Abstract: Repeated Bargaining under Uncertainty of Value Distribution

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The Coase conjecture (1972) first provides a compelling proposition that a durable goods monopolist, who sells over time and can quickly reduce prices as sales are made, will price at marginal cost. Subsequent work (Fudenberg et al, 1985) shows that there exists generically a unique subgame perfect equilibrium, when the lowest value of the buyer exceeds the seller's marginal cost (called the Gap case). In the equilibrium, when the stages occur quickly, prices over time converge to the lowest value rapidly. The intuition in these results is of the same spirit. When the monopolist is faced with a residual demand after having sold some quantity to a high type buyer, she is induced to sell at a lower price to the remaining types. Anticipating this, high type buyer will wait for future lower price. Provided that the monopolist can cut price sufficiently rapidly, the initial price will converge to the marginal cost or the lowest value of the buyer.

The rental model in Hart and Tirole (1988) looks at the case that the monopolist rents a durable good to a buyer with private value. Their result shows that for long but finite horizons, the seller is not able to price discriminate and she charges low price to both low type and high type buyer, until close to the end of the horizon. The intuition in the rental model is different from the durable goods sale model. The buyer in the rental model has a demand in each period. When the time horizon is long, the high type buyer will not accept a high price, in order to avoid being charged with a high price in all later periods. Thus the seller has to offer a low price to avoid rejection except toward the end of the game. The result of Hart and Tirole (1988)'s rental model is counterintuitive because it holds for any prior of the seller, which means the seller will charge a low price in most of the periods even if she believes the buyer is of high type with a probability of 99%.

Though the intuitions in the Gap case durable goods sale model and the rental model are different, both of them have the stark conclusion that the monopolist can hardly earn profit higher than the lowest value of the buyer, which

is counterintuitive and difficult to accept.

This paper weakens the result above by introducing a higher layer of uncertainty about the buyer's value. We investigate the rental model when the seller doesn't know the distribution of the buyer's valuation. We find when the seller has a high prior, there exists an equilibrium that the seller always charges high price and the high value buyer in each period always accepts the offer, which is contrasted with Hart and Tirole (1988)'s rental model.

There are two distributions that the buyer's value can be possibly drawn: the favorable distribution G and the unfavorable distribution F. Both distributions can draw *high* type buyer and *low* type buyer, but the favorable distribution G has a higher probability to draw a *high* type than the F distribution. The seller only knows the prior of two distributions, however, the buyer knows which distribution his value is drawn from.

The timing of the game is as follows. At the beginning of the game, the distribution of the buyer's value is decided by nature, and it is fixed throughout the game. The seller's cost is always 0. Then the buyer's value is drawn from the distribution at the beginning of each period, independently across time periods. The valuation is the buyer's private information. The seller has one unit of a perishable good to sell in each period. The buyer has multi-unit demand. In each period, the seller proposes a take-it-or-leave-it offer, and the buyer decides whether to accept or reject.

We first look at the problem in a two-period model. We find there exist multiple equilibria. The set of perfect Bayesian equilibria depends critically on the seller's prior of the favorable distribution G.

The most interesting case is when the seller has a very high prior of the G distribution. In the equilibrium the seller always asks for a *high* price in both periods, and the buyer with a *high* value in each period always accepts the offer. The intuition is the following. Assume the seller offers *high* in the first period and will offer a *high* price in the second period no matter the first offer is accepted or rejected. Then the buyer accepts the offer in each period as long as it's not greater than her value in that period. So the buyer from both distributions accepts the first offer if the value in the first period is *high* and rejects the first offer if it's *low*. Given the buyer's strategy, the seller's belief of the G distribution is lowered only if the first *high* offer is rejected, which comes

from the fact that the F distribution has a higher probability to draw a *low* type buyer. However, if the seller's prior is very high, the lowered posterior belief is still high enough for the seller to offer *high* in the second period. Therefore, the seller does offer *high* in both periods, which verifies our original assumption.

We then extend the model from two periods to finite horizon. We find there exists a similar equilibrium as in the two-period model when the seller's prior of the G distribution is high. In each period the seller always charges a *high* price and the buyer with a *high* value in that period accepts the offer. The intuition is the same as in the two-period model, except that a higher prior, which is contingent to time horizon, is required to guarantee that the seller's posterior belief is still high enough even if she observes rejection in every period.

The intuitive interpretation of the difference between Hart and Tirole (1988)'s rental model and our result is as follows. There are only *high* type and *low* type buyer in Hart and Tirole (1988)'s rental model. Since the *low* type buyer never accepts a price higher than *low*, the acceptance by the *high* type buyer of a price higher than *low* drives the seller's posterior belief either immediately to 0 (when the offer is rejected) or to 1 (when the offer is accepted), *no matter* what the seller's prior belief is. Consequently, the seller either always charges *low* or always charges *high* in all later periods. This provides a large incentive for the *high* type buyer to mimic the response of the *low* type and not to accept an offer higher than *low*, given there are many periods left. So the seller has to offer *low* in early periods correspondingly to obtain positive payoffs.

In our model, the seller's offer essentially depends on his belief of the G distribution. In the equilibrium we find, the seller is able to distinguish *high* type and *low* type buyer in each period, but not the distribution which the buyer's value is drawn from. Because the two distributions have the same support, the seller updates her posterior belief of the G distribution slowly and never has an extreme belief of 0 or 1. Thus when the seller's prior belief is very high and the time horizon is finite, it's possible for the seller to always maintain a high belief of the G distribution and offer *high* in each period. The *high* buyer will accept the offer anyway because he can't lower the seller's belief that much and change her future offer by rejecting the offer.