Using Aftermarket Power to Soften Foremarket Competition*

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Abstract

This paper studies competition among equipment sellers who each monopolize their equipment's aftermarket. However, their aftermarket power is contested by foremarket competition as equipment owners view new equipment as a substitute for their incumbent firm's aftermarket product. I show that such *constrained aftermarket power* allows a larger number of firms to sustain the monopoly profits. More strikingly, as long as existing customers have a shorter market life expectancy than incoming customers, for any discount factor, supranormal profits are sustainable among arbitrarily many firms each selling *ex ante* identical products. Ironically, if the aftermarket is isolated from foremarket competition, then aftermarket power no longer facilitates tacit collusion, suggesting the importance of distinguishing between two types of aftermarket power which are often considered to be qualitatively the same.

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1 Introduction

During the early 1980s, *Kodak* had a practice of selling the replacement parts required to service its photocopiers and micrographics equipment to a number of independent service organizations (ISOs). When *Kodak* unilaterally terminated this practice in 1985, by blocking ISOs' access to the replacement parts, a group of eighteen ISOs sued the company, alleging that its refusal to deal with them constituted an unlawful tying of the sale of its replacement parts to its service. In 1992, the U.S. Supreme Court ruled in favor of the plaintiffs, thereby deciding that *Kodak*'s blockage of sales to the ISOs was indeed illegal.[1] The Supreme Court decision has drawn the attention of — and generated much debate among — legal scholars and economists. The questions of particular interest to economists are, (i) whether a firm without substantial market power in the equipment market would in fact be able to exercise its power in a related proprietary aftermarket, and (ii) whether aftermarket power leads to substantial industry profit and consumer injury.

Borenstein, Mackie-Mason, and Netz (1995, 2000) show that equipment manufacturers who possess aftermarket power tend to set supranormal aftermarket prices even when the equipment market is competitive and customers have perfect foresight and are fully aware of the life-cycle cost. The idea is that as long as firms cannot commit to future prices, they will be tempted to raise the price of the aftermarket service as soon as they have established an installed base from sales in the equipment market.

Shapiro and Teece (1994) and Shapiro (1995) argue, however, that installed-base opportunism is unlikely if equipment manufacturers are concerned about long-term reputation and can provide protection to customers through long-term contracts.

While these studies provide different answers to question (i) concerning firms' abilities to exercise aftermarket power, the authors largely agree on question (ii), suggesting that if the equipment market is competitive, then aftermarket power is irrelevant to firm profits in the sense that with or without aftermarket power, they cannot earn supernormal *overall* profits (i.e., profits from sales of both, the equipment and related aftermarket service). The idea is that even if equipment manufacturers can hold up their customers in the aftermarket, competition in the equipment market would induce them to rebate this profit through the offers of lower equipment prices.

[1] Further details of the case are available, for example, in Hay (1993). Borenstein et al. (2000) observed that over twenty antitrust cases were under legal process against manufacturers at the time their paper was published.

In this paper, I adopt an alternative approach to analyzing the impacts of aftermarket power on firm profits and consumer welfare. Instead of studying how firms aggressively compete against each other in the equipment market, I look at how they may tacitly collude in order to preserve industry profit. I measure the competitiveness of an industry by how difficult it is for firms to conduct tacit collusion in a dynamic setting. My findings suggest that in the presence of aftermarket power, competitiveness of the foremarket should not be presumed even if the foremarket is neccessarily competitive in the absence of aftermarket power. More specifically, I identify conditions under which tacit collusion is impossible to sustain when firms do not possess aftermarket power, and thereafter demonstrate how aftermarket power can restore the tacitly collusive outcome. I further show in an extension that, strikingly, as long as customers newly arrived at the market are expected to stay in the market longer than existing customers do, for any discount factor, tacit collusion is sustainable among arbitrarily many firms possessing aftermarket power. My findings also provide a meaning distinction between two types of aftermarket power which are treated identically in the existing literature.

My analysis exploits the temporal structure of customers' demands for the equipment and aftermarket products and the substitutability between these two products for existing equipment owners. I consider (potentially many) oligopolistic firms competing in the equipment market, each of them the sole provider in the aftermarket for services (or refill supplies) that maintain the continued functionality of their equipment. New customers arrive in the market every period, each staying for multiple periods.[2] Each customer purchases the equipment in the first period of his/her market life and the equipment needs servicing to maintain its continued functionality in the future periods. Products offered by different firms are *ex ante* homogeneous to consumers in the sense that if firms set the same prices for their equipment and aftermarket services, then consumers in the first period of their life value these firms' equipment equally.

The aftermarket power of the firms is protected not only by the incompatibility between the service provided by one firm and the equipment produced by a different firm, but also by the cost advantage in service provision over outright replacement of the equipment. For example, if it cost the same amount to produce a printer cartridge as a printer, or to repair a photo copier as to produce a new one, then manufacturers would have no advantage over others in servicing their established customers in the aftermarket. What is implicit in the preceding discussion is that in the markets I consider, existing

[2] In the main model, consumers stay in the market for two periods. In the extended model, consumers exit the market with a certain probability each period and can potentially stay in the market for any number of periods.

equipment owners can choose to replace previously purchased equipment by buying it afresh from any firm instead of getting it serviced by the incumbent firm. They will do so if maintenance and repair are more expensive than brand new equipment.

To illustrate the impact of aftermarket power on firm profits, first imagine as a *benchmark* situation in which the aftermarket is perfectly competitive, so the repair services are provided at marginal cost. When the aftermarket is competitive, firms earn profits solely from the equipment sale. As a result, they may tacitly collude only in the equipment market. The maximum price firms can potentially charge for the equipment is the present value of a customer's life-cycle consumer surplus net the discounted marginal costs of services she would have to pay in the aftermarket. If a firm deviates by undercutting the equipment price, it will capture the entire industry profit from the incoming generation of customers. However, by triggering a price war, it will lose its share of profits from all future generations of customers. This leads to the result that the industry can sustain any profit between zero and the monopoly profit if the number of firms is no larger than a critical value, but if the number of firms exceeds this critical value, then the unique equilibrium outcome is zero profit. In particular, it is not easier to sustain tacit collusion in an equipment market which is accompanied by a competitive aftermarket than in a single-product market.

Now suppose firms have aftermarket power. When firms possess aftermarket power, equilibrium profits may come from both equipment and aftermarket sales. First consider the case where equipment is sold at a price significantly higher than that of the aftermarket service, yet both the equipment and service sales are profitable. By undercutting the equipment price, a deviating firm is able to capture the entire industry's equipment sales revenue from the incoming generation of customers. However, it will not be able to capture the entire industry's aftermarket sales revenue from this generation of customers' life-cycle demands, for the following reason. By the time the deviating firm sells aftermarket services to these customers, the price war in the equipment market will have begun. Since existing equipment owners consider new equipment and aftermarket service as substitutes, competition from the equipment market will bring down the price of service. It remains true that the deviating firm loses its share of profits from all future generations of customers. The fact that the deviating firm is unable to capture the entire industry profit from a generation of customers before losing its profits from future generations of customers explains why tacit collusion is generally easier to sustain when firms possess aftermarket power.

As the number of firms becomes sufficiently large, successful tacit collusion necessarily entails selling the equipment at a loss and relying exclusively on aftermarket sale for overall profitability. Otherwise the benefit from stealing competitors' equipment-market shares will eventually dominate a firm's future equilibrium profits. Now suppose firms sell both the equipment and aftermarket supply at the same price, where the equipment serves as a loss leader, yet firms each earn a positive life-cycle profit from every customer. Selling both the equipment and services below the marginal cost of equipment can be profitable as long as it costs less to produce the aftermarket supply than the equipment. Since the equipment is priced below cost, any deviation to undercut the equipment price necessarily leads to an immediate loss. Furthermore, in the following period, the price war will bring down both the equipment and aftermarket prices to a level that the overall profit from a customer's life-cycle consumption is zero. As a result, the deviating firm will not be able to sell aftermarket services at the collusive price. More importantly, since the equipment and aftermarket supply are priced at the same level, a deviating firm trying to steal new customers will simultaneously induce its competitors' existing customers to abandon their usable equipment and purchase a new one from it. This deepens the deviating firm's up-front loss from equipment sale. I formally show in an extension that as long as existing customers have a shorter market life expectancy than new customers do, a positive industry profit can be supported among any number of firms.

It is important to point out that in my analysis firms' aftermarket power is subject to competition from the equipment market. I call this type of aftermarket power *constrained aftermarket power*. The constraint on the aftermarket power comes from the substitutability between new equipment and the aftermarket product to existing equipment owners. For example, printers manufacturers who sell cartridges and equipment manufacturers who sell proprietary repair parts for their equipment enjoy constrained aftermarket power. Ironically, the potential competition between the equipment market and the aftermarket plays a key role in facilitating tacit collusion. It is due to this competition that the price war in the equipment market will dampen the portion of the deviation profit coming from aftermarket sales. It is also due to this competition that when both the equipment and aftermarket product are priced at the same level and below the marginal cost of the equipment, a deviating firm who undercuts the foremarket price will attract undesirable established customers of its competitors.

In fact, I formally show that aftermarket power will not help sustain tacit collusion if the aftermarket is completely shielded from the competition from the primary market. I call this type of aftermarket power *unconstrained aftermarket power.* For example, hotel owners who may charge high prices for room service and mini-bar items enjoy unconstrained aftermarket power. The fact that only constrained aftermarket power facilitates tacit collusion suggests it is important to distinguish between these two types of aftermarket power.

In most existing analyses of aftermarket power, the competitiveness of the equipment market is treated as unaffected by firms' aftermarket power. Moreover, whenever the related equipment market is unconcentrated, it is presumed to be competitive. Although my analysis leaves open the possibility that under intense competition in the equipment market aftermarket profits may be rebated to customers, my findings caution that diffusion of market shares in the equipment market does not by itself warrant competition among firms if these firms possess aftermarket power.

The paper is structured as follows. In Section 2, I survey the related literature. In Section 3, I describe my model and present the zero profit equilibrium. In Section 4, I look at how firms support tacit collusion in the absence of aftermarket power. In Section 5, I look at how firms support tacit collusion when they possess aftermarket power; I derive the most effective ways to sustain tacit collusion and characterize the set of equilibrium profits sustainable by tacit collusion. In Section 6, I discuss certain aspects of my model and some driving forces behind my findings. In the section, I also formally prove that *unconstrained aftermarket power* does not facilitate tacit collusion. In Section 7, I generalize my model to show that my main finding that aftermarket power facilitates tacit collusion is robust. Section 8 concludes the paper. Most of the proofs appear to an appendix.

2 Literature

Studies of competition among firms possessing aftermarket power are by now quite voluminous. Here I review some recent contributions that I did not cover in the Introduction.[3] Chen and Ross (1999) show that when aftermarkets of repair are monopolized, manufacturers can use price discrimination to more efficiently serve a market in which customers use the equipment with different intensities and thus value the equipment differently. Aftermarket power allows firms to charge a low price in the primary market and ensures that high-intensity, high-valuation customers pay more in the aftermarket. Introducing competition into the aftermarket removes the industry's ability to price-discriminate and thus may lead to lower consumer welfare. A result of somewhat similar spirit is presented by Carlton and Waldman

[3] Shapiro (1995) and Chen, Ross, and Stanbury (1998) offer detailed reviews of earlier aftermarket theories.

(2001), who point out that if the equipment is priced above marginal cost, either due to monopoly power or brand switching costs, then it is inefficient to have a competitive aftermarket for maintenance. This is because customers tend to repair the equipment when it is socially efficient to replace it with a new one. Carlton (2001) also argues that in the absence of scale effects, it is unlikely for monopolization of the aftermarket (through for e.g., refusal to deal) to be harmful to customers.

By focusing on the anti-competitive potential of aftermarket power, I find that aftermarket power can cause significant consumer injury; I demonstrate that markets, whose equilibria would necessarily entail competitive properties in the absence of aftermarket power, have collusive equilibria once firms possess aftermarket power. My result that an industry can achieve supranormal profits even in the presence of a large number of firms, each selling *ex-ante* homogeneous products in the equipment market challenges the conventional wisdom that when market concentration is low, the market outcome can be presumed to be competitive.

The current study is not the first to explore the anti-competitive effects of aftermarket power. Ellison (2005) provides an interesting theory for high add-on prices that seeks to explain firms' incentives to conceal high add-on prices from customers. He analyzes duopolist firms, each selling a low-quality and a high-quality good, who serve customers of different price elasticities, valuation for qualities, and brand preferences. The high-quality good can be interpreted as the low-quality good coupled with an upgrade add-on. Customers of low price elasticity are assumed to also value quality more. When firms compete by posting two prices, in a separating equilibrium, customers of low price elasticity purchase the high quality good and those with high price elasticity purchase the low-quality good. When firms compete by posting only the price of the low-quality good, customers infer that firms will charge a high fixed markup for a quality upgrade. In the latter case, due to the fixed markup, a firm that wants to cut prices to attract customers is forced to offer equal price cuts for both goods. Such equal price cuts on high-quality and low-quality goods tend to attract disproportionately more price sensitive customers who are less likely to purchase the high-quality good and thus are less profitable. As a result, when firms compete by posting one price instead of two, their incentives to cut prices are weakened and they each earn higher profits.

In a model with naïve customers who systematically underestimate their aftermarket consumption and are not aware of their bias, Gabaix and Laibson (2006) demonstrate why competitive forces may fail to incentivize firms to undercut the industry's high aftermarket prices and inform the naïve customers of its competitors' high aftermarket prices. The rationale is that once a firm has educated a particular group of customers, this group will exert an effort to avoid purchasing any firm's expensive aftermarket product, including the deviating firm's. Therefore, no firm will blow the whistle.

My aftermarket theory differs from those by Ellison and Gabaix and Laibson in several significant ways. First, in their analyses, the high aftermarket prices are supported by the concealment of these prices by firms at the time of equipment sale while in my theory, the high aftermarket prices are publicly observable. Second, in their analyses, high aftermarket prices impact firms' profitability only when products are heterogeneous.[4] In contrast, in my analysis, aftermarket power impacts firms' profitability even when many firms sell homogeneous products. There is also an important difference between the add-on analyzed by Ellison and the aftermarket product studied in this paper. In Ellison's analysis, customers consume the primary product and add-on simultaneously. In contrast, the fact that the aftermarket product are consumed later than the equipment and that the two are substitutes for existing equipment owners play a critical role in my analysis. Consequently, our theories are complementary, each suitable for different types of aftermarkets. My assumption that consumers are fully rational and forward looking also distinguishes my analysis from Gabaix and Laibson's.

Morita and Waldman (2004) show that, by also monopolizing the maintenance market, a durable-goods monopolist can commit not to cut product price after having sold it to the customers with the highest willingness to pay. The commitment is credible because cutting product price will harm the monopolist's profit from maintenance. When consumers anticipate the monopolist's lack of temptation to cut price in the future, early adopters are willingness to pay a higher price for the durable good. As a result, monopolization of maintenance market enhances a durable-goods monopolist's profit like leasing contracts do. In the current paper, aftermarket power impacts firm profits by softening competition among firms instead of helping an individual firm overcome the Coase conjecture. In fact, firms' time-inconsistency problem is absent in my model because of the constant inflow of new demands and my simplifying assumption of unit demands; if the market in my model was served by a monopolist, the monopolist would have been able to sell both the equipment and aftermarket product at consumers' reservation values.

The literature has also studied brand switching cost [first introduced by Klemperer (1987a, 1987b)] as a potential source of aftermarket power. Padilla (1995) and Anderson, Kumar, and Rajiv (2004) show

[4]By extending Gabaix and Laibson's model with behavioral consumers to a dynamic setting, Miao (2006) shows that duopoly firms can earn overall positive profits by exploiting naïve consumers even when products are homogenous.

that switching costs make tacit collusion harder to sustain. This is because brand switching costs protect a deviating firm in the punishment phase.[5] Since consumers do not have to incur a brand switching cost in my analysis, this effect is absent in my model. Another difference is that in these authors' models, firms each sell only one product to both new and existing customers and always charge them the same price. In my setting, firms sell equipment to new customers but induce existing customers to purchase only the aftermarket product, which cost less to produce than the equipment. For tacit collusion to be sustainable among a large number of firms each with a small discount factor, it requires that firms offer the equipment as a loss-leader and earn more than enough profit to compensate for this loss in the sales of the aftermarket product. In the switching cost literature, since new and existing customers pay the same price and the marginal costs of the goods sold to the new and existing customers are identical, profitable loss-leader strategies are impossible.

Finally, this paper is also related to the literature that studies outcomes sustained by tacit collusion in order to evaluate market performance in various other contexts. For instance, Bernheim and Whinston (1990) analyze how multi-market contracts affect firms' ability to sustain high prices; Ausubel and Deneckere (1987), Gul (1987), and Dutta, Matros, and Weibull (2003) analyze how firms use tacit collusion to sustain high prices for durable goods; Nocke and White (forthcoming) show that vertical integration of a single firm can help all firms in an industry sustain tacit collusion; Bernhardt and Chambers (forthcoming) show that profit sharing with workers allow firms to tacitly collude more effectively when demand is uncertain; and Kühn and Rimler (2006) provide a general analysis of how product differentiation impacts firms' ability to tacitly collude. Also see Ivaldi, Jullien, Rey, Seabright, and Tirole (2003) for an excellent review of theories built on the assumption that firms tacitly collude.

3 Environment

There are $n \geq 2$ infinitely lived sellers who each produce two products, the *equipment* and the *repair service/refill supply*, at constant marginal costs C and c, where $0 \leq c < C$. I call the equipment market the *foremarket* and the market for service/supply the *aftermarket*. Throughout the paper, I use the printer – containing an initial cartridge – as a working example of the foremarket product, and

[5] Klemperer (1987a) informally argues that, when monitoring is imperfect, switching costs may facilitate tacit collusion. The idea is that when there are brand-specific switching costs, to successfully steal competitors' customers, a firm has to offer a large price cut, which is necessarily more easily detected by competitors.

the replacement cartridge as a working example of the aftermarket product, but my analysis applies more broadly to markets in which refill supplies or maintenance and repair services of equipment are a significant part of the business.

Consumers arrive in overlapping generations. In each period, a continuum of consumers of measure *one* enter the market and each of them stays in the market for two periods.[6] Firms and consumers have a common discount factor $\delta \in (0,1)$. A consumer in the first period of his life is called a *new consumer*. A consumer in the second period of his life is called an *established customer* (or sometimes *existing customers*) if he already owns a printer. Every consumer demands up to one functional printer in each period of his life, where a functional printer is either a brand new printer or an old one with a replacement cartridge installed. Each printer produced by any firm provides a new consumer a utility of U. Cartridges are firm specific in the sense that a cartridge produced by firm i is compatible only with firm i's printer but not with any other firm's. An established customer values a cartridge compatible with the printer he owns at U but an incompatible cartridge has no value to him. A new printer of any brand is valued by an established customer identically to a cartridge compatible with his existing printer. So although firms have market power in the cartridge market, they may still face competition from the printer market. For this reason, I call firms' market power in the cartridge market *constrained aftermarket power*.

I assume that the production of printers and cartridges is socially efficient:

$$C + \delta c < (1+\delta)\, U. \tag{E}$$

Note that although established customers only value the cartridges of the same brand as their printers, products produced by different firms are *ex ante homogeneous* to new customers.

Firms individually maximize the discounted values of their profits. Established customers maximize their instantaneous consumer surplus and new customers maximize the discounted value of their life-time consumer surpluses.

In each period $t \in \mathbb{N}$, each firm $i \in \mathbb{N}$ simultaneously announces the price of its printer $P_{i,t}$ and if a firm has established customers, it announces the price of its cartridge $p_{i,t}$. In other words, I assume that when a firm sells its printer to a customer, it cannot commit to the future price of a cartridge to this customer.[7] In each period, after the prices are announced, new and established customers make

[6] In Section 7, I check the robustness of my findings and draw more general conclusions by modifying the model to allow consumers to exit the market at a hazard rate strictly between zero and one every period.

[7] This assumption is commonly adopted in the durable goods and switching costs literatures. This assumption is

their purchase decisions.

Throughout the paper, I restrict my attention to symmetric, stationary subgame perfect Nash equilibria. For this reason, the time and firm subscripts for prices are dropped for ease of exposition.

3.1 Zero-Profit Equilibrium

Let (P, p) be an arbitrary printer-cartridge price pair and π be the profit the industry earns from a generation of customers, which I call *per-generation industry profit*. First, in any equilibrium in which firms earn zero profit from each consumer's life-cycle demands, $p = P$ must hold for the following reasons. Suppose $p > P$. Then no cartridges would be sold and the per-generation industry profit would be $\pi = P - C$. Zero profit would imply $P = C$. In this case, a firm could earn a positive profit by lowering its cartridge price to some $p' \in (c, C)$. Next, if $p < P$, then a firm could raise its profit above zero by charging its established customers a higher price $p'' \in (p, P)$ for the cartridge. The deviating firm's established customers will continue to purchase its cartridge because a new printer costs more.

Furthermore, in any zero-profit equilibrium all established customers purchase a compatible cartridge. If some established customers purchased new printers, then some firms could earn a positive profit by lowering the cartridge price by an infinitessimal amount to induce these established customers to purchase the cartridge instead of the printer. Let p^C denote the common price in a zero-profit equilibrium. Then $\left(p^C - C\right) + \delta\left(p^C - c\right) = 0$, or

$$c < p = P = p^C \equiv \frac{C + \delta c}{1 + \delta} < C. \tag{1}$$

It is easy to verify that no firm has an incentive to deviate. While firms earn zero profit overall, the aftermarket price is above marginal cost: $(C + \delta c) / (1 + \delta) > c$. This equilibrium is qualitatively similar to that characterized in Borenstein et al. (2000).[8] The main difference is that the demand for the aftermarket product is downward sloping in Borenstein et al. (2000) so there is consumer injury in their equilibrium but not in mine.

One noteworthy observation at this stage is that while firms earn zero profits from each generation

more reasonable in a more realistic setting where consumers' demand for, and/or the production cost of, the cartridge is uncertain, and/or the quality of the cartridge is not verifiable. For a detailed discussion of other situations under which a long-term contract is infeasible, please see, e.g., Borenstein et al. (1995). In section 6 we discuss how our findings are modified if firms are able to commit to future aftermarket price at the time of equipment sale.

[8] Equilibria with this property have also been reported in the switching cost literature.

of customer, their per-period profits are positive beginning with the second period:

$$2\frac{C+\delta c}{1+\delta}-C-c=\frac{(1-\delta)(C-c)}{1+\delta}.$$

This profit is exactly offset by the loss incurred in the first period of the game so that firms indeed earn zero profit overall:

$$\left(\frac{C+\delta c}{1+\delta}-C\right)+\frac{\delta}{1-\delta}\left(\frac{(1-\delta)(C-c)}{1+\delta}\right)=0.$$

4 Benchmark: Tacit Collusion in the Absence of Aftermarket Power

In this section, I artificially remove firms' aftermarket power and identify the necessary and sufficient condition for tacit collusion to be sustainable. This condition serves as a useful benchmark for later comparison to illustrate how aftermarket power facilitates tacit collusion. In a competitive aftermarket, $p=c$. One can interpret this as being achieved by having sufficiently many sellers selling the cartridge compatible with each printer. Knowing that they only have to pay c for the cartridge and are able to gain a surplus of $(U-c)$ in the second period of their life, new consumers are willing to pay up to $U+\delta(U-c)$ $(>U>0)$ for a printer. As a result, firms may still collude in the market for printers. Suppose firms collude on a printer price $P\in(C,U+\delta(U-c)]$. In this setting, since firms only earn a profit from the sale of printers, the per-generation industry profit becomes $\pi=P-C$. Also, zero-profit pricing means $P=C$. The discounted value of the stream of profits to a firm is $(P-C)/n(1-\delta)$, where n is the number of firms. By undercutting the printer price, a firm can gain an instantaneous profit arbitrarily close to $(P-C)$. Therefore, the condition for the collusive outcome to be sustainable is

$$\begin{aligned}\frac{P-C}{n(1-\delta)} &\geq P-C,\\ n &\leq \frac{1}{1-\delta}.\end{aligned}$$

Let

$$\pi^M\equiv U-C+\delta(U-c)$$

denote the highest possible per-generation industry profit, achievable if firms set $P=U+\delta(U-c)$. This is also the profit a firm monopolizing both the printer and cartridge markets would be able to earn. I can summarize the main observation of this section as follows:

Lemma 1 *Suppose the cartridge price is exogenously fixed at* c. *Then any per-generation industry profit* $\pi \in [0, \pi^M]$ *is sustainable by tacit collusion if*

$$n \leq \frac{1}{1-\delta}.$$

Firms necessarily earn zero profit otherwise.

The condition for sustainability of tacit collusion reported in Lemma 1 is identical to the well known condition for tacit collusion among firms competing in a single-product market. In the next section, I show that with constrained aftermarket power, however, profitable tacit collusion is sustainable among a larger number of firms. In particular, the industry can achieve positive profits by tacit collusion among any number of firms and for any discount factor, suggesting that low market concentration does not guarantee competition when firms possess constrained aftermarket power.

5 Tacit Collusion Facilitated by Constrained Aftermarket Power

My main objective in this section is to identify, for all $\delta \in (0, 1)$ and for all $n \geq 2$, the range of *steady state per-generation industry profits* that can be supported by tacit collusion. In my analysis, I assume that any deviation from tacit collusion is punished by all firms reverting to the zero-profit equilibrium prices $P = p = \frac{C+\delta c}{1+\delta}$ as stated in (1) forever,[9] where the common zero-profit price for printer and cartridge is below the cost of a printer but above the cost of a cartridge, $c < \frac{C+\delta c}{1+\delta} < C$. Maintaining this assumption on the punishment path of tacit collusion, I define the *most effective collusive prices* as follows:

Definition 1 *For any given per-generation industry profit* π, *a printer-cartridge price pair* (P, p) *that yields the per-generation industry profit* π *are the most effective collusive prices if and only if they minimize the deviation payoff.*

While any given per-generation industry profit may be achieved by many combinations of printer and cartridge prices, it is obvious that if the most effective collusive prices fail to sustain this per-generation industry profit, then there exists no other price pair which can support such profit, as the alternative

[9] This may not constitute the maximal punishment. Therefore, our results might be strengthened if we required firms to implement the maximal punishment.

prices necessarily lead to a higher deviation payoff. Therefore, for the purpose of characterizing the set of industry profits sustainable by tacit collusion, there is no loss of generality in assuming that firms always adopt the most effective collusive prices. For this reason, I adopt this assumption throughout.

5.1 Identifying the Most Effective Collusive Prices

I would first like to begin the derivation of the most effective collusive prices by pointing out one intuitive observation:

Lemma 2 *The most effective collusive prices must satisfy $p \leq P$.*

Proof. Suppose instead that $p > P$ in equilibrium. Every established customer would strictly prefer purchasing a new printer to purchasing a replacement cartridge. So no cartridges would be sold and the per-generation industry profit would be $\pi = (P - C) + \delta (P - C)$. By cutting the printer price below P, a deviating firm could steal all the new and established customers.

Suppose firms instead coordinate on the common printer and cartridge price p', where

$$p' = P - \frac{\delta (C - c)}{1 + \delta} < P < p,$$

so that cartridges will be sold in equilibrium. One can verify that the equilibrium per-generation industry profit would remain at $(P - C) + \delta (P - C)$. However, it would now require a deviating firm to cut the printer price below p' to steal the new and established customers. This would lower the deviation profit and weaken the incentives to deviate. ■

It is clear that for established customers to be willing to purchase the cartridge, the cartridge price must not exceed their reservation value U. However, new customers may still purchase the printer even if its price exceeds their reservation value, as long as they expect to earn a positive surplus from the consumption of the cartridge. Suppose consumers expect to pay $p \leq U$ for the cartridge and earn a surplus of $(U - p)$ in the second period of their life. Then they are willing to pay up to $U + \delta (U - p)$ for the printer. In other words, for both the printer and cartridge to be purchased, it is necessary that $p \leq U$ and $P \leq U + \delta (U - p)$. Suppose firms collude on the price pair (P, p) such that the industry is earning a profit of $\pi \equiv (P - C) + \delta (p - c) > 0$ from each generation of customers. In the steady state, by staying on the equilibrium path, each firm will earn a discounted profit of

$$\frac{\pi}{n (1 - \delta)} = \frac{(P - C) + \delta (p - c)}{n (1 - \delta)}$$

from customers entering the market in the current and all the future periods. Note that a profit of $(p - c)/n$ which comes from the established customers who already purchased the printer in the previous period is excluded from this expression.

Now consider a firm's deviation payoff. First, look at the case where $p < P$. Since consumers are rational, they can anticipate both the printer and cartridge prices to become $\frac{C+\delta c}{1+\delta}$ according to (1) in the period following a unilateral deviation. Because consumers can purchase either the printer or the cartridge at the same price once the price war begins, ownership of an old printer does not affect the second-period consumer surplus. This implies that the deviating firm has to cut the printer price below U to attract new consumers, or otherwise these consumers will respond to the deviation by abstaining from consumption for one period. Summing up, the deviating firm can attracts a whole generation of new customers by setting a printer price P' arbitrarily close to but below $\min\{P, U\}$.

Since $\min\{P, U\} > p$, the deviating firm can also simultaneously raise the cartridge price up to P' without losing its measure $1/n$ of established customers or inducing them to purchase its new printer which costs more than the cartridge. This leads to an instantaneous deviation profit arbitrarily close to

$$(\min\{P, U\} - C) + \frac{\min\{P, U\} - p}{n}.$$

If the firm cuts the printer price further so that P' is arbitrarily close to but less than p, then it also attracts a measure $(n-1)/n$ of established customers from its competitors. By doing so, it will earn an instantaneous profit arbitrarily close to $(2n-1)(p - C)/n$. Note that the deviating firm has to lower its cartridge price to P' as well to avoid having its existing customers replace its old printer with a new one. However, since P' is arbitrarily close to p, this price cut does not affect the deviation profit.

Whether the deviating firm undercuts $\min\{P, U\}$ or p, the new consumers it attracts will continue to purchase the cartridge from it at the price of $p^C = (C + \delta c)/(1+\delta)$ in the following period, allowing it to earn an additional discounted profit of $\delta(C - c)/(1+\delta)$. The deviating firm does not earn additional profits from the competitors' existing customers it has attracted because they will leave the market in the following period.

Due to the ensuing price war, the deviating firm will not earn any more profit from future generations of customers. This gives rise to the following incentive constraint for firms to stay collusive:[10]

$$\frac{\pi}{n(1-\delta)} \geq \max\left\{(\min\{P,U\}-C)+\frac{\min\{P,U\}-p}{n}+\delta\frac{(C-c)}{1+\delta},\frac{(2n-1)(p-C)}{n}+\delta\frac{(C-c)}{1+\delta}\right\}, \text{ for } p<P. \tag{2}$$

Next, look at the case where firms collude by setting $P=p$. When a deviating firm undercuts the equilibrium printer price, it attracts the whole generation of new customers as well as all the established customers of its competitors. By cutting the price of its cartridge by the same infinitesimal amount it can avoid inducing its own established customers to purchase its new printer. Therefore, the incentive constraint becomes

$$\frac{\pi}{n(1-\delta)} \geq \frac{(2n-1)(p-C)}{n}+\delta\frac{(C-c)}{1+\delta}, \text{ for } p=P. \tag{3}$$

To most effectively collude, for any given per-generation industry profit level π that the firms target to achieve, firms choose a price pair (P,p) satisfying $(P-C)+\delta(p-c)=\pi$ such that the deviation profit is minimized. This transforms the identification of the most effective collusive prices into the following problem of minimizing a firm's deviation payoff:

$$\begin{gathered}\min_{(p,P)} D = \begin{cases} \max\left\{\frac{(n+1)\min\{P,U\}-p-nC}{n}+\delta\frac{(C-c)}{1+\delta},\frac{(2n-1)(p-C)}{n}+\delta\frac{(C-c)}{1+\delta}\right\} & \text{if } p<P, \\ \frac{(2n-1)(p-C)}{n}+\delta\frac{(C-c)}{1+\delta} & \text{if } p=P, \end{cases} \\ \text{subject to } \begin{array}{c}(P-C)+\delta(p-c)=\pi, \\ p\leq P.\end{array}\end{gathered} \tag{4}$$

The following proposition characterizes the most effective collusive prices that solves Problem (4):

Proposition 1 *Let* $\tilde{\pi}=U+\delta\frac{(n+1)U+(n-1)C}{2n}-C-\delta c$. (i) *If* $U>C$, *then* $\delta(C-c)<\tilde{\pi}<\pi^M$, *and the most effective prices are*

$$(P,p)=\begin{cases}\left(\frac{\pi+C+\delta c}{1+\delta},\frac{\pi+C+\delta c}{1+\delta}\right) & \textit{if } \pi\in[0,\delta(C-c)), \\ \left(\frac{2n\pi+(2n-\delta(n-1))C+2n\delta c}{2n+n\delta+\delta},\frac{(n+1)\pi+2nC+(n+1)\delta c}{2n+n\delta+\delta}\right) & \textit{if } \pi\in[\delta(C-c),\tilde{\pi}), \\ \left(\pi+\frac{(2n-\delta(n-1))C+2n\delta c-(n+1)\delta U}{2n},\frac{(n+1)U+(n-1)C}{2n}\right) & \textit{if } \pi\in[\tilde{\pi},\pi^M],\end{cases} \tag{5}$$

(ii) *If* $U\leq C$, *then* $\pi^M\leq\tilde{\pi}\leq\delta(C-c)$, *and, for all* $\pi\in[0,\pi^M]$, *the most effective collusive prices are*

$$(P,p)=\left(\frac{\pi+C+\delta c}{1+\delta},\frac{\pi+C+\delta c}{1+\delta}\right).$$

[10] Recall that the profit from the established customers who already purchased the printer in the previous period is excluded from both sides of the inequality sign.

Proof. See Appendix. Figure A1 is included to enhance the exposition of the proof. ■

Here I discuss some properties of the most effective collusive prices as derived in Proposition 1. First, when the industry intends to support a relatively low profit level, as measured by $\pi < \delta (C - c)$, firms will charge the same price for both the printer and cartridge. These prices are both below the marginal cost of a printer:

$$\frac{\pi + C + \delta c}{1 + \delta} < C \Leftrightarrow \pi < \delta (C - c).$$

The advantage of setting identical prices both below the marginal cost of the printer is that when a firm deviates by undercutting the printer price, it necessarily attract its competitors' existing customers to abandon their old printers and buy new ones from the firm, forcing the firm to incur an immediate loss on every printer sold to these established customers. Since these established customers will leave the market in the following period, the deviating firm is unable to recoup this loss. The net loss on competitors' established customers will offset some of the deviation profit the deviating firm obtains by capturing the printer sale to a whole generation of the new customers.

As firms try to support a larger per-generation industry profit, as measured by $\pi \in [\delta (C - c), \tilde{\pi})$, the printer and cartridge prices necessarily have to be raised above the marginal cost of a printer. In this case, it becomes profitable to steal competitors' established customers. Any given π can be achieved by either a low p with a high P or a high p with a low P. The first option will lead to a high deviation payoff from undercutting just the printer price; the second option will lead to a high deviation payoff from undercutting both the printer and cartridge prices. Since the deviating firm is free to choose either option to deviate, the deviation incentive is minimized when firms post prices such that a deviating firm feels indifferent between undercutting the printer price and undercutting both prices. The equalization of deviation payoffs happens at

$$(P, p) = (\frac{2n\pi + (2n - \delta (n-1)) C + 2n\delta c}{2n + n\delta + \delta}, \frac{(n+1)\pi + 2nC + (n+1)\delta c}{2n + n\delta + \delta}).$$

As the targeted per-generation industry profit is set higher, the printer price will be pushed beyond consumers' reservation value for a printer, U. This happens when $\pi > \tilde{\pi}$. In this case, a deviating firm has to discretely lower the printer price to below U in order to steal new customers. Furthermore, the deviating firm can also choose to undercut the cartridge price in order to steal its competitors' existing customers. The deviation profit from cutting the printer price to just below U still decreases in p because the deviating firm will simultaneously raise its cartridge price from p to just below U, and a lower p allows the deviating firm to raise the cartridge price by a greater amount. By the same logic adopted in the previous paragraph, the price pair

$$(P,p) = (\pi + \frac{(2n - \delta(n-1))C + 2n\delta c - (n+1)\delta U}{2n}, \frac{(n+1)U + (n-1)C}{2n})$$

is chosen such that a deviating firm is indifferent between the two deviation options. Note that p is independent of π for $\pi \geq \tilde{\pi}$.

In the case where $C \geq U$, π^M is sufficiently small that it falls below $\delta(C-c)$. Therefore, any feasible profit is most effectively supported by identical printer and cartridge prices.

5.2 Characterization of Profits Sustainable by Tacit Collusion

Building on Proposition 1, I proceed to characterize the set of per-generation profits sustainable by tacit collusion. In the discussion following Proposition 1, I pointed out that when the industry tacitly colludes on the most effective collusive prices, the firm choosing to deviate either has no choice but to undercut the cartridge price since $p = P$, as when $\pi < \delta(C-c)$, or is indifferent between undercutting $\min\{P, U\}$ and undercutting p, as when $\pi \geq \delta(C-c)$. Therefore, the characterization of the equilibrium profit set can be accomplished by plugging (5) into (3), assuming that that the deviating firm always undercuts the cartridge price.

Theorem 1 (i) *First, suppose $U > C$. Then $\pi^M > \delta(C-c)$. In this case, there exist $\hat{n}_1$, $\hat{n}_2$, and $\hat{n}_3$, where*

$$\frac{1}{1-\delta} < \hat{n}_1 < \hat{n}_2 < \hat{n}_3,$$

such that the following hold. (i.i) *If the number of firms is no larger than $\hat{n}_1$, then any per-generation industry profit $\pi \in [0, \pi^M]$ can be supported by tacit collusion.* (i.ii) *If the number of firms is in the interval $(\hat{n}_1, \hat{n}_2]$, then any per-generation industry profit*

$$\pi \in \left[0, \tfrac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}\right] \cup \left[\tfrac{(1-\delta)(2n-1)(n+1)(U-C)}{2n} + \tfrac{\delta n(1-\delta)(C-c)}{1+\delta}, \pi^M\right]$$

can be supported by tacit collusion. (i.iii) *If the number of firms is in the interval $(\hat{n}_2, \hat{n}_3]$, then any per-generation industry profit*

$$\pi \in \left[0, \tfrac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}\right]$$

can be supported by tacit collusion. (i.iv) *If the number of firms exceeds $\hat{n}_3$, then any per-generation industry profit*

$$\pi \in \left[0, \tfrac{\delta(1-\delta)(n-1)(C-c)}{2(n(1-\delta)-1)}\right]$$

can be supported by tacit collusion.

(ii) *Suppose* $U \in \left(\frac{(2+\delta)C+\delta c}{2(1+\delta)}, C\right]$. *Then* $\pi^M \in \left(\frac{\delta(C-c)}{2}, \delta(C-c)\right]$. *In this case, there exists* $\hat{n}_4 \geq \hat{n}_3$ *such that* (ii.i) *if* $n \leq \hat{n}_4$, *then any per-generation industry profit* $\pi \in \left[0, \pi^M\right]$ *can be supported by tacit collusion.* (ii.ii) *If* $n > \hat{n}_4$, *then any per-generation industry profit*

$$\pi \in \left[0, \frac{\delta(1-\delta)(n-1)(C-c)}{2(n(1-\delta)-1)}\right]$$

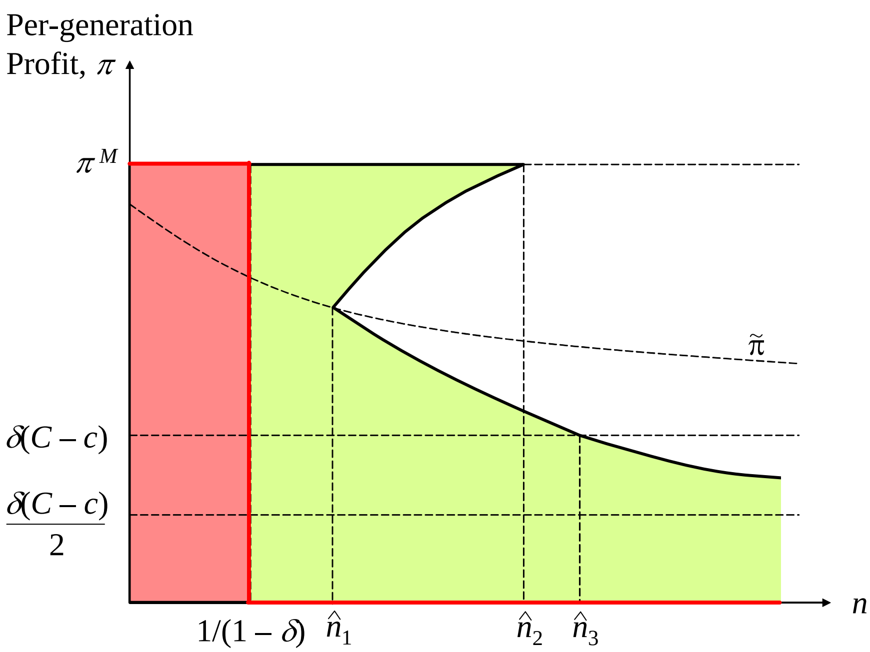
can be supported by tacit collusion.

(iii) *Suppose* $U \in \left(\frac{C+\delta c}{1+\delta}, \frac{(2+\delta)C+\delta c}{2(1+\delta)}\right]$. *Then* $\pi^M \in \left(0, \frac{\delta(C-c)}{2}\right]$. *In this case, for all* n, *any per-generation industry profit* $\pi \in \left[0, \pi^M\right]$ *can be supported by tacit collusion.*

Proof. See Appendix. ■

Figure 1 depicts the set of per-generation industry profits that can be supported by tacit collusion in the case where $U > C$. This corresponds to the characterization reported in part (i) of Theorem 1.

Figure 1: Set of collusive per-generation industry profits, *C* < *U*



The curves $\pi = \delta(C-c)$ and $\pi = \tilde{\pi}$ in Figure 1 divide the set of feasible profits into three regions as they are divided into three cases in Proposition 1: $[0, \delta(C-c))$, $\pi \in [\delta(C-c), \tilde{\pi})$, and $\pi \in [\tilde{\pi}, \pi^M]$. The per-generation profits in different regions are most effectively supported by prices with different expressions reported in the proposition.

Recall that when the aftermarket is perfectly competitive, any profit between zero and the monopoly profit can be supported by tacit collusion among equipment manufacturers whenever $n \leq 1/(1-\delta)$, but firms necessarily earn zero profit otherwise. According to Theorem 1, when firms possess aftermarket powers, the full set of feasible profits is sustainable among a larger number of firms, up to $n = \hat{n}_1$ if $U > C$, up to $n = \frac{1+\delta}{1-\delta}\frac{2(U-C)+\delta(C-c)}{2(1+\delta)(U-C)+\delta(C-c)}$ if $U \in \left(\frac{(2+\delta)C+\delta c}{2(1+\delta)}, C\right]$, and for any number of firms if $U \in \left(\frac{C+\delta c}{1+\delta}, \frac{(2+\delta)C+\delta c}{2(1+\delta)}\right]$. Moreover, even when the number of firms exceeds these values, the industry can still maintain a positive profit in the presence of aftermarket power.

It is obvious that aftermarket power does not enhance the industry's profit potential when $n \leq (1-\delta)^{-1}$ because in this case the market is concentrated enough so that firms are able to use tacit collusion to achieve any profit between zero and the monopoly profit with or without aftermarket power. As the number of firms becomes sufficiently large, as indicated by $n > (1-\delta)^{-1}$, however, firms can sustain supranormal profits only if they possess aftermarket power. This yields the implication that equipment manufacturers may be willing to accommodate competition in the aftermarket if the equipment market is relatively concentrated, but monopolization of the aftermarket becomes essential if the equipment market shares are sufficiently diffuse.

Now I provide some intuition as to why it is easier to sustain tacit collusion when firms possess aftermarket power than when they do not. When the aftermarket is perfectly competitive, firms earn profits from each generation of customers only in the first period of their life through the sale of printers. In this case, any deviation allows a deviating firm to capture the entire industry profit from a generation of customers, or in other words scale up its profit from one generation of customers for one period by n times. The consequence is the loss of its share of profits from all future generations of customers.

When firms each possess constrained aftermarket power, however, the profit from each generation of customers is split into two parts. The first part comes from the sale of printers and the remainder comes from the sale of cartridges which takes place one period later. Suppose for the time being $p^C < p < P \leq U$ and that the printer price is sufficiently larger than the cartridge price so that the deviating firm (weakly) prefers stealing only new customers' business. By undercutting the printer price, the deviating firm captures the entire industry's printer sale from the incoming new customers. However, when it sells its cartridge to these customers in the following period, the price war will already have begun and consequently caused the cartridge price to drop to p^C. As a result, the deviating firm is unable to capture the whole industry's life-cycle profit from a generation of customers before it loses its equilibrium profits from all future generations of consumers. This comparison makes clear that constrained aftermarket power facilitates tacit collusion.

When the industry targets a profit higher than $\tilde{\pi}$, the profit is most effectively supported by setting the printer price above consumers' per-period reservation value, i.e., $P > U$. When $P > U$, there is another effect that limits the incentive to deviate. Because consumers are rational and anticipate a price war upon observing a deviation, a deviating firm has to cut the printer price below the reservation value, which is discretely below the printer price, to attract new consumers. If the deviating firm set any price above the reservation value, new customers would choose not to consume for one period. In other words, the equilibrium profit increases in P over $(U, U + \delta (U - p)]$ but the deviation profit does not. This explains why, as illustrated in Figure 1, when $n \in (\hat{n}_1, \hat{n}_2]$, it is possible to sustain high but not moderate profits.

Firms can earn a positive per-generation industry profit by charging the same price for the printer and cartridge, with the common price set below the marginal cost of the printer. This happens when the targeted profit is sufficiently modest ($\pi \leq \delta (C - c)$). In this case, the printer serves as a loss leader and firms rely on the sale of cartridges to earn an overall positive profit. Such a pricing strategy further weakens the incentive to deviate. When a firm deviates by undercutting the common price for the printer and cartridge, it has to incur an immediate loss to serve the demand of all the customers it attracts which includes competitors' established customers. The deviating firm can recoup the up-front loss on the printer sale to the new consumers by selling to them the cartridge in the following period, although at a lowered price. What makes deviation particularly inefficient and unprofitable is the fact that the deviating firm has to produce a new printer for every established customer of its competitors at a loss, who otherwise would have bought a cartridge instead, yet these customers will leave the market in the following period. The smaller deviation profit renders tacit collusion easier to sustain.

Part (ii) and part (iii) of Theorem 1 characterize the equilibrium profit set in cases where consumers' reservation value is relatively low, leading to a moderate monopoly profit. Part (ii) of the theorem shows that as the monopoly profit falls below $\delta (C - c)$, which happens when $U \leq C$, the number of firms among which the monopoly profit can be support increases to $\hat{n}_4$. Part (iii) of the theorem points out that if the monopoly profit falls below $\delta (C - c)/2$, which happens when $U \leq \frac{(2+\delta)C+\delta c}{2(1+\delta)}$, then the monopoly profit is sustainable among any number of firms.

A corollary of Theorem 1 is that profitable tacit collusion is sustainable even when there are arbitrarily many firms.

Corollary 1 *For all* $\delta \in (0,1)$*, as* n *approaches infinity, the set of per-generation industry profit sustainable by tacit collusion converges to* $\left[0, \min\{\pi^M, \frac{\delta(C-c)}{2}\}\right]$.

Proof. When $U \leq \frac{(2+\delta)C+\delta c}{2(1+\delta)}$ so that $\pi^M \leq \frac{\delta(C-c)}{2}$, according to part (iii) of Theorem 1, π^M remains the upper bound of the sustainable profit as n goes to infinity. When $U > \frac{(2+\delta)C+\delta c}{2(1+\delta)}$ so that $\frac{\delta(C-c)}{2} < \pi^M$, according to parts (i) and (ii) of Theorem 1, for sufficiently large n, the upper bound the of the sustainable profit becomes $\frac{\delta(n-1)(1-\delta)(C-c)}{2(n(1-\delta)-1)}$, where $\lim_{n\to\infty} \frac{\delta(n-1)(1-\delta)(C-c)}{2(n(1-\delta)-1)} = \frac{\delta(C-c)}{2}$. ■

It is important to note that the assumption that established customers have no future cartridge demands is not crucial for profitable tacit collusion to be sustainable among any number of firms. I show in Section 7 with an extended model that profitable tacit collusion remains sustainable among arbitrarily many firms as long as new customers have a longer market life expectancy than existing customers. Put differently, this striking result holds as long as serving existing customers is not as profitable as serving new ones.

5.3 First-Period Profit

In the main portion of the paper, I focus on identifying the set of sustainable collusive profits in the steady state. I have shown that, for all n, the set of collusive equilibrium profits in the steady state is (weakly) larger when firms possess aftermarket power. However, to complete the argument that aftermarket power makes it easier to tacitly collude, I have to show that the same result holds if the profits for comparison are evaluated in the first period of the game. For this purpose, I assume that firms possessing aftermarket power coordinate according to Theorem 1 to achieve the largest set of steady state profits starting from the second period. This is sufficient because the set of collusive equilibrium profits evaluated in the first period derived based on this assumption is in general a subset of the full set of profits sustainable by tacit collusion. If this subset is larger than what is achievable in the absent of aftermarket power, then so is the full set.

The main difference between the first period and any other period is that there are no established customers in the first period so the deviating firm does not get to steal its competitors' established customers. Suppose firms charge P_1 for the printer in the first period. Since consumers are rational and anticipate both the printer and cartridge prices to fall to $(C + \delta c) / (1 + \delta)$ upon seeing a deviation, a deviating firm attracts all first generation customers if and only if it set a printer price lower than $\min\{P_1, U\}$. This leads to the first-period incentive constraint:

$$\frac{1}{n}\left(P_1 - C + \delta(p-c) + \delta\frac{P - C + \delta(p-c)}{(1-\delta)}\right) \geq \min\{P_1, U\} - C + \delta\frac{(C-c)}{1+\delta}. \tag{6}$$

Now I show that constrained aftermarket power helps sustain a larger set of first period discounted industry profit.

Lemma 3 (i) *For any targeted discounted industry profit in the first period* $\Pi \in [0, \frac{\pi^M}{1-\delta}]$*, there exists* $\bar{n}(\Pi) > (1-\delta)^{-1}$ *such that* Π *can be supported by tacit collusion for all* $n \leq \bar{n}(\Pi)$*. (ii) Furthermore, there exists* $\overline{\Pi} > 0$ *such that any discounted industry profit* $\Pi \in [0, \overline{\Pi}]$ *can be supported by tacit collusion for all* $n \geq 2$*.*

Proof. See Appendix. ■

6 Discussion

6.1 Potential competition between the equipment market and aftermarket

In the Introduction, I argued that the potential competition between the equipment market and the aftermarket plays a crucial role in supporting tacit collusion, and that aftermarket power would *not* facilitate tacit collusion if the aftermarket is *not* subject to competition from the equipment market. To formalize my argument, here I consider the following modification to the model.

6.1.1 Unconstrained Aftermarket Power

Now I modify the main model to eliminate the competition between the foremarket and aftermarket products. Here I assume that an established customer can derive utility only from the aftermarket product provided by his equipment manufacturer but not from any new equipment. Because of the change in the nature of the aftermarket product, I call it an *add-on*. Let the utility from the add-on be $V > c$ which may or may not be equal to U. Without facing competition from the equipment market, firms can charge their established customers up to $p = V$ for the aftermarket product both in a collusive equilibrium and on the punishment path.

In a zero-profit equilibrium, which is also assumed to be the punishment path of tacit collusion, firms necessarily charge $p^C = V$ for the add-on; otherwise a firm can generate a positive profit by raising its add-on price. The zero-profit condition $\left(P^C - C\right) + \delta(V - c) = 0$ also implies that $P^C = C - \delta(V - c)$.

Now suppose firms tacitly collude on a price pair (P, p). Expecting to pay $p \in [c, V]$ for the add-on in the second period of their life, new customers are willing to pay up to $U + \delta(V - p)$ for the equipment. For firms to earn non-negative profits from a customer's lifetime demand, it is required that $P \geq C - \delta(p - c)$. For any $(P, p) \in [C - \delta(p - c), U + \delta(V - p)] \times [c, V]$, the discounted value of the stream of profits to a firm from incoming customers is $\frac{[P - C + \delta(p - c)]}{n(1-\delta)}$. If any firm deviates, in the following period, *all* firms which have established customers will raise its price of add-on to V. Given firms' identical treatments of established customers on the punishment path, a deviating firm is able to capture all the incoming new customers by undercutting the equilibrium printer price by an infinitesimal amount. This implies that a deviating firm is able to earn a deviation profit of $P - C + \delta(V - c)$ by capturing the entire incoming generation of customers. It will then lose all the profits from future generations. Therefore, it is incentive compatible for firms to charge the equilibrium prices if and only if

$$\frac{P - C + \delta(p - c)}{n(1 - \delta)} \geq P - C + \delta(V - c).$$

By raising p and lowering P while keeping the equilibrium profit $\pi = P - C + \delta(p - c)$ constant, firms can lower the deviation profit. Therefore, to most effectively sustain tacit collusion, firms must set $p = V$. Plugging this back into the incentive constraint, tacit collusion is sustainable if and only if

$$n \leq \frac{1}{1 - \delta}.$$

When firms' aftermarket power is not contested by the equipment market, the onset of a price war in the foremarket does not prevent the deviating firm from selling its add-on to the customers it has stolen at the equilibrium price. Also, the deviating firm does not attract undesirable customers from its competitors. In other words, a deviating firm can capture the entire industry profit from one generation of customers, just as in the case of a competitive aftermarket or a single product market, before losing the profits from all future generations of customers. This explains why unconstrained aftermarket power does not facilitate tacit collusion.

6.1.2 Distinction between Two Types of Aftermarket Power

In the main model of this paper, firms' aftermarket power originates from the incumbent firm's cost advantage in *restoring* the functionality of the equipment, and the aftermarket product and equipment are *substitutes* for existing customers. Refill supplies and repair services for equipment fall into this category. Another type of aftermarket power is built upon the incumbent firm's unique position to provide add-on products or services to *enhance* the functionality of the equipment. In the latter case, the foremarket product is *not a substitute* for the add-on product to an established customer. In other words, the aftermarket power of an add-on is not constrained by competition from the foremarket. As I have shown in the preceding analysis, monopolization of the add-on products does not facilitate tacit collusion either. Examples of such add-on products include room service and minibar items sold in a hotel room or memory upgrade for a PC. I believe the observation that the competition softening effect of aftermarket power arises only among firms selling refill supplies but not among firms selling add-on products provides a meaningful distinction between these two types of aftermarket power which are treated identically in existing studies.

6.2 Long-term Contracts/Bundling

Firms' ability to commit to future prices often significantly impacts market outcomes. For example, commitment to future prices can solve the hold-up problem arising from brand switching cost and help a durable-good monopolist overcome the Coase conjecture (Farrell and Shapiro 1988, p. 123). In my model, one possible strategy firms can adopt to commit to future cartridge prices is to issue to printer buyers a coupon with which they can later purchase from the firm the replacement cartridge at a discounted price. I call this bundling although literally selling the printer and cartridge in bundle is less efficient than issuing a coupon, as bundling requires the firm to incur the production cost of the cartridge one period before it is consumed.

Given my assumption that firms cannot sell bundles, in an equilibrium with $P = p < C$, a deviating firm undercutting the printer market unavoidably attracts its competitors' established customers who do not need the replacement cartridge in the future. However, if bundle offers are allowed, then the deviating firm can offer a bundle to target new customers for business stealing and avoid attracting the undesirable established customers. So allowing firms to offer bundles will eliminate the strongest result that tacit collusion is sustainable among arbitrarily many firms.

Here I argue, however, allowing firms to offer bundles will not affect the general finding that tacit collusion is easier to sustain when firms possess aftermarket power. More precisely, with aftermarket power, profitable collusive outcomes in which no firms offer bundles can be supported among more than $1/(1-\delta)$ firms even if they are free to offer bundles. To illustrate this point, suppose the industry targets to support some per-generation industry profit $\pi > \delta(C-c)$ in the case where $n \in (1/(1-\delta), \hat{n}_3]$. Also suppose the industry coordinates on a printer-cartridge price pair (P, p) as specified according to Proposition 1. Recall that even when firms cannot offer bundles, a deviating firm is still free to choose between undercutting P just to attract new customers and undercutting p to attract established as well as new customers and it can sell to every new customer it attracts a replacement cartridge in the following period at the price p^C. Now suppose the deviating firm can bundle a coupon for a replacement cartridge with a printer. Since consumers are forward looking and can anticipate the cartridge price to drop to p^C in the following period, the maximum they are willing to pay for the bundle is $P + \delta p^C$. Therefore, the deviating firm can attract all new customers by selling the bundle at this price. If it wants to attract competitors' established customers, it has to sell the printer at p. In that case, new customers will accept the bundle only if the bundle is priced no higher than $p + \delta p^C$. Otherwise, they will purchase the printer at the price p and wait until the following period to purchase the cartridge. As a result, the deviating firm will steal exactly the same profit as in the case when bundling is not allow. This allows us to apply Theorem 1 to argue that for all $n \in (1/(1-\delta), \hat{n}_3]$, firms can support some per-generation industry profit $\pi > \delta(C-c)$.

It is also important to point out that in reality, bundling simply is not feasible in some markets because consumers' future demand and firms' production costs of the aftermarket product are uncertain, and/or the quality of the aftermarket is nonverifiable. Finally, although firms' ability to commit to future prices renders tacit collusion among arbitrarily many firms infeasible in my analysis, it is not generally true that price commitment weakens firms' ability to tacitly collude. For instance, Dana and Fong (2006) show that long-term contracts can actually facilitate tacit collusion in markets where firms do not possess aftermarket power.

7 Extension: Generalized Flow of Consumers

In the main body of my analysis, I assume that consumers live exactly two periods and exit the market with certainty afterward. A more realistic model would allow consumers to potentially stay in the market for longer and not to exit in such an abrupt manner. In this section, I modify the main model to allow for these features. By doing so, I demonstrate that the facilitation of tacit collusion owing to firms' aftermarket power is a general property that extends to markets wherein consumers exhibit a exit rate between zero and one. The extended model to be presented here includes, as special cases, the model analyzed in the previous sections as well as markets where the exit of established consumers exhibits a constant exit rate property. It will also be clear from my presentation in this section that the striking result that tacit collusion can be sustained among arbitrarily many firms applies as long as the market life expectancies of established customers are lower than that of new customers.

7.1 Model Modification

I first describe the entry and exit of consumers in periods starting from the second period. In every period $t \geq 2$, a measure one of consumers arrives. New consumers remain in the market in the following period with probability θ. All established customers, regardless of when they arrived, remain in the market in the following period with probability $\phi \in [0, \theta]$. To ensure that the market arrives at a steady state in the second period, I assume that $\theta + \phi = 1$ and that in the first period, $1/\theta$ new customers enter the market. It is clear that starting from the second period, there are a measure one of new consumers and a measure one of established customers in every period. Note that this model captures two polar cases: (i) $\theta = \phi = 0.5$ (constant exit rate) and (ii) $\theta = 1$, $\phi = 0$ (the main model). When $\theta = \phi$, new customers and established customers have the same *market life expectancy*. When $\theta > \phi$, established customers have a shorter market life expectancy than new customers do.

7.2 Zero Profit Equilibrium

Following the same procedure as in Section 3, I can compute the zero profit equilibrium prices. With the zero profit condition

$$(P - C) + \delta\theta \left[1 + \frac{\phi\delta}{1 - \delta\phi}\right](p - c) = 0$$

and the requirement that $P = p$, I can pin down the zero profit equilibrium prices to be

$$P^C = p^C = \frac{(1 - \delta\phi)\, C + \delta\theta c}{1 - \delta\phi + \delta\theta}. \tag{7}$$

7.3 Tacit Collusion

By serving both the foremarket and aftermarket, a monopolist could earn from each generation of consumers

$$\pi^M = U - C + \delta\theta \frac{(U-c)}{1-\delta\phi}.$$

As in Section 3, if the cartridge price were fixed at c, possibly caused by competitive supply in aftermarkets, then collusive pricing and full (consumer) surplus extraction could be sustained in equilibrium if and only if $n \leq (1-\delta)^{-1}$.

I focus the remainder of this section on deriving the maximum steady-state profit firms can sustain through tacit collusion, without imposing a fixed cartridge price. The tacit collusion I consider is supported by trigger strategies in which firms revert to the zero profit equilibrium pricing (7) as soon as any firm deviates. I prove the following result:

Proposition 2 (i) *For all* $(\theta, \phi) \in [0,1]^2$ *such that* $\phi = 1-\theta$ *and* $\phi \leq \theta$*, there exists* $\tilde{n} > \frac{1}{1-\delta}$ *such that any per-generation industry profit* $\pi \in [0, \pi^M]$ *is sustainable if and only if* $n \leq \tilde{n}$*. Moreover,* (ii) *for all* $n \geq 2$*, any per-generation industry profit*

$$\pi \in \left[0, \min\{\pi^M, \tfrac{\delta(\theta-\phi)(C-c)}{2(1-\delta\phi)}\}\right]$$

can be supported by tacit collusion.

Proof. See Appendix. ■

Proposition 2 shows that aftermarket power allows a larger number of firms to sustain any profit between zero and the monopoly profit, whether established customers have a shorter market life expectancy or not. This is because regardless of the rates at which customers exit the market, it remains true that after a deviating firm steals the new generation of customers from its competitors, it will not be able to charge these customers the equilibrium cartridge price in the following period. In other words, aftermarket power still prevents a deviating firm from stealing the entire industry profit from one generation of customers.

However, since $\frac{\delta(\theta-\phi)(C-c)}{2(1-\delta\phi)} > 0$ if and only if $\theta > \phi$, the proposition above also clarifies the fact that the sustainability of profitable tacit collusion among arbitrarily many firms of any discount factor relies on the property that established customers exit at a higher hazard rate than new customers, i.e., established customers have a shorter market life expectancy. This property can be a consequence of consumers' market lifetimes being finite or due to the fact that the products are targeted to consumers of a particular age group (e.g. entry level printers targeted to college students).

To see why the industry can sustain positive profits as long as established customers have shorter market life expectancies than new customers, suppose the industry tacitly collude on some identical printer and cartridge prices whereby firms earn a small but positive expected equilibrium profit in serving each generation of customers' market lifetime demands. Because the printer and cartridge are priced at the same level, a deviating firm necessarily attracts the other firms' established customers when it tries to steal new customers. The deviating firm has to provide new equipment to every established customer it steals, just as it does to new customers. It is true that the deviating firm is able to steal the entire industry profit from the new customers. However, as established customers demand fewer cartridges in their life time than new customers, the profit from an established customer it steals is strictly less than the profit from a new customer. Because of this, one can always find a low enough, yet positive, equilibrium profit from each generation of customers whereby a deviating firm takes a loss on the established consumers it steals and this loss dominates the profit it earns on its stolen new customers.

8 Conclusion

In this paper, I illustrate how aftermarket power can soften competition among firms. The time lag between foremarket consumption and aftermarket consumption and the substitutability between the foremarket and the aftermarket products for established customers prevents a deviating firm from capturing the entire industry profit from a generation of customers before losing the profits from all future generations of customers. This remains true even if a firm can deviate by offering a bundle, as long as consumers are rational and can anticipate a price war upon seeing a price cut. I believe this competition softening effect is very general.

I also prove the stronger result that when firms possess aftermarket power, a supranormal industry profit is sustainable among any number of firms and for any discount factor. This result hinges on the assumptions that firms do not sell foremarket and aftermarket products in a bundle and that established customers have a shorter market life expectancy. Although these assumptions can be justified in many real-world settings, this strong result should not be accepted wholesale. There are other extraneous factors that can prevent a large number of firms from tacitly colluding. For one, the monitoring of deviation may become difficult when the number of firms becomes sufficiently large.

Another implication of my analysis is that it is important to distinguish aftermarket power originating from monopolization of addons from aftermarket power originating from monopolization of refill supplies. This is because according to my aftermarket theory, only the latter source of aftermarket power softens competition among firms.

I make the important assumption that firms revert to the zero-profit equilibrium following a deviation. In general this is not the maximal punishment. Future research should investigate what the maximal punishment should be and whether tacit collusion can sustain larger set of profits than what I have identify when firms enforce harsher punishment.

Another obvious and important extension for future research is to allow for downward-sloping demand functions. While I believe my analysis provides useful insights on how aftermarket power impacts firm profitability and consumer welfare, my model with unit demands is not suited to the analysis of the overall welfare of the market. In my analysis, consumers always consume both the equipment and aftermarket products so the first best is always achieved; any consumer injury caused by aftermarket power is captured by the industry as profit.

Appendix

Proof of Proposition 1. By substituting the rearranged constraint

$$P = \pi + C - \delta(p - c) \tag{8}$$

into the minimization problem (4), the latter can be rewritten as:

$$\min_{p\in[0,\bar{p}]} D(p,\pi) = \begin{cases} \max\{\min\{f_1(p,\pi), f_2(p)\}, f_3(p)\} & \text{if } p < \bar{p}, \\ f_3(p) & \text{if } p = \bar{p}, \end{cases} \tag{9}$$

where $\bar{p} \equiv \frac{\pi+C+\delta c}{1+\delta}$ and

$$\begin{aligned} f_1(p,\pi) &= \frac{(n+1)(\pi + C - \delta(p-c)) - p - nC}{n} + \frac{\delta(C-c)}{1+\delta}, \\ f_2(p) &= \frac{(n+1)U - p - nC}{n} + \frac{\delta(C-c)}{1+\delta}, \\ f_3(p) &= \frac{(2n-1)(p-C)}{n} + \frac{\delta(C-c)}{1+\delta}. \end{aligned}$$

Notice that

$$\frac{\partial f_1}{\partial p} = -\frac{(n+1)\delta + 1}{n} < \frac{\partial f_2}{\partial p} = -\frac{1}{n} < 0 < \frac{\partial f_3}{\partial p} = \frac{2n-1}{n}. \tag{10}$$

Let $p = \hat{p}_{12}$ solve $f_1(p,\pi) = f_2(p)$, $p = \hat{p}_{13}$ solve $f_1(p,\pi) = f_2(p)$, and $p = \hat{p}_{23}$ solve $f_2(p) = f_3(p)$. It can be verified that

$$\hat{p}_{12} = \frac{\pi - U + C + \delta c}{\delta}, \tag{11}$$

$$\hat{p}_{13} = \frac{(n+1)\pi + 2nC + (n+1)\delta c}{2n + n\delta + \delta}, \tag{12}$$

$$\hat{p}_{23} = \frac{(n+1)U + (n-1)C}{2n}. \tag{13}$$

By applying (10), we can also obtain that

$$\begin{aligned} f_1(p,\pi) < f_2(p) &\qquad \text{if and only if } p > \hat{p}_{12}, \\ f_1(p,\pi) < f_3(p) &\qquad \text{if and only if } p > \hat{p}_{13}, \\ f_2(p) < f_3(p) &\qquad \text{if and only if } p > \hat{p}_{23}. \end{aligned} \tag{14}$$

Next, it can be verified that $f_1 = f_2 = f_3$ and $\hat{p}_{12} = \hat{p}_{13} = \hat{p}_{23}$ if and only if

$$\pi = \tilde{\pi} \equiv U + \delta\frac{(n+1)U + (n-1)C}{2n} - C - \delta c.$$

Since

$$\frac{\partial \hat{p}_{23}}{\partial \pi} = 0 < \frac{\partial \hat{p}_{13}}{\partial \pi} = \frac{n+1}{2n+\delta n+\delta} < \frac{\partial \hat{p}_{12}}{\partial \pi} = \frac{1}{\delta},$$

$$\begin{aligned} &\hat{p}_{12} < \hat{p}_{13} < \hat{p}_{23} && \text{if } \pi < \tilde{\pi}, \\ &\hat{p}_{12} \geq \hat{p}_{13} \geq \hat{p}_{23} && \text{if } \pi \geq \tilde{\pi}. \end{aligned} \tag{15}$$

It can be verified that

$$\begin{aligned} \pi^M - \tilde{\pi} &= \frac{\delta(n-1)}{2n}(U-C), \\ \tilde{\pi} - \delta(C-c) &= \frac{2n+\delta(n+1)}{2n}(U-C). \end{aligned}$$

So,

$$\begin{aligned} &\delta(C-c) < \tilde{\pi} < \pi^M \quad \text{if } C < U, \\ &\delta(C-c) \geq \tilde{\pi} \geq \pi^M \quad \text{if } C \geq U. \end{aligned} \tag{16}$$

(i) First consider the case that $C < U$.

(i.i) Also suppose for now $\pi < \tilde{\pi}$. Then according to (15), $\hat{p}_{12} < \hat{p}_{13} < \hat{p}_{23}$. Applying (14), it follows that

$$\max\{\min\{f_1(p,\pi), f_2(p)\}, f_3(p)\} = \begin{cases} f_2(p) & \text{if } p < \hat{p}_{12}, \\ f_1(p,\pi) & \text{if } p \in [\hat{p}_{12}, \hat{p}_{13}), \\ f_3(p) & \text{if } p \geq \hat{p}_{13}. \end{cases}$$

According to (10),

Lemma A1 *If* $\pi < \tilde{\pi}$*, then* $\max\{\min\{f_1(p,\pi), f_2(p)\}, f_3(p)\}$ *is decreasing in* p *for* $p < \hat{p}_{13}$ *and increasing in* p *for* $p \geq \hat{p}_{13}$.

If $\bar{p} < \hat{p}_{13}$, which holds if and only if

$$\begin{aligned} \frac{\pi + C + \delta c}{1+\delta} &< \frac{(n+1)\pi + 2nC + (n+1)\delta c}{2n+n\delta+\delta} \\ \Leftrightarrow \quad & \pi < \delta(C-c), \end{aligned}$$

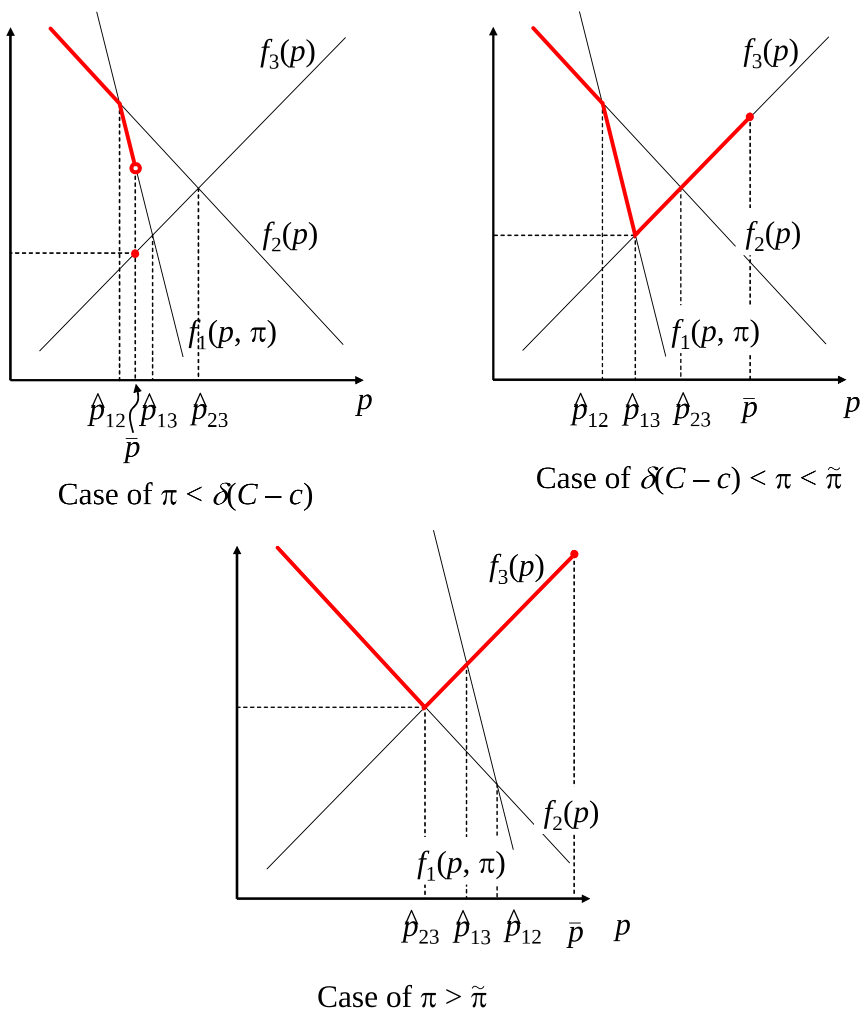
then

$$\begin{aligned} f_3(\bar{p}) &\leq \max\{\min\{f_1(\bar{p},\pi), f_2(\bar{p})\}, f_3(\bar{p})\} \\ &= \min\{f_1(\bar{p},\pi), f_2(\bar{p})\} \\ &\leq \min\{f_1(p,\pi), f_2(p)\}. \end{aligned}$$

The equality is implied by (14) and (15) and the second inequality follows Lemma A1. So, the deviation profit $D(p, \pi)$ is minimized at $p = \bar{p}$.

Figure A1 provides a graphical illustration of the identification of the most effective collusive prices for the case of $C < U$ which covers the sub-cases considered in parts (i.i)-(i.iii), although the formal proof does not utilize the figure. The deviation profit $D(p, \pi)$ is depicted by bolded lines in the figure.

Figure A1: Deviation Profit *D*(*p*, *π*), *C* < *U*



(i.ii) Now look at the case where $\pi < \tilde{\pi}$ and $\hat{p}_{13} \leq \bar{p}$; the latter inequality holds if and only if $\pi \geq \delta (C - c)$. According to Lemma A1, $\max \{\min \{f_1(p, \pi), f_2(p)\}, f_3(p)\}$ is minimized at $p = \hat{p}_{13}$. Since $f_3(\hat{p}_{13}) < f_3(\bar{p})$, as implied by $f_3(\cdot)$ being increasing,

$$\arg \min_{p \in [0, \bar{p}]} D(p, \pi) = \arg \min_{p \in [0, \bar{p}]} \max \{\min \{f_1(p, \pi), f_2(p)\}, f_3(p)\} = \hat{p}_{13}.$$

(i.iii) Now look at the case where $\pi \geq \tilde{\pi}$. According to (15), $\hat{p}_{12} \geq \hat{p}_{13} \geq \hat{p}_{23}$. Applying (14), it follows that

$$\max\{\min\{f_1(p,\pi), f_2(p)\}, f_3(p)\} = \begin{cases} f_2(p,\pi) & \text{if } p < \hat{p}_{23}, \\ f_3(p) & \text{if } p \in [\hat{p}_{23}, \hat{p}_{12}), \end{cases}$$

which is decreasing in p for $p < \hat{p}_{23}$ and increasing in p for $p \geq \hat{p}_{23}$. Thus, $\max\{\min\{f_1(p,\pi), f_2(p)\}, f_3(p)\}$ is minimized at $p = \hat{p}_{23}$. Since $\pi \geq \tilde{\pi} > \delta(C-c)$, it also follows that $\hat{p}_{23} < \bar{p}$; the latter and the fact that $f_3(\cdot)$ is increasing imply $f_3(\hat{p}_{23}) < f_3(\bar{p})$. Therefore,

$$\arg\min_{p\in[0,\bar{p}]} D(p,\pi) = \arg\min_{p\in[0,\bar{p}]} \max\{\min\{f_1(p,\pi), f_2(p)\}, f_3(p)\} = \hat{p}_{23}.$$

(ii) Now, consider the case where $U \leq C$. In this case, for all $\pi \in [0, \pi^M]$, $\pi \leq \delta(C-c)$. Therefore, we can apply part (i.i) of the proof to establish that

$$\arg\min_{p\in[0,\bar{p}]} D(p,\pi) = \bar{p}.$$

Finally, the corresponding printer prices are easily obtained by using (8). This completes the proof of the proposition. ■

Proof of Theorem 1. From the proof of Proposition 1, we can see that when firms charge the most effective collusive prices, a deviating firm is either forced to undercut the cartridge price (when $p = P$) or indifferent between undercutting the printer price and undercutting the cartridge price (when $p < P$). In other words, given that we assume that firms post the most effective collusive prices, the deviation profit is always

$$f_3(p) = \frac{(2n-1)(p-C)}{n} + \frac{\delta(C-c)}{1+\delta}.$$

This result will be applied repeated in this proof.

Since $(U-C)+\delta(U-c) > \delta(C-c)$ if and only if $U > C$ and $(U-C)+\delta(U-c) > \frac{\delta(C-c)}{2}$ if and only if $U > \frac{(2+\delta)C+\delta c}{2(1+\delta)}$, we have

$$\begin{array}{ll} \pi^M > \delta(C-c) & \text{if } U > C, \\ \pi^M \in \left(\frac{\delta(C-c)}{2}, \delta(C-c)\right] & \text{if } U \in \left(\frac{(2+\delta)C+\delta c}{2(1+\delta)}, C\right], \\ \pi^M \in \left(0, \frac{\delta(C-c)}{2}\right] & \text{if } U \in \left(\frac{C+\delta c}{1+\delta}, \frac{(2+\delta)C+\delta c}{2(1+\delta)}\right]. \end{array} \tag{17}$$

(i) The first part of the theorem focuses on the case of $U > C$, i.e., $\pi^M > \delta(C-c)$.

Suppose for now the industry targets a per-generation industry profit of $\pi \leq \delta(C-c)$. Applying Proposition 1, the deviation profit is minimized at $P = p = \bar{p} = \frac{\pi+C+\delta c}{1+\delta}$. So, for $\pi \leq \delta(C-c)$, firms' incentive constraint reduces to

$$\frac{\pi}{n(1-\delta)} \geq \frac{(2n-1)}{n}\left(\frac{\pi+C+\delta c}{1+\delta}-C\right)+\delta\frac{C-c}{1+\delta}, \tag{18}$$

which can be rewritten as

$$2(n(1-\delta)-1)\pi \leq \delta(n-1)(1-\delta)(C-c). \tag{19}$$

This incentive constraint is obviously satisfied if $n \leq 1/(1-\delta)$. And for $n > 1/(1-\delta)$, it is easier to satisfy with a lower π. Therefore, the incentive constraint is satisfied for all $\pi \leq \delta(C-c)$ if it is satisfied at $\pi = \delta(C-c)$, i.e.,

$$\begin{aligned} & 2(n(1-\delta)-1)\delta(C-c) \leq \delta(n-1)(1-\delta)(C-c) \\ \Leftrightarrow \quad & n \leq \frac{1+\delta}{1-\delta} \equiv \hat{n}_3. \end{aligned}$$

And for $n > \hat{n}_3$, the set of sustainable profits is characterized by (19). By now we have established the following lemma:

Lemma A2 *For all $n \leq \hat{n}_3$, any profit $\pi \in [0, \delta(C-c)]$ can be supported by tacit collusion. For all $n > \hat{n}_3$, any profit*

$$\pi \in [0, \frac{\delta(n-1)(1-\delta)(C-c)}{2(n(1-\delta)-1)}] \tag{20}$$

can be supported by tacit collusion.

Next, suppose the industry targets a per-generation industry profit of $\pi \in [\delta(C-c), \tilde{\pi}]$. According to Proposition 1, the most effective collusive prices are $(P,p) = (\frac{2n\pi+(2n-\delta(n-1))C+2n\delta c}{2n+n\delta+\delta}, \frac{(n+1)\pi+2nC+(n+1)\delta c}{2n+n\delta+\delta})$. Therefore, tacit collusion is sustainable if and only if

$$\frac{\pi}{n(1-\delta)} \geq \frac{(2n-1)}{n}\left(\frac{(n+1)\pi+2nC+(n+1)\delta c}{2n+(n+1)\delta}-C\right)+\delta\frac{(C-c)}{1+\delta}, \tag{21}$$

which can be rewritten as

$$\left(\frac{(2n-1)(n+1)}{2n+(n+1)\delta}-\frac{1}{1-\delta}\right)\pi \leq \frac{(n-1)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2n+(n+1)\delta)}. \tag{22}$$

The incentive constraint is always satisfied if

$$\begin{aligned} & \frac{(2n-1)(n+1)}{2n+(n+1)\delta} \leq \frac{1}{1-\delta} \\ \Leftrightarrow \quad & n \leq \frac{(1+2\delta)+\sqrt{4\delta^2-4\delta+9}}{4(1-\delta)}. \end{aligned}$$

When n exceeds this critical value, the incentive constrain is easier to satisfy with a lower π. Therefore, it is satisfied for all $\pi \in [\delta(C-c), \tilde{\pi}]$ if it is satisfied at $\pi = \tilde{\pi}$, i.e.,

$$\left(\tfrac{(2n-1)(n+1)}{2n+(n+1)\delta} - \tfrac{1}{1-\delta}\right)\left(U + \delta\tfrac{(n+1)U+(n-1)C}{2n} - C - \delta c\right) \leq \tfrac{(n-1)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2n+(n+1)\delta)},$$

which can be rewritten as

$$n \leq \tfrac{(\delta+1)(U-C+2\delta(U-c))+\sqrt{(\delta+1)^2(U-C+2\delta(U-c))^2+8(1-\delta^2)(U-C+U\delta-c\delta)(U-C)}}{4(1-\delta)(U-C+U\delta-c\delta)} \equiv \hat{n}_1.$$

For $n > \hat{n}_1$, the sustainable profit is bounded from above according to (22):

$$\pi \leq \tfrac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}. \tag{23}$$

Besides, to support any $\pi \geq \delta(C-c)$, it is also necessary that

$$\left(\tfrac{(2n-1)(n+1)}{2n+(n+1)\delta} - \tfrac{1}{1-\delta}\right)\delta(C-c) \leq \tfrac{(n-1)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2n+(n+1)\delta)},$$

which can be verified to be equivalent to

$$n \leq \hat{n}_3.$$

Summing up, we have:

Lemma A3 *For $n \leq \hat{n}_1$, any profit $\pi \in [\delta(C-c), \tilde{\pi}]$ is sustainable by tacit collusion. For $n \in (\hat{n}_1, \hat{n}_3]$, any profit*

$$\pi \in \left[\delta(C-c), \tfrac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}\right] \tag{24}$$

is sustainable by tacit collusion.

To support $\pi \in [\tilde{\pi}, \pi^M]$, according to Proposition 1, the most effective collusive prices are

$$(P, p) = (\pi + \tfrac{(2n-\delta(n-1))C+2n\delta c-(n+1)\delta U}{2n}, \tfrac{(n+1)U+(n-1)C}{2n}).$$

Therefore, the incentive constraint is

$$\begin{aligned} \frac{\pi}{n(1-\delta)} &\geq \frac{(2n-1)}{n}\left(\frac{(n+1)U+(n-1)C}{2n} - C\right) + \delta\frac{(C-c)}{1+\delta} \\ &= \frac{(2n-1)(n+1)}{2n^2}(U-C) + \delta\frac{(C-c)}{1+\delta}. \end{aligned} \tag{25}$$

This is easier to satisfy with higher π because the deviation profit is independent of π. In other words, (25) is satisfied for all $\pi \in [\tilde{\pi}, \pi^M]$ if it is satisfied at $\pi = \tilde{\pi}$, i.e.,

$$\tfrac{1}{n(1-\delta)}\left(U + \delta\tfrac{(n+1)U+(n-1)C}{2n} - C - \delta c\right) \geq \tfrac{(1-\delta)(2n-1)(n+1)(U-C)}{2n} + \tfrac{\delta n(1-\delta)(C-c)}{1+\delta},$$

which can be verified to be equivalent to

$$n \leq \hat{n}_1.$$

Besides, to support any $\pi \leq \pi^M = (1+\delta)U - C - \delta c$, it is necessary that

$$\frac{(1+\delta)U - C - \delta c}{n(1-\delta)} \geq \frac{(2n-1)(n+1)}{2n^2}(U-C) + \delta\frac{(C-c)}{1+\delta},$$

which can be rewritten as

$$n \leq \frac{(1+\delta)((1+3\delta)(U-C)+2\delta(C-c))+\sqrt{(1+\delta)^2((1+3\delta)(U-C)+2\delta(C-c))^2+8(1-\delta)^2(1+\delta)(U-C)(U-C+\delta(U-c))}}{4(1-\delta)(U-C+\delta(U-c))} \equiv \hat{n}_2.$$

It can be verified that the (25) is easier to satisfy for smaller n. This, with the facts that (25) is easier to satisfy for larger π and that $\tilde{\pi} < \pi^M$, implies that $\hat{n}_1 < \hat{n}_2$. Summing up, we have:

Lemma A4 *For $n \leq \hat{n}_1$, any profit $\pi \in [\tilde{\pi}, \pi^M]$ is sustainable by tacit collusion. For $n \in (\hat{n}_1, \hat{n}_2]$, then any*

$$\pi \in \left[\tfrac{(1-\delta)(2n-1)(n+1)(U-C)}{2n} + \tfrac{\delta n(1-\delta)(C-c)}{1+\delta}, \pi^M\right] \tag{26}$$

is sustainable by tacit collusion.

Next, we show that $\frac{1}{1-\delta} < \hat{n}_1$ and $\hat{n}_2 < \hat{n}_3$. Recall that the per-generation industry profit $\tilde{\pi}$ can be supported by setting $P = U$ and $p = \frac{(n+1)U+(n-1)C}{2n}$ if and only if $n \leq \hat{n}_1$. At $\pi = \tilde{\pi}$ and $n = \frac{1}{1-\delta}$, the difference between the equilibrium profit and the deviation profit is

$$\begin{aligned}
& \tfrac{1}{n(1-\delta)}\left(U + \delta\tfrac{(n+1)U+(n-1)C}{2n} - C - \delta c\right) - \left(\tfrac{(2n-1)(n+1)}{2n^2}(U-C) + \delta\tfrac{(C-c)}{1+\delta}\right)\Big|_{n=(1-\delta)^{-1}} \\
= & \left(U + \delta\tfrac{(n+1)U+(n-1)C}{2n} - C - \delta c\right) - \left(\tfrac{(2n-1)(n+1)}{2n^2}(U-C) + \delta\tfrac{(C-c)}{1+\delta}\right)\Big|_{n=(1-\delta)^{-1}} \\
= & \tfrac{(1+\delta)(\delta n(n+1)-(n-1))(U-C)+2\delta^2 n^2(C-c)}{2n^2(1+\delta)}\Big|_{n=(1-\delta)^{-1}} \\
= & \delta\tfrac{(1+\delta)(U-C)+2\delta(C-c)}{2(1+\delta)} > 0.
\end{aligned}$$

In other words, the per-generation industry profit $\tilde{\pi}$ can be supported among more than $(1-\delta)^{-1}$ firms; so $\hat{n}_1 > (1-\delta)^{-1}$.

Next, $\hat{n}_2 < \hat{n}_3$ is established by the fact that the per-generation industry profit π^M can be supported among $\hat{n}_2$ firms but cannot be supported among $\hat{n}_3 \equiv (1+\delta)/(1-\delta)$ firms, as implied by

$$\begin{aligned}
&\frac{\pi}{n(1-\delta)} - \left(\frac{(2n-1)}{n}\left(\frac{(n+1)U+(n-1)C}{2n} - C\right) + \delta\frac{(C-c)}{1+\delta}\right) \\
= &\frac{(1+\delta)U - C - \delta c}{(1+\delta)} - \left(\frac{\left(2\frac{1+\delta}{1-\delta}-1\right)\left(\frac{1+\delta}{1-\delta}+1\right)(U-C)}{2\left(\frac{1+\delta}{1-\delta}\right)^2} + \delta\frac{(C-c)}{1+\delta}\right) \\
= &\frac{-\delta(1-\delta)(U-C)}{(1+\delta)^2} < 0.
\end{aligned}$$

Now, we are ready to summarize the characterization of the set of equilibrium profits that tacit collusion can support for the case that $C < U$. By applying Lemmas A2-A4, for all $n \leq \hat{n}_1$, any per-generation industry profit in $[0, \delta(C-c)] \cup (\delta(C-c), \tilde{\pi}] \cup (\tilde{\pi}, \pi^M] = [0, \pi^M]$ can be supported by tacit collusion; this proves part (i.i) of the theorem. By once again applying Lemmas A2-A4, the set of sustainable per-generation industry profits for $n \in (\hat{n}_1, \hat{n}_2]$ is

$$[0, \delta(C-c)] \cup \left[\delta(C-c), \tfrac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}\right] \cup \left[\tfrac{(1-\delta)(2n-1)(n+1)(U-C)}{2n} + \tfrac{\delta n(1-\delta)(C-c)}{1+\delta}, \pi^M\right].$$

This proves part (i.ii) of the theorem. Similarly, according to Lemmas A2-A4, for $n \in (\hat{n}_2, \hat{n}_3]$, the set of sustainable π is

$$[0, \delta(C-c)] \cup \left[\delta(C-c), \tfrac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}\right].$$

This proves part (i.iii) of the theorem. Finally, the range of sustainable π as listed in part (i.vi) of the theorem for $n > \hat{n}_3$ follows immediately Lemma A2.

(ii) & (iii) Now we prove the second and third parts of the theorem concerning the case that $U \leq C$. First, the value of π^M follows immediately (17). When $U \leq C$, for any $\pi \in [0, \pi^M]$, according to (16) in the proof of Proposition 1, $\pi \leq \tilde{\pi} \leq \delta(C-c)$. According to Proposition 1, the most effective way to support the profit levels is to set $P = p = \bar{p} = \frac{\pi + C + \delta c}{1+\delta}$ for all $\pi \in [0, \pi^M]$. Therefore, tacit collusion is sustainable for all $\pi \in [0, \pi^M]$ if

$$\frac{\pi^M}{n(1-\delta)} \geq \frac{(2n-1)}{n}\left(\frac{\pi^M + C + \delta c}{1+\delta} - C\right) + \delta\frac{C-c}{1+\delta},$$

which is equivalent to

$$n(2(1+\delta)(U-C) + \delta(C-c)) \leq \frac{1+\delta}{1-\delta}(2(U-C) + \delta(C-c)). \tag{27}$$

This incentive constraint is satisfied for all n if $2(1+\delta)(U-C) + \delta(C-c) \leq 0$, i.e.,

$$U \leq \frac{(2+\delta)C + \delta c}{2(1+\delta)}.$$

This proves part (iii) of the theorem.

For $U \in (\frac{(2+\delta)C+\delta c}{2(1+\delta)}, C]$, (27) is satisfied if and only if

$$n \leq \hat{n}_4 \equiv \frac{1+\delta}{1-\delta} \frac{2(U-C)+\delta(C-c)}{2(1+\delta)(U-C)+\delta(C-c)}. \tag{28}$$

Since $U \leq C$,

$$\begin{aligned} \hat{n}_4 - \hat{n}_3 &= \frac{1+\delta}{1-\delta} \frac{2(U-C)+\delta(C-c)}{2(1+\delta)(U-C)+\delta(C-c)} - \frac{1+\delta}{1-\delta} \\ &= \frac{2\delta(1+\delta)(C-U)}{(1-\delta)(2(1+\delta)(U-C)+\delta(C-c))} > 0. \end{aligned}$$

This proves part (ii.i) of the theorem. If $n > \hat{n}_4$, then, according to part (i) of this proof, (20) characterizes the range of per-generation industry profit sustainable. This proves part (ii.ii) of the theorem and completes the proof for the theorem. ■

Proof of Lemma 3. (i) To prove the first part of the lemma, it is sufficient to assume that the industry sets the same printer price in the first period as in the steady state. In this case, the incentive constraint will become

$$\frac{P-C+\delta(p-c)}{n(1-\delta)} \geq \min\{P, U\} - C + \delta\frac{(C-c)}{1+\delta},$$

which can be rewritten as

$$\frac{\pi}{n(1-\delta)} \geq \min\{f_1(p,\pi), f_2(p)\}.$$

From the proof of Proposition 1, we learn that $f_3(\bar{p}) < \min\{f_1(\bar{p},\pi), f_2(\bar{p})\}$ only if $\pi < \delta(C-c)$. First, suppose the industry targets a per-generation industry profit higher than $\delta(C-c)$, which is possible only if $U > C$. In this case, the first period deviation profit $\min\{f_1(p,\pi), f_2(p)\}$ is less than the steady state deviation profit, due to the absence of established customers. As a result, as implied by Theorem 1, any profit $\pi \in [\delta(C-c), \pi^M]$ from the first generation of customers can be sustained among any $n \leq \hat{n}_1$, where $\hat{n}_1 > 1/(1-\delta)$.

Next, suppose firms target a per-generation industry profit of less than $\delta(C-c)$. In this case tacit collusion is most effectively supported by setting $P = p$ in the steady state and the corresponding first period incentive constraint becomes

$$\frac{p-C+\delta(p-c)}{n(1-\delta)} \geq (p-C) + \delta\frac{C-c}{1+\delta},$$

or

$$n \leq \frac{1}{1-\delta} \frac{p-C+\delta(p-c)}{(p-C)+\delta\frac{C-c}{1+\delta}} \equiv \bar{n}_3.$$

Notice that to support any positive profit in the steady state, it requires that $p > (C + \delta c) / (1 + \delta)$. This implies that $\frac{p - C + \delta(p-c)}{(p-C) + \delta \frac{C-c}{1+\delta}} > 1$:

$$\begin{aligned} & (p - C + \delta(p - c)) - \left((p - C) + \delta \frac{C - c}{1 + \delta}\right) \\ = \quad & \delta \left(p - \frac{C + \delta c}{1 + \delta}\right) > 0. \end{aligned}$$

Therefore,

$$\bar{n}_3 > \frac{1}{1 - \delta}.$$

By setting $\bar{n} = \min\{\hat{n}_1, \bar{n}_3\}$, we can complete the proof of part (i).

(ii) To prove the second part of the lemma, we suppose that firms set the first period printer price at $P_1 = (C + \delta c) / (1 + \delta)$. By doing so, no firm has any incentive to deviate in the first period. This printer price and the steady state cartridge price allow the industry to earn from the first generation of consumers a non-negative profit:

$$\begin{aligned} & \frac{(C + \delta c)}{1 + \delta} - C + \delta(p - c) \\ \geq \quad & \frac{(C + \delta c)}{1 + \delta} - C + \delta\left(\frac{(C + \delta c)}{1 + \delta} - c\right) = 0. \end{aligned}$$

According to Corollary 1, any steady state per-generation industry profit $\pi \in \left[0, \min\{\pi^M, \frac{\delta(C-c)}{2}\}\right]$ can be supported for all n. By setting $\overline{\Pi} = \min\{\frac{\pi^M}{1-\delta}, \frac{\delta(C-c)}{2(1-\delta)}\}$ we can complete the proof of part (ii) of the lemma. ■

Proof of Proposition 2. First, we prove that any profit ranging from zero to the monopoly profit can be supported among a larger number of firms than $1/(1 - \delta)$. For this purpose, we do not have to fully characterize the conditions under which profitable tacit collusion is sustainable. Instead, we will just derive *sufficient conditions* for sustainability of tacit collusion.

Given a print-cartridge price pair (P, p), where $p \leq P$, the equilibrium per-generation industry profit will be

$$\pi = (P - C) + \delta\theta \frac{p - c}{1 - \delta\phi},$$

which can be rewritten as

$$P = C + \pi - \delta\theta \frac{p - c}{1 - \delta\phi}. \tag{29}$$

If firms charge the same price for the printer and cartridge, then

$$P = p = \bar{p} \equiv \frac{(1 - \delta\phi)(\pi + C) + \delta\theta c}{1 + \delta(\theta - \phi)}. \tag{30}$$

Note that, however, the steady state profit each firm earns *per-period* will be $(P - C + p - c)/n$ instead of π/n. This is because in every period starting from the second period, each firm will serve a measure $1/n$ of established and a measure $1/n$ of new customers. Therefore, the discounted profit of each firm, inclusive of profit from its established customers is

$$\begin{aligned}\frac{P - C + p - c}{n(1-\delta)} &= \frac{C + \pi - \delta\theta\frac{p-c}{1-\delta\phi} - C + p - c}{n(1-\delta)} \\ &= \frac{\pi + \frac{1-\delta\theta-\delta\phi}{1-\delta\phi}(p-c)}{n(1-\delta)}. \qquad (31)\end{aligned}$$

If a firm deviates by setting the printer price arbitrarily close to but less than $\min\{P, U\}$, then it will capture a measure one of new consumers, earning from them an immediate profit of $(\min\{P, U\} - C)$ and, in all future periods, a discounted profit of $\delta\theta\left(\frac{p^C-c}{1-\delta\phi}\right)$, based on the expectation that a price war will begin in the following period. When the firm deviates, it will also raise its cartridge price to arbitrarily close to $\min\{U, P\}$ and earn from its $1/n$ established customers an immediate profit arbitrarily close to $(\min\{U, P\} - c)/n$ [instead of $(p - c)/n$ as in equilibrium] and a discounted future profit of $\frac{\delta\phi}{n}\left(\frac{p^C-c}{1-\delta\phi}\right)$. This gives rise to a deviation profit of

$$\begin{aligned}&(\min\{P, U\} - C) + \frac{(\min\{P, U\} - c)}{n} + \delta\left(\theta + \frac{\phi}{n}\right)\left(\frac{p^C - c}{1-\delta\phi}\right) \\ \leq\ &(P - C) + \frac{(P-c)}{n} + \delta\left(\theta + \frac{\phi}{n}\right)\left(\frac{p^C - c}{1-\delta\phi}\right) \\ =\ &\frac{(n+1)P - nC - c}{n} + \delta\left(\theta + \frac{\phi}{n}\right)\frac{(C-c)}{1-\delta\phi+\delta\theta} \\ =\ &\frac{(n+1)}{n}\left(C + \pi - \delta\theta\frac{p-c}{1-\delta\phi}\right) - C - \frac{c}{n} + \delta\left(\theta + \frac{\phi}{n}\right)\frac{(C-c)}{1-\delta\phi+\delta\theta} \equiv g_1(p, \pi).\end{aligned}$$

The inequality is trivial, the first equality follows (7), and the second equality follows (29).

If the deviating firm instead sets the printer price arbitrarily close to but less than p, then it will earn a profit of $(p - C)$ from the new consumers and a profit of $\frac{(p-c)}{n}$ from its own established customers. It will also steal a measure $(1-\frac{1}{n})$ of established customers from its competitors, earning from them a profit of $(1 - \frac{1}{n})(p - C)$. The deviating firm will also earn a discounted profit of $\delta\theta\left(\frac{p^C-c}{1-\delta\phi}\right) = \frac{\delta\theta(C-c)}{1-\delta\phi+\delta\theta}$ from the new customers and a discounted profit of $\delta\phi\left(\frac{p^C-c}{1-\delta\phi}\right) = \frac{\delta\phi(C-c)}{1-\delta\phi+\delta\theta}$ from the established customers. Therefore, such deviation leads to a profit of

$$g_2(p) \equiv \left(2 - \frac{1}{n}\right)(p - C) + \frac{p-c}{n} + \delta\frac{(\theta+\phi)(C-c)}{1-\delta\phi+\delta\theta}.$$

Therefore, for $p < P$ (i.e., $p < \bar{p}$), the deviation profit is no larger than $\max\{g_1(p, \pi), g_2(p)\}$. Suppose

$P = p$, then the deviation profit is necessarily $g_2(p)$. Summing up, the deviation profit does not exceed

$$\hat{D}(p,\pi) = \begin{cases} \max\{g_1(p,\pi), g_2(p)\} & \text{if } p < \bar{p}, \\ g_2(p) & \text{if } p = \bar{p}. \end{cases}$$

Let $p = \tilde{p}_{12}$ solve $g_1(p,\pi) = g_2(p)$. It can be verified that

$$\tilde{p}_{12} = \frac{1-\delta\phi}{2n(1-\delta\phi)+\delta\theta(n+1)}\left((n+1)\pi + 2nC + \frac{(n+1)\delta\theta c}{1-\delta\phi} - \delta\phi(n-1)\frac{C-c}{1+\delta(\theta-\phi)}\right). \tag{32}$$

Also, we know that g_1 is decreasing in p and g_2 is increasing in p and thus

$$\max\{g_1(p,\pi), g_2(p)\} = \begin{cases} g_1(p,\pi) & \text{if } p \le \tilde{p}_{12}, \\ g_2(p) & \text{if } p > \tilde{p}_{12}, \end{cases}$$

and $\max\{g_1(p,\pi), g_2(p)\}$ is minimized at $p = \tilde{p}_{12}$.

By comparing (30) with (32), it can be verified that

$$\tilde{p}_{12} < \bar{p} \text{ if and only if } \pi > \frac{\delta(\theta-\phi)(C-c)}{1-\delta\phi}. \tag{33}$$

First suppose the industry targets some $\pi \ge \frac{\delta(\theta-\phi)(C-c)}{1-\delta\phi}$ so that $\tilde{p}_{12} \le \bar{p}$. In this case,

$$\hat{D}(p,\pi) = \begin{cases} g_1(p,\pi) & \text{if } p < \tilde{p}_{12}, \\ g_2(p) & \text{if } p \in [\tilde{p}_{12}, \bar{p}], \end{cases}$$

and $\hat{D}(p,\pi)$ is minimized at $p = \tilde{p}_{12}$. Suppose that firms support this profit by setting $p = \tilde{p}_{12}$ and setting P according to (29). In this case, the sufficient condition for sustainability of tacit collusion is

$$\frac{\pi + \frac{1-\delta\theta-\delta\phi}{1-\delta\phi}(\tilde{p}_{12}-c)}{n(1-\delta)} \ge \hat{D}(\tilde{p}_{12},\pi).$$

By plugging (32) into $\hat{D}(\tilde{p}_{12},\pi)$, we can show that

$$\begin{aligned} & \frac{\pi + \frac{1-\delta\theta-\delta\phi}{1-\delta\phi}(\tilde{p}-c)}{n(1-\delta)} - \hat{D}(\tilde{p}_{12},\pi) \\ = \quad & \frac{(1-\delta\phi)K_1\pi}{(1-\delta)n(n(2-2\delta\phi+\delta\theta)+\delta\theta)} + \frac{\delta K_2(C-c)}{(1-\delta)n(1+\delta(\theta-\phi))(n(2-2\delta\phi+\delta\theta)+\delta\theta)} \end{aligned}$$

where

$$\begin{aligned} K_1 &= (1+2\delta)n + 1 - 2(1-\delta)n^2 \\ K_2 &= (1-\delta)(\theta-\phi)\delta\theta n^2 - \left(\theta + 3\phi - \theta\delta + 2\delta\phi + 2\theta^2\delta - 3\delta\phi^2 - \theta\delta\phi - 2\right)n \\ &\quad - \left(\theta - \phi - \theta^2\delta^2 - \theta\delta + \theta^2\delta + \delta\phi^2 + \theta\delta^2\phi\right) \end{aligned}$$

By plugging $\theta = 1 - \phi$ into K_2 and recalling that $\phi \leq 0.5$, we have

$$K_2 = (1-\delta)(1-2\phi)(n-1)((n+1)\delta(1-\phi)+1) \geq 0.$$

Moreover, it can be verified that

$$\begin{aligned} & K_1 \geq 0 \\ \Leftrightarrow \quad & n \leq \tilde{n}_1 \equiv \frac{\sqrt{4\delta^2 - 4\delta + 9} + 2\delta + 1}{4(1-\delta)}, \end{aligned}$$

In other words, if $\theta \geq \phi$, then any profit in the range $[\frac{\delta(\theta-\phi)(C-c)}{1-\delta\phi}, \pi^M]$ is sustainable for all $n \leq \tilde{n}_1$, and $\tilde{n}_1 > \frac{1}{1-\delta}$ because

$$\begin{aligned} & \tilde{n}_1 - \frac{1}{1-\delta} \\ = \quad & \frac{\sqrt{4\delta^2 - 4\delta + 9} + 2\delta + 1 - 4}{4(1-\delta)} = \frac{\sqrt{4\delta^2 - 4\delta + 9} - (3 - 2\delta)}{4(1-\delta)} \\ = \quad & \frac{\sqrt{4\delta^2 - 4\delta + 9} - \sqrt{4\delta^2 - 12\delta + 9}}{4(1-\delta)} > 0. \end{aligned}$$

In the case that $\theta = \phi$, the lower bound of $[\frac{\delta(\theta-\phi)(C-c)}{1-\delta\phi}, \pi^M]$ is 0. Then the proof of part (i) of the proposition can be completed by choosing $\tilde{n} = \tilde{n}_1$.

Now, consider the case that $\theta > 0.5 > \phi$ so that $\frac{\delta(\theta-\phi)(C-c)}{1-\delta\phi} > 0$. Suppose the industry targets some $\pi < \frac{\delta(\theta-\phi)(C-c)}{1-\delta\phi}$. In this case $\bar{p} < \tilde{p}_{12}$ and

$$\hat{D}(p, \pi) = \begin{cases} g_1(p, \pi) & \text{if } p < \bar{p}, \\ g_2(p) & \text{if } p = \bar{p}, \end{cases}$$

where $g_2(\bar{p}) < g_1(\bar{p}, \pi)$ because $\bar{p} < \tilde{p}_{12}$. Since $g_1(p, \pi)$ is decreasing, $\hat{D}(p, \pi)$ is minimized at $p = \bar{p}$. Suppose the industry sets $P = p = \bar{p}$ to support the targeted π. In this case, the sufficient condition for sustainability of tacit collusion is

$$\frac{2\bar{p} - (C+c)}{n(1-\delta)} \geq \left(2 - \frac{1}{n}\right)(\bar{p} - C) + \frac{(\bar{p} - c)}{n} + \delta \frac{(\theta + \phi)(C - c)}{1 - \delta\phi + \delta\theta}. \tag{34}$$

By plugging (30) and $\theta = 1 - \phi$ into (34), the latter can be simplified as

$$n\left(\pi - \frac{\delta(1-2\phi)(C-c)}{2(1-\delta\phi)}\right) \leq \frac{1}{(1-\delta)}\left(\pi - \frac{\delta(1-\delta)(1-2\phi)(C-c)}{2(1-\delta\phi)}\right). \tag{35}$$

For $\pi \in \left(\frac{\delta(1-2\phi)(C-c)}{2(1-\delta\phi)}, \frac{\delta(\theta-\phi)(C-c)}{1-\delta\phi}\right]$, (35) can be further rewritten as

$$n \leq \tilde{n}_2 \equiv \frac{1}{(1-\delta)} \frac{2(1-\delta\phi)\pi - \delta(1-\delta)(1-2\phi)(C-c)}{2(1-\delta\phi)\pi - \delta(1-2\phi)(C-c)}.$$

It can be verified that $\phi < 0.5$ implies

$$\frac{2(1-\delta\phi)\pi - \delta(1-\delta)(1-2\phi)(C-c)}{2(1-\delta\phi)\pi - \delta(1-2\phi)(C-c)} > 1.$$

Therefore,

$$\tilde{n}_2 > \frac{1}{1-\delta}.$$

For $\pi \in \left(\frac{\delta(1-\delta)(1-2\phi)(C-c)}{2(1-\delta\phi)}, \frac{\delta(1-2\phi)(C-c)}{2(1-\delta\phi)}\right]$, (35) holds for all n because the LHS of the equation is negative while its RHS is positive for all n. For $\pi \in \left[0, \frac{\delta(1-\delta)(1-2\phi)(C-c)}{2(1-\delta\phi)}\right]$, (35) becomes

$$\begin{aligned} n &\geq \frac{1}{(1-\delta)} \frac{\delta(1-\delta)(1-2\phi)(C-c) - 2(1-\delta\phi)\pi}{\delta(1-2\phi)(C-c) - 2(1-\delta\phi)\pi} \\ &= \frac{\delta(1-2\phi)(C-c) - \frac{2(1-\delta\phi)\pi}{(1-\delta)}}{\delta(1-2\phi)(C-c) - 2(1-\delta\phi)\pi} \ (<1), \end{aligned}$$

which is always satisfied. So, in the case that $\theta > 0.5 > \phi$, any profit $\pi \in [0, \pi^M]$ can be supported by tacit collusion for all $n \leq \min\{\tilde{n}_1, \tilde{n}_2\}$, where $\min\{\tilde{n}_1, \tilde{n}_2\} > 1/(1-\delta)$. Therefore, we can complete the proof of part (i) of the proposition by setting $\tilde{n} = \min\{\tilde{n}_1, \tilde{n}_2\}$.

Note that (35) can be rewritten as

$$2(1-\delta\phi)((1-\delta)n-1)\pi \leq \delta(1-\delta)(n-1)(1-2\phi)(C-c). \tag{36}$$

This holds for all $n \leq 1/(1-\delta)$ and when $n > 1/(1-\delta)$, it hold if and only if

$$\begin{aligned} \pi &\leq \frac{\delta(1-\delta)(n-1)(1-2\phi)(C-c)}{2(1-\delta\phi)((1-\delta)n-1)} \\ &= \frac{\delta(1-\delta)(n-1)(\theta-\phi)(C-c)}{2(1-\delta\phi)((1-\delta)n-1)}. \end{aligned}$$

$$\begin{aligned} &\frac{d}{dn}\left(\frac{\delta(1-\delta)(n-1)(\theta-\phi)(C-c)}{2(1-\delta\phi)((1-\delta)n-1)}\right) \\ = \ &-\frac{\delta^2(1-\delta)(\theta-\phi)(C-c)}{2(1-\delta\phi)(n(1-\delta)-1)^2} < 0 \end{aligned}$$

As n approaches infinity, the upper bound on π becomes

$$\lim_{n\to\infty} \frac{\delta(1-\delta)(n-1)(\theta-\phi)(C-c)}{2(1-\delta\phi)((1-\delta)n-1)} = \frac{\delta(\theta-\phi)(C-c)}{2(1-\delta\phi)}.$$

Therefore, for all $n \geq 2$, any $\pi \in \left[0, \frac{\delta(\theta-\phi)(C-c)}{2(1-\delta\phi)}\right]$ can be supported by tacit collusion. Obviously, π cannot exceed π^M as well. This completes the proof of part (ii) of the proposition. ■

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