Who abstains in equilibrium?*

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Abstract

We study abstention when each voter selects the quality of information. We introduce conflict among committee members using two dimensions of heterogeneity: ideology and concern. In equilibrium, 1) voters collect information of different qualities, 2) there are informed voters that abstain, and 3) information and abstention need not be inversely correlated for all voters. The existence of an equilibrium in which voters collect information of different quality does not follow from a straightforward application of fixed point arguments. Instead of looking for a fixed point in the (infinite) space of best response functions, we construct a transformation with domain in a suitable finite-dimensional space. We show that differences in the level of concern are crucial in determining whether abstention occurs in equilibrium and why models that assume away this dimension can not capture the positive correlation between information and abstention.

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1 Introduction

1.1 Motivation

Although many organizations rely on committees for decision-making, we lack a complete understanding of how these bodies operate. In particular, while in many committees like electorates, corporate board, etc., members do not always vote, our understanding of abstention is relatively narrow. At the same time, a common feature behind the variety of committees we observe is that in order to make an informed decision, committee members must *collect* and *interpret* information. The literature provides results about the strategic interaction in committees when information is exogenously given and communication among members is not allowed.[1] However, the process of information gathering and its impact on the final decision has been relatively neglected.

While some voters decide to stay home during an election, other voters decide to attend the booth and abstain selectively on some issues. For example, in the United States, even though national and local elections often share the same ballot, the number of votes cast in each usually differs (Blais (2000)). The first phenomenon, "absenteeism", is usually studied assuming that voting is costly[2] while the second one, usually referred to as "roll-off", cannot be explained by assuming that the act of voting is costly when the voter is already in the booth.

Feddersen and Pesendorfer (1996) provide an explanation for abstention based on the level of information that a voter exogenously receives.[3] They argue that uninformed voters can rely on their peers for a better decision: abstention is a method of delegation when a voter is poorly informed. In this sense, abstention might not damage the committee's effectiveness since only votes that do not carry much information are not being counted. Abstention is an endogenous technology that improves the

[1] See Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997) and Feddersen and Pesendorfer (1998).

[2] See Piketty (1999), Grossman and Helpman (2002) or Dhillon and Peralta (2002) for a survey. See Downs (1957) and Riker and Ordeshook (1968) for seminal *calculus of voting models*, and Palfrey and Rosenthal (1983), Ledyard (1984) and Palfrey and Rosenthal (1985) for seminal contributions on *endogenous pivotal probabilities*. Borgers (2004), Castanheira (2002), Lockwood and Ghosal (2004) and Krishna and Morgan (2005) are more modern studies.

[3] See also Feddersen and Pesendorfer (1999) and Kim and Fey (2006). Matsusaka (1995), Ghirardato and Katz (2006), and Larcinese (2005) are decision theoretic models that introduce differences in the priors in order to create heterogeneity. Shotts (2005) uses abstention as a signalling device to affect the outcome in a second election.

average information conveyed by each cast vote.

Feddersen and Pesendorfer (1996) insight about roll-off being an informational phenomenon raises the question of how and when is information acquired: without a theory of information acquisition in committees is not possible to fully understand abstention.[4] In this paper we build that bridge and answer a simple question: *Who abstains?* We construct a model of committees and study the equilibrium incentives to abstain when each committee member can select the precision of the signal she will use to decide her vote. We assume a common values set up but we introduce conflict (heterogeneity) in the committee.

We show that endogenous information affects the way we should think about abstention. Abstention is driven by indifference between candidates and this indifference is not completely determined by lack of information when information is endogenous. Contrary to previous results we show that the correlation between information and abstention depends on the voter's type. In our model, there are some voters that abstain with higher probability the less informed they are but there are also other voters that *abstain with higher probability the more informed they are*. The positive correlation between information and abstention found in empirical papers (Coupé and Noury (2004) and Lassen (2005)) is derived by performing a test that does not condition on the voter's ideology. The fact that some highly informed voters decide to abstain questions the effectiveness of abstention as a way endogenously select the more informative votes.

1.2 Information Acquisition, Ideology and Concern

Information acquisition in committees has recently become an important issue in the literature.[5] Persico (2004) shows that optimal committees with homogenous members are finite when information comes from a common source and its cost is fixed. Li (2001), Gerardi and Yariv (2004) and Gershkov and Szentes (2004) show that in order to generate incentives for information acquisition in committees with homogenous members, the committee's decision rule may need to inefficiently aggregate information. Cai (2002) allows for heterogeneity that may create incentives for information

[4]Endogenous information as an effect on abstention is used in George and Waldfogel (2002) and Gentzkow (2006). Benz and Stutzer (2004) find correlation between information and "political participation rights"; they suggest that information is endogenous and is inversely correlated with pivotal probabilities.

[5]See Gerling et al. (2003) for a survey.

gathering. Finally, Martinelli (2006) study information aggregation, showing that the shape of the cost function for information acquisition plays an important role.[6]

All these models share an equilibrium feature: every informed member holds the same quality of information.[7] This result directly relates to their assumption about preferences: they all assume a particular level of conflict by restricting preferences to differ only on the ideological level.[8] This restricted conflict, or heterogeneity among members, results in all members having the same incentives to collect information.

We model conflict in our committee by assuming two dimensional preferences and we allow committee members to be heterogenous in both dimensions. The first dimension captures the traditional ideological diversity while the second dimension allows committee members to differ also in terms of their level of concern about possible outcomes.

Why do we need the extra dimension? Traditionally, preferences in committees are modeled with a single parameter: a relative ranking of alternatives (ideology) is enough to describe all the incentives to vote. This restricted heterogeneity captures the relevant conflict at the voting stage. This approach ignores the fact that incentives to acquire information depend on the absolute level of utility losses that a voter suffers for mistaken decisions: there are voters with the same ideology that collect information of different precision depending how much they care about possible mistakes. Therefore, in order to capture that voters endogenously collect information of different qualities we need to introduce this extra dimension: when preferences differ in terms of the levels of concern all incentives to collect information are unleashed. This allows us to fully understand information acquisition and, therefore, abstention.

We show that restricting preferences to be single dimensional is not innocuous when information is endogenous. Some strategies that are optimal in the model with richer conflict are strictly dominated for all members when restrictions on preferences are assumed. If those strategies that are now dominated use abstention as part of an optimal voting rule, restricted models fail to capture abstention as an equilibrium behavior. Therefore, restricting preferences may give misleading characterizations of

[6]Feddersen and Sandroni (2004) incorporate information acquisition in a model with ethical voters and study its aggregation properties.

[7]Li (2001) provides an example with a very particular type of heterogeneity in a two-member committee in which he derives differences in collected information.

[8]Moreover, Persico (2004), Gerardi and Yariv (2004) and Gershkov and Szentes (2004) assume that every committee member who acquires information receives a signal drawn from a *common* distribution.

abstention.

1.3 Information Acquisition and Heterogeneity

We allow each committee member to select the precision of the signal to be received, with increased precision entailing an increased marginal cost. Together with the specification of preferences, this implies that information will be unequally distributed among voters: there are heterogeneously informed voters. This contrasts with the standard endogenous-information voting models, in which the equilibrium yields homogeneously-informed voters.

Our characterization of equilibrium is simple and intuitive, and its geometric properties are presented in such a way that all the main forces are easy to understand and compare. Five classes of voters emerge in the voting stage: *strong partisans* and *weak partisans* for each option, and *abstainers*. Strong partisans do not collect information and always vote for the same option; weak partisans collect information and vote in favor of their most preferred candidate only if the information supports this candidate; abstainers do not collect any information and they never vote.

These groups of voters are ordered on the ideological spectrum. Strong partisans are the most biased voters. Their ideological preference is so extreme that the precision of the information they are willing to collect is not enough to make them change their mind. Weak partisans are biased but not as much as strong partisans. They are willing to collect information in order to confirm they should vote in favor of their ideologically preferred candidate. If the information goes against their ideology they are not willing to support any candidate. Abstainers are moderate voters and the information they are willing to collect is not good enough for them to decide their vote based on that information.

Note that information can be poor either because it is too costly or because voters are not willing to collect much of it (low level of concern). Therefore, a sixth group of voters might emerge in equilibrium: moderate voters that are willing to collect a highly precise signal and are willing to vote according to this signal. We call these voters *independents*. Because behaving as an independent requires a certain minimum level of information, the cost of information acquisition is crucial for this behavior to be optimal. The existence of independents depends on the type of equilibrium that emerges.[9]

In our model rational ignorance takes two different forms: extremists and moderate voters who are not willing to collect information of high precision. On the other hand, information collected differ among groups. Independents and weak partisans are not homogeneously informed: in the voting stage there is a continuum of types of voters.

The existence of an equilibrium with voters endogenously collecting information of different quality does not follow from a straightforward application of fixed point arguments. First, the quality of information may be a discontinuous mapping of the preference parameters, even among voters who decide to collect information. The best response function is only a C^0 function (almost everywhere) which precludes the application of fixed point arguments for infinite dimensional spaces.[10] Second, because behaving as an independent might not be optimal, the equilibrium characterization takes very different forms and fixed point arguments need to keep track of all these forms.

We overcome this difficulty by exploiting the geometric properties of the equilibrium. Instead of looking for a fixed point in the infinite-dimensional space of best response functions, we characterize the equilibrium and use the best response functions to construct a transformation with domain in a suitable finite-dimensional space. Since the best response functions are embedded in this transformation, we can show that a fixed point of this transformation is an equilibrium of the game.

The rest of the paper is organized as follows. In the next section we present the model and section (3) presents the main characterization and existence results. In section (4) we focus on the simple majority rule and discuss the incentives to abstain and the importance of our assumption about preferences. Conclusions are provided in the last section.

2 The model

There are n potential voters that must decide between two options A and Q; there are two equally likely states of nature $\omega \in \{a, q\}$. The set of possible actions for a

[9] Although our characterization is unique there are different classes of equilibria that depend on the existence of independents, and how likely it is that a voter is independent.

[10] See Rudin (1973), in particular, the equicontinutity requirement in Schauder's Fixed Point Theorem.

voter is $\mathbf{X}$=$\{Q, \varnothing, A\}$ where Q (A) is a vote for candidate Q (A) and $\varnothing$ stands for abstention. We refer to a generic decision rule as $\mathbf{R}$.

2.1 Voters

There are two classes of voters: **responsive** and **stubborn**. Stubborn voters are described in terms of their behavior: with probability $\xi_x \in (0,1)$, a stubborn voter is type $x \in\mathbf{X}$ in which case she votes for option $x \in\mathbf{X}$, where $\sum_{x\in\mathbf{X}} \xi_x = 1$. Responsive voters have contingent preferences described by $\theta = \{\theta_q, \theta_a\} \in [0,1]^2$: if A (Q) is selected in state q (a) then the voter type $\theta = \{\theta_q, \theta_a\}$ suffers a utility loss of θ_q (θ_a) and there is no utility loss for selecting A (Q) in state a (q). We refer to responsive voter i's preferences as her type, and to a "responsive voter type θ" simply as a "type θ". Voter's preferences are private information. With probability $\alpha \in (0,1)$ a voter i is stubborn and with the remaining probability the voter is responsive. If the voter is responsive her preferences are drawn independently from a distribution with cumulative distribution function F on $[0,1]^2$ with no mass points. We assume that F and α are common knowledge.

Definition 1 *A* ***symmetric committee*** *is characterized by* $\xi_A = \xi_Q < \frac{1}{2}$, *and* $F(x,y) = F(y,x)$ *for all* $(x,y) \in [0,1]^2$.

After knowing their types, each voter i can select the precision of the information they will receive: $p \in \left[\frac{1}{2}, 1\right]$ where p is the parameter of a Bernoulli random variable S that takes values on the set $\{s_q, s_a\}$. We assume that the probability of signal $s = s_\omega$ in state $\omega \in \{a, q\}$ is equal to p: $\Pr(s_\omega \mid p, \omega) = p$ for $\omega \in \{a, q\}$. The precision cost is given by $C : \left[\frac{1}{2}, 1\right] \to \mathcal{R}_+$ where we assume that:

Assumption 1 *The cost function* C *is twice continuously differentiable everywhere in* $\left[\frac{1}{2}, 1\right]$ *and satisfies 1)* $C'(p) > 0$ *and* $C''(p) > 0$ *for all* $p > \frac{1}{2}$, *2)* $C''\left(\frac{1}{2}\right) \geq C\left(\frac{1}{2}\right) = C'\left(\frac{1}{2}\right) = 0$, *3)* $\lim_{p\to 1} C'(p) \to \infty$.

2.2 Decision rule

The sum of votes in favor of A and Q (effective votes) is now a random variable that depends on how many players decide to abstain. The decision rule in a committee in which abstention is allowed is a contingent decision rule. Define m as the number of effective votes in the committee; implicitly $m \in \{0, 1...n\}$ is a function of the strategies used by all players, where n is the number of potential voters.

Let T_n^m as the number of votes in favor of A when there are m effective votes and n potential votes. We describe the decision rule as a function $N(m)$ for $m \in \{0, 1, ...n\}$, with $N(m) \geq \frac{m}{2}$,[11] and a tie breaking rule (r, r_0). For all $m \in \{0, 1...n\}$, A wins if $T_n^m > N(m)$ and Q wins if $T_n^m < N(m)$. If $T_n^m = N(m)$ and $m \geq 1$, then A wins with probability $1 - r$, and Q wins with probability r. If all n voters abstain, A is selected with probability $1 - r_0$ and Q is selected with probability r_0.[12]

Assumption 2 ***1)*** *"Smooth monotonicity": $0 \leq N(m+1) - N(m) \leq 1$, and* ***2)*** *"Non triviality": $N(m) \leq m$.*

Assumption (2) ensures that, given the other votes, an incremental vote in favor of A (Q) can never decrease the likelihood that A (Q) wins.[13] The second part precludes rules that require quorum for A to win: to change the status quo a minimum number of voters is required.[14]

Assumption 3 *If the decision rule is such that $N(m) \neq m$ for some $m \in \{0, 1, ...n\}$, then $r_0 \in (0, 1)$, and if $N(m) = m$ for all $m \in \{0, 1, ..n\}$, then $r_0 > 0$ and $r < 1$.*

When the decision rule is not the unanimity decision rule, both Q and A have some chance of winning if all n voters abstain and, when the decision rule is the unanimity decision rule, A has some chance of winning if there are some effective votes and Q has some chance of winning if there are no effective votes. We are in position to define more formally the type of committees we analyze in this paper:

Definition 2 *A committee with abstention is a* ***regular committee*** *if 1) the decision rule* ***R****= $(N(m), r, r_0)$ satisfies the assumptions (2) and (3), 2) there are $n \geq 2$ members whose preferences are determined by the probability of being stubborn $\alpha \in$*

[11] The case with $N(m) < \frac{m}{2}$ can be studied by inverting the roles of Q and A.

[12] Fixing the tie breaking rule for all $m \geq 1$ simplifies expressions, but also plays a role for abstention to be an equilibrium behavior for some types. It can be easily replaced by some other assumption like $r_m \in (0, 1)$.

[13] If $N(m+1) > N(m) + 1$, when there are m effective votes, and $N(m) - 1$ votes for A, another vote for A makes it even harder for A to be the winner. On the other hand, if $N(m+1) < N(m)$, the same situation occurs for Q.

[14] The characterization and existence results are not affected by requiring quorum.

$(0,1)$, *stubborn behavior given by* $(\xi_A, \xi_Q, \xi_\varnothing) \in (0,1)^3$ *with* $\xi_\varnothing = 1 - \xi_A - \xi_Q$, *and a type drawn for each responsive voter from a distribution* F *on* $[0,1]^2$ *with no mass points, and 3) each member information acquisition cost is characterized by the cost technology* C *satisfying assumption (1).*

2.3 Strategies and equilibrium

Since voters decide the precision of the signal and how they vote after receiving it, strategies are composed of two elements.

Definition 3 *A pure strategy of responsive voter* i *is an investment rule* $P^i : [0,1]^2 \to \left[\frac{1}{2}, 1\right]$ *and a voting rule* $V^i : [0,1]^2 \times \{s_q, s_a\} \to X$, *such that* $P^i(\theta)$ *is the investment level of responsive voter* i *with type* θ, *and* $V^i(\theta, S) = (V^i(\theta, s_q), V^i(\theta, s_a))$ *is the contingent voting rule used by responsive voter* i *with type* θ *who receives the signal* $s \in \{s_q, s_a\}$.[15]

When we refer to a generic voting rule, investment rule or strategy, we omit the superscript indicating types.

The voting rule $V(\theta, S)$ is an ordered pair, where the first (second) element describes how the player votes after receiving $s = s_q$ $(s = s_a)$. [16] We will refer to a profile of strategies as $\left(\widetilde{P}, \widetilde{V}\right)$ where $\widetilde{P} = (P^1, ...P^n)$ and $\widetilde{V} = (V^1, ...V^n)$ are the profile of investment rules and voting rules for the whole committee. Analogously $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$ is the profile of strategies for all players but player i. We will say that, if $V^i(\theta, s) = v$ for all $s \in \{s_q, s_a\}$ player i of type θ uses an **uninformed** voting rule, and if $V^i(\theta, s_q) \neq V^i(\theta, s_a)$ player i of type θ uses an **informed** voting rule. We therefore identify strategies by the voting rule they employ.

The timing of the game is as follows: 1) Nature selects the profile of types, 2) Each player i observes her own type (stubborn or responsive) and preferences, 3) responsive player i privately decides whether or not to acquire information by selecting

[15] The reader may argue that voting rules should be contingent in the level of investment performed by each voter so $V^i : [0,1]^2 \times \left[\frac{1}{2}, 1\right]^2 \times \{s_q, s_a\} \to \mathbf{X}$. This approach substantially complicates the model without affecting any of the results. That results are unaffected follows by the fact that between the investment decision and voting decision no other public information is revealed to the voters.

[16] $V(\theta, S)$ describes the voter's behavior and $v_q v_a \in \mathbf{X}^2$ is notation to describe arbitrary strategies (vote v_q after receiving s_q and vote v_a after receiving s_a) . When we want to refer to a particular vote we use just v.

$p^i \in \left[\frac{1}{2}, 1\right]$, 4) each player draws a private signal from the selected distribution parameterized by p^i, 5) players vote after signals are observed and the winner is elected according to the rule $\mathbf{R}$.

Conditional on the profile of strategies of all voters but i, $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$, we define the probability that the winner is $x \in \{Q, A\}$ in state $\omega \in \{q, a\}$, when voter i casts vote $v \in \mathbf{X}$, as

$$\Pr\left(x \mid \omega, v, \left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)\right) \tag{1}$$

The expected utility of player i of type $\theta \in [0,1]^2$ when she votes $v \in \mathbf{X}$, and the state is $\omega \in \{q, a\}$, is

$$u^i\left(v \mid \theta, \omega\right) \equiv -\theta_\omega \Pr\left((-\omega) \mid \omega, v, \left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)\right) \tag{2}$$

where we let $(-\omega) = Q$ if $\omega = a$ and $(-\omega) = A$ if $\omega = q$. This expression is just the product of the disutility of a mistake $(-\theta_\omega)$ and the probability of a mistake in the state $\omega \in \{q, a\}$, given vote v. We define the expected utility of player i of type $\theta \in [0,1]^2$ and investment choice $p \in \left[\frac{1}{2}, 1\right]$, when she votes $v \in \mathbf{X}$ after receiving the signal $s \in \{s_q, s_a\}$ as

$$U^i\left(p, v \mid \theta, s\right) \equiv \sum_{\omega \in \{q,a\}} u^i\left(v \mid \theta, \omega\right) \Pr\left(\omega \mid s, p\right) \tag{3}$$

Using (3), the gross expected utility of player i of type $\theta \in [0,1]^2$ and investment choice $p \in \left[\frac{1}{2}, 1\right]$, for a voting rule (v_q, v_a) is

$$\mathcal{U}^i\left(p, (v_q, v_a)) \mid \theta\right) \equiv \sum_{x \in \{q,a\}} \frac{U^i\left(p, v_x \mid \theta, s_x\right)}{2} \tag{4}$$

where we used Bayes rule and the fact that both states are equally likely for $\Pr\left(s_\omega\right) = \frac{1}{2}$.

Voting decisions are based on expression (3). Investment decisions are based on expression (4) which aggregates over realizations of expression (3). Although we omit other players' strategies in definitions (2), (3) and (4), the reader should understand that player i's payoffs depend on $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$.

We study symmetric Bayesian equilibria in pure strategies.

Definition 4 *A symmetric Bayesian equilibrium for the voting game with decision* *rule **R** and voting alternatives **X** is a strategy* $(P^*(\theta), V^*(\theta, S))$ *such that:* ***1)*** *for all* $j = 1, ...n$, $V^j(\theta, S) = V^*(\theta, S)$ *and* $P^j(\theta) = P^*(\theta)$ *for every* $\theta \in [0,1]^2$ *,**2)** for every* $\theta \in [0,1]^2$, *for all* $s \in \{s_q, s_a\}$, *and for any other feasible* $v' \in$***X****, the strategy* $(P^*(\theta), V^*(\theta, S))$ *satisfies*

$$U^i(P^*(\theta), V^*(\theta, s) \mid \theta, s) \geq U^i(P^*(\theta), v' \mid \theta, s) \quad (5)$$

and ***3)*** *for every* $\theta \in [0,1]^2$, *and for any other feasible* (v_q, v_a) *and* p, *the strategy* $(P^*(\theta), V^*(\theta, S))$ *satisfies*

$$\mathcal{U}^i(P^*(\theta), V^*(\theta, S) \mid \theta) - C(P^*(\theta)) \geq \mathcal{U}^i(p, (v_q, v_a) \mid \theta) - C(p) \quad (6)$$

Inequality (5) is a consistency condition (the player follows the plan) and inequality (6) is an optimization condition (the player gets the highest expected utility). [17] From now on, we omit the strategy profile of all other players as an argument of endogenous variables.

The probability that an arbitrary voter $j \neq i$ votes for $v \in$**X**, in state ω, when all other players but i are using the strategy $(P(\theta), V(\theta, S))$ is

$$\Pr(v \mid \omega) = (1-\alpha) \int\limits_{\theta \in [0,1]^2} \sum_{s \in \{s_q, s_a\}} I(V(\theta, s) = v) \Pr(s \mid P(\theta), \omega) \, dF(\theta) + \alpha \xi_v \quad (7)$$

The first part of the right side is just the probability that a voter is responsive multiplied by the probability that a responsive voter votes for $v \in$**X**. The second part is the probability that a voter is stubborn, multiplied by the probability that a stubborn member votes for $v \in$**X**. This expression aggregates over the two sources of private information present in the model: the type of player and the signal received after investment.

[17] Since $(P^*(\theta), V^*(\theta, S))$ maximize expected utility type by type, it also maximizes ex-ante expected utility

$$\int\limits_{\theta \in [0,1]^2}^{i} \mathcal{U}^i(P^*(\theta), V^*(\theta, S) \mid \theta) \, dF(\theta)$$

3 Solving the Model

3.1 Voting Incentives

Let $\Pr(x \mid \omega, v)$ be the probability of a particular outcome $x \in \{Q, A\}$, in state ω, after player i cast a vote $v \in \mathbf{X}$.[18] Define the change in the probability of A winning when voter i switches her vote from $X \in \{Q, \varnothing\}$ to A in state ω as,

$$\Delta \Pr(\omega, X) \equiv \Pr(A \mid \omega, A) - \Pr(A \mid \omega, X) \tag{8}$$

$\Delta \Pr(\omega, Q)$ and $\Delta \Pr(\omega, \varnothing)$ are not the only expressions that reflect how chances of A winning change when a voter switches votes. Indeed, if the voter switches her vote from Q to $\varnothing$, A's chances of winning will also increase. That term can be described by $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \varnothing)$, for $\omega \in \{q, a\}$.

When abstention is allowed not necessarily all voters cast a positive vote. This creates some difficulty in order to show that "pivotal probabilities" are positive. For example, imagine a committee with only two members that decide under the unanimity decision rule. Player 1 switching the vote from abstention to A, is only relevant when player 2 does not vote. To see this note that, if player 2 votes for Q, Q is the sure winner while if player 2 votes for A, there is no change in the winner if player 1 votes for A or abstains. Now assume the tie breaking rule states that A wins if there are no votes ($r_0 = 0$). In this case, player 1 switching the vote from $\varnothing$ to A does not affect the winner. This implies that $\Delta \Pr(\omega, \varnothing) = 0$, A and $\varnothing$ are equivalent and endogenous abstention is ruled out from the beginning. Assumption (3) deals with this situation.

Lemma 1 *In any regular committee,* $\Delta \Pr(\omega, Q)$*,* $\Delta \Pr(\omega, \varnothing)$ *and* $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \varnothing)$ *are positive for each* $\omega \in \{q, a\}$*.*

Proof. Before proving the lemma we need to define some objects. Assume that every player but i use the strategy $\left(\widetilde{P}, \widetilde{V}\right)$ and player i uses (P^i, V^i).

Let $\Pr(T_n^m = l \mid \omega)$ be the probability that there are m effective votes out of n possible voters, and exactly l of the m positive votes are in favor of A in state

[18] As the reader suspects $\Pr(x \mid \omega, v)$ is a combination of $\Pr(v \mid \omega)$, for $v \in \mathbf{X}$, $x \in \{Q, A\}$ and $\omega \in \{q, a\}$. We provide the result below.

$\omega \in \{a, q\}$. For $l \leq m \leq k$ we have

$$\Pr\left(T_k^m = l \mid \omega\right) \equiv \frac{k!}{l!\,(m-l)!\,(k-m)!} \Pr\left(Q \mid \omega\right)^{m-l} \Pr\left(A \mid \omega\right)^l \Pr\left(\varnothing \mid \omega\right)^{k-N} \tag{9}$$

Using the definitions of $\Pr\left(A \mid \omega, v\right)$ for $v \in \{Q, A, \varnothing\}$

$$\begin{aligned} \Delta \Pr\left(\omega, Q\right) &= \Pr\left(T_{n-1}^0 = 0 \mid \omega\right) + r \sum_{m=1}^{n-1} \Pr\left(T_{n-1}^m = N\left(m+1\right) \mid \omega\right) \\ &\quad + (1-r) \sum_{m=1}^{n-1} \Pr\left(T_{n-1}^m = N\left(m+1\right) - 1 \mid \omega\right) \end{aligned}$$

$$\begin{aligned} \Delta \Pr\left(\omega, \varnothing\right) &= \Pr\left(T_{n-1}^0 = 0 \mid \omega\right) r_0 + r \sum_{m=1}^{n-1} \Pr\left(T_{n-1}^m = N\left(m\right) \mid \omega\right) I_{N(m+1)}^{N(m)} \\ &\quad + (1-r) \sum_{m=1}^{n-1} \Pr\left(T_{n-1}^m = N\left(m\right) - 1 \mid \omega\right) I_{N(m+1)}^{N(m)} \end{aligned}$$

where we use that, $N\left(m+1\right) \neq N\left(m\right)$ implies $N\left(m+1\right) = N\left(m\right) + 1$ and where $I_x^y = 1$ if $x = y$ and 0 otherwise. Recalling that $\Pr\left(v \mid \omega\right) \geq \alpha \xi_v$ for $v \in \{Q, A, \varnothing\}$, it is straightforward to see that, if $n \geq 2$ then $\Delta \Pr\left(\omega, Q\right) \geq \zeta_2\left(\omega\right)$, $\Delta \Pr\left(\omega, \varnothing\right) \geq \zeta_3\left(\omega\right)$ and $\Delta \Pr\left(\omega, Q\right) - \Delta \Pr\left(\omega, \varnothing\right) \geq \zeta_4\left(\omega\right)$ if $r_0 \in (0, 1)$, for some $\zeta_i\left(\omega\right) > 0$, $i = 2, 3, 4$. If $N\left(m\right) = m$, the result follows because $r < 1$ and $r_0 > 0$. ■

Using the definition of expected utility in (4) and equation (5), a necessary condition for a responsive voter with preferences $\theta \in [0, 1]^2$ to vote for A after receiving the signal $s \in \{s_q, s_a\}$ is

$$\frac{\theta_q}{\theta_a} \frac{\Pr\left(q \mid s, p\right)}{\Pr\left(a \mid s, p\right)} \leq \min\left\{ \frac{\Delta \Pr\left(a, Q\right)}{\Delta \Pr\left(q, Q\right)}, \frac{\Delta \Pr\left(a, \varnothing\right)}{\Delta \Pr\left(q, \varnothing\right)} \right\} \tag{10}$$

and a necessary condition for her to vote for Q is

$$\frac{\theta_q}{\theta_a} \frac{\Pr\left(q \mid s, p\right)}{\Pr\left(a \mid s, p\right)} \geq \max\left\{ \frac{\Delta \Pr\left(a, Q\right)}{\Delta \Pr\left(q, Q\right)}, \frac{\Delta \Pr\left(a, Q\right) - \Delta \Pr\left(a, \varnothing\right)}{\Delta \Pr\left(q, Q\right) - \Delta \Pr\left(q, \varnothing\right)} \right\} \tag{11}$$

Strict inequalities give sufficient conditions.

It is immediate to see that the set of uninformed voters with type θ using $V(\theta, s_a) \neq V(\theta, s_q)$ have probability zero. Therefore, only uninformed strategies with $V(\theta, s_a) = V(\theta, s_q)$ and informed strategies with $P(\theta) > \frac{1}{2}$ and $V(\theta, s_a) \neq V(\theta, s_q)$, need to be studied. Under which conditions is abstention an optimal action for a responsive voter?

Lemma 2 *A necessary condition for abstention to be part of an optimal strategy for some responsive voter θ in any regular committee is*

$$\frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \geq \frac{\Delta \Pr(a, \varnothing)}{\Delta \Pr(q, \varnothing)} \tag{12}$$

Proof. The condition $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \geq \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$ is equivalent to $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \leq \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)}$. Assume then that inequality (12) does not hold. Then (10) and (11) become

$$\frac{\Pr(q \mid s, p)}{\Pr(a \mid s, p)} \leq \frac{\theta_a}{\theta_q} \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \leq \frac{\Pr(q \mid s, p)}{\Pr(a \mid s, p)}$$

which implies that either for almost all types, a positive vote, either for A or Q, is preferred to abstaining. ■

Recalling that a voting rule is a pair (v_q, v_a) where $v_\omega \in \{Q, A, \varnothing\}$ for $\omega \in \{q, a\}$, there are 9 possible voting rules. Six of them may be part of an informed strategy: AQ, $A\varnothing$, QA, $\varnothing A$, $\varnothing Q$, and $\varnothing Q$.[19] Some of them can not be optimal with positive probability.

Lemma 3 *In any regular committee, strategies that use the voting rules AQ, $A\varnothing$ or $\varnothing Q$ are not optimal for almost all types. Moreover, if abstention occurs with positive probability, then there are no types that use these voting rules.*

Proof. We will show the proof for the case $A\varnothing$; the cases $\varnothing Q$ and AQ are analogous.

If a responsive voter uses $A\varnothing$, (10) gives

$$\frac{\Pr(q \mid s_q, p)}{\Pr(a \mid s_q, p)} \leq \frac{\theta_a}{\theta_q} \min\left\{\frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)}, \frac{\Delta \Pr(a, \varnothing)}{\Delta \Pr(q, \varnothing)}\right\} \leq \frac{\Pr(q \mid s_a, p)}{\Pr(a \mid s_a, p)}$$

[19] We simplify notation: the ordered pair $v_q v_a$ describes the strategy of voting v_q after receiving s_q and voting v_a after receiving s_a.

Therefore, we must have that $\frac{\Pr(q|s_q,p)}{\Pr(a|s_q,p)} \leq \frac{\Pr(q|s_a,p)}{\Pr(a|s_a,p)}$ which is a contradiction since $\Pr(\omega \mid s_\omega, p) > \Pr(\omega \mid s_{-\omega}, p)$ for $p > \frac{1}{2}$. If $p = \frac{1}{2}$, it is optimal only for types that satisfy $\frac{\theta_q}{\theta_a} = \min\left\{\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}, \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}\right\}$.

Assume now that abstention occurs with positive probability. Using (11) we require that $\frac{\theta_q}{\theta_a} \geq \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$ which is a contradiction since $\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} > \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$.
■

Now we need to consider only six strategies with different voting rules that may occur in equilibrium with positive probability. Voters can be separated in six different groups: **strong partisans for each candidate** ($\mathcal{SP}^A$ for A, and $\mathcal{SP}^Q$ for Q), **weak partisans for each candidate** ($\mathcal{WP}^A$ for A, and $\mathcal{WP}^Q$ for Q), **abstainers** ($\mathcal{A}$) and **independents** ($\mathcal{I}$). Weak partisans for A (Q) vote for A (Q) if $s = s_a$ ($s = s_q$) and abstain if $s = s_q$ ($s = s_a$) while strong partisans for A (Q) vote for A (Q) without collecting information. Abstainers abstain no matter the signal received and independents collect information and follow the signal they receive.

3.2 Information acquisition

It is straightforward to see that abstainers do not invest,[20] while the probability that a type uses a weak partisan's strategy without performing any investment is 0. Now there are three investment rules for each group that collects information (independents and weak partisans for A and Q). We define

Definition 5 *Let $P^x : [0,1]^2 \to \left[\frac{1}{2}, 1\right]$ for $x \in \{QA, \varnothing A, Q\varnothing\}$ be such that $P^{\varnothing A}(\theta)$, $P^{Q\varnothing}(\theta)$ and $P^{QA}(\theta)$ are the investment rule of weak partisans for A, weak partisans for Q, and independents, respectively.*

When abstention is not allowed voters collect information of different qualities because of differences in preferences (θ). When abstention is allowed different informed strategies make different use of the information collected. All information collected by independents reach the electorate, while weak partisans that abstain hold some information back from the electorate.

[20]Otherwise, if investment were positive, either abstention after any signal $s \in \{s_q, s_a\}$ is not optimal, or payoffs could improve by saving on information acquisition.

Using (4) for each of the possible optimal strategies with investment and the information technology, we derive the optimal investment policy implicitly as:

$$\begin{aligned} C'\left(P^{XA}(\theta)\right) &= \sum_{\omega\in\{q,a\}} \theta_\omega \frac{\Delta \Pr(\omega, X)}{2}, X \in \{Q, \varnothing\} \\ C'\left(P^{Q\varnothing}(\theta)\right) &= C'\left(P^{QA}(\theta)\right) - C'\left(P^{\varnothing A}(\theta)\right) \end{aligned} \tag{13}$$

Since $\lim_{p\to 1} C'(p) \to \infty$, there is some $\eta < 1$ such that $P^x(\theta) \leq \eta$ for all informed voting rules with $x \in \{QA, \varnothing A, Q\varnothing\}$. The second equation in (13) illustrates that a player type θ using the voting rule QA collects more information than she would have collected if she were a weak partisan. It is worth noticing that the restriction of P to the domain $[0,1]^2$ is not needed. This will play an important role when we show that an equilibrium exists.

For the independent behavior to be optimal, the level of investment required must be high. The next lemma states formally that whenever there are incentives to abstain, independents must invest a positive amount so the precision of information must be strictly bigger than $\frac{1}{2}$.

Lemma 4 *In any regular committee a necessary condition for the independent behavior to be optimal with investment level p, is*

$$\left(\frac{p}{1-p}\right)^2 \geq \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(a,\varnothing)} \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} \tag{14}$$

Moreover, if there is endogenous abstention with positive probability ((12) holds with strict inequality) independents must invest a strictly positive amount.

Proof. Using the optimal conditions for voting, (10) and (11), we have that it is necessary for independents that

$$\frac{\Pr(a \mid s_q, p)}{\Pr(q \mid s_q, p)} \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} \leq \frac{\theta_q}{\theta_a} \leq \frac{\Pr(a \mid s_a, p)}{\Pr(q \mid s_a, p)} \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$$

Using that $\frac{\Pr(q|s_q,p)}{\Pr(a|s_q,p)} = \frac{\Pr(a|s_a,p)}{\Pr(q|s_a,p)} = \frac{p}{1-p}$, it is necessary that

$$\frac{1-p}{p} \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} \leq \frac{\theta_q}{\theta_a} \leq \frac{p}{1-p} \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$$

which gives (14). Now assume that there is endogenous abstention with positive probability. Lemma (2) gives that (12) holds with strict inequality, and therefore $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} > \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$. Using (14) gives that $\left(\frac{p}{1-p}\right)^2 > 1$ and, $p > \frac{1}{2}$ is necessary. ■

Therefore, there are no independents close to the type $(0,0)$.This creates some technical problems when we prove existence of equilibrium: there can be very different classes of equilibria and the characterization depends on "how many" independents are.

Assume that θ_a and θ_q are low so there is little investment in information. If they are about equal, the risk of introducing noise in the electorate plus the cost of investment is too high. Since preferences are balanced, the responsive voter prefers delegating to the electorate rather than voting for one or the other candidate with very weak evidence: being an abstainer is a better strategy than being independent because it saves on investment. When θ_a and θ_q are further apart, the argument is valid for the signal that favors the candidate the voter is biased against: abstention when that signal is received must be preferred to any positive vote. Basically the signal does not convey enough evidence to overturn the bias. Therefore, a weak partisan strategy is better than being independent. We discussed earlier when θ_a and θ_q are very different, strong partisanship dominates independence.

3.3 Characterization and existence of equilibrium

It is common to see in the literature existence results before characterizations results. In order for us to be able to follow that strategy, our best responses must behave well enough. In particular our investment functions should belong to a equicontinuous family of real functions (see Rudin (1973)). We know that the investment functions are not even continuous so we are forced to develop a new strategy in order to show existence. We first characterize the equilibrium and then use its geometric properties to actually show that there is one. Our strategy is motivated by a parametric approximation of functions: instead of looking in a space of functions we look in the space of parameters that define a function of that space.[21]

In a generic setting, players (say player $i = 0$) derive (expected) utility (U) from

[21] The idea can be traced back to finding a value function in recursive problems by approximating the function using a simplified parametrization. We do not suffer this approximation/simplification problem because we are able to fully characterize that space.

their behavior (b_0), their types (θ_0), and the other players' behavior ($\{b_i\}_{i=1}^n$). A best response is a function b_0 that verifies $U(b_0;\theta,\{b_i\}_{i=1}^n) \geq U\left(\widetilde{b};\theta,\{b_i\}_{i=1}^n\right)$ for all θ and all feasible $\widetilde{b}$. A symmetric equilibrium is a function b^* such that $U(b^*;\theta,\{b^*\}_{i=1}^n) \geq U\left(\widetilde{b};\theta,\{b^*\}_{i=1}^n\right)$. Note that, actually a best response depends on the player's type and the other players strategies: $b_0(\theta;\{b_i\}_{i=1}^n)$. In our model b_0 and U do not behave well enough to apply traditional fixed point arguments. In our model $\{b_i\}_{i=1}^n$ affect utility of player 0 indirectly through a set of "market" parameters, $\{x_1,x_2\}$: that is there is a function G such that $G(\{b_i\}_{i=1}^n) = \{x_1,x_2\}$ and $U(b;\theta,\{b_i\}_{i=1}^n) = \widetilde{U}(b;\theta,\{x_1,x_2\})$. These "market" parameters uniquely defined the best response function through the optimization process $b_0(\theta;\{x_1,x_2\}) = \arg\max_b \widetilde{U}(b;\theta,\{x_1,x_2\})$. We prove that a pair $\{x_1,x_2\}$ gives a unique best response function, and that a set of other player's behavior ($\{b_i\}_{i=1}^n$) gives a unique pair $\{x_1,x_2\}$.

We first characterize the best response function in terms of the parameters $\{x_1,x_2\}$: $b^*(\theta;\{x_1,x_2\}) = \arg\max_b \widetilde{U}(b;\theta,\{x_1,x_2\})$. Then we use this best response function, now parametrized by $\{x_1,x_2\}$ to construct $\widetilde{G}(\{x_1,x_2\}) = G(\{b^*(\theta;\{x_1,x_2\})\}_{i=1}^n)$. We prove then that a pair $\{x_1^*,x_2^*\}$ such that $\widetilde{G}(\{x_1^*,x_2^*\}) = \{x_1^*,x_2^*\}$ is an equilibrium with best response functions determined by $b^*(\theta;\{x_1^*,x_2^*\})$. It is important to note that by characterizing $b^*(\theta;\{x_1,x_2\})$ we can avoid the infinite dimensional space of real functions and search for an equilibrium in the finite dimensional space of parameters that define our characterized best response functions.

3.3.1 Characterization

In order to formally describe the equilibrium we need to define cutoff functions that separate types according to the strategy they use. There are six possibly optimal strategies which implies that a particular type θ must perform 15 comparisons in order to decide which strategy to use. Fortunately, there are some strategies that do not intersect. For example, the strategy AA and QQ requires consistency conditions that rule each other out: if a voter is considering AA so (10) holds for $s \in \{s_a,s_q\}$ then (11) does not hold for $s \in \{s_a,s_q\}$. This reduces the number of comparisons to 10. Each cutoff function will de described by a superscript.

Let $v_q v_a$ and $v'_q v'_a$ be a pair of voting rules such that $v_\omega \in \{A,Q,\varnothing\}$ for $\omega \in \{q,a\}$, $v'_\omega \in \{A,Q,\varnothing\}$ for $\omega \in \{q,a\}$. Using the expression for expected utilities (4), an uninformed strategy that always uses $v_q = v_a = v$ for $v \in \{Q,A,\varnothing\}$ gives expected utility

$$\mathcal{U}^i\left(\frac{1}{2},(v_q,v_a)\mid\theta\right)=-\frac{\theta_a\Pr(d=Q\mid a,v)+\theta_q\Pr(d=A\mid q,v)}{2}$$

while an informed strategy with $v_q\neq v_a$ gives expected utility

$$\begin{aligned}\mathcal{U}^i\left(P^{v_qv_a}(\theta),(v_q,v_a)\mid\theta\right)-C\left(P^{v_qv_a}(\theta)\right) &= C'\left(P^{v_qv_a}(\theta)\right)P^{v_qv_a}(\theta)\\ &\quad-\frac{\theta_a\Pr(d=Q\mid a,v_q)+\theta_q\Pr(d=A\mid q,v_a)}{2}\end{aligned}$$

Using this expression for every pair v_qv_a and $v'_qv'_a$ we can define the function $g^j(\theta_a)$ implicitly by

$$\begin{aligned}&\mathcal{U}^i\left(P^{v_qv_a}(\theta),v_qv_a\mid\theta\right)-C\left(P^{v_qv_a}(\theta)\right)\\ =\ &\mathcal{U}^i\left(P^{v'_qv'_a}(\theta),v'_qv'_a\mid\theta\right)-C\left(P^{v'_qv'_a}(\theta)\right)\end{aligned}$$

where j corresponds to the cutoff function for the strategies that use the voting rule v_qv_a and $v'_qv'_a$.

For example, the function $g^3:\mathcal{R}\to\mathcal{R}$ is such that the type $\left(g^3(\theta_a),\theta_a\right)\in[0,1]^2$ is indifferent between the strategy that uses the voting rule $Q\varnothing$ and QA; g^3 is implicitly defined by

$$\begin{aligned}&\mathcal{U}^i\left(P^{(Q,\varnothing)}\left(g^3(\theta_a),\theta_a\right),Q\varnothing\mid g^3(\theta_a),\theta_a\right)-C\left(P^{(Q,\varnothing)}\left(g^3(\theta_a),\theta_a\right)\right)\\ =\ &\mathcal{U}^i\left(P^{(Q,A)}\left(g^3(\theta_a),\theta_a\right),QA\mid g^3(\theta_a),\theta_a\right)-C\left(P^{(Q,A)}\left(g^3(\theta_a),\theta_a\right)\right)\end{aligned}$$

Figure (1) shows which numbers correspond to which pair of strategies. For example, number 5 is the superscript that identifies the cutoff function for types that are indifferent between the strategy that uses the voting rule $Q\varnothing$ and the strategy that uses the voting rule $\varnothing A$. In Appendix A we present relations between g^i, $i\in\{1,2,...10\}$ that are used in the characterization.[22]

Three important comments are in order. First, these functions are defined beyond $[0,1]^2$. Second, we cannot show that, $g_1^{10}(\theta_a)$ (a function that maps $\theta_a\in[0,1]$ into $\theta_q\in[0,1]$) or $g_2^{10}(\theta_q)$ (a function that maps $\theta_q\in[0,1]$ into $\theta_a\in[0,1]$) always exist. Nevertheless, we can show that, at least one of them exists and, when both are

[22] The proofs involve the use of the implicit function theorem repeatedly and some algebraic manipulations. Details are provided in Oliveros (2006).

Number	Strategy 1	Strategy 2
1	*QQ*	*QØ*
2	*QQ*	*QA*
3	*QØ*	*QA*
4	*QØ*	*ØØ*
5	*QØ*	*ØA*
6	*ØA*	*ØØ*
7	*ØA*	*QA*
8	*AA*	*QA*
9	*AA*	*ØA*
10	*ØØ*	*QA*

Figure 1: Number assigned to cut off functions according to the strategies that yield the same expected utilities.

properly defined, they are each other's inverse: $g_2^{10}\left(g_1^{10}\left(x\right)\right) = x$. Third, contrary to all other cases, it may be that $g_1^{10}\left(\theta_a\right) > 1$ (or $g_2^{10}\left(\theta_q\right) > 1$) for all $\theta_a \in [0,1]$ (or $\theta_q \in [0,1]$). In that case, being an abstainer is always better than following an independent behavior.

Using the cutoff functions described previously, we can define the set of strong partisans as[23]

$$\begin{aligned} \mathcal{SP}^A &\equiv \left\{\theta \in [0,1]^2 : \theta_q \leq \min\left\{g^9\left(\theta_a\right), g^8\left(\theta_a\right)\right\}\right\} \\ \mathcal{SP}^Q &\equiv \left\{\theta \in [0,1]^2 : \theta_q \geq \max\left\{g^1\left(\theta_a\right), g^2\left(\theta_a\right)\right\}\right\} \end{aligned}$$

Strong partisans are located where $\frac{\theta_a}{\theta_q}$ is extremely low or extremely high.

The sets of weak partisans are defined as:

$$\begin{aligned} \mathcal{WP}^A &\equiv \left\{\theta \in [0,1]^2 : \min\left\{g^7\left(\theta_a\right), g^6\left(\theta_a\right)\right\} \geq \theta_q, \theta_q > g^9\left(\theta_a\right)\right\} \\ \mathcal{WP}^Q &\equiv \left\{\theta \in [0,1]^2 : g^4\left(\theta_a\right) \leq \theta_q < g^1\left(\theta_a\right), \theta_a \leq g^3\left(\theta_q\right)\right\} \end{aligned}$$

Weak partisans for A (Q) are located exactly above (below) strong partisans for A (Q).

[23] Since its measure is zero we can assign types that are indifferent to any of the groups that provides the same expected utility.

The case of independents and abstainers is more delicate because they are separated by the function $g_1^{10}(\theta_a)$ or $g_2^{10}(\theta_q)$ depending on which one is properly defined. We define the set of abstainers $\mathcal{A}$, when $1 \geq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ as

$$\mathcal{A} \equiv \left\{\theta \in [0,1]^2 : g^6(\theta_a) < \theta_q < g^4(\theta_a), \theta_q \leq g_1^{10}(\theta_a)\right\}$$

while if $1 < \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ the set of abstainers $\mathcal{A}$ is defined by

$$\mathcal{A} \equiv \left\{(\theta_q,\theta_a) \in [0,1]^2 : g^6(\theta_a) < \theta_q < g^4(\theta_a), \theta_a \leq g_2^{10}(\theta_q)\right\}$$

Independents are defined as the complement of all these groups in $[0,1]^2$. If $1 \geq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$, independents are

$$\mathcal{I} \equiv \left\{ \begin{array}{c} \theta \in [0,1]^2 : \theta_q > \max\left\{g^7(\theta_a), g^8(\theta_a)\right\} \\ g^2(\theta_a) > \theta_q > g_1^{10}(\theta_a), \theta_a > g^3(\theta_q) \end{array} \right\}$$

while if $1 < \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$, independents are

$$\mathcal{I} \equiv \left\{ \begin{array}{c} \theta \in [0,1]^2 : \theta_q > \max\left\{g^7(\theta_a), g^8(\theta_a)\right\}, g^2(\theta_a) > \theta_q \\ , \theta_a > \max\left\{g^3(\theta_q), g_2^{10}(\theta_q)\right\} \end{array} \right\}$$

Proposition 1 *In any regular committee the strategy $(P^*(\theta), V^*(\theta,S))$ with*

1. *the investment rule $P^*(\theta)$ that prescribes $P^{\varnothing A}(\theta)$ for $\theta \in \mathcal{WP}^A$, $P^{Q\varnothing}(\theta)$ for $\theta \in \mathcal{WP}^Q$, $P^{QA}(\theta)$ for $\theta \in \mathcal{I}$, and $P^*(\theta) = \frac{1}{2}$ otherwise,*

2. *and the voting rule $V^*(\theta,S)$ that prescribes the uninformative behavior $\varnothing\varnothing$ for $\theta \in \mathcal{A}$, XX for $\theta \in \mathcal{SP}^X$ with $X \in \{Q,A\}$, and the informative behavior $\varnothing A$ for $\theta \in \mathcal{WP}^A$, $Q\varnothing$ for $\theta \in \mathcal{WP}^Q$, and QA for $\theta \in \mathcal{I}$,*

is a symmetric Bayesian equilibrium.

Proof. Along the proof, "consistency" refers to (10) and (11) with the proper use of s_a and s_q. "Optimality", on the other hand, refers to the proper expression of (6). It must be clear that inconsistent strategies can not be optimal. All proofs regarding properties for the cutoffs functions are provided in Oliveros (2006); they are not mathematically demanding but just the application of the implicit function theorem and some manipulations of the proper terms.

First we are going to prove that the strategies are consistent and optimal. Then we are going to show that they actually cover all the space of types without intersecting each other.

Strong partisans

Since every pair with $\theta \in \mathcal{SP}^A$ satisfies $\theta_q \leq \min\{g^9(\theta_a), g^8(\theta_a)\}$ we must have that $\varnothing A$ and QA are not optimal by definition of $g^9(\theta_a)$ and $g^8(\theta_a)$. Using that $g^9(\theta_a) < \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}\theta_a$, the strategies that involve voting rule QQ (inequality (11)) and $\varnothing\varnothing$ (converse of inequality (10)) are not consistent for $\theta \in \mathcal{SP}^A$.Recalling (11), consistency of $Q\varnothing$ requires

$$\frac{\theta_q}{\theta_a} \geq \frac{P^{Q\varnothing}(\theta_q,\theta_a)}{1-P^{Q\varnothing}(\theta_q,\theta_a)}\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} \geq \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$$

which does not hold since $g^9(\theta_a) < \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}\theta_a$.

For $\theta \in \mathcal{SP}^Q$, it holds that $\theta_q \geq \max\{g^1(\theta_a), g^2(\theta_a)\}$ which implies that QA and $Q\varnothing$ are not optimal either by definition of $g^1(\theta_a)$ and $g^2(\theta_a)$. Using $g^1(\theta_a) > \theta_a\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$, the converse of inequality (11) gives that $\varnothing\varnothing$ is not consistent for $\theta \in \mathcal{SP}^Q$ and $g^2(\theta_a) > \theta_a\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}$ with (10) gives that AA is not consistent for $\theta \in \mathcal{SP}^Q$. Now recalling that consistency of $\varnothing A$ requires

$$\frac{\theta_q}{\theta_a} \leq \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}\frac{1-P^{\varnothing A}(\theta_q,\theta_a)}{P^{\varnothing A}(\theta_q,\theta_a)}$$

which does not hold since $\theta_q > g^1(\theta_a)$ and $g^1(\theta_a) > \theta_a\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$.

It remains to see if $\mathcal{SP}^A$ and $\mathcal{SP}^Q$ are using consistent strategies. Using that $g^9(\theta_a) < \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}\theta_a$ and $g^8(\theta_a) < \frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}\theta_a$ we get the result for $\mathcal{SP}^A$; $g^1(\theta_a) > \theta_a\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$ and $g^2(\theta_a) > \theta_a\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}$ give the result for $\mathcal{SP}^Q$.

Weak Partisans.

Let $\theta \in \mathcal{WP}^A$ which implies that $\min\{g^7(\theta_a), g^6(\theta_a)\} \geq \theta_q$. By definition of $g^7(\theta_a)$ we have that QA is not optimal and by definition of $g^6(\theta_a)$ we have that $\varnothing\varnothing$ is not optimal. Since $g^7(\theta_a) < \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$, (11) gives that QQ is not consistent. Using (2), $g^4(\theta_a) \geq g^7(\theta_a)$, it must be that $g^4(\theta_a) > \theta_q$ so $Q\varnothing$ is worse than $\varnothing\varnothing$ by definition of $g^4(\theta_a)$ and since $\varnothing A$ is better than $\varnothing\varnothing$, we have that $\varnothing A$ is preferred to $Q\varnothing$. By definition of $g^9(\theta_a)$, $\varnothing A$ is preferred to AA.

Let $\theta \in \mathcal{WP}^Q$ so $g^4(\theta_a) \le \theta_q$ and it follows directly that $Q\varnothing$ is preferred to $\varnothing\varnothing$ by definition of $g^4(\theta_a)$. At the same time, $\theta_a \le g^3(\theta_q)$ gives directly that it is also better than QA by definition of $g^3(\theta_q)$. Since $\frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(a,\varnothing)} > \frac{g^3(\theta_q)}{\theta_q}$, the uninformative strategy AA is not consistent (see (10)).

Using that $\theta_a \le g^3(\theta_q)$ implies that $\theta_q > g^6(\theta_a)$ (by relation (1)) we have that $\varnothing\varnothing$ is preferred to $\varnothing A$ (by definition of $g^6(\theta_a)$) and since $Q\varnothing$ is preferred to $\varnothing\varnothing$ (by definition of $g^4(\theta_a)$), it must be that $Q\varnothing$ is also preferred to $\varnothing A$. By definition of $g^1(\theta_a)$ we get that $Q\varnothing$ is preferred to QQ.

Consistency of the voting rule $\varnothing A$ follows by the properties

$$\begin{aligned}
\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)} \frac{1 - P^{\varnothing A}(\theta_a, g^9(\theta_a))}{P^{\varnothing A}(\theta_a, g^9(\theta_a))} &< \frac{g^9(\theta_a)}{\theta_a} \\
\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} \frac{1 - P^{\varnothing A}(\theta_a, g^7(\theta_a))}{P^{\varnothing A}(\theta_a, g^7(\theta_a))} &> \frac{g^7(\theta_a)}{\theta_a} \\
\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)} \frac{P^{\varnothing A}(\theta_a, g^6(\theta_a))}{1 - P^{\varnothing A}(\theta_a, g^6(\theta_a))} &> \frac{g^6(\theta_a)}{\theta_a}
\end{aligned}$$

and consistency of $Q\varnothing$ follows by the properties

$$\begin{aligned}
\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} \frac{1 - P^{Q\varnothing}(\theta_a, g^4(\theta_a))}{p^{Q\varnothing}(\theta_a, g^4(\theta_a))} &< \frac{g^4(\theta_a)}{\theta_a} \\
\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} \frac{P^{Q\varnothing}(\theta_a, g^1(\theta_a))}{1 - P^{Q\varnothing}(\theta_a, g^1(\theta_a))} &> \frac{g^1(\theta_a)}{\theta_a} \\
\frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(a,\varnothing)} \frac{1 - P^{Q\varnothing}(g^3(\theta_q), \theta_q)}{P^{Q\varnothing}(g^3(\theta_q), \theta_q)} &> \frac{g^3(\theta_q)}{\theta_q}
\end{aligned}$$

Abstainers.

The constraint that $\theta_q \in (g^6(\theta_a), g^4(\theta_a))$ ensures that either $g_1^{10}(\theta_a)$ or $g_2^{10}(\theta_q)$ is well defined as proven in Oliveros (2006).

Using $\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} > \frac{g^4(\theta_a)}{\theta_a}$ and $\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)} < \frac{g^6(\theta_a)}{\theta_a}$, we have that AA and QQ are not consistent by (10) and (11), respectively. By definition of $g^6(\theta_a)$, the relation $g^6(\theta_a) < \theta_q$ implies that $\varnothing\varnothing$ is preferred to $\varnothing A$; the same argument applies for $\theta_q < g^4(\theta_a)$ which ensures that $\varnothing\varnothing$ is preferred to $Q\varnothing$. Now assume that $1 \ge \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$ and recall that $\theta_q \le g_1^{10}(\theta_a)$ which implies that $\varnothing\varnothing$ is preferred to QA by definition of $g_1^{10}(\theta_a)$. On the other hand, if $1 \le \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$ the definition of $g_2^{10}(\theta_q)$ gives that all types that satisfy $\theta_a \le g_2^{10}(\theta_q)$ prefers the uninformed strategy with $\varnothing\varnothing$ to the informed strategy with QA.

Consistency of $\varnothing\varnothing$ follows by $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} > \frac{g^4(\theta_a)}{\theta_a}$ and $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} < \frac{g^6(\theta_a)}{\theta_a}$ which reverse the inequalities (10) and (11).

Independents.

If there are no independents we are done, so let $g_1^{10}(\theta_a) < 1$ for some $\theta_a \leq 1$ or $g_2^{10}(\theta_q) < 1$ for some $\theta_q \leq 1$ when appropriate. The condition $\theta_q > \max\{g^7(\theta_a), g^8(\theta_a)\}$ gives that QA is preferred to $\varnothing A$ and AA by definition of $g^7(\theta_a)$ and $g^8(\theta_a)$ respectively. By definition of $g^3(\theta_q)$ and $g^2(\theta_a)$, if $\theta_a > g^3(\theta_q)$ we have that QA is preferred to $Q\varnothing$ and if $g^2(\theta_a) > \theta_q$ we have that QA is preferred to QQ. If $1 \geq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ by definition of $g_1^{10}(\theta_a)$ we have that QA is preferred to $\varnothing\varnothing$. The case $1 \leq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ follows by the same arguments.

Consistency of QA follows because (11) for $s = s_q$ is verified by $\theta \in \mathcal{I}$ since

$$\begin{aligned} \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} &< \frac{P^{QA}(g^7(\theta_a),\theta_a)}{1-P^{QA}(g^7(\theta_a),\theta_a)}\frac{g^7(\theta_a)}{\theta_a} \\ \frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} &< \frac{P^{QA}(g^8(\theta_a),\theta_a)}{1-P^{QA}(g^8(\theta_a),\theta_a)}\frac{g^8(\theta_a)}{\theta_a} \end{aligned}$$

while (10) for $s = s_a$ is verified by $\theta \in \mathcal{I}$ since

$$\begin{aligned} \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} &> \frac{\theta_q}{g^3(\theta_q)}\frac{1-P^{QA}(\theta_q,g^3(\theta_q))}{P^{QA}(\theta_q,g^3(\theta_q))} \\ \frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} &> \frac{1-P^{QA}(g^2(\theta_a),\theta_a)}{P^{QA}(g^2(\theta_a),\theta_a)}\frac{g^2(\theta_a)}{\theta_a} \end{aligned}$$

Now we are going to show that it actually covers all types in $[0,1]^2$ without intersecting each other.

Intersection of voters

Since each uninformed strategy is consistent for those types that use it, it is clear that: $\mathcal{SP}^A \cap \mathcal{SP}^Q = \varnothing$, $\mathcal{SP}^A \cap \mathcal{A} = \varnothing$ and $\mathcal{A} \cap \mathcal{SP}^Q = \varnothing$.

Since weak partisans for A satisfy $\theta_q > g^9(\theta_a)$ and strong partisans for A satisfy that $\theta_q \leq \min\{g^9(\theta_a), g^8(\theta_a)\}$, we have that $\mathcal{SP}^A \cap \mathcal{WP}^A = \varnothing$; the same holds for $\mathcal{SP}^Q$ and $\mathcal{WP}^Q$ since the former satisfy $\theta_q \geq \max\{g^1(\theta_a), g^2(\theta_a)\}$ and the latter $\theta_q < g^1(\theta_a)$. Using (1) if there is a type θ with $\theta_q \leq g^9(\theta_a)$ ($\mathcal{SP}^A$) it is also true that $\theta_q \leq g^6(\theta_a)$ and that $\theta_a \geq g^3(\theta_q)$ so it can be that $\theta_q \in \mathcal{WP}^Q$ because it is necessary that $\theta_a < g^3(\theta_q)$. Using (2) if $g^7(\theta_a) > \theta_q$ ($\mathcal{WP}^A$) it must hold that $g^1(\theta_a) > \theta_q$ and $\mathcal{WP}^A \cap \mathcal{SP}^Q = \varnothing$.

Since $\mathcal{SP}^A$ satisfy $\min\{g^9(\theta_a), g^8(\theta_a)\} \geq \theta_q$ and $\mathcal{SP}^Q$ satisfy $\theta_q \geq \max\{g^1(\theta_a), g^2(\theta_a)\}$, the fact that $\mathcal{I}$ satisfy $g^2(\theta_a) > \theta_q$ and $\theta_q > g^8(\theta_a)$ is enough to show that $\mathcal{I} \cap \mathcal{SP}^A = \varnothing$ and $\mathcal{I} \cap \mathcal{SP}^Q = \varnothing$.

Since $\mathcal{WP}^A$ satisfy $\min\{g^7(\theta_a), g^6(\theta_a)\} \geq \theta_q$ and $\mathcal{WP}^Q$ satisfy $g^4(\theta_a) \leq \theta_q$, using the relation (2) $g^4(\theta_a) \geq g^7(\theta_a)$, we get $\mathcal{WP}^Q \cap \mathcal{WP}^A = \varnothing$.

$\mathcal{A} \cap \mathcal{WP}^A = \varnothing$ follows by the fact that $\mathcal{A}$ satisfies $g^6(\theta_a) < \theta_q$ and $\mathcal{WP}^A$ satisfies $\min\{g^7(\theta_a), g^6(\theta_a)\} \geq \theta_q$. $\mathcal{A} \cap \mathcal{WP}^Q = \varnothing$ follows since $\mathcal{WP}^Q$ satisfy $g^4(\theta_a) \leq \theta_q$ while $\mathcal{A}$ satisfy $\theta_q < g^4(\theta_a)$.

Finally, $\theta_q > g^7(\theta_a)$ and $\theta_a > g^3(\theta_q)$ gives $\mathcal{I} \cap \mathcal{WP}^A = \varnothing$ and $\mathcal{I} \cap \mathcal{WP}^Q = \varnothing$

Now, for $\mathcal{I}$ and $\mathcal{A}$ the definition of $g_1^{10}(\theta_a)$ (when $1 \geq \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$) and $g_2^{10}(\theta_q)$ (when $1 \leq \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$) gives the separation.

We need to show now that all the space of types if following some strategy.

Cover all $[0,1]^2$.

Assume that $1 \geq \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$ (the other case is analogous). First note that

$$\begin{aligned} \mathcal{SP}^A \cup \mathcal{WP}^A &= \left\{\theta \in [0,1]^2 : \min\left\{g^7(\theta_a), g^6(\theta_a), g^8(\theta_a)\right\} \geq \theta_q\right\} \\ \mathcal{SP}^Q \cup \mathcal{WP}^Q &= \left\{\theta \in [0,1]^2 : \theta_q \geq \max\left\{g^2(\theta_a), g^4(\theta_a)\right\}, \theta_a \leq g^3(\theta_q)\right\} \end{aligned}$$

Now adding independents to the first group and abstainers to the second group, we have

$$\mathcal{SP}^A \cup \mathcal{WP}^A \cup \mathcal{I} = \left\{ \begin{array}{c} \theta \in [0,1]^2 : g^6(\theta_a) \geq \theta_q, \\ g^2(\theta_a) > \theta_q > g_1^{10}(\theta_a), \theta_a > g^3(\theta_q) \end{array} \right\}$$

$$\mathcal{SP}^Q \cup \mathcal{WP}^Q \cup \mathcal{A} = \left\{ \begin{array}{c} \theta \in [0,1]^2 : g^6(\theta_a) < \theta_q, \\ g^2(\theta_a) \leq \theta_q \leq g_1^{10}(\theta_a), \theta_a \leq g^3(\theta_q) \end{array} \right\}$$

which covers all $[0,1]^2$. ■

Again, although we can not prove uniqueness of equilibrium, our characterization is valid for *all* symmetric Bayesian equilibria.

It is important to note that, for low values of θ_a and θ_q, we know that the investment condition (14) does not hold so the only restriction for abstainers to exists in equilibrium is that there is a pair $(\theta_q, \theta_a) \in [0,1]^2$ such that $\theta_q \in (g^6(\theta_a), g^4(\theta_a))$. If (12) holds with strict inequality, $g^6(\theta_a) < g^4(\theta_a)$ for low values of θ_a, so

Lemma 5 *In any regular committee a sufficient condition for some responsive voters* *to strictly prefer abstention rather than any other voting option after some signal is that (12) holds with strict inequality*

3.3.2 Existence

Once the characterization is complete we are ready to prove existence. We have to consider that there are two possible configurations of equilibria. On one hand, if $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} > \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$, the equilibrium involves some responsive voters that strictly prefer to abstain in equilibrium after some signal (endogenous abstention). On the other hand, if $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \leq \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$ the equilibrium involves abstention only by stubborn voters (exogenous abstention).

We first need to show that the equilibrium with endogenous abstention approaches smoothly the equilibrium with only exogenous abstention when $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \searrow \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$. Here is where the transformation that uses all best responses as arguments plays a crucial role. The result will follow by considering that the set of abstainers and weak partisans disappear as soon as abstention is not part of an optimal voting rule. In a sense, all cutoff functions and investment rules change smoothly when we move slowly from an equilibrium with endogenous abstention to an equilibrium without endogenous abstention.

Proposition 2 *In any regular committee there exists a symmetric Bayesian equilibrium. Moreover, this equilibrium is characterized by the strategy* $(P^*(\theta), V^*(\theta, S))$ *in Proposition (1).*

Proof. Let $\phi = \left(1 - \left(\xi_A + \xi_Q\right)\alpha\right)^{n-1}$ and define the spaces

$$X_1 \equiv \left\{(x,y) \in \left[\xi_A \alpha, 1 - \left(\xi_\varnothing + \xi_Q\right)\alpha\right] \times \left[\xi_Q \alpha, 1 - \left(\xi_\varnothing + \xi_A\right)\alpha\right]\right\}$$

$$X_2(\phi) \equiv \left\{(x,y,v,z) \in [\phi r_0, 1]^2 \times [\phi, 1]^2 : x + \phi(1-r_0) \leq v, y + \phi(1-r_0) \leq z\right\}$$

$$X_3(\phi) \equiv \left\{(x,y) \in \left[\phi(1-r_0), \frac{1}{\phi(1-r_0)}\right] \times \left[\phi r_0, \frac{1}{\phi r_0}\right]\right\}$$

Let $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right)$ by a generic element of the space $X_2(\phi)$ and $\left(\Pi^{Q-\varnothing}, \Pi^\varnothing\right)$ a generic element of the space $X_3(\phi)$. Note that $y^\omega_\varnothing$ plays the role of $\Delta \Pr(\omega, \varnothing)$ and y^ω_Q plays the role of $\Delta \Pr(\omega, Q)$ for $\omega \in \{a, q\}$. On the other hand, $\Pi^{Q-\varnothing}$ plays the role of $\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)}$ and $\Pi^\varnothing$ plays the role of $\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$.

Let $p^i : [0,1]^2 \times X_2(\phi) \to \left[\frac{1}{2}, 1-\eta\right], i = 1,2,3$ be implicitly defined by $C'(p^1) = \frac{\theta_a y^a_\varnothing + \theta_q y^q_\varnothing}{2}$, $C'(p^2) = \frac{\theta_a y^a_Q + \theta_q y^q_Q}{2}$, and $C'(p^3) = \frac{\theta_a \left(y^a_Q - y^a_\varnothing\right) + \theta_q \left(y^q_Q - y^q_\varnothing\right)}{2}$, and let η be such that $C'(1-\eta) > 1$. So p^1 plays the role of $P^{\varnothing A}$, p^2 plays the role of P^{QA} and p^3 plays the role of $P^{Q\varnothing}$.

Now consider an element $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right) \in X_2(\phi)$ and using (p^1, p^2, p^3), we can define the cutoff functions used in the characterization of equilibrium . Therefore, the sets of strong and weak partisans, independents and abstainers are well defined. Using Proposition (1) we have that $P(X^\omega)$, the probability of a vote for $X \in \{Q, A\}$ in state $\omega \in \{q, a\}$, is

$$\begin{aligned}
\Pr(A^a) &\equiv \int_{\theta \in \mathcal{WP}^A} p^1(\theta)\, dF(\theta) + \int_{\theta \in \mathcal{SP}^A} dF(\theta) + \int_{\theta \in \mathcal{I}} p^2(\theta)\, dF(\theta) \qquad (15) \\
\Pr(A^q) &\equiv \int_{\theta \in \mathcal{WP}^A} \left(1 - p^1(\theta)\right) dF(\theta) + \int_{\theta \in \mathcal{SP}^A} dF(\theta) + \int_{\theta \in \mathcal{I}} \left(1 - p^2(\theta)\right) dF(\theta)
\end{aligned}$$

$$\begin{aligned}
\Pr(Q^q) &\equiv \int_{\theta \in \mathcal{WP}^Q} p^3(\theta)\, dF(\theta) + \int_{\theta \in \mathcal{SP}^Q} dF(\theta) + \int_{\theta \in \mathcal{I}} p^2(\theta)\, dF(\theta) \qquad (16) \\
\Pr(Q^a) &\equiv \int_{\theta \in \mathcal{WP}^Q} \left(1 - p^3(\theta)\right) dF(\theta) + \int_{\theta \in \mathcal{SP}^Q} dF(\theta) + \int_{\theta \in \mathcal{I}} \left(1 - p^2(\theta)\right) dF(\theta)
\end{aligned}$$

For functions (p^1, p^2, p^3) and $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right) \in X_2(\phi)$ and $\left(\Pi^{Q-\varnothing}, \Pi^\varnothing\right) \in X_3(\phi)$, we define the functions $G^\omega_X : X_2(\phi) \times X_3(\phi) \to X_1$ for $X = A, Q$ such that

$$\begin{aligned}
G^\omega_A\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q, \Pi^{Q-\varnothing}, \Pi^\varnothing\right) &\equiv \xi_A \alpha + (1-\alpha) \Pr(A^\omega)\, I\left(\Pi^{Q-\varnothing} > \Pi^\varnothing\right) \\
&\quad + (1-\alpha) \Pr(A \mid \omega)\, I\left(\Pi^{Q-\varnothing} \leq \Pi^\varnothing\right) \\
G^\omega_Q\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q, \Pi^{Q-\varnothing}, \Pi^\varnothing\right) &\equiv \xi_Q \alpha + (1-\alpha) \Pr(Q^\omega)\, I\left(\Pi^{Q-\varnothing} > \Pi^\varnothing\right) \\
&\quad + (1-\alpha) \Pr(Q \mid \omega)\, I\left(\Pi^{Q-\varnothing} \leq \Pi^\varnothing\right)
\end{aligned}$$

where $\Pr(A \mid \omega)$ and $\Pr(Q \mid \omega)$ are defined for the case where no responsive voter abstain. That is

$$\Pr(A \mid a) \equiv \int_0^1 \int_0^{\min\left\{1, g^8(\theta_a)\right\}} dF(\theta) + \int_0^1 \int_{\min\{1, g^8(\theta_a)\}}^{\min\left\{1, g^2(\theta_a)\right\}} P^{QA}(\theta)\, dF(\theta)$$

$$\Pr(A \mid q) \equiv \int_0^1 \int_0^{\min\left\{1, g^8(\theta_a)\right\}} dF(\theta) + \int_0^1 \int_{\min\{1, g^8(\theta_a)\}}^{\min\left\{1, g^2(\theta_a)\right\}} \left(1 - P^{QA}(\theta)\right) dF(\theta)$$

and $\Pr(Q \mid \omega) + \Pr(A \mid \omega) = 1$. Now, for a pair $(x_1^\omega, x_2^\omega) \in X_1$ we can define $\Pr\left(T_{n-1}^m = l \mid \omega\right)$ in terms of (x_1^ω, x_2^ω) as

$$\begin{aligned} \Pr(m, l \mid (x_1^\omega, x_2^\omega), \omega) \equiv\ & \frac{(n-1)!}{l!\,(m-l)!\,(n-1-m)} \\ & (x_1^\omega)^l (x_2^\omega)^{m-l} \left(1 - (x_1^\omega + x_2^\omega)\right)^{n-1-m} \end{aligned}$$

Recalling the expressions for $\Delta \Pr(\omega, \varnothing)$ and $\Delta \Pr(\omega, Q)$, we define the function $K_i : X_1 \times X_1 \to X_2(\phi)$, such that for $i \in \{1, 2\}$ we let

$$\begin{aligned} K_i(x_1^a, x_2^a, x_1^q, x_2^q) \equiv\ & \Pr(0, 0 \mid (x_1^\omega, x_2^\omega), \omega)\, r_0 + r \sum_{m=1}^{n-1} \Pr(m, N(m) \mid (x_1^\omega, x_2^\omega), \omega)\, I_{N(m+1)}^{N(m)} \\ & + (1-r) \sum_{m=1}^{n-1} \Pr(m, N(m) - 1 \mid (x_1^\omega, x_2^\omega), \omega)\, I_{N(m+1)}^{N(m)} \end{aligned}$$

and for $i \in \{3, 4\}$, we let

$$\begin{aligned} K_i(x_1^a, x_2^a, x_1^q, x_2^q) \equiv\ & \Pr(0, 0 \mid (x_1^\omega, x_2^\omega), \omega) + r \sum_{m=1}^{n-1} \Pr(m, N(m+1) \mid (x_1^\omega, x_2^\omega), \omega) \\ & + (1-r) \sum_{m=1}^{n-1} \Pr(m, N(m+1) - 1 \mid (x_1^\omega, x_2^\omega), \omega) \end{aligned}$$

So, if we let $\omega = a$ for $i \in \{1, 3\}$ and $\omega = q$ for $i \in \{2, 4\}$, $(x_1^a, x_2^a, x_1^q, x_2^q)$ are the probabilities of voting for A or Q in different states, and K_1 gives $\Delta \Pr(a, \varnothing)$, K_2 gives $\Delta \Pr(q, \varnothing)$, K_3 gives $\Delta \Pr(a, Q)$, and K_4 gives $\Delta \Pr(q, Q)$.

We also define the function $L : X_2(\phi) \rightarrow X_3(\phi)$ such that

$$L\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right) \equiv \left(\frac{y^a_\varnothing}{y^q_\varnothing}, \frac{y^a_Q - y^a_\varnothing}{y^q_Q - y^q_\varnothing}\right)$$

which maps the probabilities of changing the election according to the change in the vote ($\Delta \Pr(a, \varnothing)$, $\Delta \Pr(q, \varnothing)$, $\Delta \Pr(a, Q)$, and $\Delta \Pr(q, Q)$), into the ratios that gives the incentives to abstain: $\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$ and $\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)}$.

Now we have all the elements to show that an equilibrium actually exists.

Take an arbitrary element of $\mathcal{S} \equiv (X_1)^2 \times X_2(\phi) \times X_3(\phi)$, define the function $\Gamma : \mathcal{S} \rightarrow \mathcal{S}$ such that $\Gamma \equiv \left\{G^a_A, G^a_Q, G^q_A, G^q_Q, K, L\right\}$, where the components are defined above.

We are going to show first that actually Γ is a continuous function.

For continuity of $\left(G^a_A, G^a_Q, G^q_A, G^q_Q\right)$ we first observe that all the cutoff functions that determine the types (weak and strong partisans, abstainers and independents), are well defined and continuous for $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right)$ and (p^1, p^2, p^3) as defined above. Therefore $\Pr(A^\omega)$ and $\Pr(Q^\omega)$, are continuous on $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right)$ when we consider that (p^1, p^2, p^3) are also continuous and well defined for $y^\omega_\varnothing \in [\phi r_0, 1]$, $y^\omega_Q \in [\phi, 1]$ and $r_0 \in (0, 1)$. We only need to prove that $\Pr(X^\omega) \rightarrow \Pr(X \mid \omega), X \in \{A, Q\}$ when $\Pi^{Q-\varnothing} \rightarrow \Pi^\varnothing$.

Note that $\mathcal{WP}^X \rightarrow \varnothing$ for $X \in \{A, Q\}$ when $\Pi^{Q-\varnothing} \rightarrow \Pi^\varnothing$. We show the case of $X = A$. Since

$$\begin{array}{ccc}
\dfrac{\Delta \Pr(a, \varnothing)}{\Delta \Pr(q, \varnothing)} \dfrac{1 - P^{\varnothing A}\left(g^9(\theta_a), \theta_a\right)}{P^{\varnothing A}\left(g^9(\theta_a), \theta_a\right)} & < & \dfrac{g^9(\theta_a)}{\theta_a} \\
\dfrac{1 - P^{\varnothing A}\left(g^7(\theta_a), \theta_a\right)}{P^{\varnothing A}\left(g^7(\theta_a), \theta_a\right)} \dfrac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{\Delta \Pr(q, Q) - \Delta \Pr(q, \varnothing)} & > & \dfrac{g^7(\theta_a)}{\theta_a}
\end{array}$$

and recalling that $\theta \in \mathcal{WP}^A$ verifies $g^7(\theta_a) \geq \theta_q > g^9(\theta_a)$, it must hold that

$$\frac{1 - P^{\varnothing A}\left(g^7(\theta_a), \theta_a\right)}{P^{\varnothing A}\left(g^7(\theta_a), \theta_a\right)} \frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{\Delta \Pr(q, Q) - \Delta \Pr(q, \varnothing)} > \frac{\Delta \Pr(a, \varnothing)}{\Delta \Pr(q, \varnothing)} \frac{1 - P^{\varnothing A}\left(g^9(\theta_a), \theta_a\right)}{P^{\varnothing A}\left(g^9(\theta_a), \theta_a\right)}$$

When $\Pi^{Q-\varnothing} \rightarrow \Pi^\varnothing$ the previous inequality is just

$$\frac{1 - P^{\varnothing A}\left(g^7(\theta_a), \theta_a\right)}{P^{\varnothing A}\left(g^7(\theta_a), \theta_a\right)} \geq \frac{1 - P^{\varnothing A}\left(g^9(\theta_a), \theta_a\right)}{P^{\varnothing A}\left(g^9(\theta_a), \theta_a\right)}$$

which implies that $g^7(\theta_a) \leq g^9(\theta_a)$ and therefore $\mathcal{WP}^A = \varnothing$.

Using that abstainers must satisfy that $\theta_q \in (g^6(\theta_a), g^4(\theta_a))$ and

$$\begin{aligned} \frac{\Delta \Pr(a, \varnothing)}{\Delta \Pr(q, \varnothing)} &< \frac{g^6(\theta_a)}{\theta_a} \\ \frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{\Delta \Pr(q, Q) - \Delta \Pr(q, \varnothing)} &> \frac{g^4(\theta_a)}{\theta_a} \end{aligned}$$

it must be that $\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)} < \frac{\Delta \Pr(a,Q)-\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q)-\Delta \Pr(q,\varnothing)}$ which implies that if $\frac{\Delta \Pr(a,Q)-\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q)-\Delta \Pr(q,\varnothing)} \to \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$ then $\mathcal{A} \to \varnothing$.

Therefore only strong partisans and independents survive and

$$\mathcal{I} \to \left\{ (\theta_q, \theta_a) \in [0,1]^2 : g^8(\theta_a) < \theta_q < g^2(\theta_a) \right\}$$

which implies the desire result.

The fact that K is continuous in $(x_1^a, x_2^a, x_1^q, x_2^q)$ follows trivially by continuity of $\Pr(N, l \mid (x_1^\omega, x_2^\omega), \omega)$ in $(x_1^a, x_2^a, x_1^q, x_2^q)$.

The same applies for continuity of L when we consider that $y_Q^q - y_\varnothing^q \geq \phi(1 - r_0)$ and $y_\varnothing^q \geq \phi r_0$.

X_1, $X_2(\phi)$ and $X_3(\phi)$ are convex and compact, so Brouwer's fixed point theorem holds (Border (1985)) and there is some $x \in \mathcal{S}$ such that $\Gamma(x) = x$. ■

4 Applications

4.1 Abstention under simple majority rule

Early models of abstention[24] associated this phenomenon to turnout: abstention means that the voter does not show up. The reason why the voter decides not to vote is that voting is costly. This turnout justification is valid only to explain direct absence, but fails to explain why some voters who are already in the booth decide not to vote on some issues, while voting on others. In costly voting models when voting is free, abstention is never strictly optimal. This is because abstention is not a feasible voting action.

Feddersen and Pesendorfer (1996) is the first general equilibrium model that stud-

[24] See Dhillon and Peralta (2002) and Feddersen (2004) for surveys.

ies optimal abstention as a strategy available to voters in the booth. They provide an informational theory for this phenomenon: uninformed voters abstain as a method of delegation to more informed voters. Therefore for abstention to be part of a pure strategy in equilibrium, there should be unequally informed voters with a common preference component.[25]

We use the characterization of the equilibrium provided previously and impose the plurality decision rule as the decision rule. Because the exact characterization of equilibrium is needed we specify the uncertainty that voters face by introducing symmetry. We can show that the plurality decision rule induces optimal abstention by exploiting the fact that the equilibrium under this decision rule is symmetric in the sense that the following conditions hold.

Condition 1 ***a)*** $\Delta\Pr(a,Q) = \Delta\Pr(q,Q)$*,* ***b)*** $\Delta\Pr(\omega,Q) - \Delta\Pr(\omega,\varnothing) = \Delta\Pr(-\omega,\varnothing)$ *for* $(\omega,-\omega) \in \{(q,a),(a,q)\}$

Condition 2 ***a)*** $\Pr(\varnothing \mid a) = \Pr(\varnothing \mid q)$*,* ***b)*** $\Pr(A \mid a) = \Pr(Q \mid q)$[26]

Proposition 3 *In any regular and symmetric committee with rule of election* $N(m) = \frac{m}{2}$ *if* m *is even and* $N(m) = \frac{m+1}{2}$ *if* m *is odd for all* $m \in \{0,1,...n\}$*,* $r = r_0 = \frac{1}{2}$*, there is an informative equilibrium in which responsive voters abstain with positive probability.*

Proof. We are going to show first that condition (1) is necessary and sufficient for condition (2).

First we derive the expressions for $\Delta\Pr(\omega,\varnothing)$ and $\Delta\Pr(\omega,Q)$. Using the plurality decision,

$$\begin{aligned}\Delta\Pr(\omega,Q) &= \Delta\Pr(\omega,\varnothing) + \frac{\tau_1(\omega) + \tau_2\left(\frac{m+1}{2},\omega\right)}{2} \\ \Delta\Pr(\omega,\varnothing) &= \frac{\tau_1(\omega) + \tau_2\left(\frac{m-1}{2},\omega\right)}{2}\end{aligned} \tag{17}$$

[25] See Ghirardato and Katz (2006) for a discussion in terms of the equilibrium beliefs needed for abstention ot be optimal.

[26] This implies that $\Pr(Q \mid a) = \Pr(A \mid q)$.

where

$$\tau_1(\omega) \equiv \Pr\left(T^0_{n-1} = 0 \mid \omega\right) + \sum_{m=1}^{n-1} \Pr\left(T^m_{n-1} = \frac{m}{2} \mid \omega\right) I(m \text{ is even})$$

$$\tau_2(k,\omega) \equiv \sum_{m=1}^{n-1} \Pr\left(T^m_{n-1} = k \mid \omega\right) I(m \text{ is odd})$$

Necessity

Assume first that condition (2) holds. Using expression (9), it is straightforward to see that $\tau_1(a) = \tau_1(q)$, $\tau_2\left(\frac{m-1}{2}, a\right) = \tau_2\left(\frac{m+1}{2}, q\right)$ and $\tau_2\left(\frac{m+1}{2}, a\right) = \tau_2\left(\frac{m-1}{2}, q\right)$. Using these equalities in (17) we have $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \varnothing) = \Delta \Pr(-\omega, \varnothing)$ and $\Delta \Pr(\omega, Q) = \Delta \Pr(-\omega, Q)$ where $-\omega = a$ if $\omega = q$ and $-\omega = q$ if $\omega = a$.

Sufficiency

Assume now that condition (1) holds.

Take any type $(\theta_q = y, \theta_a = x)$. Using (13) and $\Delta \Pr(a, Q) = \Delta \Pr(q, Q)$, for $\omega \in \{q, a\}$, it follows that $P^{QA}(x,y) = P^{QA}(y,x)$. Using (13) and $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \varnothing) = \Delta \Pr(-\omega, \varnothing)$, it follows that $P^{\varnothing A}(x,y) = P^{Q\varnothing}(y,x)$.

In general the cutoff functions, are derived implicitly from an expression where

$$L\left(P^X(x,y)\right) \equiv C'\left(P^X(x,y)\right) P^X(x,y) - C'\left(P^X(x,y)\right)$$

for $X \in \{QA, \varnothing A, Q\varnothing\}$ and $(x,y) \in [0,1]^2$, and $L\left(P^X(x,y)\right)$ is equal to the Cartesian product of (θ_q, θ_a) with some subset of $\Delta \Pr(\omega, \varnothing)$, $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \varnothing)$ and $\Delta \Pr(\omega, Q)$. For example, $g^4(\theta_a)$ and $g^6(\theta_a)$ are implicitly defined by:

$$\theta_a \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{2} = L\left(P^{Q\varnothing}\left(g^4(\theta_a), \theta_a\right)\right)$$

$$g^6(\theta_a) \frac{\Delta \Pr(q,\varnothing)}{2} = L\left(P^{\varnothing A}\left(g^6(\theta_a), \theta_a\right)\right)$$

Using $P^{Q\varnothing}(x,y) = P^{\varnothing A}(y,x)$ we get that $L\left(P^{Q\varnothing}(x,y)\right) = L\left(P^{\varnothing A}(y,x)\right)$; recalling that $\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{2} = \frac{\Delta \Pr(q,\varnothing)}{2}$, if $g^4(x) = y$ it must be true that $g^6(y) = x$. Using the same argument we have

1. $g^9(x) = y$ iff $g^1(y) = x$
2. $g^8(x) = y$ iff $g^2(y) = x$

3. $g^{7}(x) = y$ iff $g^{3}(y) = x$

Take now a type (x, y) such that $x > g^{4}(y)$, so $Q\varnothing$ is preferred to $\varnothing\varnothing$, and we must have that $g^{6}(x) > y$, so the type (y, x) must prefer the strategy with $\varnothing A$ to the one with $\varnothing\varnothing$. This result extends to all functions presented above:

1. $g^{9}(x) > y$ iff $x > g^{1}(y)$
2. $g^{8}(x) > y$ iff $x > g^{2}(y)$
3. $g^{7}(x) > y$ iff $x > g^{3}(y)$

so $(x, y) \in \mathcal{SP}^{A}$ iff $(y, x) \in \mathcal{SP}^{Q}$ and $(x, y) \in \mathcal{WP}^{A}$ iff $(y, x) \in \mathcal{WP}^{Q}$. The problem arises when we want to compare independents and abstainers, since we do not have a "mirror" function. In this case, (x, y) prefer being independent rather than being abstainer if:

$$L\left(P^{QA}(x, y)\right) \geq x\frac{\Delta \Pr(q, \varnothing)}{2} + y\frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{2}$$

Using that $L\left(P^{QA}(y, x)\right) = L\left(P^{QA}(x, y)\right)$, $\frac{\Delta \Pr(q,\varnothing)}{2} = \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{2}$ we must have that

$$L\left(P^{QA}(y, x)\right) \geq x\frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{2} + y\frac{\Delta \Pr(q, \varnothing)}{2}$$

so the type (y, x) also prefers the informed strategy with QA to the uninformed strategy with $\varnothing\varnothing$: $(x, y) \in \mathcal{I}$ iff $(y, x) \in \mathcal{I}$. Given that $\mathcal{A}$ is the complement of the previous groups (weak partisans, strong partisans and independents) in $[0, 1]^{2}$, it is true that $(x, y) \in \mathcal{A}$ iff $(y, x) \in \mathcal{A}$.

Using the symmetry of F, and the previous results we get $\int\limits_{\theta \in \mathcal{WP}^{A}} dF(\theta) = \int\limits_{\theta \in \mathcal{WP}^{Q}} dF(\theta)$ and $\int\limits_{\theta \in \mathcal{SP}^{Q}} dF(\theta) = \int\limits_{\theta \in \mathcal{SP}^{A}} dF(\theta)$. Now, with the symmetry of F, the result that $(x, y) \in \mathcal{WP}^{A}$ iff $(y, x) \in \mathcal{WP}^{Q}$ and $p^{\varnothing A}(x, y) = p^{Q\varnothing}(x, y)$, implies that

$$\int\limits_{\theta \in \mathcal{WP}^{Q}} P^{Q\varnothing}(\theta)\, dF(\theta) = \int\limits_{\theta \in \mathcal{WP}^{A}} P^{\varnothing A}(\theta)\, dF(\theta)$$

Therefore, using that $(x, y) \in \mathcal{I}$ iff $(y, x) \in \mathcal{I}$ and that $P^{QA}(x, y) = P^{QA}(x, y)$, and recalling the characterization when abstention occurs in equilibrium of $\Pr(Q^{\omega})$ and $\Pr(A^{\omega})$ in (15) and (16), the condition (2) follows as desired.

Note that condition (1) and condition (2) define a closed and convex subset of $\mathcal{S} = (X_1)^2 \times X_2(\phi) \times X_3(\phi)$, as defined in Proposition (2) so we can apply Brouwer's fixed point theorem (Border (1985)) and there is some $x \in \mathcal{S}$ such that $\Gamma(x) = x$ where x verifies both set of condition (1) and condition (2).

Because $\Delta \Pr(\omega, \varnothing) > 0$ and $\Delta \Pr(\omega, Q) > 0$ it must be that information is collected. This implies that $\Pr(A \mid a) > \Pr(Q \mid a)$ so $\tau_2\left(\frac{m+1}{2}, a\right) > \tau_2\left(\frac{m-1}{2}, a\right)$. It follows that $\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing) > \Delta \Pr(a, \varnothing)$, while using $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \varnothing) = \Delta \Pr(-\omega, \varnothing)$

$$\frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{\Delta \Pr(a, \varnothing)} > 1 > \frac{\Delta \Pr(q, Q) - \Delta \Pr(q, \varnothing)}{\Delta \Pr(q, \varnothing)}$$

so responsive voters abstain in equilibrium with positive probability (see (12)). ■

Is abstention inversely correlated with information? Consider a majority election in which the symmetry assumption holds. Now assume that the true state of nature is $\omega = a$. In this case, weak partisans for Q will receive $s = s_a$ the more informed they are: the more information they collect the higher it is the probability of receiving a signal that goes against her own preferences. Therefore, the more likely it is that they will end up abstaining. In this case information and abstention are positively correlated. Weak partisans for A still are voting with higher probability the more informed they are.[27]

4.2 The role of flexible preferences

In the model presented here preferences are described by two parameters. It is traditional in voting models to assume that utility losses are perfectly and inversely correlated ($\theta_q + \theta_a = \delta_1$).[28] This assumption is sufficient to describe voting behavior (see (10) and (11)) but the levels of these losses are relevant in terms of information acquisition (see (13)). We have already discussed the behavioral motivation for allow-

[27] Note that conditioning on the ratio $\frac{\theta_a}{\theta_q}$ yields the same results although the relation between information and abstention is not everywhere monotonic.

[28] Assumptions presenting heterogeneity as $\theta_q - \theta_a = \phi$ or $\frac{\theta_q}{\theta_a} = \phi$ suffer the same drawback presented her.

ing θ_q and θ_a not be perfectly related: introducing voters that care about both types of mistakes (false positives and true negatives). We now illustrate why allowing for flexible preferences matters theoretically, and why restricting preferences may lead to undesirable conclusions and predictions about abstention in committees.

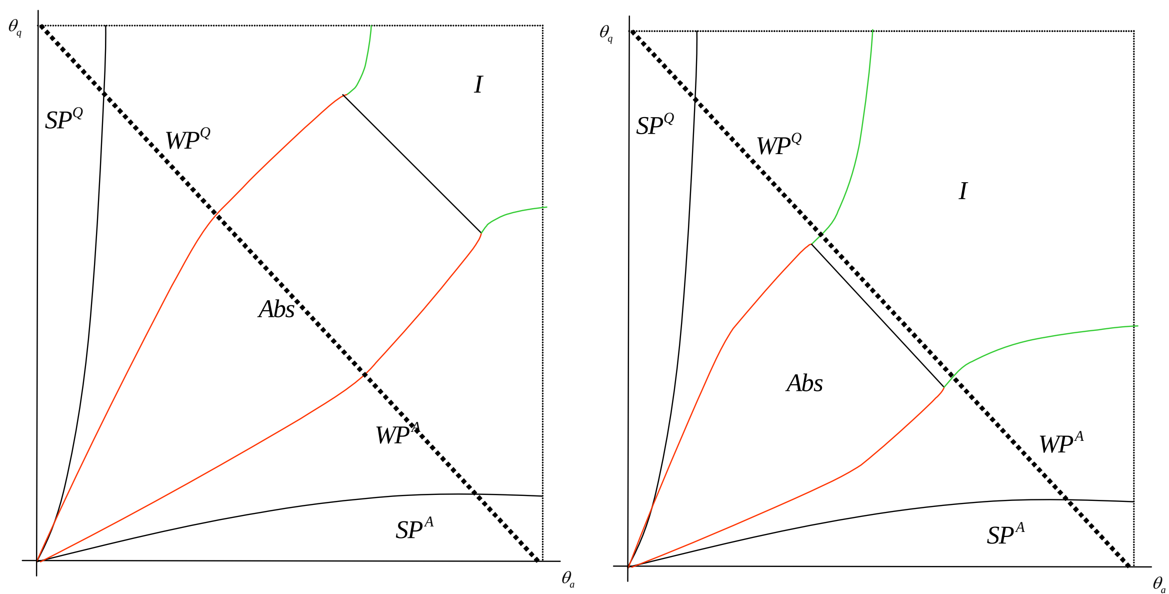

Figure 2: Possible equilibria when preferences are restricted to lie in the manifold $\theta_a + \theta_q = 1$. On the left hand side the restriction eliminiates independents while on the right hand side abstainers are eliminated.

Assume that F is such that $F(\theta_a, \theta_q) = 0$ if $\theta_q + \theta_a < 1$ and $F(\theta_a, \theta_q) = 1$ if $\theta_q + \theta_a > 1$. This implies that F places all the mass on the manifold $\theta_q + \theta_a = 1$; using the assumption that F has no mass points we have that preferences can be described by $\widetilde{F}(\theta_a)$ where $\widetilde{F}$ has no mass points and $\widetilde{F}(\theta_a) = F(\theta_a, 1 - \theta_a)$. Note that our characterization of equilibrium is still valid; in Figure (2) we illustrate two possible configurations of equilibrium.

Using that if $1 \geq \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$, $g_1^{10}(\theta_a)$ verifies that

$$\frac{\partial g_1^{10}(\theta_a)}{\partial \theta_a} = \frac{\Delta \Pr(a,\varnothing) - \Delta \Pr(a,Q)\left(1 - P^{QA}\left(g_1^{10}(\theta_a), \theta_a\right)\right)}{\Delta \Pr(q,\varnothing) - \Delta \Pr(q,Q)\, P^{QA}\left(g_1^{10}(\theta_a), \theta_a\right)}$$

and the symmetry of equilibrium under the plurality decision rule (condition (1)), it follows that $\frac{\partial g_1^{10}(\theta_a)}{\partial \theta_a} = -1$. Then, player type θ with $\theta_q + \theta_a = 1$ prefers being independent than being abstainer, if and only if, any other type θ' with $\theta'_q + \theta'_a = \theta_q + \theta_a$ prefers being independent rather than being abstainer. If there is a pure strategy equilibrium with optimal abstention (either by weak partisans or abstainers), it is not possible to observe both abstainers and independents together. These cases are illustrated in Figure (2).

If F, α or $(\xi_A, \xi_Q, \xi_\varnothing)$ are such that the equilibrium is described by the right picture in Figure (2) we conclude that there are no voters that abstain always although voting is not costly. By restricting preferences we would conclude that the rational ignorance hypothesis only holds for strongly ideological voters. If F, α or $(\xi_A, \xi_Q, \xi_\varnothing)$ are such that the equilibrium is described by the left picture in Figure (2) we conclude that there are no informed voters that always vote. By restricting preferences we would conclude that the rational ignorance hypothesis holds also for unbiased voters who never collect information.[29]

Moreover, it is possible that there is no equilibrium with optimal abstention. If F, α or $(\xi_A, \xi_Q, \xi_\varnothing)$ are such that the equilibrium is described in Figure (3) weak partisans are driven away and only stubborn voters abstain. This leads us to conclude that abstention is not an equilibrium phenomenon: no responsive voter ever abstain. Restricting preferences diminishes the model's capacity of properly capturing optimal abstention as a social phenomenon. Restricting preferences is not innocuous when information is endogenous.

[29] If we let $\theta_a + \theta_q = h$ and we increase h on the equilibrium illustrated on the right hand side of Figure (2) strong partisans may disappear and all responsive voters collect information. The possibility arises because changing h affects the equilibrium behavior and the cut off functions change with it.

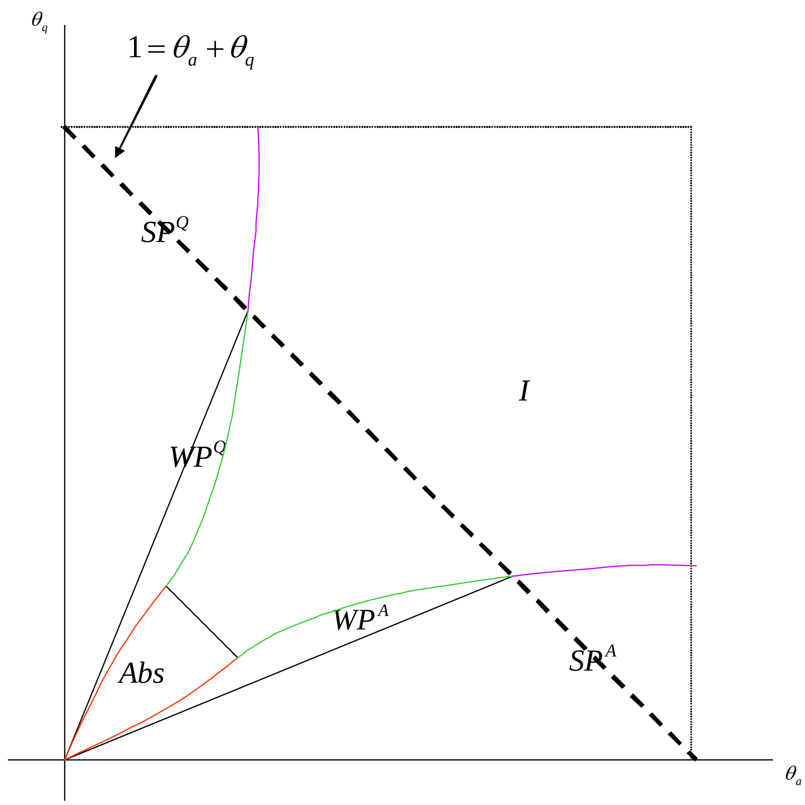

Figure 3: F, α and $(\xi_A, \xi_Q, \xi_\varnothing)$ yield and equilibrium in which no responsive voter abstain.

5 Conclusions

Many organizations rely on committees for efficient decision-making. A common argument in favor of these bodies is their capacity to aggregate information dispersed among its members. The literature focuses mainly on the strategic use of information (Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997)), neglecting the impact of information acquisition on the final outcome of the decision-making process. Since information is a public good in voting models, free riding incentives may reduce the committee's ability to select the right candidate. It was not until recently that the cost of information started playing an important role when studying the behavior of committee members (see (Mukhopadhaya (2003), Gerardi and Yariv (2004), Gershkov and Szentes (2004), Persico (2004), Martinelli (2006)).

Allowing committee members to abstain also affects the information that each vote conveys. Indeed, if abstention is part of equilibrium behavior then some information is lost when a voter decides not to vote. Feddersen and Pesendorfer (1996) show that only poor information is lost due to abstention when information is provided to the voters exogenously. Therefore allowing for abstention (in exogenous information set ups) might not diminish the capacity of the committee to reach an optimal decision. Indeed, Feddersen and Pesendorfer (1999) show that information aggregation is possible with abstention.

There are very few papers that study abstention as optimal behavior and none of them allow for information gathering. This contrasts with the result that abstention is indeed an informational phenomenon (Feddersen and Pesendorfer (1996)). Following this idea, we present a model of committees with *abstention and endogenous information acquisition* using two interdependent innovations: we allow voters to *select the precision of the signal* they will receive and committee members' preferences incorporate differences on the levels of *both ideology and concern*.

In equilibrium there are three classes of uninformed voters (abstainers and strong partisans for each candidate), two classes of informed voters (weak partisans for each candidate) and there might be another class of informed voter (independents). The level of investment differs dramatically even among informed voters. Indeed, small changes that make a voter change her behavior from an independent to a weak partisan create jumps in the level of investment. At the same time rational ignorance takes two different forms. On one side, abstainers decide not to collect information and delegate on the other members by abstaining; in a sense, they do not introduce noise in the election. On the other side, strong partisans vote always although they are not adding any information to the electorate.

In our set up, the plurality decision rule generates abstention as an equilibrium behavior. Our model predicts that voters abstain without assuming a random number of voters as it is common in the literature with heterogenous preferences (Feddersen and Pesendorfer (1999)). Surprisingly, in our model there are some voters abstaining although they have a lot at stake in the election and received strong evidence in favor of one candidate. We show that abstention is not simply the result of poor information but it is a more complex interaction between preferences and information. In our model some well informed voters may abstain precluding this good information to reach the electorate.

Moreover, our results show that when committee members can select the quality of the information to be received, traditional restrictions on preferences may give a misleading understanding of abstention. This is because modelling preferences only on the ideological axis constrains the incentives to collect information and, depending on the class of equilibrium that emerges, abstention might not be an equilibrium behavior. On top of that, restricting preferences affects also our predictions about who is rationally ignorant in equilibrium.

Although we base all our analysis on roll off (selective abstention when the voter is already in the booth) our model gives insightful results about absence. Indeed, if voters collect information before they approach the booth we would predict absence although voting is not costly. Therefore, our model can also provide links between information and turnout. Unlike other studies we show that correlation patterns between information and turnout are present as long as we condition these patterns on particular groups of voters: some voters are *more likely to vote* the more informed they are, while some other voters are *more likely to abstain* the more informed they are.

Empirical models that study abstention and information either test $\Pr(v(\theta) = \varnothing)$ across different electorates (e.g. Coupé and Noury (2004)) or try to determine whether $\Pr(v \neq \varnothing \mid P)$ is increasing or decreasing in P (e.g. Larcinese (2005) and Lassen (2005)). These tests only capture that independents vote since the effect of information on weak partisans for one candidate cancel out with the effect of information on the other group of weak partisans. In essence, the strength of the test depends on the percentage of independents in the electorate.

Our model suggests that this is not the whole story and that empirical tests need to consider the ideological axis in order to capture the differential effect of information acquisition on voting. Palfrey and Poole (1987) is an example of an empirical paper that uses this strategy. They use voters that actually voted[30] and our model suggests that a more direct test of information and turnout must condition on ideology among those that did not vote: weak partisans that abstained. For example, our model predicts that moderate/conservatives identified with the republican party were more likely to abstain in the 2006 election according to the (negative) information they

[30]They find differences in the level of information: those that voted for the winner (Reagan) were more informed than those that supported the loser (Carter). According to our model this correlation might be driven by independents.

collected about the war in Irak in 2006 while moderate/liberals identified with the democratic party were more likely to vote in the 2006 election in response to the same information.

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A Appendix: Relation between cutoff functions

Here we summarize some useful relations between different cutoff functions.

Fact 1 *For every pair* $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)$ *satisfying* $\widetilde{\theta}_q = g^6\left(\widetilde{\theta}_a\right)$ *we have* $\widetilde{\theta}_a \geq g^3\left(\widetilde{\theta}_q\right)$ *and* $g^6(\theta_a) \geq g^9(\theta_a)$ *for all* θ_a.

Fact 2 $g^1(\theta_a) \geq g^4(\theta_a) \geq g^7(\theta_a)$ *for all* θ_a.

Fact 3 $g^6(\theta_a) < g^5(\theta_a)$ *iff* $g^5(\theta_a) < g^4(\theta_a)$ *and* $g^6(\theta_a) > g^5(\theta_a)$ *iff* $g^5(\theta_a) > g^4(\theta_a)$. *Moreover, there is some* $\overline{\theta}_a \in (0,1]$ *such that, for all* $\theta_a \in \left(0, \overline{\theta}_a\right)$, *the relation* $g^4(\theta_a) > g^6(\theta_a)$ *holds.*

Fact 4 *From the previous results, the uninformed strategy that calls for abstention and no collection of information is optimal only for types such that* $g^4(\theta_a) \geq g^6(\theta_a)$.

Fact 5 $g^8(\theta_a) < g^9(\theta_a)$ *iff* $g^7(\theta_a) < g^8(\theta_a)$ *and* $\theta_q^9(\theta_a) < g^8(\theta_a)$ *iff* $g^8(\theta_a) < g^7(\theta_a)$.

Fact 6 *for every* $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)$ *that satisfies* $\widetilde{\theta}_a = g^3\left(\widetilde{\theta}_q\right)$, *it also holds that* $\theta_q^1\left(\widetilde{\theta}_a\right) > g^2\left(\widetilde{\theta}_a\right)$ *iff* $g^2\left(\widetilde{\theta}_a\right) > \widetilde{\theta}_q$ *and* $g^1\left(\widetilde{\theta}_a\right) < g^2\left(\widetilde{\theta}_a\right)$ *iff* $g^2\left(\widetilde{\theta}_a\right) < \widetilde{\theta}_q$.