A Model of the "Reasonable Man"

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Preliminary and Incomplete.
Comments welcome.

1 Introduction

In the United States and other common law jurisdictions, many legal rules are decided according to the view of the "reasonable man". The most prominent example is found in criminal law, where a defendant may be found guilty only if there is "proof beyond a reasonable doubt." Other famous examples include the law of torts, where an unwanted touching is a *battery* only if it would be deemed offensive by a reasonable man, and where an act is deemed negligent only if the tortfeasor took less than a 'reasonable' amount of precaution.

A natural question is whether the "reasonable man" can be derived from the opinions of actual agents. The first axiomatic model of the reasonable man is due to Rubinstein [3] who understood reasonableness as a relationship between an agent's preference ordering of outcomes and "realization function" which represents beliefs over the relationship between actions and outcomes. Rubinstein sought to independently aggregate preference orderings and realization functions and reached an impossibility result.

I introduce a different model of "reasonableness". In this model, reasonableness is a characteristic of *responses*, or actions taken upon receipt of observable signals. For every possible signal, each individual gives an opinion as to which actions (out of a very large set) are reasonable responses. These opinions are then aggregated according to some rule.

I introduce four standard axioms, Pareto, monotonicity, anonymity, and neutrality, and show that the only rule satisfying these axioms is the "single-supporter" rule,

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in which a response is deemed reasonable if at least one agent considers it reasonable. This rule is used in some legal settings. For example, in tort law, a defendant can be found liable for negligence only if the jury believes that he did not act as would a "reasonable man". The single-supporter rule corresponds to the unanimous jury rule, found in some jurisdictions, where every member of the jury must agree that the defendant's actions were not reasonable.

2 The Model

2.1 Notation and the Model

There is a set $N \equiv \{1, ..., n\}$ of agents. The space of **actions** is denoted by (A, Σ, μ), where A is the set of actions, Σ is the σ-algebra of subsets of actions, and μ is a measure on (A, Σ). The space (A, Σ) is assumed to be isomorphic to $([0, 1], \mathscr{B})$, where such $\mathscr{B}$ is the set of Borel subsets of $[0, 1]$. I assume that μ is countably additive, non-atomic, finite, non-negative, and that $\mu(A) < +\infty$.[1] Let $\mathscr{A} \equiv \{F \in \mathscr{B} : \mu(F) > 0\}$ be the set of subsets of $\mathscr{A}$ of positive measure.

Let $\Omega \equiv \{\omega_1, ..., \omega_k, ...\}$ denote a finite or countable set of **signals**.

A **reaction** $(\omega, \alpha) \in \Omega \times A$ is a pair of a signal and an action. Each individual has an opinion as to which reactions are reasonable.

A **view** of reasonableness is a mapping $R_i : \Omega \to \mathscr{A}$ from signals to subsets of A of positive measure. The requirement that the signals must map to subsets of positive measure reflects the idea that reasonableness is not perfection. The collection of all views is denoted $\mathscr{R}$. For any two views $R_i, R_i' \in \mathscr{R}$, $R_i \subset R_i'$ if $R_i(\omega) \subset R_i'(\omega)$ for all $\omega \in \Omega$.

For each $i \in N$ there exists an element $R_i \in \mathscr{R}$. A **profile** of views is a vector $R \in \mathscr{R}^n$. A reasonableness rule $f : \mathscr{R}^N \to \mathscr{R}$ is a mapping from a profiles to the **social view** of reasonableness, which will also be denoted R_0.

2.2 Axioms

The first axiom, *Pareto*, requires the social view to consider a reaction unreasonable if it is not considered reasonable by any member of the society.

Pareto: For any $(\omega, \alpha) \in \Omega \times A$ such that $\alpha \notin R_i(\omega)$ for all $i \in N$, $\alpha \notin R_0(\omega)$.

Consider two profiles which are identical except that, in the second profile, one individual changes her opinion and decides that additional reactions are reasonable. The second axiom, monotonicity, requires that any reaction considered reasonable by the social view in the first profile is also considered reasonable in the second.

[1] The space of actions is taken from the model of non-atomic games studied in [1] and [2].

Monotonicity: If $R \subset R^*$ then $f(R) \subset f(R^*)$.

The principle of anonymity requires all agents' views to be treated equally. Individuals' names are switched through a permutation π of N. For a given permutation, $\pi(i)$ is the new name of the individual formerly known as i. For a given profile R, $\pi R \equiv \left(R_{\pi(1)}, ..., R_{\pi(n)}\right)$ is the profile which results once names are switched. The third axiom, anonymity, requires that permutations of the agents' names do not affect whether certain reactions are deemed reasonable.

Anonymity: For every permutation π of N, $f(R) = f(\pi R)$

The principle of neutrality is similar. It requires that a reasonableness rule not treat some actions differently from others on the basis of their names, but only on the basis of the views. Let Φ_μ be the set of all automorphisms of (A, Σ) which preserve the measure μ. Actions' names switched through an automorphism $\phi \in \Phi_\mu$. For a given profile R, $\phi R \equiv (\phi(R_1), ..., \phi(R_n))$ is the profile where the actions' names are switched.

Neutrality: For every automorphism $\phi \in \Phi_\mu$, $\phi\left(f(R)\right) = f(\phi R)$.

2.3 The Single-Supporter Rule

This leads to the single-supporter rule, in which a reaction is considered reasonable if it is considered reasonable by at least one individual.

Single-Supporter Rule: For every $\omega \in \Omega$, $\alpha \in R_0(\omega)$ if $\alpha \in R_i(\omega)$ for some $i \in N$.

Theorem 2.1. *The single-supporter rule is the only reasonableness rule which satisfies Pareto, monotonicity, anonymity, and neutrality. Moreover, all four axioms are independent.*

Proof. Let $R \in \mathscr{R}^N$. By the definition of Pareto, for all $\omega \in \Omega$, if $\alpha \notin R_i(\omega)$ for all $i \in N$, then $\alpha \notin R_0(\omega)$. To prove the claim I must show that if $\alpha \in R_i(\omega)$ for some $i \in N$, then $\alpha \in R_0(\omega)$. Suppose, contrariwise, that $\hat{\alpha} \in R_1(\hat{\omega})$ but that $\hat{\alpha} \notin R_0(\hat{\omega})$. Let $\varepsilon \equiv \min_{i \in N,\, \omega \in \Omega} \mu\left(R_i(\omega)\right)$.

Let $\tilde{R}$ be a profile such that: (1) $\hat{\alpha} \in \tilde{R}_1(\hat{\omega})$, (2) $\tilde{R}_i(\omega_k) \cap \tilde{R}_j(\omega_l) = \varnothing$ unless $i = j$ and $k = l$, (3) $\mu(\tilde{R}_i(\omega_k)) = \frac{\varepsilon}{2^{k+1}n}$ for all $i \in N$ and $\omega_k \in \Omega$, and (4) $\tilde{R} \subset R$.

Anonymity and neutrality imply that $\tilde{R}(\omega) = \cup_{i \in N} \tilde{R}_i(\omega)$ for all $\omega \in \Omega$.

To prove this, suppose, contrariwise, that $\dot{\alpha} \in \tilde{R}_1(\dot{\omega})$ but that $\dot{\alpha} \notin \tilde{R}_0(\dot{\omega})$ It follows from neutrality that $\tilde{R}_1(\dot{\omega}) \cap \tilde{R}_0(\dot{\omega}) = \varnothing$. From neutrality and anonymity it follows that, for all $i \in N$, $\tilde{R}_i(\dot{\omega}) \cap \tilde{R}_0(\dot{\omega}) = \varnothing$, which together with Pareto implies that $\tilde{R}_0(\hat{\omega}) = \varnothing$. But this is a contradiction, as $\{\varnothing\} \notin \mathscr{A}$.

Thus $\hat{\alpha} \in \tilde{R}_0(\hat{\omega})$. By monotonicity, $\tilde{R} \subset R$ implies that $f(\tilde{R}) \subset f(R)$, and therefore, $\hat{\alpha} \in R_0(\hat{\omega})$. This contradiction proves the claim. $\square$

Appendices

A Independence of the Axioms

Claim 1. *The Pareto, monotonicity, anonymity, and neutrality axioms are independent.*

Proof. I present four rules. Each violates one axiom while satisfying the remaining three. This is sufficient to prove the claim.

Rule 1: Consider the degenerate rule in which $R_0(\omega) = A$ for all $\omega \in \Omega$ and all $R \in \mathscr{R}^N$. This satisfies montonicity, anonymity, and neutrality but violates Pareto.

Rule 2: Consider the rule in which $\alpha \in R_0(\omega)$ if $|\{i \in N : \alpha \in R_i(\omega)\}| \geq 2$ or if $|\{i \in N : \alpha \in R_i(\omega)\}| = 1$ and there is no $\alpha' \in A$ such that $|\{i \in N : \alpha' \in R_i(\omega)\}| \geq 2$. This satisfies Pareto, anonymity, and neutrality but violates montonicity.

Rule 3: Consider the rule in which $R_0(\omega) = R_1(\omega)$ for all $\omega \in \Omega$ and all $R \in \mathscr{R}^N$. This satisfies Pareto, monotonicity, and neutrality but violates anonymity.

Rule 4: Let $\alpha^* \in A$. Consider the rule in which $R_0(\omega) = \bigcup_{i \in N} R_i(\omega) \cup \{\alpha^*\} \setminus \{\alpha^*\}$ where for all $\omega \in \Omega$ and all $R \in \mathscr{R}^N$. This satisfies Pareto, monotonicity, and anonymity but not neutrality.

□

References

[1] R. J. Aumann and L. S. Shapley. *Values of Non-Atomic Games.* Princeton University Press, 1974.

[2] P. Dubey and A. Neyman. Payoffs in nonatomic economies: An axiomatic approach. *Econometrica*, 52:1129–1150, 1984.

[3] A. Rubinstein. The reasonable man – a social choice approach. *Theory and Decision*, 15:151–159, 1983.