Efficient and inefficient durable-goods monopolies*

João V. Montez†

Preliminary draft: *all* comments welcome.

Abstract

We study Markov Perfect Equilibria (MPE) of a durable-goods monopoly model with a finite number of buyers. We show that, while all pure-strategy MPE are asymptotically efficient (e.g. Pacman and Coasian), there also exist previously unstudied asymptotically inefficient MPE. In these equilibria high valuation buyers randomize their purchase decision while trying to benefit from low prices which are offered once a critical mass has purchased. Due to an attrition behavior the market takes real time to clear even for discount factors arbitrarily close to one, an unusual monopoly distortion.

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1 Introduction

Much of the literature on durable-goods monopolies pricing behavior has focused on the time inconsistency problem, first discussed in Coase (1972), which can be summarized as follows: once high-valuation buyers have purchased, a monopolist seller has an incentive to lower its price and sell to low-valuation buyers as well. High-valuation buyers anticipating this behavior are reluctant to accept high prices, which forces the monopolist to offer low prices at the outset of the game. When the time between offers is short (frictionless markets), opening prices should be close to the lowest valuation and the market should clear in a "twinkle of an eye". All Markov-Perfect-Equilibrium (MPE) of the infinite-horizon model with a continuum of buyers satisfy these properties, a result known as the Coase conjecture (e.g. Stokey, 1981 and Gul et al., 1986).

It is now known that rational expectations alone are not sufficient for the Coase conjecture. Arguing that a finite number of buyers provide a better description of both natural and laboratory economies, several authors have shown that, since each single buyer has in this case a nonnegligible effect on profits, the monopolist can also credibly condition price reductions on single purchases to "eat" down the demand curve and virtually achieve the profits of a perfect discriminating monopolist (Bagnoli et al., 1989 and von der Fehr and Kuhn, 1995). When the time between offers is short, this outcome—known as Pacman—is a MPE of the infinite-horizon model with a finite number of buyers.

Distributive considerations aside, this literature seems to support a captivating idea: that markets served by durable-goods monopolies are efficient—since in the studied MPE all gains from

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†Département d'Econométrie et d'Economie Politique, Université de Lausanne, Ecole des HEC. CH-1015 Lausanne, Switzerland; e-mail adress: Joao.VieiraMontez@unil.ch.

trade are realized in the "twinkle of an eye"—and inefficiencies are only explained by technological constraints—such as infrequent offers, imperfect durability or limited capacity (see e.g. Deneckere and Liang, 2006 and McAfee and Wiseman, 2006)—or reputational concerns (see Ausubel and Deneckere, 1989). This insight has played an important role in public policy towards durable goods markets. It seems therefore important to know wether *all* MPE of the game with a finite number of buyers are asymptotically efficient.

We find that in durable goods markets with a finite number of buyers, while all pure-strategy MPE are asymptotically efficient, there also exist asymptotically *inefficient* MPE where the market clearing date remains bounded away from zero even as the interval between offers shrinks to zero. In the equilibria we construct prices become low once a critical mass of high-valuation buyers has purchased, early buyers thus create a positive externality for the remaining ones in the form of lower prices. As in a war of attrition, high-valuation buyers delay purchases in the hope some other buyers purchases first. The monopolist is able to resist the temptation to cut prices—and increase the acceptance rate—when intermediate-valuation buyers create a discontinuity in his payoff function. We call these *attrition equilibria.*

Observe that randomization is necessary but not sufficient for asymptotic inefficiency: the market will still clear in the "twinkle of an eye" if, along the equilibrium path, the equilibrium probability some buyer accepts in each single period remains strictly positive as the interval between offers shrinks to zero. A stronger condition is required: in some state on the equilibrium path the joint per period probability of acceptance has to converge to zero—in the attrition equilibrium we study the cumulative probability each high valuation buyer purchase before a certain date converges to an exponential distribution and it is zero for the remaining buyers.

A major contribution of our work is to rationalize monopoly inefficiency in frictionless durable good markets with a finite number of buyers as Ausubel and Deneckere (1989) did for markets with a continuum of buyers by constructing reputational equilibria. Their common feature is that deviations from the equilibrium price path lead to low continuation profits. There are however several important differences. Non-stationarity is critical in their work while in our setting asymptotic inefficiency arises even in Markov strategies.[1] Moreover, in our setting the market always clears in finite time with probability one while in the equilibrium they construct sales necessarily occur over infinite time.

The remainder of the paper is organized as follows. In section 2 we present the model and in section 3 look at pure strategies. In section 4 we characterize and study an asymptotically inefficient attrition MPE. We conclude in section 5.

2 The model

We consider the standard setting with a finite number of buyers: A monopolist seller, indexed by m, can produce any amount of a durable good at zero marginal cost in any period $t = 0, 1, 2, ...$ There is a set $N = \{1, .., n\}$ of buyers and each buyer has a valuation $v(i) > 0$ for a single unit of the good. Buyers' valuations are common knowledge and $L = \min\{v(i)|\, i \in N\}$ while $H = \max\{v(i)|\, i \in N\}$.

In each period t the monopolist offers a price $p_t \in R$ which then buyers simultaneously accept (and leave the game) or reject and continue to the next period—action $a_t^i = 0$ denotes a rejection and $a_t^i = 1$ an acceptance in period t by buyer i; each buyer i may choose $a_t^i = 1$ at most once.

The discount factor is $\delta \equiv e^{-\rho\triangle} \in (0, 1)$, where $\triangle > 0$ denotes the real time between two

[1] Recall that in the setting with a continum of buyers all stationary equilibria are efficient and verify the Coase conjecture.

successive offers and $\rho > 0$ the discount rate. The payoff function u^i of buyer i is

$$u^i = \begin{cases} \delta^t \left[v(i) - p_t\right] \text{ if } a_t^i = 1 \text{ for some } t \\ 0 \text{ if } a_t^i = 0 \text{ for all } t \end{cases},$$

and the monopolist's payoff is

$$u^m = \sum_{t=0}^{\infty} \delta^t \left(p_t \sum_{i \in N} a_t^i \right).$$

A t-period history of the game $F(t)$ is a list of prices and acceptance decisions from period 0 to $t-1$. A pure strategy is a function specifying a player's action plan at each period for each history of the game prior to that period. Denote the vector of strategies by players other than j by $\mathbf{s}^{-j}$ and the vector of all strategies by $\mathbf{s} = (s^j, \mathbf{s}^{-j})$. For a given $\mathbf{s}$, player j's expected payoff is

$$\mu^j(\mathbf{s}) \equiv E\left[u^j \left| \mathbf{s}\right.\right].$$

Let $\mu^j(\mathbf{s}|\, F(t))$ denote player j's expected payoff if after history $F(t)$ players behave according to $\mathbf{s}$. A strategy is Markov if it only depends on the payoff relevant history, in our game the set of buyers remaining in the market at t which we denote by $I(t) \subseteq N$. A Markov Perfect Equilibrium (MPE) is a strategy vector $\mathbf{s}^*$ such that for all $I(t)$ and for any alternative strategy s^j satisfies

$$\mu^j(s^{j*}, \mathbf{s}^{-j*} \big| \, I(t)) \geq \mu^j(s^j, \mathbf{s}^{-j*} \big| \, I(t)) \text{ for all } j \in N \cup m.$$

In line with the literature we focus on the case where the time between offers $\triangle$ is close to zero, i.e. δ arbitrarily close to 1. Our main question is wether all MPE are asymptotically efficient:

Definition 1. *Asymptotic efficiency*: As $\delta \to 1$ all gains from trade are realized, i.e.

$$\lim_{\delta \to 1} \sum_{j \in N \cup m} \mu^j = \sum_{i \in N} v(i).$$

We conclude this section with an important lemma that holds for any MPE.

Lemma 2.1. i) The monopolist's relevant action space is $p_t \in [L, H]$ and ii) the market clears at t if and only if $p_t = L$, i.e. $p_t = L \Leftrightarrow I(t+1) = \emptyset$ when $I(t) \neq \emptyset$.

Proof. See appendix.

So either all L-buyers are in the market or the game is over and, although L-buyers use a simple cut-off strategy by buying whenever the price does not exceed L, these buyers affect the monopolist's cost of waiting which is an important determinant of equilibrium behavior.

3 Pure strategies and efficiency

This section is organized as follows: We first introduce the Pacman and Coasian outcomes, which can both be sustained as pure-strategy equilibria of the game and are asymptotically efficient. We then show that *all* pure-strategy equilibria of the game are asymptotically efficient.

Bagnoli et al. (1989) showed that:

Lemma 3.1. For δ close to 1 the *Pacman* strategy, where for all $I(t) \neq \emptyset$

$$p_t = \max\left\{v(i) : i \in I(t)\right\} \text{ and } a_t^i = \begin{cases} 1 \text{ if } p_t \leq v(i) \\ 0 \text{ otherwise} \end{cases},$$

always forms a MPE where the market clears in at most n periods and as $\delta \to 1$ the monopolist's payoff becomes that of a perfectly discriminating monopolist, i.e.

$$\lim_{\delta \to 1} \mu^m = \sum_{i \in N} v(i) \text{ and } \lim_{\delta \to 1} \mu^i = 0 \text{ for all } i \in N.$$

The Pacman equilibrium is unique only if every buyer has a valuation which is "large relative to the sum of valuations of all buyers with a lower willingness to pay" (von der Fehr and Kuhn, 1995 pp. 791). Otherwise there exist other MPE and some may be Coasian. In a *Coasian* outcome equilibrium prices are never significantly higher than L, i.e. $\lim_{\delta \to 1} p_t^* = L$ for all $I(t) \neq \emptyset$, and the market is asymptotically efficient with

$$\lim_{\delta \to 1} \mu^m = nL \text{ and } \lim_{\delta \to 1} \mu^i = v(i) - L \text{ for all } i \in N.$$

So the benefit from price discrimination vanishes when the time between offers is arbitrarily close to zero. For example, if no buyer has a valuation which is large relative to the lowest valuation of all buyers there always exists a Coasian outcome with $p_0^* = L$ which all buyers accept.[2] More generally (but trivially) we have:

Proposition 1. All pure-strategy MPE are asymptotically efficient.

Proof. If we restrict attention to pure-strategies, a price which all buyers refuse can not be an equilibrium price offer since the expected payoff of all players in that subgame would be zero. So a non empty subset of the remaining buyers has to accept every equilibrium price with probability one. In equilibrium the market always clears in at most n periods and it is therefore asymptotically efficient.

Any pure strategy MPE remains an MPE if players are allowed to use mixed strategies. As we argued in the introduction, the existence of mixed strategy MPE does not however guarantee asymptotic inefficiency. A game is asymptotic inefficient only if in the limit, as the interval between offers converges to zero, the equilibrium probability of acceptance at each period is zero for all buyers in at least some state reached with positive probability.

4 Attrition and delay

For simplicity of exposition we study here only a simple example of asymptotic inefficiency.[3] We let consumers' valuations take three values, i.e $v(i) \in \{L, M, H\}$ with $0 < L < M < H$, and we focus on symmetric equilibria—buyers with the same valuation use the same strategy. In this case the payoff relevant history of the game $I(t)$ can be summarized by

$$h(t) = (n_t^H, n_t^M, n_t^L),$$

where $n_t^v \in \mathbb{N}$ is the number of buyers with valuation v in the market at time t. The simple setting with delay we study has

$$h(0) = (2, 1, z).$$

We first study some of its subgames in more detail before characterizing an attrition MPE. This exercise provides a simple review of the forces behind the Pacman and Coasian outcomes and is useful to trace the source of delays. For brevity, we henceforth make reference to the state $h(t)$ in the text and, since Markov play depends on h alone, we denote equilibrium actions (not strategies) by p_h^* and a_h^{v*}.

[2] In the appendix the proof of Claim 1 shows this is the case if e.g. $1/(n-1) \geq x \equiv (H-L)/L$.

[3] For our purpose, an example is all that is needed to show that not all equilibria are asymptotically efficient.

4.1 The subgames

From Lemma 2.1 we know that the equilibrium actions of the subgame $h(0) = (0, 0, z)$ (and any of its subgames) is $p_h^* = L$ and $a_h^{L*} = 1$. We can therefore restrict our attention to the subgames where $n_t^L = z$.

Lemma 4.1. Each of the states $h(t) = (1, 0, z)$ and $h(t) = (0, 1, z)$ has two MPE for δ close to 1: the Pacman and a Coasian with $p_h^* = L$. The Pacman is unique if $z < x \equiv (H - L)/L$ and $z < (M - L)/L$ respectively.

Proof: For the state $h(t) = (1, 0, z)$ we look in step 1 at $p_h^* > L$ and in step 2 at $p_h^* = L$ (the proof for the state $h(t) = (0, 1, z)$ is analogous).

Step 1: Suppose that in state $h(t) = (1, 0, z)$ some price $p \in (L, H]$ is part of a (mixed) equilibrium price strategy and that $E[p_h^*]$ is the expectation of this strategy profile. L-buyers refuse p and, since strategies are Markovian, the H-buyer accepts this price with positive probability if the payoff of accepting p is higher than the payoff of waiting an additional period, i.e.

$$H - p \geq \delta(H - E[p_h^*]).$$

For all $\delta \in (0, 1)$ the H-buyer thus accepts with probability 1 any price $p \leq E[p_h^*]$ and the monopolist therefore chooses only prices $p_t \geq E[p_h^*]$. This means the monopolist uses a pure strategy: charges one price p_h^* which the H-buyer accepts. Moreover $p_h^* \notin (L, H)$ since there always exists some higher price p' which the H-buyer always accepts, i.e.

$$\forall p \in (L, H)\ \exists p' \in (p, H] : H - p' > \delta(H - p),$$

and which generates a higher profit, i.e.

$$p' + \delta z L > p + \delta z L.$$

The price $p_h^* = H$ is however an equilibrium price (the Pacman outcome) since no price p' exists and for δ sufficiently close to 1 it generates a profit larger than the non-discriminating profit since

$$H + \delta z L > (z + 1)L.$$

Step 2: The Coasian price $p_h = L$ is also an equilibrium price if the premium the H-buyer is willing to pay to avoid a one period delay in consumption when he expects the next period price to be L (i.e. $[(1 - \delta)H + \delta L] - L$), is lower than the interest the monopolist loses by delaying the sales to the L-buyers (which is $(1 - \delta)Lz$), since then the monopolist prefers to sell immediately to all buyers immediately rather than price discriminate the H-buyer, i.e.

$$(z + 1)L \geq [(1 - \delta)H + \delta L] + \delta z L \Leftrightarrow z \geq x.$$

If however the premium on the H-buyer is significant—the condition above does not hold—then $p = L$ can not be an equilibrium price and it follows from step 1 and Lemma 2.1 that the Pacman equilibrium is also unique. Q.E.D.

The next Lemma looks at a case where the monopolist is unable to credibly threat to wait for *all* H-buyers before reducing its price to sell to L-buyers. In this case the monopolist looses the benefit of price discriminate.

In fact, a Markov pricing strategy creates a payoff structure similar to a war of attrition for H-buyers and their randomized acceptance decisions create potential delay. To increase the acceptance rate of H-buyers and advance sales to all consumers (therefore reducing the lost interest), the monopolist is tempted to offer a one period price reduction. This ultimately undermines his ability to price discriminate. The last part of the argument provides an intuition similar to Gul et al. (1986) explanation of the Coase conjecture in a setting with a continuum of buyers.

Lemma 4.2. When $p_h^* = L$ in state $\widetilde{h}(t) = (1, 0, z)$, for δ sufficiently close to 1, the play of the unique symmetric MPE in state $h(t) = (2, 0, z)$ is Coasian with the following equilibrium actions:

$$\begin{cases} p_h^* = W \equiv (1-\delta)H + \delta L,\ a_h^{H*} = 1 \text{ and } a_h^{L*} = 0 \text{ if } z \leq 2x \\ \\ p_h^* = L \text{ and } a_h^{H*} = a_h^{L*} = 1 \text{ if } z > 2x. \end{cases}$$

Proof. In step 1 we show by contradiction that there can be no equilibrium with $p_h^* \in (W, H]$. In step 2 we look at $p_h^* \in [L, W]$.

Step 1: Suppose in state $h(t) = (2, 0, z)$ there exists $p_h^* \in (W, H]$ when $p_h^* = L$ in state $h(t) = (1, 0, z)$. Then the first buyer creates a positive externality for the remaining one in the form of a low price: if H-buyer i accepts the offer p_t^* he gets $\mu^i = H - p_t^*$ while the other H-buyer j gets $\mu^j = \delta(H - L)$ if he waits for $p_h^* = L$ in state $h(t+1) = (1, 0, z)$. From H-buyers perspective the game resembles a war of attrition: the returns to buying decrease with time but, at any time, each buyer is better off if the other buys first.[4]

For this reason for any p_h^* there are several MPE but a unique symmetric equilibrium which involves mixed strategies. We denote by $q^i(p_t)$ the probability a typical H-buyer i accepts an offer p_t when the monopolist uses the Markovian strategy $p_t^* \in (W, H]$, i.e. $q^i(p_t) = \Pr(a_t^i(p_t) = 1 \big| p_h^*)$ for $i : v(i) = H$. H-buyer i's best response function is:

$$\left.\begin{array}{r} q^i(p_t) = 1 \text{ if } H - p_t > \\ \\ q^i(p_t) \in [0,1] \text{ if } H - p_t = \\ \\ q^i(p_t) = 0 \text{ if } H - p_t < \end{array}\right\} \delta\left[q^j(p_t)(H-L) + (1 - q^j(p_t))(H - p_h^*)\right].$$

In the symmetric equilibrium, for a given p_t and $p_h^* \in (W, H]$, we have

$$q^i(p_t) = q^j(p_t) = q(p_t) \begin{cases} = 1 \text{ if } p_t \leq W \\ \\ = \frac{(H-p_t)-\delta(H-p_h^*)}{\delta(p_h^*-L)} \in (0,1) \text{ if } p_t \in (W, (1-\delta)H + \delta p_h^*) \\ \\ = 0 \text{ if } p_t \geq (1-\delta)H + \delta p_h^* \end{cases} \tag{1}$$

So suppose there exists indeed an equilibrium price $p_h^* \in (W, H]$ in state $\widetilde{h}(t) = (2, 0, z)$: the monopolist's expected payoff in this subgame $\mu^m(p_h^*)$ is the solution to

$$\mu^m(p_h^*) = q(p_h^*)^2\left[2p_h^* + \delta z L\right] + q(p_h^*)(1 - q(p_h^*))\left[p_h^* + \delta(z+1)L\right] + (1 - q(p_h^*))^2 \delta \mu^m(p_h^*)$$

or

$$\mu^m(p_h^*) = \frac{1}{q(p_h^*)(2 - q(p_h^*))}\left[q(p_h^*)^2\left[2p_h^* + \delta z L\right] + q(p_h^*)(1 - q(p_h^*))\left[p_h^* + \delta(z+1)L\right]\right],$$

which converges to

$$\lim_{\delta \to 1} \mu^m(p_h^*) = \frac{2(H - p_h^*)(p_h^* + (z+1)L)}{2H - p_h^* - L}.$$

[4] For a textbook presentation of the war of attrition see e.g. Fudenberg and Tirole (1991).

Given the opportunity, the monopolist would want to commit to some price $p_h^* \in (W, H)$ in state $h(t) = (2, 0, n)$ if $z < 2(H - 2L)/L$.

However, for δ arbitrarily close to 1, each H-buyer accepts in each period with *almost* zero probability—in the limit each H-buyer acceptance follows a Poisson process with parameter $\rho(H - p)/H - L$. When buyers' strategies are Markovian the monopolist can increase the acceptance rate by offering a slightly lower price at t without affecting the future. By doing this i) he losses profits on buyers who accept p_t but ii) he gains the additional interest on the sales to all remaining buyers that are done earlier.

For δ sufficiently close to 1, i) is smaller than ii) and therefore it always pays to undercut any price $p^* \in (W, H]$, implying that no such price can be part of an MPE. Formally, the monopolist's payoff when he offers a price $p_t \in (W, (1-\delta)H + \delta p_h^*)$ is

$$\mu^m(p_t) \equiv q(p_t)^2(2p_t + \delta zL) + 2q(p_t)(1 - q(p_t))(p_t + \delta(z+1)L) + (1 - q(p_t))^2 \delta \mu^m(p_h^*),$$

and with (1) we have

$$\begin{aligned}
\lim_{\delta \to 1} \left. \frac{d\mu^m(p_t)}{dp_t} \right|_{p_t = p_h^*} &= \lim_{\delta \to 1} \left[\frac{\partial \mu^m(p_t)}{\partial p_t} + \frac{\partial \mu^m(p_t)}{\partial q(p_t)} \frac{\partial q(p_t)}{\partial p_t} \right]_{p_t = p_h^*} \\
&= \frac{2(H - p_h^*)}{2H - p_h^* - L} - \infty \text{ for all } p_h^* \in (W, H]\,.
\end{aligned}$$

Step 2: Now we focus on $p_h^* \in [L, W]$ which, by Lemma 2.1, both H-buyers accept with probability 1 (L-buyers only accept $p_h^* = L$). Depending on the number of L-buyers z the price which maximizes the seller's payoff in that range is either L or W. Q.E.D.

Lemma 4.3. When $h(t) = (1, 1, z)$ the actions $p_h^* = M$, $a_h^{M*} = 1$ and $a_h^{H*} = a_h^{L*} = 0$ forms the equilibrium play of a MPE if $z \geq x$ and δ close to 1.

Proof. Suppose that $z \geq x$, that the outcome of the game $h(t) = (1, 0, z)$ is coasian while the game $h(t) = (0, 1, z)$ has a Pacman outcome. The Lemma follows from the one-stage-deviation principle: The M-buyer has no advantage in refusing the price M since he would still get the same price in the future. The H-buyer should refuse M since he expects a payoff $\delta(H - L) > H - M$ for δ close to 1, and for the same reason should refuse any price $p > (1-\delta)H + \delta L$. The monopolist could deviate to any $p \in [L, (1-\delta)H + \delta L]$, but such deviation would give him a payoff lower than his equilibrium payoff $M + \delta(z+1)L$ for all δ close to 1.

The previous Lemma presented a MPE where the M-buyer accepts an offer before the H-buyer. There are thus MPE of the durable-goods monopoly game with a finite number of buyers which violates what Fudenberg et al (1985) termed *skimming property*, which says that higher valuation buyers purchase no later than lower valuation buyers and is satisfied by the Pacman and Coasian outcomes.

4.2 Attrition equilibria

From the previous subsection we have that, with $x \equiv (H - L)/L$ and $W \equiv (1-\delta)H + \delta L$, when $z \geq x$ and δ sufficiently close to one the actions in table 1 are the equilibrium play of a MPE in each proper subgame.

$h(t)$	p_h^*	a_h^{H*}	a_h^{M*}	a_h^{L*}
$(0,0,z)$	L	–	–	1
$(0,1,z)$	M	–	1	0
$(1,0,z)$	L	1	–	1
$(1,1,z)$	M	0	1	0
$(2,0,z)$	L if $z > 2x$	1	–	1
	W if $z \in [x, 2x]$	1	–	0

We say buyers with valuation v are *soft* if for all $h(t)$ where $n^v \neq 0$ we have $p_h^* \geq v$, and are *tough* if $p_h^* < v$ for some $h(t)$ with $n^v \neq 0$.[5] This table corresponds to the case where the M-buyer is soft while H-buyers are tough.

The next result shows that a soft M-buyer can (partially) restore the monopolist's ability to price discriminate tough H-buyers when that a soft M-buyer can create a discontinuity in the monopolist's payoff function which allows him to withstand the Coasian temptation to cut prices we identified in Lemma 4.2.

In this equilibrium the monopolist sets a price exactly equal to M to avoid the acceptance by the M-buyer and the subsequent transition to the low profit state $h(t) = (2, 0, z)$. The H-buyers engage in a war of attrition trying to purchase at the low price which the remaining H-buyer gets in the play of subgame $h(t) = (1, 1, z)$. The transition to the latter state—and subsequent market clearing—takes real time even for discount factors arbitrarily close to 1.

Proposition 2. For δ arbitrarily close to one, there exists an asymptotic inefficient MPE of the game with $h(0) = (2, 1, z)$ when

$$M \in \left(W, \frac{1}{3}(2H - (z+2)L)\right) \text{ and } z > x \tag{2}$$

such that in state $h(t) = (2, 1, n)$ we have $p_h^* = M$ which the M-buyer refuses while each H-buyer accepts with probability

$$\frac{(H-M)(1-\delta)}{\delta\left[(1-\delta)H + M - \delta L\right]} \tag{3}$$

In the limit, as $\delta \to 1$, each H-buyer acceptance follows a Poisson process with parameter $\rho(H - M)/(H - L)$ and the monopolist expected payoff is

$$\frac{2(H-M)(2M + (z+1)L)}{2H - M - L}. \tag{4}$$

Proof: Suppose table 1 describes the equilibrium play in all proper subgames. An argument similar to the proof of Lemma 4.2 shows that a $p_t \in (M, H]$ can not be part of a symmetric MPE of the game. Consider now $p_t \in (W, M)$. The M-buyer accepts the price with probability 1 since no lower price is offered in any other state he is present. H-buyers should then reject p_t since $h(t+1) = (2, 0, z)$ with has $p_h^* \leq W$ and

$$H - p_t < \lim_{\delta \to 1} \delta(H - W) \ \forall p_t \in (W, M)\,.$$

For δ close to 1, $p_t \in (W, M)$ gives the monopolist a sure payoff

$$p_t + \delta(z+2)L, \tag{5}$$

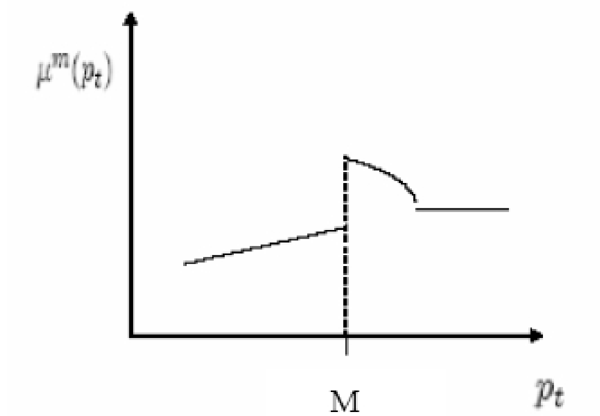
[5]For example, in the Pacman outcome all buyers are soft and in the Coasian outcome all buyers with $v(i) > L$ are tough while buyers with $v(i) = L$ are soft.

which has the supremum for $p_t = M$, where the M-buyer is indifferent between accepting or rejecting.

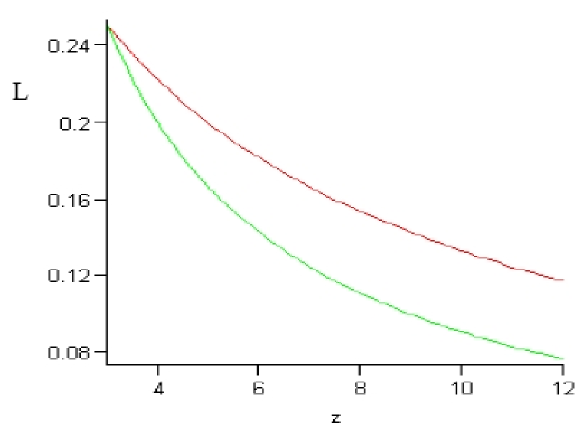
Suppose the M-buyer rejects $p = M$ when $h(t) = (2, 1, z)$. Why does not the monopolist charge a slightly lower price and obtain (5) immediately? As in Lemma 4.2, from the H-buyers' perspective the game resembles a war of attrition. In a symmetric equilibrium H-buyers accept the monopolist's offer with probability (3) and the monopolist gets, in the limit, the payoff (4). The monopolist credible holds to $p_h^* = M$ in state $h(t) = (2, 1, z)$ if

$$\frac{2(H-M)(2M+(z+1)L)}{2H-M-L} > M + (z+2)L, \tag{6}$$

i.e. when the soft M-buyer creates a positive jump in the monopolist payoff function at $p_t = M$. This jump restores the monopolist's credibility to wait out for additional purchases before lowering the price to sell to the remaining buyers (see figure below).



For δ arbitrarily close to 1, a positive jump exists if (2) is satisfied—the upper bound on M is the solution to (6) and $z > x$ insures the actions in table 1 are part of an MPE. The region between the two lines in the next figure gives the values of L as a function of z for which both conditions are satisfied when $H = 1$ (thus values are normalized by H): the upper line $(2/(z+5))$ insures that $M > L$ and the lower line $(1/(z+1))$ that $z > x$. For a given L in this region the range of admissible values of M can be obtained with (2).



Q.E.D.

In all MPE the market clears in finite time with probability one but in the MPE presented above this may take considerable real time even for discount factors arbitrarily close to one. Real time delay is here an unusual distortion of the *lost cake* type.

When the ratio $(H-M)/(H-L)$ is high the H-buyers' hazard rate is low and the expected delay is therefore long. In this case waiting for the H-buyers to self-select would result in substantial delay and a significant loss in the profits made by selling to the remaining buyers and the monopolist will choose to precipitate market clearing by selling to the M-buyer. An attrition equilibrium exist when both the profits on the remaining buyers and the ratio above is not too high (so that the H-buyers acceptance rate is sufficiently high).

5 Conclusion

We first showed that all pure strategy MPE of the standard game with a finite number of buyers are asymptotically efficient. Then, using a tractable example, we showed the existence of MPE where two standard results from the durable-goods literature can fail: asymptotic efficiency and skimming. The mechanism we identified can provide an explanation of monopoly inefficiency in frictionless durable goods markets without relaxing the restriction to Markov strategies.

The mechanism we explored in this paper may also be interesting to the literature of coalitional bargaining with externalities—they are endogenous here and a result of equilibrium play. For example, Gomes (2005) showed that with exogenous positive externalities there can be real time delay when the grand coalition is inefficient or unfeasible: attrition behavior is sustained by the belief of each player that a coalition (not involving themselves) will form. Our game can be seen as a coalitional game where one agent (the monopolist) makes non discriminating offers to the remaining agents (the buyers). Attrition behavior is here sustained by the belief some other buyer may accept the current offer and trigger a price reduction (a better offer), which is consistent when the monopolist makes a single offer to all buyers which they then simultaneously accept or reject. This suggests that non-discriminating offers coupled with simultaneous acceptance can lead to asymptotic inefficient outcomes in coalitional bargaining even when the grand coalition is efficient.

Appendix

Proof of Lemma 2.1. We prove i) in step 1 and ii) in step 2.

Step 1: Let $\overline{p} = \sup\left\{p : a_t^i = 1\ \forall i \in I(t) \subseteq N\right\}$, i.e. the highest price any buyer would accept with probability 1 in all subgames. By profit maximization the monopolist will always offer $p_t \geq \overline{p} \geq 0$ and, for any $\mathbf{S}^*$ we have $\mu^i \leq v(i) - \overline{p}$ for all $i \in N$. Buyer i will accept with probability 1 any other price $\widetilde{p}$ such that

$$v(i) - \widetilde{p} \geq \delta(v(i) - \overline{p}) \Leftrightarrow \widetilde{p} \leq (1-\delta)v(i) + \delta\overline{p}.$$

Since buyer i refuses prices larger than $v(i)$, $\overline{p} = L$ by the definition of $\overline{p}$ and without loss of generality we can restrict the monopolist's action space to $p_t \in [L, H]$.

Step 2: by i) all buyers accept with probability 1 a price $p_t = L$ when it is offered, i.e. $a_t^i = 1$ for all i since

$$v(i) - L \geq \delta(v(i) - p_t) \text{ for all } v(i) \in [L, H] \text{ and } p_t \in [L, H]\,.$$

Q.E.D.

Proof of Claim 1. Suppose that $p_t^* = L$ for every payoff relevant history $I(t)$. If the monopolist offers a price $p_t > L$ then

$$a_t^i = 1 \text{ for all } i : v(i) - p_t \geq \delta(v(i) - p_t) \Leftrightarrow p_t \leq v(i)(1-\delta) + \delta L. \tag{7}$$

The proposed strategy forms an equilibrium if the lost interest from delaying sales is larger than the total premium $l\,(p_t - L)$ extracted on those l buyers which satisfy (7), i.e. if

$$l\,[v(i)(1-\delta) + \delta L] + \delta\,[(n-l)L] \leq nL \Leftrightarrow lv(i) \leq nL \tag{8}$$

Since $l \leq n-1$ and $v(i) \leq H$, (8) is always satisfied if

$$(n-1)H \leq nL \Leftrightarrow \frac{H-L}{L} \leq \frac{1}{n-1}.$$

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