On multiple-principal multiple-agent models of moral hazard *

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Abstract

In multiple principal multiple agent models of moral hazard, we consider whether the outcomes of equilibria in direct mechanisms are preserved when principals can offer indirect communication schemes. We first discuss the role of random allocations and recommendations in a context with a single principal and two agents, and show by example that, in the absence of either, indirect communication schemes may be preferred to direct ones. We then provide two conditions on direct mechanism equilibria under which these equilibria survive the possibility that a principal can deviate to an indirect mechanism. Finally, we provide a method to check robustness of equilibria in direct mechanisms with no communication between principals and agents (the typical case in current literature): It is sufficient to rule out deviations by a principal that induce incentive compatible responses from agents.

Key words: Moral hazard, multiple principal, multiple agent, direct mechanisms.
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1 Introduction

We consider multiple principal, multiple agent models of pure moral hazard. That is, there is complete information about the types of principals and agents, but agents' effort is not contractible. Our goal is to establish conditions under which equilibria in which principals offer direct mechanisms are robust to the possibility that any principal may deviate to a richer communication scheme (i.e., an indirect mechanism) to interact with agents.

With multiple principals, it is well-established that there is a loss of generality in focusing on simple incentive compatible direct mechanisms (see, for example, Peck [1997], Martimort and Stole [2002], and Peters [2001]). That is, there exist equilibrium outcomes with richer communication schemes that are not replicable in direct mechanisms. In such contexts, it is important to understand whether there can be some rationale for a restriction to simple mechanisms. It turns out (see Theorems 1 and 2 in Peters [2003]) that, whenever multiple principals interact in the presence of a single agent, pure strategy equilibria in direct mechanisms remain equilibria when richer communication schemes are feasible. We aim to establish a similar result for multiple principal, multiple agent games. To properly analyze this scenario, one has to consider the difficulties related to the competition among many principals together with the coordination issues induced by the strategic interaction among agents.

The papers cited in the previous paragraph all allow for the general case of incomplete information, where an important role of communication schemes is to allow an agent to convey private information to the principal. Our focus in this paper is only on complete information with non-contractible effort. At first glance, the idea of a communication scheme in a complete information setting may seem strange. In Section 3, we show via example that, in a single-principal two-agent setting, communicating with one agent allows the principal to create private information between the agents before the agents choose their efforts, and thereby improve his payoff compared to a mechanism that does not allow for communication. In Myerson's [1982] construction, such private communication is made possible by the principal sending recommendations on actions to the agents. Thus, our examples show that recommendations are a necessary feature to sustain the robustness of direct mechanism outcomes to indirect communication schemes. Of course, private communication with agents itself is of no value unless there is uncertainty in the environment. The principal, in fact, creates this uncertainty by offering stochastic allocation schemes.

Even with a single principal, therefore, stochastic allocations and recommendations are necessary to establish the revelation principle in a *pure moral hazard* scenario. This is the first result of the paper and develops for the moral hazard environment the same intuition suggested by Strausz [2003] for the pure incomplete information case. The applied literature on moral hazard with multiple agents usually does not consider any form of communication between players. Potentially, such communication may be beneficial in the contexts of hierarchies, moral hazard in teams, or collusion between agents.[1]

Stochastic allocations and recommendations will continue to be required when we consider multiple principals. We turn to the multiple principal case in Section 4. An additional difficulty is created here, since incentive compatibility in terms of obeying a principal's recommendations need not hold with respect to each principal. For example, if two principals are each choosing probability distributions over allocations and recommendations on effort, an agent will sometimes receive contradictory recommendations from the principals. Which one should he obey? Rather than requiring incentive compatibility in the strict sense, we allow for agents to play a Nash equilibrium of the efforts game, conditional on the private recommendations they have received. As Peters [2004] shows, multiplicity of equilibria in the agents' game can be an issue. In particular, agents may play different equilibria when a principal offers an indirect instead of a direct mechanism.

To overcome this possibility, we introduce an obedient deviations property that specifies that a principal switching from an indirect mechanism to a direct mechanism, keeping other principals' strategies the same, can induce agents to play the same equilibrium in the continuation game. We also provide a no-correlation condition that specifies that, in the equilibrium of the game in which all principals choose direct mechanisms, each principal sends recommendations that are uncorrelated with his chosen allocations. Equilibria in direct mechanisms that satisfy our conditions remain robust to a deviation by a principal to other communication schemes.

Our conditions further allow us to identify a clear methodology to check whether equilibria in mechanisms with no recommendations remain robust to a unilateral devi-

[1]The role of multiple agent moral hazard models in these settings is discussed in Bolton and Dewatripont [2005], Chapter 8.

ation by a principal to a mechanism with communication. In principle, there could be infinitely many deviations that involve some form of communication. Our final result shows that one can restrict without loss of generality to those deviations that are incentive compatible following a unilateral deviation by a principal to a mechanism with recommendations. If an equilibrium of a simple game in mechanisms with no recommendations is robust to such deviations, it will remain robust to the introduction of any form of communication.

As yet, little is known about multi-principal multi-agent models. The menu theorems of Martimort and Stole [2002] and Peters [2001] do not extend straightforwardly to a general multi-principal setting.[2] The methodology proposed by Pavan and Calzolari [2006] has also not yet been extended to multi-principal multi-agent games. Our theorem represents one step toward a more general characterization of equilibria in this framework. Our result, based on non-contractible effort, complements the work of Han [2006b], who considers complete information with contractible effort in a model similar to that of Prat and Rustichini [2003].

2 The Model

There are n principals dealing with k agents, where $n \geq 1$ and $k \geq 2$. That is, we consider a model with multiple agent. While the general model allows for multiple principal as well, the single principal case is a special case of some interest, and will be the focus of Section 3.

Let Y_j be a set of deterministic allocations available to principal j, with typical element $y_j \in Y_j$. An allocation can be, for example, monetary transfers, tax rates, prices, or quantities, depending on the particular interpretation of the model. Each principal j chooses an allocation in the set $\Delta\left(Y_j\right)$, the set of lotteries that can be generated over the set of deterministic allocations Y_j.

There is complete information about agent types. Each agent i chooses an unobservable effort $e^i \in E^i$, where E^i is a finite set. Therefore, the model is one of pure moral hazard. We denote the vector of efforts as $e = \left(e^1, e^2, ..., e^k\right) \in E = \times_{i=1}^{k} E^i$.

[2] Han [2006a] extends the menu theorems to a restricted class of multi-principal multi-agent games, in which the contract between a principal and agent is essentially bilateral, and separate from the contract with any other principal or agent.

We use the general communication structure for principal-agent models introduced by Myerson [1982]. Each principal j chooses a message space M_j^i and a recommendation space R_j^i for each agent. To avoid measure-theoretic issues that arise with continuum spaces, we restrict M_j^i and R_j^i to be finite (possibly empty) for each i and j. Let $R_j = \times_{i=1}^k R_j^i$ denote the set of recommendations principal j can make, and $M_j = \times_{i=1}^k M_j^i$ the set of messages he can receive. The allocations and recommendations chosen by principal j depend on the messages received from the agents.

As in Myerson [1982], principal j's behavior is described by the choice rule $\pi_j : M_j \rightarrow \Delta\left(Y_j \times R_j\right)$. That is, principal j may choose a stochastic mechanism, which provides a lottery over allocations and recommendations for some message array m_j. When the choice rule π_j is stochastic, principal j chooses a realization from the lottery π_j, and communicates the realized recommendations r_j to the agents. Conditional on observing r_j^i, agent i updates her belief about the allocation y_j, but need not know the actual realization. Since recommendations are private, two agents i and i' may have different posterior beliefs about principal j's chosen allocation, y_j. Potentially, this allows a principal to induce a correlated equilibrium in the continuation game in which agents choose efforts.

A mechanism offered by principal j is thus given by $\left(M_j, R_j, \pi_j\right)$. Mechanisms are publicly observed, but a message from agent i to principal j, and a recommendation from principal j to agent i, are observed only by i and j. As is usual in the literature, principals commit to their mechanisms before agents send messages.

There are two stages at which agent i moves in the game. First, she sends a message array $m^i = \left(m_1^i, \ldots, m_n^i\right)$ to the principals. Then, after observing only her private recommendations $r^i = \left(r_1^i, \ldots, r_n^i\right)$, she chooses an effort $e^i \in E^i$. Given the offered mechanisms, let

$$\mu^i \in \Delta\left(M^i\right) \tag{1}$$

denote the message strategy of agent i, where $M^i = \times_{j=1}^n M_j^i$, and let

$$\delta^i : M^i \times R^i \rightarrow \Delta\left(E^i\right) \tag{2}$$

be her strategy in the effort game, where $R^i = \times_{j=1}^n R_j^i$.

The time structure of the interaction is provided in Figure 1 and follows the one considered by Myerson [1982].

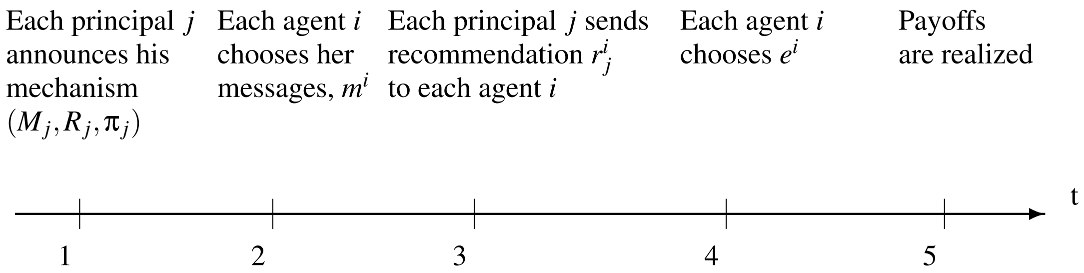

Figure 1: Timing of the generalized communication game

Agent i's payoff from a final outcome (y,e) is given by the von Neumann–Morgenstern utility function $U^i(y,e)$ and principal j's payoff is given by $V_j(y,e)$. Let $\pi = \times_{j=1}^{n}\pi_j$ denote the strategies of the principals, $\mu = \times_{i=1}^{k}\mu^i$ the message reporting strategies of the agents, and $\delta = \times_{i=1}^{k}\delta^i$ the agents' strategies in the effort game.

In this complete information framework, a direct mechanism is defined as follows. Principals do not solicit messages from agents, and directly suggest the actions they should take. That is, $M^i_j = \emptyset$ and $R^i_j = E^i$ for every $j = 1,\ldots,n$ and for every $i = 1,\ldots,k$. Finally, $\pi_j \in \Delta\left(Y_j \times E\right)$. A mixed strategy for an agent in a direct mechanism is given by $\delta^i : \left(E^i\right)^n \to \Delta\left(E^i\right)$. Any mechanism in which, for any principal j and any agent i, either $M^i_j \neq \emptyset$ or $R^i_j \neq E^i$, or both, is an indirect mechanism.

3 Single Principal

To isolate the issues introduced by multiple agents, we first consider the model with a single principal. In this case, a revelation principle holds (see, e.g., Myerson [1982]). That is, any outcome (i.e., a joint distribution over allocations and efforts) that can be sustained as an equilibrium in the agents' effort game in an indirect mechanism can also be sustained as an equilibrium of the agents' effort game in an incentive compatible direct mechanism. In the direct mechanism, it is a best response for an agent to "obey" the recommendation received from the principal.

It follows that the optimal direct mechanism is optimal in the class of all communication mechanisms. That is, an equilibrium of the game depicted in Figure 1 in which the principal is restricted to choosing among direct mechanisms remains an equilibrium of the game even when the principal is allowed to choose indirect mechanisms.

In this section, we provide examples to show that two features of the construction of Myerson [1982], stochastic allocations and recommendations, are necessary to sustain the robustness of equilibria in direct mechanisms to a possible deviation by a principal to an indirect mechanism. Each of the two examples we consider has one principal and two agents. In each example, we first exhibit the optimal direct mechanism, subject to the restriction that allocations must be deterministic in Example 1, and that no recommendations are permitted in Example 2. We then show that by offering a non-empty message space to one of the two agents, the principal can sustain outcomes in an indirect mechanism that are not feasible in a direct mechanism.

Example 1 (Necessity of stochastic allocations):

First, suppose that only deterministic allocations are permissible. Consider a game with one principal and two agents. The principal can choose between two allocations, and each agent chooses between two efforts levels. Following our notation, we have $Y = \{y_1, y_2\}$, $E^1 = \{a_1, a_2\}$ and $E^2 = \{b_1, b_2\}$.

The payoffs are given by the following matrices. In each cell, the first element corresponds to the utility of the principal, and the second and third to the utilities of agents 1 and 2 respectively.

$y = y_1$

	b_1	b_2
a_1	$(1,-4,4)$	$(1,4,-4)$
a_2	$(1,4,-4)$	$(1,-4,4)$

$y = y_2$

	b_1	b_2
a_1	$(1,1,1)$	$(x,0,0)$
a_2	$(x,0,0)$	$(1,-1,-1)$

where $x > 1$.

With deterministic allocations, a mechanism is characterized by

$$\tilde{\pi} : M \to Y \times \Delta(R),$$

where M is the message space and $R = R^1 \times R^2$ the recommendation space. Hence, a direct mechanism with deterministic allocations is defined by a lottery in $Y \times \Delta\left(E^1 \times E^2\right)$.

In a direct mechanism with deterministic allocations, recommendations have no role: For each of the allocations y_1, y_2, there is a unique correlated equilibrium (which is also the unique Nash equilibrium) in the agents' effort game. Regardless of the recommendations the principal sends, the agents will play the unique equilibrium of the game.

If the allocation is y_1, the payoff of the principal is trivially 1, regardless of agents' efforts. Agents play the following effort game:

$y = y_1$

	b_1	b_2
a_1	$(1, -4, 4)$	$(1, 4, -4)$
a_2	$(1, 4, -4)$	$(1, -4, 4)$

In the unique correlated equilibrium of this game, agents equally randomize between their two strategies.

If the principal chooses the allocation y_2, the effort game is given by

$y = y_2$

	b_1	b_2
a_1	$(1, 1, 1)$	$(x, 0, 0)$
a_2	$(x, 0, 0)$	$(1, -1, -1)$

Each agent has a strictly dominant strategy (a_1 for agent 1 and b_1 for agent 2). Hence, there is again a unique correlated equilibrium in which the principal's payoff is still 1.

Hence, using a direct mechanism with deterministic allocations, the payoff of the principal is 1. We now show that using an indirect mechanism with deterministic allocations, he can do better.

Consider the following indirect mechanism: the principal communicates only with agent 1, and offers no recommendations. That is, $M^1 = \{m_1, m_2\}$, and $M^2 = R^1 = R^2 = \emptyset$. The principal uses the following deterministic allocation rule $\tilde{\pi}$: if agent 1 sends message m_k, the allocation is y_k, for $k = 1, 2$.

Since agent 1 does not observe any new information (i.e., a recommendation) after sending her message, and agent 2 observes no information before choosing her effort, the following simultaneous-move game is induced between the agents:

	b_1	b_2
(m_1,a_1)	$(1,-4,4)$	$(1,4,-4)$
(m_1,a_2)	$(1,4,-4)$	$(1,-4,4)$
(m_2,a_1)	$(1,1,1)$	$(x,0,0)$
(m_2,a_2)	$(x,0,0)$	$(1,-1,-1)$

In the absence of recommendations, agents play a Nash equilibrium of this game. We now show that every Nash equilibrium of this game places a positive probability on the outcome $(x,0,0)$.

First, by inspection, we observe that there is no pure strategy equilibrium in this game. Therefore, in any Nash equilibrium, agent 2 must play both b_1 and b_2 with strictly positive probability. Further, strategy (m_2,a_2) for agent 1 is strictly dominated by (m_2,a_1).

Now, agent 1 must also be mixing in equilibrium (else agent 2 will not mix over $\{b_1,b_2\}$). Suppose that in equilibrium agent 1 mixes over only (m_1,a_1) and (m_1,a_2). Then, agent 2 must be playing each of b_1 and b_2 with probability $\frac{1}{2}$ (else agent 1 is not indifferent between his two strategies). But, against this strategy of agent 2, (m_2,a_1) is a strict best response for agent 1.

Hence, in every Nash equilibrium of this game, agent 1 must play (m_2,a_1) with positive probability, and agent 2 must play both b_1 and b_2 with positive probability. Therefore, the outcome $(x,0,0)$ has positive probability.

However, the outcome $(x,0,0)$ provides a payoff $x > 1$ to the principal. Since the payoff from every other outcome is 1, the expected payoff of the principal from any equilibrium of the indirect mechanism strictly exceeds 1. That is, the principal does strictly better with an indirect mechanism than with a direct mechanism. ■

Therefore, in this example, the optimal direct mechanism with deterministic allocations is dominated by an indirect mechanism with deterministic allocations. Hence, stochastic allocations are necessary for the revelation principle to go through. In the example, the principal uses an indirect mechanism to provide agent 1 with private information about allocations, and create uncertainty about allocations for agent 2. This uncertainty, in turn, affects the equilibrium of the agents' effort game, leading to an eventual outcome that is not sustainable in a direct mechanism with deterministic allocations.

Strausz [2003] provides an example to show that, in a setting of pure adverse selection with one principal and two agents it is no longer true that any payoff implementable by a deterministic indirect mechanism can be replicated by a deterministic direct mechanism. In his example, an agent with veto power may veto a direct deterministic mechanism and prefer an indirect one. Our example above shows that, with pure moral hazard and two agents, the principal may strictly prefer a deterministic indirect mechanism to a deterministic direct mechanism.

Next, we provide an example to show that recommendations are necessary to sustain the revelation principle. In their absence, a principal can again do better with an indirect mechanism than with a direct mechanism.

Example 2 (Necessity of recommendations):

Again, $n = 1$ and $k = 2$. As before, $Y = \{y_1, y_2\}$, $E^1 = \{a_1, a_2\}$ and $E^2 = \{b_1, b_2\}$.

The payoffs are given by the following matrices. In each cell, the first element corresponds to the utility of the principal, and the second and third to the utilities of agents 1 and 2 respectively.

$y = y_1$

	b_1	b_2
a_1	$(0,0,10)$	$(50,6,6)$
a_2	$(0,-10,-10)$	$(-10,0,10)$

$y = y_2$

	b_1	b_2
a_1	$(0,0,-10)$	$(-200,0,0)$
a_2	$(0,10,0)$	$(4,1,6)$

Suppose that the principal offers no recommendations (so that $R^1 = R^2 = \emptyset$), but can choose a lottery over allocations, so that $y = py_1 + (1-p)y_2$.

In a direct mechanism, the agents' effort game is as follows:

	b_1	b_2
a_1	$(0,\, 0,\, 20p-10)$	$(250p-200,\, 6p,\, 6p)$
a_2	$(0,\, 10-20p,\, -10p)$	$(4-14p,\, 1-p,\, 4p+6)$

For $p < \frac{5}{7}$, b_2 strictly dominates b_1. We first analyze the equilibrium possibilities that result if the principal chooses p in this range. For $p \leq \frac{1}{7}$, the action a_2 of agent 1 is a best response to b_2, and results in a utility of $(4-14p)$ for the principal. Thus, if the principal induces agent 1 to play a_2, his utility is maximized at $p=0$ at a value of 4. For $p \in [\frac{1}{7}, \frac{5}{7})$, the action a_1 of agent 1 is a best response to b_2, and results in a utility of $(250p-200)$ for the principal. This has a supremum in this range of p at $p=\frac{5}{7}$, and a value of $-\frac{150}{7}$.

Thus, over all $p < \frac{5}{7}$, the maximal payoff the principal achieves in equilibrium is 4, obtained when $p=0$.

When $p=\frac{5}{7}$, for agent 1, a_1 strictly dominates a_2, and agent 2 is indifferent over b_1, b_2. The maximal utility the principal can obtain is 0, when agent 2 plays b_1.

Finally, for $p > \frac{5}{7}$, the unique equilibrium of the agents' subgame is (a_1, b_1), with principal utility being 0.

Hence, the optimal allocation for the principal is y_1 (i.e., choosing $p=0$), with resultant equilibrium (a_2, b_2) in the agents' game. The principal achieves a utility of 4.

Now, consider the following indirect mechanism. The principal communicates with agent 1, with the message space being $M_1 = \{m_1, m_2\}$. Set $M^2 = R^1 = R^2 = \emptyset$. The allocation rule, as in Example 1, is $\tilde{\pi}(m_k) = y_k$ for $k = 1, 2$.

As in example 1, a simultaneous-move game is induced between the agents, and can be represented as follows.

	b_1	b_2
(m_1, a_1)	(0, 0, 10)	(50, 6, 6)
(m_2, a_1)	(0, 0, -10)	(-200, 0, 0)
(m_1, a_2)	(0, -10, -10)	(-10, 0, 10)
(m_2, a_2)	(0,10,0)	(4,1,6)

Notice that (m_2, a_1) and (m_1, a_2) are both strictly dominated by (m_2, a_2). Further, there is no pure strategy equilibrium in this game. The game has the following unique Nash equilibrium:

- Agent 1 mixes between (m_1, a_1) and (m_2, a_2), with probabilities $3/5$ and $2/5$.
- Agent 2 mixes between b_1 and b_2 with probabilities, $1/3$ and $2/3$.

Thus, the principal's expected payoff from the indirect mechanism is $316/15 > 4$. That is, the principal has a higher payoff from the indirect mechanism than is achievable in a direct mechanism.

Allowing for recommendations, we can resurrect the equilibrium of the indirect mechanism in a direct mechanism. A direct mechanism with recommendations in this example may be characterized as a function $\pi : Y \times E^1 \times E^2 \rightarrow [0,1]$, where $\pi(y,a,b)$ is the probability that the principal chooses allocation y and recommends effort a to agent 1 and b to agent 2.

In the equilibrium of the indirect mechanism above, the resultant distribution over allocations and efforts is $\pi(y_1,a_1,b_1) = 1/5$, $\pi(y_1,a_1,b_2) = 2/5$, $\pi(y_2,a_2,b_1) = 2/15$ and $\pi(y_2,a_2,b_2) = 4/15$. Suppose the principal plays this strategy in the direct mechanism. That is, the principal chooses allocations and efforts according to $\pi(\cdot)$, and announces the resulting recommendations to the agents.

It is straightforward to check that neither agent has an incentive to deviate, so the mechanism is incentive compatible. For example, when agent 2 is told "b_2", his posterior beliefs place probability $3/5$ on (y_1,a_1) and $2/5$ on (y_2,a_2). Given these beliefs, b_2 is a (weak) best response. The principal obtains the utility $\frac{316}{15}$, as before. ■

In Example 2, the principal uses an indirect mechanism to communicate privately with agent 1, thereby sustaining a correlated outcome over allocations and efforts. Such correlation can be replicated in the direct mechanism only if the principal sends recommendations.

Thus, even in a pure moral hazard setting, if there are multiple agent, private communication between the principal and an agent can allow the principal to achieve superior outcomes. Therefore, to characterize optimal mechanisms in a setting with complete information and pure moral hazard, it is important to allow for such communication.

This result is potentially relevant in several institutional contexts where multiple agents strategically interact in the presence of moral hazard. We show here that the explicit introduction of a simple form of communication from the principal to the agents could be welfare enhancing. Consider as an example the literature on moral hazard in teams. In a context where first best efficiency is not implementable, this literature has traditionally suggested several devices to improve efficiency: monitoring, yardstick competition, or repetition.[3] Our last example suggests that, in such a context, explicitly allowing the principal to communicate and contract with the agents can constitute a simple instrument to perform this task.

4 Multiple Principals

We consider here the additional difficulties created when there are multiple principal. The principals are now playing a game with each other, and their choices of mechanisms must correspond to a Nash equilibrium of this game. Further, agents' choices of messages and efforts must represent continuation equilibria of the game, given the mechanisms chosen by the principals and recommendations received by the agents.

We first observe that, with multiple principal and stochastic mechanisms, agent's obedience of principals' recommendations is a troublesome notion. An agent may be recommended different actions by different principals. For example, if two principals are both randomizing over recommendations, since principals choose their strategies independently, there is a strictly positive probability that an agent will receive different recommendations from the principals. Given this difficulty, we bypass the issue of agents obeying recommendations received from principals, and require only that, given the strategies of principals and other agents, agents play an equilibrium of the effort game.

Ex ante, this is a complete information game: no participant has a non-trivial type. However, since agents receive private recommendations from principals, agents may have private information when they play the effort game. Hence, in the spirit of perfect Bayesian equilibrium, we require that each agent i plays a best response following any recommendation array $r^i = \left(r^i_1, \ldots, r^i_n\right)$ she may receive.

Recall that a mechanism offered by principal j is defined by (M_j, R_j, π_j). A direct mechanism is defined by $(\emptyset, E, \pi_j)$, where $\pi_j \in \Delta(Y_j \times E)$. If the probabilities over allocations and recommendations in a direct mechanism are independent, we say the recommendations are uncorrelated with allocations.

[3] See d'Aspremont and Gérard-Varet [1998] for an overview.

Definition 1 *In a direct mechanism, a strategy* π_j *of principal* j *has no correlation between recommendations and allocations if there exist marginal densities* $\pi_{y_j} \in \Delta(Y_j)$ *and* $\pi_{e_j} \in \Delta(E)$ *such that* $\pi_j(y,e) = \pi_{y_j}(y)\,\pi_{e_j}(e)$ *for each* $y \in Y_j$ *and* $e \in E$.

Note that even when recommendations are uncorrelated with allocations, principals can still induce a correlated equilibrium in the agents' effort game, since the recommendations are private. In this setting, agents have symmetric information about allocations, so that the context is similar to Aumann [1974].

A special case of recommendations uncorrelated with allocations is when recommendations are deterministic rather than stochastic. For example, suppose that each agent can put in a binary effort, say high or low. In addition, suppose that in equilibrium, each principal recommends that each agent should choose high effort. Then, recommendations are deterministic, and regardless of allocation strategies, satisfy our definition of being uncorrelated with allocations.

Let $\Gamma_{\mathcal{D}}$ be the direct mechanism game among the principals. In this game, each principal j chooses a direct mechanism $\pi_j \in \Delta(Y_j \times E)$ at stage 1 (see Figure 1). Let $\Gamma_{\mathcal{G}}$ be the indirect mechanism game, in which each principal j chooses (M_j, R_j, π_j), where (with a slight abuse of notation) $\pi_j : M_j \to \Delta(Y_j \times R_j)$.

In an equilibrium of either $\Gamma_{\mathcal{D}}$ or $\Gamma_{\mathcal{G}}$, we require that (i) each principal plays a best response, given other principals' strategies and agents' strategies, and (ii) each agent i plays a best response for every recommendation array r^i she may receive, given principals' strategies and other agents' strategies.

We now consider the robustness of equilibria in direct mechanisms to the possibility that a principal may offer an indirect communication scheme instead. When there are multiple principal, the issue of possible multiple equilibria in the agents' continuation game is particularly vexing.

Suppose, for example, each principal j offers a direct mechanism $(\emptyset, E, \pi_j)$, and a continuation equilibrium δ_1 is played in the agents' effort game. Now, suppose principal 1 alone deviates to an indirect mechanism $(M_1, R_1, \tilde{\pi}_1)$, which leads to another continuation equilibrium δ_2 in the agents' effort game. To show that the original equilibrium in direct mechanisms remains an equilibrium, we need to show that principal 1 can induce the same continuation equilibrium δ_2 via a direct mechanism.

Suppose, in fact, principal 1 offers a direct mechanism $(\emptyset, E, \hat{\pi}_1)$ to induce the agents to achieve δ_2 as an equilibrium. This new direct mechanism may well induce other equilibria in the agents' game. In particular, suppose there is another continuation equilibrium δ_3 which leads to a worse outcome for principal 1 than in the original setting, in which he offered $(\emptyset, E, \pi_1)$, and agents played the continuation equilibrium δ_1.

If there is a single principal, incentive compatibility would ensure that agents obey the recommendation of the principal. However, as mentioned above, with multiple principal, an agent may receive conflicting recommendations from different principals. Thus, while it is a best response for agents to obey principal 1 in the new direct mechanism, they may instead coordinate on the other equilibrium. Suppose agents play equilibrium δ_2 when principal 1 offers the indirect mechanism $(M_1, R_1, \tilde{\pi}_1)$, but coordinate on δ_3 if principal switches to the corresponding direct mechanism $(\emptyset, E, \hat{\pi}_1)$. Then, the deviation to an indirect mechanism may be strictly beneficial for principal 1.

Note that this issue is relevant even when there is a single agent. Of course, with a single agent, multiple equilibria in the agent's continuation game is equivalent to the agent being indifferent between two actions at that stage. With a single agent (as in, for example, Peters [2003]), this issue can be resolved in at least one of two ways: (i) by assuming that the agent is never indifferent across any two actions, given the mechanisms she is offered by the principals,[4] or (ii) by requiring that a principal can induce the desired continuation action from the agent in the event of a deviation at the mechanism design stage. With multiple agent, (i) is not an option, since there may still be multiple equilibria in the agents' continuation game. We therefore define (ii) explicitly.

Definition 2 *An equilibrium (π^*, δ^*) of the direct mechanism game satisfies the obedient deviations property if π_j^* remains a best response for principal j when, by making a unilateral deviation, principal j could induce all agents to follow his recommendations if it is incentive compatible for them to do so.*

This property is a natural extension to multiple principal of the notion of incentive compatibility in a single-principal model. Even with only one principal, a mechanism

[4] For a discussion on indifference across actions and its impact on the revelation principle in common agency games with moral hazard, see Attar et al. [2006] and Peters [2006].

may induce multiple equilibria in the agents' effort game, and coordination on a particular equilibrium is achieved via incentive compatibility.

We show that, with multiple principal, equilibria of the direct mechanism game that exhibit the two properties defined above, recommendations uncorrelated with allocations and obedient deviations, survive possible deviations by a principal to an indirect mechanism. Formally,

Theorem 1 *Suppose the direct mechanism game $\Gamma_{\mathcal{D}}$ has an equilibrium (π^*,δ^*) that satisfies the obedient deviations property, and in which, for each principal j, π_j^* has no correlation between allocations and recommendations. Then, in the indirect mechanism game $\Gamma_{\mathcal{G}}$, it remains an equilibrium for each principal j to offer the mechanism $(\emptyset,E,\pi_j^*)$ and for each agent i to play δ^{i*}. Thus, the joint distribution over allocations and efforts that obtains in the equilibrium of the direct mechanism game remains an equilibrium outcome of the indirect mechanism game.*

Proof.

Consider the game $\Gamma_{\mathcal{G}}$. Suppose that, in this game, every principal j offers a mechanism $\left(M_j,R_j,\pi_j\right) = (\emptyset,E,\pi_j^*)$, where π_j^* is his equilibrium strategy in the direct mechanism game $\Gamma_{\mathcal{D}}$. It is immediate that $\delta^* = \times_{i=1}^k \delta^{i*}$ must remain a continuation equilibrium in the agents' efforts game.

Hence, we need only to show that no principal j' has an incentive to unilaterally deviate from the mechanism $\left(\emptyset,E,\pi_{j'}^*\right)$. Suppose, therefore, that some principal j' has an incentive to deviate to $\left(\tilde{M}_{j'},\tilde{R}_{j'},\tilde{\pi}_{j'}\right) \neq \left(\emptyset,E,\pi_j^*\right)$, while all other principals $j \neq j'$ continue to offer mechanisms $\left(\emptyset,E,\pi_j^*\right)$. Suppose the agents play $\left(\tilde{\mu},\tilde{\delta}\right)$ in response to these mechanisms, and the mechanisms and the agents' effort strategies $\tilde{\delta}$ induce a (possibly correlated) distribution over allocations y and efforts e. Let $\tilde{\nu}\left(y,e\right)$ denote this distribution. Since principal j' has an incentive to deviate to the indirect mechanism, his utility from such a deviation, $V_{j'}(\tilde{\nu}(y,e))$ must exceed his utility from the equilibrium of the direct mechanism.

Now, every principal $j \neq j'$ is using recommendations uncorrelated with allocations. Since each agent i observes only the mechanisms, his own message $m_{j'}^i$ to principal j', and his own recommendation array $r^i = (e_1^i,\ldots,r_{j'}^i,\ldots,e_n^i)$, the efforts chosen must remain uncorrelated with the allocations of principals $j \neq j'$. Hence, we can write

$\tilde{\nu}(y,e) = \tilde{\nu}_{j'}(y_{j'},e) \cdot \prod_{j \neq j'} \pi^*_{y_j}(y_j)$, where $\pi^*_{y_j}(\cdot)$ is the marginal distribution over the allocations of a principal $j \neq j'$, given his strategy π^*_j.

The remainder of the proof replicates the arguments in Myerson [1982] for the single-principal case. It is now straightforward for principal j' to induce the same joint distribution in the direct mechanism game. Rather than play the strategy $\pi^*_{j'}$, he plays the strategy $\tilde{\nu}_{j'}$. Since this strategy induces the same joint distribution over efforts and allocations as in the continuation equilibrium of the indirect mechanism game, it must be a best response for each agent i to obey the recommendation of principal j', and to ignore the recommendations of the others (else agent i would have a profitable deviation in the indirect mechanism game, rather than playing $\tilde{\delta}$). But if every agent i obeys the recommendation of principal j', and principal j' plays $\tilde{\nu}_{j'}(y,e)$ in the direct mechanism game, the same joint distribution over allocations and efforts is induced as in the indirect mechanism game. Hence, if principal j' has a profitable deviation in the indirect mechanism game, he has a profitable deviation in the direct mechanism game as well, contradicting the assumption that (π^*, δ^*) satisfies the obedient deviations property. ■

The obedient deviation and no correlation properties both play critical roles in our construction. As mentioned earlier, without the obedient deviation property, a multiple equilibrium problem may result, with agents co-ordinating on one equilibrium in the direct mechanism game, and on another one if a principal deviates to an indirect mechanism. Similarly, suppose, in some equilibrium of $\Gamma_{\mathcal{D}}$, a principal $\tilde{j}$ offers recommendations correlated with his allocations. If some other principal j were to deviate, the resulting distribution over allocations and efforts may continue to exhibit such correlation. Thus, principal j alone cannot replicate the correlation with his own strategy.

Standard models of moral hazard with multiple principal or agent typically do not consider communication. Instead, principals offer just allocations, and agents take a non-contractible effort. We call such a game a "game without recommendations."

If the equilibrium outcomes generated in a game without recommendations were replicable in direct mechanism games with recommendations, with strategies that satisfied the conditions of our theorem, we would be confident that no principal could gain by a unilateral deviation to an indirect mechanism. In principle, checking robustness to every feasible deviation to a direct mechanism with recommendations can be a complicated task. However, we show that it is sufficient to only consider those deviations by a principal that induce obedient behavior by the agents in the continuation game.

In a game without recommendations, let $\sigma_j \in \Delta(Y_j)$ denote the (mixed) strategy of principal j, and let $\rho^i : \prod_{j=1}^k \Delta(Y_j^i) \to \Delta(E^i)$ denote the strategy of agent i. Let $\sigma = \times_{j=1}^n \sigma_j$, and $\rho = \times_{i=1}^k \rho^i$.

Theorem 2 *Consider an equilibrium (σ^*, ρ^*) of a game without recommendations. Suppose, following any unilateral deviation by some principal $\tilde{j}$ to a direct mechanism in $\Delta(Y_{\tilde{j}} \times E)$, there does not exist a continuation equilibrium in which agents obey the recommendations received from principal $\tilde{j}$, and principal $\tilde{j}$ has a higher expected payoff. Then, the equilibrium outcome induced by (σ^*, ρ^*) remains an equilibrium outcome of $\Gamma_{\mathcal{G}}$.*

Proof.

Consider an equilibrium of the game without recommendations. Suppose the condition in the statement of the corollary is satisfied, and no principal $\tilde{j}$ can gain by a unilateral deviation to a direct mechanism in $\Delta(Y_{\tilde{j}} \times E)$, even if, in the continuation equilibrium, agents were to obey the recommendations received from principal $\tilde{j}$.

Then, it is straightforward to construct strategies (π^*, δ^*) in the game $\Gamma_{\mathcal{D}}$ that constitute an equilibrium of $\Gamma_{\mathcal{D}}$ and replicate the outcome of the equilibrium in the game with no recommendations. For example, let $\pi_j = \sigma_j^* \times e_1^1 \times \cdot \times e_1^k$ for each principal j. That is, each principal offers the allocation lottery σ_j and the recommendation array $(e_1^i, \cdots, e_1^k)$ to the agents. Set $\delta^i(\pi_1^i, \ldots, \pi_n^i) = \rho^{i*}(\sigma_j^i, \ldots, \sigma_j^n)$ for each i. Then, (π, δ) constitutes an equilibrium of $\Gamma_{\mathcal{D}}$.

By construction, π satisfies the property that recommendations are uncorrelated with allocations for each principal. By assumption, (π, δ) satisfies the obedient deviations property. Therefore, from Theorem 1, it remains an equilibrium in $\Gamma_{\mathcal{G}}$ for each principal j to offer a direct mechanism π_j, and for each agent i to play δ^i.

However, (π, δ) induces the same distribution over terminal payoffs as (σ^*, ρ^*). Thus, the outcome induced by (σ^*, ρ^*) remains an equilibrium outcome of $\Gamma_{\mathcal{G}}$. ■

Theorem 2 identifies a methodology to evaluate the robustness of equilibrium outcomes in multi-principal, multi-agent models of moral hazard to the introduction of communication. It shows that there is no loss of generality in considering principals' unilateral deviations to recommendation mechanisms, and ignoring mechanisms in which a principal asks agents to send messages to the principal. Importantly, it is sufficient to analyze incentive compatible (i.e., obedient) behavior by the agents at the deviation stage, since only one principal deviates to a mechanism with recommendations. That is, we only need to consider deviations by each principal involving recommendations that are followed by agents in the continuation game. Even in a multi-principal context where the notion of obedience is not helpful to characterize equilibria, there is therefore a rationale for considering incentive compatibility at the deviation stage of a game without recommendations.

Our results cannot be straightforwardly extended to games with incomplete information. The intuition is the following: even if recommendations are uncorrelated with allocations, a recommendation from principal j to agent i may communicate information about the type of some other agent i'. This may lead to a correlation between agents' efforts and principals' allocations, which is difficult for a single principal to replicate in a direct mechanism.

5 Discussion

In a recent paper, Peters [2004] provides two thought-provoking examples in a setting with two principals and two agents who are taking some non contractible effort. His first example suggests that "In a multiple agency environment [...] pure strategy equilibria are not robust against the possibility that principals might deviate to more complex indirect mechanisms".[5] We note that Peters restricts attention to deterministic allocations with no recommendations. By introducing lotteries over allocations in his example, one can recover the robustness of direct mechanism equilibria.[6]

The second example shows that a "no externality" assumption [see Peters, 2003] sufficient to imply the revelation principle in a multi-principal, single agent context fails to do so when there are many agents. This example clearly demonstrates the problems created by multiple equilibria in the agents' continuation game. When one principal offers

[5]Peters [2004, p. 184].

[6]Details are available from the authors on request.

an indirect mechanism, agents play one equilibrium of their effort game. If the principal deviates to an "equivalent" direct mechanism, agents play a different equilibrium. Thus, the outcome of an indirect mechanism cannot be replicated in direct mechanisms.

Consider our obedient deviations condition in the light of Peters' second example. This condition is exactly an attempt to ensure that the same equilibrium is played by the agents when a principal who was offering an indirect mechanism instead offers a corresponding direct mechanism. Since we do not impose a no externality condition, we cannot recover a revelation principle. Instead, we focus on whether equilibria in direct mechanisms are robust to a principal deviating to an indirect mechanism.

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