Unbundled Auction Procurement over Multiple Periods (Extended abstract)

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1 Introduction

Consider a firm, called the *buyer*, which assigns orders to its suppliers via a Vickrey auction. We will refer to this type of procurement auction as a *bundle auction*, since the demand might in fact be an aggregate of demand over multiple time periods. In the scenario we consider, the buyer is alternatively considering auctioning this demand by breaking it down by time period and permitting bids for one or more periods, either as independent bids or as package bids. In this alternative auction, which we call the *unbundled auction*, the buyer will assign orders to his suppliers via the VCG mechanism [4][1].

Choosing the unbundled auction over the bundle auction will have three effects. First, this will increase supplier production efficiency, as the unbundled auction will allow each supplier to more fully incorporate his cost and capacity information when submitting his bids [2]. Second, suppliers can focus their bids on a specific period or periods, and consequently will bid more competitively against each other. Third, the buyer might be able to (i) combine bids from different suppliers to further lower his purchase cost, and (ii) purchase units closer to the period in which he requires them to lower his inventory cost. We call these three effects, respectively, the *supplier efficiency effect*, the *competition effect*, and the *buyer flexibility effect*. These effects might lead one to expect that the unbundled auction would always be preferred by the buyer, as it appears as though each one can only lower the buyer's purchase cost.

We will show that, to the contrary, there are cases in which the buyer will be worse off with the unbundled auction; thus, in these cases, the buyer needs to pay more to satisfy his demand, despite the fact that the suppliers will be producing more cheaply. This situation arises from the fact that the competition effect has another side to it, viz., although the suppliers can bid more competitively against each other, they can also bid more competitively *against the buyer*. Thus, our result shows that the negative aspect (from the buyer's point of view) of the competition effect can dominate the positive aspect of this effect plus the production efficiency effect plus the buyer flexibility effect. (This somewhat counter-intuitive result is reminiscent of the well-known result of Hart [3], who considers the consequences in a market structure of permitting trades that were previously prohibited, where "our intuition tell us that the introduction of additional markets ought to make people better off," but provides an example in which this is not the case.)

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We also show that, when suppliers are not restricted by capacities, the buyer will always do at least as well in the unbundled auction as compared with the bundle auction, i.e., will not increase his total cost and might lower it. (Again, there is an analogy with [3], where Hart points out that if enough new markets are opened to make the market structure complete, then his counter-intuitive result cannot occur.) Moreover, we delimit a set of problem instances for which the unbundled auction is never more expensive than the bundle auction. We also provide an upper bound on how much greater the buyer's total cost can be in the unbundled auction as compared with the bundle auction.

2 The Two Auctions

We consider a planning horizon of T periods, where the buyer faces demand D_t in period t. If supplier s produces in period t, then he faces a setup cost f_t and a unit production cost p_{st}, as well as a production capacity b_{st}. If supplier s carries inventory over from period t to period $t+1$ that he had produced in period $\hat{t}$, then he faces a unit inventory holding cost $h_{s\hat{t}}$. We will denote by $p_t^{(n)}|_\alpha$ the n-th cheapest unit production costs among all those suppliers who have sufficient capacity to produce α units in period t. The buyer pays each supplier at the end of the auction, which occurs before the beginning of period 1. If the buyer carries inventory over from period t to period $t+1$, then he faces a unit inventory holding cost H_t. The two auctions are defined as follows:

The Bundle Auction. The buyer announces to the suppliers his total demand $\sum_{t=1}^T D_t$ which is to be provided by a single supplier. The suppliers individually submit bids in the form of a bid price representing an offer to supply the total demand to be delivered at the beginning of period 1. Here, supplier s is restricted by his production capacity b_{s1} in period 1, the only period in which production is available to him. Let w^{B} denote the winner in the bundle auction, and C_S^{B} denote the cost to w^{B} associated with delivering the total demand. Because the setup costs in each period, and in particular in period 1, are supplier-independent, the winner of the bundle auction will be the one with the cheapest unit production cost in period 1 among all those suppliers who have sufficient capacity to produce $\sum_{t=1}^T D_t$ units in period 1. The *buyer's total cost in the bundle auction*, π^{B}, is given by

$$\pi^{\mathrm{B}} \;=\; C^{\mathrm{B}}_{\mathcal{S}\setminus\{w^{\mathrm{B}}\}} + \sum_{t=2}^{T}(\sum_{\tau=1}^{t-1} H_\tau)D_t = f_1 + p^{(2)}|_{\sum_{t=1}^T D_t} \sum_{t=1}^{T} D_t + \sum_{t=2}^{T}(\sum_{\tau=1}^{t-1} H_\tau)D_t.$$

This expresses the lowest cost at which any one of the suppliers in $\mathcal{S}\setminus\{w^{\mathrm{B}}\}$ can supply the total demand $\sum_{t=1}^T D_t$, plus the sum of the buyer's inventory holding costs over the entire planning horizon. (Since all production occurs in period 1, inventory is only held at the buyer level.)

The Unbundled Auction. The buyer announces to the suppliers a demand D_t for each period t. Let $D = (D_t) \in \mathbb{R}^T$ denote the vector of demands, which can be provided by one or more suppliers. The suppliers individually submit bids in the form of a bid price together with a T-vector representing an offer to supply specific quantities of units in periods $1, \ldots, T$, where an offer in period t is not restricted to being zero or D_t, but can be the sum of any subset of the buyer's demands D_τ for $\tau \in \{t, t+1, \ldots, T\}$. However, each supplier s is restricted by his production capacity b_{st} in period t. Let $\mathcal{W}$ be the set of winners in the unbundled auction. Let T_s be the set of demands, indexed by period, won by $s \in \mathcal{W}$, and let $C^{\mathrm{U}}_{\mathcal{S},s}$ be the cost of supplying T_s incurred by supplier s. Finally, let w_t denote the winner of demand D_t, for $t = 1, 2, \ldots, T$. Note that while the members of $\{s \mid s \in \mathcal{W}\}$ are distinct, the members of $\{w_t \mid t = 1, 2, \ldots, T\}$

might not be. The *buyer's total cost in the unbundled auction*, π^{U}, is given by the buyer's sum total payment to the suppliers:

$$\pi^{\mathrm{U}} \;=\; C^{\mathrm{U}}_{\mathcal{S}} + \sum_{s\in\mathcal{W}} (C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}} - C^{\mathrm{U}}_{\mathcal{S}}), \tag{1}$$

where $C^{\mathrm{U}}_{\mathcal{S}}$ is the lowest cost at which the set of suppliers $\mathcal{S}$ can supply the vector of demands D.

3 Results

We will consider, under various assumptions, the difference in total cost in the bundle and unbundled auctions, $\pi^{\mathrm{B}} - \pi^{\mathrm{U}}$, which we call the *savings.* However, in general, the savings can be negative.

Proposition 3.1 *If the capacities are not binding,* $\pi^{\mathrm{U}} \leq \pi^{\mathrm{B}}$. □

In the lemma below, we will derive two conditions under which the unbundled auction is at least as good as the bundle auction, depending on the winners in the auctions.

Lemma 3.2 *We have that* $\pi^{\mathrm{U}} \leq \pi^{\mathrm{B}}$ *in the following cases:*

(i) the unbundled auction has one winner, or

(ii) the unbundled auction has at least two winners where $w_1 = w^{\mathrm{B}}$. □

We can show by means of an example that the bundle auction can be better than the unbundled auction. In the following, we will show that the buyer's total cost in the unbundled auction cannot be more than $f_1(|\mathcal{W}| - 1)$ plus the buyer's total cost in the bundle auction.

Proposition 3.3 *We have that* $\pi^{\mathrm{B}} - \pi^{\mathrm{U}} \geq -f_1(|\mathcal{W}| - 1)$. □

References

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