The competitive outcome in a dynamic entry and price game with capacity indivisibility

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Abstract

Strategic market interaction is here modelled as a two-stage game in which potential entrants choose capacities and active firms compete in prices. Due to capital indivisibility, the capacity choice is made from a finite grid and there are substantial economies of scale. In the simplest version of the model assuming a single production technique, the equilibrium of the game is shown to depend on the level of total demand at a price equal to the minimum of average cost: with a sufficiently large market, the competitive price (a price equal to the minimum of average cost) emerges at a subgame-perfect equilibrium of the game; failing the large market condition, the firms randomize in prices on the equilibrium path of the game. Generalizations are provided for the case of two techniques.

1 Introduction

Recent research on Bertrand-Edgeworth competition with endogenous capacity determination has achieved interesting results. Under the efficient rationing rule, the subgame-perfect equilibrium of a duopolistic two-stage capacity and price game (henceforth, CPG) yields the Cournot outcome (Kreps and Scheinkman, 1983). This result may not hold, though, under alternative rationing rules, where a mixed strategy equilibrium of the price subgame can arise on the equilibrium path (Davidson and Deneckere, 1986). More recently, Madden (1998) has established that a uniformly elastic demand curve is sufficient for the Cournot outcome under oligopoly, regardless of the rationing rule. Boccard and Wauthy (2000 and 2004) have shown that, while the Cournot result extends to oligopoly under KS's assumption on cost and the efficient rationing rule, this need not be so if, in the short run, the firms can produce above "capacity" at a finite extra-cost.

Throughout most of this literature the cost of capacity has been viewed as a continuous and convex function. As a consequence, at an equilibrium of the CPG identical firms choose a positive capacity (all choose to "enter"). Quite differently, this paper allows for economies of scale over some range of output. More specifically, because of capital indivisibility, potential entrants are taken to have a discrete capacity choice set and the long-run cost function exhibits discontinuities and nonconvexities.[1] We analyze a two-stage CPG under the efficient rationing rule and assuming (short-run) average variable cost to be constant until capacity. It turns out that the equilibrium may yield the long-run competitive outcome. The competitive outcome is also obtained by Yano (2008), although the price game under free entry being envisaged here is quite different from Yano's. Yano is concerned with the case of U-shaped average cost curves and, secondly, a strategy for a firm is a "price-set of quantities being offered" pair, each element in the set containing any profit maximizing output (at the chosen price).[2]

The paper is organized as follows. Section 2 presents a model with a single production technique. At any equilibrium of the CPG, total capacity turns out to be equal to the competitive one (the quantity demanded at a price equal to the minimum average cost), while pricing on the equilibrium path depends on the size of the market compared to the firm minimum efficient scale: with a sufficiently large market, the competitive price (the minimum of long-run average cost) is charged, otherwise the price subgame has a mixed strategy equilibrium. To test robustness of these results under a plurality of techniques, Section 3 shows how the competitive outcome may also arise when two production techniques are available. Section 4 points out the crucial role of capacity indivisibility for the possible emergence of the competitive outcome.

[1] The role of indivisibility of productive factors (especially of capital equipment) for economies of scale has long been recognized (see Kaldor, 1934, and Koopmans, 1957). For a recent discussion see Tone and Sahoo (2003).

[2] Hence, at a price equal to the minimum average cost, the set of quantities includes 0 as well as the average-cost minimizing output. By now, it is clear why the price game under free entry yields the competitive outcome - all active firms producing the average cost minimizing output (q^*) and charging a price equal to the minimum of average cost (c). Suppose any firm producing q^* deviates to a higher price. It could not sell anything since previous customers would now turn elsewhere and have their demand met by an "inactive" firm (any firm announcing a price of c but previously producing nothing).

2 A single technique

In a homogeneous-product industry, let $D(p)$ and $P(Q)$ be the demand and the inverse demand function, respectively, p the market price, and Q the total quantity. $D'(p) < 0$ and $D''(p) \leq 0$ for $p \in (0, \overline{p})$, where $D(p) = 0$ at $p \geq \overline{p}$ and $D(p) > 0$ at $p < \overline{p}$. A set $\mathcal{Z} = \{1, ..., i, ..., z\}$ of potential entrants choose capacity at stage 1, while active firms (each i with capacity $\overline{q}_i > 0$) set prices at stage 2. Capacity is chosen from a finite grid, due to indivisibility of capital. $\mathcal{F}_+ = \{f\}$ and $\mathbb{R}_+$ are the sets of nonnegative integers and reals, respectively. In this section where a single technique is assumed to be available, we let $\mathcal{F}_+$ be the capacity choice set faced by each firm and c the cost per unit of capacity. Given $\overline{q}_i$, i's cost is $c(q_i) = c\overline{q}_i$ for output $q_i \leq \overline{q}_i$ (we let 0 be the (constant) unit variable cost), while q_i cannot exceed $\overline{q}_i$. Long-run cost is thus $C(q_i) = c\overline{q}_i$, with $\overline{q}_i = [q_i, q_i + 1) \cap \mathcal{F}_+$ for any $q_i \in \mathbb{R}_+$: $C(q_i)$ is constant at any $q_i \in (f, f + 1]$ - hence not everywhere convex - and right-discontinuous at any f, where it increases by c. At any $q_i \in \mathcal{F}_+$, the average cost function equals c and is right-discontinuous (with an upward jump of c/f), while being decreasing, from $c(f + 1)/f$ to c, for $q_i \in (f, f + 1]$: capacity indivisibility results in scale economies over any such range of output.

A deterministic capacity choice is made by each $i \in \mathcal{Z}$ to maximize the expectation of profits $\pi_i = p_i q_i - c\overline{q}_i$. We denote by $\overline{\mathcal{Q}} = \mathcal{F}_+^z = \{\overline{q}\}$ the set of feasible capacity configurations, where $\overline{q} = (\overline{q}_1, ..., \overline{q}_z)$ is a capacity vector resulting from stage-1 decisions. Further, let $\overline{q}_{-i}$ denote the capacity configuration of i's rivals, $\overline{Q}$ total capacity, $\mathcal{A} = \{i \mid \overline{q}_i > 0\}$ and $n = \#\mathcal{A}$ the set and number of active firms at $\overline{q}$, respectively, and g any firm with the largest capacity. At stage 2 every $i \in \mathcal{A}$ knows $\overline{q}$.

We want to compare the outcome of strategic capacity and price setting with the long-run competitive equilibrium, namely, the equilibrium of an industry where price-taking potential entrants make simultaneous capacity and quantity decisions. Unfortunately, the competitive equilibrium may not exist.[3] In fact, total supply $S(p)$ is indefinitely large at $p > c$, and zero at $p < c$, while $S(c) \in \mathcal{F}_+$ (at $p = c$ entrants choose any feasible capacity and supply it entirely). Thus it can be $S(c) = D(c)$ only if $D(c) \in \mathcal{F}_+$.[4] We overcome nonexistence by restricting ourselves to demand curves such that $D(c) \in \mathcal{F}_+$; thus the "competitive" price (p^*) and output (Q^*) are, respectively, $p^* = c$ and $Q^* = D(c)$.[5]

[3] For nonexistence under U-shaped average cost, see Mas-Colell et al., 1995, pp. 337-8.
[4] Even so, some coordination is needed for the firms to exactly supply $D(c)$.
[5] De Francesco (2006) provides an analysis of the $D(c) \notin \mathcal{F}_+$ case under linear demand.

We denote by $\overline{\mathcal{Q}}^*$ the set of the least concentrated capacity configurations (active firms have the minimum feasible capacity) consistent with the long-run competitive capacity: $\overline{\mathcal{Q}}^* = \{\overline{q}^*\}$, where each $\overline{q}^*$ is such that $n^* = \overline{Q}^* = D(c)$. Further, at any $\overline{q}$, let $p^w(\overline{q})$ and $Q^w(\overline{q})$ be, respectively, the market-clearing price and total output with price-taking firms: $p^w(\overline{q}) = P(\overline{Q})$ and $Q^w(\overline{q}) = \overline{Q}$ if $\overline{Q} \leq D(0)$, while $p^w(\overline{q}) = 0$ and $Q^w(\overline{q}) = D(0)$ if $\overline{Q} \geq D(0)$. Henceforth $\pi_i^w(\overline{q}) = (p^w(\overline{q}) - c)\overline{q}_i$ denotes i's profit at $\overline{q}$ under market clearing and $\pi_i^w(\overline{q}_i, \overline{q}_{-i}) = (p^w(\overline{q}_i, \overline{q}_{-i}) - c)\overline{q}_i$ denotes i's profit under market clearing as a function of $\overline{q}_i$, given $\overline{q}_{-i}$. If $\overline{q}_i$ were continuous, then concavity of $\pi_i^w(\overline{q}_i, \overline{q}_{-i})$ would follow straightforwardly from $D''(p) \leq 0$.

A price subgame is played at any $\overline{q}$. Let $\mathbf{p} = (p_1, ..., p_n) = (p_i, p_{-i})$ be a (pure) strategy profile in the subgame, p_{-i} being the strategy profile of i's rivals, and let $d_i(p_i, p_{-i}, \overline{q})$, $q_i(p_i, p_{-i}, \overline{q})$, $\pi_i(p_i, p_{-i}, \overline{q})$ and $\Pi_i(p_i, p_{-i}, \overline{q})$ be, respectively, firm i's demand, output, profit and revenue in subgame $\overline{q}$ at strategy profile $\mathbf{p}$: $\pi_i(p_i, p_{-i}, \overline{q}) = p_i q_i(p_i, p_{-i}, \overline{q}) - c\overline{q}_i = p_i \min\{d_i(p_i, p_{-i}, \overline{q}), \overline{q}_i\} - c\overline{q}_i$. Under efficient rationing, $d_i(p_i, p_{-i}, \overline{q}) = \max\{0, D(p_i) - \sum_{j \neq i} \overline{q}_j\}$ when $p_i > p_j$ for any $j \neq i$. Let $\widetilde{q}_i = \widetilde{q}(\sum_{j \neq i} \overline{q}_j) = \arg\max_{q_i} P(q_i + \sum_{j \neq i} \overline{q}_j)q_i$ and $\widetilde{\Pi}_i = P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j)\widetilde{q}_i$.[6] So long as $\widetilde{q}_i \leq \overline{q}_i$, $\widetilde{q}_i$ is i's (short-run) Cournot best response to an output of $\sum_{j \neq i} \overline{q}_j$ by rivals. With $D'' \leq 0$, $\widetilde{q}'(\cdot) < 0$ for $\sum_{j \neq i} \overline{q}_j < D(0)$. With $\sum_{j \neq i} \overline{q}_j < D(0)$, we also let $\widetilde{p}_i = \widetilde{p}(\sum_{j \neq i} \overline{q}_j) = \arg\max_p p[D(p) - \sum_{j \neq i} \overline{q}_j]$. Clearly, $\widetilde{p}_i = P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j)$ and $\widetilde{\Pi}_i = \widetilde{p}_i \widetilde{q}_i$; also, $\max_i \widetilde{p}_i = \widetilde{p}_g$ because $\widetilde{p}' < 0$ for $\sum_{j \neq i} \overline{q}_j < D(0)$. Let $\pi_i(\overline{q})$ and $\Pi_i(\overline{q})$ be, respectively, i's expected profit and revenue at an equilibrium of the price subgame. The following result is easily established.

Lemma 1. *For any $i \in \mathcal{A}$, $\pi_i(\overline{q}) \geq \pi_i^w(\overline{q})$.*

Proof. This is obvious if $p^w(\overline{q}) = 0$. With $p^w(\overline{q}) > 0$, by charging $p^w(\overline{q})$ firm i fully utilizes capacity and hence earns $\pi_i^w(\overline{q})$, regardless of p_{-i}. ■

With $\overline{Q} \neq D(0)$, for the market-clearing price to obtain at an equilibrium of the price subgame individual capacities must be sufficiently small compared to industry capacity (for the symmetric case, see Vives, 1986).

Lemma 2. *(i) With $p^w(\overline{q}) = 0$, $(p^w, ..., p^w)$ is an equilibrium of price subgame $\overline{q}$ iff $\overline{q}_g/\overline{Q} \leq 1 - D(0)/\overline{Q}$. (ii) With $p^w(\overline{q}) > 0$, $(p^w, ..., p^w)$ is the equilibrium of price subgame $\overline{q}$ iff*

$$-p^w D'(p^w) \geq \overline{q}_g. \tag{1}$$

[6] With $\sum_{j \neq i} \overline{q}_j < D(0)$, $\widetilde{q}_i$ is the unique solution to $\frac{\partial}{\partial q_i}\left[P(q_i + \sum_{j \neq i} \overline{q}_j)q_i\right] = 0$; with $\sum_{j \neq i} \overline{q}_j \geq D(0)$, $P(q_i + \sum_{j \neq i} \overline{q}_j)q_i = 0$ at any $q_i \geq 0$.

Proof. (i) All prices equal to zero is an equilibrium if and only if $\sum_{j \neq g} \overline{q}_j \geq D(0)$, which leads to the stated condition.[7]

(ii) $(p^w, ..., p^w)$ is an equilibrium if and only if $\left[\frac{\partial(p(D(p)-\sum_{j\neq i}\overline{q}_j)}{\partial p}\right]_{p=p^w(+)} \leq 0$ for all $i \in \mathcal{A}$, which leads to $-p^w D'(p^w) \geq \overline{q}$ and hence to (1). Uniqueness of equilibrium can be established straightforwardly. ■

Inequality $-p^w D'(p^w) \geq \overline{q}_i$ has a clear meaning. A uniform price $p^w > 0$ is an equilibrium if and only if, for any firm, residual demand has elasticity not less than 1 when its price is raised above p^w. Inequality (1) can also be written $\overline{q}_g/\overline{Q} \leq \eta_{p=p^w}$, where $\eta_{p=p^w}$ is total demand elasticity at price p^w, or $\widetilde{p}_g \leq p^w$. A pure-strategy equilibrium (pse) does not exist when $\overline{Q} \geq D(0)$ and $\sum_{j\neq g} \overline{q}_j < D(0)$ or when $\overline{Q} < D(0)$ and $\widetilde{p}_g > p^w$. Then a mixed-strategy equilibrium (mse) exists: all the sufficient conditions of Theorem 5 of Dasgupta and Maskin (1986) for equilibrium existence are satisfied. At a mse, expected revenue for the largest firm equals the revenue of the Stackelberg follower when rivals supply their capacity. (For this property, see Kreps and Scheinkman (1983) for the duopoly; and see Boccard and Wauthy, 2000, and De Francesco, 2003, for the oligopoly.).

Lemma 3. *At any $\overline{q}$ for which no pse exists, $\Pi_g(\overline{q}) = \widetilde{\Pi}_g = \widetilde{p}_g \widetilde{q}_g$.*
Proof. See De Francesco (2003). ■

Let $\pi_i^w(\overline{q}_i = \widetilde{q}_i, \overline{q}_{-i}) \equiv \pi_i^w(\widetilde{q}_i, \overline{q}_{-i})$, where $\pi_i^w(\widetilde{q}_i, \overline{q}_{-i}) = (P(\widetilde{q}_i + \sum_{j\neq i} \overline{q}_j) - c)\widetilde{q}_i$. In light of Lemma 3, firm g's expected profit at a mse of price subgame $\overline{q}$ can be written $\pi_g(\overline{q}) = \pi_g^w(\widetilde{q}_g, \overline{q}_{-g}) - c(\overline{q}_g - \widetilde{q}_g)$. One main result of the paper is the following.

Proposition 1 *(i) If $-cD'(c) \geq 1$, then any $\overline{q}^*$ is (part of) an equilibrium of the CPG in which the competitive price c is charged on the equilibrium path; (ii) if $-cD'(c) < 1$, then any $\overline{q}^*$ is an equilibrium of the CPG in which the firms randomize over prices on the equilibrium path. (iii) $\overline{Q} = D(c)$ at any equilibrium of the CPG.*

Proof. (i) At any $\overline{q}^*$ inequality (1) reads $-cD'(c) \geq 1$: holding it, $(c, ..., c)$ is the equilibrium of the price subgame. In all cases any active firm (any $i \in \mathcal{A}^*$) has made a best capacity response to $\overline{q}^*_{-i}$. If $\widetilde{p}_i^* > P(D(c)+1)$,[8] then a mse obtains when i deviates to $\overline{q}'_i \geq 2$, resulting in $\pi_i(\overline{q}'_i, \overline{q}^*_{-i}) = \widetilde{p}_i^* \widetilde{q}_i^* - c\overline{q}'_i$. This is negative because $\widetilde{p}_i^* \leq c$ and $1 \leq \widetilde{q}_i^* < 2 \leq \overline{q}'_i$. If $\widetilde{p}_i^* \leq P(D(c)+1)$, then deviating to $\overline{q}'_i = 2$ leads to a pse, hence to a loss.

[7] Other equilibria are such that $\sum_{j\neq i: p_j = 0} \overline{q}_j \geq D(0)$ for any $i : p_i = 0$.

[8] According to our notation, $\widetilde{p}_i^* = P(\widetilde{q}_i^* + \sum_{j\neq i} \overline{q}_j^*)$ and $\widetilde{q}_i^* = \arg\max_{q_i} P(q_i + \sum_{j\neq i} \overline{q}_j^*) q_i$.

A fortiori losses arise if deviating to $\overline{q}_i' > 2$. Finally, at $\overline{q}^*$ any inactive firm (any $u \notin \mathcal{A}^*$) has made a best response. Denote by $(\overline{q}_u', \overline{q}_{-u}^*)$ the capacity configuration when u deviates to $\overline{q}_u' > 0$. Obviously $\pi_u(\overline{q}_u', \overline{q}_{-u}^*) < 0$ if a pse obtains. If a mse obtains, then $\pi_u(\overline{q}_u', \overline{q}_{-u}^*) = \widetilde{p}_u \widetilde{q}_u - c\overline{q}_u'$; this is negative because $\widetilde{p}_u = P(\widetilde{q}_u + D(c)) < c$ and $\widetilde{q}_u < \overline{q}_u'$.

(ii) A mse obtains at $\overline{q}^*$, hence $\pi_i(\overline{q}^*) = \widetilde{p}_i^* \widetilde{q}_i^* - c > 0$. Any $i \in \mathcal{A}^*$ has replied optimally: deviating to $\overline{q}_i' > 1$ raises cost without affecting expected revenue. Any $u \notin \mathcal{A}^*$ has also made a best response: deviating to $\overline{q}_u' > 0$ leads to a mse,[9] hence $\pi_u(\overline{q}_u', \overline{q}_{-u}^*) = \widetilde{p}_u \widetilde{q}_u - c\overline{q}_u' < 0$ since $\widetilde{p}_u = P(\widetilde{q}_u + D(c)) < c$ and $\widetilde{q}_u < \overline{q}_u'$.

(iii) With $\overline{Q} < D(c)$, any $u \notin \mathcal{A}$ will profit by deviating to $\overline{q}_u' = 1$ and charging $p_u = P(\overline{Q}+1)$.[10] With $\overline{Q} > D(c)$ and holding (1), any $i \in \mathcal{A}$ make losses. If $\overline{Q} > D(c)$ and (1) does not hold, then g will profit by reducing capacity by one unit. This is immediately seen if $\overline{q}_i = 1$ for all $i \in A$, in which case $\pi_i(\overline{q}) = \widetilde{p}_i \widetilde{q}_i - c < 0$ since $\widetilde{p}_i = P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j) < c$. With $\overline{q}_g > 1$, $\pi_g(\overline{q}) = \widetilde{p}_g \widetilde{q}_g - c\overline{q}_g$, with $\widetilde{q}_g < \overline{q}_g$. Let $\widetilde{p}_g > c$ (otherwise the point is obvious). Since $\widetilde{p}_g = P(\widetilde{q}_g + \sum_{j \neq g} \overline{q}_j)$, it follows that $\widetilde{q}_g + \sum_{j \neq g} \overline{q}_j < D(c)$. On the other hand, $\overline{q}_g + \sum_{j \neq g} \overline{q}_j - D(c)$ is a positive *integer*, hence $\widetilde{q}_g < \overline{q}_g - 1$. Clearly, firm g will profit by deviating to $\overline{q}_g' = \overline{q}_g - 1$: this lowers costs while affording an expected revenue not less than $\widetilde{p}_g \widetilde{q}_g$. ■

Remarks. (a) The condition of statement (i) can also be written $\frac{1}{D(c)} \leq \eta_{p=c}$: for the competitive outcome to arise at an equilibrium of the CPG the market has to be sufficiently "large", in the sense that the ratio between the firm's minimum efficient size and competitive industry output must not exceed demand elasticity.

(b) According to statement (iii), equilibrium total capacity *always* equals the long-run competitive output $D(c)$. (As will be seen in the next section, this result is not robust to the introduction of a plurality of techniques.)

Computing the mse obtaining at $\overline{q}^*$ when $-cD'(c) < 1$ is standard. There is an equilibrium distribution $\phi(p)$ over support $S = [\underline{p}^*, \overline{p}^*]$; since expected revenue is $\widetilde{\Pi}_i^* = \widetilde{p}_i^* \widetilde{q}_i^*$, $\overline{p}^* = \widetilde{p}_i^*$ and $\underline{p}^* = \widetilde{\Pi}_i^*$. For $p \in S$, $p\phi^{n-1}[D(p) - (D(c) - 1)] + p(1 - \phi^{n-1}) = \widetilde{\Pi}_i^*$ and hence $\phi(p) = \sqrt[D(c)-1]{\frac{p - \widetilde{\Pi}_i^*}{p[D(c) - D(p)]}}$.

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[9] This fact is immediate if $\overline{q}_u + D(c) \geq D(0)$ since then $p^w(\overline{q}_u, \overline{q}_{-u}) = 0$ while $\sum_{j \neq u} \overline{q}_j = D(c) < D(0)$ (see statement (i) of Lemma 2). With $\overline{q}_u + D(c) < D(0)$, then $d[p(D(p) - D(c))]/dp > 0$ at $p = p^w(\overline{q}_u, \overline{q}_{-u}) = P(\overline{q}_u + D(c))$: this follows from $D'' \leq 0$ and the fact that $d[p(D(p) - \sum_{j \neq i} \overline{q}_j)]/dp > 0$ at $p = c$ and $\sum_{j \neq i} \overline{q}_j = D(c) - 1$.

[10] With $\overline{Q} = \overline{Q}^* - 1$, this would result in zero profit if the resulting subgame has a p.s.e.. Any such $\overline{q}$ is disposed of if, at zero profit, entering is preferred to not entering.

Examples. 1. $D(p) = 15 - p$ and $c = 2$. At any equilibrium, $n = \overline{Q} = D(c) = 13$, and the competitive price c obtains.

2. $D(p) = (10.5 - p)/3$ and $c = 1.5$. At any equilibrium, $n = \overline{Q} = D(c) = 3$, $\Pi_i = \widetilde{\Pi}_i^* = 1.6875$, and $\pi_i = .1875$. On the equilibrium path, $\phi(p) = \sqrt[2]{\frac{3(1.6875-p)}{p(1.5-p)}}$ over $S = [1.6875, 2.25]$. $\diamond$

Having shown that capacity configurations $\overline{q}^*$ are always part of an equilibrium, one might ask whether the converse is also true: can capacity configurations $\overline{q} : \overline{q_g} > 1; \overline{Q} = D(c)$ be always ruled out as equilibria? The answer is definitely yes under linear demand.

Proposition 2 *If $D'' = 0$, then $\overline{q} \in \overline{\mathcal{Q}}^*$ at any equilibrium of the CPG.*

Proof. In the Appendix. ■

Unlike with linear demand, with $D'' < 0$ there might be equilibria with some active firms having more than the minimum capacity. For example, let $p = 16.01 - Q^2$ and $c = 0.01$. Then $D(c) = 4$ and $-cD'(c) < 1$, hence any $\overline{q}^*$ (any configuration with $\overline{Q} = 4; \overline{q}_i = 1$ for all $j \in \mathcal{A}^*$) is an equilibrium where active firms randomize on the equilibrium path. However, one can check that any $\overline{q}$ such that $\overline{Q} = 4, n = 3, \overline{q}_g = 2$ is an equilibrium too.

3 The case of two production techniques

The competitive outcome may also arise under a plurality of available techniques. Suppose that potential entrants can choose among two production techniques, α and β, entailing a cost per capacity unit of c_α and c_β and capacity choice sets $\overline{\alpha}\mathcal{F}_+$ and $\overline{\beta}\mathcal{F}_+$, respectively. We let $\overline{\beta} < \overline{\alpha}$, $c_\beta\overline{\beta} < c_\alpha\overline{\alpha}$, and $c_\alpha < c_\beta$: the average-cost minimizing technique is α, although β, by entailing a lower minimum capacity, is cheaper at a sufficiently low output. Similarly as before, we assume $D(c_\alpha) \in \overline{\alpha}\mathcal{F}_+$ and $D(c_\beta) \in \overline{\beta}\mathcal{F}_+$. Note that, at the competitive equilibrium, α is adopted, $p = c_\alpha$, and $\overline{Q} = D(c_\alpha)$. We let $\overline{\mathcal{Q}}^{(\alpha)} = \{\overline{q}^{(\alpha)} : n^{(\alpha)} = D(c_\alpha)/\overline{\alpha}, \overline{q}_i^{(\alpha)} = \overline{\alpha}\ \forall i \in \mathcal{A}^{(\alpha)}\}$, i.e., the set of the least concentrated capacity configurations consistent with the competitive capacity. We also let $\overline{\mathcal{Q}}^{(\beta)} = \{\overline{q}^{(\alpha)} : n^{(\beta)} = D(c_\beta)/\overline{\beta}, \overline{q}_i^{(\beta)} = \overline{\beta}\ \forall i \in \mathcal{A}^{(\beta)}\}$. Further, we let $\widetilde{q}_i^{(\alpha)} = \widetilde{q}(\sum_{j \neq i} \overline{q}_j^{(\alpha)})$ and $\widetilde{p}_i^{(\alpha)} = P(\widetilde{q}_i^{(\alpha)} + \sum_{j \neq i} \overline{q}_j^{(\alpha)})$, $\widetilde{q}_i^{(\beta)} = \widetilde{q}(\sum_{j \neq i} \overline{q}_j^{(\beta)})$ and $\widetilde{p}_i^{(\beta)} = P(\widetilde{q}_i^{(\beta)} + \sum_{j \neq i} \overline{q}_j^{(\beta)})$. Though not pursuing a complete characterization of equilibria, we provide necessary and sufficient conditions for the competitive outcome to possibly arise at an equilibrium of the CPG.

Proposition 3 *(i) Let* $-c_\alpha D'(c_\alpha) \geq \overline{\alpha}$ *and* $P(D(c_\alpha) - \overline{\alpha} + \overline{\beta}) \leq c_\beta$*. Then* $\overline{q}^{(\alpha)}$ *is an equilibrium of the CPG, with the competitive price* c_α *being charged on the equilibrium path. (ii) Let* $-c_\alpha D'(c_\alpha) \geq \overline{\alpha}$ *and* $P(D(c_\alpha) - \overline{\alpha} + \overline{\beta}) > c_\beta$*. Then* $\overline{q}^{(\alpha)}$ *is not an equilibrium of the CPG; any* $\overline{q}_\beta$ *is an equilibrium, with the market-clearing price* c_β *being charged on the equilibrium path.*

Proof. (i) With $-c_\alpha D'(c_\alpha) \geq \overline{\alpha}$, a pse arises at $\overline{q}^{(\alpha)}$. Further, for any $i \in \mathcal{A}^{(\alpha)}$ it does not pay to reduce capacity, what might be done by deviating to technique β and installing, say, capacity $\overline{\beta}$:[11] at the new pse, [12] the deviant will sell $\overline{\beta}$ at price $P(D(c_\alpha) - \overline{\alpha} + \overline{\beta}) \leq c_\beta$, hence losses (or no gains).

(ii) Since $D'' \leq 0$, it is also $-c_\beta D'(c_\beta) > \overline{\beta}$: a pse obtains at $\overline{q}^{(\beta)}$. While $\overline{q}^{(\alpha)}$ is not an equilibrium (it pays any $i \in \mathcal{A}^{(\alpha)}$ to deviate to technique β and capacity $\overline{\beta}$), $\overline{q}^{(\beta)}$ is an equilibrium. Suppose any $i \in \mathcal{A}^{(\beta)}$ deviates to technique α and capacity $\overline{\alpha}$. The market-clearing price falls to $P(D(c_\beta) + \overline{\alpha} - \overline{\beta})$, less than c_α since $P(D(c_\alpha) - \overline{\alpha} + \overline{\beta}) > c_\beta$: hence a loss, if a pse arises. A loss would also arise at a mse, i.e., $\widetilde{p}_i^{(\beta)} \widetilde{q}_i^{(\beta)} - c_\alpha \overline{\alpha} < 0$: in fact, $\widetilde{q}_i^{(\beta)} < \overline{\alpha}$ while, on the other hand, $\widetilde{p}_i^{(\beta)} < c_\alpha$ (since $\widetilde{p}_i^{(\alpha)} \leq c_\alpha$, $\widetilde{p}'(\cdot) < 0$, and $\sum_{j \neq i} \overline{q}_j^{(\beta)} = D(c_\beta) - \overline{\beta} > \sum_{j \neq i} \overline{q}_j^{(\alpha)} = D(c_\alpha) - \overline{\alpha}$). ■

Note that, in the circumstances of statement (ii), equilibrium total capacity is below the competitive level; firms make no profit but average cost is above the minimum.

Examples. 1: $D(p) = 32 - 2p$, $c_\alpha = 1$ and $\overline{\alpha} = 1$, $c_\beta = 1.2$ and $\overline{\beta} = 0.8$. Statement (i) applies: any $\overline{q}^{(\alpha)}$ is an equilibrium of the CPG, with the firms charging the competitive price c_α on the equilibrium path.

2. $D(p) = 16 - p$, $c_\alpha = 2$ and $\overline{\alpha} = 2$, $c_\beta = 2.2$ and $\overline{\beta} = 0.6$. Statement (ii) applies. At $\overline{q}^{(\alpha)}$ (7 active firms adopting α, each with capacity $\overline{\alpha}$), it would pay for any active firm to deviate to technique β and install $\overline{\beta}$. Any $\overline{q}^{(\beta)}$ (23 active firms adopting β, each with capacity $\overline{\beta}$) is an equilibrium, with the firms charging 2.2 on the equilibrium path. At $\overline{q}^{(\beta)}$ it does not pay any active firm to raise capacity and adopt α (it would make a loss at the mse obtaining at the new price subgame).

Note that demand elasticity matters in two ways for the possibility of the competitive outcome. First, as in the preceding section, demand must be sufficiently elastic for the price subgame to have a pse at $\overline{q}^{(\alpha)}$: inequality

[11] Installing any $\overline{q}_i \geq \overline{\alpha}$ while deviating to β is immediately discarded.

[12] From the fact that $-c_\alpha D'(c_\alpha) \geq \overline{\alpha}$ and since $D'' \leq 0$, it follows that (1) also holds at the new subgame, namely, $-pD'(p) \geq \overline{\alpha}$ at $p = P(D(c_\alpha) - \overline{\alpha} + \overline{\beta})$.

$-c_{\alpha}D'(c_{\alpha}) \geq \overline{\alpha}$ can in fact be expressed as $\overline{\alpha}/D(c_{\alpha}) \leq \eta_{p=c_{\alpha}}$, hence the "large market" condition is all the less restrictive the higher demand elasticity at the competitive price. Secondly, given $D(c_{\alpha})$, a sufficiently high demand elasticity makes it unworthy, at $\overline{q}^{(\alpha)}$, to any active firm to deviate to capacity $\overline{\beta}$ (the price increase at the new subgame is less than the increase in the firm's unit cost).

One might wonder about generalization of previous results to the case of several techniques. While not dwelling with this issue at lenght, we will show how statement (i) of Proposition 3 would generalize. Let available techniques be $\alpha, \beta, \gamma, \delta, ...$, with capacity choice sets $\overline{\alpha}\mathcal{F}_+, \overline{\beta}\mathcal{F}_+, \overline{\gamma}\mathcal{F}_+, \overline{\delta}\mathcal{F}_+, ...$, respectively, where $\overline{\alpha} > \overline{\beta} > \overline{\gamma} > \overline{\delta} > ....$ As before, we let $c_{\alpha} < c_{\beta} < c_{\gamma} < c_{\delta}, ...,$ and $c_{\alpha}\overline{\alpha} > c_{\beta}\overline{\beta} > c_{\gamma}\overline{\gamma} > c_{\delta}\overline{\delta} > ....$ Again, at the competitive equilibrium, technique α is adopted, $p = c_{\alpha}$, and $\overline{Q} = D(c_{\alpha})$. Statement (i) would now read as follows: Let $-c_{\alpha}D'(c_{\alpha}) \geq \overline{\alpha}$, $P(D(c_{\alpha}) - \overline{\alpha} + \overline{\beta}) \leq c_{\beta}$, $P(D(c_{\alpha}) - \overline{\alpha} + \overline{\gamma}) \leq c_{\gamma}$, $P(D(c_{\alpha}) - \overline{\alpha} + \overline{\delta}) \leq c_{\delta}$, and so on. Then $\overline{q}^{(\alpha)}$ is an equilibrium of the CPG, with the competitive price c_{α} being charged on the equilibrium path.

4 Final remarks

We have seen how the long-run competitive outcome can arise at an equilibrium of a CPG. Capacity indivisibility plays a key role in this connection. Suppose that capacity were instead a continuous variable. Then the competitive price c cannot arise at an equilibrium of the CPG.[13] Consider any capacity configuration with total capacity equal to $D(c)$ and such that the market-clearing price c is charged at an equilibrium of the price subgame. Clearly, any active firm has not made a best capacity response: by lowering its capacity the market clearing price would rise above c, resulting in positive profits at an equilibrium of the price subgame.

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[13] We are here referring to the single-technique version of our model but the argument extends straightforwardly to the case of several techniques.

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Proof of Proposition 2. Let $P(Q) = a - bQ$ for $Q \leq \overline{p} = a/b$ $(a, b > 0)$. Then $D(c) = (a-c)/b$, $\widetilde{q}_i = \frac{a-b\sum_{j\neq i}\overline{q}_j}{2b}$ and $\widetilde{p}_i = \frac{a-b\sum_{j\neq i}\overline{q}_j}{2}$. Further, $\partial^2\pi_i^w(\overline{q}_i, \overline{q}_{-i})/\partial\overline{q}_i^2 = -2b$ when $\overline{q}_i + \sum_{j\neq i}\overline{q}_j < D(0)$. Given Prop. 1, we just need to rule out any $\overline{q} : \overline{Q} = (a-c)/b; \overline{q}_g > 1$. In fact, firm g would benefit from deviating to $\overline{q}_g - 1$. This is immediate when price subgame $\overline{q}$ has a p.s.e: then $\pi_i(\overline{q}) = 0$ for any $i \in \mathcal{A}$, hence g would profit from deviating to $\overline{q}_g - 1$ and then charging $P(D(c) - 1) = c + b$. If price subgame $\overline{q}$ has a mse, then $\widetilde{p}_g > c$ and $\pi_g(\overline{q}) = \widetilde{\Pi}_g - c\overline{q}_g$. There are two possibilities: either $\widetilde{p}_g \geq c+b$ or $\widetilde{p}_g < c+b$. In the former case deviating to $\overline{q}_g - 1$ would raise g's expected profit at least to $\widetilde{\Pi}_g - c(\overline{q}_g - 1)$: since rivals can produce $\sum_{j \neq g} \overline{q}_j$ at most, firm g will sell at least $\widetilde{q}_g = D(\widetilde{p}_g) - \sum_{j \neq g} \overline{q}_j \leq \overline{q}_g - 1$ when charging $\widetilde{p}_g$. If $\widetilde{p}_g < c + b$, then $\overline{q}_g - 1 < \widetilde{q}_g < \overline{q}_g$. Let $\overline{q}_i^{\dagger} = argmax_{\overline{q}_i \in \mathbb{R}_+} \pi_i^w(\overline{q}_i, \overline{q}_{-i})$: with $\sum_{j \neq i} \overline{q}_j \leq (a - c)/b$, then $\overline{q}_i^{\dagger} = 0.5[\frac{a-c)}{b} - \sum_{j \neq i} \overline{q}_j]$ and one can write $\pi_i^w(\overline{q}_i, \overline{q}_{-i}) = \pi_i^w(\overline{q}_i^{\dagger}, \overline{q}_{-i}) - b\left(\overline{q}_i - \overline{q}_i^{\dagger}\right)^2$. The capacity reduction can be broken down in two virtual reductions, from $\overline{q}_g$ to $\widetilde{q}_g$ and then from $\widetilde{q}_g$ to $\overline{q}_g - 1$. It suffices to prove that g's profit will rise if, at each step, g is charging the (short-run) market-clearing price. Assuming so, then g's profit will rise to $\pi_g^w(\widetilde{q}_g, \overline{q}_{-g})$ in the first step. After the second step, g's profit will be $\pi_g^w(\overline{q}_g - 1, \overline{q}_{-g})$: this is larger than $\pi_g^w(\widetilde{q}_g, \overline{q}_{-g})$ because $\overline{q}_g^{\dagger} \leq \overline{q}_g - 1 < \overline{q}_g$ at any $\overline{q} : n < \overline{Q} = (a - c)/b$. ■