costs ($\gamma < 1$)	*under-investment in trade links*
Buyers and sellers negotiate over costs	*over-investment in non-trade links*

TABLE 1. Accounting for the possible 100% BCEL identified in Proposition 1.

When costs are shared in exogenous proportions under-investment in trade links is sufficient to generate the result of Proposition 1. In contrast inefficiency due to over-investment in non-trade links is bounded. Moving to endogenous cost sharing resolves the problem of under-investment in trade links. However, over-investment in non-trade links is now sufficient to generate 100% inefficiency, as measured by the best case efficiency loss.

This trade-off between inefficiency due to under-investment in trade links when cost sharing is exogenous and over-investment in non-trade links when it is endogenous is also reflected in the conditions under which the efficient network is stable.[48] In the case of vertically differentiated suppliers,[49] when costs are shared exogenously the efficient network is only ever unstable because of insufficient incentives to retain traded over links whilst when costs are shared endogenously the efficient network is only ever unstable because of incentives to form outside option links.

5.6. **Proving the Main Results.** Propositions 1 and 2 and Corollary 1 follow immediately from three lemmas that are presented in this section.

Lemma 3. *When cost shares are exogenous the best case efficiency loss due to only under-investment in trade links can be 100% $\forall \beta \neq \gamma$. In contrast there is no inefficiency loss due to under-investment in trade links when $\beta = \gamma$: When $\beta \neq \gamma$ there always exists some potential gains from trade α where in the best stable network all the net gains from trade generated by the efficient network are lost despite there only being inefficiency due to under investment in trade links. When $\beta = \gamma$ there is no under-investment inefficiency in any stable network for any potential gains from trade α.*

[48]See Section AM-4 of the additional material.
[49]See Section AM-6 of the additional material

The proof of Lemma 3 is in Appendix C. It is intuitive that there will be no under-investment in efficiency when $\beta = \gamma$. In this case each party's revenue and cost shares from joint production are aligned.[50]

The next lemma will bound the worst case efficiency cost due to over-investment in trade links when cost shares are exogenous. However, it is first necessary to define the worst case efficiency loss due to over-investment in trade links.

Definition: The *worst case efficiency loss due to over-investment* in non-trade links, given gains from trade α is given by: $\frac{(\tilde{Q}-\tilde{K})c}{NGT(L)-\tilde{K}c}$, where L is the stable network that maximizes this expression (and implements $\tilde{K}$ matches and has $\tilde{Q}$ links in total).

The worst case efficiency loss searches over all stable networks. It finds the stable network with the highest lost gains from trade due over-investment in non trade links where this loss is expressed as a percentage of the net gains from trade that could be realized on the network were there no over-investment inefficiency.

Lemma 4. *The worst case efficiency loss due to over-investment in non-trade links is bounded by the amounts shown in the table below and this bound is tight: There does not exist a stable network for any potential gains form trade that results in a higher loss in efficiency due to over-investment in non-trade links, but there does exist a stable network for some potential gains from trade with this level of inefficiency.*[51]

β	γ	*Bound on WCEL caused by over-investment*
$\in (0,1)$	$\in (0,1)$	$h(\tilde{K},\gamma,\beta)^* \leq \frac{1}{2}$
$\in \{0,1\}$	$\in (0,1)$	0^{**}
0	0	0
$\in (0,0.5)$	0	$\frac{\beta}{1-\beta}$
$\in [0.5,1]$	0	1
$\in [0,0.5]$	1	1
$\in (0.5,1)$	1	$\frac{1-\beta}{\beta}$
1	1	0

Table 2. Bounding the worst case efficiency lost due to over-investment in non-trade links

$$* \; h(\tilde{K},\gamma,\beta) \equiv \frac{\tilde{K}-1}{max\left\{\frac{(\tilde{K}-1)+(1-\gamma)}{1-\beta}+\frac{(\tilde{K}-1)(1-\gamma)}{\beta},\frac{(\tilde{K}-1)+(\gamma)}{\beta}+\frac{(\tilde{K}-1)(\gamma)}{1-\beta}\right\}-\tilde{K}}$$

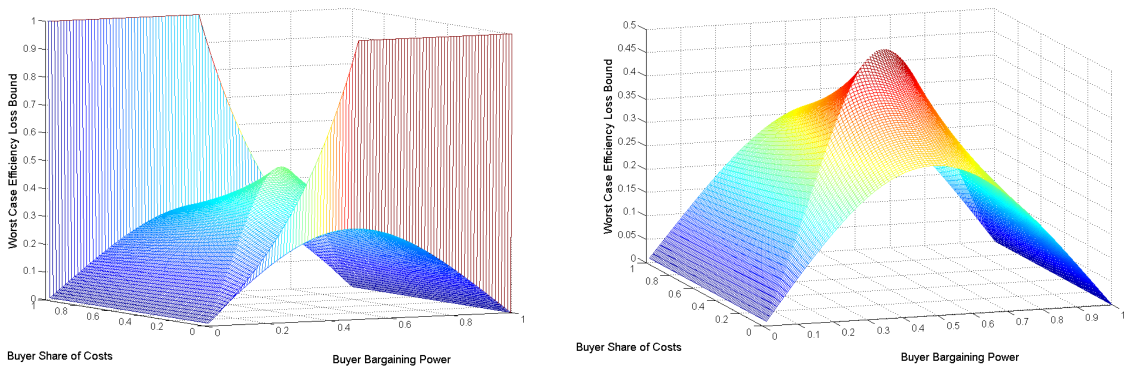
*** The unique stable network is the empty network so the worst case efficiency loss due to over-investment in non-trade links is 0.*

[50]Similar conditions are also found in Hosios (1990) and Caballero and Hammour (1998).

[51]The size of the cost of link formation is normalized away: the values of α can always be scaled to compensate for a higher or lower c.

where $\tilde{K}$ is the number of matches on the component in the network with the most matches when $\beta \leq (1-\gamma)$; the number of matches on the component in the network with the least number of matches greater than one when $\beta > (1-\gamma)$ and there is a component with more than one match; and 1 when all components have only one match.

The proof of Lemma 4 is in Appendix C. These bounds are shown in Figure 6 below.



(A) All parameters (B) Interior points

FIGURE 4. Over-investment inefficiency bound ($\tilde{K} = 10$)

Figure 4a shows the bound on the worst case efficiency loss due to over-investment in non-trade links for $\gamma \in [0,1]$, $\beta \in [0,1]$, $\tilde{K} = 10$ and c normalized to 1. There are similar bounds for other values of $\tilde{K}$. In particular the bound on the worst case efficiency loss, for $\gamma \in (0,1)$, is always largest for $\gamma = \beta = \frac{1}{2}$ and at this point is always $\frac{1}{2}$ (50% of the net gains form trade generated by the efficient network) $\forall \tilde{K} > 1$.

From Figure 4a the discontinuity in the bound at the extreme values of γ is apparent. This is because when $\gamma \in \{0,1\}$ either buyers or sellers cannot contribute towards the cost of link formation. This relaxes a number of constraints on the stable network: outside option links no longer have to benefit both the parties they connect, they only need to benefit the party who has to pay for them.

As can be seen in Figure 4b, and is easily shown directly from the $h()$ function, for any β the interior bound is highest when $\gamma = \beta$.[52] Recall that the it is precisely when $\gamma = \beta$ that there is no under-investment inefficiency. Thus, it is when there is no under-investment inefficiency that the problem of over-investment is most acute.

[52]This is proved as part of the proof of Lemma 4 in Appendix C.

The final Lemma that is required for the proof of the main results shows that the best case inefficiency loss due to over-investment in non-trade links is unbounded when cost shares are endogenous.

Lemma 5. *When cost sharing is endogenous the worst case inefficiency due to over-investment in outside trade links can be 100%: There exist potential gains from trade where the most efficient stable network consumes all the net gains from trade generated on the efficient network due to over-investment inefficiency.*

The proof of Lemma 5 is in Appendix C.

In is proved in the additional material that the efficient network is stable when buyers (sellers) have all the bargaining power and only they can form links.[53] Propositions 1 and 2 and Corollary 1 then follow immediately from Lemmas 3, 4 and 5.

6. Related Literature

The section places the models of Sections 3 and 5 in the context of three related literatures: the literatures on bargaining over fixed networks, forming trade networks, and the considerable literature on making relationship specific investments when contacts are incomplete.[54]

6.1. **Bargaining over fixed networks.** There is a growing literature on bargaining over a fixed network. An important difference between much of this literature and the model built in this paper is the ability to model heterogenous gains from trade. In many situations allowing for heterogeneous agents is more realistic and it also leads to a richer model of bargaining over a network that permits new results to be derived. Most importantly, modeling heterogeneous buyers and sellers is essential for a thorough analysis of inefficiencies in network formation.

The role of networks in affecting bargaining outcomes has long been recognized (e.g. Emerson 1967).[55] Models of bargaining over a network tend to have taken one of two

[53]Section AM-4.

[54]Some related literatures are not discussed. Network formation not in the context of trade networks is considered by Bala and Goyal (2000), Galeotti et al (2006) and Goyal and Vega-Redondo (2007) amongst others. Jackson (2008) provides an overview. Perhaps most relevant in this literature is Currarini and Morelli (2000) who consider a non-cooperative game of network formation in which players form links and propose surplus splits thereby endogenzing these splits. This is bit similar to permitting buyers and sellers to negotiate their investment shares. In contrast to model of Section 5, they find that the efficient network is stable under the condition of size monotonicity. The search literature is also related but not discussed. See Rogerson, Shimer and Wright (2005) for an excellent survey or Pissarides (2000).

[55]In experiments Cook and Emerson (1983) have investigated the role of networks in determining the terms of trade and found that the network mattered.

approaches: A non-cooperative game theoretic approach or a cooperative game theoretic approach. There are a number of papers that model non-cooperative alternating offer bargaining games over the network. Corominas-Bosch (2004), Navarro and Perea (2001), Polanski (2007), Manea (2008) and Abreu and Manea (2008) all pursue this methodology. The Corominas-Bosch model (2004) is closest to the model considered in this paper. As in this paper, the Corominas-Bosch model can be viewed as affecting bargaining outcomes by providing outside options.[56] Further, if the environment of the model presented in this paper is restricted to only permit homogeneous gains from trade, as is modeled in Corominas-Bosch (2004),[57] the alternative methodology employed in this paper replicates the Corominas-Bosch equilibrium payoffs.[58] This provides some non-cooperative justification for the cooperative approach pursued in this paper.[59]

The second approach to modeling bargaining over a network employs cooperative game theory. Myerson (1977) pioneered this approach by applying the Shapley value over a network.[60] Unlike many other bargaining models, the Shapley value can be applied to networks with heterogenous gains from trade. However in the context of buyers and sellers bargaining over a bipartite network, it seems unreasonable to constrain links, especially those that generate outside options, to benefit both connected parties equally. Thus the core is considered a more appealing solution concept. The core is also widely used in the matching literature.[61]

The analysis of bargaining over a fixed network in this paper is closest to Kranton and Minehart (2000a) who also generate outcomes in the core. Kranton and Minehart consider competitive bargaining outcomes (which correspond to core bargaining outcomes) and identify chains of links between buyers and sellers which in effect determine outside options. However, they are only able to deal with the case of identical sellers such that

[56]This contrasts with Manea (2008) and Abreu and Manea (2008) where the network can be interpreted as affecting parties bargaining power. The approximate intuition is that when a party has more links (although it also matters who these links are to) the bargaining protocol selects this party more often to receive and make offers reducing the time they wait between making or receiving offers. This has the affect of making parties with more links more patient, increasing their bargaining power.

[57]It is not straight forward to directly extend Corominas-Bosch's model to include heterogeneous gains from trade. Indeed she concludes that although it is "natural to ask for the introduction of a little bit of heterogeneity in the model [. . .] [w]e believe that this line of research is unlikely to lead to fruitful results."

[58]See Section AM-2 of the additional material.

[59]Charness and Corominas-Bosch (2007) provide experimental support for the qualitative predictions of Corominas-Bosch (2004).

[60]Navarro and Perea (2001) replicate the payoffs identified by Myerson with a non-cooperative bargaining protocol. Kranton and Minehart (2000b), Segal and Whinston (2000) and Rajan and Zingales (1998) all consider investments prior to surpluses being split in accordance with the Shapley value.

[61]See Roth and Sotomayor (1990).

it does not matter which seller a buyer trades with, only whether they trades with a seller or not. Their analysis exploits the homogeneity of sellers, and in particular the fact that they provide a common reference price to all buyers connected to them. To consider heterogenous gains from trade it is necessary to take a different, albeit related approach building on their core insights. This new approach generates new results. For example, a key result in the bargaining over a fixed network section of this paper is the characterization of payoffs as a weighted sum of the potential gains from trade over links in the network (Lemma 2). This result has no counterpart in Kranton and Minehart (2000a).[62]

6.2. **Trade network formation.** The main focus of this paper is modeling the formation of trade networks and analyzing inefficiencies in these networks. The paper closest to the methodology pursued in this paper is Kranton and Minehart (2001) and has already been described in Section 1.

Kranton and Minehart (2000b) consider supply networks as an alternative to vertical integration. Buyers make a decision whether to form their own integrated production or to form links to potential external suppliers. Suppliers must decide whether to invest in productive capacity and produce a good that can be sold to any connected buyer. There is uncertainty in buyers' valuations for inputs. Kranton and Minehart consider splits of the surplus generated by the network according to both the Shapley value and the buyer optimal point of the core. When surpluses are shared according to the Shapley value buyers can both form too many or too few links. When surpluses are shared according to the buyer optimal point of the core buyers invest efficiently, as in Kranton and Minehart (2001), but sellers may under-invest.[63]

Manea (2008) considers both pairwise stable and unilaterally stable networks where gains from trade are split according to his non-cooperative bargaining game and there are no costs to forming links. With no costs of link formation links can no longer be interpreted as relationship specific investments, indeed there are no investments at all. Thus the network formation problem considered by Manea is fundamentally different form the problem considered in this paper: Were $c = 0$ in the model of Section 5 there would be no inefficiencies in network formation.

[62]Considering heterogenous gains from trade also permits new situations to be analyzed. For example, the special case of vertically differentiated sellers is considered in Section AM-6 of the additional material, whilst in Kranton and Minehart (2000a) sellers are assumed identical.

[63]Whilst this is suggestive of inefficiency being more of a problem with the Shapley value than the buyer-optimal core, at other points of the core buyers will receive less than their marginal social contribution and so may not invest efficiently.

6.3. **Other models of relationship specific investments.** Outside of the formation of trade networks there is a very large literature considering the efficiency of relationship specific investments. This section considers a few of the most relevant papers in this literature. In much of this literature relationship specific investment take a very different form to those considered in Section 5 (where parties make joint investments that matter only insofar as they enable trade). In the case of fixed exogenous investments, considered in Section 5.3, when $\gamma \in (0, 1)$ both parties must make positive investments to enable trade, investment decisions are binary and highly interdependent: A link is not formed unless both parties invest. When investments are negotiated endogenously, as considered in Section 5.4, parties' investments are again highly interdependent: Only joint investment greater than c generate a link. In contrast much of the relationship specific investment literature models the returns to investments as increasing at a diminishing marginal rate. This difference is important. Many of the solutions for restoring efficiency discussed in the relationship specific investment literature cannot be applied given the set up of this paper.

Caballero and Hammour (1998) are an exception and model investments in a similar way to how fixed exogenous investments are modeled in Section 5.3. In their model capital and labour must be combined in fixed proportions to be productive and parties split the surplus generated from investment equally. When capital and labour must be combined in equal proportions to generate a productive unit efficient investment decisions are made. This follows from Hosios (1990) and is a special case of the condition, $\beta = \gamma$, identified in this paper, under which there is no under-investment inefficiency. By construction there is no over-investment inefficiency in Caballero and Hammour as only two parties are modeled. The condition that there will be no under-investment inefficiency when $\beta = \gamma$, in both Caballero and Hammour and this paper, contrasts with the standard condition that parties must have all the bargaining power to make efficient relationship specific investments.[64] This difference arises because investments are, in effect, jointly made by the parties rather than separately as in the majority of the literature. As far as I am aware the case of endogenizing investment decisions of the form considered in this paper, by allowing parties to negotiate their cost contributions such that the Hosios condition can always be met, has not previously been studied.

Although the investments made in this paper differ from those made in much of the contract theory literature, inefficiencies due to under-investment for fear of hold up and over-investment in outside options, have been widely identified and analyzed. However,

[64]See Malcomson (1997).

the approaches identified to restore efficiency cannot typically be applied when relationship specific investments enable trade.[65] Grossman and Hart (1986) and Hart and Moore (1990) consider the affect of adjusting asset ownership to improve investment decisions where investments are asset specific. When another party owns an asset (has residual control rights over it), a specific investment in that asset is translated into a relationship specific investment and is subject to hold up. In contrast when a party owns the asset they are investing in, their investment is no longer relationship specific: they can obtain the full benefits of the investment outside of the relationship. Adjusting asset ownership can therefore affect the efficiency of investment decision. Cole, Mailath and Postlewaite (2001b) also remove the specificity of investments, but do so by modeling a continuum of potential trade partners such that there are outside trade partners who benefit from investments as much as trade partners do. However, when investments create links that *enable* trade they are often fundamentally relationship specific. These investments are the subject of this paper.

Cole, Mailath and Postlewaite (2001b) consider a finite economy and look for conditions under which relationship specific investments will be efficient. They show it is possible to construct bargaining protocols such that there is efficiency. By conditioning the equilibrium selected from the core on parties' investments, incentives for efficient investment are restored. This approach might also yield efficiency when investments are in relationships that enable trade. However, the bargained outcomes require careful manipulation to generate the correct incentives for efficient investment and if bargaining power is determined institutionally, the efficient outcomes cannot be reached in this way.

7. Conclusions

This paper aims to provide a thorough and systematic investigation of inefficiencies in relationship specific investment decisions that enable trade when there are many potential trade partners for each party. Contributions are made to the trade network formation literature by: (i) extending the environment these models have considered, (ii) introducing new tools for measuring the size of inefficiencies in general and by the different types of inefficiency, and (iii) generating new results that (a) illustrate the potential significance of

[65] An exception, and perhaps the simplest approach towards reducing inefficiency, is to ensure only one party can make a relationship specific investment and this party has all the bargaining power. See, for example, Hart and Moore (1988). This is also the approach successfully taken, by assumption, by Kranton and Minehart (2000b), (2001) as already discussed. However, situations in which only buyers, or only sellers, make relationship specific investments and have all the bargaining power are a relatively special case.

inefficiencies (Proposition 1) and (b) identify the impact of investments being negotiated endogenously (Proposition 2).

Previous models of network formation have not considered environments in which links are formed between buyers and sellers with different ex-ante potential gains from trade, bargaining power is varied and investment protocols include both exogenous fixed contributions to forming links and endogenous negotiated individual contributions towards forming links. This relatively rich environment permits a fuller analysis of inefficiencies than would otherwise be possible. To analyze inefficiencies new tools are introduced to measure the size of inefficiencies, the best and worst case efficiency loss, and inefficiencies are partitioned into different types: under-investment in links that should be used for trade, over-investment in links that are not used for trade and so provide no contribution to total surplus, and coordination inefficiencies. Analysis of these inefficiencies generated new and interesting results.

Kranton and Minehart (2001) show that there is no inefficiency when investments are exogenous, only buyers contribute towards link formation costs, and buyers have all the bargaining power. It is striking that efficiency can be achieved even in this relatively specialized environment and already well understood that in environments close to this inefficiencies may be present. However what was not well understood is whether inefficiencies in close by environments could be significant. It is shown that even on the best stable network *all* the gains from trade can be lost to inefficiencies in environments even arbitrarily close to this special case. This then raises additional questions about what these inefficiencies are and when they are important. When buyers' bargaining powers is not in proportion to their share of the link formation costs under-investment inefficiency can consume all the gains from trade. Under-investment inefficiency is eliminated by permitting endogenous negotiations over costs shares. However, permitting endogenous negotiations also makes it easier to form outside options links. Whilst over-investment inefficiency is bounded on any stable network with exogenous cost shares it becomes unbounded on even the best stable network when cost shares are endogenously determined. The trade off between under investment inefficiency with exogenous cost sharing and over-investment inefficiency with endogenous cost sharing is also reflected in the necessary and sufficiency conditions under which the efficient network is stable.[66] In particular, when there is vertical differentiation, with exogenous investments the efficient network is only ever unstable due to incentives to delete trade links whilst when investments are endogenous the efficient network is only ever unstable due to incentives to form outside

[66]See Section AM-4 of the additional material.

option links.[67] To the best of my knowledge the impact of endogenizing cost shares in this way has not been considered before.

To permit the analysis of trade network formation in environments where there are heterogenous gains from trade and it matters who is matched to who, it was necessary to first model bargaining over networks with heterogenous gains from trade. An intuitive methodology for determining bargaining outcomes was presented that emphasized the role of networks in generating outside options. Further, this process identified precisely how the structure of any network impacted all parties' payoffs. It is hoped this tool will also be useful for future work.

In many applications it may be unrealistic to think of all parties knowing the potential gains from trade between all buyer-seller pairs when forming relationships. Indeed, in some applications it may be the case that relationship specific investments are undertaken precisely for the purpose of determining the gains from trade. Modeling this imperfect information environment in the presence of ex-ante heterogeneous expected gains from trade is left to future work. Nonetheless this paper should provide a foundation for further work in this direction as well as some immediate incremental improvement to our understanding of these markets.

[67]See Section AM-6 of the additional material.

Appendix A. Bargaining over a Fixed Network- Additional Results

This section supports Section 3 by providing additional detail and results referred to therein.

It is useful to first formally define the core for bargaining outcomes over a network:

Definition: Payoff vectors π^S, π^B are feasible for a coalition $\Psi \subseteq B \cup S$ if $\sum_{i\in\Psi} \pi_i^B + \sum_{j\in\Psi} \pi_j^S \leq V(\Psi)$.

Definition: Payoff vectors π^S, π^B are implementable for μ by a coalition $\Psi \subseteq B \cup S$ if $\sum_{i\in\Psi} \pi_i^B + \sum_{j\in\Psi} \pi_j^S \leq \sum_{i\in\Psi} \alpha_{i\mu(i)}$ and $\mu(i) \in \Psi$, $\forall i \in \Psi$.

Definition: Payoff vectors π^B, π^S and match μ are in the core for a network L if the payoff vectors are feasible for $B \cup S$, implementable by μ for $B \cup S$, and there do not exist a payoff vectors $\tilde{\pi}^B$ and $\tilde{\pi}^S$ for any coalition $\Psi \subseteq B \cup S$ that is weakly preferred by all members of the coalition, strictly preferred by one member and feasible: There do not exist a payoff vectors $\tilde{\pi}^B$ and $\tilde{\pi}^S$ such that:

- $\tilde{\pi}_i^B \geq \pi_i^B$, $\forall i \in \Psi$ and $\tilde{\pi}_j^S \geq \pi_j^S$, $\forall j \in \Psi$;
- $\tilde{\pi}_i^B > \pi_i^B$, for some $i \in \Psi$ or $\tilde{\pi}_j^S > \pi_j^S$, for some $j \in \Psi$; and
- $\sum_{i\in\Psi} \tilde{\pi}_i^B + \sum_{j\in\Psi} \tilde{\pi}_j^S \leq V(\Psi)$.

A.1. **Negotiated Rematching.** Having formally defined the core it can be seen that the negotiated rematching process terminates in the core. By construction there is no buyer-seller pair that could form a deviating coalition and this is necessary and sufficient for no coalition of buyers and sellers being able to form a deviating coalition.[68]

Payoffs can be pinned down by the negotiated rematching process:

Lemma 6. *Suppose a seller $s_{j'}$ is removed from a network (L) leaving their trade partner $b_{i'} = \mu(j', L)^*$ unmatched. Following a negotiated rematching process remaining sellers' payoffs are given by:*

$$\pi_j^S(L - j') = max\{\pi_j^S(L), \pi_j^S(L - j')^{BO}\}, \qquad \forall j \neq j'$$

where $\pi_j^S(L - j')^{BO}$ is seller s_j's payoff at the buyer optimal point of the core (the lowest payoff s_j can receive for any core outcome) on the network $L - s_{j'}$.

Proof. The negotiated rematching process bids up the payoffs of sellers following the removal of seller $s_{j'}$ form a network. This proof will proceed in three steps. First it will be shown that all sellers $s_j \neq s_{j'}$ such that $\pi_j^S(L) > \pi_j^S(L - j')^{BO}$ never have their payoff bid up further. Second it will be shown that all sellers $s_j \neq s_{j'}$ such that $\pi_j^S(L) \leq \pi_j^S(L - j')^{BO}$ never have their payoff bid up above $\pi_j^S(L - j')^{BO}$. Third it will be shown that all sellers $s_j \neq s_{j'}$ such that $\pi_j^S(L) \leq \pi_j^S(L - j')^{BO}$ must have their payoff bid up to at least $\pi_j^S(L - j')^{BO}$. It then follows that $\pi_j^S(L - j') = max\{\pi_j^S(L), \pi_j^S(L - j')^{BO}\}$, $\forall s_j \neq s_{j'}$.

Partition the set of sellers S on the network $L - j'$ such that $\hat{S} = \{j : \pi_j^S(L) \leq \pi_j^S(L - j')^{BO}\}$ and $\tilde{S} = \{j : \pi_j^S(L) > \pi_j^S(L - j')^{BO}\}$. As sellers' payoffs are simply the prices they receive

[68] Shapley and Shubik (1972); Roth and Sotomayor (1990).

these terms will be used interchangeably. Label the set of trade partners of sellers $s_j \in \hat{S}$ on the initial network L, $\hat{B} = \mu(\hat{S})^* \equiv \{b_i \in B : \mu(b_i, L)^* \in \hat{S}\}$ and label the set of trade partners of sellers $s \in \tilde{S}$ on the initial network L, $\tilde{B} = \mu(\tilde{S})^* \equiv \{b_i \in B : \mu(b_i, L)^* \in \tilde{S}\}$ (this is not necessarily a partition of all buyers).

Suppose that the negotiated rematching process has reached any point where $\pi_j^S \leq \pi_j^S(L-j')^{BO}$, $\forall s_j \in \hat{S}$ and $\pi_j^S = \pi_j^S(L) > \pi_j^S(L-j')^{BO}$, $\forall s_j \in \tilde{S}$. It will be shown that any buyer $b_i \notin \tilde{B}$ will not demand trade with any seller $s_j \in \tilde{S}$ at these prices such that the price received by sellers $s_j \in \tilde{S}$ cannot be bid up. Suppose, in contradiction, that a buyer $b_{i'} \notin \tilde{B}$ demanded trade with some seller $s_j \in \tilde{S}$ at these prices. Now weakly increase the price of all sellers $s_j \in \hat{S}$ such that $\pi_j^S = \pi_j^S(L-j')^{BO}$ and decrease the price of all sellers $s_j \in \tilde{S}$ such that $\pi_j^S = \pi_j^S(L-j')^{BO}$. Buyer $b_{i'}$ must still demand trade with a seller $s_j \in \tilde{S}$. Further all buyers $b_i \in \tilde{B}$ must still demand trade with a seller $s_j \in \tilde{S}$. There must then be some seller $s_j \in \tilde{S}$ with whom buyers over demand trade. Thus the prices at the buyer optimal point of the core for network $L-j'$ are not stable. This is a contradiction.

Continue to consider payoffs such that $\pi_j^S \leq \pi_j^S(L-j')^{BO}$, $\forall s_j \in \hat{S}$ and $\pi_j^S = \pi_j^S(L) > \pi_j^S(L-j')^{BO}$, $\forall s_j \in \tilde{S}$. It will be shown that the payoff of any seller $s_j \in \hat{S}$ will never be bid up above the buyer optimal point. Suppose in contradiction that a seller $s_{j''} \in \hat{S}$'s payoff would be bid up above the buyer optimal point in the next round of the negotiated rematching process. The payoffs of sellers $s_j \in \tilde{S}$ were stable (in the core) on the network L. Thus increasing the relative price of trade with other sellers will not result in any buyer $b_i \in \tilde{B}$ demanding trade with any seller $s_j \in \hat{S}$, including $s_{j''}$. For a $s_{j''}$'s payoff to be bid above the buyer optimal payoff on network $L-j'$ it must then be that trade with $s_{j''}$ is over-demanded by buyers $B' \subseteq \{i : i \notin \tilde{B}\}$ at current prices. Suppose that the prices of all sellers $s_j \in \hat{S}$ were weakly increased such that $\pi_j^S = \pi_j^S(L-j')^{BO}$. Demand for trade with $s_{j''}$ must have weakly increased and still be demanded by all buyers $b_i \in B'$. Further, buyers $b_i \in \tilde{B}$ must still demand trade with sellers $s_j \in \tilde{S}$. Suppose that the prices of sellers $s \in \tilde{S}$ are decreased such that $\pi_j^S = \pi_j^S(L-j')^{BO}$. Buyers $B' \cup \tilde{B}$ must then demand trade with sellers $s_{j''} \cup \tilde{S}$ such that trade with at least one of these sellers is over-demanded. As all prices are now at the buyer optimal point of the core this is a contradiction.

It is has been shown that the payoff of a seller $s_j \in \tilde{S}$ cannot be bid up above $\pi_j^S(L-j')^{BO}$ by the negotiated rematching process unless there is some seller $s_j \in \hat{S}$ with a payoff greater than $\pi_j^S(L-j')^{BO}$. It has also been shown that the payoff of a seller $s_j \in \hat{S}$ cannot be bid up above $\pi_j^S(L-j')^{BO}$ by the negotiated rematching process unless there is some seller $s_j \in \tilde{S}$ with a payoff greater than $\pi_j^S(L)$. As at the beginning of the negotiated rematching process $\pi_j^S \leq \pi_j^S(L-j')^{BO}$, $\forall s_j \in \hat{S}$ and $\pi_j^S = \pi_j^S(L) > \pi_j^S(L-j')^{BO}$, $\forall s_j \in \tilde{S}$ this means that the payoff of a seller $s_j \in \tilde{S}$ cannot be bid up above $\pi_j^S(L)$ and the payoff of a seller $s_j \in \hat{S}$ cannot be bid up above $\pi_j^S(L-j')^{BO}$.

Finally, as the negotiated rematching process cannot terminate at a point where trade with a seller is over-demanded, sellers' prices must be bid up to at least the buyer optimal point of the core. Thus $\pi_j^S(L-j') = max\{\pi_j^S(L), \pi_j^S(L-j')^{BO}\}$. □

A.2. **Optimal Rematches.** The optimal rematch can be represented as a chain where each party is sequentially rematched to their optimal rematch along connected links:

Lemma 7. *Suppose $\mu(i')^*$ is removed from the network L. The optimal rematch can be represented as a chain (a sequence of links each sharing a node in common). This chain never cycles and only ever rematches parties to their optimal rematch. The chain is of the following form:*

(i) *Set $b_i = b_{i'}$.*
(ii) *If $\mu(i, L - \mu(i')^*)^* = b_i$ such that b_i is matched to himself in the optimal rematch following the removal of $\mu(i')^*$ the process ends. If not rematch b_i to $\mu(i, L - \mu(i')^*)^* = \nu(i)$.*
(iii) *If seller $\nu(i)$ was unmatched on the initial network ($\mu(\nu(i), L)^* = \nu(i)$), the process ends. If not, then a new buyer $\mu(\nu(i), L)^*$ is displaced.*
(iv) *Set b_i equal to this displaced buyer and go back to step ii).*

Proof. As all rematches in the chain are to parties' unique optimal rematches, rematching each buyer at most once, the chain must terminate having only matched parties to their optimal rematch and the only way the chain could fail to implement the optimal rematch would be if it terminated before all optimal rematches had been made. Denote by Ψ the set of buyers and sellers that are not rematched in the chain but are rematched in the optimal rematch. The buyers and sellers in Ψ could also have been rematched on the original network L without affecting the matchings of any buyers and sellers outside of Ψ. As the initial matching maximized the gains from trade on L, $\Psi = \emptyset$ and all rematches implemented by the optimal rematch must also be implemented by the chain identified. □

As the chain identified in Lemma 7 implements the optimal rematch it will be referred to as the optimal rematch chain. Further, as the negotiated rematching process implements the optimal rematch, the rematches it implements can also be represented by the optimal rematch chain.[69] In optimal rematch chains (and therefore outside option chains) all the parties that are rematched at any point of the chain must be rematched to their outside trade partner. This is proved in Lemma 8:

Lemma 8. *$\forall s_j$ displaced by the optimal rematch chain on $L - b_{i'}$, their optimal rematch is to their outside trade partner, $\mu(s_j, L - b_{i'})^* = \mu(s_j, L - \mu(s_j, L)^*)^* = \nu(s_j)^*$, and they receive their outside option value, $\pi_j^S(L - b_{i'}) = \pi_j^S(L - \mu(s_j, L)^*)^{SO} = \underline{\pi}_j^S(L)$.*

Proof. It will then be shown that if a party is displaced and rematched to someone other than their outside trade partner in the optimal rematch chain, then the optimal rematch chain cannot yield matches that maximized the gains from trade on the reduced network. Thus each displaced party must be rematched to their outside trade partner.

To complete this proof it is useful to first develop some additional notation: Define $\mu_{RC}(L-i')^*$ as the sequence of rematches that occur in the optimal rematch chain following the removal of $b_{i'}$ from the network L. This sequence of rematches can be partitioned into two subsequences of rematches in the following way. Abusing notation and letting $\mu_{RC}(L - i')$ also be the set of matches in the sequence $\mu_{RC}(L - i')$, suppose that the match $\{i'', \mu(i'', L - i')^*\} \in \mu_{RC}(L-i')^*$. Then the sequence $\mu_{RC}(L-i')^*$ can be partitioned into those matches before and

[69]In the non-generic case where there are multiple optimal rematches each of these rematches will have a corresponding optimal rematch chain which can all be implemented by the negotiated rematching process.

including $\{i'', \mu(i'', L-i')^*\}$ and those matches after $\{i'', \mu(i'', L-i')^*\}$. The subsequence (and substring) of matches before and including $\{i'', \mu(i'', L-i')^*\}$ will be denoted $\mu_{RC}^{i''}(L-i')^*$ and the subsequence (and substring) of matches after and not including $\{i'', \mu(i'', L-i')^*\}$ will be denoted $\mu_{RC}^{\bar{i}''}(L-i')^*$. Thus $\mu_{RC}(L-i') = \mu_{RC}^{i''}(L-i')^*, \mu_{RC}^{\bar{i}''}(L-i')^*$. It is assumed that each party has a unique outside trade partner in this proof. This is generically true and even if it were not true the proof could be extended to allow for multiple optimal trade partners and / or multiple optimal outside trade partners with a little additional work.

Consider an optimal rematching chain $\mu_{RC}(L-i')^*$. By definition, if rematched, $\mu(i', L)^*$ must be rematched to their outside trade partner $\nu(\mu(i', L)^*)$. Let i'' be this buyer ($i'' = \nu(\mu(i', L)^*)$). Suppose, in contradiction, that i'''s trade partner on L, $\mu(i'', L)^*$ is not rematched to their outside trade partner: $\mu(\mu(i'', L)^*, L-i')^* \neq \nu(\mu(i'', L)^*)$.

There are two cases to consider. Suppose that were i'' removed from the network L the optimal rematch $\mu_{RC}(L-i'')^*$ would *not* displace i'. Consider again the optimal rematch following the removal of i' and in particular the sequence of rematches after $\mu(i'', L)^*$ is reached. At this point both sequence of rematches $\mu_{RC}^{\bar{i}''}(L-i')^*$ and $\mu_{RC}(L-i'')^*$ are possible. By definition of the optimal rematch when i'' is removed from the network the total additional surplus generated by the rematch $\mu_{RC}(L-i'')^*$ must be greater than for any other rematch. The total additional value generated by the rematch $\mu_{RC}(L-i'')^*$ must then be weakly greater than the total additional value generated by remainder of the optimal rematch $\mu_{RC}^{\bar{i}''}(L-i')^*$. Thus $\mu_{RC}(L-i'')^* = \mu_{RC}^{\bar{i}''}(L-i')^*$. This is a contradiction.

Consider now the second case where i' is rematched in the sequence $\mu_{RC}(L-i'')^*$. As optimal rematches never rematch the same party twice the subsequence of optimal rematches $\mu_{RC}^{\bar{i}'}(L-i'')^*$ is a possible rematch on the network $L-i'$. Similarly the subsequence of optimal rematches $\mu_{RC}^{\bar{i}''}(L-i')^*$ is a possible rematch on the network $L-i''$. The optimal rematch must implement matches that maximize the gains from trade on the reduced network. Abusing notation such that μ' represents a matched buyer and seller pair:

$$\sum_{\mu' \in \mu_{RC}(L-i')^*} \alpha_{\mu'} + \sum_{j \notin \mu_{RC}(L-i')^*} \alpha_{\mu(j,L)^* j} > \sum_{\mu' \in \mu_{RC}^{\bar{i}'}(L-i'')^*} \alpha_{\mu'} + \sum_{j \notin \mu_{RC}^{\bar{i}'}(L-i'')^*} \alpha_{\mu(j,L)^* j} \quad (3)$$

$$\sum_{\mu' \in \mu_{RC}(L-i'')^*} \alpha_{\mu'} + \sum_{j \notin \mu_{RC}(L-i'')^*} \alpha_{\mu(j,L)^* j} > \sum_{\mu' \in \mu_{RC}^{\bar{i}''}(L-i')^*} \alpha_{\mu'} + \sum_{j \notin \mu_{RC}^{\bar{i}''}(L-i')^*} \alpha_{\mu(j,L)^* j} \quad (4)$$

Combining equations 3 and 4:

$$\sum_{\mu' \in \mu_{RC}^{i'}(L-i'')^*} \alpha_{\mu'} + \sum_{\mu' \in \mu_{RC}^{i''}(L-i')^*} \alpha_{\mu'} > \sum_{j \in \mu_{RC}^{i'}(L-i'')^*} \alpha_{\mu(j,L)^* j} + \sum_{j \in \mu_{RC}^{i''}(L-i')^*} \alpha_{\mu(j,L)^* j} \quad (5)$$

Consider the left hand side of equation 5. The two rematching chains summed over provide a sequence of links that goes from $\mu(i'', L)^*$ to i', and then from $\mu(i', L)^*$ back to i''. This is a cycle. Consider now the right hand side of equation 5. The rematching chains on this side of the equation rematches exactly the same set of buyers and sellers as are rematched on the left hand side of the equation. However, these buyers and sellers are matched to their optimal matches on L. As the matches implemented on L must have maximized the gains from trade this is a contradiction.

It has been shown that $\mu(i'', L)^*$ cannot be rematched to someone other than their outside trade partner on the network $L - i'$, where $i'' = \nu(\mu(i', L))$. Consider now the next buyer reached on $\mu_{RC}(L - i')^*$: $i''' = \nu(\mu(i'', L))$. As $\mu(i'', L)^*$ was matched to their outside trade partner, $\mu(i''', L)^*$ must also be matched to their outside trade partner (repeat the arguments above replacing i' with i'' and i'' with i'''). Thus by induction every seller displaced in the optimal rematching chain $\mu_{RC}(L - i')^*$ must be rematched to their outside trade partner. Using the symmetry of buyers and sellers, all parties displaced by any rematching chain must be rematched to their outside trade partner.

By construction of the negotiated rematching process, the terms of trade a party receives from the negotiated rematching when they have been displaced at any point is equivalent to the terms of trade they would have received were their trade partner removed from the network. Consider a displaced buyer b_1 and suppose that $\nu(b_1) = s_2$. It has already been shown that b_1 must be rematched to s_2. b_1 therefore has to just induce s_2 to trade with them instead of their current trade partner. Further, s_2's current trade partner must be s_2's trade partner on the initial network ($\mu(s_2, L)^*$) as the optimal rematch rematches a party at most once. b_1 therefore negotiates the same terms of trade to just incentivize s_2 to trade with them as they would have in the negotiated rematching process were $\mu(b_1, L)^*$ initially removed from the network. Displaced parties therefore receive their outside option payoff. □

A.3. **Proof of Lemma 2.** For convenience Lemma 2 is replicated here:

Consider a seller $s_{j'}$ and denote the set of non-traded over links in their outside option chain $L^{j'}_{s\to b}$. Denote the set of traded over links in their outside option link $L^{j'}_{b\to s}$.

(i) *Seller $s_{j'}$'s outside option value is $\underline{\pi}^S_{j'} = \sum_{l\in L^{j'}_{s\to b}} \alpha_l - \sum_{l\in L^{j'}_{b\to s}} \alpha_l$;*
(ii) *Buyer $\mu(j', L)^*$'s payoff when their trade partner $s_{j'}$ receives only their outside option is their Vickrey payoff (their marginal contribution to the grand coalition); and*
(iii) *These payoffs correspond to seller $s_{j'}$'s and buyer $\mu(j', L)^*$'s buyer optimal core payoffs.*

Proof. **Part i):** Consider a seller $s_{j'}$'s outside option value. This is the payoff they would obtain from the negotiated rematching process were buyer $\mu(j', L)^*$ removed from the network. Without loss of generality suppose that there are k rematches that occur on this chain. Relabel buyers and sellers as follows: Label buyer $\mu(j', L)^*$, b_1, seller $s_{j'}$, s_1, their outside trade partner $\nu(s_1, L) = b_2$, the second displaced seller $\mu(b_2, L)^* = s_2$, their outside trade partner $\nu(s_2, L) = b_3$ and so on until the kth displaced seller is labeled s_k. Consider first the last seller, s_k, to be displaced by the outside option chain. This seller is either rematched to an unmatched buyer $\nu(s_k, L) = b_{k+1}$, in which case their outside option payoff is $\alpha_{k+1,k}$ or they remain unmatched in which case their outside option payoff is zero. Consider now the $k' < k$th rematched seller to be displaced ($s_{k'}$). This seller is rematched to buyer $b_{k'+1}$. However for $s_{k'}$ to induce $b_{k'+1}$ to trade with them they will offer (under the negotiated rematching process) $b_{k'+1}$ a payoff just sufficient to prevent $b_{k'+1}$ from continuing to trade with $s_{k'+1}$: $\pi^B_{k'+1} = \alpha_{k'+1,k'+1} - \underline{\pi}^S_{k'+1}$. By Lemma

8[70] the outside option chain on the network $L - i'$ must give each rematched seller their outside option value. Thus $s_{k'}$ receives a payoff $\pi^S_{k'}(L - i') = \underline{\pi}^S_{k'}(L) = \alpha_{k'+1,k'} - \alpha_{k'+1,k'+1} + \underline{\pi}^S_{k'+1}$. By induction $\underline{\pi}^S_{j'} = \sum_{l \in L^{j'}_{s \to b}} \alpha_l - \sum_{l \in L^{j'}_{b \to s}} \alpha_l$.

Part ii): Recall that the coalitional value of a subset of buyers and sellers $V(\Psi \subseteq B \cup S)$ is defined as the maximum gains from trade the coalition Ψ can generate. Suppose that a buyer $b_{i'}$ is removed from a network L. The negotiated rematching process maximizes the gains from trade on the network $L - i'$, therefore generating gains from trade $V(B \cup S/i')$. The initial match on the network L must also maximize the gains from trade and therefore generate gains from trade $V(B \cup S)$. The difference in these gains from trade is the value of the matches broken under the negotiated rematching process when moving from L to $L - i'$, less the value of matches formed in the negotiated rematching process when moving from L to $L - i'$:[71]

$$\begin{aligned} V(B \cup S) - V(B \cup S/i) &= \alpha_{i'\mu(i',L)^*} - \left(\sum_{l \in L^{\mu(i',L)^*}_{s \to b}} \alpha_l - \sum_{l \in L^{\mu(i',L)^*}_{b \to s}} \alpha_l \right) \\ &= \alpha_{i'\mu(i',L)^*} - \underline{\pi}^S_{\mu(i',L)^*} = \overline{\pi}^B_{i'} \end{aligned}$$

Part iii): Buyer $b_{i'}$ can never receive a core payoff above their Vickrey payoff. This is because, by definition, this would leave the coalition of all buyers and sellers other than $b_{i'}$ with a profitable deviation. To show that the vector of Vickrey payoffs is in the core for this environment Theorem 7 of Ausubel and Milgrom (2002) can be employed. First however it must be shown that the coalitional value function in this environment is buyer-submodular:

Definition: The coalitional value function V is *buyer-submodular* if for all $i \in B$ and all coalitions $\{\Psi_B \cup \Psi_S\} \subset \{\hat{\Psi}_B \cup \Psi_S\}$ such that $\Psi_S \subset S$ and $\Psi_B \subset \hat{\Psi}_B \subset B$, $V(\Psi_B \cup \Psi_S \cup i) - V(\Psi_B \cup \Psi_S/i) \geq V(\hat{\Psi}_B \cup \Psi_S \cup i) - V(\hat{\Psi}_B \cup \Psi_S/i)$.

The coalitional value function is buyer-submodular when the marginal contribution of a buyer to a coalition, for a given set of sellers, is weakly greater when some other buyers are removed from the coalition. From Corollary 9 in the additional material, a buyer's payoff must weakly increase following the removal of another buyer from the network. This is true for all levels of bargaining power including $\beta = 1$. For this level of bargaining power, by part ii), buyers' payoffs are equal to their marginal contribution to total surplus. Thus the marginal contribution of a buyer to total surplus must be higher when there are fewer other buyers present (in the strong set order): The coalitional value function is buyer-submodular. In then follows from Theorem 7 of Ausubel and Milgrom that buyer $b_{i'}$'s payoff when their trade partner $s_{j'}$ receives their outside option value is their payoff at the buyer optimal point of the core. Correspondingly seller $s_{j'}$'s outside option value is also their payoff at the buyer optimal point of the core. □

[70]See Section A

[71]Equivalently the difference is the value of the gains from trade generated by i''s match, less the additional gains from trade generated by the optimal rematch.

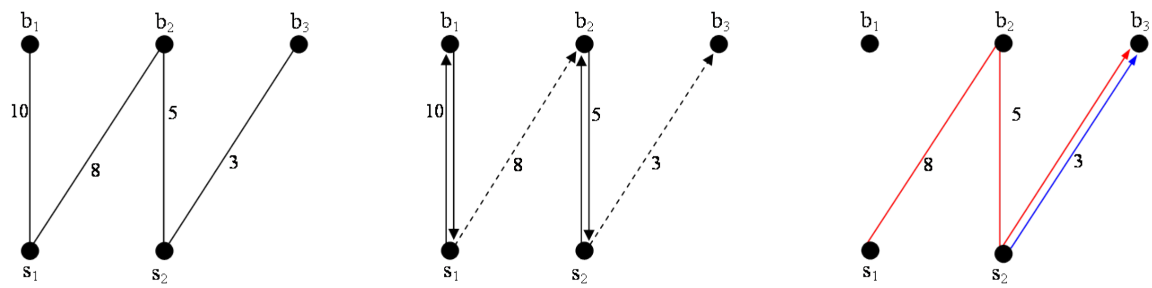
Appendix B. Network Decomposition

As outside option chains only rematch displaced parties to their outside trade partner outside options chains can be found on a network by identifying each party's trade partner and outside trade partner. This can be done using the process below which represents this information on a directed graph:[72]

(i) Identify trade partners. The matches on a network that are in the core must maximize the gains from trade. In the non-generic case any such match can be selected.
(ii) Identify outside trade partners. To find seller $s_{j'}$'s outside trade partner remove their trade partner $\mu(j', L)^*$ from the network and identify the match that maximizes the gains from trade on this new network.[73] $s_{j'}$ is now matched to their outside trade partner. If $s_{j'}$ is left unmatched they have no outside trade partner. Repeat for all sellers and all buyers.
(iii) Construct directed graph. This graph consists of two types of directed links. Links to trade partners and links to outside trade partners. For each buyer and seller construct a solid directed link to trade partners and a dashed directed link to outside trade partners.

The following relatively simple example shows how this process can be implemented. A more complex example is given in Section AM-1 of the additional material.

Example 4. *This example will show how a network can be decomposed to identify sellers' outside option chains. An equivalent process could be used to identify buyers' outside option chains. Consider the network shown in Figure 5a.*



(A) Underlying network (B) Outside options (C) Outside Option Chains

Figure 5. Directed Network Decomposition for Sellers

Matching b_1 to s_1 and b_2 to s_2 maximizes the possible gains from trade. This is shown below in match $\mu(L)^*$. Each seller's outside trade partner is found by removing their trade partner from the network and considering who they would be rematched to in order to maximize the gains form trade over this new network. For example to find s_1's outside trade partner we remove

[72]Generically this directed graph will be unique. However, if it is not unique, any directed graph can be selected and the results of this section will continue to hold.

[73]In the non generic case any such match can be selected. Although the directed network representation will not be unique the same outside option payoffs will ultimately be derived.

their trade partner b_1 from the network. On this reduced network $(L - b_1)$ match $\mu(L - b_1)^*$ maximizes the gains from trade by matching s_1 to b_2. b_2 is therefore s_1's outside trade partner. Outside trade partners for the other sellers are found in a similar way. These outside trade partners are represented by dashed arrows in Figure 5b whilst trade partners are shown by the solid arrows. Buyers' outside trade partners could be found in a similar way to generate the full directed network representation of L although this not shown in Figure 5. In Figure 5b parties benefit from a directed link that originates from them. For example s_1 has a directed trade link to b_1 and a directed non-trade (outside option) link to b_2.

$$\mu(L)^* = \begin{matrix} \{b_1, s_1\} \\ \{b_2, s_2\} \\ \{b_3, \emptyset\} \end{matrix} \quad \mu(L - b_1)^* = \begin{matrix} \{\boldsymbol{b_2, s_1}\} \\ \{b_3, s_2\} \end{matrix} \quad \mu(L - b_2)^* = \begin{matrix} \{b_1, s_1\} \\ \{\boldsymbol{b_3, s_2}\} \end{matrix}$$

Sellers' outside options chains can then be found from Figure 5b.[74] This is done by following the set of directed links originating at them and alternating between outside option links and trade links. Sellers' outside option chains are highlighted in Figure 5c. Seller s_1's outside option chain is in red and s_2's is in blue.

As shown in Lemma 2 each sellers' outside option value can be found by alternately adding and then subtracting the value of the links in their outside option chain:

$$\underline{\pi}^S = \begin{pmatrix} \alpha_{2,1} - \alpha_{2,2} + \alpha_{3,2} \\ \alpha_{3,2} \end{pmatrix} = \begin{pmatrix} 8 - 5 + 3 = 6 \\ 3 \end{pmatrix}$$

This representation of sellers' outside option payoffs identifies exactly how each link in the network affects each seller's outside option.

Appendix C. Network Formation Proofs

C.1. **Proof of Lemma 3.** *The best case efficiency loss due to only under-investment in trade links can be 100% $\forall \beta \neq \gamma$. In contrast there is no inefficiency loss due to under-investment in trade links when $\beta = \gamma$: When $\beta \neq \gamma$ there always exists some potential gains from trade α where in the best stable network all the net gains from trade generated by the efficient network are lost despite there only being inefficiency due to under investment in trade links. When $\beta = \gamma$ there is no under-investment inefficiency in any stable network for any potential gains from trade α.*

Proof. It will be shown that for $\beta \neq \gamma$ there always exists gains from trade such that the empty network is the unique stable network and a non-empty network is efficient.

Consider gains from trade α such that there is a unique non-empty efficient network.[75] For every link formed on the efficient network there must be gains from trade greater than the cost of link formation: $\alpha_{i'\mu(i',L^E)^*} > c$, $\forall i' : \mu(i', L^E)^* \neq i'$.

For the empty network to be stable there must be no incentives to form any trade links. For the links constructed on the efficient network to not be formed either the buyer or seller

[74]To find buyers' outside option chains buyers outside trade partners would have to be added to Figure 5b.

[75]Generically this is true anyway.

must be unwilling to pay their share of the cost of link formation: either $\beta\alpha_{i\mu(i,L^E)^*} < \gamma c$ or $(1-\beta)\alpha_{i\mu(i,L^E)^*} < (1-\gamma)c$.[76] Thus links that would be formed on the efficient network are not formed when:

$$max\left\{\frac{\gamma c}{\beta}, \frac{(1-\gamma)c}{(1-\beta)}\right\} > \alpha_{i\mu(i,L^E)^*} > c$$

Then $\forall\alpha_{i\mu(i,L^E)^*} \in (max\{\frac{\gamma c}{\beta}, \frac{(1-\gamma)c}{(1-\beta)}\}, c)$ there can exist links that are efficient to form that are not formed.

Consider now gains from trade α such that there is a unique efficient network L^E that implements $k > 0$ matches such that for each of these k matches $(\{i, \mu(i, L^E)^*\})$, $\alpha_{i\mu(i,L^E)^*} \in (max\{\frac{\gamma c}{\beta}, \frac{(1-\gamma)c}{(1-\beta)}\}, c)$ and all other potential gains from trade are zero ($\alpha_{ij} = 0$, $\forall j \neq \mu(i, L^E)^*$). With these potential gains from trade the unique stable network is the empty network but the efficient network generates positive net gains from trade: the best case efficiency loss is 100%. Such gains from trade exist if and only if $max\{\frac{\gamma c}{\beta}, \frac{(1-\gamma)c}{(1-\beta)}\} > c$ or equivalently if and only if $\beta \neq \gamma$.

In any stable network when $\gamma = \beta$ there can not exist an unmatched buyer i' and unmatched seller j' such that $\alpha_{i'j'} > c$. If such an unmatched pair existed buyer i' would be willing to form the link $l_{i'j'}$ (as $\beta\alpha_{i'j'} > \gamma c$) and seller j' would also be willing to form to this link (as $(1-\beta)\alpha_{i'j'} > (1-\gamma)c$). Any network with any under-investment inefficiency cannot therefore be stable. □

C.2. **Proof of Lemma 4.** *The worst case efficiency loss due to over-investment in non-trade links is bounded by the amounts shown in the table below and this bound is tight: There does not exist a stable network for any potential gains form trade that results in a higher loss in efficiency due to over-investment in non-trade links, but there does exist a stable network for some potential gains from trade with this level of inefficiency.*[77]

β	γ	Bound on $WCEL$ caused by over-investment
$\in (0,1)$	$\in (0,1)$	$h(\tilde{K}, \gamma, \beta)^* \leq \frac{1}{2}$
$\in \{0,1\}$	$\in (0,1)$	0^{**}
0	0	0
$\in (0, 0.5)$	0	$\frac{\beta}{1-\beta}$
$\in [0.5, 1]$	0	1
$\in [0, 0.5]$	1	1
$\in (0.5, 1)$	1	$\frac{1-\beta}{\beta}$
1	1	0

TABLE 3. Bounding the worst case efficiency lost due to over-investment in non-trade links

[76]If $\beta\alpha_{i\mu(i,L^E)^*} = \gamma c$ and $(1-\beta)\alpha_{i\mu(i,L^E)^*} = (1-\gamma)c$ then there would also be no incentives for the link $l_{i'\mu(i',L^E)^*}$ to be formed on the efficient network, but the efficient network would not be unique

[77]The size of the cost of link formation is normalized away: the values of alpha can always be scaled to compensate for a higher or lower c.

* $h(\tilde{K},\gamma,\beta) \equiv \frac{\tilde{K}-1}{max\left\{\frac{(\tilde{K}-1)+(1-\gamma)}{1-\beta}+\frac{(\tilde{K}-1)(1-\gamma)}{\beta},\frac{(\tilde{K}-1)+(\gamma)}{\beta}+\frac{(\tilde{K}-1)(\gamma)}{1-\beta}\right\}-\tilde{K}}$

*** The unique stable network is the empty network so the worst case efficiency loss due to over-investment in non-trade links is 0.*

where $\tilde{K}$ is the number of matches on the component in the network with the most matches when $\beta \leq (1-\gamma)$; the number of matches on the component in the network with the least number of matches greater than one when $\beta > (1-\gamma)$ and there is a component with more than one match; and 1 when all components have only one match.

Proof. This proof is undertaken in four parts. Part i) corresponds to the first row in the above table, Part ii) to the second row, Part iii) to rows five and six and Part iv) to rows three, four, seven and eight. In each part the existence of the bound and it tightness is proved.

Part i): First consider inefficiency due to the excessive formation of non-trade links when $\beta \in (0,1)$ and $\gamma \in (0,1)$. When $\gamma \in (0,1)$ all non-trade links must benefit *both* parties and as outside option chains cannot cycle (Lemma 7) each component of the network *must* take the form of a chain, as shown below for four trade links.[78]

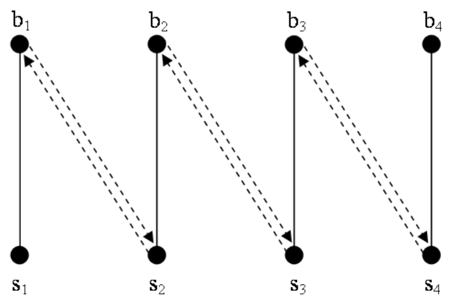


FIGURE 6. Component structure: 4 trade links

First it will be shown that on each component of the network with $\tilde{K}$ matches the level of inefficiency due to the excessive formation of non-trade links is limited.

Suppose buyers and sellers are labeled in the same way as the component in Figure 6, but with $\tilde{K}$ buyers and $\tilde{K}$ sellers. Buyer $b_{\tilde{K}}$ and seller s_1 must have sufficient incentives to form their trade links: $\beta\overline{\pi}^B_{\tilde{K}} \geq \gamma c$ and $(1-\beta)\overline{\pi}^S_1 \geq (1-\gamma)c$. Applying Lemma 2:

$$\begin{aligned} \overline{\pi}^S_1 &= \sum_{k=2}^{k=j}(\alpha_{k-1,k-1}-\alpha_{k-1,k}) + \overline{\pi}^S_j \geq \frac{(1-\gamma)c}{(1-\beta)} \\ \overline{\pi}^B_{\tilde{K}} &= \sum_{k=i}^{k=\tilde{K}-1}(\alpha_{k+1,k+1}-\alpha_{k,k+1}) + \overline{\pi}^B_i \geq \frac{\gamma c}{\beta} \qquad \forall j \in \{2,\ldots,\tilde{K}\};\ \forall i \in \{1,\ldots,\tilde{K}-1\} \end{aligned}$$

[78]To illustrate the structure of a component Figure 6 is shown for four traded over links and three non-traded over links.

As neither $b_{\tilde{K}}$ or s_1 have an outside trade partner $\underline{\pi}^B_{\tilde{K}} = 0$ and $\underline{\pi}^S_1 = 0$. All other buyers and sellers must have sufficient incentives to form their outside option links. For the other sellers $\beta\underline{\pi}^S_j \geq (1-\gamma)c$ and for the other buyers $(1-\beta)\underline{\pi}^B_i \geq \gamma c$. Applying Lemma 2 again:

$$\begin{aligned}
\underline{\pi}^B_i &= \sum_{k=i}^{k=\tilde{K}-1} (\alpha_{k,k+1} - \alpha_{k+1,k+1}) \geq \frac{\gamma c}{(1-\beta)} \\
\underline{\pi}^S_j &= \sum_{k=2}^{k=j} (\alpha_{k-1,k} - \alpha_{k-1,k-1}) \geq \frac{(1-\gamma)c}{\beta} \qquad \forall j \in \{2,\ldots,\tilde{K}\};\ \forall i \in \{1,\ldots,\tilde{K}-1\}
\end{aligned}$$

Combining the above expressions:

$$\begin{aligned}
\overline{\pi}^B i &\geq \frac{\gamma c}{\beta} + \sum_{k=i}^{k=\tilde{K}-1} (\alpha_{k,k+1} - \alpha_{k+1,k+1}) \geq \frac{\gamma c}{\beta} + \frac{\gamma c}{(1-\beta)} \\
\overline{\pi}^S j &\geq \frac{(1-\gamma)c}{(1-\beta)} + \sum_{k=2}^{k=j} (\alpha_{k-1,k} - \alpha_{k-1,k-1}) \geq \frac{(1-\gamma)c}{(1-\beta)} + \frac{(1-\gamma)c}{\beta} \\
&\qquad \forall j \in \{2,\ldots,\tilde{K}\};\ \forall i \in \{1,\ldots,\tilde{K}-1\}
\end{aligned}$$

The gains from trade on a network are equal to the total payoffs received by all parties. This is true at the buyer and seller optimal points:

$$\begin{aligned}
\sum_{k=1}^{\tilde{K}} \alpha_{k,k} &= \sum_{k=1}^{\tilde{K}} (\overline{\pi}^B_k + \underline{\pi}^S_k) \geq (\tilde{K}-1)\left(\frac{1}{\beta} + \frac{\gamma}{(1-\beta)}\right) c + \frac{\gamma}{\beta} c \\
\sum_{k=1}^{\tilde{K}} \alpha_{k,k} &= \sum_{k=1}^{\tilde{K}} (\underline{\pi}^B_k + \overline{\pi}^S_k) \geq (\tilde{K}-1)\left(\frac{1}{(1-\beta)} + \frac{1-\gamma}{\beta}\right) c + \frac{1-\gamma}{1-\beta} c
\end{aligned}$$

This provides a lower bound on the gains from trade reached with $\tilde{K}-1$ outside option links.

Applying the definition for the worst case efficient loss due to over-investment in non-trade links: $WCEL = \frac{(\tilde{K}-1)c}{\sum_{k=1}^{\tilde{K}} \alpha_{k,k} - \tilde{K}c}$. The denominator of this expression is bounded from below and this expression is bounded from above. Define the following functions:

$$\begin{aligned}
f_1(\tilde{K},\gamma,\beta) &\equiv (\tilde{K}-1)\left(\frac{1}{(1-\beta)} + \frac{1-\gamma}{\beta}\right) + \frac{1-\gamma}{1-\beta} \\
f_2(\tilde{K},\gamma,\beta) &\equiv (\tilde{K}-1)\left(\frac{1}{\beta} + \frac{\gamma}{(1-\beta)}\right) + \frac{\gamma}{\beta} \\
f(\tilde{K},\gamma,\beta) &\equiv max\{f_1(\tilde{K},\gamma,\beta), f_2(\tilde{K},\gamma,\beta)\} \\
h(\tilde{K},\gamma,\beta) &\equiv \frac{\tilde{K}-1}{f(\tilde{K},\gamma,\beta) - \tilde{K}}
\end{aligned}$$

Thus we have that on any component with $\tilde{K}$ matches, buyer bargaining power β and a buyer cost share of γ: $WCEL \leq h(\tilde{K}, \gamma, \beta)$.

In a stable network however, there may be multiple components. First note that the WCEL on the network must lie between the WCEL on the component with the highest WCEL and the component with the lowest WCEL. When $\beta \leq 1 - \gamma$ the upper bound on the WCEL is weakly increasing in the size of the component. Therefore the WCEL on the network must be less than or equal to the WCEL on the largest component. When $\beta > 1 - \gamma$ the bound on the worst case efficiency loss is weakly decreasing in the size of the component (for components involving at least two buyers and sellers- smaller components are necessarily efficient when stable). Therefore the WCEL on the network must be less than the WCEL on the smallest component with at least two buyers and two sellers. The existence of the bound follows.

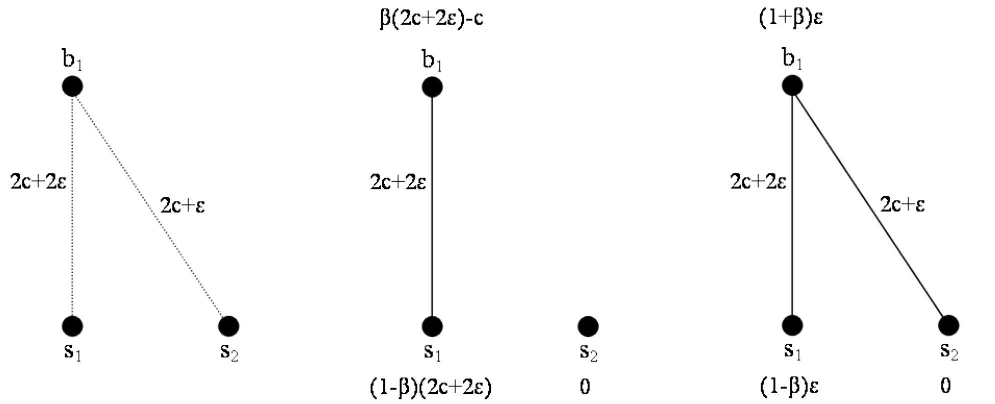
To show that this bound is always less that 50% the worst case efficiency loss bound can be maximized over β and γ: $WCEL^* \equiv max_{\beta,\gamma}\left\{h(\tilde{K}, \gamma, \beta)\right\}$. It is straightforward to show that the worst case efficient loss bound is maximized at $\beta = \gamma = \frac{1}{2}$. Thus $WCEL^* = \frac{1}{2}$. This is independent of $\tilde{K}$.

To show that the derived bound is tight an example will be constructed such that the binding constraints are just satisfied. Suppose that $\beta \geq \gamma$, such that $f_1 > f_2$ and buyers have more bargaining power relative to their cost share than sellers. In f_1 the binding constraints are on sellers' incentives to form trade links and on buyers' incentives to form outside option links. An example will be constructed such that these constraints are tight, assuming that the other constraints are slack. It will then be verified that these other constraints are slack.

For s_1 to be just willing to construct their trade link $\overline{\pi}_1^S = \alpha_{11} - \underline{\pi}_1^B = \frac{(1-\gamma)c}{1-\beta}$. For b_1 to be just willing to construct their outside option link $\underline{\pi}_1^B = \frac{\gamma c}{1-\beta}$. Combining these equations $\alpha_{11} = \frac{c}{1-\beta}$. For s_2 to be just willing to construct their trade link $\overline{\pi}_2^S = \alpha_{22} - \underline{\pi}_2^B = \frac{(1-\gamma)c}{1-\beta} + \frac{(1-\gamma)c}{\beta}$. For b_2 to be just willing to construct their outside option link $\underline{\pi}_2^B = \frac{\gamma c}{1-\beta}$. Thus $\alpha_{22} = \frac{c}{1-\beta} + \frac{(1-\gamma)c}{\beta}$. Applying the same logic $\alpha_{k,k} = \frac{c}{1-\beta} + \frac{(1-\gamma)c}{\beta}$, $\forall k \in \{2, \tilde{K}-1\}$. For $s_{\tilde{K}}$ to be just willing to construct their trade link $\overline{\pi}_{\tilde{K}}^S = \alpha_{\tilde{K}\tilde{K}} - \underline{\pi}_{\tilde{K}}^B = \alpha_{\tilde{K}\tilde{K}} = \frac{(1-\gamma)c}{1-\beta} + \frac{(1-\gamma)c}{\beta}$. This pins down the value of all trade links. For $b_{\tilde{K}-1}$ to just form their outside option link $\underline{\pi}_{\tilde{K}}^B = \alpha_{\tilde{K}-1,\tilde{K}} - \alpha_{\tilde{K}\tilde{K}} = \frac{\gamma c}{1-\beta}$. Thus $\alpha_{\tilde{K}-1,\tilde{K}} = \frac{c}{1-\beta} + \frac{(1-\gamma)c}{\beta}$. Working backwards it is straight forward to shown that all outside option links must have (at least) this value: $\alpha_{k-1,k} = \frac{c}{1-\beta} + \frac{(1-\gamma)c}{\beta}$, $\forall k \in \{2, \ldots \tilde{K}$. The minimum values of all traded links such that sellers are just willing to contribute to their formation have now been found, in conjunction with outside option links which buyers are just willing to form. The value of all other links can be set to zero and it can be verified that all other constraints are satisfied. The constructed network is therefore stable, by construction achieves the inefficiency bound and all inefficiency is due to over-investment in non-trade links.

Part ii): When $\beta = 1$ and $\gamma \in (0, 1)$ the seller has no incentive to form a trade link unless they also have an outside option link. Further a buyer will not pay towards the cost of forming any link unless they trade with a seller (because their payoff will be zero). Suppose the stable network was non-empty. As outside option chains cannot cycle there must exist a connected seller in the network without a valuable non-trade link, or a connected buyer without a trade partner. This party will not contribute towards the cost of forming any link and so cannot be connected to any other party as $\gamma \in (0,1)$. This is a contradiction and so the unique stable network must be the empty network. Thus there is never any efficiency loss due to over-investment in non-trade links.

Part iii): When $\beta \geq \frac{1}{2}$ and $\gamma = 0$ or $\beta \leq \frac{1}{2}$ and $\gamma = 1$ there exist potential gains form trade where the best case efficiency loss is 100%. This is shown for $\beta \leq \frac{1}{2}$ and $\gamma = 1$. Consider the gains from trade shown in the example below.



(A) Potential links (B) Efficient network (C) Unique stable network

FIGURE 7. Unbounded over-investment inefficiency

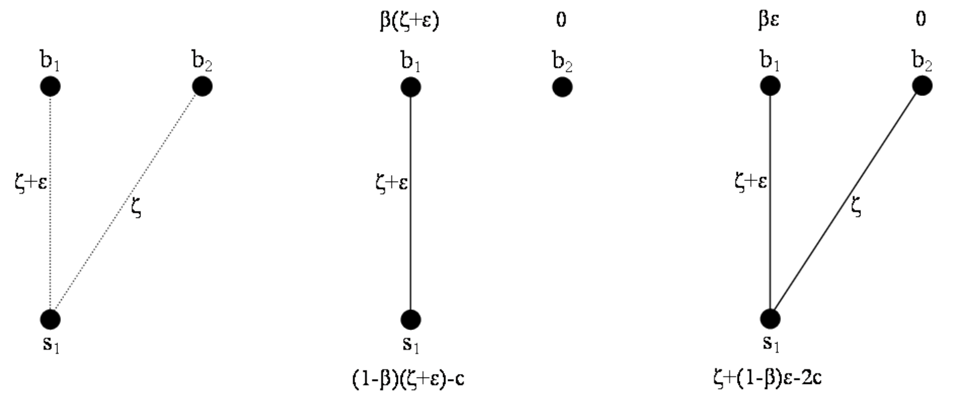
Here the unique stable network is the network shown in Figure 7c. The efficient network shown in Figure 7b is not stable because b_1 always wants to form an outside option link to s_2: $(1+\beta)\varepsilon \geq \beta(2c+2\varepsilon) - c$ for $\beta \leq \frac{1}{2}$. As buyer b_1 can form links on their own ($\gamma = 1$) and as b_1 receives a positive payoff on the network shown in Figure 7c, the empty network cannot be stable: To be stable it must be a Nash equilibrium of the simultaneous link formation game and b_1 would not be playing a best response. The network shown in Figure 7c is therefore the unique stable network. The best case efficiency loss is therefore: $\frac{c}{2c+2\varepsilon-c} \rightarrow 1$, as $\varepsilon \rightarrow 0$.

Part iv): In this proof it will be argued that the worst case efficiency loss due to over-investment in non-trade links can be largest on a subset of possible networks. This subset of networks will then be parameterized and largest possible worst case efficiency loss on any such network found.

Let $\beta < \frac{1}{2}$ and $\gamma = 0$. As $\gamma = 0$ only sellers can form outside option links. By Lemma 2 seller s_j's incentives to form an outside option link to buyer b_i are given by $\beta(\alpha_{\nu(j)j} - \overline{\pi}^B_{\nu(j)}) \leq \beta\alpha_{\nu(j)j}$. Thus the maximum incentives to form outside option links can always occur on networks where sellers' outside trade partners do not trade with anyone and $\beta(\alpha_{\nu(j)j} - \overline{\pi}^B_{\nu(j)}) = \beta\alpha_{\nu(j)j}$. Without loss of generality therefore, and as sellers will form at most one outside option link, inefficiency on network components consisting of two buyers and one seller can be considered: If a bound is found for the worst case efficiency loss on these networks it applies to all networks.

Consider then, without loss of generality,[79] the following network component:

[79]Varying $\varepsilon \geq 0$ and $\zeta > 0$ this example accounts for all possible two buyers one seller networks, for any c.



(A) Potential links (B) Efficient network (C) Stable network

FIGURE 8. Bounded over-investment inefficiency

On this network component for any efficiency loss due to over-investment in non-trade links s_1 must form links to both b_1 and b_2. To be incentivised to do this s_1's payoff on the network shown in Figure 8c must be greater than their payoff on the efficient network: $\zeta+(1-\beta)\varepsilon-2c \geq (1-\beta)(\zeta+\varepsilon)-c$. This holds if and only if $\zeta\beta \geq c$. Let $c = \zeta\beta - \xi$, $\xi \geq 0$, such that this constraint is satisfied. For this cost, and for the gains from trade above, $WCEL = \frac{\zeta\beta-\xi}{\zeta(1-\beta)+\varepsilon+\xi}$. The worst case efficiency loss is maximized by setting $\xi = \epsilon = 0$. Thus the worst case efficiency loss due to over-investment in non-trade links must be less than or equal to $\frac{\beta}{1-\beta}$ for any network with $\beta < \frac{1}{2}$ and $\gamma = 0$. This bound is achieved in the above example with $\xi = \epsilon = 0$. □

C.3. **Proof of Lemma 5.** *When cost sharing is endogenous worst case inefficiency due to over-investment in outside trade links can be 100%: There exist potential gains from trade where the most efficient stable network consumes all the net gains from trade generated on the efficient network.*

Proof. This proof is by counter example. Potential gains form trade (for all $\beta \in [0,1]$) such that the unique stable network is the complete network and on the complete network there are zero net gains from trade are identified. The proof proceeds first by considering $\beta \in [0, \frac{1}{2}]$ and then $\beta \in [\frac{1}{2}, 1]$.

Consider again the potential gains from trade shown in Figure 1a in Example 1. Suppose now that instead of sellers having all the bargaining power $\beta \in [0, \frac{1}{2}]$. The efficient network will not be stable for any such β: b_1 will receive an increase in their share of the surplus generated from trade with s_1 equal to $(1-\beta)\alpha_{b_1 s_2} = (1-\beta)(1-\varepsilon)$, at a cost of $c = \frac{1}{2} - \varepsilon$, if they form the link to s_2. This increases b_1's net payoff if and only if $(1-\beta)(1-\varepsilon) > \frac{1}{2} - \varepsilon$, or equivalently $\beta < \frac{1}{2-2\varepsilon} \leq \frac{1}{2}$). It is then straightforward to show that the unique efficient network is the complete network with b_1 paying for the link $l_{b_1 s_2}$ and paying between $\frac{1}{2} - (2-\beta)\varepsilon$ and $\frac{1}{2} - \varepsilon$ towards the formation of link $l_{b_1 s_1}$. The best case efficiency loss is therefore $\frac{\frac{1}{2}-\varepsilon}{\frac{1}{2}+\varepsilon}$. As $\varepsilon \to 0$ the best case efficiency loss goes to 100%. To show that the best case efficiency loss can go to 100% for $\beta \in [\frac{1}{2}, 1]$ relabel b_1 as s_1, s_1 as b_1 and s_2 as b_2. □

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