Pre-electoral Debate: The Case of a Large Election*

(Preliminary and Incomplete)

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Abstract

The model presented in this paper captures some of the effects of a pre-electoral debate on the incentives for information acquisition of voters that belong to different ideological strands. We introduce the option to publicly share information into a fairly standard model of information aggregation through an election with costly information acquisition. We find that this option dramatically changes the incentive to acquire information. Without the option to share one's signal no extremist has any incentive to acquire information. With this option present the extremists' incentive to acquire information is even stronger than the independents' incentive. In equilibrium this extra incentive leads the extremists acquire more information than the independents. We use this to explain the empirically observed correlation between extremism and information.

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1 Introduction

One of the main conclusions of "The Relationship between Information, Ideology and Voting Behavior" by Palfrey and Poole (1987) is that "Highly informed voters consist of a significantly more polarized subset of the electorate than uninformed voters." Similarly a correlation between political extremism and investment in information acquisition has been observed in the literature on political psychology see for example Sidanius (1988) and the references cited therein.

While pre-electoral debates have been for the most part overlooked in the political economics literature it seems that they play an important role in the democratic process. Debates and discussions about the running candidates are among the most common features of political life as we know it. In fact, according to The American National Election Studies (ANES) poll in 2004, 34% of respondents reported yes to the question, "During the campaign, did you talk to any people and try to show them why they should vote for or against one of the parties or candidates?"(The National Election Studies 2004). This desire to persuade could lead to the desire to be more informed, and so a model of information acquisition before an election without some form of communication between voters seems incomplete. Introducing such an option to share one's information before an election should change the incentive structure of voter's decisions to acquire information. Our aim in this paper is to highlight the effects that communication might have on a model of information acquisition in an election.

This is not the first study that suggests that political information has other uses than just the guidance it provides for a voter's private voting decision. In Baron (2004)'s model of media bias voters use political information for private investment and consumption decisions. In Bernhardt, Krasa and Polborn (2006) information is modeled as a consumption good: people read or watch for fun. Neither one of these two rationales can explain the correlation between extremism and information. In Bernhardt et al. (2006) it is simply assumed that extremists assign a higher consumption value to information. It is unclear how one could generate a correlation between information and extremism when political information is acquired for the sake of private decisions as in Baron (2004).

To examine at the effects of a pre-electoral debate we incorporate a communication stage into a standard Condorcetian model of an election with costly information acquisition.[1] There are two equally probably states l, r and two candidates L, R, such that every voter (weakly) prefers candidate X to candidate Y in state x. To make any statements about the relation between information and extremism it is assumed that information is endogenous and that there are voters of varying degrees of extremism in the electorate.[2] There are three different camps of voters in the model: left extremists, right extremists and independents. There can be two types of mistakes that an electorate can make: choosing candidate L in state r, and vice versa. The voters in this paper differ not by an variance in their willingness to reach their goals but rather by their views of the mistakes. We call a left wing extremist a voter who suffers serious disutility from the mistake of choosing the right wing candidate when the correct decision would have been to choose the left wing candidate, and loses no utility from the opposite mistake. We call a right wing extremist a voter who is characterized by exactly the opposite preference structure. These voters are extremists because they will always weakly prefer the electorate to choose the extremist's candidate as they do not care if the that candidate is a mistake or not. We call independents those that have an average loss if either type of mistake is made.[3] These voters bring no ideological bias to the table and only wish to minimize the probability of a mistake.

In the model voters first have to decide whether to acquire a costly signal on the state of the world. It is assumed that signals are independent conditionally on the state. None of the extremists would have any incentive to acquire costly information if voters had to only decide whether to acquire (private) information and how to vote. In such a model extremists would always vote for their preferred candidate and would consequently have no

[1]By standard Condorcetian model we refer to the model set forth by Austin-Smith and Banks (1996) that has been the basis of much work in the literature.

[2]An alternative and very different approach to explain the correlation would be to let extremism be endogenously explained by varying levels of information. Palfrey and Poole (1987) suggest such an explanation. Sidanius' "context theory" sees in "cognitive orientation" an exogenous factor that explains both a persons extremism and his or her investment in information acquisition (Sidanius 1988)

[3]That the independents suffer only an average loss from either mistake assures that we are not driving the results of the model by giving some voters undue incentive for action compared to the rest of the electorate.

incentive to acquire information.[4]

The introduction of a pre-electoral debate is the crucial innovation in this model. Once a voter has acquired a signal they then decide whether to reveal their private signal. A “debate” consists of the simultaneous publication of the signals from those voters that choose to reveal their information. This option to share information significantly changes the incentive structure for information acquisition. The optimal voting strategy of any extremist remains unchanged. However, they now have a chance to use their knowledge of politics to convince some of the independent voters to vote for their preferred candidate.

To close the model an assumption on the number of voters is needed. It is assumed that the total number of voters is a Poisson random variable.[5] Creating a situation in which voters neither know the number of other voters nor do they know the exact proportion of voters in any of the three camps.

Our main result is the existence of equilibria in which extremists have strictly higher benefit from information acquisition than independents and therefore are always at least as likely as independents to acquire information. Indeed, under certain parameters the only voters that acquire information are extremists, and the rest of the electorate just free rides.[6] This stands in stark contrast to the political economics literature which has had a hard time explaining any acquisition of information by extremists. The first result gives some intuition as to why Palfrey and Poole (1987) find that extremism is correlated with being informed, and we believe that we are the first to explore this result theoretically.[7]

[4]This is in fact a result in models with endogenous information acquisition and voters of varying degrees of extremism, see Oliveros (2006), Degan (2006).

[5]Formally this places the model in the literature on Poisson games as developed by Myerson (1998) and Myerson (1997).It turns out that the analysis of the present model is considerably simplified by the setup as a Poisson game. This will be described in the section on the solution of this game.

[6]This result is related to Dewatripont and Tirole (1999) in which an interested party may prefer having opposing extremist advocates instead of investing in information herself (especially the section on verifiable information). The setup of that model however is quite different insofar as the advocates have no decision making power and no intrinsic interest in the outcome of the principal’s decision. Instead the principal has to optimally select advocates and devise a payment scheme to motivate the advocates to action.

[7]As far as we could discern the only work that allows for endogenous information acquisition as well as pre-electoral communication is Gerardi and Yariv (2004). They show

Another result of Palfrey and Poole (1987) is that more informed voters are less likely to abstain, which has been theoretically explained by Feddersen and Pesendorfer (1996) and a large literature following that. Another result of our paper is that the existence of what we call the "swing voters boon" in which under the condition that an independent voter is pivotal, that voter has even more incentive to vote for the candidate that the other independents are voting for. Since this feature of our model exists and there is no cost of voting, voters never want to abstain. As such we cannot tie the two results of Palfrey and Poole (1987) together into one unified theory.

2 The Model

We define an extended Poisson game following Myerson (1998) $\{\Omega, \alpha, T, n, p, C, U\}$ with the state space $\Omega = \{l, r\}$ and a common prior of $\alpha = \frac{1}{2}$ that the state is l. The parameter T denotes the set of voter types $T = T_1 \times T_2$ where $t_1 \in T_1 = \{l, i, r\}$ denotes the preference type of a voter and $t_2 \in T_2 = \{l, r\}$ denotes the informational type of a voter. The size of the electorate is a Poisson random variable with mean n. For each state of the world $\omega \in \{l, r\}$ the preference and information type of a voter are drawn from independent distributions $p_1(\cdot|\omega)$ and $p_2(\cdot|\omega)$ respectively. We assume that the preference type of a voter is independent of the state of the world, and that any voter is equally likely to be or preference type l or r, we let $p_1(t_1 = l|\omega) = p_1(t_1 = r|\omega) = \eta < \frac{1}{2}$ for $\omega = l, r$. We assume that $n(1 - 2\eta) > 1$ which amounts to a very weak statement of the requirement that the electorate is "large". Finally the informational type t_2 of a voter depends on the state of the world, we assume that $p_2(t_2 = l|\omega = l) = p_2(t_2 = r|\omega = r) = p > \frac{1}{2}$, t_2 can be interpreted as an informative signal about the state of the world.

The game proceeds according to the following time-line: First Nature draws a state of the world and an electorate. Secondly voters choose whether to acquire information. Thirdly voters choose whether to publish their information. Fourth voters learn the value of all the information that has been published. Fifth the Supreme Court throws a fair coin which can come up

that incentives for information acquisition have to be traded off against efficient information aggregation when designing an optimal voting rule. Their work differs from ours in their goal (they are interested in optimal committee design whereas we are interested in informational dynamics of an electorate) as well as their communication structure.

R or L. Sixth voters vote for L or R.

The exchange of information is modeled as a persuasion game following Glaezer and Rubenstein (2001), Milgrom and Roberts (1986) and Shin (1994). A voter that chooses to acquire information learns his informational type t_2. The voter that knows his signal t_2 can decide to either share this with the electorate as a whole, or he can stay silent. He cannot lie, reveal partial truths or communicate with subsets of the entire electorate. The electorate learns the total number of l and r signals that have been published, denoted by the vector $\vec{s} = (s_r, s_l)$. No voter learns anything beyond that. This implies in particular that voters do not learn the total number of voters, or the number of voters in any of the camps. They also do not learn how many signals have been acquired.

In the voting stage voter have to pick between R and L.[8] The vote is decided by majority rule. In case of a tie the Supreme Court picks on the behalf of independent voters, meaning that the Supreme Court picks the candidate, whom an independent would like better given all the information contained in the public signal vector $\vec{s}$. If this does not yield a clear cut result then the Supreme Court picks the candidate that was determined by the fair coin toss in the fifth stage.

The utility of a voter depends on the outcome of the vote $W = L$ or R (which in turn depends the votes of all voters and in case of a tie on $\vec{s}$) his decision whether to acquire information or not ($x_1 \in \{0, 1\}$), the state of the world and his preference type. We have that: $U : \{L, R\} \times \{0, 1\} \times T_1 \times \Omega \rightarrow \mathbb{R}$:

$$U(W, x_1, t_1, \omega) = \begin{cases} -\delta(t_1) - x_1 c & \text{if } \ \omega = r \text{ and } \ W = L \\ -(1 - \delta(t_1)) - x_1 c & \text{if } \ \omega = l \text{ and } \ W = R \\ -x_1 c & \text{otherwise} \end{cases}$$

The preference type of a voter, t_1, determines the disutility a voter incurs when candiate X is picked in state Y. For leftist voters we have that $\delta(l) = 0$. A lefstist does not receive any disutility from a wrong choice of candidate L however he receives the maximal disutility when candidate R wins the election in state l. Conversely we set $\delta(r) = 1$, so a rightist's utility is

[8]We will show in section 7 that not considering abstention is without loss of generality in the context of the present model, in the sense that we would obtain the same equilibrium result if we where to allow voter's to abstain.

minimized when L wins in state r. Finally we assume that independents suffer an equal amount of disutility from either mistake, $\delta(i) = \frac{1}{2}$.

3 Strategies and Equilibrium

The strategy set of a voter $C = C_1 \times C_2 \times C_3$ is composed of an *information acquisition*-strategy $C_1 = \{0, 1\}$ a *broadcasting* strategy $C_2 = (\mathcal{L} \times \mathcal{R})$ and a *voting*-strategy $C_2 = \{f : \mathbb{N}_0 \times \mathbb{N}_0 \times T \rightarrow \{L, R\}\}$ where f is allowed to depend on T_2 only if the voter chose to acquire information in the first step. The broadcasting strategy consists of two elements: $\mathcal{L} = \{\emptyset, l\}$ is the choice to broadcast a signal with value l, and $\mathcal{R} = \{\emptyset, r\}$ is the decision to broadcast a signal with value r, in each case $\emptyset$ stands for the suppression of the signal. C_2 is the set of all voting rules given vector of broadcast signals s and $(t_1, x_1 t_2)$.

We denote a mixed strategy of a a voter of type t_1 by τ_{t_1}. We define $\tau = (\tau_l, \tau_i, \tau_r)$ as a mixed strategy profile for the game. So $\tau_l(1, \emptyset, r, f)$, for example, denotes the probability that a leftist acquires information, only broadcasts right wing signals and follows the voting rule f. We define $\tau_{t_1}(\cdot, l, r, f)$ as the probability that a voter of type t_1 passes on both signals and plays voting strategy f. Formally $\tau_{t_1}(\cdot, l, r, f) = \tau_{t_1}(0, l, r, f) + \tau_{t_1}(1, l, r, f)$, the expressions $\tau_{t_1}(\cdot, \cdot, r, f)$, $\tau_{t_1}(\cdot, l, r, \cdot)$ and so forth are defined analogously. We define $EU_{t_1}(\tau, x)$ as the expected utility of a voter of preference type t_1 when this voter uses the pure strategy x while all other voters follow the profile τ. The probability that candidate L wins in state r when all other voters follow strategy τ and the voter under consideration follows strategy x is denoted by $Pr(L, r|\tau, x)$. Conversely the probability that R wins in state l when the voter uses the pure strategy x and all other voters follow the strategy profile τ is denoted by $Pr(R, l|\tau, x)$. Given these definitions we can express the expected utility of a voter of type t_1 $EU_{t_1}(\tau, x)$ as follows:

$$EU_{t_1}(\tau, x) = -\delta(t_1) Pr(L, r|\tau, x) - (1 - \delta(t_1)) Pr(R, l|\tau, x) - cx_1. \quad (1)$$

where x_1 denotes the first component of the player's strategy, x_1 equals 1 if and only if the voter acquired information.

Definition 1 *A strategy profile τ is an equilibrium if*

1. *$EU_{t_1}(\tau, x) \geq EU_{t_1}(\tau, y)$ for all $x \in C : \tau_{t_1}(x) > 0$ and all $y \in C$ for all $t_1 \in T_1$.*

2. *The strategy profile is symmetric in the sense that extremists of both camps are equally likely not to acquire any information, we have that $\tau_l(0, \cdot, \cdot, \cdot) = \tau_r(0, \cdot, \cdot, \cdot)$.*

3. *The independents will pass on any signal that they acquire, we have that $\tau_i(1, l, \emptyset, f) = \tau_i(1, \emptyset, r, f) = \tau_i(1, \emptyset, \emptyset, f) = 0$ for all $f \in C_2$.*

4. *Voters vote sincerely.*

The first condition says that any strategy that a voter plays with a positive probability in τ has to be a best response to the strategy profile τ. This is Roger Myerson's definition of an equilibrium in an extended Poisson game (Myerson 1998). This definition alone does not rule out certain odd behaviors. A profile τ that prescribes that all voters vote L if more than 2 signals have been sent can be an equilibrium profile. In this case voters can infer that there are more than 2 voters in the electorate. So no voter has a chance to change the outcome of the vote.

To deal with such problems we require conditions 2,3 and 4. An important question arises: is the set of all 4 conditions compatible? It is, for example, well known that sincere and strategic voting need not coincide. It is easy to show that the second requirement is compatible with the first. It turns out that the most involved proofs in this paper are devoted to showing that the third and fourth requirements are compatible with the first see section 7 and 8.

4 Voting Strategies

4.1 Sincere Voting

Sincere voting requires a voter to vote for the candidate whom he would choose if he alone had to determine the winner based on all information available to him. We define $Pr[\vec{s}, x_1 t_2 | \omega]$ as the probability that the signal vector and the voters own information is $\vec{s}, x_1 t_2$ is state ω, where $x_1 t_2$ is equal to 0 if $x_1 = 0$ meaning that the voter has no private information beyond the public vector of signals and $x_1 t_2 \in \{l, r\}$ for the case that the voter has private information that is the case $x_1 = 1$. We calculate a voters expected utility of candidates L, R when the information available to the voter is the signal vector $\vec{s}$ and the voter's own private information δt_2 respectively as

$$-\delta(t_1) \frac{Pr[\vec{s}, x_1 t_2 | r]}{Pr[\vec{s}, x_1 t_2 | l] + Pr[\vec{s}, x_1 t_2 | r]} \tag{2}$$

$$-(1 - \delta(t_1)) \frac{Pr[\vec{s}, x_1 t_2 | l]}{Pr[\vec{s}, x_1 t_2 | l] + Pr[\vec{s}, x_1 t_2 | r]}. \tag{3}$$

If candidate L is chosen only one type of mistake matters: namely the case that L is chosen in state r. To obtain a voter's expected utility the probability of this mistake has to be multiplied with the disutility that this voter receives if L is chosen in state r, this disutility is $-\delta(t_1)$. Analogously we obtain the expected utility of candidate R given $\vec{s}, x_1 t_2, t_1$.

Voting sincerely requires a voter to vote for candidate L if expression 2 is larger than expression 3. If the opposite inequality holds true a voter that votes sincerely has to vote for candidate R. If the two expressions are equal the voter is indifferent between the two candidates. We assume that indifferent voters base their vote on the coin thrown by the Supreme Court, if the coin comes up on X they vote for X.

4.2 Extremists

Observe that $Pr[\vec{s}, x_1 t_2 | \omega] \neq 0$ for all possible signal profiles $(\vec{s}, x_1 t_2)$ and both states $\omega = l, r$. Our assumption that extremists vote sincerely implies the following Lemma:

Lemma 1 *In equilibrium all leftists vote for candidate L and rightists vote for candidate R.*

Proof Expression 2 equals to 0 if $\delta(t_1) = 0$. On the other hand for no vector $(\vec{s}, x_1 t_2)$ does the probability $Pr(\vec{s}, x_1 t_2 | l)$ equal 0. So expression 3 is always negative for a leftist. Consequently expression 2 is larger than expression 3 for any $(\vec{s}, x_1 t_2)$. A leftist votes sincerely if he votes for L. The same argument holds mutatis mutandum for for a rightist. □

4.3 Independents

We show next that in equilibrium the independent voters vote according to a simple cutoff rule.

Lemma 2 *There exists a cutoff $g(n,p,\tau)$ such that an independent voter votes for L if $s_l > s_r + g(n,p,\tau)$ and votes for R if $s_l < s_r + g(n,p,\tau)$, otherwise the independent is indifferent.*

Proof We need to show that

$$Pr[\vec{s}, x_1t_2|l] > Pr[\vec{s}, x_1t_2|r] \tag{4}$$

if and only if $s_l > s_r + g(n,p,\tau)$ for some expression $g(n,p,\tau)$.

If the voter has not invested in information acquisition, that is if $x_1t_2 = 0$, then we have that

$$Pr[\vec{s}, tx_2|l] = \frac{e^{-npS_l}[npS_l]^{s_l}}{s_l!} \cdot \frac{e^{-n(1-p)S_r}[n(1-p)S_r]^{s_r}}{s_r!}$$

$$Pr[\vec{s}, tx_2|r] = \frac{e^{-n(1-p)S_l}[n(1-p)S_l]^{s_l}}{s_l!} \cdot \frac{e^{-npS_r}[npS_r]^{s_r}}{s_r!}$$

for[9]

$$S_l = \eta[\tau_l(1,l,\emptyset,\cdot) + \tau_l(1,l,r,\cdot) + \tau_r(1,l,\emptyset,\cdot) + \tau_r(1,l,r,\cdot)] \\ +(1-2\eta)[\tau_i(1,l,\emptyset,\cdot) + \tau_i(1,l,r,\cdot)]$$

$$S_r = \eta[\tau_l(1,\emptyset,r,\cdot) + \tau_l(1,l,r,\cdot) + \tau_r(1,\emptyset,r,\cdot) + \tau_r(1,l,r,\cdot)] \\ +(1-2\eta)[\tau_i(1,\emptyset,r,\cdot) + \tau_i(1,l,r,\cdot)]$$

Observe that expression 2 is larger than expression 3 if and only if

$$\frac{Pr[\vec{s}|l]}{Pr[\vec{s}|r]} = (\frac{p}{1-p})^{[s_l-s_r]} \cdot e^{[n(1-2p)(S_l-S_r)]} < 1 \tag{5}$$

[9]One might ask: isn't it obvious that $\tau_r(1,l,\cdot,\cdot) = \tau_l(1,\cdot,r,\cdot) = 0$? Yes, this is obvious but we believe that there should be no loose ends in a proof. At this point we believe that this setup helps us to show most straightforwardly that this "obvious" statement indeed holds true

we can rearrange to find the cutoff condition

$$s_l < \frac{n(2p-1)}{\ln(\frac{p}{1-p})}(S_l - S_r) + s_r \tag{6}$$

which is always well defined in the given environement. If equation 6 holds with equality then the voter is indifferent.

We next need to show that an independent voter that has observed a signal follows the same rule when voting sincerely. To see this assume that the voter has observed an l-signal. So the public signal count without his signal then becomes $s_l - 1$ and s_r. In this case the voter estimates the probabilities $Pr[\vec{s}, tx_2|l]$ and $Pr[\vec{s}, tx_2|r]$ as

$$Pr[\vec{s}, tx_2|l] = p\frac{e^{-npS_l}[npS_l]^{(s_l-1)}}{(s_l-1)!} \cdot \frac{e^{-n(1-p)S_r}[n(1-p)S_r]^{s_r}}{s_r!}$$

$$Pr[\vec{s}, tx_2|r] = (1-p)\frac{e^{-n(1-p)S_l}[n(1-p)S_l]^{(s_l-1)}}{(s_l-1)!} \cdot \frac{e^{-npS_r}[npS_r]^{s_r}}{s_r!}$$

which yields the exact same condition on the voter's behavior. So we find that the statement of the Lemma holds true for $g(n,p,\tau) = \frac{n(2p-1)}{\ln(\frac{p}{1-p})}(S_l - S_r)$ $\square$

The assumption of the Poisson distribution is essential for the proof to hold. Under the assumption of the Poisson distribution a voter's own own signal carries as much information as anyone else's. For other assumptions the case in which the independent has not acquired any information differs significantly from the alternative case in which he has acquired information.

In the next section on optimal communication we will formally show that no extremists will ever send evidence that favors the opposing candidates or $\tau_l(1,\cdot,r,\cdot) = \tau_r(1,l,\cdot,\cdot) = 0$. This implies that $S_l = S_r$ and the independents decision rule becomes vote for R if $s_r > s_l$, vote for L if $s_l > s_r$. [10]

[10] An interesting comparison with Shin (1994) arises. The cutoff x can be interpreted as the burden of proof: if x is large the leftists face a large burden of proof, in the sense that they need to provide more signals for any given amount of right signals to convince an independent that L is the better candidate. This result is stronger than Shin's as x can be completely characterized in terms of n, p and τ.

Sincere voting does not imply anything for the case that $s_l = s_r + g(n, p, \tau)$. We assume that independent voters follow the coin throw of the Supreme Court in this particular case.

We conclude this section by defining the voting rules $f_l, f_i, f_r \in C_2$ by $f_l(x) = L$, $f_r(x) = R$ and $f_i(x) = L$ if $s_l > s_r$, $f_i(x) = R$ if $s_l > s_r$ and finally if $s_l = s_r$ then $f_i(x) = R$ if and only if the publicly thrown coin came up R. In the two preceding Lemmata 1 and 2 we have shown that in any equilibrium profile τ the voters will only follow these strategies. In short, we have shown that $\tau_{t_1}(\cdot, \cdot, \cdot, f_{t_1}) = 1$ for $t_1 \in \{l, i, r\}$.

5 Optimal Communication

In this section we show that in equilibrium extremists will pass on signals in their favor and suppress signals in favor of the other candidate. The broadcasting behavior of the independents is determined by equilibrium assumption 3: in equilibrium independents broadcast all signals. We will show in section 8 that this assumption is consistent with strategic behavior. We start this section by showing that extremists will never acquire information without any plans to broadcast it.[11]

Lemma 3 *In equilibrium we have that* $\tau_l(1, \emptyset, \emptyset, f) = \tau_r(1, \emptyset, \emptyset, f) = 0$ *for all voting strategies* f.

Proof Suppose in equilibrium some $x = (1, \emptyset, \emptyset, f)$ was played with positive probability by the left wing extremists. As f_l is a weakly dominant voting strategy for left wing extremists we have that $EU_l(\tau, x) \leq EU_l(\tau, x^*)$ for $x^* = (1, \emptyset, \emptyset, f_l)$. Now consider the alternative strategy $x' = (0, \emptyset, \emptyset, f_l)$, which differs from x^* only insofar as that the voter does not acquire information. Remember that a voter's expected utility can be expressed as

$$EU_{t_1}(\tau, y) = -\delta(t_1)Pr(L, r|\tau, y) - (1 - \delta(t_1))Pr(R, l|\tau, y) - cy_1.$$

for $t_1 \in \{l, r, i\}$. The signal acquired by the extremist is only known to him, since this signal will never sway his vote we have that $Pr(Y, \omega|\tau, y)$ does

[11] This result is reminiscent of some results in Degan (2006) and Oliveros (2006) insofar as that given that information sending is never an option in their respective papers, both obtain that extremists will never acquire information in equilibrium

not depend on this signal. So we obtain that $EU_l(\tau, x^*) - EU_l(\tau, x') = -c$. Consequently the voter strictly prefers x' to x^* and x, we must have that $\tau_l(1, \emptyset, \emptyset, f) = 0$ for all voting strategies f. An analogous argument holds for right wing extremists. $\square$

Note that this proof cannot be used to show that $\tau_i(1, \emptyset, \emptyset, f) = 0$ in equilibrium as the independents do not have a dominant voting strategy.

To analyze the optimal broadcasting strategies we need to find out how the broadcasts of l and r-signals change the winning probabilities of the two candidates in the two states. We define $Pr(E|\tau, \omega)$ as the probability that event E occurs in state ω when all voters follow strategy τ. Let us assume the stance of one particular voter, and let us define the events R and L such that in these events candidates R and L win the election, when the voter under consideration does not send a signal. Additionally we define the events $X + r$ $(X + l)$ as the event that the candidate X wins the election given that the voter sent an r (l)-signal .

The effect of an additional signal l or r being sent on the expected utility comes through the effect of that additional signal on the winning probabilities of the two candidates in the two states. We define define $\Delta^z_{Y,\omega}$ as the change in probability that candidate Y is being chosen in state ω when another z signal is being sent. Formally

$$\Delta^z_{Y,\omega} := Pr(Y + z|\tau, \omega) - Pr(Y|\tau, \omega).$$

Observe that $\Delta^r_{R,\omega}, \Delta^l_{L,\omega} > 0$. That is, observe that in either state the probability that candidate Y wins is increasing in the number of signals sent in his favor.

We are now ready to state and prove a Lemma about the equilibrium information acquisition strategies of extremists

Lemma 4 *In equilibrium we have that* $\tau_l(1, \cdot, r, f_l) = \tau_r(1, l, \cdot, f_r) = 0$.

Proof Conditional on having received an l-signal the following holds true for a left wing extremist. $EU_l(\tau, x) - EU_l(\tau, x') = p(Pr(L+l|\tau, l) - Pr(L|\tau, l)) = p\Delta^l_{L,l} > 0$ for $x = (1, l, \cdot, f_l)$ and $x' = (1, \emptyset, \cdot, f_l)$. To see this observe that the utility of a left wing extremist is always 0 in state r, so utility changes only happen in state l. Given that the extremist has observed an l signal he thinks that state l is the true state with probability p. So any left wing extremist strictly prefers sending an l signal strictly to being silent as $\Delta^l_{L,l} > 0$. Let us check next that no leftist extremist will send an r signal in equilibrium. To see this observe that, conditional on having received an r we have that $EU_l(\tau, x) - EU_l(\tau, x') = -(1-p)((Pr(R+r|\tau) - Pr(R|\tau, l)) = -(1-p)\Delta^r_{R,l} < 0$ for $x = (1, \cdot, r, f_l)$ and $x' = (1, \cdot, 0, f_l)$, so a left wing extremist strictly prefers to stay silent. Analogous arguments hold for right wing extremists. □

Lemma 4 together with equilibrium assumption 2 that $\tau_l(0, \cdot, \cdot, \cdot) = \tau_r(0, \cdot, \cdot, \cdot)$ implies that $\tau_l(1, l, \emptyset, f_l) = \tau_r(1, \emptyset, r, f_r)$ as $\tau_l(0, \cdot, \cdot, f_l) = 1 - \tau_l(1, l, \emptyset, f_l)$ and $\tau_r(0, \cdot, \cdot, f_r) = 1 - \tau_r(1, \emptyset, r, f_r)$. To save on notation we say from now on that λ is the probability that an extremist acquires information in the equilibrium strategy profile τ. Lemmata 3 and 4 imply that the extremists always pass one information that supports their case.

Lemma 4 together with the symmetry assumption also implies that $S_l = S_r$ and consequently the independents equilibrium rule for sincere voting becomes: vote for the candidate for whom more signals have been broadcast, in short, $g(n, p, \tau) = 0$. The symmetry of the equilibrium also implies that $\Delta^r_{R,r} = \Delta^l_{L,l}$ and $\Delta^l_{L,r} = \Delta^r_{R,l}$. To save on notation we define $\Delta^r_{L,r} = \Delta^l_{R,l} := \Delta^+$ as the increase in probability of a correct (or good) choice Y in state $\omega = y$ given that another signal in favor of Y is being sent. Analogously we define $\Delta^l_{L,r} = \Delta^r_{R,l} := \Delta^-$ as the increase in the probability of wrong (or bad) choice of candidate Y in state $\omega = y$ given that another signal in Y's favor is being sent.

Economizing further on notation we say that π is the probability that an independent acquires information in the equilibrium strategy profile τ. Our assumption that independents will share any signal they acquired implies that $\tau_i(1, l, r, f_i) = \pi$.

We summarize that up until now we know that in any equilibrium τ left wing extremists vote for the left candidate and right wing extremists vote for the right candidate. Independents vote according to a very simple cutoff rule: if more l-signals have been published they vote for candidate L, conversely if more r-signals have been published they vote for candidate R, otherwise they base their vote on the coin of the Supreme Court. We also know that extremists would only broadcast signals in their favor. We show that our assumption that independents broadcast all their signals is consistent with equilibrium in section 8. We are now ready to prove our main result, namely the fact that in equilibrium that extremists never acquire less information than the independents.

6 Extremists Are More Informed

Theorem 1 *In an equilibrium the utility gain from acquiring information for an extremist is always greater than that of an independent.*

The intuition for this proof is simple. In any equilibrium τ the extremists only broadcast information in their favor. A left wing extremist, for example, would only broadcast an l signal. This reduces the probability that candidate R is chosen in state l, at the same time this increases the probability of the alternative mistake namely that candidate L is chosen in state r. A left wing extremist does not care about the increase in the probability of the second mistake. An independent also broadcasts an l signal if he receives one. However, differently from the extremists both effects are felt for the independent. The independent appreciates the fact that the probability of an erroneous choice of R is being reduced. At the same time an independent suffers from the fact that the additional l signal increases the probability that L is chosen in state r.

Proof We compare the expected utility of right wing extremist voter for the pure strategies $x_r = (1, \emptyset, r, f_r)$ and $x'_r = (0, \emptyset, \emptyset, f_r)$ to each other. From the proof of Lemma 4 we know that a right wing extremist values a right wing signal at $p\Delta^+$. The acquisition of signal costs c, and the extremists obtains a right wing signal with probability $\frac{1}{2}$, so we obtain that:

$$EU_r(\tau, x_r) - EU_r(\tau, x'_r) \quad = \tfrac{p}{2}\Delta^+ - c.$$

Analogously we have for a left wing extremist, where $x_l = (1, l, \emptyset, f_l)$ and $x'_l = (0, \emptyset, \emptyset, f_l)$:

$$EU_l(\tau, x_l) - EU_l(\tau, x'_l) \quad = \quad \frac{p}{2}\Delta^+ - c.$$

Finally let us investigate the utility difference for an independent, let $x_i = (1, l, r, f_i)$ and $x'_i = (0, \emptyset, \emptyset, f_i)$, we have that

$$\begin{aligned} EU_i(\tau, x_i) &= -\frac{1}{4}pPr(L + r|\tau, r) - \frac{1}{4}(1-p)Pr(R + r|\tau, l) \\ &\quad -\frac{1}{4}(1-p)Pr(L + l|\tau, r) - \frac{1}{4}pPr(R + l|\tau, l) - c \\ EU_i(\tau, x'_i) &= -\frac{1}{4}pPr(L|\tau, r) - \frac{1}{4}(1-p)Pr(R|\tau, l) \\ &\quad -\frac{1}{4}(1-p)Pr(L|\tau, r) - \frac{1}{4}pPr(R|\tau, l) \end{aligned}$$

So the expected utility difference between acquiring and not acquiring becomes:

$$\begin{aligned} EU_i(\tau, x_i) - EU_i(\tau, x'_i) &= \frac{p}{2}\Delta^+ - \frac{1-p}{2}\Delta^- - c \\ &< \frac{p}{2}\Delta^+ - c = EU_l(\tau, x_l) - EU_l(\tau, x'_l) = EU_r(\tau, x_r) - EU_r(\tau, x'_r). \end{aligned}$$

Where the inequality follows from the observation that $\Delta^- > 0$. We conclude that for any equilibrium strategy profile τ information is more valuable for extremists than for independents. □

We observe in passing that the proof of Theorem 1 implies that the left and right wing extremists benefit equally much from the acquisition of information. Consequently the second assumption in our equilibrium concept, namely that all extremists are equally likely not to acquire information is consistent with our first assumption on τ that all players are best responding.

Our model generates a positive correlation between information and extremism. We state this result as a simple corollary of Theorem 1.

Corollary 1 *If the equilibrium probability that independents acquire information is positive then all extremists acquire information in equilibrium. If the equilibrium probability that an extremist acquires no information is positive, then no independent acquires any information in equilibrium.*

Proof We can express Corollary 1 as:

$$\pi > 0 \Longrightarrow \lambda = 1 \text{ and } \lambda < 1 \Longrightarrow \pi = 0. \tag{7}$$

Using the results derived in Theorem 1:

$$\begin{aligned}
\pi > 0 \quad &\Rightarrow \quad EU_i(\tau, x_i) - EU_i(\tau, x_i') = 0 \\
&\Rightarrow \quad \frac{p}{2}\Delta^+ - \frac{1-p}{2}\Delta^- = c \\
&\Rightarrow \quad \frac{p}{2}\Delta^+ > c \\
&\Rightarrow \quad EU_l(\tau, x_l) - EU_l(\tau, x_l') > c \Rightarrow \lambda = 1
\end{aligned}$$

and

$$\begin{aligned}
\lambda < 1 \quad &\Rightarrow \quad EU_l(\tau, x_l) - EU_l(\tau, x_l') = 0 \\
&\Rightarrow \quad \frac{p}{2}\Delta^+ = c \\
&\Rightarrow \quad \frac{p}{2}\Delta^+ - \frac{1-p}{2}\Delta^- < c \\
&\Rightarrow \quad EU_i(\tau, x_i) - EU_i(\tau, x_i') < 0 \Rightarrow \pi = 0
\end{aligned}$$

□

7 Sincere Voting is Strategic Voting

Equilibrium requirement 4 imposes that all voters vote sincerely in any equilibrium τ. In section 4 we describe the sincere voting strategies of the three voter types. It remains to be shown that no voter has an incentive to deviate from the sincere strategies given that all other voters vote sincerely. In other words, it remains to be shown that Equilibrium requirement 1 is consistent with sincere voting. We need to show that a voter that votes for L following the sincere strategy, would like to vote for L if he knew that he was the pivotal voter. For the extremists this is easy to see, as a left wing extremists strictly prefers L to R in any case in which l could happen with a positive probability.

It is somewhat more difficult to see that an independent would like to vote sincerely. Consider the case of an equilibrium strategy profile τ and a signal structure $\vec{s}$ such that τ prescribes that independents votes L. If the expected utility of R is higher than the expected utility of L given $\vec{s}$ *and given that the voter is pivotal* then a strategic independent should vote R instead of L. The crucial question is whether an independent who knows that he is pivotal thinks that the state r is more likely than the state l. We will show that the information contained in the pivotality event never overturns the information in the public signal $\vec{s}$, in fact in most cases the information in the event of being pivotal strengthens the information contained in the public signal $\vec{s}$.

The intuition goes as follows: Suppose we have that $s_l \geq s_r$, and that the coin of the Supreme Court came up L (which is relevant only in the case that $s_l = s_r$). All independents will vote L according to the strategy profile τ. An independent would be pivotal if L and R receive equally as many votes. The fact that all independents vote for L implies that there must be more right wing extremists in the electorate than there are left wing extremists. Now consider the fact that more l signals have been sent. Taken together this implies it is likely that more right wing extremists decided to hide an l signal than there are a left wing extremist who hid an r signal.

The following Lemma will prove useful in this context.

Lemma 5 *Suppose that $Pr(l|\vec{n},\vec{s}) \geq Pr(r|\vec{n},\vec{s})$. Let $\vec{n}', \vec{s}'$ be such that $n'_r \geq n_r$, $n'_l \leq n_l$, $n'_i = n_i$, $s'_r \leq s_r$, $s'_l \geq s_l$ and either $\vec{n}' \neq \vec{n}$ or $\vec{s}' \neq \vec{s}$ or both, then we have that $Pr(l|\vec{n}',\vec{s}') > Pr(r|\vec{n}',\vec{s}')$.*

Proof Observe that $Pr(l|\vec{n},\vec{s}) \geq Pr(r|\vec{n},\vec{s})$ holds if and only if $Pr(\vec{n},\vec{s}|l) \geq Pr(\vec{n},\vec{s}|r)$. We proceed by distinguishing two cases: 1. $\pi = 0$ and 2. $\pi > 0$.

Case 1. In this case the signals s_l, s_r are drawn from binomial distributions $B(n_l, \lambda p), B(n_r, 1-\lambda p)$ in state l and $B(n_l, 1-\lambda p), B(n_r, \lambda p)$ in state r. The inequality $Pr(\vec{n},\vec{s}|l) \geq Pr(\vec{n},\vec{s}|r)$ holds if and only if

$$\binom{n_l}{s_l}(\lambda p)^{s_l}(1-\lambda p)^{n_l-s_l}\binom{n_r}{s_r}(\lambda(1-p))^{s_r}(1-\lambda(1-p))^{n_r-s_r} \geq$$
$$\binom{n_l}{s_l}(\lambda(1-p))^{s_l}(1-\lambda(1-p))^{n_l-s_l}\binom{n_r}{s_r}(\lambda p)^{s_r}(1-\lambda p)^{n_r-s_r} \Leftrightarrow$$
$$\left(\frac{p}{1-p}\right)^{s_l-s_r} \geq \left(\frac{1-\lambda p}{1-\lambda(1-p)}\right)^{n_r-n_l+s_l-s_r}$$

As $1-\lambda p > 1-\lambda(1-p)$ and $p > (1-p)$ we have that

$$\left(\frac{p}{1-p}\right)^{(s'_l-s'_r)} > \left(\frac{1-\lambda p}{1-\lambda(1-p)}\right)^{(n'_r-n'_l)+(s'_l-s'_r)}.$$

for $s_l' - s_r' \geq s_l - s_r$ and $n_r' - n_l' \geq n_r - n_l$ with at least one of the inequalities strict. We conclude that $Pr(l|\vec{n}', \vec{s}') > Pr(r|\vec{n}', \vec{s}')$ for $\vec{s}', \vec{n}'$ described in the statement of the Lemma.

Case 2. The state l is more likely if more signals have been sent in its favor. The independent needs to compare the total number of signals in favor of l, s_l^*, with the total number of signals in favor of r, s_r^*. Not knowing these numbers the independent can calculate the expected difference between the two numbers for a fixed $\vec{s}, \vec{n}$ as $E(s_l^* - s_r^* | \vec{s}, \vec{n})$; $Pr(l|\vec{n}, \vec{s}) \geq Pr(r|\vec{n}, \vec{s})$ holds if and only if $E(s_l^* - s_r^* | \vec{s}, \vec{n}) \geq 0$.

Define s_l^i, s_r^i as the number of l and r signals sent by the independents. Since $\pi > 0$, we have by Corollary 1 that all extremists acquire information. Consequently every silent extremist hides a signal in favor of the opposite candidate. We can calculate s_l^*, s_r^* as:

$$s_l^* = s_l + n_r - (s_r - s_r^i) \qquad s_r^* = s_r + n_l - (s_l - s_l^i)$$

The expected difference becomes:

$$\begin{aligned} E(s_l^* - s_r^* | \vec{s}, \vec{n}) = E(2(s_l - s_r) + (n_r - n_l) + (s_l^i - s_r^i) | \vec{s}, \vec{n}) = \\ 2(s_l - s_r) + (n_r - n_l) + E(s_r^i - s_l^i | \vec{s}, n_i) \end{aligned}$$

Clearly $E(s_l^* - s_r^* | \vec{s}, \vec{n})$ is increasing in $n_r - n_l$. The expression is also increasing in $s_l - s_r$ as an increase of $s_l - s_r$ by 1 increases $s_l - s_r$ by 2 while it decreases $E(s_r^i - s_l^i | \vec{s}, \vec{n})$ by at most 1. □

Theorem 2 *Equilibrium requirement 4 and 1 are consistent.*

Proof Suppose that $\vec{s}$ is such that τ prescribes for an independent to vote L. Would this independent want to vote L if he knew that he was pivotal? In other words, if T is the event that i is pivotal i.e. $T = \{\vec{n} | n_r = n_l + n_i, n_r = n_l + n_i + 1\}$, is it true that $Pr(l|\vec{s}, T) \geq Pr(r|\vec{s}, T)$. We know from Lemma 5 that $Pr(l|\vec{s}, T) \geq Pr(r|\vec{s}, T)$ holds for all cases in which independents are supposed to pick L if it holds for the case in which $s_l - s_r$ and $n_r - n_l$ are being minimized given $s_l \geq s_r$ and the independent is pivotal, namely $s_l = s_r$ and $n_r - n_l = n_i$. In the sequel we will only investigate this case and show that the independent has an incentive to vote for L even in this worst case scenario.

Case 1. By the argument given in the proof of Lemma 5 we know that $Pr(l|\vec{n},\vec{s}) \geq Pr(r|\vec{n},\vec{s})$ if and only if

$$\left(\frac{p}{1-p}\right)^{s_l-s_r} \geq \left(\frac{1-\lambda p}{1-\lambda(1-p)}\right)^{n_r-n_l+s_l-s_r}$$

Observe that for $s_l = s_r$ and $n_r - n_l = n_i \geq 0$ the above in equality always holds so we are done.

Case 2: By Lemma 5 we need to show that $E(s_l^* - s_r^*|s_r = s_l, n_r - n_l = n_i) \geq 0$. Following the arguments given in the proof of Lemma 5 we can calculate $E(s_l^* - s_r^*|s_r = s_l, n_r - n_l = n_i)$ as $E(n_i + s_r^i - s_l^i|s_r = s_l, n_r - n_l = n_i)$.

If k out of the n_i independents acquired information then we can calculate a lower bound on $E(s_r^i - s_l^i) = (1-p)k - pk - 1 = (1-2p)k - 1$ (assuming that the state is l, the independent under consideration observed an l signal, and $s_l = s_r > 2k$ which in turn implies that the constraint of the total number of signals does not bind). This expression is bounded from below by $(1-2p)n_i - 1$ as $k = 0, ..., n_i$. So a lower bound for $E(n_i + s_r^i - s_l^i|s_r = s_l, n_r - n_l = n_i)$ is $E(n_i - n_i(1-2p) - 1) = n(1-2\eta)2p - 1$. This expression is non-negative as $n(1-2\eta) \geq \frac{1}{2p} \geq 1$ as was assumed in the setup of the model.

□

8 Independents never want to acquire signals for private use only

Equilibrium requirement 3 says that no independent would acquire a signal and not send it. The requirement states that $\tau_i(1, l, \emptyset, f) = \tau_i(1, \emptyset, r, f) = \tau_i(1, \emptyset, \emptyset, f) = 0$ for all $f \in C_2$. In our solution we used this requirement. We now need to show that no strategic voter would like to deviate from this behavior. We need to show that $EU_i(\tau, (1, \emptyset, \cdot, \cdot)) \leq EU_i(\tau)$.

Lemma 6 *Equilibrium requirement 1 and 3 are consistent.*

Proof We know from the prior section that:

$$\forall f \in C_2,\ EU_i(\tau,(0,\cdot,\cdot,f_i)) \geq EU_i(\tau,(0,\cdot,\cdot,f))$$

We will show

$$\text{if } \exists\ f \in C_2\ s.t.\ EU_i(\tau,(1,\emptyset,\cdot,f)) \geq EU_i(\tau,(0,\cdot,\cdot,f_i)) \\ \Rightarrow EU_i(\tau,(1,l,.,f_i)) > EU_i(\tau,(1,\emptyset,.,f))$$

So $(1,\emptyset,.,.)$ is never a best reply for an independent. For this proof let us assume the role of an informed independent, i^*, that has received an l signal. Observe that there are $n_i - 1$ other independents in the electorate. We define $\vec{s}^* = (s_l^*, s_r^*)$ as the vector of all public signals when i^* does not send his signal and $\vec{s}^* + l$ as the set of all signals available to voter i^*, which consists of the signals sent by all voters but him and his own signal. Observe that f_i prescribes to vote L if $s_l^* > s_r^*$ when i^* does not send his signal and $s_l^* + 1 > s_r^*$ when i^* does send his signal. In the sequel assume that f^* maximizes $EU_i(\tau,(1,\emptyset,\cdot,f))$.

For $EU_i(\tau,(1,\emptyset,.,f^*)) \geq EU_i(\tau,(0,.,.,f_i))$ we must have that $f^* \neq f_i$. If these two would coincide then the independent would be strictly better off not to incur any cost for the private signal which he would never use.[12] We know from Theorem 2 that sincere voting is strategic voting: If $\vec{s}^*$ and $(s_l^* + 1, s_r^*)$ yield the same voting recommendation under f_i, then it must be true that $f_i(\vec{s}^*) = f^*(\vec{s}^* + l)$. So we need to only consider the cases in which the signal of the independent could possibly sway the vote of all independents. This happens either if $s_l^* = s_r^*$ or if $s_l^* = s_r^* - 1$.

In the case that $s_l^* = s_r^* - 1$ all independents except for i^* will vote R ($f_i(\vec{s}^*) = R$). Together with i^*'s signal l there would be equally many signals in favor of each candidate. Observe that the same voting behavior is prescribed if i^* does not send his signal and i^* publicizes his signal if the supreme court issues a a recommendation to vote R. By Theorem 2 an independent prefers to vote R in the latter case. So the independent would prefer to vote R in the former case. We conclude that $f_i(\vec{s}^*) = f^*(\vec{s}^* + l)$. If i^* has an incentive to vote according to his private signal this must happen when $s_l^* = s_r^*$.

[12] We used this argument in Lemma 3 to show that the extremists would never buy a signal without any plans to send it.

Now let us look at the this case.[13] In case the supreme court's coin shows L, i^* would not want to deviate from the public recommendation to vote L. So we only need to consider the case in which the Court's coin shows R. Since this is our last chance to find a $\vec{s}^*$ such that $f_i(\vec{s}^*) \neq f^*(\vec{s}^* + l)$ it must be true that $f^*(\vec{s}^* + l) = L$ for $s_l^* = s_r^*$.

To fully understand the incentives of the privately informed voter we need to find out when this voter would be pivotal. In the case under study all independents except for i^* ($n_i - 1$ independents) vote R. Without i^*'s vote R receives $n_r + n_i - 1$ votes whereas l receives n_l votes. Voter i^* is pivotal in exactly two cases namely $n_r + n_i = n_l$ and $n_r + n_i = n_l + 1$ (Remember that we are only looking at the case in which the Court's coin came up R, so R wins the election in case of a tie). So we have found a necessary condition for $EU_i(\tau, (1, \emptyset, ., f^*)) \geq EU_i(\tau, (0, ., ., f_i))$ to hold. This condition is that i^* prefers L to R conditional on having himself an l signal, $s_l^* = s_r^*$ and $n_l - n_r \in \{n_i - 1, n_i\}$. Using Lemma 5 we obtain that i^* strictly prefers L to R for $n_l - n_r < n_i - 1$ keeping all else (namely $\vec{s}^*$ and i^*'s signal) equal.

We need to show next that if i^* prefers L to R under the condition named above, then he will prefer sending his signal l to keeping it secret. To show this we need to first identify the range of cases in which a switch from silence to sending changes the outcome. As above for all $\vec{s}^*$ with either $s_l^* > s_r^*$ or $s_l^* < s_r^* - 1$ the outcome remains the same. So let us investigate how the outcome changes in the remaining cases that $s_l^* = s_r^* - 1$ and $s_l^* = s_r^*$. As before the case that $s_l^* = s_r^* - 1$ is easier to deal with, and we attack this case first.

If $s_l^* = s_r^* - 1$ and i^* remains silent then all independents will vote for R, by our arguments above we know that this includes the i^*. To the contrary if i^* publishes his signal a publicly thrown coin will decide whom the independents will vote for. Under the assumption that $s_l^* = s_r^* - 1$ and that i^* holds an additional l signal, the signals do not reveal anything about the state of the world. The symmetry of the problem implies that the

[13] A plausible case in which i^* would have an incentive to vote according to the hidden signal is that n is very small and $s_l = s_r = 0$. In this case it is very unlikely there there are any other voters out there, so pivotality concerns do not matter much, i^* should follow his signal

decision whether to reveal the signal or not does not entail a utility change for i^*.

So let us now have a look at the alternative case $s_l^* = s_r^*$. If the public lottery directs the independents to vote for L, it does not matter whether i^* sends his signal or keeps it secret. So we only need to look at the case that the public lottery directs all independents to vote R. In this case i^* will sway the vote of the independents by sending his l signal. To evaluate whether he should do so i^* has to come up with a list of cases in which his sending of the signal changes the outcome of the election (switch from R to L). If i^* does not send his signal the right wing candidate receives $n_i + n_r - 1$ votes whereas the left wing candidate receives $n_l + 1$ votes. On the other hand if i^* sends the signal the right wing candidate receives n_r votes whereas the left wing candidate receives $n_l + n_i$ votes. The signal of the independent is pivotal if $n_l - n_r \leq n_i - 2$ and $n_l - n_r > -n_i$. In other words the voter signal is pivotal if $n_l - n_r \in \{-n_i + 1, ..., n_i - 2\}$. By our above arguments we know that i^* strictly prefers L to R for any single one of these cases. It is therefore true that $EU_i(\tau, (1, l, ., f_i)) > EU_i(\tau, (1, \emptyset, ., f^*))$ for $EU_i(\tau, (1, \emptyset, ., f^*)) = \max_f EU_i(\tau, (1, \emptyset, ., f)) \geq EU_i(\tau, (0, ., ., f_i)$. An analogous argument holds for the case that the independent observed an r signal. □

9 Existence of Equilibrium

We have now reduce the problem of showing that an equilibrium exists to a problem of showing that there exist probabilities of information acquisition λ, π such that neither the extremists nor the independents would change their information acquisition behavior given every one else's information acquisition behavior and that all voters follow τ after the information has been acquired. For the proofs in this section it is convenient to define Δ^+ and Δ^- as functions of the information acquisition probabilities. We write $\Delta^+(\pi, \lambda)$ and $\Delta^-(\pi, \lambda)$ for the probability increases of the correct choice being made when one more correct or incorrect signal is being sent given that the information acquisition probabilities of the independents and extremists are π and λ.

Theorem 3 *An equilibrium τ exists.*

Proof Define a correspondence $f : [0,1]\times[0,1] \to [0,1]\times[0,1]$, by $f = h \circ g$ with g being a function and $g : [0,1] \times [0,1] \to \mathbb{R} \times \mathbb{R}$ such that

$$g(\pi, \lambda) = \begin{bmatrix} \frac{p}{2}\Delta^{+}(\pi, \lambda) \\ \frac{1}{2}[p\Delta^{+}(\pi, \lambda) - (1-p)\Delta^{-}(\pi, \lambda)] \end{bmatrix}$$

and h being a correspondence $h : \mathbb{R} \times \mathbb{R} \to [0,1] \times [0,1]$ with

$$h_i(x,y) = \begin{cases} 1 \text{ if } g_i(x,y) > c \\ [0,1] \text{ if } g_i(x,y) = c \\ 0 \text{ if } g_i(x,y) < c \end{cases}$$

Observe that g is continuous and h is upperhemicontinuous. So f is upperhemicontinuous. We conclude by Kakutani's fixed point theorem that a fixed point exists. □

The reason why we could not apply Myerson (1997) existence result is that strategy spaces in our game are not finite: the set of voting rules is infinite. However, we reduce the our exsitence problem early on to a problem that would fit Myerson's existence result. As soon as we know that players will play f_{t_1} in equilibrium, we could apply Myerson's result. The reason why we proved existence here is that this proof introduces the necessary terminology for the a uniqueness discussion.

We have a strong intuition that the equilibrium is unique. Indeed if $\frac{p}{2}\Delta^{+}(\pi, \lambda$ and $\frac{1}{2}[p\Delta^{+}(\pi, \lambda) - (1-p)\Delta^{-}(\pi, \lambda)]$ are strictly decreasing over certain intervals of π and λ that are consistent with corollary (1) we can show that there is a unique equilibrium. The intuition is most clear using a diagram.

By corollary (1) the only possible values of $[\pi, \lambda]$ in equilibrium are $\pi = 0$ when $\lambda < 1$ and $\lambda = 1$ when $\pi > 1$. This allows us to reduce π and λ to one dimension, as in figure (1). In the portion of the independent axis labeled λ only $-\frac{p}{2}\Delta^{-}(\pi, \lambda$ is relevant and in the portion labeled π only $-\frac{1}{2}[p\Delta^{-}(\pi, \lambda) - (1-p)\Delta^{+}(\pi, \lambda)]$ is relevant. The bold areas represent the marginal benefit for an extremist and an independent on the left and right side of the graph respectively. The constant marginal cost of information acquisition can cross these curves at most once if they are decreasing. If

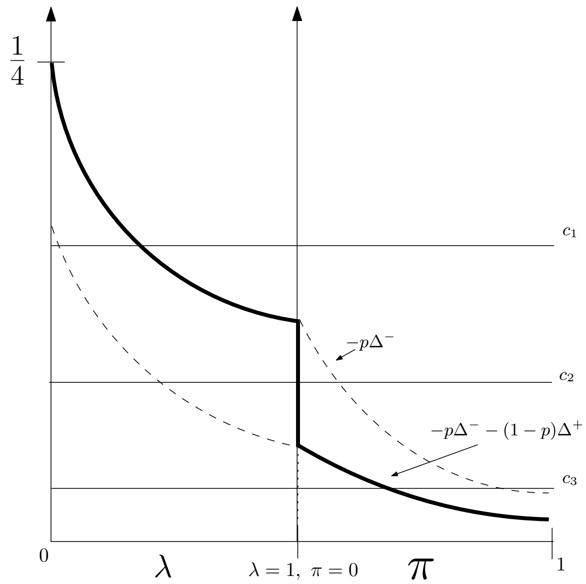

Figure 1: MB and MC

c does not cross, either information is so cheap that everyone acquires the signal, it is so expensive that no one acquires a signal, or it is between the extremist and independent curves so that extremists always acquire and independents never do. If c does cross one of the curves, that uniquely defines λ and π.

If it were the case that the extremists had no vote, but were just information gatherers we can show that this intuition is is indeed fact as the Δ functions are decreasing (and look like the above figure). Unfortunately once we attempt to take into account the small probability that an extremist group outnumbers the rest of the electorate, manipulations of the Δ function become intractable. To see this consider that just Δ^+ can be written as:

$$\sum_{i=0}^{\infty} \frac{e^N N^i}{i!} \sum_{j=0}^{i} \binom{i/2}{j} \eta^j (1-\eta)^{i/2-j} \sum_{k=0}^{i-j} \binom{i/2}{k} \eta^k (1-\eta)^{1/2-k}$$
$$\cdot [\Pr(s_l = s_r | \tau, i, j, k) + \Pr(s_l = s_r - 1 | \tau, i, j, k)]$$

As such uniqueness is left uncertain.

10 Asymmetries

The model as well as the equilibrium concept contains a set of symmetry assumptions. There are 3 symmetry assumptions in the model: The two states are assumed to happen with equal probability, both signals are equally strong, the expected share of right and left wingers is equal. In the solution concept we impose two further symmetry assumptions: in equilibrium both types of extremists are equally likely not to acquire information and the independents will pass on both signals. What are these assumptions needed for? How would the results change if we were to drop some (or all) of these assumptions?

As an answer to the first question observe that our proof that strategic voters do not have an incentive to deviate from sincere voting depend our symmetry assumptions. To see this consider an electorate in which there are twice as many leftists as rightists on average. Consider the case of a prior that slightly favors state l and assume that both signals are equally strong and in equilibrium all extremists acquire information. Now let us consider the case in which no signals have been sent: $s_r = s_l = 0$. A sincere independent should vote for R. He should expect that there are about twice as many leftists than rightists in the electorate. Since all extremists stayed silent, this means that there are about twice as many right wing signals than there are left wing signals. Since the prior favors l only slightly the information contained in $s_l = s_r = 0$ should outweigh the prior. Now let us reconsider this question from the point of view of a strategic independent. Assume that the electorate is small and assume that independents are "nearly never" born. So the pivotality event contains the information that there are approximately equally many rightists and leftists. This implies that they hide approximately equally many right and left signals. So a strategic voter should vote according to the prior for L since $s_r = s_l = 0$ together with the pivotality event do not reveal any information about the state of the world.

The consistency between Equilibrium requirements 1 and 3 and 4 will be given for certain parameter ranges. Proofs of this would certainly messy. In this section we follow a different route. We complement the relaxation of the symmetry assumptions with the imposition of two behavioral assumptions. In this section we modify the model insofar as that we assume that voters vote sincerely and that independents share all their information, we do not require that these two assumption be compatible with strategic voting and strategic information transmission. With these behavioral assumptions we are able to establish a version of our main result for the case of asymmetric equilibria in asymmetric models. To deal with the proposed asymmetries we need more notation. We will introduce all this information as part of the setup of our the modified model. For brevity we refer the reader to the prior sections for a discussion of our setup.

10.1 Asymmetric Model

Take the Poisson $\{\Omega, \alpha, T, n, p, C, U\}$ game defined in Section 2 and modify it such that the common prior that the state is l is $\alpha \in (0,1)$. Assume that $p_1(t_1 = l|\omega) = \eta_l,\ p_1(t_1 = r|\omega) = \eta_r$ and $\eta_l + \eta_r < 1$ for $\omega \in \{l, r\}$ which implies that any voter is a leftist with probability η_l and a rightist with probability η_r. Finally let $p_2(t_2 = l|\omega = l) = p_l;\ p_2(t_2 = r|\omega = r) = p_r$, this implies that the signals l and r might have different strength as $p_r,\ p_l$ can be viewed as measures of their strength.

The prior model arises as a special case for $\alpha = \frac{1}{2},\ p_l = p_r,\ \eta_r = \eta_l$ and the assumption that the electorate in "large" $((1-2\eta)n > 1)$. The last assumption was only used to establish that independents have no strategic incentive to deviate from sincere voting. We now have the behavioral assumption of sincere voting. Consequently the assumption that the electorate is large is no longer needed.

10.2 Strategies and Equilibrium

With the behavioral assumptions strategies can be described by shorter vectors in the modified game. In fact the strategy of independents can be described by their probability to acquire information π alone (their voting and information sharing behavior is fixed according to our modelling assumption.) Strictly speaking the strategy vectors of extremists should have two entries as we did not make any behavioral assumption on their information sharing behavior. However, just as in the above model it is straightforward to show that extremists will always broadcast signals in favor of their candidate and never broadcast signals in favor of the other candidate. We therefore suppress this strategy variable and define the strategy of leftists by λ_l the probability that leftists acquire information and λ_r respectively for rightists.

We solve this game for an equilibrium following Myerson's definition of an equilibrium in an extended Poisson game (Myerson 1998). So a vector $(\lambda_l, \lambda_r, \pi)$ is an equilibrium if no voter has an incentive to deviate from their (information acquisition) strategy given everyone else's (information acquisition) strategy. It is important to note that this definition of equilibrium does allow for asymmetric strategies, we do not impose that $\lambda_r = \lambda_l$. Also note that Myerson's (Myerson 1998) equilibrium existence result applies directly to the games in this family.

10.3 Solution

Voters are assumed to vote sincerely. Just as above this implies that all extremists will vote for their preferred candidate. Let us calculate an asymmetric analog of the independents cutoff rule. Just as in the prior setup s_r and s_l are Poisson distributed random variables in the current setup. The parameters of s_l and s_r in states l and r are called x_l^l, x_l^r, x_r^r and x_l^r, where the superscript denotes the state and the subscript denotes the signal:

$$\begin{aligned} x_l^l &= n\eta_l\lambda_l p_l + n(1-\eta_l-\eta_r)\pi p_l \\ x_r^l &= n\eta_r\lambda_r(1-p_l) + n(1-\eta_l-\eta_r)\pi(1-p_l) \\ x_l^r &= n\eta_l\lambda_l(1-p_r) + n(1-\eta_l-\eta_r)\pi(1-p_r) \\ x_r^r &= n\eta_r\lambda_r p_r + n(1-\eta_l-\eta_r)\pi p_r \end{aligned}$$

So state l is at least as likely as state r if and only

$$\alpha\frac{e^{-x_l^l}[x_l^l]^{s_l}}{s_l!}\cdot\frac{e^{-x_r^l}[x_r^l]^{s_r}}{s_r!} \geq (1-\alpha)\frac{e^{-x_r^l}[x_r^l]^{s_l}}{s_l!}\cdot\frac{e^{-x_r^r}[x_r^r]^{s_r}}{s_r!}$$

Observe this holds expression if and only if

$$ln(\frac{\alpha}{1-\alpha}) + x_r^r + x_l^r - x_l^l - x_r^l \geq s_l ln\left(\frac{x_l^r}{x_l^l}\right) - s_r ln\left(\frac{x_r^l}{x_r^r}\right)$$

Substituting in the values for x_l^l, x_l^r, x_r^r and x_l^r we obtain that

$$s_l ln\left(\frac{p_l}{1-p_r}\right) \geq ln(\frac{1-\alpha}{\alpha}) + n(p_r+p_l-1)(\eta_l\lambda_l - \eta_r\lambda_r) + s_r ln\left(\frac{p_r}{1-p_l}\right)$$

First of all observe that this expression reduces to the known cutoff rule if $\alpha = \frac{1}{2}$, $p_l = p_r$, $\eta_r = \eta_l$ and $\lambda_r = \lambda_l$. Let us now discuss this rule in some detail: to make their decision the independents will multiply the raw data s_l, s_r with factors $ln\left(\frac{p_l}{1-p_r}\right)$ and $ln\left(\frac{p_r}{1-p_l}\right)$ that reflect the relative ease or difficulty to obtain l or r signals in either state. The larger is $\frac{p_r}{1-p_l}$ the more meaningful are the r signals. In the expression the total amount of r signals is multiplied by this indicator of the informativeness. In the extremely noninformative case that $p_r = p_l = \frac{1}{2}$ the signals are meaningless, in this case the decision will be based on the prior alone, the above inequality is either always true or never true, depending on the value of α.

If r is more likely following the prior more l-signals are needed to sway the independents to vote L. This is reflected by the term $ln\left(\frac{\alpha}{1-\alpha}\right)$. Finally the term $n(p_r+p_l-1)(\eta_r\lambda_r - \eta_l\lambda_l)$ is equal to zero if $\eta_r\lambda_r = \eta_l\lambda_l$. In this case any voter is equally likely to be either a right winger that acquires information or a left winger that acquires information. In this case the only asymmetries in the decision rule should be attributable to the prior (α) and to the different signal strength (the factors $ln\left(\frac{p_r}{1-p_l}\right)$ and $ln\left(\frac{p_l}{1-p_r}\right)$ as discussed above). If this does not hold, say if $\eta_r\lambda_r > \eta_l\lambda_l$, then independents need more r signals to be convinced in R's favor, as there are on average more right wingers that send such signals than there are left wingers. This intuition holds true as $\eta_r\lambda_r - \eta_l\lambda_l$ is multiplied by a positive factor $n(p_l + p_r - 1)$ in this equation. As expected this factor depends on the average size of the electorate (n) and a measure of the value of signals.

Recall that $\Delta^z_{Y,\omega}$ was defined as the change in probability that candidate Y is being chosen in state ω when another z signal is being sent. Just as in the asymmetric case it is true that the probability that a candidate wins is increasing in the number of signals sent in his favor: $\Delta^l_{L,\omega}, \Delta^r_{R,\omega}$ for $\omega = l, r$ Without the various symmetry assumptions it is however not necessarily true that $\Delta^r_{R,l} = \Delta^l_{L,r}$ and $\Delta^l_{L,l} = \Delta^r_{R,r}$.

Following the arguments given in section 6 we can establish a weaker version of the result that in equilibrium extremists have stronger incentives to acquire information. We state this result as a separate Theorem.

Theorem 4 *In an equilibrium the utility gain from acquiring information for an independent is never greater than the utility gain of both extremists.*

The proof follows along the same lines as the proof of Theorem 1. It is therefore kept very short.

Proof The utility of acquiring information for the right and left wing extremists are respectively:

$$X = \frac{p}{2}\Delta^r_{R,r} - c \text{ and } Y = \frac{p}{2}\Delta^l_{L,l} - -c.$$

Finally we calculate the utility of information acquisition for an independent as

$$Z = \frac{p}{4}(\Delta^l_{L,l} + \Delta^r_{R,r}) - \frac{1-p}{4}(\Delta^r_{R,l} + \Delta^l_{L,r}) - c = \frac{X+Y}{2} - \frac{1-p}{4}(\Delta^r_{R,l} + \Delta^l_{L,r}).$$

Finally observe that $\Delta^r_{R,l}, \Delta^l_{L,r} > 0$ so we have that $Z < \frac{X+Y}{2}$ and it cannot be true that both extremists receive a lower utility from information acquisition than the independent does. □

Corollary 2 *If the equilibrium probability that independents acquire information is positive then at least one set of extremists extremists acquires information in equilibrium. If the equilibrium probability that extremists acquire no information is positive for both camps of extremists, then no independent acquires any information in equilibrium.*

Proof In analogy to the proof of corollary 1 for the symmetric case. □

References

Austin-Smith, D. and J.S. Banks, "Information Aggregation, Rationality, and the Condorcet Jury Theorem," *American Political Science Review*, March 1996, (90), 34–45.

Baron, David, "Persistent Media Bias," August 2004. Stanford University GSB Research Paper No. 1845, Mimeo.

Bernhardt, Dan, Stefan Krasa, and Mattias K. Polborn, "Political Polarization and the Electoral Effects of Media Bias," March 2006. CESifo Working Paper Series No. 1798, Mimeo.

Degan, Arianna, "Policy Position, Information Acquisition, and Turnout," *Scandinavian Journal of Economics*, December 2006, *108* (4), 669–682.

Dewatripont, Mathias and Jean Tirole, "Advocates," *The Journal of Political Economy*, February 1999, *107* (1), 1–39.

Feddersen, Timothy J. and Wolfgang Pesendorfer, "The Swing Voter's Curse," *The American Economic Review*, June 1996, *86* (3), 408–424.

Gerardi, Dino and Leeat Yariv, "Information Acquisition in Committees," November 2004. Mimeo.

Glaezer, Jacob and Ariel Rubenstein, "Debates and Decisions: On a Rationale of Argumentation Rules," *Games and Economic Behavior*, 2001, *36*, 158–173.

Milgrom, Paul and John Roberts, "Relying on the Information of Interested Parties," *Rand Journal of Economics*, 1986, *17*, 18–32.

Myerson, Roger B., "Large Poisson Games," Discussion Papers 1189, Northwestern University, Center for Mathematical Studies in Economics and Management Science Jun 1997. available at http://ideas.repec.org/p/nwu/cmsems/1189.html.

———, "Extended Poisson Games and the Condorcet Jury Theorem," *Games and Economic Behavior*, 1998, pp. 111–131.

Oliveros, Santiago, "Aggregation of Endogenous Information in Mandatory Elections," 2006. Haas School of Business, UC: Berkeley, Mimeo.

Palfrey, Thomas R. and Keith T. Poole, "The Relationship Between Information, Ideology, and Voting Behavior," *American Journal of Political Science*, August 1987, *31* (3), 511–530.

Shin, Hyun-Sung, "The Burden of Proof in a Game of Persuasion," *Journal of Economic Theory*, 1994, *64*, 253–264.

Sidanius, Jim, "Political Sophistication and Political Deviance: A Structural Equation Examination of Context Theory," *Journal of Personality and Social Psychology*, 1988, *55* (1), 37–51.

The National Election Studies, "2004 National Election Study," Ann Arbor, MI: University of Michigan, Center for Political Studies [producer and distributor] 2004.