Two Experts Are Better Than One

Multi-Sender Cheap Talk Under Simultaneous Disclosure

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Abstract

We propose a communication mechanism between a receiver and two perfectly informed senders which are biased in the same direction. By recurring to his priors when receiving inconsistent reports, the receiver can design a simultaneous mechanism that is more informative than a game in which only one sender is consulted. The findings are robust to a wide range of informative bias combinations and differ from the results of the literature on sequential disclosure.

JEL classification: C72, D72, D74, D80, D82.

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"*When [the] experts are agreed, the opposite opinion cannot be held to be certain. When they are not agreed, no opinion can be regarded as certain by a non-expert.*"

Bertrand Russell.

1 Introduction[1]

That there is typically *some* information transmission between a perfectly informed expert (sender) and an uninformed decision maker (receiver) has been highlighted by Crawford and Sobel (1982), henceforth CS. Their seminal work on costless and unverifiable cheap talk has been applied to several different fields, from accounting to political science. A question studied more recently is how the presence of more than one sender may be beneficial for the receiver since it may permit additional communication gains and increase the informativeness of the setting.[2]

Krishna and Morgan (2001a), KM hereafter, have extended and refined this analysis toward a general theory of expertise. In particular, they explain how political decision makers should optimally choose their cabinet of advisors. It is their merit to have characterized a typical *sequential* disclosure mechanism. Since their paper, we speak of a two-sender setting showing opposed biases when the receiver's ideal point is in between the two senders, and of like biases when the two senders are positioned on the same side.

Recent extensions to the literature have introduced multidimensional state and action spaces. Complex decisions often exceed one-dimensional expertise, even if only one expert is consulted, and multidimensional cheap talk, besides permitting the use of different probability distributions, has the advantage of adding substantial communication gains through restricting an expert's ability to exaggerate, as Chakraborty and Harbaugh (2005) have shown. In extending KM's findings on opposing biases, Battaglini (2002) illustrates that with two senders and a two-dimensional state space there may even be *full* information transmission.[3]

Although Battaglini's (2002) idea to introduce two dimensions is applicable to the case of like biases, our paper uses a different approach. We agree with KM that under like biases, sequential disclosure captures well the situation of political talk. What is missing in this literature is an analysis of how the presence of two similarly biased experts can be beneficial in the case of *simultaneous disclosure*, which in our view is the defining characteristic for a variety of examples outside the legislative realm. Think of a corporate CEO who calls two of his subordinates into his office to hear their advice. Business meetings are typical settings where information is disclosed simultaneously; they make use of visual presentations, joint discussions and brainstorming. It furthermore makes perfect sense to assume that teamplayers in the corporate world know their different biases, and how they play their cards when joining in the CEOs office to disclose their view, as we argue to be the case in our model.

The main result of our paper is that simultaneous disclosure permits more informative expertise combinations than sequential disclosure. Our mechanism is not only more informative compared to KM's solution, it is also more informative than the CS equilibrium with the less

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[2]Their work has extended the seminal contributions of Austen-Smith (1990 and 1993).

[3]For an overview see also Ambrus and Takahashi (2005).

biased sender. We thus shed some light on how institutions like business meetings reach a more informative combination of message because of the mutual presence of two experts.

Our paper closes a particular lacuna in the strand of multi-sender cheap talk games. KM have already referred to simultaneous disclosure but limited their analysis to a setting with two experts who are supposed to not know of each other's existence. In what KM call the "coarsest common refinement" (KM, p. 757), they refer to the option of directly combining break points of two CS disclosure profiles. This view relates to some thought modeled implementation literature where "the planner is a player."[4] While KM abandon the underlying idea because of possible strategic interaction between the two senders, our paper uses a different way to combine two simultaneously disclosed messages in a unidimensional setting. Besides KM's seminal contribution, Li (2003) is the paper closest in spirit to ours. Li as well assumes that both experts are perfectly informed, but he limits the state space to three states. In our model, the senders' biases can take any value between 0 and 1, so can the state variable θ. Furthermore, Li's option of the receiver to "do nothing" is similar to our idea to take a default action whenever observing disjunct messages.

Our paper is organized as follows. Section two illustrates the basic properties of the basic CS model and illustrates the KM setting under sequential disclosure and like biases. Section three describes the mechanism that this paper proposes. Section four offers a parametric analysis, section five a generalization, and section six concludes. Similar to Krishna and Morgan (2001b), the existing nonmonotonic cases are discussed in the appendix.

2 Preliminaries

The classic CS model of strategic information disclosure uses two players, a sender and a receiver. The sender has private information on a one dimensional uncertainty space, he observes a state of nature θ that is uniformly distributed on $[0, 1]$. After the sender discloses a message that usually conveys *some* information to the receiver, the latter takes action y that affects the utility of both. The receiver's utility function is

$$U^R(y, \theta) = -(y - \theta)^2. \tag{1}$$

The optimal action y can be found for any θ.by using the maximizer $y^*(\theta) = \underset{y}{arg\,max}\; U_R = (y, \theta)$. Both player's utility functions are twice continuously differentiable. Using KM's notation, we assume the existence of two senders, $i = 1, 2$, that observe θ and are aware of the receiver's maximization problem. Each sender tries to persuade the latter to take an action that comes closest to her own bias b_i, with

Body Math

$$U^S(y, \theta, b_i) = -(y - (\theta + b_i))^2. \tag{2}$$

[4]See e.g. Baliga et al. (1997).

In general, information transmission takes forms between two extremes, typically entailing some information loss. Besides those equilibria, there is one truthful disclosure equilibrium that occurs if the senders' position coincide with the one of the receiver, and the bias is $b_i = 0$. In turn, as CS have shown, there is no meaningful communication if the preferred positions diverge too much. This is the case if $b_i \geq \frac{1}{4}$. In this case, a "babbling" equilibrium emerges in which the sender "babbles" and the receiver does not learn anything, and thus θ remains uniformly distributed.

Let us first assume the receiver interacts with one of the two senders only and that this sender's bias is between $\frac{1}{24} \leq b_1 < \frac{1}{12}$. The CS game has an equilibrium in which the sender chooses one out of three reports, and the break points a_1 and a_2 are $a_1 = \frac{1}{3} - 4b_1$ and $a_2 = \frac{2}{3} - 4b_1$. These equations follow from the equilibrium construction in CS (equation 23).[5] *Fig. 1* depicts the most informative equilibrium with partitions for $b = \frac{1}{18}$. The sender is indifferent at any break point between two adjacent actions y and truth-telling occurs within the intervals:

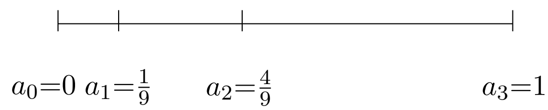

Fig. 1: Most-informative partitions with b=$\frac{1}{18}$

If the receiver interacts only with sender 2, her bias being between $\frac{1}{12} \leq b_2 < \frac{1}{4}$, the break point is now different, using $a_1 = \frac{1}{2} - 2b_2$:

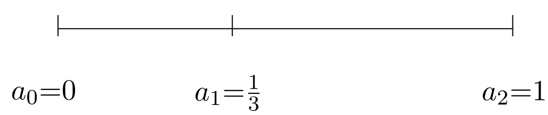

Fig. 2: Most-informative partitions with b=$\frac{1}{12}$

When consulting one sender with her bias being between $\frac{1}{24} > b_1 \geq \frac{1}{40}$, a 4-partition equilibrium occurs. Higher-order equilibria can be computed using equation (23) in CS (p. 1441).

The standard approach to tackle informativeness of partitioning equilibria follows CS: an equilibrium is ex-ante Pareto superior to a given one if the bias decreases since the equilibria are monotonic and a lower bias creates "finer" partition equilibria (CS, p. 1441). Informativeness and informationally superior equilibria follow the CS terminology in this literature. With a "quadratic loss" utility function, the loss over each interval is found using the interval's midpoint as bliss

[5]See the original equilibrium construction for the uniform quadratic case in CS (1982, p. 1441). For an overview see also Krishna and Morgan (2005) and Osborne (2003, ch. 10.8).

point and integrating (1) over all interval borders. The expected utility or ex-ante payoff for the receiver is found by adding all losses from the leftmost to the rightmost interval.[6]

KM have shown that there is a way to combine the messages of both experts that have like biases $b_2 > b_1 > 0$ by asking them *sequentially* for their advice. Since each sender is aware of the other's existence, the receiver now consulting one sender at a time, starting with sender 1. Following the CS no-arbitrage condition, break points are offered that divide the state space into additional partitions.[7] Under this construction, the biases $b_1 = \frac{1}{18}$ and $b_2 = \frac{1}{12}$ lead to the most informative equilibrium with break points at $a_1 = \frac{1}{27}$ and $a_2 = \frac{11}{27}$. The sender's strategy space includes $y_1 = \frac{1}{54}$, $y_2 = \frac{12}{54}$, and $y_3 = \frac{38}{54}$. This equilibrium under sequential talk is illustrated in the state-action space below (*Fig.* 3).

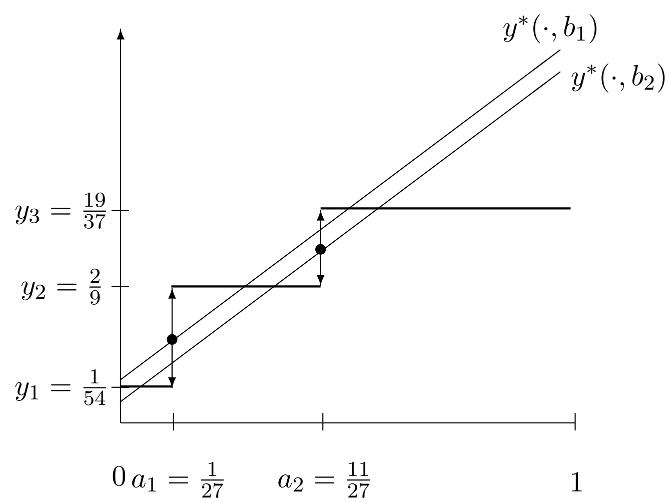

Fig. 3: KM Equilibrium with three partitions and sequential disclosure

3 A hybrid PBE under simultaneous disclosure

We now characterize a game with two senders and like biases, with $b_2 > b_1 > 0$. Both senders prefer "higher actions" compared to the decision maker, whose bias is normalized to zero.

The receiver does not ask one sender at a time but requires that the disclosed messages m_1 and m_2. As in the mechanism design and implementation literature with two senders, the action that receiver chooses is now also dependent on whether both messages are disjunct or not. If they are, the receiver commits to take a default action $\overline{y}$ that in our basic setup is normalize to $\frac{1}{2}$.

[6]An excellent introduction is found in Osborne (2004, ch. 10.8).
[7]See KM (p. 755) and Krishna and Morgan (2005) for a brief overview.

The timing of the game reads as follows:

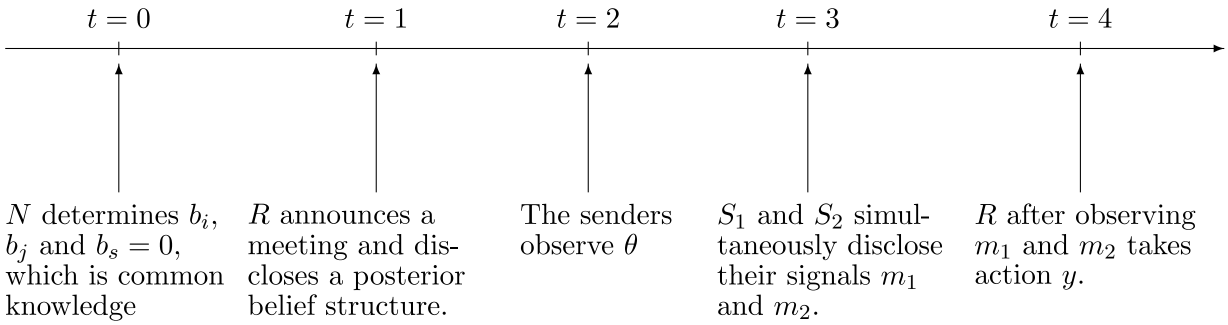

Fig. 4: Timing of the game

Definition 1 *A pure Perfect Bayesian Equilibrium (PBE) with one receiver and two senders S_1* and S_2 *consists of*

(i) *the pure strategy of the receiver as a function $y(m_1, m_2)$, mapping messages m_1* and *m_2 into actions,*

(ii) *of the pure strategies of sender S_1* and S_2 *as a function $\mu(\theta, b_1, b_2)$, mapping states into messages, depending on the own and the opponent sender's bias b_1* and *b_2,*

(iii) *and of the c.d.f. $P(\cdot \mid m_1, m_2)$ specifying the posterior beliefs of the receiver such that:*

(*a*) *For all m_i, $m_j \in [0, 1]^2$, $y(m_1, m_2) = \underset{m_1, m_2}{\arg\max}\ EU^R(y|P(\cdot|m_1, m_2))$,*

(*b*) *Given $y(m_1, m_2)$, for all messages m_1* and *m_2,*

$$m_{1,2}(\theta, b_1, b_2) = \underset{y}{\arg\max}\ E[U^S_{1,2}(y|\theta, b_1, b_2)].$$

(*c*) *The receiver's beliefs $P(\cdot \mid m_1, m_2)$ are derived from senders' strategies (m_1, m_2) using Bayes' rule whenever possible. This requires in particular that the two messages m_1 and m_2 are not disjunct.*

Proposition 1 For all $\frac{1}{24} \leq b_1 < \frac{1}{12}$ there exists a hybrid equilibrium with the following strategies and belief structures*:*

$\cdot$ Sender 1's strategy:

$$\mu_1(\theta, b_1, b_2) = \begin{cases} m_1 \in [0, a_1^1] \text{ if } \theta \in [0, a_1^1] \text{ and } U_1^S(y_1^*(\theta)) \geq U_1^S(\bar{y}), \\ m_1 \in (a_1^1, a_1^2] \text{ if } \theta \in (a_1^1, a_1^2] \text{ and } U_1^S(y_1^*(\theta)) \geq U_1^S(\bar{y}), \\ \quad m_1 \notin (a_1^1, a_1^2] \text{ else}, \\ m_1 \in [a_1^2, 1] \text{ if } \theta \in [a_1^2, 1] \text{ and } U_1^S(y_1^*(\theta)) \geq U_1^S(\bar{y}), \\ \quad m_1 \notin [a_1^2, 1] \text{ else}. \end{cases} \tag{3}$$

$\cdot$ Sender 2's strategy:

$$\mu_2(\theta, b_1, b_2) = \begin{cases} m_2 \in [0, a_2^1] \text{ if } \theta \in [0, a_2^1] \text{ and } U_2^S(y_2^*(\theta)) \geq U_2^S(\bar{y}), \\ \quad m_2 \in (a_2^1, 1] \text{ else}, \\ m_2 \in (a_2^1, 1] \text{ if } \theta \in (a_2^1, 1] \text{ and } U_2^S(y_2^*(\theta)) \geq U_2^S(\bar{y}), \\ \quad m_2 \in [0, a_2^1] \text{ else}. \end{cases} \tag{4}$$

$\cdot$ The receiver's posterior beliefs are

$$P(\cdot \mid m_1, m_2) = \begin{cases} \theta \in [0, \text{a}_1^1] \text{ if } m_1 \in [0, a_1^1] \text{ and } m_2 \in [0, a_2^1] \\ \theta \in [a_1^1, a_2^1] \text{ if } m_1 \in (a_1^1, a_1^2] \text{ and } m_2 \in [0, a_2^1] \\ \theta \in [a_1^2, 1] \text{ if } m_1 \in (a_1^2, 1] \text{ and } m_2 \in (a_2^1, 1] \end{cases} \tag{5}$$

Whenever the messages are disjunct, the receiver takes the default action $\overline{y} = \frac{1}{2}$,which is known to all players. Because of this property being non-Bayesian, the default action is not part of the receiver's posterior beliefs, it however enters the receiver's strategy, which is specified as follows.
$\cdot$ Receiver's strategy:

$$y(m_1, m_2) = \begin{cases} y_1^1 \text{ if } m_1 \in [0, a_1^1] \text{ and } m_2 \in [0, a_2^1] \\ y_1^2 \text{ if } m_1 \in (a_1^1, a_1^2] \text{ and } m_2 \in [0, a_2^1] \\ y_1^3 \text{ if } m_1 \in (a_1^2, 1] \text{ and } m_2 \in (a_2^1, 1] \\ \bar{y} \text{ otherwise}. \end{cases} \tag{6}$$

Using the terminology of KM, the possible break points now depend on the number of partitions and thus on the bias of the more loyal sender. While Definition 1 holds in general for all possible bias settings, note that Proposition 1 above is valid for all 3-partition equilibria, that is for any $\frac{1}{12} > b_1 \geq \frac{1}{24}$ and any informative b_2.

Fig. 5 illustrates the properties of the equilibrium for a three-partition equilibrium using $b_1 = \frac{1}{18}$ and $b_2 = \frac{1}{12}$; the proof to Proposition 1 is given below. Note that when consulting the less biased sender, the break points are a_1^1 and a_1^2, while the hybrid equilibrium creates a fourth partition, with the break points a_1^1, e_2^1, and e_1^2, within the [0,1] interval.[8]

[8] Similar to KM we refrain from using a general notation for all possible partitions for reasons of complexity and legibility. Covering 2- and 3- partition equilibria counts for a wide range of possible equilibria and is common in the literature that rarely illustrates biases of $\frac{1}{40}$ and less (See also Osborne (2004) for an overview of other models).

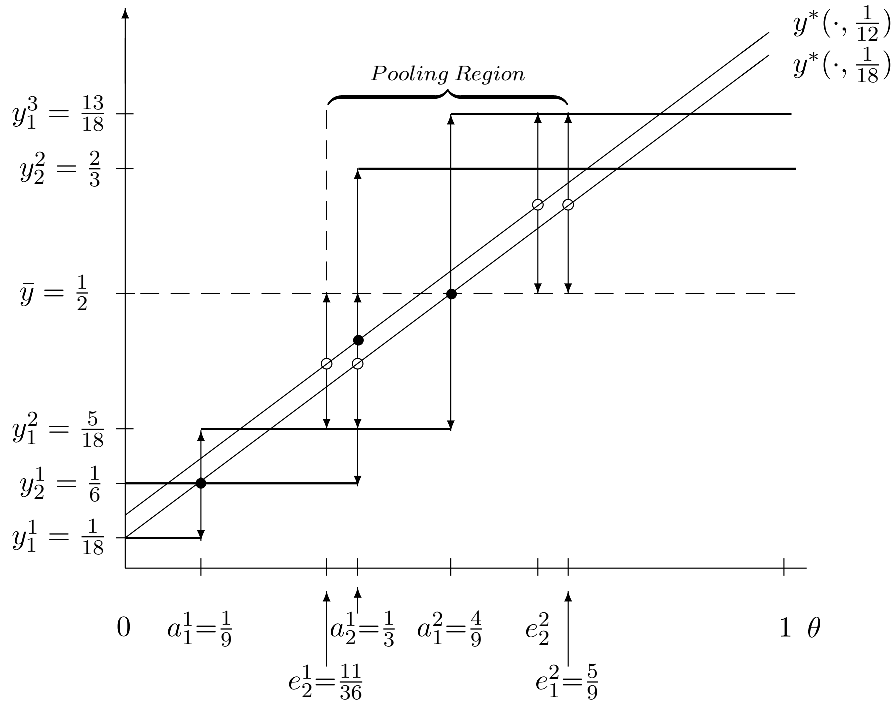

Fig. 5: Hybrid equilibrium under simultaneous disclosure with $b_1 = \frac{1}{18}, b_2 = \frac{1}{12}$

r **Proof of Proposition 1.** To start the analysis it is helpful to use Krishna and Morgan's (2005) suggested method and to define the outcome function for a PBE. In our case this function is defined $Y(\theta, b_1, b_2) = y(m_1(\theta, b_1, b_2), m_2(\theta, b_1, b_2))$, and its inverse $Y^{-1}(y) = \{(\theta, b_1, b_2) : Y(\theta, b_1, b_2) = y\}$. This permits to calculate a partition P that itself specifies y for this partition. Using Y^{-1} we can check whether y is indeed an equilibrium action, given (θ, b_1, b_2).

We first check the receiver's behavior: following Definition 1 we see that he is optimizing given his beliefs. Second, we examine the sender's equilibrium actions following (3) and (4) and ask
(1) Given the receiver's beliefs, is there any chance that one of the two senders deviates?
(2) If upon observation of θ it turns out that at least one sender would prefer action $\bar{y}$, how does the disclosure of the other sender matter? In particular, is it possible that one sender holds interim beliefs about the other sender such that pooling does not anymore occur in the suggested region?

We discuss the following cases:

Case 1. $\theta \in [0, a_1^1]$.

Sending $m_1 \in [0, a_1^1]$ is optimal for sender 1 upon observation of $\theta \in [0, a_1^1]$, and the receiver takes action y_1^1 as long as sender 2 discloses $m_2 \in [0, a_2^1]$. None of the senders will deviate and trigger pooling at $\overline{y}$, since $U_{1,2}^S(y^*(\theta)) \geq U_{1,2}^S(\overline{y})$ is always fulfilled.

Case 2. $\theta \in (a_1^1, e_2^1]$.

Observing $\theta \in (a_1^1, e_2^1]$ makes it optimal for sender 1 to disclose $m_1 \in [a_1^1, e_2^1)$. As in Case 1, sender 2 discloses $m_2 \in [0, a_2^1]$ and the receiver, following his posterior beliefs, takes action y_2^1. Note that at break point a_1^1 this is the optimal action for sender 1. Sender 2, by deviating to

$m_2 \in (a_2^1, 1]$, would again trigger $\overline{y}$. As before, this will not occur since either sender is better off when disclosing truthfully and obtaining y_2^1.

Case 3. $\theta \in (e_2^1, e_1^2]$.

In this interval, at least one sender will deviate.

• We first consider the subinterval $(e_2^1, a_2^1]$. Once θ has reached the value of $e_2^1 = \frac{11}{36}$, sender 2 will prefer to deviate and to disclose discloses $m_2 \in (a_2^1, 1]$, which induces $\overline{y} = \frac{1}{2}$. Sender 1 cannot do better than using her equilibrium strategy and to disclose in $(a_1^2, a_1^2]$.

By contradiction check if sender 1 could improve her payoff by disclosing a message belonging to a different interval. Instead, assume she would disclose in $[0, a_1^1]$. Then, the receiver would observe two messages in disjunct intervals and again implement $\overline{y}$. In turn, should she disclose $m_1 \in (a_1^2, 1]$, this would trigger y_1^3, which is the least preferred action for sender 1. Following this reasoning, sender 2 can be sure that sender 1 will not challenge her own disclosure of sending $m_2 \in (a_2^1, 1]$ and to accept pooling. Deviation to the pooling equilibrium cannot be reverted.

• In the subinterval $(a_2^1, e_2^2]$ both senders are better off under $\overline{y} = \frac{1}{2}$ compared to any other action y offered by the receiver. To accomplish this, one could e.g. assume that both senders would send babbling messages that alone and independently are recognized as meaningless by the receiver.

However, it can be shown that both senders can disclose meaningful messages that belong to disjunct message spaces and so trigger $\overline{y}$. Any disclosure $m_2 \in (a_2^1, 1]$ is understood by the receiver as a meaningful message when matched with any message coming from sender 1 other than $m_1 \in (a_1^2, 1]$. Thus, to disclose $m_2 \in (a_2^1, 1]$ is the dominant strategy for sender 2. As long as sender 1 holds beliefs that sender 2 will disclose $m_2 \in (a_2^1, 1]$, it is sufficient for sender 1 to not disclose $m_1 \in (a_1^2, 1]$ to induce pooling.

• We continue with the next subinterval, namely $(e_1^1, e_2^2]$. Here sender 2 prefers y_1^3 over $\overline{y}$ and will disclose $m_2 \in (a_2^1, 1]$, while sender 1 will trigger $\overline{y}$ through disclosing any other message than $m_1 \in (a_1^2, 1]$.

Assume not. Can sender 2 do better by triggering any other action than $\overline{y} = \frac{1}{2}$? Deviating from this disclosure will make him worse off. Similar to the subcase before, sender 1 holds beliefs that sender 2 will disclose $m_2 \in [a_2^1, 1]$, and it is sufficient for sender 1 to not disclose $m_1 \in (a_1^2, 1]$ to induce pooling.

• The analysis of the third subinterval, namely $(e_2^2, e_1^2]$ is analog to the first subinterval $(e_2^1, e_1^1]$. Here however sender 1 has an incentive to disclose any messages $m_1 \notin (a_1^2, 1]$ since she is better off when $\bar{y}$ is implemented instead of y_1^3.

Assume again that sender 2 would not accept the expected disclosure triggering $\bar{y}$. Deviating to $m_2 \in (0, a_2^1]$ would trigger either y_1^1 or y_1^2, both of them worse outcomes for sender 2.

Case 4. $\theta \in (e_1^2, 1]$.

In this interval, it remains a dominant strategy for sender 1 to disclose $m_1 \in (a_2, 1]$ and for sender 2 to disclose $m_2 \in (a_3, 1]$. Action y_1^3 is implemented. Any unilateral deviation would make the deviating sender worse off. This completes the proof. ■

Proposition 2 For $b_1 = \frac{1}{18}$ and $b_2 = \frac{1}{12}$, the hybrid PBE is more informative than the CS equilibrium in which only the less biased is consulted.

Proof. Note first that the break points for the 3-partition equilibrium are a_1^1, e_2^1, e_1^2, and 1. The expected utility of the receiver in the hybrid equilibrium with $b_1 = \frac{1}{18}$ and $b_2 = \frac{1}{12}$ is

$$EU^R = -\left[\int_0^{a_1^1} \left(\frac{a_1^1}{2}\right)^2 + \int_{a_1^1}^{e_2^1} \left(\frac{e_2^1 - a_1^1}{2}\right)^2 + \int_{e_2^1}^{e_1^2} \left(\frac{e_1^2 - e_2^1}{2}\right)^2 + \int_{e_1^2}^{1} \left(\frac{1 - e_1^2}{2}\right)^2 \right] = -0.0093449. \quad (7)$$

The expected utility in the CS equilibrium with $b_1 = \frac{1}{18}$ is

$$EU^R = -\left[\int_0^{a_1^1} \left(\frac{a_1^1}{2}\right)^2 + \int_{a_1^1}^{a_1^2} \left(\frac{a_1^2 - a_1^1}{2}\right)^2 + \int_{a_1^2}^{1} \left(\frac{1 - e_1^2}{2}\right)^2 \right] = -\frac{17}{972}. \quad (8)$$

■

4 Parametric analysis

We now offer a parametric analysis of different partition equilibria with different biases.

• 2-partition equilibria

We first analyze 2-partition equilibria, with biases of $\frac{1}{4} > b_{1,2} \geq \frac{1}{12}$, $b_1 \neq b_2$. For the entire range of biases, there is always a pooling region around the first break point of the CS model. It is easy to see that replacing the break point a_1^1 through two new break points e_2^1 and e_1^2 always improves the information structure.[9]

To see that the pooling region can never become too large for values $\frac{1}{4} > b_{1,2} \geq \frac{1}{12}$ it is sufficient to check extreme bias differences. The borderline case with $b_1 = \frac{1}{12}$ and b_2 close to $\frac{1}{4}$ reveals the borders of the pooling region to be $e_2^1 = y_1^1 + \left[\frac{1}{2}(\frac{1}{2} - y_1^1)\right] - b_2 = 0.15\overline{8}$ and $e_1^2 = \frac{1}{2} + \left[\frac{1}{2}(y_1^2 - \frac{1}{2})\right] - b_1 = \frac{5}{9}$, rendering the length of the pooling region $e_2^1 - e_1^2 = 0.39\overline{6}$.[10]

• 3-partition equilibria

As in CS we call 3-partition equilibria those for which $\frac{1}{12} > b_{1,2} \geq \frac{1}{24}$, with $b_1 \neq b_2 < \frac{1}{4}$. The CS equilibrium with three partitions shows the first two actions of the receiver y_1^1 and y_1^2 being now always below $\overline{y} = \frac{1}{2}$. Nonmonotonic equilibria now become possible, similar to those discussed in Krishna and Morgan (2001b).

We continue the parametric discussion by first keeping $b_1 = \frac{1}{18}$ as in the first example and increase b_2. The case with $b_1 = \frac{1}{18}, b_2 = \frac{1}{8}$ is sketched in *Fig.* 6 below (the actions y_2^* are removed to keep the graph simple). The break points for the CS equilibrium remain a_1^1 and a_1^2, and for the hybrid equilibrium a_1^1, e_2^1, and e_1^2, the values for the latter have changed.

[9] This follows the general argument in CS, p. 1442.

[10] It is left to the reader to find the values of EU^R both in the hybrid and the CS equilibrium.

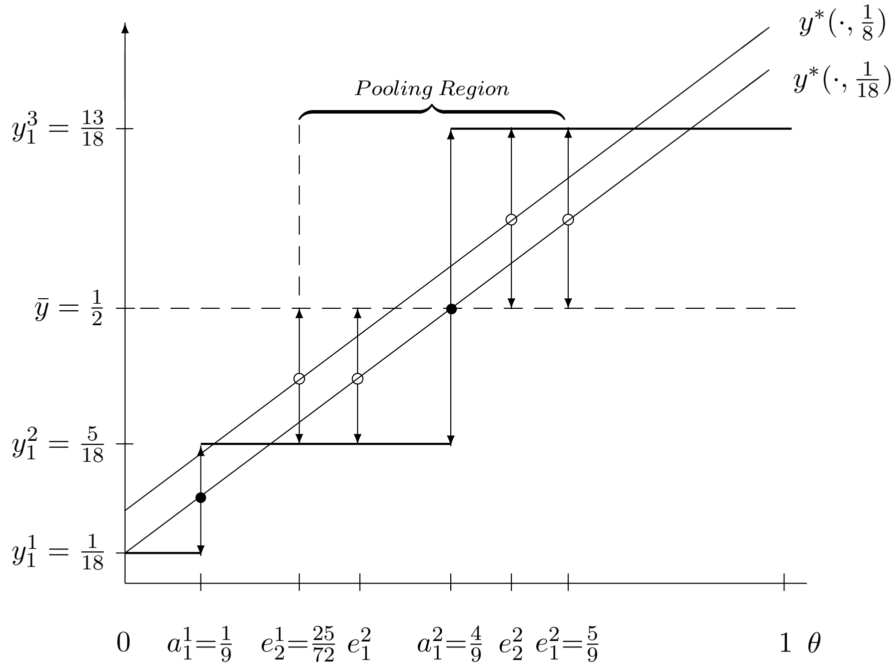

Fig. 6: The same hybrid equilibrium with $b_1 = \frac{1}{18}, b_2 = \frac{1}{8}$

By repeating the steps used in Proposition 2 it is easy to show that again the hybrid equilibrium is more informative, yielding an expected utility for the receiver of

$$EU^R = -\left[\int_0^{a_1^1} \left(\frac{a_1^1}{2}\right)^2 + \int_{a_1^1}^{e_1^2} \left(\frac{e_1^2 - a_1^1}{2}\right)^2 + \int_{e_1^2}^{1} \left(\frac{1 - e_1^2}{2}\right)^2\right] = -0.0097951.$$

Comparing this value to the already computed result in the CS equilibrium in (8) reveals that the hybrid equilibrium leads again to a communication gain.

We now search for the highest possible value for b_2 that satisfies a monotonic equilibrium under the 3-partition setting. This is $b_1 = \frac{1}{18}$. Note that y_1^2 is here *exactly* between y_1^1 and $\overline{y}$ in this particular situation. The cutoff $b_2^{\max}$ is found by again using the formula for $e_2^1 = y_1^1 + \left[\frac{1}{2}(\frac{1}{2} - y_1^1)\right] - b_2$, now setting e_2^1 equal to a_1^1. This leads to $\frac{1}{9} = \frac{1}{18} + \frac{\frac{1}{2} - \frac{1}{18}}{2} - b_2^{\max}$, thus $b_2^{\max}(b_1 = \frac{1}{18}) = \frac{1}{6}$. Any higher value for b_2, together with $b_1 = \frac{1}{18}$, would induce a second pooling region and therefore lead to a nonmonotonic equilibrium.[11] *Fig.* 7 illustrates this case.

[11] A further discussion of nonmonotonic cases similar to KM's supplement in Krishna and Morgan (2001b) is provided in the appendix.

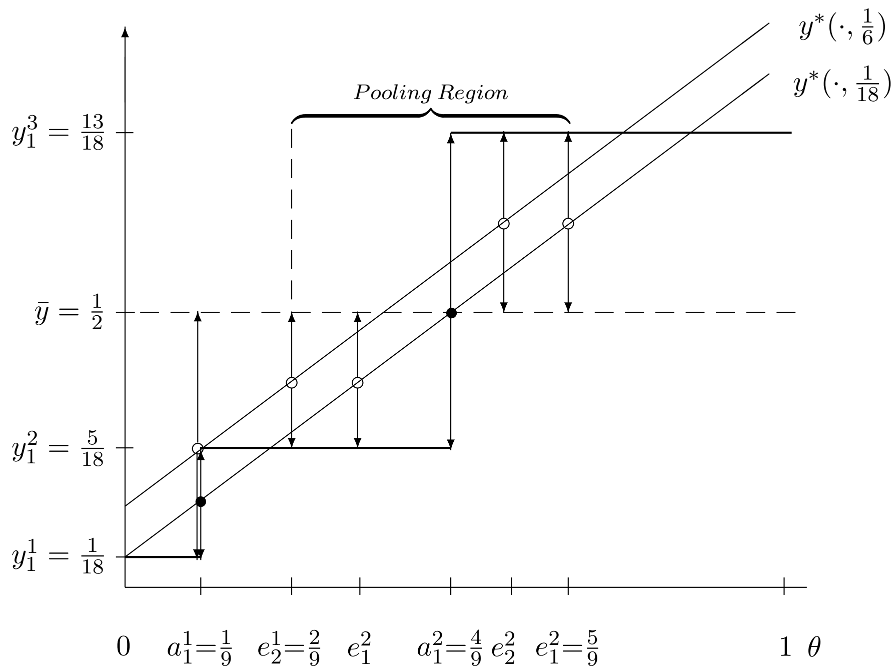

Fig. 7: Borderline case with with $b_1 = \frac{1}{18}, b_2 = \frac{1}{6}$

• **Equilibria with 4 and more partitions**

The parametric discussion can be extended to cover the existence of hybrid equilibria with biases $\frac{1}{32} < b_1 \leq \frac{1}{40}$ (4 partitions) and higher, using the same principle. There are two special cases that emerge for equilibria with 5 and higher partitions: $b_1 = \frac{1}{50}$ has a value of $y_1^{n-1}$of exactly $\frac{1}{2}$, the same holds for the special case $b_1 = \frac{1}{144}$ and y_1^{n-2}. Above $b_1 = \frac{1}{50}$ there are two actions of the receiver above $\bar{y}$, thus the next highest action is y_1^{n-2} for all $\frac{1}{50} > b_1 > \frac{1}{144}$. As in KM we refrain to sketch a general and nonparametric solution, for reasons of complexity and legibility. A simulation in Mathematica is available from the author.[12]

5 A General Setup

{to be written, should contain an analysis of all possible bias combinations and monotonic/nonmonotonic cases.}

[12]The author wants to thank Bibek Dhital for help, time and effort to program this simulation.

6 Conclusion

In this paper we have characterized a cheap talk communication mechanism in which a CEO (receiver) consults two subordinate experts (senders) simultaneously. We find that the uninformed CEO typically profits from simultaneously consulting two informed senders. To add a second expert that is more biased than the first, improves the information structure.

The equilibrium construction permits to generally reward the less biased sender, but it uses the presence of the second experts to profit from any unilateral deviation. We find that the results are robust; our equilibrium concept is typically superior to the CS game for a wide range of informative bias combinations. Our result both contrasts and complements the findings of KM on sequential disclosure mechanisms. It also shows that commitment to refuse expertise altogether does not need to be antithetical to the spirit of studying cheap talk. We have shown that the decision maker can profit from the fact that whenever there are two experts deliver conflicting mesages, the option to remain uninformed enhances the information structure of the setting.

The mechanism we proposed is relatively simple and intuitive and thus opens several possibilities for extensions. Additional research could follow Battaglini's (2002) concept, extending the discussion toward the existence of fully revealing equilibria when the senders' biases become arbitrarily close. Another worthwhile extension could be a scenario with different default actions in case when disjunct messages are observed. Such a setting would share some similarities with the literature on veto power in political science, with a decision maker having the option to change the status quo after observing the experts' opinion. A third research direction that could be pursued lies in the exploitation of the robustness of the mechanism when the two senders do not anymore observe the identical state of nature, e.g., when their observations are noisy. So far, the paper has set the stage for further research into simultaneous disclosure mechanisms in the growing multisender-expertise literature.

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8 Appendix

Two more cases are discussed, starting with $b_1 = \frac{1}{24}$ and $b_2 \sim \frac{1}{4}$ (see *Fig.* 8):

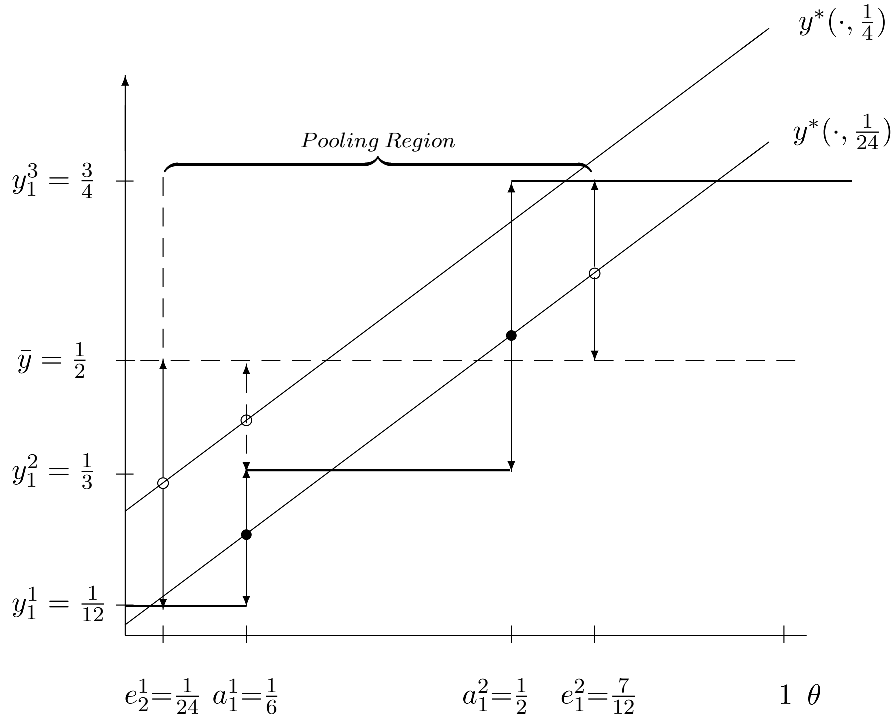

Fig. 8: Borderline case with $b_1 = \frac{1}{24}$ and b_2 close to $\frac{1}{4}$

Note first that the left pooling breakpoint e_2^1 is left of a_1^1. This case nevertheless leads to a monotonic equilibrium: sender 2 induces pooling preferring $\overline{y}$ over y_1^1 right of e_2^1. Once a_1^1 is reached, she still prefers $\overline{y}$ over y_1^2. The pooling region now extends from $e_2^1 = \frac{1}{24}$ to $e_1^2 = \frac{7}{12}$.

Last we consider a nonmonotonic equilibrium under $b_1 = \frac{1}{22}$ and $b_2 \sim \frac{1}{4}$ in which the upper bound $b_2^{\max}$ has been exceeded. Two pooling regions now emerge, the first between $e_2^1 = 0.03\overline{78}$ and $a_1^1 = 0.\overline{15}$, the second between $e_2^2 = 0.159$ and $e_2^2 = 0.575$. Since the payoffs triggered in the two pooling regions are $\overline{y} > y_1^2$ there is a nonmonotonic interval between $a_1^1 = 0.\overline{15}$ and $e_2^2 = 0.15$, as illustrated in Fig. 9 below:

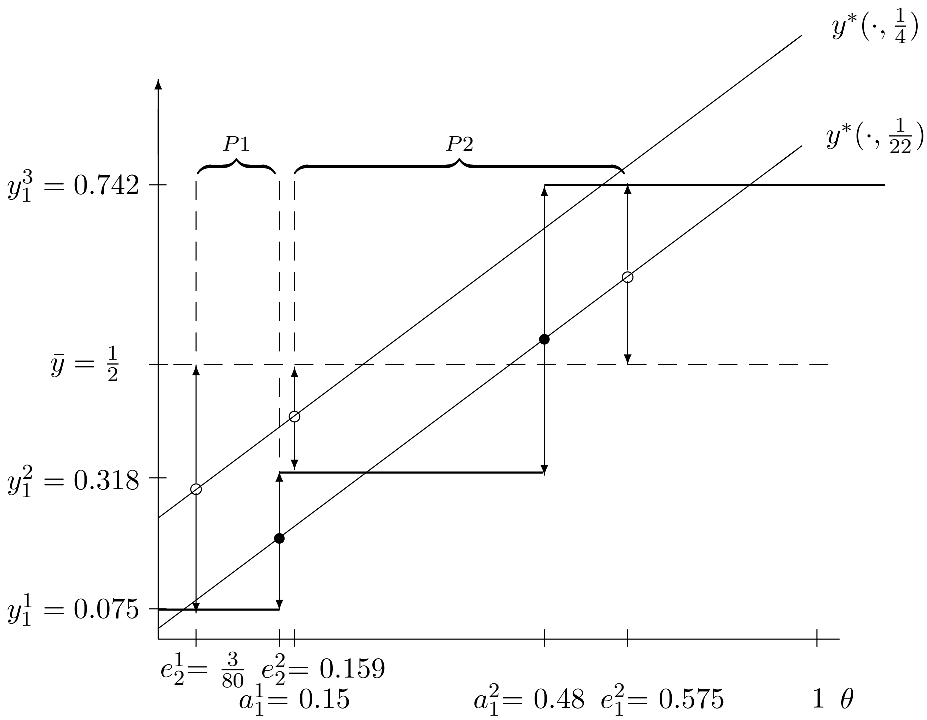

Fig. 9: Nonmonotonic case with $b_1 = \frac{1}{22}$ and b_2 close to $\frac{1}{4}$

This concludes our parametric analysis of nonmonotonic and borderline cases. ■