Strategic Market Games with Cyclic Production

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Extended Abstract

We consider a simple stochastic economy with production in each stage modeled by independent random variables from cyclically varying, known distributions. We prove the existence of a neutral stationary equilibrium for the economy both when there is no bank and when a central bank is available to the agents for borrowing and investing at a set interest rate ρ. In each case, we derive explicit formulae for the equilibrium profile of strategies.

When there is no bank, the total money supply remains constant. Additionally, when there is no bank and the agents are in equilibrium, the prices are independent random variables from known, cyclically changing distributions.

If a central bank is included in the economy, it is possible for the economy to inflate and/or deflate. For the equilibrium context, we calculate the expected inflation rate across a cycle of length k as a function of the interest rate ρ, the cyclically varying endowment distributions, and the agents' discount factor β. Interestingly, the rate of inflation is greater when agents face real uncertainty than when endowments are deterministic.

In an economy with deterministic cyclic production, the rate of inflation across a cycle of length k depends only on the nominal interest rate ρ and the discount rate of the agents β. That is,

$$\frac{p_{n+k}}{p_n} = \frac{m_{n+k}}{m_n} = [\beta(1+\rho)]^k$$

where p_n is the price and m_n is the total amount of money in the economy in stage n. This relationship is as prescribed by the famous Fisher equation.

We show that in an economy with random cyclic production and well-informed agents who know the value of their random endowment before choosing how much to spend, the

expected one-cycle inflation rate is strictly greater:

$$E[\frac{p_{n+k}}{p_n}] > [\beta(1+\rho)]^k$$

However, a Fisher-like equation does hold when the one-cycle inflation rate is replaced by its harmonic mean. That is,

$$\frac{1}{E[\frac{p_n}{p_{n+k}}]} = [\beta(1+\rho)]^k$$

The economic context of our analysis involves a continuum of symmetric agents. We will consider a representative agent with an arbitrary concave utility function, a single nondurable consumer good, and independent endowments from cyclically varying distributions.

This work is a direct extension of Karatzas, et al. (2006) which considered an economy with iid endowments rather than independent endowments from cyclically varying distributions. This particular extension was proposed by Martin Shubik and William Sudderth.

References

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