Chinese Auctions

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A Chinese Auction is one of the most popular mechanisms at charity or other fund-raising events. In a Chinese Auction, bidders buy lottery tickets, which are essentially chances to win items. Bidders may buy as many tickets as they like, and bid them on any item(s) they want by placing them in a basket or other container in front of the item(s) they are trying to win. At the conclusion of bidding, the winning ticket is drawn from the tickets bid on each item, and the item is given to the owner of that ticket.[1]

Chinese Auctions have not been properly analyzed in the literature so far. This paper aims to fill in this gap.

We consider a model where K bidders are competing for N items in the Chinese Auction. Bidders have to decide how to allocate their budgets across all the items. The *main assumption* of this paper is that *the winner for each item is determined stochastically*. In other words, each item contest is a lottery where higher wager means higher chance to win.

We analyze four situations: bidders can have *given* (like in Elections, Blotto games, R&D), or *costly* (like in Chinese auctions) budgets and aim to maximize *the total prize* (like in Chinese Auctions and R&D), or maximize a chance to win the *majority* (like in Elections and Blotto games).

[1] It is unclear whether this type of auction actually originates in China; it is much more likely that the term derives from "chance auction," which is also another name for this type of auction. For more information see http://en.wikipedia.org/wiki/Chinese_auction

How should bidders allocate their budgets? How different the behavior of bidders with different budgets should be? Will all bidders compete for all items? These and similar questions are analyzed in this paper.

Given our main assumption we find a unique symmetric equilibrium of the game in all four situations. It turns out that the players allocate their budgets in the same proportion in the symmetric Nash equilibria. Moreover, the individual equilibrium strategy depends on the contests' values and the individual budget, but it is independent from the budgets of all other players. The equilibria have a monotonic property: a player with higher budget has higher chance to win each contest. It is interesting to note that each player competes in each contest in the symmetric Nash equilibria.

We consider also a situation when individual budgets are private information. It turns out that there exists a unique monotonic Bayesian equilibrium. Each player believes that all players allocate their budgets in the same proportion as it is in the case of complete information, and allocates her own budget in the same way. This is the main result of the paper. Since in many elections candidates indeed do not know budgets of other candidates, our result suggests a campaign budget allocation in these cases. This is the first result of this kind in the literature.

There are two directions in the literature which are closed to the topic of this paper. The first one is different Colonel Blotto games, see Borel (1921), Blackett (1958), Laslier and Picard (2002), Kvasov (2006), and Weinstein (2006) among other. The main difference between our approach and this literature is that our contest winners are determined stochastically (which is almost always the case in the applications) and their contest winners are chosen deterministically: the player with the highest spending in a contest is the winner of this contest. We consider a general case: K players, different budgets, and different values in different contests. We find a unique Nash equilibrium in pure strategies in the general case. All papers on Colonel Blotto games have two competing players and analyze different mixed-strategy equilibria. Borel (1921), Blackett (1958), Weinstein (2006) consider identical budgets, three contests with the same values. Laslier and Picard (2002) consider N contests with the same values. Kvasov (2006) introduces a cost function of the budget and considers identical values and budgets; identical budgets and different values; identical values and different budgets. Our main assumption makes payoff functions continuous and allows obtaining a unique prediction. Classic Colonel Blotto games have multiplicity of equilibria. Our paper is the first which considers a situation when budgets are private information.

The second direction is a contest literature. This literature considers one contest (see Tullock, 1980; Nitzan, 1994, among other) or a sequence of contests (Rosen, 1986; Matros, 2006, among other) when the winner is determined stochastically. The only paper in which players have fixed resources, Matros (2006), considers elimination tournaments. This paper is a natural extension of the contest literature.

There are many applications of our model, such as R&D, arm races, military conflicts, simultaneous rent-seeking activities, and so on.